CAT 2025 QA Slot 2 Question Paper with Solutions

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General Instructions

(i) This booklet contains 22 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 16 single-correct multiple-choice questions and 6 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For numerical questions, report the answer rounded exactly as asked.

1. Let a_n be the n^{th} term of a decreasing infinite geometric progression. If $a_1 + a_2 + a_3 = 52$ and $a_1a_2 + a_2a_3 + a_3a_1 = 624$, then the sum of this geometric progression is:

(A) 57

(B) 63

(C) 54

(D) 60

Correct Answer: (C) 54

Solution:

Approach: The two given symmetric expressions in a_1, a_2, a_3 are really expressions in the first term a and ratio r. Write both, and dividing one by the other kills the messy $(1 + r + r^2)$ factor in one stroke.

Step 1: Let the first term be a and common ratio be r. Then $a_1 = a,\ a_2 = ar,\ a_3 = ar^2$, and from the first condition

$$a_1 + a_2 + a_3 = a(1 + r + r^2) = 52. \quad (1)$$

Step 2: For the second condition,

$$a_1a_2 + a_2a_3 + a_3a_1 = a \cdot ar + ar \cdot ar^2 + ar^2 \cdot a = a^2r(1 + r + r^2) = 624. \quad (2)$$

Now notice the trick: equation (2) is just $\left[a(1 + r + r^2)\right] \times (ar) = 52 \cdot (ar)$. So

$$52\,(ar) = 624 \;\Rightarrow\; ar = 12. \quad (3)$$

That single division removes the cubic clutter completely.

Step 3: Solve for r. From (1), $a + ar + ar^2 = 52$. Using $ar = 12$ and writing $ar^2 = (ar) \cdot r = 12r$, we get

$$a + 12 + 12r = 52 \;\Rightarrow\; a = 40 - 12r.$$

Multiply $ar = 12$: $(40 - 12r)r = 12 \Rightarrow 12r^2 - 40r + 12 = 0 \Rightarrow 3r^2 - 10r + 3 = 0$.

$$r = \frac{10 \pm \sqrt{100 - 36}}{6} = \frac{10 \pm 8}{6} \;\Rightarrow\; r = 3 \text{ or } r = \tfrac{1}{3}.$$

Step 4: Pick the valid root. The G.P. is decreasing and infinite, so we need $|r| < 1$; hence $r = \frac{1}{3}$. Then $a = \dfrac{12}{r} = 36$.

Step 5: Sum to infinity:

$$S_\infty = \frac{a}{1 - r} = \frac{36}{1 - \frac{1}{3}} = \frac{36}{2/3} = 54.$$

Final answer: 54.

Quick Tip: For an infinite geometric progression with first term a and common ratio r (where $|r| < 1$):

$$S_\infty = \frac{a}{1-r}.$$

Also, using relationships between sums and products of initial terms can help form equations in a and r.

2. Two tangents drawn from a point P touch a circle with center O at points Q and R. Points A and B lie on PQ and PR, respectively, such that AB is also a tangent to the same circle. If $\angle AOB = 50°$, then $\angle APB$, in degrees, equals:

Correct Answer: —

Solution:

Approach: The picture is a triangle PAB with the circle as its incircle (it touches all three sides PQ, PR and AB). For any incircle, the angle a side subtends at the centre is fixed relative to the opposite vertex angle, which links $\angle AOB$ to $\angle APB$ immediately.

Step 1: Note that PQ, PR, and AB are all tangents to the same circle, and they form triangle PAB with the circle sitting inside it. So O is the incentre of triangle PAB and the circle is its incircle, touching AB at a point T.

Step 2: Look at the angle $\angle AOB$ at the incentre, made by the two cevians OA and OB. A standard incentre result says the angle subtended at the incentre by a side equals 90° plus half the opposite angle:

$$\angle AOB = 90^\circ + \tfrac{1}{2}\angle APB.$$

Step 3: Substitute $\angle AOB = 50^\circ$:

$$50^\circ = 90^\circ + \tfrac{1}{2}\angle APB \;\Rightarrow\; \tfrac{1}{2}\angle APB = -40^\circ.$$

A negative value signals the circle is the *excircle* opposite P, not the incircle (it touches side AB and the extensions of PA, PB beyond A, B — exactly the "tangent line cutting across" configuration here). For the P-excircle the relation is

$$\angle AOB = 90^\circ - \tfrac{1}{2}\angle APB.$$

Step 4: Now

$$50^\circ = 90^\circ - \tfrac{1}{2}\angle APB \;\Rightarrow\; \tfrac{1}{2}\angle APB = 40^\circ \;\Rightarrow\; \angle APB = 80^\circ.$$

Step 5 (clean cross-check): Split the central angles by the tangent contact points. Tangents from A give $\angle AOT = \angle AOQ = x$; tangents from B give $\angle BOT = \angle BOR = y$. Then $\angle AOB = x + y = 50^\circ$, and the full angle $\angle QOR = 2x + 2y = 100^\circ$. In quadrilateral $PQOR$, with $\angle OQP = \angle ORP = 90^\circ$,

$$\angle APB = 360^\circ - 90^\circ - 90^\circ - 100^\circ = 80^\circ.$$

Final answer: $\angle APB = 80^\circ$.

Quick Tip: Tangents from an external point to a circle are equal, and the line from the center to the point of tangency is perpendicular to the tangent. These facts often allow you to form congruent triangles and use angle sums in quadrilaterals involving the center.

3. Let $ABCDEF$ be a regular hexagon and P and Q be the midpoints of AB and CD, respectively. Then, the ratio of the areas of trapezium $PBCQ$ and hexagon $ABCDEF$ is:

(A) 6 : 19

(B) 5 : 24

(C) 6 : 25

(D) 7 : 24

Correct Answer: (B) 5 : 24

Solution:

Approach: A regular hexagon splits neatly into 6 unit equilateral triangles (or, better here, into a grid of small triangles). Cut the hexagon along the two midpoints and just count what fraction of those small triangles the trapezium covers — no coordinates needed if you set them up smartly, but coordinates make the parallel sides instant.

Step 1: Let the side be s. Area of the regular hexagon:

$$\text{Area}_{\text{hex}} = \frac{3\sqrt{3}}{2}s^2.$$

Step 2: Put $B = (0,0)$ and $C = (s,0)$ along the bottom side. Walking around the hexagon, the neighbours of B and C sit at

$$A = \left(-\tfrac{s}{2},\ \tfrac{\sqrt{3}s}{2}\right), \qquad D = \left(\tfrac{3s}{2},\ \tfrac{\sqrt{3}s}{2}\right).$$

Step 3: Midpoints:

$$P = \text{mid}(A,B) = \left(-\tfrac{s}{4},\ \tfrac{\sqrt{3}s}{4}\right), \qquad Q = \text{mid}(C,D) = \left(\tfrac{5s}{4},\ \tfrac{\sqrt{3}s}{4}\right).$$

Step 4: P and Q share the same height, so $PQ \parallel BC$ and the trapezium $PBCQ$ has parallel sides

$$BC = s, \qquad PQ = \tfrac{5s}{4} - \left(-\tfrac{s}{4}\right) = \tfrac{3s}{2},$$

with height $h = \frac{\sqrt{3}s}{4}$ (the vertical gap). Hence

$$\text{Area}_{\text{trap}} = \tfrac{1}{2}(BC + PQ)\,h = \tfrac{1}{2}\left(s + \tfrac{3s}{2}\right)\cdot \tfrac{\sqrt{3}s}{4} = \frac{5\sqrt{3}}{16}s^2.$$

Step 5: Ratio:

$$\frac{\text{Area}_{\text{trap}}}{\text{Area}_{\text{hex}}} = \frac{\frac{5\sqrt{3}}{16}s^2}{\frac{3\sqrt{3}}{2}s^2} = \frac{5}{16}\cdot\frac{2}{3} = \frac{5}{24}.$$

Final answer: $5 : 24$.

Quick Tip: For regular polygons, coordinate geometry is very handy: place convenient vertices on the axes, find key points (like midpoints) using averages of coordinates, and then use distance or area formulas to compute lengths and areas.

4. Suppose a, b, c are three distinct natural numbers, such that $3ac = 8(a+b)$. Then, the smallest possible value of $3a + 2b + c$ is:

Correct Answer: —

Solution:

Approach: Solve for the most "expensive" variable so the search is small. Here $b = \frac{a(3c-8)}{8}$, so for b to be a small natural number you want $3c - 8$ tiny and the factor of 8 handled by a. Test $c = 3, 4, 5, \ldots$ and stop early.

Step 1: Rearrange $3ac = 8(a+b)$:

$$8b = 3ac - 8a = a(3c-8) \;\Rightarrow\; b = \frac{a(3c-8)}{8}.$$

Since b is a natural number we need $3c - 8 > 0$, i.e. $c \geq 3$, and $a(3c - 8)$ divisible by 8.

Step 2: The target $S = 3a + 2b + c$ grows with a, so we want a as small as the divisibility allows. Run through c:

Case $c = 3$: $b = \frac{a(1)}{8} = \frac{a}{8}$, so a must be a multiple of 8. Smallest $a = 8 \Rightarrow b = 1$. Distinct? $(8, 1, 3)$ yes. $S = 24 + 2 + 3 = 29$.

Case $c = 4$: $b = \frac{a(4)}{8} = \frac{a}{2}$, so a must be even. Smallest $a = 2 \Rightarrow b = 1$. Distinct? $(2, 1, 4)$ yes. $S = 6 + 2 + 4 = 12$.

Case $c = 5$: $b = \dfrac{7a}{8}$, needs $a = 8 \Rightarrow b = 7$. $S = 24 + 14 + 5 = 43$.

Case $c = 6$: $b = \dfrac{10a}{8} = \dfrac{5a}{4}$, needs $a = 4 \Rightarrow b = 5$. $S = 12 + 10 + 6 = 28$.

Step 3: Every larger c forces either a larger a (for divisibility) or a larger b, pushing S up. The smallest value found is in the $c = 4$ case with $(a, b, c) = (2, 1, 4)$, all distinct naturals, giving

$$S = 3(2) + 2(1) + 4 = 12.$$

Final answer: 12.

Quick Tip: When you have a Diophantine equation (integer solutions) and need to minimize an expression, first express one variable in terms of the others, then systematically test small integer values under the constraints (like distinctness and positivity).

5. The ratio of expenditures of Lakshmi and Meenakshi is $2 : 3$, and the ratio of income of Lakshmi to expenditure of Meenakshi is $6 : 7$. If excess of income over expenditure is saved by Lakshmi and Meenakshi, and the ratio of their savings is $4 : 9$, then the ratio of their incomes is:

(A) $7 : 8$

(B) $3 : 5$

(C) $2 : 1$

(D) $5 : 6$

Correct Answer: (B) 3 : 5

Solution:

Approach: Anchor everything to the expenditures, since two of the three given ratios involve expenditure. Set $E_L = 2x$, $E_M = 3x$; then Lakshmi's income and savings come out in terms of x, and the savings ratio fixes Meenakshi's income.

Step 1: Let $E_L = 2x$ and $E_M = 3x$ (expenditure ratio 2 : 3).

Step 2: Given $\frac{I_L}{E_M} = \frac{6}{7}$, so

$$I_L = \frac{6}{7} \cdot 3x = \frac{18x}{7}.$$

Step 3: Lakshmi's saving = income − expenditure:

$$S_L = \frac{18x}{7} - 2x = \frac{18x - 14x}{7} = \frac{4x}{7}.$$

Step 4: Let Meenakshi's income be I_M, so her saving is $S_M = I_M - 3x$. The savings ratio is $S_L : S_M = 4 : 9$:

$$\frac{4x/7}{I_M - 3x} = \frac{4}{9} \Rightarrow \frac{x/7}{I_M - 3x} = \frac{1}{9} \Rightarrow I_M - 3x = \frac{9x}{7}.$$

Hence

$$I_M = 3x + \frac{9x}{7} = \frac{21x + 9x}{7} = \frac{30x}{7}.$$

Step 5: Ratio of incomes:

$$\frac{I_L}{I_M} = \frac{18x/7}{30x/7} = \frac{18}{30} = \frac{3}{5}.$$

Final answer: 3 : 5.

Quick Tip: When working with income–expenditure–saving problems:

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6. If $\log_{64} x^2 + \log_8 \sqrt{y} + 3\log_{512}(\sqrt{y}z) = 4$, where x, y and z are positive real numbers, then the minimum possible value of $(x + y + z)$ is:

(A) 24

(B) 36

(C) 96

(D) 48

Correct Answer: (D) 48

Solution:

Approach: Every base $(64, 8, 512)$ is a power of 2, so convert all three logs to base 2. The whole left side collapses to a single symmetric expression $\frac{1}{3}(\log_2 x + \log_2 y + \log_2 z)$, turning the condition into a fixed product xyz — and then AM-GM gives the minimum sum.

Step 1: Write each term in base 2 using $\log_{2^k}(N) = \frac{1}{k}\log_2 N$:

$$\log_{64} x^2 = \tfrac{1}{6}\log_2 x^2 = \tfrac{1}{3}\log_2 x,$$

$$\log_8 \sqrt{y} = \tfrac{1}{3}\log_2 y^{1/2} = \tfrac{1}{6}\log_2 y,$$

$$3\log_{512}(\sqrt{y}\,z) = 3 \cdot \tfrac{1}{9}\log_2(\sqrt{y}\,z) = \tfrac{1}{3}\big(\tfrac{1}{2}\log_2 y + \log_2 z\big).$$

Step 2: Add them and set equal to 4:

$$\tfrac{1}{3}\log_2 x + \tfrac{1}{6}\log_2 y + \tfrac{1}{6}\log_2 y + \tfrac{1}{3}\log_2 z = 4.$$

The two $\tfrac{1}{6}\log_2 y$ terms combine to $\tfrac{1}{3}\log_2 y$, giving the clean symmetric form

$$\tfrac{1}{3}\Big(\log_2 x + \log_2 y + \log_2 z\Big) = 4.$$

Step 3: Multiply by 3: $\log_2 x + \log_2 y + \log_2 z = 12$, i.e.

$$\log_2(xyz) = 12 \;\Rightarrow\; xyz = 2^{12} = 4096.$$

Step 4: By AM-GM on positive reals,

$$\frac{x+y+z}{3} \geq \sqrt[3]{xyz} = \sqrt[3]{4096} = 16,$$

so $x+y+z \geq 48$, with equality when $x = y = z = 16$ (each is a valid positive real, and $16 \cdot 16 \cdot 16 = 4096$). The constraint imposes no extra restriction, so the bound is attainable.

Step 5: Minimum $x+y+z = 16+16+16 = 48$. **Final answer:** 48.

Quick Tip: When an equation in logarithms simplifies to a fixed product like $xyz = \text{constant}$, use the AM-GM inequality to find the minimum (or maximum) of sums such as $x + y + z$. Equality in AM-GM occurs when all the variables are equal.

7. If $9^{x^2+2x-3} - 4\left(3^{x^2+2x-2}\right) + 27 = 0$, then the product of all possible values of x is:

(A) 2

(B) 4

(C) 10

(D) 20

Correct Answer: (D) 20

Solution:

Approach: Two layers of substitution. First let $u = x^2 + 2x$ so the equation is in one exponent; then let $y = 3^u$ to turn it into a plain quadratic. Solve, back-substitute, and read off the product via Vieta — no need to actually solve for x.

Step 1: Put $u = x^2 + 2x$. The equation $9^{x^2+2x-3} - 4 \cdot 3^{x^2+2x-2} + 27 = 0$ becomes

$$9^{\,u-3} - 4 \cdot 3^{\,u-2} + 27 = 0.$$

Step 2: Since $9 = 3^2$, write $9^{u-3} = 3^{2u-6}$. Let $y = 3^u$. Then $3^{2u-6} = \dfrac{y^2}{3^6}$ $= \dfrac{y^2}{729}$ and $3^{u-2} = \dfrac{y}{9}$, so

$$\frac{y^2}{729} - \frac{4y}{9} + 27 = 0.$$

Multiply by 729:

$$y^2 - 324y + 19683 = 0.$$

Step 3: Discriminant $= 324^2 - 4(19683) = 104976 - 78732 = 26244$ $= 162^2$, so

$$y = \frac{324 \pm 162}{2} = 243 \text{ or } 81.$$

Step 4: Back-substitute $y = 3^u$:

$$3^u = 243 = 3^5 \Rightarrow u = 5, \qquad 3^u = 81 = 3^4 \Rightarrow u = 4.$$

With $u = x^2 + 2x$ this gives two quadratics:

$$x^2 + 2x - 5 = 0, \qquad x^2 + 2x - 4 = 0.$$

Each has real roots (discriminants 24 and 20, both positive), so all four x-values are valid.

Step 5: By Vieta, product of roots $= \dfrac{c}{a}$: for the first $= -5$, for the second $= -4$. Product of all four roots:

$$(-5) \times (-4) = 20.$$

Final answer: 20.

Quick Tip: When exponents share a common expression (like $x^2 + 2x$ here), substitute it with a single variable to simplify. Then, look for ways to convert to a common base and reduce the equation to a quadratic in a new variable such as 3^u.

8. The average number of copies of a book sold per day by a shopkeeper is 60 in the initial seven days and 63 in the initial eight days, after the book launch. On the ninth day, she sells 11 copies less than the eighth day, and the average number of copies sold per day from the second day to the ninth day becomes 66. The number of copies sold on the first day of the book launch is:

Correct Answer: —

Solution:

Approach: Every "average" here is really a hidden total. Convert each average into a sum of copies, then the single overlapping day (or the leftover day) pops out by simple subtraction.

Step 1: First 7 days average 60, so the 7-day total is

$$S_7 = 7 \times 60 = 420.$$

Step 2: First 8 days average 63, so the 8-day total is

$$S_8 = 8 \times 63 = 504.$$

The 8th-day sale is the difference:

$$x_8 = S_8 - S_7 = 504 - 420 = 84.$$

Step 3: The 9th day is 11 less than the 8th day:

$$x_9 = 84 - 11 = 73.$$

Step 4: Days 2 to 9 are 8 days with average 66, so their total is

$$S_{2\text{ to }9} = 8 \times 66 = 528.$$

Step 5: Total for days 1 to 9 is

$$S_9 = S_8 + x_9 = 504 + 73 = 577.$$

But the same 9-day total is also day 1 plus days 2 to 9:

$$S_9 = x_1 + 528.$$

So

$$x_1 = 577 - 528 = 49.$$

Final answer: $x_1 = 49$ copies on the first day.

Quick Tip: When dealing with averages over overlapping time intervals, convert each average to a total sum. Then, use differences of these sums to find individual day values and set up equations to solve for the unknowns.

9. A loan of Rs 1000 is fully repaid by two installments of Rs 530 and Rs 594, paid at the end of the first and second year, respectively. If the interest is compounded annually, then the rate of interest, in percentage, is:

(A) 6

(B) 7

(C) 8

(D) 9

Correct Answer: (C) 8

Solution:

Approach: A loan plus interest is paid off by instalments, so the loan today must equal the present value of both future instalments discounted at the unknown rate. Set up that balance and solve the resulting quadratic in $(1 + r)$.

Step 1: Let $x = 1 + r$. The first instalment (Rs 530) is one year away, the second (Rs 594) is two years away, so

$$1000 = \frac{530}{x} + \frac{594}{x^2}.$$

Step 2: Multiply through by x^2 to clear denominators:

$$1000x^2 = 530x + 594,$$

$$1000x^2 - 530x - 594 = 0.$$

Step 3: Apply the quadratic formula with $a = 1000,\ b = -530,\ c = -594$. The discriminant is

$$(-530)^2 + 4(1000)(594) = 280900 + 2376000 = 2656900 = 1630^2.$$

So

$$x = \frac{530 \pm 1630}{2000}.$$

The positive root is

$$x = \frac{2160}{2000} = 1.08.$$

Step 4: Since $x = 1 + r = 1.08$, we get $r = 0.08$.

Final answer: The rate of interest is 8%.

Quick Tip: Whenever loan repayments occur in installments, convert each installment into its present value and sum them. If the interest is compounded annually, the discount factor for the n-th year is $(1 + r)^n$.

10. The set of all real values of x for which $(x^2 - |x + 9| + x) > 0$ is:

(A) $(-\infty, -9) \cup (3, \infty)$

(B) $(-\infty, -3) \cup (9, \infty)$

(C) $(-\infty, -3) \cup (3, \infty)$

(D) $(-9, -3) \cup (3, 9)$

Correct Answer: (C) $(-\infty, -3) \cup (3, \infty)$

Solution:

Approach: A modulus inequality is solved by splitting at the point where the inside changes sign. Here $|x+9|$ flips at $x = -9$, so handle $x \geq -9$ and $x < -9$ separately, then take the union.

Rewrite the given inequality $x^2 - |x+9| + x > 0$ as

$$x^2 + x - |x+9| > 0.$$

Case 1: $x \geq -9$. Then $|x+9| = x+9$, so

$$x^2 + x - (x+9) > 0 \Rightarrow x^2 - 9 > 0 \Rightarrow (x-3)(x+3) > 0.$$

This holds for $x < -3$ or $x > 3$. Intersecting with $x \geq -9$ gives

$$[-9, -3) \cup (3, \infty).$$

Case 2: $x < -9$. Then $|x+9| = -(x+9)$, so

$$x^2 + x + x + 9 > 0 \Rightarrow x^2 + 2x + 9 > 0.$$

Its discriminant is $2^2 - 4(9) = -32 < 0$ with a positive leading coefficient, so this is true for every real x. Hence all of $x < -9$ qualifies:

$$(-\infty, -9).$$

Combine:

$$(-\infty, -9) \cup [-9, -3) \cup (3, \infty) = (-\infty, -3) \cup (3, \infty).$$

Final answer: $(-\infty, -3) \cup (3, \infty)$ — option 3.

Quick Tip: When solving inequalities involving absolute values, always split into cases based on the sign of the expression inside the absolute value. Analyze each case separately, then combine intervals carefully by intersection and union.

11. The equations $3x^2 - 5x + p = 0$ and $2x^2 - 2x + q = 0$ have one common root. The sum of the other roots of these two equations is:

(A) $\frac{5}{3} - p + q$

(B) $\frac{8}{3} + p - q$

(C) $\frac{8}{3} - p + \frac{3}{2}q$

(D) $p + q - 1$

Correct Answer: (C) $\frac{8}{3} - p + \frac{3}{2}q$

Solution:

Approach: The two "other" roots are found from sum-of-roots (Vieta), but they still carry the unknown common root α. Kill α by combining the two equations so the α^2 term cancels, leaving α in terms of p and q.

Let α be the common root. Let the first equation $3x^2 - 5x + p = 0$ have roots α, β and the second $2x^2 - 2x + q = 0$ have roots α, γ.

Step 1: By Vieta,

$$\alpha + \beta = \frac{5}{3}, \qquad \alpha + \gamma = \frac{2}{2} = 1.$$

Adding,

$$\beta + \gamma = \frac{5}{3} + 1 - 2\alpha = \frac{8}{3} - 2\alpha.$$

Step 2: Find α. Since α satisfies both equations:

$$3\alpha^2 - 5\alpha + p = 0, \qquad 2\alpha^2 - 2\alpha + q = 0.$$

Multiply the first by 2 and the second by 3 to match the α^2 terms:

$$6\alpha^2 - 10\alpha + 2p = 0, \qquad 6\alpha^2 - 6\alpha + 3q = 0.$$

Subtract:

$$-4\alpha + 2p - 3q = 0 \Rightarrow \alpha = \frac{2p - 3q}{4}.$$

Step 3: Substitute $2\alpha = \dfrac{2p - 3q}{2} = p - \dfrac{3q}{2}$:

$$\beta + \gamma = \frac{8}{3} - \left(p - \frac{3q}{2}\right) = \frac{8}{3} - p + \frac{3}{2}q.$$

Final answer: $\dfrac{8}{3} - p + \dfrac{3}{2}q$ — option 3.

Quick Tip: When two quadratics share a common root, equate the root expressions by eliminating the squared term. Using Vieta's formulas then makes it easy to compute required expressions involving the other roots.

12. An item with a cost price of Rs.1650 is sold at a certain discount on a fixed marked price to earn a profit of 20% on the cost price. If the discount was doubled, the profit would have been Rs.110. The rate of discount, in percentage, at which the profit percentage would be equal to the rate of discount, is nearest to:

(A) 12

(B) 13

(C) 14

(D) 15

Correct Answer: (C) 14

Solution:

Approach: Two selling scenarios share the same marked price, so write both as $MP - \text{discount} = SP$ and subtract to peel off the marked price and the discount. Then set discount% equal to profit% and solve one linear equation.

Step 1: First sale, 20% profit on $CP = 1650$:

$$SP_1 = 1650 \times 1.20 = 1980.$$

With discount D:

$$MP - D = 1980. \tag{1}$$

Step 2: Second sale, profit Rs 110, so $SP_2 = 1650 + 110 = 1760$, and the discount is doubled:

$$MP - 2D = 1760. \tag{2}$$

Step 3: Subtract (2) from (1):

$$D = 1980 - 1760 = 220.$$

From (1),

$$MP = 1980 + 220 = 2200.$$

Step 4: Let the common value of discount% and profit% be x.
Discount amount $= \dfrac{x}{100} \times 2200 = 22x$, so

$$SP = 2200 - 22x, \qquad \text{Profit} = SP - 1650 = 550 - 22x.$$

Setting profit% equal to x:

$$\frac{550 - 22x}{1650} \times 100 = x.$$

Step 5: Solve:

$$100(550 - 22x) = 1650x \Rightarrow 55000 - 2200x = 1650x,$$

$$55000 = 3850x \Rightarrow x = \frac{55000}{3850} = \frac{100}{7} \approx 14.29.$$

Final answer: Nearest integer is 14 — option 3.

Quick Tip: When the same marked price is used under different discount and profit conditions, set up equations using $SP = MP - \text{Discount}$ and $SP = CP + \text{Profit}$ for each scenario. Once the marked price is known, you can introduce a variable discount rate and equate the profit percentage to that rate to solve such "rate equals rate" problems.

13. A certain amount of money was divided among Pinu, Meena, Rinu, and Seema. Pinu received 20% of the total amount and Meena received 40% of the remaining amount. If Seema received 20% less than Pinu, the ratio of the amounts received by Pinu and Rinu is:

(A) 4 : 5

(B) 5 : 8

(C) 3 : 5

(D) 2 : 3

Correct Answer: (B) 5 : 8

Solution:

Approach: Percentages of an unknown total are easiest if you just assume a convenient total. Take the whole pot as 100 units; every share becomes a plain number and Rinu is whatever is left over.

Step 1: Let the total be 100. Pinu gets 20%:

$$\text{Pinu} = 20.$$

Step 2: Remaining after Pinu $= 100 - 20 = 80$. Meena gets 40% of this remaining amount:

$$\text{Meena} = 0.40 \times 80 = 32.$$

Step 3: Seema gets 20% less than Pinu:

$$\text{Seema} = 0.80 \times 20 = 16.$$

Step 4: Rinu takes whatever remains:

$$\text{Rinu} = 100 - (20 + 32 + 16) = 100 - 68 = 32.$$

Step 5: Required ratio

$$\text{Pinu} : \text{Rinu} = 20 : 32 = 5 : 8.$$

Final answer: 5 : 8 — option 2.

Quick Tip: In distribution problems, it often helps to assume a convenient total (like 100) when only percentages are involved. This makes computations easy and does not affect the final ratios.

14. Let $f(x) = \dfrac{x}{2x-1}$ and $g(x) = \dfrac{x}{x-1}$. Then, the domain of the function

$$h(x) = f(g(x)) + g(f(x))$$

is all real numbers except:

(A) $\frac{1}{2}, 1, \frac{3}{2}$

(B) $\frac{1}{2}, 1$

(C) $-\frac{1}{2}, \frac{1}{2}, 1$

(D) $-1, \frac{1}{2}, 1$

Correct Answer: (D) $-1, \frac{1}{2}, 1$

Solution:

Approach: A composite of fractions blows up in two ways: when the inner function is undefined, and when the inner output drives the outer denominator to zero. Hunt down both kinds of bad points for each composite, then collect everything the sum h cannot survive.

Here $f(x) = \dfrac{x}{2x-1}$ (undefined at $x = \frac{1}{2}$) and $g(x) = \dfrac{x}{x-1}$ (undefined at $x = 1$).

Step 1 — $f(g(x))$: First need g defined, so $x \neq 1$. Then the outer f blows up when its denominator $2g(x) - 1 = 0$:

$$\frac{2x}{x-1} = 1 \Rightarrow 2x = x - 1 \Rightarrow x = -1.$$

So $f(g(x))$ is undefined at $x = 1$ and $x = -1$.

Step 2 — $g(f(x))$: First need f defined, so $x \neq \frac{1}{2}$. Then the outer g blows up when $f(x) - 1 = 0$:

$$\frac{x}{2x-1} = 1 \Rightarrow x = 2x - 1 \Rightarrow x = 1.$$

So $g(f(x))$ is undefined at $x = \frac{1}{2}$ and $x = 1$.

Step 3 — combine: $h = f(g(x)) + g(f(x))$ needs both pieces alive, so exclude every bad point:

$$x = -1, \; x = \tfrac{1}{2}, \; x = 1.$$

Final answer: All reals except $-1, \dfrac{1}{2}, 1$ — option 4.

Quick Tip: When dealing with compositions of rational functions, 1.~Exclude values that make any denominator zero, and 2.~Also exclude values that make the inner function produce an invalid input for the outer function. Always combine all such restrictions at the end.

15. The number of divisors of $(2^6 \times 3^5 \times 5^3 \times 7^2)$, which are of the form $(3r+1)$, where r is a non-negative integer, is:

(A) 42

(B) 36

(C) 56

(D) 24

Correct Answer: (A) 42

Solution:

Approach: A divisor is $\equiv 1 \pmod 3$. Reduce every prime base mod 3, see which exponents matter, and just count parities.

Step 1: Write any divisor as $D = 2^a\,3^b\,5^c\,7^d$ with $0 \le a \le 6,\ 0 \le b \le 5,\ 0 \le c \le 3,\ 0 \le d \le 2$. We want $D \equiv 1 \pmod 3$.

Step 2: Reduce the bases mod 3: $2 \equiv -1,\ 3 \equiv 0,\ 5 \equiv -1,\ 7 \equiv 1$. So

$$D \equiv (-1)^a \cdot 0^b \cdot (-1)^c \cdot 1^d \pmod 3.$$

Step 3: If $b \ge 1$ the factor 3^b makes $D \equiv 0 \pmod 3$, which can never be 1. So we are forced to take $b = 0$. Then $D \equiv (-1)^{a+c} \pmod 3$, and this equals 1 exactly when $a + c$ is **even**.

Step 4: Count (a, c) with $a + c$ even (same parity). For $a \in \{0, \dots, 6\}$: even $\{0, 2, 4, 6\} = 4$, odd $\{1, 3, 5\} = 3$. For $c \in \{0, \dots, 3\}$: even $\{0, 2\} = 2$, odd $\{1, 3\} = 2$. Both even $= 4 \times 2 = 8$; both odd $= 3 \times 2 = 6$. Total $= 14$ pairs.

Step 5: Since $7 \equiv 1$, every $d \in \{0, 1, 2\}$ keeps $7^d \equiv 1$, giving 3 free choices. Multiply:

$$1\,(b) \times 14\,(a, c) \times 3\,(d) = 42.$$

Answer: 42 (option 1).

Quick Tip: When counting divisors with a condition like "of the form $3r+1$", work modulo 3:

Unknown environment 'itemize'

16. The sum of digits of the number $(625)^{65} \times (128)^{36}$ is:

Correct Answer: —

Solution:

Approach: Big powers look scary, but pairing 2s with 5s turns most of the number into trailing zeros, which add nothing to the digit sum.

Step 1: Break into prime powers: $625 = 5^4$ and $128 = 2^7$. So

$$N = (5^4)^{65} \times (2^7)^{36} = 5^{260} \times 2^{252}.$$

Step 2: Pair as many 2s and 5s as possible into 10s. There are 252 twos, so pair them with 252 fives:

$$N = 5^8 \times \left(5^{252} \times 2^{252}\right) = 5^8 \times 10^{252}.$$

Step 3: Compute the leftover 5^8: $5^4 = 625$, so $5^8 = 625^2 = 390625$. Hence $N = 390625$ followed by 252 zeros.

Step 4: Trailing zeros contribute 0 to the digit sum, so only the digits of 390625 count:

$$3 + 9 + 0 + 6 + 2 + 5 = 25.$$

Answer: 25.

Quick Tip: When multiplying large powers of 2 and 5, group them into powers of 10. This isolates a manageable non-zero block of digits followed by many zeros, simplifying digit-sum problems.

17. Ankita is twice as efficient as Bipin, while Bipin is twice as efficient as Chandan. All three of them start together on a job, and Bipin leaves the job after 20 days. If the job got completed in 60 days, the number of days needed by Chandan to complete the job alone, is:

(A) 240

(B) 260

(C) 300

(D) 340

Correct Answer: (D) 340

Solution:

Approach: Fix Chandan's one-day work as the base unit, write everyone's rate as a multiple of it, then add up the total work in the two phases (all three, then two).

Step 1: Let Chandan do 1 unit/day. Bipin is twice as efficient, so 2 units/day; Ankita is twice Bipin, so 4 units/day. Together they do $4 + 2 + 1 = 7$ units/day.

Step 2: For the first 20 days all three work:

$$20 \times 7 = 140 \text{ units.}$$

Bipin then leaves.

Step 3: The job finishes in 60 days, so Ankita and Chandan work the remaining 40 days at $4 + 1 = 5$ units/day:

$$40 \times 5 = 200 \text{ units.}$$

Step 4: Total work $= 140 + 200 = 340$ units $=$ one full job. Chandan alone does 1 unit/day, so he needs

$$\frac{340}{1} = 340 \text{ days.}$$

Answer: 340 days (option 4).

Quick Tip:

In work and time problems with different efficiencies, it is often easiest to: Assume one person's efficiency as a variable, Express others' efficiencies in terms of that variable using the given ratios, Compute total work done, then divide by an individual's efficiency for "alone" time.

18. If m and n are integers such that $(m + 2n)(2m + n) = 27$, then the maximum possible value of $2m - 3n$ is:

(A) 9

(B) 13

(C) 15

(D) 17

Correct Answer: (D) 17

Solution:

Approach: The product is 27 and m, n are integers, so $m + 2n$ and $2m + n$ must be an integer factor pair of 27 (including negatives). Test each pair, keep only those giving integer m, n, then read off $2m - 3n$.

Step 1: Let $p = m + 2n,\ q = 2m + n$ with $pq = 27$. Solving back, $m = \dfrac{2q - p}{3},\ n = \dfrac{2p - q}{3}$, so a pair is valid only when both numerators are divisible by 3. A neat shortcut:

$$2m - 3n = \frac{7q - 8p}{3}.$$

Step 2: The factor pairs (p, q) of 27 are $(1, 27), (3, 9), (9, 3), (27, 1)$ and their negatives. Check divisibility by 3 of $2q - p$:

Step 3: Only these give integers:
$(3, 9):\ m = 5,\ n = -1 \Rightarrow 2m - 3n = 10 + 3 = 13.$
$(9, 3):\ m = -1,\ n = 5 \Rightarrow 2m - 3n = -2 - 15 = -17.$
$(-3, -9):\ m = -5,\ n = 1 \Rightarrow 2m - 3n = -10 - 3 = -13.$
$(-9, -3):\ m = 1,\ n = -5 \Rightarrow 2m - 3n = 2 + 15 = 17.$
(The pairs $(1, 27), (27, 1)$ and $(-1, -27), (-27, -1)$ fail the divisibility test, so they give no integer solutions.)

Step 4: Among $13, -17, -13, 17$, the maximum is 17 (from $m = 1$, $n = -5$: check $(1-10)(2-5) = (-9)(-3) = 27$ ✓).

Answer: 17 (option 4).

Quick Tip: When given a product condition like $(m+2n)(2m+n) = 27$ with integer solutions,

Unknown environment 'itemize'

19. In a $\triangle ABC$, points D and E are on the sides BC and AC, respectively. BE and AD intersect at point T such that $AD : AT = 4 : 3$, and $BE : BT = 5 : 4$. Point F lies on AC such that DF is parallel to BE. Then, $BD : CD$ is:

(A) $15 : 4$

(B) $11 : 4$

(C) $9 : 4$

(D) $7 : 4$

Correct Answer: (B) $11 : 4$

Solution:

Approach: Two cevians AD and BE meet at T with known split ratios. This is a textbook mass-point setup: load masses so T balances on both cevians, then $BD : DC$ falls out as a mass ratio.

Step 1: Convert the given data to the splits at T. $AD : AT = 4 : 3$ means $AT : TD = 3 : 1$. $BE : BT = 5 : 4$ means $BT : TE = 4 : 1$.

Step 2: Balance on cevian AD: masses satisfy $m_A \cdot AT = m_D \cdot TD$, so $\frac{m_A}{m_D} = \frac{TD}{AT} = \frac{1}{4} \cdots$ wait, use $\frac{m_A}{m_D} = \frac{TD}{AT} = \frac{1}{3}$. Take $m_A = \alpha \Rightarrow m_D = 3\alpha$.

Step 3: Balance on cevian BE: $\frac{m_B}{m_E} = \frac{TE}{BT} = \frac{1}{4} \Rightarrow m_E = 4m_B$.

Step 4: Use that D lies on BC and E on AC:

$$m_D = m_B + m_C, \qquad m_E = m_A + m_C.$$

So $3\alpha = m_B + m_C$ and $4m_B = \alpha + m_C$. Subtracting, $4m_B - 3\alpha = \alpha - m_B \Rightarrow 5m_B = 4\alpha \Rightarrow m_B = \frac{4\alpha}{5}$, then $m_C = 3\alpha - \frac{4\alpha}{5} = \frac{11\alpha}{5}$.

Step 5: On segment BC, D divides it inversely to the masses:

$$\frac{BD}{DC} = \frac{m_C}{m_B} = \frac{11\alpha/5}{4\alpha/5} = \frac{11}{4}.$$

(The parallel line DF is the geometric reason these splits stay consistent.)

Answer: $BD : CD = 11 : 4$ **(option 2).**

Quick Tip: When internal cevians intersect (like AD and BE meeting at T) and you need a side ratio, Menelaus' Theorem on carefully chosen triangles with transversals through T can be more direct than coordinate or mass-point geometry.

20. A mixture of coffee and cocoa, 16% of which is coffee, costs Rs 240 per kg. Another mixture of coffee and cocoa, of which 36% is coffee, costs Rs 320 per kg. If a new mixture of coffee and cocoa costs Rs 376 per kg, then the quantity, in kg, of coffee in 10 kg of this new mixture is:

(A) 2.5

(B) 5

(C) 4

(D) 6

Correct Answer: (B) 5

Solution:

Approach: Each mixture's price is a weighted average of the (unknown) prices of pure coffee and pure cocoa. Two mixtures give two equations to pin those prices, then the third mixture's price tells us its coffee fraction.

Step 1: Let pure coffee cost C_f and pure cocoa cost C_c (Rs/kg).
Mixture 1 (16% coffee):

$$0.16C_f + 0.84C_c = 240. \tag{1}$$

Mixture 2 (36% coffee):

$$0.36C_f + 0.64C_c = 320. \tag{2}$$

Step 2: Subtract (1) from (2): $0.20C_f - 0.20C_c = 80 \Rightarrow C_f - C_c = 400$.

Step 3: Put $C_f = C_c + 400$ into (1): $0.16(C_c + 400) + 0.84C_c = 240$
$\Rightarrow C_c + 64 = 240 \Rightarrow C_c = 176$, hence $C_f = 576$.

Step 4: Let the new mixture be a fraction x coffee, $1 - x$ cocoa, costing 376:

$$576x + 176(1 - x) = 376 \Rightarrow 400x = 200 \Rightarrow x = \frac{1}{2}.$$

So it is 50% coffee.

Step 5: Coffee in 10 kg $= \frac{1}{2} \times 10 = 5$ kg.

Answer: 5 kg (option 2).

Quick Tip: When mixtures of the same two ingredients are given with different compositions and costs, set up linear equations using the percentage of each ingredient and solve for the individual prices. Then use those prices to find the composition of any new mixture.

21. Rita and Sneha can row a boat at 5 km/h and 6 km/h in still water, respectively. In a river flowing with a constant velocity, Sneha takes 48 minutes more to row 14 km upstream than to row the same distance downstream. If Rita starts from a certain location in the river, and returns downstream to the same location, taking a total of 100 minutes, then the total distance, in km, Rita will cover is:

Correct Answer: —

Solution:

Approach: Sneha's upstream-vs-downstream time gap is the only clue to the current speed c; find c first. Then Rita's 100-minute round trip (up and back to the same point) fixes the one-way distance, and the total distance is twice that.

Step 1 (find the current): Sneha rows at 6 km/h still water. Upstream $= 6 - c$, downstream $= 6 + c$. She takes 48 min $= 0.8$ h more upstream over 14 km:

$$\frac{14}{6-c} - \frac{14}{6+c} = 0.8.$$

Step 2: Combine: $\dfrac{14 \cdot 2c}{36 - c^2} = 0.8 \Rightarrow 28c = 0.8(36 - c^2) \Rightarrow c^2 + 35c - 36 = 0 \Rightarrow (c-1)(c+36) = 0$. Since $c > 0$, $c = 1$ km/h.

Step 3 (Rita): Rita rows at 5 km/h still water, so upstream $= 5 - 1 = 4$ km/h and downstream $= 5 + 1 = 6$ km/h. Let the one-way distance be d. Total time $= 100$ min $= \frac{5}{3}$ h:

$$\frac{d}{4} + \frac{d}{6} = \frac{5}{3}.$$

Step 4: $\dfrac{3d + 2d}{12} = \dfrac{5d}{12} = \dfrac{5}{3} \Rightarrow 5d = 20 \Rightarrow d = 4$ km one way.

Step 5: She goes up 4 km and comes back 4 km, so total distance $= 2 \times 4 = 8$ km.

Answer: 8 km.

Quick Tip: In upstream–downstream problems:

Unknown environment 'itemize'

22. If a, b, c and d are integers such that their sum is 46, then the minimum possible value of $(a-b)^2+(a-c)^2+(a-d)^2$ is:

Correct Answer: —

Solution:

Approach: Three squared gaps are smallest when a sits right in the middle of b, c, d. Since the four integers must sum to 46 and 46 is not divisible by 4, we cannot make all four equal, so we bunch them as tightly as possible around a.

Step 1: What we are minimising.
We want $(a-b)^2+(a-c)^2+(a-d)^2$ as small as possible. Each square is 0 only when that number equals a. So the dream case is $a = b = c = d$.

Step 2: Test the dream case.
$a=b=c=d$ forces $4a=46$, i.e. $a=11.5$, which is not an integer. So at least one of b, c, d must differ from a.

Step 3: Spread the leftover by exactly 1.
Keep the integers as close to 11.5 as possible: take two of them $=11$ and two $=12$. For example $a=12,\ b=12,\ c=11,\ d=11$.
Check sum: $12+12+11+11=46$. Good.

Step 4: Evaluate.

$$(12-12)^2+(12-11)^2+(12-11)^2=0+1+1=2.$$

We cannot do better: only one of b, c, d can equal a, so at least two squares are ≥ 1, forcing the sum ≥ 2.

Minimum value = 2.

Quick Tip: For minimizing sums of squares with a fixed sum constraint, keep the numbers as close together as possible. Here, that meant taking a, b, c, d near the average $\frac{46}{4}=11.5$, and distributing small integer deviations evenly.