CAT 2023 QA Slot 1 Question Paper with Solutions

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General Instructions

(i) This booklet contains 22 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 19 single-correct multiple-choice questions.

(iii) Attempt each question on your own before reviewing the given solution.

1. If x and y are real numbers such that x^2 + (x-2y-1)^2 = 4y(x+y), here the value x-2y is?

(A) 1

(B) 2

(C) 0

(D) -1

Correct Answer: (A) 1

Solution:

We have,

$x^2 + (x - 2y - 1)^2 = -4y(x + y)$

$\Rightarrow x^2 + 4xy + 4y^2 + (x - 2y - 1)^2 = 0$

$\Rightarrow (x + 2y)^2 + (x - 2y - 1)^2 = 0$

Since squares cannot be negative, all of the square terms in the equation must be zero for the L.H.S. to be 0.

$x - 2y - 1 = 0$

$\Rightarrow x - 2y = 1$

2. Let n be the least positive integer such that 168 is a factor of 1134^n . If m is the least positive integer such that 1134^n is a factor of 168^m , then $m+n$ equals

(A) 15

(B) 12

(C) 24

(D) 9

Correct Answer: (A) 15

Solution:

The following are the prime factorizations of 1134 and 168:

$168 = 2^3 \times 3 \times 7$

$1134 = 2 \times 3^4 \times 7$

Clearly, 3 is the least positive integral number of n that allows 168 to be a factor of 1134^n.

$1134^3 = 2^3 \times 3^{12} \times 7^3 = 1134^n$

It is evident that 12 is the least positive integral value of m that allows 1134^3 to be a factor of 168^m.

It follows that $m+n = 12+3 = 15$

The correct option is (A): 15

3. If $\sqrt{5x+9}+\sqrt{5x-9}=3(2-\sqrt{2})$ then $\sqrt{10x+9}$ is equal to

(A) $3\sqrt{7}$

(B) $4\sqrt{5}$

(C) $3\sqrt{31}$

(D) $2\sqrt{7}$

Correct Answer: (A) $3\sqrt{7}$

Solution:

To solve this problem, we start with the given equation: $\sqrt{5x+9} + \sqrt{5x-9} = 3(2-\sqrt{2})$.

Let $a = \sqrt{5x+9}$ and $b = \sqrt{5x-9}$, so we have:

$a + b = 3(2-\sqrt{2})$.

Next, square both sides to eliminate the square roots:

$(a+b)^2 = [3(2-\sqrt{2})]^2$.

This gives:

$a^2 + 2ab + b^2 = 9(4 - 4\sqrt{2} + 2)$.

$a^2 + 2ab + b^2 = 54 - 36\sqrt{2}$.

Now, using the values $a^2 = 5x + 9$ and $b^2 = 5x - 9$, we find:

$a^2 + b^2 = (5x+9) + (5x-9) = 10x$.

Substituting back gives:

$10x + 2ab = 54 - 36\sqrt{2}$.

From ab, apply the difference of squares:

$(\sqrt{5x+9} - \sqrt{5x-9})(\sqrt{5x+9} + \sqrt{5x-9}) = a^2 - b^2$.

$a^2 - b^2 = 18$.

Given $a = 3 + 3\sqrt{2}$, and using $b = 3 - 3\sqrt{2}$, calculate ab:

$ab = (3+3\sqrt{2})(3-3\sqrt{2}) = 9 - 18 + 18 = 9$.

Replacing in the equation:

$10x + 18 = 54 - 36\sqrt{2}.$

Thus:

$10x = 36.$

$x = \frac{36}{10} = 3.6.$

We need to find $\sqrt{10x + 9}$:

$\sqrt{10(3.6) + 9} = \sqrt{36 + 9} = \sqrt{45}.$

Simplifying yields $\sqrt{45} = 3\sqrt{5}.$

Therefore, the correct option is $\boxed{3\sqrt{7}}$.

4. If x and y are positive real numbers such that $log_x(x^2 + 12) = 4$ and $3\ log_y x = 1$,then $x + y$ equals

(A) 11

(B) 20

(C) 10

(D) 68

Correct Answer: (C) 10

Solution:

We have $\log_x(x^2 + 12) = 4$

$\Rightarrow x^2 + 12 = x^4$

$\Rightarrow x^4 - x^2 - 12 = 0$

$x^2(x^2 - 4) + 3(x^2 - 4) = 0$

$(x^2 - 4)(x^2 + 3) = 0$

given that x is a positive real number, then x = 2.

Given $3\log_y x = 1$

$\log_y x = \frac{1}{3}$

$\Rightarrow \quad x = y^{\frac{1}{3}}$

$\Rightarrow \quad y = x^3$

$\Rightarrow \ y = 8.$

$\Rightarrow \ x + y = 2 + 8 = 10.$

5. The number of integer solutions of equation $2|x|(x^2+1) = 5x^2$ is

(A) 0

(B) 3

(C) 5

(D) -1

Correct Answer: (B) 3

Solution:

The correct answer is 3.

6. The equation $x^3 + (2r+1)x^2 + (4r-1)x + 2 = 0$ has -2 as one of the roots. If the other two roots are real, then the minimum possible non-negative integer value of r is

(A) 2

(B) -2

(C) 3

(D) 0

Correct Answer: (A) 2

Solution:

The correct option is (A): 2.

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7. Let α and β be the two distinct roots of the equation of $2x^2$-6x+k=0, such that $(\alpha + \beta)$ and $\alpha\beta$ are the distinct roots of the equation x^2+px+p=0, then, the value of 8(k-p) ?

Correct Answer: —

Solution:

Given :

α and β are the distinct roots of the equation $2x^2 - 6x + k = 0$

⇒ αβ = $\frac{k}{2}$ ...... (Product of the roots)

⇒ α + β = $-(\frac{-6}{2})$ = 3 (Sum of the roots)

So, (α + β) and αβ are the roots of the equation $x^2 + px + p = 0$

⇒ α + β + αβ = -p

⇒ 3 + $\frac{k}{2}$ = -p ...... (i)

⇒ (α + β)(αβ) = p

⇒ $3(\frac{k}{2})$ = p ...... (ii)

Now , from eqn (i) and (ii) , we get

$3 + \frac{k}{2} = -\frac{3k}{2}$

= 2k = -3

⇒ k = $-\frac{3}{2}$

By using the value of k , we get p

p = $\frac{3k}{2} = \frac{3}{2}(-\frac{3}{2}) = -\frac{9}{4}$

Now , the value of 8(k-p) is

⇒ 8(k-p) = $8(-\frac{3}{2} + \frac{9}{4})$

= -12 + 18

= 16

So, the correct answer is 16.

8. In an examination, the average marks of 4 girls and 6 boys is 24 . Each of the girls has the same marks while each of the boys has the same marks. If the marks of any girl is at most double the marks of any boy, but not less than the marks of any boy, then the number of possible distinct integer values of the total marks of 2 girls and 6 boys is

(A) 20

(B) 22

(C) 21

(D) 19

Correct Answer: (C) 21

Solution:

Considering that the mean score for four females and six guys is 24

Assume that 'b' denotes a boy's mark and 'g' represents a girl's mark.

$4g + 6b = 10 \times 24 = 240......(1)$

We have, $b \leq g \leq 2b.$

The only possible values of $2g + 6b = 2g + 240 - 4g = 240 - 2g$ must be found.

When b = g, 10g = 240, and g = 24, we can deduce (1).

the value of 240 - 2g varies from $240 - 2 \times 24 - 240 - 2 \times \frac{240}{7}$

when $b = \frac{g}{2}$

$\Rightarrow 7g = 240$

$\Rightarrow\ g = \frac{240}{7}$

$\Rightarrow$ 171.42 to 192

$\Rightarrow$ Integer values of 172 to 192 = 21 values

9. The salaries of three friends Sita, Gita and Mita are initially in the ratio 5:6:7 respectively. In the first year, they get salary hikes of 20%, 25% and 20% , respectively. In the second year, Sita and Mita get salary hikes of 40% and 25% , respectively, and the salary of Gita becomes equal to the mean salary of the three friends. The salary hike of Gita in the second year is

Correct Answer: —

Solution:

Initially, Sita, Ggita, and Mita's salaries are in the ratio $5:6$
$:7$ respectively.

Assuming their salaries are represented by $5p, 6p,$ and $7p$.

After receiving salary hikes of $20\%, 25\%,$ and 20%, respectively, their salaries become $6p, 7.5p,$ and $8.4p$.

Now, if Sita and Mita receive further salary hikes of 40% and 25%, respectively.

Sita's salary $= 1.4 \times 6p = 8.4p$

and Mita's salary $= 1.25 \times 8.4p = 10.5p$

Let Gita's salary be g after hike.

$\Rightarrow 3g = 8.4p + g + 10.5p$

$\Rightarrow 2g = 18.9p$

$\Rightarrow g = 9.45p$

Hike percent $= \frac{9.45—7.5}{75 \times 100} = 26\%$

So, the answer is 26%.

10. The minor angle between hours hand and minutes hand of a clock was observed at 8:48 am. The minimum deviation (in min) after 8:48 am on when angle increased by 50% is?

(A) $\frac{24}{11}$

(B) 4

(C) $\frac{36}{11}$

(D) 2

Correct Answer: (A) $\frac{24}{11}$

Solution:

The Correct answer is $\frac{24}{11}$

11. Brishti went on a 8-hour trip in a car, before the trip the car had traveled a total of x kms till then, where x is a whole number and is palindromic, At the end of his trip the car had had traveled a total of 26862 km. If Bristi never drove at more than 110 km/h, then the greatest possible average speed at which see dove is?

(A) 90

(B) 80

(C) 110

(D) 100

Correct Answer: (D) 100

Solution:

Considering that the trip took place across a total of 26862 kilometers and lasted for 8 hours.
The kilometers traveled up to immediately before the trip is 26862 - 8s, which should likewise be a palindrome, if the average speed of the vehicle during the journey is's'.
Among the choices the reading will be $26862 - 110 \times 8 = 25982$ (Not a palindrome)
If s = 110 the reading will be $26862 - 100 \times 8 = 26062$
If s = 100 it's a palindrome.
=> s = 100 is the right choice.
The correct answer is 100.

12. Geeta sells A $\rightarrow$ 20% (Profit) and B $\rightarrow$ 10% (Loss) at the same sp. If she increases SP such that A and B still sold at an equal price and profit of 10% made on B, then profit made on A will be?

(A) 45%

(B) 47%

(C) 42 %

(D) 49%

Correct Answer: (B) 47%

Solution:

Let the starting selling prices of A and B be p.
Given that, she made profit of 20% on A,
$\Rightarrow 1.2 \times c = p$
$\Rightarrow c = \frac{5p}{6}$

$\Rightarrow$ cost of $A = \frac{5p}{6}$

Given that, she made a loss of 10% on B.

$\Rightarrow 0.9 \times c = p$

$\Rightarrow c = \frac{10p}{9}$

$\Rightarrow$ cost of $B = \frac{10p}{9}$

Now, she sold them at a price such that a 10% profit is made on B.

Selling price $= \frac{11}{10} \times \frac{10p}{9} = 9p$

Now,

Profit % on A,

$= \frac{\frac{11}{9} - \frac{5}{6}}{\frac{5}{6}} \times 100$

$= 46.66\%$

$\simeq 47\%$

So, the correct option is (B): 47%

13. A mixture P is formed by removing a certain amount of coffee from a coffee jar and replacing the same amount with cocoa powder and the same amount is again removed from P and replaced with the same amount of cocoa powder to form a mixture Q. If the ratio of coffee and cocoa in Q=16:9. Then the ratio of coca in with P:Q is?

(A) 1:3

(B) 4:9

(C) 1:2

(D) 5:9

Correct Answer: (D) 5:9

Solution:

Let, coffee comprises 16 units and cocoa comprises 9 units.

There are 25 units of coffee and 0 units of cocoa.

Let, x units of the mixture are removed and replaced with cocoa.

If x units of the mixture are removed then (25 - x) units of coffee left.

According to the question,

$(25 - x)(1 - \frac{x}{25}) = 16$

$(25 - x)\frac{(25-x)}{25} = 16$

$(25 - x)^2 = 25 \times 16$

$(25 - x)^2 = 400$

$(25 - x) = \sqrt{400}$

$25 - x = 20$

$x = 5$

Now, in mixture P, cocoa = 5 units

And, in mixture Q, cocoa = 9 units

The required ratio = 5:9

So, the correct option is (D): 5:9

14. Anil invests Rs. 22000 for 6 years in a certain scheme with 4% interest per annum, compounded half-yearly. Sunil invests in the same scheme for 5 years, and then reinvests the entire amount received at the end of 5 years for one year at 10% simple interest. If the amounts received by both at the end of 6 years are same, then the initial investment made by Sunil, in rupees, is

(A) 40008

(B) 20008

(C) 20808

(D) 10808

Correct Answer: (C) 20808

Solution:

Anil invested 22000 for 6 years at 4% interest compounded half-yearly.

Amount = $22000(1+\frac{2}{100})^{12}$

$= 22000(1.02)^{12}$

Suppose, Sunil invest P rupees for 5 years at 4% C.I. half-yearly and 10% S.I. for 1 additional year.

Amount = $P(1+\frac{2}{100})^{10}\times 1.1$

$= P(1.02)^{10}\times 1.1$

Given, both amounts are same,

$22000(1.02)^{12} = P(1.02)^{10}\times 1.1$

$P = \frac{22000(1.02)^{12}}{(1.02)^{10}\times 1.1}$

$\Rightarrow P = \frac{22000(1.02)^{2}}{1.1}$

$\Rightarrow P = 20808$

So, the correct option is (C): 20808

15. The amount of job that Amal, Sunil and Kamal can individually do in a day, are in harmonic progression. Kamal takes twice as much time as Amal to do the same amount of job. If Amal and Sunil work for 4 days and 9 days, respectively, Kamal needs to work for 16 days to finish the remaining job. Then the number of days Sunil will take to finish the job working alone, is

Correct Answer: —

Solution:

Given: Amal, Sunil, and Kamal can individually do a job in such a way that the amount of work they do per day is in **Harmonic Progression (H.P.)**.

This implies that the **time taken** by them to do the full job is in **Arithmetic Progression (A.P.)**.

We are also told that:

- Kamal takes **twice** the time Amal takes to complete the job.

Let's assume:

- Amal takes t days,
- Then Kamal takes $2t$ days.
- Since time is in A.P., Sunil must take $\frac{t+2t}{2} = 1.5t$ days.

So, the ratio of time taken by Amal : Sunil : Kamal is:

$$t : 1.5t : 2t = 2 : 3 : 4$$

Now given actual times taken:

- Amal: 4 days
- Sunil: 9 days
- Kamal: 16 days

Let us now determine how much work Sunil does compared to Amal and Kamal.

Comparison with Amal:

- Amal completes 1 job in 4 days $\Rightarrow$ Amal's rate $= \frac{1}{4}$ job/day
- Sunil completes 1 job in 9 days $\Rightarrow$ Sunil's rate $= \frac{1}{9}$ job/day

Compare their work rates:

$$\frac{1/9}{1/4} = \frac{4}{9} \Rightarrow \text{In 3 days, Sunil does } \frac{3}{9} = \frac{1}{3} \text{ of the job}$$

$$\text{In 2 days, Amal does } \frac{2}{4} = \frac{1}{2} \text{ of the job}$$

So to match Amal's full job (4 days), Sunil will need:

$$\text{If } \frac{1}{2} \text{ job takes 3 days} \Rightarrow \text{full job takes 6 days}$$

Comparison with Kamal:

- Kamal's rate $= \frac{1}{16}$ job/day
- In 4 days, Kamal does $\frac{1}{4}$ job
- Sunil's rate $= \frac{1}{9}$ job/day $\Rightarrow$ In 3 days, Sunil does $\frac{1}{3}$ job

So, to do Kamal's full job (16 days), Sunil would need:

$$\text{If } \frac{1}{4} \text{ job takes 3 days} \Rightarrow \text{full job takes 12 days}$$

Thus, Sunil would take:

- 6 days to do what Amal does in 4 days
- 9 days to do what he normally does himself
- 12 days to do what Kamal does in 16 days

Total time = 6 + 9 + 12 = 27 days

Final Answer: 27 days

16. Arvind travels from town A to town B, and Surbhi from town B to town A, both starting at the same time along the same route. After meeting each other, Arvind takes 6 hours to reach town B while Surbhi takes 24 hours to reach town A. If Arvind travelled at a speed of 54 km/h, then the distance, in km, between town A and town B is [This question was asked as TITA]

(A) 982 Kms

(B) 970 Kms

(C) 974 Kms

(D) 972 Kms

Correct Answer: (D) 972 Kms

Solution:

To solve this problem, let's denote the distance Arvind and Surbhi have traveled to meet each other as x km from their respective starting points.

Given that Arvind travels at a speed of 54 km/h, he takes 6 hours to cover the remaining distance after meeting Surbhi. Therefore, the remaining distance Arvind travels is:

$$\text{Distance}_{\text{remaining\_Arvind}} = 54 \text{ km/h} \times 6 \text{ h} = 324 \text{ km}$$

Thus, this implies:

$$x = \text{Total Distance} - 324$$

Similarly, Surbhi takes 24 hours to cover the remaining distance after meeting Arvind. Let Surbhi's speed be v km/h. Then the distance she travels after meeting Arvind is:

$$\text{Distance}_{\text{remaining\_Surbhi}} = v \text{ km/h} \times 24 \text{ h}$$

According to the problem, both Arvind and Surbhi meet at the same point, meaning Arvind's x km and Surbhi's total distance to meet Arvind after this is used in the equation:

$$x + v \times 24 \text{ h} = \text{Total Distance}$$

Since Arvind and Surbhi meet exactly halfway in terms of time spent, Surbhi would take four times as long to complete her remaining journey as Arvind does:

$$\frac{v}{54} = \frac{1}{4}$$

Thus, the speed of Surbhi is:

$$v = \frac{54}{4} = 13.5 \text{ km/h}$$

Now apply this to Surbhi's remaining distance:

$$\text{Distance}_{\text{remaining\_Surbhi}} = 13.5 \text{ km/h} \times 24 \text{ h} = 324 \text{ km}$$

Thus:

$$\text{Total Distance} = 324 \text{ km} + 324 \text{ km} = 972 \text{ km}$$

Therefore, the distance between town A and town B is 972 km.

17. A quadrilateral $ABCD$ is inscribed in a circle such that $AB : CD = 2 : 1$ and $BC : AD = 5 : 4$. If AC and BD intersect at the point E, then $AE : CE$ equals

(A) 2 :1

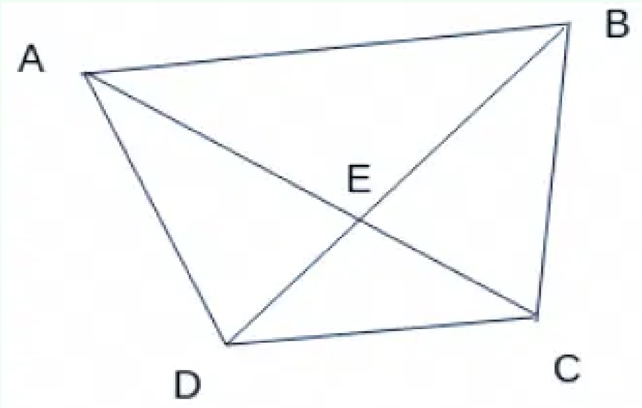
(B) 5: 8

(C) 8 : 5

(D) 1: 2

Correct Answer: (C) 8 : 5

Solution:



Given that $ABCD$ is a cyclic quadrilateral.

$\angle ADB = \angle ACB$ (Angle subtended by chord on the same side of arc)

$\angle DAC = \angle DBC$ (Angle subtended by chord on the same side of arc

$\Rightarrow \triangle AED$ and $\triangle BEC$ are similar triangles.

Similarly, $\triangle AEB$ and $\triangle DEC$ are also similar using AA similarity property.

Given that,

$AB : CD = 2 : 1$

and $BC : AD = 5 : 4$

$\frac{AE}{BE} = \frac{AD}{BC} = \frac{4}{5}$ (Similar Triangles $\triangle AED$ and $\triangle BEC$)

$\frac{BE}{CE} = \frac{AB}{CD} = \frac{2}{1}$ (Similar Triangles $\triangle AEB$ and $\triangle DEC$)

On multiplying both,

$\frac{AE}{CE} = \frac{8}{5}$

So, the correct option is (C): 8 : 5

18. Let C be the circle $x^2 + y^2 + 4x - 6y - 3 = 0$ and L be the locus of the point of intersection of a pair of tangents to C with the angle between the two tangents equal to 60^o. Then, the point at which L touches the line $x = 6$ is

(A) (6, 6)

(B) (6, 8)

(C) (6, 4)

(D) (6, 3)

Correct Answer: (D) (6, 3)

Solution:

Given :

Equation of circle = $x^2 + y^2 + 4x - 6y - 3 = 0$

Radius of the circle = $\sqrt{g^2 + f^2 - c}$

$= \sqrt{4 + 9 + 3} = \sqrt{16}$

= 4

Center of the circle : (2, -3)

Suppose the point of intersection of the tangents is (h, k)

Now, the angle created by the line joining (h, k) to the centre makes an angle of 30° with the tangent and sin(30) will be the ratio of radius and distance between the center and (h, k)

$\Rightarrow \sin(30) = \frac{4}{\sqrt{(h+2)^2+(k-3)^2}}$

Now, by squaring on both sides , we get :

$\frac{1}{4} = \frac{16}{(h+2)^2+(k-3)^2}$

$\Rightarrow (h + 2)^2 + (k - 3)^2 = 64$

Now , when x = 6 ⇒ h = 6 , we get

⇒ $(6 + 2)^2 + (k - 3)^2 = 64$

⇒ $64 + (k - 3)^2 = 64$

k = 3.

Therefore, the required point is (6, 3)

So, the correct option is (D) : (6, 3)

19. In a right-angled triangle Δ ABC , the altitude AB is 5 cm , and the base BC is 12 cm . P and Q are two points on BC such that the areas of ΔABP, ΔABQ , and Δ ABC are in arithmetic progression. If the area of Δ ABC is 1.5 times the area of Δ ABP , the length of PQ , in cm , is

(A) 2

(B) 1

(C) 0

(D) 3

Correct Answer: (A) 2

Solution:

The correct answer is 2.

20. For some positive and distinct real numbers $x, y,$ and z , if $\frac{1}{\sqrt{y}+\sqrt{z}}$ is the arithmetic mean of $\frac{1}{\sqrt{x}+\sqrt{z}}$ and $\frac{1}{\sqrt{x}+\sqrt{y}}$, then the relationship which will always hold true, is

(A) y, x, and z are in arithmetic progression

(B) $\sqrt{x}, \sqrt{y}$, and $\sqrt{z}$ are in arithmetic progression

(C) x, y, and z are in arithmetic progression

(D) $\sqrt{x}, \sqrt{z}$, and $\sqrt{y}$ are in arithmetic progression

Correct Answer: (A) y, x, and z are in arithmetic progression

Solution:

Given

$\frac{1}{\sqrt{y}+\sqrt{z}}$ is the arithmetic mean of $\frac{1}{\sqrt{x}+\sqrt{z}}$ and $\frac{1}{\sqrt{x}+\sqrt{y}}$

$\frac{2}{\sqrt{y}+\sqrt{z}} = \frac{1}{\sqrt{x}+\sqrt{z}} + \frac{1}{\sqrt{x}+\sqrt{y}}$

$\Rightarrow 2(\sqrt{x}+\sqrt{z})(\sqrt{x}+\sqrt{y}) = (\sqrt{y}+\sqrt{z})(\sqrt{x}+\sqrt{y}+\sqrt{x}+\sqrt{z})$

$\Rightarrow 2(x+\sqrt{xy}+\sqrt{xz}+\sqrt{yz}) = 2\sqrt{xy}+y+\sqrt{yz}+2\sqrt{xz}+\sqrt{yz}+z$

$\Rightarrow\ 2x = y+z$

Therefore, x is the arithmetic mean of y and z, y, x, and z are in A.P

The correct option is (A): y, x and z are in arithmetic progression.

21. The number of all natural numbers up to 1000 with non-repeating digits is

(A) 648

(B) 585

(C) 504

(D) 738

Correct Answer: (D) 738

Solution:

The correct option is (D): 738

22. A lab experiment measures the number of organisms at 8 am every day. Starting with 2 organisms on the first day, the number of organisms on any day is equal to 3 more than twice the number on the previous day. If the number of organisms on the nth day exceeds one million, then the lowest possible value of n is

(A) 15

(B) 17

(C) 19

(D) 23

Correct Answer: (C) 19

Solution:

Starting with 2 organisms on day-1, each day the population grows.
On day-2, it's $22 + 3 = 7, and\ on\ day\text{-}3, it's\ 27 + 3 = 17$.
Following the pattern, we see T(n) $= 2n + 3(2n-1-1)$.
To find when the population exceeds 1 million,
we check for n = 19, where we get $2^{19} + 3(2^{18}-1) = 2^{20} + 2^{18}-3$,
which is indeed over a million. For n = 18, it's $2^{18} + 3(2^{17}-1) = 2^{19} + 2^{17}-3$,
which is not over a million.
So, the population surpasses a million at n = 19.