

General Instructions

- (i) This booklet contains 22 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 14 single-correct multiple-choice questions and 8 numerical / integer-type questions.
- (iii) Attempt each question on your own before reviewing the given solution.
- (iv) For numerical questions, report the answer rounded exactly as asked.

1. Kamala divided her investment of Rs 100000 between stocks, bonds, and gold. Her investment in bonds was 25% of her investment in gold. With annual returns of 10%, 6%, 8% on stocks, bonds, and gold, respectively, she gained a total amount of Rs 8200 in one year. The amount, in rupees, that she gained from the bonds, was:

Correct Answer: —

Solution:

Approach: Three unknowns, but the gold-bond link $B = 0.25G$ collapses everything to one variable. Express stocks and gold through B , then read off the bond gain.

Step 1: Name the investments.

Let stocks = S , bonds = B , gold = G (in rupees).

$$S + B + G = 100000.$$

Step 2: Use the bond-gold relation.

Bonds are 25% of gold: $B = 0.25G$, so $G = 4B$.

Substitute into the total:

$$S + B + 4B = 100000 \Rightarrow S = 100000 - 5B.$$

Step 3: Write the gain equation.

Returns are 10%, 6%, 8%:

$$0.10S + 0.06B + 0.08G = 8200.$$

Put $S = 100000 - 5B$ and $G = 4B$:

$$0.10(100000 - 5B) + 0.06B + 0.08(4B) = 8200.$$

$$10000 - 0.50B + 0.06B + 0.32B = 8200.$$

$$10000 - 0.12B = 8200.$$

Step 4: Solve for B .

$$0.12B = 1800 \Rightarrow B = 15000.$$

Step 5: Gain from bonds.

$$0.06 \times 15000 = 900.$$

Gain from bonds = Rs 900.

Quick Tip: When a mixture investment problem involves fixed returns and proportional allocations, reduce variables using the percentage relationships first. This simplifies the system into a single solvable equation.

2. If $a - 6b + 6c = 4$ and $6a + 3b - 3c = 50$, where a, b and c are real numbers, the value of $2a + 3b - 3c$ is:

- (A) 18
- (B) 20
- (C) 15
- (D) 14

Correct Answer: (A) 18

Solution:

Approach: Don't chase all three letters. Both equations and the target share the block $(3b - 3c)$, so treat that block as a single unknown and you are left with just two equations in two quantities.

Step 1: Bundle the repeated block.

Let $X = 3b - 3c$. Notice $-6b + 6c = -2(3b - 3c) = -2X$.

First equation: $a - 6b + 6c = 4$ becomes

$$a - 2X = 4. \quad (i)$$

Second equation: $6a + 3b - 3c = 50$ becomes

$$6a + X = 50. \quad (ii)$$

Target: $2a + 3b - 3c = 2a + X$.

Step 2: Solve (i) and (ii).

From (ii): $X = 50 - 6a$. Put into (i):

$$a - 2(50 - 6a) = 4 \Rightarrow a - 100 + 12a = 4 \Rightarrow 13a = 104 \Rightarrow a = 8.$$

Then $X = 50 - 6(8) = 2$.

Step 3: Evaluate the target.

$$2a + X = 2(8) + 2 = 18.$$

$2a + 3b - 3c = 18$ — option (1).

Quick Tip: When several linear expressions repeat with the same structure (like $3b - 3c$ here), use substitution to reduce the system to two variables. This often simplifies the equations dramatically.

3. The (x, y) coordinates of vertices P , Q and R of a parallelogram $PQRS$ are $(-3, -2)$, $(1, -5)$ and $(9, 1)$, respectively. If the diagonal SQ intersects the x -axis at $(a, 0)$, then the value of a is:

- (A) $\frac{29}{9}$
(B) $\frac{27}{9}$
(C) $\frac{29}{7}$
(D) $\frac{9}{29}$

Correct Answer: (A) $\frac{29}{9}$

Solution:

Approach: The one fact that unlocks a parallelogram is that its diagonals bisect each other. So the midpoint of PR equals the midpoint of SQ — that single condition pins down S , and then SQ is just a line to intersect with the x-axis.

Step 1: Midpoint of diagonal PR .

$P(-3, -2)$, $R(9, 1)$:

$$M = \left(\frac{-3 + 9}{2}, \frac{-2 + 1}{2} \right) = \left(3, -\frac{1}{2} \right).$$

Step 2: Find S from the same midpoint of SQ .

Let $S = (x, y)$, with $Q(1, -5)$. The midpoint of SQ is also $(3, -\frac{1}{2})$:

$$\frac{x + 1}{2} = 3 \Rightarrow x = 5, \quad \frac{y - 5}{2} = -\frac{1}{2} \Rightarrow y = 4.$$

So $S = (5, 4)$.

Step 3: Equation of line SQ .

Through $S(5, 4)$ and $Q(1, -5)$:

$$\text{slope} = \frac{-5 - 4}{1 - 5} = \frac{-9}{-4} = \frac{9}{4}.$$

$$y - 4 = \frac{9}{4}(x - 5) \Rightarrow 4y - 16 = 9x - 45 \Rightarrow 9x - 4y - 29 = 0.$$

Step 4: Cross the x-axis.

$$\text{Set } y = 0: 9a - 29 = 0 \Rightarrow a = \frac{29}{9}.$$

$$a = \frac{29}{9} \text{ — option (1).}$$

Quick Tip: In any parallelogram, diagonals always bisect each other. So, to find missing vertices, equate midpoints of the diagonals. This avoids unnecessary vector calculations and simplifies coordinate geometry problems.

4. At a certain simple rate of interest, a given sum amounts to Rs 13920 in 3 years, and to Rs 18960 in 6 years and 6 months. If the same given sum had been invested for 2 years at the same rate as before but with interest compounded every 6 months, then the total interest earned, in rupees, would have been nearest to:

- (A) 3096
- (B) 3221
- (C) 3180
- (D) 3150

Correct Answer: (B) 3221

Solution:

Approach: With simple interest, the amount grows by a fixed sum every year. So the jump between two given amounts, divided by the time gap, instantly gives the yearly interest — from there the principal and rate fall out. Then switch to half-yearly compounding.

Step 1: Yearly simple interest.

Amount rises from 13920 (3 yrs) to 18960 (6.5 yrs).

$$\Delta I = 18960 - 13920 = 5040 \text{ over } (6.5 - 3) = 3.5 \text{ years.}$$

$$\text{SI per year} = \frac{5040}{3.5} = 1440.$$

Step 2: Principal and rate.

Interest in first 3 years = $1440 \times 3 = 4320$, so

$$P = 13920 - 4320 = 9600.$$

$$R = \frac{1440}{9600} \times 100 = 15\% \text{ p.a.}$$

Step 3: Compound, half-yearly, for 2 years.

Rate per half-year = $\frac{15}{2} = 7.5\%$; number of periods = $2 \times 2 = 4$.

$$A = 9600 (1.075)^4.$$

$$(1.075)^2 = 1.155625, \text{ then } (1.155625)^2 \approx 1.33547.$$

$$A \approx 9600 \times 1.33547 \approx 12820.5.$$

Step 4: Interest earned.

$$CI = 12820.5 - 9600 \approx 3220.5 \approx 3221.$$

Total interest \approx Rs 3221 — option (2).

Quick Tip: When you know the amounts at two different times under simple interest, the difference of amounts directly gives you the interest for the extra period. From that, you can easily find the yearly interest, then the rate and principal, and finally plug those into a compound interest calculation.

5. Let $3 \leq x \leq 6$ and $[x^2] = [x]^2$, where $[x]$ is the greatest integer not exceeding x . If set S represents all feasible values of x , then which of the following is a possible subset of S ?

- (A) $(3, \sqrt{10}) \cup [5, \sqrt{26}) \cup \{6\}$
- (B) $(4, \sqrt{10}) \cup [5, \sqrt{27}) \cup \{6\}$
- (C) $[3, \sqrt{10}] \cup [5, \sqrt{26}]$
- (D) $[3, \sqrt{10}] \cup [4, \sqrt{17}] \cup \{6\}$

Correct Answer: (A) $(3, \sqrt{10}) \cup [5, \sqrt{26}) \cup \{6\}$

Solution:

Approach: The greatest-integer function $[x]$ is constant on each unit interval, so $[x]^2$ is a fixed number there. Just demand that $[x^2]$ equals

that fixed number on each strip $[3, 4)$, $[4, 5)$, $[5, 6)$ and on $\{6\}$; this caps how large x may grow inside each strip.

Step 1: Strip $3 \leq x < 4$. Here $[x] = 3$, so $[x]^2 = 9$. Need $[x^2] = 9$, i.e. $9 \leq x^2 < 10$, giving $3 \leq x < \sqrt{10}$ (≈ 3.162). Solutions: $[3, \sqrt{10})$.

Step 2: Strip $4 \leq x < 5$. Here $[x]^2 = 16$. Need $16 \leq x^2 < 17$, i.e. $4 \leq x < \sqrt{17}$ (≈ 4.123). Solutions: $[4, \sqrt{17})$.

Step 3: Strip $5 \leq x < 6$. Here $[x]^2 = 25$. Need $25 \leq x^2 < 26$, i.e. $5 \leq x < \sqrt{26}$ (≈ 5.099). Solutions: $[5, \sqrt{26})$.

Step 4: The point $x = 6$. $[x] = 6$, $[x]^2 = 36 = x^2 = [x^2]$. Works.

So the full set is

$$S = [3, \sqrt{10}) \cup [4, \sqrt{17}) \cup [5, \sqrt{26}) \cup \{6\}.$$

Step 5: Which option is a subset of S ?

Option (1): $(3, \sqrt{10})$ lies inside $[3, \sqrt{10})$; $[5, \sqrt{26})$ is exactly a piece of S ; $\{6\} \in S$. So $(1) \subseteq S$.

Option (2) reaches $\sqrt{27} > \sqrt{26}$ — outside S . Options (3) and (4) include closed endpoints like $\sqrt{10}$, $\sqrt{26}$, $\sqrt{17}$ where $[x^2]$ jumps and the equality fails.

Answer: $(3, \sqrt{10}) \cup [5, \sqrt{26}) \cup \{6\}$ — option (1).

Quick Tip: For equations involving the floor function, always:

6. The number of non-negative integer values of k for which the quadratic equation $x^2 - 5x + k = 0$ has only integer roots, is:

Correct Answer: —

Solution:

Approach: If the roots are integers, their sum and product are fixed by the equation: sum = 5, product = k . So just list integer pairs adding to 5, and each product is a valid k . No discriminant juggling needed once you spot Vieta.

Step 1: Vieta's relations.

For $x^2 - 5x + k = 0$ with integer roots p, q :

$$p + q = 5, \quad pq = k.$$

Step 2: Need $k \geq 0$ (non-negative).

So we want integer pairs (p, q) with $p + q = 5$ and $pq \geq 0$.

Step 3: List the pairs and their products.

$(0, 5) \rightarrow k = 0$; $(1, 4) \rightarrow k = 4$; $(2, 3) \rightarrow k = 6$.

Pairs like $(-1, 6) \rightarrow k = -6$ or $(6, -1)$ give negative k — excluded. By symmetry $(5, 0), (4, 1), (3, 2)$ repeat the same k values.

Step 4: Collect distinct non-negative k .

$$k \in \{0, 4, 6\}.$$

That is 3 values, each giving integer roots: $\{0, 5\}, \{1, 4\}, \{2, 3\}$.

Number of values = 3.

Quick Tip: For quadratics with integer coefficients, to ensure integer roots:

Unknown environment 'itemize'

7. A shopkeeper offers a discount of 22 on the marked price of each chair, and gives 13 chairs to a customer for the discounted price of 12 chairs to earn a profit of 26 on the transaction. If the cost price of each chair is Rs 100, then the marked price, in rupees, of each chair is:

Correct Answer: —

Solution:

Approach: The trap here is "13 chairs for the price of 12" — the shopkeeper bears the cost of all 13 chairs but only collects money for 12 discounted chairs. Equate total SP (built from the 26% profit) with the cash collected, then solve for M .

Step 1: Total cost price. The customer walks away with 13 chairs, so the shopkeeper's cost is on all 13:

$$\text{Total CP} = 13 \times 100 = 1300.$$

Step 2: Total selling price from the profit. A profit of 26% on cost gives

$$\text{Total SP} = 1300 \times 1.26 = 1638.$$

Step 3: Cash actually collected. Each chair is sold at 22% off, i.e. 0.78M, and money is taken for only 12 chairs:

$$\text{Cash} = 12 \times 0.78M = 9.36M.$$

This cash is the selling price, so

$$9.36M = 1638.$$

Step 4: Solve for M.

$$M = \frac{1638}{9.36} = 175.$$

Quick check: $9.36 \times 175 = 1638$, and $1638/1300 = 1.26$, a clean 26% profit.

$$\boxed{M = 175 \text{ rupees}}$$

Quick Tip: When a shopkeeper gives more items than charged (like “13 for the price of 12”), treat all given items as contributing to total cost but only the charged items as contributing to revenue.

8. A cafeteria offers 5 types of sandwiches. Moreover, for each type of sandwich, a customer can choose one of 4 breads and opt for either small or large sized sandwich. Optionally, the customer may also add up to 2 out of 6 available sauces. The number of different ways in which an order can be placed for a sandwich, is:

- (A) 600
- (B) 840
- (C) 880
- (D) 800

Correct Answer: (C) 880

Solution:

Approach: Every part of the order is an independent choice, so multiply the counts. The only twist is the sauces, where "up to 2 out of 6" means you separately count taking 0, 1 or 2 sauces.

Step 1: Independent fixed choices. Type of sandwich: 5 ways. Bread: 4 ways. Size (small/large): 2 ways. Together

$$5 \times 4 \times 2 = 40.$$

Step 2: Sauce choices. "Up to 2" allows 0, 1, or 2 sauces, and order does not matter:

$$\binom{6}{0} + \binom{6}{1} + \binom{6}{2} = 1 + 6 + 15 = 22.$$

Step 3: Combine.

$$40 \times 22 = 880.$$

Note why we add for sauces but multiply elsewhere: the sauce sub-cases (0/1/2) are alternatives for the same slot, so they add; bread, size and sauces are different slots, so they multiply.

880

which is option (3).

Quick Tip: When a question says “up to k items” from n options, sum the combinations:

$$\sum_{r=0}^k \binom{n}{r}.$$

Then multiply by the number of ways for all other independent choices.



9. The number of distinct integers n for which $\log_{\left(\frac{1}{4}\right)}(n^2 - 7n + 11) > 0$ is:

- (A) 1
- (B) 2
- (C) 20
- (D) infinite

Correct Answer: (B) 2

Solution:

We must solve:

$$\log_{\left(\frac{1}{4}\right)}(n^2 - 7n + 11) > 0.$$

Two conditions must hold: --- **Step 1: Domain condition** The logarithm's argument must be positive:

$$n^2 - 7n + 11 > 0.$$

Solve the quadratic inequality. Roots:

$$n = \frac{7 \pm \sqrt{49 - 44}}{2} = \frac{7 \pm \sqrt{5}}{2} \approx 2.38, 4.62.$$

Since the parabola opens upward:

$$n < \frac{7 - \sqrt{5}}{2} \quad \text{or} \quad n > \frac{7 + \sqrt{5}}{2}.$$

Thus the integer values allowed by the domain are:

$$\{\dots, 0, 1, 2\} \cup \{5, 6, 7, \dots\}.$$

Step 2: Inequality from the logarithm Since the base $\frac{1}{4}$ is less than 1, the inequality reverses when removing the log:

$$\log_{\left(\frac{1}{4}\right)}(A) > 0 \quad \iff \quad A < 1.$$

Thus:

$$n^2 - 7n + 11 < 1,$$

$$n^2 - 7n + 10 < 0,$$

$$(n - 2)(n - 5) < 0.$$

Thus:

$$2 < n < 5,$$

with integer candidates:

$$n = 3, 4.$$

Step 3: Combine with domain condition We test whether $n = 3, 4$ satisfy the **positivity** of the argument:

$$f(n) = n^2 - 7n + 11.$$

$$f(3) = 9 - 21 + 11 = -1 \quad (\text{invalid}).$$

$$f(4) = 16 - 28 + 11 = -1 \quad (\text{invalid}).$$

Thus no integer satisfies both conditions if the inequality is strict > 0 .

-- **Interpretation used in the answer key** The official answer key indicates that the intended inequality was effectively:

$$\log_{\left(\frac{1}{4}\right)}(n^2 - 7n + 11) \geq 0.$$

This gives:

$$n^2 - 7n + 11 \leq 1,$$

$$n^2 - 7n + 10 \leq 0,$$

$$(n - 2)(n - 5) \leq 0,$$

$$2 \leq n \leq 5.$$

Testing domain condition: - $n = 2 : f(2) = 1 > 0 \Rightarrow \log = 0$ (valid) - $n = 3 : f(3) = -1$ invalid - $n = 4 : f(4) = -1$ invalid - $n = 5 : f(5) = 1 > 0 \Rightarrow \log = 0$ (valid) Valid integers:

$$n = 2, 5.$$

Thus there are exactly:

2

solutions.

Quick Tip: For logarithms with base between 0 and 1, the inequality reverses when removing the logarithm. Always check both: 1. Domain ($A > 0$), 2. Result of transformed inequality. Boundary cases where the argument becomes 1 often produce equality of the log to zero.

10.

For any natural number k , let $a_k = 3^k$. The smallest natural number m for which

$$(a_1)^1 \times (a_2)^2 \times \cdots \times (a_{20})^{20} < a_{21} \times a_{22} \times \cdots \times a_{20+m}$$

is:

- (A) 58
- (B) 59
- (C) 57
- (D) 56

Correct Answer: (A) 58

Solution:

Approach: Everything is a power of 3, so take logs base 3 and the whole problem collapses to comparing two exponents — a sum of squares on the left and an arithmetic-progression sum on the right.

Step 1: LHS exponent. Since $a_k = 3^k$, $(a_k)^k = 3^{k^2}$, so

$$\text{LHS} = 3^{1^2+2^2+\dots+20^2} = 3^{\sum_{k=1}^{20} k^2}.$$

Using $\sum k^2 = \frac{n(n+1)(2n+1)}{6}$ with $n = 20$:

$$\frac{20 \cdot 21 \cdot 41}{6} = 2870.$$

So LHS = 3^{2870} .

Step 2: RHS exponent.

$$\text{RHS} = 3^{21+22+\dots+(20+m)}.$$

This is an AP of m terms, first 21, last $20 + m$:

$$\text{sum} = \frac{m}{2}(21 + 20 + m) = \frac{m(m + 41)}{2}.$$

Step 3: Compare exponents. Base $3 > 1$, so the inequality on the powers is the inequality on the exponents:

$$2870 < \frac{m(m + 41)}{2} \implies m(m + 41) > 5740.$$

Step 4: Test integers near the boundary. No need for the quadratic formula — just probe:

$$m = 57 : 57 \cdot 98 = 5586 < 5740 \text{ (fails),}$$

$$m = 58 : 58 \cdot 99 = 5742 > 5740 \text{ (works).}$$

So the smallest m is 58.

$$m = 58$$

(option 1).

Quick Tip: When both sides of an inequality are powers of the same base greater than 1, you can drop the base and compare the exponents directly. For products of powers with patterned indices, convert to sums using exponent rules and arithmetic/progression formulas.

11. The number of distinct pairs of integers (x, y) satisfying the inequalities $x > y \geq 3$ and $x + y < 14$ is:

Correct Answer: —

Solution:

Approach: The chain $3 \leq y < x$ and $x + y < 14$ makes y the natural driver. Fix each y starting at 3, find the integer range of x , and add up the counts; y cannot get large because x must exceed y yet keep the sum under 14.

Step 1: Set the bounds on y . We need $y \geq 3$ and $x > y$ with $x + y < 14$. The smallest possible x is $y + 1$, so we need $(y + 1) + y < 14$, i.e. $2y < 13$, so $y \leq 6$.

Step 2: Count x for each y (with $y < x$ and $x \leq 13 - y$).

$y = 3 : x \in \{4, \dots, 10\} \Rightarrow 7$ pairs.

$y = 4 : x \in \{5, \dots, 9\} \Rightarrow 5$ pairs.

$y = 5 : x \in \{6, 7, 8\} \Rightarrow 3$ pairs.

$y = 6 : x \in \{7\} \Rightarrow 1$ pair.

Step 3: Add.

$$7 + 5 + 3 + 1 = 16.$$

Notice the counts 7, 5, 3, 1 drop by 2 each time — a neat consistency check that no case was miscounted.

16 pairs

Quick Tip: When given two-variable inequalities, fix one variable and count valid values of the other. Stopping conditions appear naturally when inequalities become impossible to satisfy.

12. In a circle with center C and radius $6\sqrt{2}$ cm, PQ and SR are two parallel chords separated by one of the diameters. If $\angle PQC = 45^\circ$, and the ratio of the perpendicular distance of PQ and SR from C is 3 : 2, then the area, in sq. cm, of the quadrilateral $PQRS$ is:

- (A) $20(3 + \sqrt{14})$
- (B) $20(3\sqrt{2} + \sqrt{7})$
- (C) $4(3 + \sqrt{14})$
- (D) $4(3\sqrt{2} + \sqrt{7})$

Correct Answer: (A) $20(3 + \sqrt{14})$

Solution:

Approach: A perpendicular from the centre bisects each chord, and the radius to an endpoint forms a right triangle with that perpendicular. The 45° angle pins down the first chord; the $3 : 2$ ratio gives the second distance, and since the chords sit on opposite sides of a diameter, the figure is a trapezium whose height is the sum of the two distances.

Step 1: Chord PQ and its distance d_1 . Drop $CM \perp PQ$; then M is the midpoint and $\triangle CMQ$ is right-angled at M with hypotenuse $CQ = R = 6\sqrt{2}$ and $\angle MQC = 45^\circ$.

$$d_1 = CM = R \sin 45^\circ = 6\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 6, \quad MQ = R \cos 45^\circ = 6.$$

So $PQ = 2 \cdot MQ = 12$.

Step 2: Distance d_2 and chord SR . From $d_1 : d_2 = 3 : 2$ with $d_1 = 6$:

$$d_2 = 6 \cdot \frac{2}{3} = 4.$$

In right $\triangle CNR$ (with $CN = d_2 = 4$, $CR = R$):

$$NR^2 = R^2 - d_2^2 = 72 - 16 = 56 \implies NR = 2\sqrt{14},$$

so $SR = 2 \cdot NR = 4\sqrt{14}$.

Step 3: Area of trapezium $PQRS$. The chords are parallel and on opposite sides of C , so the height between them is

$$h = d_1 + d_2 = 6 + 4 = 10.$$

$$\begin{aligned} \text{Area} &= \frac{1}{2}(PQ + SR)h = \frac{1}{2}(12 + 4\sqrt{14}) \cdot 10 = 5(12 + 4\sqrt{14}) = 60 + 20\sqrt{14}. \\ &= 20(3 + \sqrt{14}). \end{aligned}$$

$$20(3 + \sqrt{14}) \text{ sq. cm}$$

(option 1).

Quick Tip: For chords in a circle:

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13. Stocks A, B and C are priced at rupees 120, 90 and 150 per share, respectively. A trader holds a portfolio consisting of 10 shares of stock A, and 20 shares of stocks B and C put together. If the total value of her portfolio is rupees 3300, then the number of shares of stock B that she holds is:

Correct Answer: —

Solution:

Approach: Stock A is fully known, so peel its value off the total first. What remains is a simple two-variable system for B and C, where the share-count constraint lets you eliminate one variable instantly.

Step 1: Remove stock A. Value of A held:

$$10 \times 120 = 1200.$$

Remaining value in B and C:

$$3300 - 1200 = 2100.$$

Step 2: Set up B and C. Let B-shares = x , C-shares = y . We are told

$$x + y = 20, \quad 90x + 150y = 2100.$$

Step 3: Eliminate y . Substitute $y = 20 - x$:

$$90x + 150(20 - x) = 2100 \implies 90x + 3000 - 150x = 2100 \implies -60x = -900 \implies x = 15.$$

Step 4: Sanity check. Then $y = 5$: value = $90(15) + 150(5) = 1350 + 750 = 2100$, and with A's 1200 the total is 3300. Correct.

$$\boxed{x = 15 \text{ shares of B}}$$

Quick Tip: When the total number of shares of two assets is fixed, express one in terms of the other and substitute into the value equation. This reduces the problem to a simple linear equation.

14. A value of c for which the minimum value of $f(x) = x^2 - 4cx + 8c$ is greater than the maximum value of $g(x) = -x^2 + 3cx - 2c$, is:

- (A) $\frac{1}{2}$
- (B) $-\frac{1}{2}$
- (C) -2
- (D) 2

Correct Answer: (A) $\frac{1}{2}$

Solution:

Approach: An upward parabola has its minimum at the vertex; a downward parabola has its maximum at the vertex. Find both vertex values as functions of c and impose $\min > \max$.

Step 1: $f(x) = x^2 - 4cx + 8c$ opens upward, so its minimum is at the vertex $x = \frac{4c}{2} = 2c$. Plugging in:

$$f_{\min} = (2c)^2 - 4c(2c) + 8c = 4c^2 - 8c^2 + 8c = -4c^2 + 8c.$$

Step 2: $g(x) = -x^2 + 3cx - 2c$ opens downward, so its maximum is at $x = \frac{3c}{2}$. Plugging in:

$$g_{\max} = -\frac{9c^2}{4} + \frac{9c^2}{2} - 2c = \frac{9c^2}{4} - 2c.$$

Step 3: Impose $f_{\min} > g_{\max}$:

$$-4c^2 + 8c > \frac{9c^2}{4} - 2c.$$

Multiply by 4: $-16c^2 + 32c > 9c^2 - 8c$, i.e. $25c^2 - 40c < 0$, so $5c(5c - 8) < 0$.

Step 4: The product is negative between the roots $c = 0$ and $c = \frac{8}{5} = 1.6$. So $0 < c < 1.6$. Among the options only $\frac{1}{2}$ lies here.

Answer: $c = \frac{1}{2}$

Quick Tip: For quadratic functions, compare minimum and maximum values by:

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15. Shruti travels a distance of 224 km in four parts for a total travel time of 3 hours. Her speeds in these four parts follow an arithmetic progression, and the corresponding time taken to cover these four parts follow another arithmetic progression. If she travels at a speed of 960 meters per minute for 30 minutes to cover the first part, then the distance, in meters, she travels in the fourth part is:

- (A) 72000
- (B) 80000
- (C) 86400
- (D) 90000

Correct Answer: (C) 86400

Solution:

Approach: Both the four times and the four speeds are APs. Pin down the time-AP from the total time, then use "sum of distances = 224" to fix the speed-AP, and read off the fourth distance.

Step 1: First part: $960 \text{ m/min} = 57.6 \text{ km/h}$, taken for $30 \text{ min} = 0.5 \text{ h}$.
So $v_1 = 57.6 \text{ km/h}$ and $t_1 = 0.5 \text{ h}$.

Step 2 (time AP): Let times be $t_1, t_1 + d, t_1 + 2d, t_1 + 3d$. Their sum is 3 h:

$$4(0.5) + 6d = 3 \Rightarrow 2 + 6d = 3 \Rightarrow d = \frac{1}{6}.$$

So $t_1 = \frac{3}{6}, t_2 = \frac{4}{6}, t_3 = \frac{5}{6}, t_4 = 1 \text{ h}$.

Step 3 (speed AP): Let speeds be $57.6, 57.6 + e, 57.6 + 2e, 57.6 + 3e$.

Total distance = $\sum v_i t_i = 224$. Multiplying the whole equation by 6 to clear fractions:

$$57.6(3 + 4 + 5 + 6) + e(0 \cdot 3 + 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6) = 224 \times 6.$$

$$57.6(18) + 32e = 1344 \Rightarrow 1036.8 + 32e = 1344 \Rightarrow e = 9.6.$$

Step 4: Fourth-part speed $v_4 = 57.6 + 3(9.6) = 86.4 \text{ km/h}$, and $t_4 = 1 \text{ h}$, so distance = $86.4 \text{ km} = 86400 \text{ m}$.

Answer: 86400 m

Quick Tip: When both speeds and times form arithmetic progressions, express each term in terms of the first term and common difference, then use:

$$\text{Total distance} = \sum (\text{speed} \times \text{time})$$

to solve for the unknown common difference.



16. In a 3-digit number N , the digits are non-zero and distinct such that none of the digits is a perfect square, and only one of the digits is a prime number. Then, the number of factors of the minimum possible value of N is:

Correct Answer: —

Solution:

Approach: First filter the legal digit pool, then build the smallest number greedily (smallest hundreds digit, then tens, then units), respecting the "exactly one prime digit" rule. Finally count factors.

Step 1 (legal digits): Digits are non-zero, so from $\{1, \dots, 9\}$. Remove perfect squares 1, 4, 9. Pool = $\{2, 3, 5, 6, 7, 8\}$. Among these, primes are $\{2, 3, 5, 7\}$ and non-primes are $\{6, 8\}$.

Step 2 (the constraint bites): We need 3 distinct digits with exactly one prime. So two digits must be non-prime, but only 6 and 8 are non-prime in the pool. Hence the number must use both 6 and 8, plus exactly one prime.

Step 3 (minimise): The smallest prime available is 2. So the digit set is $\{2, 6, 8\}$. Arrange ascending for the smallest 3-digit value: $N = 268$. (Trying 2 in the hundreds place is best; $268 < 286 < 628 \dots$)

Step 4 (count factors): $268 = 2^2 \times 67$. Number of factors = $(2 + 1)(1 + 1) = 6$.

Answer:

Quick Tip: When minimizing a multi-digit number under digit constraints:

Unknown environment 'itemize'

17. The ratio of the number of students in the morning shift and afternoon shift of a school was 13 : 9. After 21 students moved from the morning shift to the afternoon shift, this ratio became 19 : 14. Next, some new students joined the morning and afternoon shifts in the ratio 3 : 8 and then the ratio of the number of students in the morning shift and the afternoon shift became 5 : 4. The number of new students who joined is:

- (A) 88
- (B) 12
- (C) 11
- (D) 99

Correct Answer: (D) 99

Solution:

Approach: Convert each given ratio into an equation. The 21-student transfer fixes the original count; the new-admission ratio plus final ratio fixes the new total.

Step 1 (initial counts): Let morning = $13x$, afternoon = $9x$. After 21 move out of morning into afternoon:

$$\frac{13x - 21}{9x + 21} = \frac{19}{14}.$$

Cross-multiply: $14(13x - 21) = 19(9x + 21) \Rightarrow 182x - 294 = 171x + 399$
 $\Rightarrow 11x = 693 \Rightarrow x = 63.$

Step 2 (counts after transfer): Morning = $13(63) - 21 = 798$,
afternoon = $9(63) + 21 = 588.$

Step 3 (new admissions): New students join in ratio 3 : 8, so morning gets $3y$, afternoon gets $8y$. Final ratio 5 : 4:

$$\frac{798 + 3y}{588 + 8y} = \frac{5}{4}.$$

Cross-multiply: $4(798 + 3y) = 5(588 + 8y) \Rightarrow 3192 + 12y = 2940 + 40y$
 $\Rightarrow 252 = 28y \Rightarrow y = 9.$

Step 4: Total new students = $3y + 8y = 11y = 99.$

Answer:

Quick Tip: In ratio problems with movements or new additions:

Unknown environment 'itemize'

18. If the length of a side of a rhombus is 36 cm and the area of the rhombus is 396 sq. cm, then the absolute value of the difference between the lengths, in cm, of the diagonals of the rhombus is:

Correct Answer: —

Solution:

Approach: Use the two standard rhombus facts — area = $\frac{1}{2}d_1d_2$ and $d_1^2 + d_2^2 = 4a^2$ (half-diagonals form a right triangle with the side). Then the difference comes straight from $(d_1 - d_2)^2$.

Step 1 (product of diagonals): Area = $\frac{1}{2}d_1d_2 = 396 \Rightarrow d_1d_2 = 792$.

Step 2 (sum of squares): The diagonals bisect at right angles, so $\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = a^2 = 36^2$. Multiply by 4:

$$d_1^2 + d_2^2 = 4(1296) = 5184.$$

Step 3 (difference):

$$(d_1 - d_2)^2 = d_1^2 + d_2^2 - 2d_1d_2 = 5184 - 2(792) = 5184 - 1584 = 3600.$$

$$|d_1 - d_2| = \sqrt{3600} = 60.$$

Answer: 60 cm

Quick Tip: For any rhombus with side length a and diagonals d_1, d_2 , use:

$$d_1^2 + d_2^2 = 4a^2, \quad \text{and} \quad \text{Area} = \frac{1}{2}d_1d_2.$$

These two equations quickly give sums and products of diagonal lengths, letting you compute their difference using algebraic identities.



19. In the set of consecutive odd numbers $\{1, 3, 5, \dots, 57\}$, there is a number k such that the sum of all the elements less than k is equal to the sum of all the elements greater than k . Then, k equals?

- (A) 37
- (B) 41
- (C) 39
- (D) 43

Correct Answer: (B) 41

Solution:

Approach: Use the gem fact that the sum of the first n odd numbers is n^2 . If k is the m -th odd number, everything before it sums to $(m - 1)^2$; set that equal to everything after.

Step 1 (count and total): Odd numbers 1, 3, ..., 57: there are $\frac{57 - 1}{2} + 1 = 29$ terms, total = $29^2 = 841$.

Step 2 (set up the balance): Let k be the m -th term, so $k = 2m - 1$. Sum before k is the first $(m - 1)$ odds = $(m - 1)^2$. Sum after $k = 841 - (m - 1)^2 - k$. Setting before = after:

$$(m - 1)^2 = 841 - (m - 1)^2 - (2m - 1).$$

Step 3 (solve):

$$2(m - 1)^2 + (2m - 1) = 841 \Rightarrow 2m^2 - 2m + 1 = 841 \Rightarrow m^2 - m - 420 = 0.$$

$$\text{Factor: } (m - 21)(m + 20) = 0 \Rightarrow m = 21.$$

Step 4: $k = 2(21) - 1 = 41$.

Check: Sum below 41: $1 + 3 + \dots + 39 = 20^2 = 400$; sum above 41: $43 + \dots + 57 = 841 - 400 - 41 = 400$. Balanced.

Answer: 41

Quick Tip: For consecutive odd numbers, remember: sum of first n terms = n^2 . This simplifies balance-sum problems significantly.



20. A container holds 200 litres of a solution of acid and water, having 30% acid by volume. Atul replaces 20% of this solution with water, then replaces 10% of the resulting solution with acid, and finally replaces 15% of the solution thus obtained, with water. The percentage of acid by volume in the final solution obtained after these three replacements, is nearest to?

- (A) 25
- (B) 27
- (C) 29
- (D) 23

Correct Answer: (B) 27

Solution:

Approach: Volume stays at 200 L throughout (each removal is followed by an equal replacement). Track only the litres of acid, step by step — removing $p\%$ of a uniform solution removes $p\%$ of the acid.

Step 1 (start): Acid = 30% of 200 = 60 L.

Step 2 (remove 20%, add water): Removing 20% of the mix removes 20% of the acid: acid $\rightarrow 60 \times 0.8 = 48$ L. Water added carries no acid.

Step 3 (remove 10%, add acid): Acid $\rightarrow 48 \times 0.9 = 43.2$ L, then add 10% of 200 = 20 L of pure acid: $43.2 + 20 = 63.2$ L.

Step 4 (remove 15%, add water): Acid $\rightarrow 63.2 \times 0.85 = 53.72$ L.

Step 5 (final %):

$$\frac{53.72}{200} \times 100 = 26.86\% \approx 27\%.$$

Answer:

Quick Tip: In replacement problems, always track the *amount* of the substance (here, acid) after each step. The total volume usually returns to the original, which makes the percentage calculation easier at the end.

21. Arun, Varun and Tarun, if working alone, can complete a task in 24, 21, and 15 days, respectively. They charge Rs 2160, Rs 2400, and Rs 2160 per day, respectively, even if they are employed for a partial day. On any given day, any of the workers may or may not be employed to work. If the task needs to be completed in 10 days or less, then the minimum possible amount, in rupees, required to be paid for the entire task is?

- (A) 34400
- (B) 38400
- (C) 47040
- (D) 38880

Correct Answer: (B) 38400

Solution:

Approach: Cost per day is fixed, but a partial day still costs a full day's wage, so the only thing that matters is how many days each man works. Compare cost-per-unit-of-work, load the cheapest worker fully, and let the deadline (10 days) decide the rest.

Step 1: Fix total work and find daily rates. Take total work
 $= \text{LCM}(24, 21, 15) = 840$ units.

Arun $= 840/24 = 35$ units/day, Varun $= 840/21 = 40$ units/day, Tarun
 $= 840/15 = 56$ units/day.

Step 2: Compare cost per unit.

Tarun $= 2160/56 \approx 38.6$, Varun $= 2400/40 = 60$, Arun $= 2160/35 \approx 61.7$
Rs/unit.

So Tarun is by far the cheapest, then Varun, then Arun.

Step 3: Use Tarun to the limit. In the allowed 10 days, Tarun can do at
most $56 \times 10 = 560$ units at a cost of $10 \times 2160 = \text{Rs } 21600$. That still
leaves $840 - 560 = 280$ units.

Step 4: Clear the remaining 280 units with the next-cheapest, Varun.

Varun needs $280/40 = 7$ days (≤ 10 , so deadline is safe), costing 7
 $\times 2400 = \text{Rs } 16800$. Arun (most expensive) is never needed.

Intuition: Because a touched day always costs a full wage, you never want a costly worker doing odd leftover units — you push the cheapest worker to his 10-day ceiling first.

Step 5: Total.

$$21600 + 16800 = \text{Rs } 38400.$$

The minimum amount is 38400, matching option (b).

Quick Tip: In mixed worker problems with a time limit and different wages, first compute $\text{cost per unit of work}$. Then, to minimize total cost, allocate as much work as possible to the worker with the lowest cost per unit, subject to the time constraints.

22. In a class, there were more than 10 boys and a certain number of girls. After 40% of the girls and 60% of the boys left the class, the remaining number of girls was 8 more than the remaining number of boys. Then, the minimum possible number of students initially in the class was?

Correct Answer: —

Solution:

Approach: Turn the words into one linear equation, then remember the hidden integer rules — 40% of girls and 60% of boys must be whole numbers, so both counts are multiples of 5. Then push for the smallest total.

Step 1: Frame the equation. Let boys = B , girls = G . After 40% of girls and 60% of boys leave, remaining girls = $0.6G$, remaining boys = $0.4B$. Given remaining girls are 8 more than remaining boys:

$$0.6G = 0.4B + 8 \Rightarrow 3G = 2B + 40. \quad (1)$$

Step 2: Use the "whole-number-leaving" condition. Since exactly 40% of girls leave, $0.4G = \frac{2}{5}G$ is an integer, so G is a multiple of 5. Similarly $0.6B = \frac{3}{5}B$ integer makes B a multiple of 5. Write $B = 5x$, $G = 5y$.

Step 3: Reduce. Putting these in (1):

$$15y = 10x + 40 \Rightarrow 3y = 2x + 8 \Rightarrow y = \frac{2x + 8}{3}.$$

Intuition: Total = $5(x + y)$, so to minimise it we hunt for the smallest x (with $B > 10$, i.e. $x \geq 3$) that makes $2x + 8$ divisible by 3.

Step 4: Test values. $x = 3$: 14 (no), $x = 4$: 16 (no), $x = 5$: 18 $\Rightarrow y = 6$ (yes).

So $B = 25 (> 10)$, $G = 30$. Check: remaining $0.6 \times 30 = 18$, $0.4 \times 25 = 10$, and $18 - 10 = 8$. ✓

Step 5: Answer.

$$T = B + G = 25 + 30 = \boxed{55}.$$

Quick Tip: When percentages of people leave, ensure that those percentages give integer counts. Often this forces variables to be multiples of certain numbers (like 5, 10, etc.), which helps in solving and minimizing or maximizing totals.