CAT 2025 QA Slot 3 Question Paper with Solutions

General Instructions

(i) This booklet contains 22 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 14 single-correct multiple-choice questions and 8 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For numerical questions, report the answer rounded exactly as asked.

1. For a 4-digit number (greater than 1000), sum of the digits in the thousands, hundreds, and tens places is 15. Sum of the digits in the hundreds, tens, and units places is 16. Also, the digit in the tens place is 6 more than the digit in the units place. The difference between the largest and smallest possible value of the number is

(A) 40

(B) 78

(C) 811

(D) 735

Correct Answer: (C) 811

Solution:

Approach: Three conditions tie four digits together — express every digit in terms of one variable (the tens digit), then let the digit bounds 0–9 squeeze out the only possibilities.

Step 1: Name the digits. Number $= \overline{abcd}$: a thousands, b hundreds, c tens, d units, with $a \geq 1$. Given:

Multiple \tag

Step 2: Reduce to one variable c. From (3), $d = c - 6$. Put in (2): $b + c + (c - 6) = 16 \Rightarrow b = 22 - 2c$. Put b in (1): $a + (22 - 2c) + c = 15 \Rightarrow a = c - 7$.

Step 3: Apply digit limits. Now everything depends on c:
$a = c - 7 \geq 1 \Rightarrow c \geq 8$; also $d = c - 6 \leq 9$ is automatic. With $c \leq 9$, only $c = 8$ or $c = 9$ survive.

Intuition: Subtracting (1) from (2) instantly gives $d - a = 1$, a neat cross-check on the two cases below.

Step 4: Build both numbers.
$c = 8: \ a = 1, \ b = 6, \ d = 2 \Rightarrow 1682.$
$c = 9: \ a = 2, \ b = 4, \ d = 3 \Rightarrow 2493.$
Both satisfy all three conditions (e.g. $2493: \ 2 + 4 + 9 = 15, \ 4 + 9 + 3 = 16, \ 9 = 3 + 6$). ✓

Step 5: Required difference.

$$2493 - 1682 = \boxed{811},$$

matching option (c).

Quick Tip: When dealing with digit problems, always convert the verbal conditions into equations using place-value notation, and apply the digit constraints $0 \leq \text{digit} \leq 9$ to narrow down possible values quickly.

2. ABCD is a trapezium in which AB is parallel to DC, AD is perpendicular to AB, and $AB = 3DC$. If a circle inscribed in the trapezium touching all the sides has a radius of 3 cm, then the area, in sq. cm, of the trapezium is

(A) 54

(B) $30\sqrt{3}$

(C) 48

(D) $36\sqrt{2}$

Correct Answer: (C) 48

Solution:

Approach: A circle inscribed in a right trapezium pins down the height immediately (height = diameter), and the tangential-polygon rule (sum of opposite sides equal) gives the slant side without coordinates.

Step 1: Get the height. The incircle touches both parallels AB and DC, so the gap between them is the diameter: $AD = 2r = 6$ cm. (Here $AD \perp AB$, so AD IS the height.) Let $DC = c$, $AB = 3c$.

Step 2: Use the tangential property. For any quadrilateral with an inscribed circle, the two pairs of opposite sides have equal sums:

$$AB + DC = AD + BC.$$

So $3c + c = 6 + BC \Rightarrow BC = 4c - 6$.

Step 3: Pin BC with the right angle at A and D. Drop the horizontal/vertical: the slant side BC spans a vertical drop of 6 (the height) and a horizontal run of $AB - DC = 3c - c = 2c$. By Pythagoras:

$$BC^2 = 6^2 + (2c)^2 = 36 + 4c^2.$$

Step 4: Solve for c. Equate the two expressions for BC:

$$(4c-6)^2 = 36 + 4c^2 \Rightarrow 16c^2 - 48c + 36 = 36 + 4c^2 \Rightarrow 12c^2 = 48c \Rightarrow c = 4.$$

So $DC = 4$, $AB = 12$, $BC = 4(4) - 6 = 10$.

Intuition: The "equal sums of opposite sides" rule is the fastest handle on any inscribed-circle quadrilateral — reach for it before coordinates.

Step 5: Area.

$$\text{Area} = \tfrac{1}{2}(AB + DC) \times AD = \tfrac{1}{2}(12+4) \times 6 = \boxed{48}\ \text{cm}^2,$$

matching option (c).

Quick Tip: For figures with an incircle:
The distance between two parallel tangents equals twice the radius.
In tangential quadrilaterals (those with an incircle), using coordinates with the incenter at convenient positions (like (r, r)) can simplify distance calculations.

3. Vessels A and B contain 60 litres of alcohol and 60 litres of water, respectively. A certain volume is taken out from A and poured into B. After stirring, the same volume is taken out from B and poured into A. If the resultant ratio of alcohol and water in A is 15 : 4, then the volume, in litres, initially taken out from A is

Correct Answer: —

Solution:

Approach: Classic double-transfer. Track only the alcohol; the key shortcut is that after the round trip vessel A is back to 60 L total, so its alcohol fraction directly gives the $15:4$ split.

Step 1: First pour, A → B. Move x L (pure alcohol) from A to B. Now A has $60 - x$ L alcohol; B has x L alcohol $+ 60$ L water $= 60 + x$ L total.

Step 2: Fraction of alcohol in B.

$$\text{alcohol fraction in B} = \frac{x}{60+x}.$$

Step 3: Second pour, B → A. Return x L of B's mixture. Alcohol returned $= x \cdot \frac{x}{60+x} = \frac{x^2}{60+x}$. Vessel A is now back to 60 L total, of which alcohol is

$$(60-x) + \frac{x^2}{60+x} = \frac{(60-x)(60+x)+x^2}{60+x} = \frac{3600}{60+x}.$$

Intuition: Since A returns to exactly 60 L, alcohol : water $= 15:4$

means alcohol $= \frac{15}{19} \times 60$. Setting $\frac{3600}{60+x}$ equal to that gives x in one line.

Step 4: Apply the ratio. Water in A $= 60 - \frac{3600}{60+x} = \frac{60x}{60+x}$. So

$$\frac{\text{alcohol}}{\text{water}} = \frac{3600/(60+x)}{60x/(60+x)} = \frac{3600}{60x} = \frac{60}{x} = \frac{15}{4}.$$

Step 5: Solve.

$$\frac{60}{x} = \frac{15}{4} \Rightarrow x = \frac{240}{15} = \boxed{16} \text{ litres.}$$

Quick Tip: In transfer-and-mix problems, track the \emph{fraction} of each component after mixing, then multiply by the transferred volume. Ratios often simplify nicely when common denominators cancel.

4. In a class of 150 students, 75 students chose physics, 111 students chose mathematics and 40 students chose chemistry. All students chose at least one of the three subjects and at least one student chose all three subjects. The number of students who chose both physics and chemistry is equal to the number of students who chose both chemistry and mathematics, and this is half the number of students who chose both physics and mathematics. The maximum possible number of students who chose physics but not mathematics, is

(A) 30

(B) 55

(C) 35

(D) 40

Correct Answer: (C) 35

Solution:

Approach: Set the pairwise overlaps in terms of one unknown, use inclusion-exclusion to link it to the triple-overlap, then minimise the relevant overlap — but only over values that keep every Venn region non-negative.

Step 1: Name the overlaps. Let $n(P \cap C) = n(C \cap M) = x$ and $n(P \cap M) = 2x$; let the triple be $z = n(P \cap M \cap C) \geq 1$. (These are full pairwise intersections, triple included.)

Step 2: Inclusion-exclusion. With union $= 150$ (everyone picks at least one):

$$150 = 75 + 111 + 40 - (2x + x + x) + z = 226 - 4x + z.$$

So $4x = 76 + z$.

Step 3: What to maximise. "Physics but not Mathematics" $= n(P) - n(P \cap M) = 75 - 2x$. To maximise this, make x as small as possible.

Step 4: Find the smallest valid x. From $4x = 76 + z$, we need $76 + z$ divisible by 4, so z is a multiple of 4; with $z \geq 1$, the smallest is $z = 4$, giving $x = 20$. ($z = 1, 2, 3$ give non-integer x, so they are impossible.)

Intuition: Always confirm the candidate produces a legal Venn diagram — here $x = 20$, $z = 4$ yields only-P = 19, only-M = 55, only-C = 4, all non-negative. ✓

Step 5: Answer.

$$75 - 2x = 75 - 2(20) = \boxed{35},$$

matching option (c).

Quick Tip: For three-set Venn diagram problems:
Convert verbal relations about overlaps into equations using $|A \cap B|$, $|A \cap B \cap C|$, etc.
Use inclusion--exclusion to connect total strength with individual set sizes and intersections.
If asked for a maximum or minimum, express the required number in terms of a single variable and then optimize it under the given constraints.

5. In ΔABC, $AB = AC = 12$ cm and D is a point on side BC such that $AD = 8$ cm. If AD is extended to a point E such that $\angle ACB = \angle AEB$, then the length, in cm, of AE is

(A) 18

(B) 16

(C) 20

(D) 14

Correct Answer: (A) 18

Solution:

Approach: The equal-angle clue secretes a similar-triangle pair. Spot that $\angle AEB = \angle ACB = \angle ABC$ (isosceles), which makes $\triangle ABD$ and $\triangle AEB$ similar and turns AB into a geometric mean.

Step 1: Use the isosceles base angles. $AB = AC = 12$ gives $\angle ABC = \angle ACB$. We are told $\angle AEB = \angle ACB$, so $\angle AEB = \angle ABC = \angle ABD$ (D lies on BC, so $\angle ABD = \angle ABC$).

Step 2: Find the similar triangles. Compare $\triangle ABD$ and $\triangle AEB$: they share $\angle A$ (the line A–D–E is one ray, so $\angle BAD = \angle BAE$), and $\angle ABD = \angle AEB$. Two equal angles $\Rightarrow \triangle ABD \sim \triangle AEB$.

Step 3: Write the proportion. Matching corresponding sides (A↔A, B↔E, D↔B):

$$\frac{AB}{AE} = \frac{AD}{AB} \Rightarrow AB^2 = AD \cdot AE.$$

Intuition: Whenever a side equals the geometric mean of two collinear segments, suspect a similar-triangle (or power-of-a-point) setup — that is exactly what the equal angles created here.

Step 4: Substitute.

$$12^2 = 8 \cdot AE \Rightarrow 144 = 8\,AE.$$

Step 5: Solve.

$$AE = \frac{144}{8} = \boxed{18}\text{ cm},$$

matching option (a).

Quick Tip: When you see a condition like $\angle ACB = \angle AEB$ involving two angles subtending the same chord, think \textbf{cyclic quadrilateral}. Once you know points are concyclic, the \textbf{intersecting chords theorem} and \textbf{Stewart's theorem} can quickly give products like $BD \cdot DC$ and lead to elegant solutions.

6. If $\left(x^2 + \frac{1}{x^2}\right) = 25$ and $x > 0$, then the value of $\left(x^7 + \frac{1}{x^7}\right)$ is

(A) $44859\sqrt{3}$

(B) $44853\sqrt{3}$

(C) $44850\sqrt{3}$

(D) $44856\sqrt{3}$

Correct Answer: (B) $44853\sqrt{3}$

Solution:

Approach: Never jump straight to the 7th power. Climb a ladder $x + \frac{1}{x} \to x^2 + \frac{1}{x}^2 \to x^3 + \frac{1}{x}^3 \to \ldots$ and combine two lower powers to reach the one you want. The key recurrence is $\left(x^a + \frac{1}{x^a}\right)\left(x^b + \frac{1}{x^b}\right) = \left(x^{a+b} + \frac{1}{x^{a+b}}\right) + \left(x^{a-b} + \frac{1}{x^{a-b}}\right)$.

Step 1: Get $x + \frac{1}{x}$. Since $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = 25 + 2 = 27$, and $x > 0$ forces a positive value, $x + \frac{1}{x} = \sqrt{27} = 3\sqrt{3}$.

Step 2: $x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right) = 3\sqrt{3}(25) - 3\sqrt{3} = 72\sqrt{3}$.

Step 3: $x^6 + \frac{1}{x^6} = \left(x^3 + \frac{1}{x^3}\right)^2 - 2 = (72\sqrt{3})^2 - 2 = 72^2 \cdot 3 - 2 = 15552$

$-2 = 15550.$

Step 4: $x^5 + \frac{1}{x^5} = \left(x^3 + \frac{1}{x^3}\right)\left(x^2 + \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right) = 72\sqrt{3}(25) - 3\sqrt{3}$
$= 1800\sqrt{3} - 3\sqrt{3} = 1797\sqrt{3}.$

Step 5: Finally use $x^7 + \frac{1}{x^7} = \left(x^6 + \frac{1}{x^6}\right)\left(x + \frac{1}{x}\right) - \left(x^5 + \frac{1}{x^5}\right).$

$$x^7 + \frac{1}{x^7} = 15550 \cdot 3\sqrt{3} - 1797\sqrt{3} = 46650\sqrt{3} - 1797\sqrt{3} = 44853\sqrt{3}.$$

Final answer: $x^7 + \dfrac{1}{x^7} = 44853\sqrt{3}.$

Quick Tip: For expressions like $x^n + \dfrac{1}{x^n}$, first find $x + \dfrac{1}{x}$ using the given data, then use the recurrence

$$S_n = S_1 S_{n-1} - S_{n-2},$$

instead of expanding powers directly. It saves a lot of time and reduces mistakes in exams.

7. Rahul starts on his journey at 5 pm at a constant speed so that he reaches his destination at 11 pm the same day. However, on his way, he stops for 20 minutes, and after that, increases his speed by 3 km per hour to reach on time. If he had stopped for 10 minutes more, he would have had to increase his speed by 5 km per hour to reach on time. His initial speed, in km per hour, was

(A) 20

(B) 15

(C) 12

(D) 18

Correct Answer: (B) 15

Solution:

Approach: The trap is to make him speed up over the whole journey — that gives contradictory equations. Reality: he drives normally, stops at one fixed point, then speeds up over the *remaining* distance to land on time. Keep two unknowns: speed x and the driving time before the stop.

Step 1: Scheduled time 5 pm to 11 pm = 6 hours, so total distance $D = 6x$. Let him drive T hours at speed x before stopping; remaining distance $= 6x - xT = x(6 - T)$.

Step 2 (20-minute stop): Wall-clock left to cover the remaining distance $= 6 - T - \frac{1}{3}$ hours at speed $x + 3$:

$$\frac{x(6-T)}{x+3} = 6 - T - \frac{1}{3}.$$

Step 3 (30-minute stop): Same remaining distance, but now speed $x + 5$ and 10 more minutes lost:

$$\frac{x(6-T)}{x+5} = 6 - T - \frac{1}{2}.$$

Step 4: Solving the two equations together gives $T = 4$ and $x = 15$.

Step 5 (sanity check): $D = 90$ km. In 4 h he covers 60 km; 30 km remain. With a 20-min stop, time left $= 6 - 4 - \frac{1}{3} = \frac{5}{3}$ h, needed speed $= 30 \div \frac{5}{3} = 18 = 15 + 3$. With a 30-min stop, time left $= \frac{3}{2}$ h, needed speed $= 30 \div \frac{3}{2} = 20 = 15 + 5$. Both fit.

Final answer: Initial speed $= 15$ km/h.

Quick Tip: In time–speed–distance problems with multiple scenarios: Express the total distance using the original (planned) speed and time. Write separate time equations for each scenario, keeping the same distance. Look for a common expression (like the remaining distance) to reduce the number of variables and solve systematically.

8. The sum of all possible real values of x for which

$$\log_{x-3}(x^2 - 9) = \log_{x-3}(x + 1) + 2,$$

is

(A) -3

(B) $\sqrt{33}$

(C) $\dfrac{3 + \sqrt{33}}{2}$

(D) 3

Correct Answer: (C) $\dfrac{3 + \sqrt{33}}{2}$

Solution:

Approach: First nail the domain (logs are picky about base and arguments), then turn the "+2" into $\log_{x-3}(x-3)^2$ so both sides become a single log and the bases cancel.

Step 1 (domain): Base $x-3>0$ and $x-3\neq 1$ give $x>3,\ x\neq 4$. Arguments: $x^2-9>0$ and $x+1>0$ are both satisfied once $x>3$. So the legal zone is $x>3$.

Step 2: Write $2=\log_{x-3}(x-3)^2$. The equation becomes

$$\log_{x-3}(x^2-9)=\log_{x-3}\left((x+1)(x-3)^2\right).$$

Step 3: Equal logs (same base) means equal arguments:

$$x^2-9=(x+1)(x-3)^2.$$

Since $x^2-9=(x-3)(x+3)$ and $x>3$ lets us cancel one $(x-3)$:

$$x+3=(x+1)(x-3).$$

Step 4: Expand: $x+3=x^2-2x-3\Rightarrow x^2-3x-6=0$. By the quadratic formula $x=\dfrac{3\pm\sqrt{9+24}}{2}=\dfrac{3\pm\sqrt{33}}{2}$.

Step 5 (domain filter): $\dfrac{3-\sqrt{33}}{2}\approx -1.37$ fails $x>3$; only $\dfrac{3+\sqrt{33}}{2}$ ≈ 4.37 survives. (The cancelled $x=3$ is also rejected – it makes the base 0.) With a single valid root, the "sum of all possible real values" is that root itself.

Final answer: $\dfrac{3+\sqrt{33}}{2}$.

Quick Tip: In logarithmic equations:
Always check the domain first: base > 0, base $\neq 1$, and argument > 0.
When the bases are the same, combine logs using properties like $\log_b A - \log_b B = \log_b\left(\frac{A}{B}\right)$, then convert to exponential form.
Don't forget to discard any solutions that fall outside the domain constraints.

9. The ratio of the number of coins in boxes A and B was 17:7. After 108 coins were shifted from box A to box B, this ratio became 37:20. The number of coins that needs to be shifted further from A to B, to make this ratio 1:1, is

Correct Answer: —

Solution:

Approach: Two ratio snapshots fix the actual counts. Use the first ratio to set variables, the second (after a known transfer) to solve for the unit, then count what's needed to level the boxes.

Step 1: Let A start with $17k$ coins and B with $7k$. After moving 108 from A to B:

$$\frac{17k - 108}{7k + 108} = \frac{37}{20}.$$

Step 2: Cross-multiply: $20(17k - 108) = 37(7k + 108) \Rightarrow 340k - 2160$

$= 259k + 3996$. So $81k = 6156 \Rightarrow k = 76$.

Step 3: Initial counts: $A = 17(76) = 1292,\ B = 7(76) = 532$. After the 108-coin shift: $A = 1184,\ B = 640$.

Step 4: The total $1184 + 640 = 1824$ is fixed. For a $1:1$ split each box must hold 912. A currently has 1184, so it must give away $1184 - 912 = 272$ more coins.

Final answer: 272 coins.

Quick Tip: For ratio problems with transfers between two containers, first express initial quantities with a variable using the given ratio, then use the new ratio after transfer to form an equation. Finally, use total quantity (which stays constant) to handle any further equalisation like making the ratio 1:1.

10. The rate of water flow through three pipes A, B and C are in the ratio 4 : 9 : 36. An empty tank can be filled up completely by pipe A in 15 hours. If all the three pipes are used simultaneously to fill up this empty tank, the time, in minutes, required to fill up the entire tank completely is nearest to

(A) 76

(B) 78

(C) 73

(D) 71

Correct Answer: (C) 73

Solution:

Approach: Convert one known fill-time into a rate, scale up to all three pipes using the ratio, then invert to get the combined time.

Step 1: Rates are in ratio $4 : 9 : 36$, so write them as $4u, 9u, 36u$ tanks/hour.

Step 2: Pipe A alone fills in 15 hours, so A's rate $= \frac{1}{15}$. Hence $4u = \frac{1}{15} \Rightarrow u = \frac{1}{60}$.

Step 3: Combined rate $= (4 + 9 + 36)u = 49u = \frac{49}{60}$ tank/hour.

Step 4: Time to fill one tank $= \frac{1}{49/60} = \frac{60}{49}$ hours $= \frac{60}{49} \times 60 = \frac{3600}{49}$ ≈ 73.47 minutes.

Final answer: Nearest to 73 minutes.

Quick Tip: In pipes and cisterns problems:
Convert ratios into actual rates using any one known filling time.
Add rates (not times) when pipes work together.
Do all calculations in hours first, and convert to minutes only at the end to avoid mistakes.

11. Teams A, B, and C consist of five, eight, and ten members, respectively, such that every member within a team is equally productive. Working separately, teams A, B, and C can complete a certain job in 40 hours, 50 hours, and 4 hours, respectively. Two members from team A, three members from team B, and one member from team C together start the job, and the member from team C leaves after 23 hours. The number of additional member(s) from team B, that would be required to replace the member from team C, to finish the job in the next one hour, is

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Approach: Reduce everything to a *per-member* rate (since members within a team are identical), add up the working members, track work done in 23 hours, then see how many B-members fill the last gap in one hour.

Step 1 (per-member rates): Team A (5 members) finishes in 40 h, so one A-member $= \frac{1}{40 \times 5} = \frac{1}{200}$ job/h. Team B (8 members, 50 h): $\frac{1}{50 \times 8} = \frac{1}{400}$. Team C (10 members, 4 h): $\frac{1}{4 \times 10} = \frac{1}{40}$.

Step 2 (starting crew): 2 A + 3 B + 1 C give rate $\frac{2}{200} + \frac{3}{400} + \frac{1}{40}$ $= \frac{4}{400} + \frac{3}{400} + \frac{10}{400} = \frac{17}{400}$ job/h.

Step 3 (after 23 hours): Work done $= 23 \times \frac{17}{400} = \frac{391}{400}$. Remaining $= 1 - \frac{391}{400} = \frac{9}{400}$.

Step 4 (C leaves): The C-member exits. Remaining crew 2 A + 3 B has rate $\frac{4}{400} + \frac{3}{400} = \frac{7}{400}$ job/h. Add n more B-members at $\frac{1}{400}$ each to finish $\frac{9}{400}$ in one hour:

$$\frac{7}{400} + \frac{n}{400} = \frac{9}{400} \Rightarrow 7 + n = 9 \Rightarrow n = 2.$$

Final answer: 2 extra members from team B.

Quick Tip: In man-hour problems, first convert team rates into \emph{individual} rates. Then:
Compute total work done in each phase.
Subtract from 1 (or total work) to find the remaining part.
Use the required time and remaining work to find how many extra workers are needed.

12. Ankita walks from A to C through B, and runs back through the same route at a speed that is 40% more than her walking speed. She takes exactly 3 hours 30 minutes to walk from B to C as well as to run from B to A. The total time, in minutes, she would take to walk from A to B and run from B to C, is

Correct Answer: —

Solution:

Approach: Two given times each isolate one leg, so use them to express both leg-distances in terms of the walking speed w. The actual value of w cancels later, so don't worry about it.

Step 1: Let walking speed $= w$, running speed $= 1.4w$ (40% faster).

Step 2: "Walk B to C in 3 h 30 min = 3.5 h": $\frac{d_{BC}}{w} = 3.5 \Rightarrow d_{BC} = 3.5w$.

Step 3: "Run B to A in 3.5 h": $\frac{d_{AB}}{1.4w} = 3.5 \Rightarrow d_{AB} = 3.5 \times 1.4w = 4.9w$.

Step 4: Required trip is walk A to B, then run B to C:

$$t = \frac{d_{AB}}{w} + \frac{d_{BC}}{1.4w} = \frac{4.9w}{w} + \frac{3.5w}{1.4w} = 4.9 + 2.5 = 7.4 \text{ h}.$$

Step 5: $7.4 \times 60 = 444$ minutes. (Notice w vanished entirely.)

Final answer: 444 minutes.

Quick Tip: When speed changes by a fixed percentage, keep everything in terms of one speed (like w), express all distances using that speed and given times, then recompute the required times with the new speed.

13. If $12^{12x} \times 4^{24x+12} \times 5^{2y} = 8^{4z} \times 20^{12x} \times 243^{3x-6}$, where x, y and z are natural numbers, then $x + y + z$ equals

Correct Answer: —

Solution:

Approach: Both sides are products of powers, so break every number into its prime factors $2, 3, 5$ and compare the exponent of each prime separately. Since x, y, z are natural numbers, equality forces each prime's exponent on the left to equal that on the right.

Step 1: Prime-factorise the left side.

$12 = 2^2 \cdot 3 \Rightarrow 12^{12x} = 2^{24x}\, 3^{12x}$

$4 = 2^2 \Rightarrow 4^{24x+12} = 2^{48x+24}$

5^{2y} stays as is.

Left $= 2^{24x+48x+24}\, 3^{12x}\, 5^{2y} = 2^{72x+24}\, 3^{12x}\, 5^{2y}$.

Step 2: Prime-factorise the right side.

$8 = 2^3 \Rightarrow 8^{4z} = 2^{12z}$

$20 = 2^2 \cdot 5 \Rightarrow 20^{12x} = 2^{24x}\, 5^{12x}$

$243 = 3^5 \Rightarrow 243^{3x-6} = 3^{15x-30}$

Right $= 2^{12z+24x}\, 3^{15x-30}\, 5^{12x}$.

Step 3: Match exponents prime by prime.

Base 3: $12x = 15x - 30 \Rightarrow 3x = 30 \Rightarrow x = 10$.

Base 5: $2y = 12x = 120 \Rightarrow y = 60$.

Base 2: $72x + 24 = 12z + 24x \Rightarrow 720 + 24 = 12z + 240 \Rightarrow 12z = 504 \Rightarrow z = 42$.

Step 4: Add up.

$x + y + z = 10 + 60 + 42 = 112$.

$$\boxed{x + y + z = 112}$$

Quick Tip: Whenever you see an equation involving different bases with exponents, convert all bases to prime factors. Then compare the exponents of each prime on both sides — it turns the problem into a simple system of linear equations.

14. The sum of all the digits of the number $(10^{50} + 10^{25} - 123)$, is

(A) 21

(B) 221

(C) 324

(D) 255

Correct Answer: (B) 221

Solution:

Approach: Don't fear the giant number. Write $10^{50} + 10^{25}$ as a string of zeros with two 1s, then subtract 123 and watch the borrowing turn a run of zeros into 9s. Only the last few digits change.

Step 1: Picture $10^{50} + 10^{25}$.

10^{50} is 1 followed by 50 zeros. Adding 10^{25} puts a 1 in the 26th position from the right:

$1\underbrace{00\ldots0}_{24}1\underbrace{00\ldots0}_{25}$ (a 51-digit number).

Step 2: Subtract 123.

The number ends in $\ldots1\underbrace{0\ldots0}_{25}$. Subtracting 123 borrows through the block of 25 trailing zeros: the right-most 1 becomes 0, the 25 zeros become 9s, and then 123 is removed from the last three of them.

The last three digits: $000 - 123$ after borrowing gives 877; the remaining 22 of the freed zeros stay as 9s.

Result:

$$1\underbrace{0\ldots0}_{24}\,0\,\underbrace{9\ldots9}_{22}\,877.$$

Step 3: Add the digits.

One 1, twenty-two 9s, then $8 + 7 + 7$:

$1 + 22 \times 9 + (8 + 7 + 7) = 1 + 198 + 22 = 221.$

$$\boxed{\text{Digit sum} = 221}$$

The correct option is 221.

Quick Tip: When dealing with expressions like $10^n + 10^m - k$, avoid direct expansion. Instead:

Treat powers of 10 as place-value blocks (leading 1 followed by zeros).

Rewrite as $10^n + (10^m - k)$ and analyze the smaller block $10^m - k$.

Use patterns like $10^n - 1 = \underbrace{99\ldots9}_{n \text{ times}}$ and adjust for the subtraction. This makes digit-sum problems much faster and cleaner.

15. If $f(x) = (x^2 + 3x)(x^2 + 3x + 2)$, then the sum of all real roots of the equation $\sqrt{f(x) + 1} = 9701$, is

(A) 6

(B) -3

(C) -6

(D) 3

Correct Answer: (B) -3

Solution:

Approach: The four-factor look is a disguise. Substitute $y = x^2 + 3x$; then $f(x) + 1$ collapses into a perfect square, killing the ugly 9701^2 into a clean linear equation in y.

Step 1: Substitute.

Let $y = x^2 + 3x$. Then $f(x) = y(y + 2) = y^2 + 2y$, so
$f(x) + 1 = y^2 + 2y + 1 = (y + 1)^2$.

Step 2: Use the equation.

$\sqrt{f(x) + 1} = 9701 \Rightarrow \sqrt{(y + 1)^2} = 9701 \Rightarrow |y + 1| = 9701$.
So $y + 1 = 9701$ or $y + 1 = -9701$, i.e. $y = 9700$ or $y = -9702$.

Step 3: Back-substitute and keep only real cases.

Case $y = 9700$: $x^2 + 3x = 9700 \Rightarrow x^2 + 3x - 9700 = 0$. Discriminant $= 9 + 4(9700) = 38809 = 197^2 > 0$, so two real roots.
Case $y = -9702$: $x^2 + 3x = -9702 \Rightarrow x^2 + 3x + 9702 = 0$. Discriminant $= 9 - 4(9702) < 0$, no real roots.

Step 4: Sum the real roots.

Only $x^2 + 3x - 9700 = 0$ gives real roots. Sum $= -\dfrac{b}{a} = -\dfrac{3}{1} = -3$.

$$\boxed{\text{Sum of all real roots} = -3}$$

The correct option is -3.

Quick Tip: When you see a product like $(x^2+3x)(x^2+3x+2)$, try a substitution such as $t = x^2+3x$. Often, the expression simplifies to a perfect square (like $(t+1)^2$ here), which makes equations with square roots much easier to solve.

16. For real values of x, the range of the function $f(x) = \dfrac{2x-3}{2x^2+4x-6}$ is

(A) $(-\infty, \frac{1}{8}] \cup [1, \infty)$

(B) $(-\infty, \frac{1}{8}] \cup [\frac{1}{2}, \infty)$

(C) $(-\infty, \frac{1}{4}] \cup [\frac{1}{2}, \infty)$

(D)

$(-\infty, \frac{1}{4}] \cup [1, \infty)$

Correct Answer: (A) $(-\infty, \frac{1}{8}] \cup [1, \infty)$

Solution:

Approach: To find the range of a rational function, set $y = f(x)$, cross-multiply to get a quadratic in x, and demand its discriminant be ≥ 0 — those are exactly the y-values the function can actually hit.

Step 1: Set $y = f(x)$ and clear the denominator.

$y = \dfrac{2x-3}{2x^2+4x-6} \Rightarrow y(2x^2+4x-6) = 2x-3.$
Rearrange into a quadratic in x:

$$2y\,x^2 + (4y-2)x + (3-6y) = 0.$$

Step 2: Demand real x (discriminant ≥ 0).

For $y \neq 0$ this is a genuine quadratic; real x needs $\Delta \geq 0$:

$\Delta = (4y-2)^2 - 4(2y)(3-6y).$

$(4y-2)^2 = 16y^2 - 16y + 4; \quad 4(2y)(3-6y) = 24y - 48y^2.$

$\Delta = 16y^2 - 16y + 4 - (24y - 48y^2) = 64y^2 - 40y + 4.$

Step 3: Solve $\Delta \geq 0$.

$64y^2 - 40y + 4 = 4(16y^2 - 10y + 1) = 4(2y-1)(8y-1).$

Roots: $y = \frac{1}{2}$ and $y = \frac{1}{8}$. The parabola opens upward, so $\Delta \geq 0$ outside the roots:

$$y \leq \tfrac{1}{8} \quad \text{or} \quad y \geq \tfrac{1}{2}.$$

Step 4: Check the boundary case $y = 0$.

At $y = 0$ the equation is linear: $-2x + 3 = 0 \Rightarrow x = \frac{3}{2}$ (real). Since $0 \leq \frac{1}{8}$, it already lies in the first branch — consistent.

Conclusion: The range is

$$\boxed{\left(-\infty, \tfrac{1}{8}\right] \cup \left[\tfrac{1}{2}, \infty\right)}$$

which is the option $\left(-\infty, \frac{1}{8}\right] \cup \left[\frac{1}{2}, \infty\right)$.

Quick Tip: To find the range of a rational function:
Set $y = f(x)$ and rearrange into a quadratic in x.
Apply the discriminant condition $\Delta \geq 0$.
Solve the resulting inequality in y.
Check that boundary values are actually achieved by plugging back. This method works reliably for all rational functions with x appearing up to degree 2 in the denominator.

17. The monthly sales of a product from January to April were 120, 135, 150 and 165 units, respectively. The cost price of the product was Rs. 240 per unit, and a fixed marked price was used for the product in all the four months. Discounts of 20%, 10% and 5% were given on the marked price per unit in January, February and March, respectively, while no discounts were given in April. If the total profit from January to April was Rs. 138825, then the marked price per unit, in rupees, was

(A) 525

(B) 510

(C) 520

(D) 515

Correct Answer: (A) 525

Solution:

Approach: Keep the marked price M as one unknown. Each month's selling price is a known multiple of M, so total revenue is (a constant) $\times M$. Profit = revenue − cost gives a single linear equation in M.

Step 1: Effective price multiplier each month.

Jan: 20% off $\Rightarrow 0.80M$, sold 120 units.

Feb: 10% off $\Rightarrow 0.90M$, sold 135.

Mar: 5% off $\Rightarrow 0.95M$, sold 150.

Apr: no discount $\Rightarrow M$, sold 165.

Step 2: Total revenue as a multiple of M.

$R = M\big(120(0.80) + 135(0.90) + 150(0.95) + 165(1)\big)$

$= M(96 + 121.5 + 142.5 + 165) = 525\,M.$

Step 3: Total cost.

Total units $= 120 + 135 + 150 + 165 = 570$. Cost $= 570 \times 240$

$= 1{,}36{,}800.$

Step 4: Use the profit.

Profit $= R -$ Cost:

$1{,}38{,}825 = 525M - 1{,}36{,}800.$

$525M = 2{,}75{,}625 \Rightarrow M = \dfrac{2{,}75{,}625}{525} = 525.$

$$\boxed{\text{Marked price} = \text{Rs. } 525}$$

The correct option is 525.

Quick Tip: In profit and loss problems over multiple periods:

Keep the marked price as a single variable M.

Express each month's profit as (SP $-$ CP) $\times$ quantity.

Add all monthly profits and equate to the given total profit. The coefficients usually simplify nicely.

18. A triangle ABC is formed with $AB = AC = 50$ cm and $BC = 80$ cm. Then, the sum of the lengths, in cm, of all three altitudes of the triangle ABC is

Correct Answer: —

Solution:

Approach: Every altitude equals $\frac{2 \times \text{Area}}{\text{its base}}$. So find the area once, then divide by each side. The 50-50-80 triangle splits into two clean 30-40-50 right triangles, so the area comes out instantly.

Step 1: Find the area.

Drop the altitude from A onto BC. It bisects BC, giving two right triangles with base 40 and hypotenuse 50. Height $= \sqrt{50^2 - 40^2}$ $= \sqrt{2500 - 1600} = \sqrt{900} = 30$.

Area $= \frac{1}{2} \times 80 \times 30 = 1200\ \text{cm}^2$.

Step 2: Altitude to BC (base $= 80$).

$h_a = \frac{2 \times 1200}{80} = 30$ cm. (Same 30 we just found — good check.)

Step 3: Altitudes to AC and AB (each base $= 50$).

$h_b = \frac{2 \times 1200}{50} = 48$ cm, and by symmetry $h_c = 48$ cm.

Step 4: Add them.

$h_a + h_b + h_c = 30 + 48 + 48 = 126$ cm.

$$\boxed{\text{Sum of altitudes} = 126\ \text{cm}}$$

Quick Tip: In an isosceles triangle, the altitude to the base not only gives the height but also splits the base into two equal parts. Once you know the area from one base–height pair, you can easily find the other altitudes using the same area with different bases.

19. Let p, q and r be three natural numbers such that their sum is 900, and r is a perfect square whose value lies between 150 and 500. If p is not less than $0.3q$ and not more than $0.7q$, then the sum of the maximum and minimum possible values of p is

Correct Answer: —

Solution:

Approach: Fix r (a perfect square in range); then $p + q = 900 - r$ is fixed, and the constraint $0.3q \leq p \leq 0.7q$ bounds p. To make p as large as possible you want a small r; to make p as small as possible you want a large r. So check the smallest and largest valid squares.

Step 1: List the perfect squares with $150 < r < 500$.
$13^2 = 169,\ 14^2 = 196,\ \ldots,\ 22^2 = 484$. Don't miss the endpoints 169 and 484.

Step 2: Turn the bound into a range for p.
With $S = p + q = 900 - r$, we have $q = S - p$. The condition $0.3q \leq p \leq 0.7q$ becomes:

Lower: $p \geq 0.3(S-p) \Rightarrow 1.3p \geq 0.3S \Rightarrow p \geq \frac{0.3}{1.3}S = \frac{3}{13}S.$

Upper: $p \leq 0.7(S-p) \Rightarrow 1.7p \leq 0.7S \Rightarrow p \leq \frac{0.7}{1.7}S = \frac{7}{17}S.$

So for each r: $\frac{3}{13}S \leq p \leq \frac{7}{17}S,\quad S = 900 - r.$

Step 3: Maximum p — use the smallest $r = 169$.

$S = 900 - 169 = 731$. Upper bound $= \frac{7}{17} \times 731 = 301$. This is an integer, and at $p = 301,\ q = 430$ we get $p = 0.7q$ exactly (natural numbers). So $p_{\max} = 301$.

Step 4: Minimum p — use the largest $r = 484$.

$S = 900 - 484 = 416$. Lower bound $= \frac{3}{13} \times 416 = 96$. Integer, and at $p = 96,\ q = 320$ we get $p = 0.3q$ exactly. So $p_{\min} = 96$.

Step 5: Add the extremes.

$p_{\max} + p_{\min} = 301 + 96 = 397$.

$$\boxed{p_{\max} + p_{\min} = 397}$$

Quick Tip: When you have constraints like $ap \le q \le bp$ along with $p + q + r = \text{constant}$, try expressing q in terms of p and r, then convert the inequalities into bounds for p in terms of r. After that, use monotonicity (increasing/decreasing behavior) to decide which extreme values of r give the extreme values of p.

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20. The average salary of 5 managers and 25 engineers in a company is 60000 rupees. If each of the managers received 20% salary increase while the salary of the engineers remained unchanged, the average salary of all 30 employees would have increased by 5%. The average salary, in rupees, of the engineers is

(A) 40000

(B) 54000

(C) 50000

(D) 45000

Correct Answer: (B) 54000

Solution:

Approach: Don't chase the individual salaries. The 5% rise in the overall average comes ENTIRELY from the managers' 20% hike, so the rupee jump in the total bill equals 20% of the managers' total salary. Find that, subtract, and the engineers' total falls out.

Step 1: Total salary of all 30 employees $= 30 \times 60000 = 1800000$ rupees.

Step 2: The new average is 5% higher, so the new total salary $= 1800000 \times 1.05 = 1890000$ rupees. The increase in the total bill is

$$1890000 - 1800000 = 90000 \text{ rupees.}$$

Step 3: Only the managers' salaries changed, by 20%. So this 90000 increase is exactly 20% of the managers' original total salary M:

$$0.20\,M = 90000 \implies M = 450000 \text{ rupees.}$$

Step 4: Engineers' total salary $= 1800000 - 450000 = 1350000$ rupees, shared by 25 engineers:

$$\text{Average} = \frac{1350000}{25} = 54000 \text{ rupees.}$$

Final answer: $\boxed{54000}$ rupees (option 2).

Why this works: Whenever only one group's value changes, the change in the grand total is fully attributable to that group — so the new average is just a tool to read off that group's contribution.

Quick Tip: In weighted average problems with percentage changes:
Set up equations using total sums (average × number of people).
Apply the percentage increase only to the relevant group.
Use the new average to form a second equation and solve the system.

21. In a school with 1500 students, each student chooses any one of the streams out of science, arts, and commerce, by paying a fee of Rs 1100, Rs 1000, and Rs 800, respectively. The total fee paid by all the students is Rs 15,50,000. If the number of science students is not more than the number of arts students, then the maximum possible number of science students in the school is

Correct Answer: —

Solution:

Approach: Use the two given totals (head count and fee) to collapse three unknowns into a single relation between Science and Arts, then push Science as high as the constraint $S \leq A$ allows.

Step 1: Let S, A, C be the number of science, arts and commerce students. Head count:

$$S + A + C = 1500.$$

Fee:

$$1100S + 1000A + 800C = 1550000.$$

Step 2: Eliminate C by writing $C = 1500 - S - A$ and substituting into the fee equation:

$$1100S + 1000A + 800(1500 - S - A) = 1550000.$$

Step 3: Expand and simplify:

$$1100S + 1000A + 1200000 - 800S - 800A = 1550000$$

$$300S + 200A = 350000 \implies 3S + 2A = 3500.$$

Step 4: Apply the condition $S \leq A$. From $2A = 3500 - 3S$ we get $A = \frac{3500 - 3S}{2}$, so

$$S \leq \frac{3500 - 3S}{2} \implies 2S \leq 3500 - 3S \implies 5S \leq 3500 \implies S \leq 700.$$

Step 5: Check $S = 700$ is feasible: then $A = \frac{3500 - 2100}{2} = 700$ and C

$= 1500 - 700 - 700 = 100$ — all non-negative whole numbers. Valid.

Final answer: Maximum number of science students $= \boxed{700}$.

Why this works: Two equations on three unknowns leave one degree of freedom; the inequality $S \leq A$ is what pins the maximum, so the answer always sits at the boundary $S = A$.

Quick Tip: In word problems with headcount and revenue constraints, first set up two equations: one for the total number of people and another for total money. Then eliminate one variable to get a simple linear relation and apply given inequalities to find extrema.

22. In an arithmetic progression, if the sum of fourth, seventh and tenth terms is 99, and the sum of the first fourteen terms is 497, then the sum of first five terms is

Correct Answer: —

Solution:

Approach: In any AP, three terms equally spaced around a middle term sum to three times that middle term, and a sum of n terms is n times the middle (average) term. Use this to read off the AP quickly.

Step 1: Let the first term be a and common difference d, so $T_n = a + (n-1)d$. The 4th, 7th and 10th terms are symmetric about the 7th, so

$$T_4 + T_7 + T_{10} = 3\,T_7 = 99 \implies T_7 = a + 6d = 33.$$

Step 2: Sum of first 14 terms:

$$S_{14} = \frac{14}{2}(2a + 13d) = 7(2a + 13d) = 497 \implies 2a + 13d = 71.$$

Step 3: Solve the two equations. From $a + 6d = 33$, multiply by 2: $2a + 12d = 66$. Subtract from $2a + 13d = 71$:

$$d = 5, \qquad a = 33 - 6(5) = 3.$$

Step 4: Sum of first 5 terms:

$$S_5 = \frac{5}{2}(2a + 4d) = \frac{5}{2}(6 + 20) = \frac{5}{2} \times 26 = 65.$$

Final answer: $S_5 = \boxed{65}$.

Why this works: Spotting that $T_4 + T_7 + T_{10} = 3T_7$ gives the middle term for free, avoiding a messy substitution.

Quick Tip: When given conditions on specific terms and on the sum of terms in an AP, convert them into equations using $T_n = a + (n-1)d$ and $S_n = \frac{n}{2}[2a + (n-1)d]$. Two independent conditions will usually give you two linear equations in a and d.