CAT 2023 QA Slot 2 Question Paper with Solutions

General Instructions

(i) This booklet contains 22 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 20 single-correct multiple-choice questions and 2 numerical / type-in-the-answer questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For numerical questions, report the answer rounded exactly as asked.

1. For any natural numbers m, n, and k, such that k divides both $m + 2n$ and $3m + 4n$, k must be a common divisor of

(A) m and n

(B) 2m and 3n

(C) 2m and n

(D) m and 2n

Correct Answer: (D) m and 2n

Solution:

Given that, k divides $m + 2n$ and $3m + 4n$.

Since k divides $m + 2n$,
$\Rightarrow k$ also divides $3(m + 2n)$
$\Rightarrow k$ divides $3m + 6n$.

Also, k divides $3m + 4n$.

Now, if k divides both $3m + 6n$ and $3m + 4n$, then it also divides their difference:

$$(3m + 6n) - (3m + 4n) = 2n$$

So, k **divides** $2n$.

Again, since k divides $m + 2n$,

$$\Rightarrow k \text{ divides } 2(m + 2n) = 2m + 4n$$

And k divides $3m + 4n$.
Taking the difference:

$$(3m + 4n) - (2m + 4n) = m$$

So, k **divides** m.

Hence, m **and** $2n$ are divisible by k.

Correct option: (D) m **and** $2n$.

2. The sum of all possible values of x satisfying the equation $2^{4x^2} - 2^{2x^2+x+16} + 2^{2x+30} = 0$, is

(A) $\frac{5}{2}$

(B) $\frac{1}{2}$

(C) 3

(D) $\frac{3}{2}$

Correct Answer: (B) $\frac{1}{2}$

Solution:

Given Equation:

$2^{4x^2} - 2^{2x^2+x+16} + 2^{2x+30} = 0$

First, rewrite the equation by breaking down the powers of 2 in terms of the base and their exponents:

$\Rightarrow (2^{2x^2})^2 - 2^{2x^2} \cdot 2^{x+15} \cdot 2^1 + (2^{x+15})^2 = 0$

This simplifies to:

$\Rightarrow (2^{2x^2} - 2^{x+15})^2 = 0$

Thus, we have the equation:

$\Rightarrow 2^{2x^2} - 2^{x+15} = 0$

Now, since the equation involves powers of 2, we can equate the exponents:

$\Rightarrow 2^{2x^2} = 2^{x+15}$

Equating the exponents of 2 gives us:

$\Rightarrow 2x^2 = x + 15$

Rearranging this into a standard quadratic form:

$\Rightarrow 2x^2 - x - 15 = 0$

Now, solve this quadratic equation by factoring:

$\Rightarrow (2x + 5)(x - 3) = 0$

Thus, the solutions for x are:

$\Rightarrow x = -\frac{5}{2}$ or $x = 3$

The sum of the possible values is:

$-\frac{5}{2} + 3 = \frac{1}{2}$

Therefore, the correct answer is (B): $\frac{1}{2}$

3. Any non-zero real numbers x, y such that $y \neq 3$ and $\frac{x}{y} < \frac{x+3}{y-3}$, will satisfy the condition

(A) $\frac{x}{y} < \frac{y}{x}$

(B) If $y > 10$, then $-x > y$

(C) If $x < 0$, then $-x < y$

(D) If $y < 0$, then $-x < y$

Correct Answer: (D) If $y < 0$, then $-x < y$

Solution:

Given:

$\frac{x}{y} < \frac{x+3}{y-3}$

This inequality can be rewritten as:

$$\frac{x}{y} - \frac{x+3}{y-3} < 0$$

Now, let's combine the two fractions:

$$\Rightarrow \frac{x(y-3) - y(x+3)}{y(y-3)} < 0$$

Expanding the numerator:

$$\Rightarrow \frac{xy - 3x - xy - 3y}{y(y-3)} < 0$$

$$\Rightarrow \frac{-3(x+y)}{y(y-3)} < 0$$

Multiplying both sides by -1 (to make the numerator positive):

$$\Rightarrow \frac{3(x+y)}{y(y-3)} > 0$$

Now, let's analyze the inequality $\frac{3(x+y)}{y(y-3)} > 0$.

For this inequality to be true, we need to consider the conditions for both the numerator and the denominator. We know that:

If $y < 0$, then $y(y-3) > 0$ (since the product of two negative numbers is positive). Thus, for the inequality $\frac{3(x+y)}{y(y-3)} > 0$ to hold true, we must have:

$$x + y > 0$$

This implies:

$$x > -y$$

Additionally, since x and y are related by their absolute values, we conclude that:

$$|x| > |y|$$

Therefore, the final conclusion is:

$$-x < y$$

Conclusion: Therefore, the correct option is (D): "If $y < 0$, then $-x < y$."

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4. Let a, b, m and n be natural numbers such that $a > 1$ and $b > 1$. If $a^m + b^n = 144^{145}$, then the largest possible value of $n - m$ is

(A) 579

(B) 289

(C) 580

(D) 290

Correct Answer: (A) 579

Solution:

Given:

We are provided with the equation: $a^m \times b^n = 144^{145}$ where $a > 1$ and $b > 1$.

Also, we know that 144 can be expressed as: $144 = 2^4 \times 3^2$.

Therefore, the equation can be rewritten as:

$$a^m \times b^n = 144^{145} = (2^4 \times 3^2)^{145}$$

By expanding the powers, we get: $a^m \times b^n = 2^{580} \times 3^{290}$

Step-by-Step Analysis:

1. **Identifying the form of the equation:**
   The equation $a^m \times b^n = 2^{580} \times 3^{290}$ suggests that the powers of the prime factors of a and b must match the powers of 2 and 3, respectively. So we can make the following observations:
   - One of the terms must involve the prime factor 2, and the other must involve the prime factor 3.
   - Thus, we conclude that a^m must be a power of 2, and b^n must be a power of 3.
2. **Considering the term involving a^m:**
   From the equation, since a^m must be a power of 2, we can write: $a^m = 2^{580}$. This means that a is a power of 2. Therefore, $a = 2$ and $m = 580$.
3. **Considering the term involving b^n:**
   Similarly, b^n must be a power of 3, so: $b^n = 3^{290}$. This means that b is a power of 3. Therefore, $b = 3$ and $n = 290$.
4. **Determining the smallest value of m and largest value of n:**
   As per the question, we need to find the least possible value of m and the largest possible value of n. We know that:

- The least possible value of m is 1, since $a = 2$ and the minimum exponent for 2^1 would make $m = 1$.
- The largest possible value of n is 580, since $b = 3$ and $b^n = 3^{580}$.

5. **Final calculation of $n - m$:**

   The difference $n - m$ is: $n - m = 580 - 1 = 579$

Conclusion:

Therefore, the correct answer is **(A): 579.**

5. Let k be the largest integer such that the equation $(x-1)^2 + 2kx + 11 = 0$ has no real roots. If y is a positive real number, then the least possible value of $\frac{k}{4y} + 9y$ is

Correct Answer: —

Solution:

Given:

$(x-1)^2 + 2kx + 11 = 0$ has no real roots, where k is the largest integer.

Step 1: Simplify the expression:

$(x-1)^2 + 2kx + 11 = 0$

$\Rightarrow x^2 - 2x + 1 + 2kx + 11 = 0$

$\Rightarrow x^2 - 2(k-1)x + 12 = 0$

Step 2: Use discriminant condition for no real roots:

For a quadratic to have no real roots, $D < 0$

$\Rightarrow b^2 - 4ac < 0$

$\Rightarrow [2(k-1)]^2 - 4 \cdot 1 \cdot 12 < 0$

$\Rightarrow 4(k-1)^2 < 48$

$\Rightarrow (k-1)^2 < 12$

As $(k-1)$ is an integer, the maximum integer satisfying this is 3.

$\Rightarrow k-1 = 3 \Rightarrow k = 4$

Step 3: Minimize the expression $\frac{k}{4y} + 9y$

$\Rightarrow \frac{4}{4y} + 9y = \frac{1}{y} + 9y$

Step 4: Use AM-GM inequality:

$\frac{1}{y} + 9y \geq 2\sqrt{9y \cdot \frac{1}{y}} = 2\sqrt{9} = 6$

Final Answer: The least possible value is **6**.

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6. The number of positive integers less than 50, having exactly two distinct factors other than 1 and itself, is

Correct Answer: —

Solution:

Objective: Find the number of positive integers less than 50 that are either:

- **Case I:** Perfect cubes of prime numbers: $N = p^3$
- **Case II:** Products of exactly two distinct prime numbers: $N = p_1 \times p_2$

Case I:

Primes whose cubes are less than 50:

- $2^3 = 8$

- $3^3 = 27$

$\Rightarrow$ **2 numbers**

Case II:

Products of two distinct primes less than 50:

$$2 \times 3 = 6, \quad 2 \times 5 = 10, \quad 2 \times 7 = 14, \quad 2 \times 11 = 22, \quad 2 \times 13 = 26, \quad 2 \times 17 = 34, \quad 2 \times 19 = 38, \quad 2 \times 23 = 46,$$

$$3 \times 5 = 15, \quad 3 \times 7 = 21, \quad 3 \times 11 = 33, \quad 3 \times 13 = 39, \quad 5 \times 7 = 35$$

$\Rightarrow$ **13 numbers**

Total = 2 + 13 = 15

Final Answer: 15

7. For some positive real number x , if $log_{\sqrt{3}}(x) + \frac{log_x(25)}{log_x(0.008)} = \frac{16}{3}$, then the value of $log_3(3x^2)$ is

(A) 4

(B) 6

(C) 7

(D) 9

Correct Answer: (C) 7

Solution:

Given Equation:

The given equation is:

$$\log_{\sqrt{3}}(x) + \frac{\log_x(25)}{\log_x(0.008)} = \frac{16}{3}$$

Step 1: Simplify the logarithmic terms

The equation can be rewritten as:

$\Rightarrow 2\log_3(x) + \log_{0.008}(25) = \frac{16}{3}$

Step 2: Express $\log_{0.008}(25)$ in terms of logarithms

We can express the second logarithmic term using the property of logarithms:

$\Rightarrow 2\log_3(x) + \log_{\frac{8}{1000}}(25) = \frac{16}{3}$

Step 3: Simplify further

We know that:

$log_{\frac{8}{1000}} 25 = \log_5 (25)^{(-3)} = \frac{2}{3}$

Substituting this value into the equation gives:

$\Rightarrow 2\log_3(x) - \frac{2}{3} = \frac{16}{3}$

Step 4: Solve for $\log_3(x)$

Adding $\frac{2}{3}$ to both sides:

$\Rightarrow 2\log_3(x) = \frac{16}{3} + \frac{2}{3} = 6$

Step 5: Solve for x

Now, divide both sides by 2:

$\Rightarrow \log_3(x^2) = 6$

This implies:

$\Rightarrow x^2 = 3^6$

Step 6: Final Expression

The final equation is:

$$log_3(3 \cdot x^2) = log_3(3 \cdot 3^6) = log_3(3^7) = 7$$

Conclusion:

Therefore, the correct option is **(C): 7**.

8. Pipes A and C are fill pipes while Pipe B is a drain pipe of a tank. Pipe B empties the full tank in one hour less than the time taken by Pipe A to fill the empty tank. When pipes A, B and C are turned on together, the empty tank is filled in two hours. If pipes B and C are turned on together when the tank is empty and Pipe B is turned off after one hour, then Pipe C takes another one hour and 15 minutes to fill the remaining tank. If Pipe A can fill the empty tank in less than five hours, then the time taken, in minutes, by Pipe C to fill the empty tank is

(A) 75

(B) 120

(C) 60

(D) 90

Correct Answer: (D) 90

Solution:

Given that A takes x hours to fill the tank alone, it follows that B (the drainage pipe) needs $(x - 1)$ hours to empty the tank alone, and C needs y hours to replenish the tank.

Step 1: First Equation Setup

When all pipes (A, B, and C) are activated simultaneously, the empty tank will fill in 2 hours. The equation representing this scenario is:

$$\frac{1}{x} - \frac{1}{x-1} + \frac{1}{y} = \frac{1}{2} \tag{1}$$

Step 2: Second Scenario Setup

In the second scenario, pipe B works for 1 hour, and pipe C works for 2 hours and 15 minutes. The work completed by each pipe is:

- Pipe B completes $-\frac{1}{x-1}$ units in 1 hour.
- Pipe C completes $\frac{9}{4y}$ units in $\frac{9}{4}$ hours.

So, the equation for this scenario is:

$$\frac{9}{4y} - \frac{1}{x-1} = 1 \tag{2}$$

Step 3: Solve the System of Equations

Now, we solve the two equations:

- Equation (1): $\frac{1}{x} - \frac{1}{x-1} + \frac{1}{y} = \frac{1}{2}$
- Equation (2): $\frac{9}{4y} - \frac{1}{x-1} = 1$

By solving these equations, we find:

- $x = 3$
- $y = \frac{3}{2}$

Step 4: Final Calculation

Now, we know that pipe C takes $3\frac{1}{2}$ hours (or 90 minutes) to complete the task. Therefore, the correct option is:

The correct answer is (D): 90 minutes.

9. Anil borrows Rs 2 lakhs at an interest rate of 8% per annum, compounded half-yearly. He repays Rs 10320 at the end of the first year and closes the loan by paying the outstanding amount at the end of the third year. Then, the total interest, in rupees, paid over the three years is nearest to

(A) 33130

(B) 40991

(C) 51311

(D) 51311

Correct Answer: (C) 51311

Solution:

Anil takes out a loan of Rs 2 lakhs, with interest compounded every six months at a rate of 8% per year.

It is also known that at the conclusion of the first year, he repays Rs 10,320, and at the end of the third year, he terminates the debt by making the final payment.

Step 1: Loan Amount After First Year

At the conclusion of the first year, the total amount can be calculated as follows:

$200000 \times \frac{104}{100} \times \frac{104}{100} = 216320$

Thus, after one year, the total amount due becomes Rs 216,320.

Step 2: Repayment at the End of Year 1

At the end of the first year, Anil repays Rs 10,320. Therefore, the remaining outstanding balance is:

Outstanding balance = Rs 216,320 - Rs 10,320 = Rs 206,000.

Step 3: Loan Amount After Two More Years

The interest will continue to be charged on the outstanding balance of Rs 206,000 for a further two years. The total amount after three years is calculated as:

$206000 \times \left(\frac{104}{100}\right)^4 = 240990.86$

So, the total amount due at the end of the third year is Rs 240,990.86.

Step 4: Interest Accrued During Two Years

The interest accrued over the next two years is:

$240990.86 - 206000 = 34990.86$

The total interest accumulated over the three years is the sum of the interest for the first year and the next two years:

$34990.86 + 16320 = 51311$

Conclusion

The total interest accumulated over the three years is Rs **51,311**.

10. Ravi is driving at a speed of 40 km/h on a road. Vijay is 54 meters behind Ravi and driving in the same direction as Ravi. Ashok is driving along the same road from the opposite direction at a speed of 50 km/h and is 225 meters away from Ravi. The speed, in km/h, at which Vijay should drive so that all the three cross each other at the same time, is

(A) 67.2

(B) 64.4

(C) 61.6

(D) 58.8

Correct Answer: (C) 61.6

Solution:

We are given the following information:

- Ravi's speed = 40 km/h, which is equivalent to $\frac{100}{9}$ meters per second.
- Ashok's speed = 50 km/h, which is equivalent to $\frac{125}{9}$ meters per second.
- The distance between Ravi and Ashok = 225 meters.

Step 1: Calculate Combined Velocity

The relative speed (combined velocity) of Ravi and Ashok when they move towards each other is the sum of their speeds:

$$\text{Combined Speed} = \frac{125}{9} + \frac{100}{9} = 25\,\text{m/s}$$

Step 2: Time to Meet

The time taken for Ravi and Ashok to meet is calculated using the formula:

$$\text{Time} = \frac{\text{Distance}}{\text{Relative Speed}} = \frac{225}{25} = 9\,\text{seconds}$$

Step 3: Distance Covered by Ravi

In these 9 seconds, Ravi will cover:

$$\text{Distance} = \frac{100}{9} \times 9 = 100 \, \text{meters}$$

Step 4: Distance Covered by Vijay

Vijay starts 54 meters behind Ravi. Therefore, Vijay needs to cover a total distance of:

$$\text{Distance} = 100 + 54 = 154 \, \text{meters}$$

Step 5: Calculate Vijay's Speed

Vijay's speed is calculated as:

$$\text{Speed} = \frac{154}{9} \, \text{m/s}$$

Next, we convert Vijay's speed into kilometers per hour:

$$\text{Speed} = \frac{154}{9} \times \frac{18}{5} = \frac{308}{5} = 61.6 \, \text{km/h}$$

Conclusion

Thus, Vijay's speed is **61.6 km/h**.

The correct option is **(C): 61.6**.

11. Minu purchases a pair of sunglasses at Rs.1000 and sells to Kanu at 20% profit. Then, Kanu sells it back to Minu at 20% loss. Finally, Minu sells the same pair of sunglasses to Tanu. If the total profit made by Minu from all her transactions is Rs.500, then the percentage of profit made by Minu when she sold the pair of sunglasses to Tanu is

(A) 26%

(B) 35.42%

(C) 52%

(D) 31.25%

Correct Answer: (D) 31.25%

Solution:

Minu purchases a pair of sunglasses for Rs. 1000. She sells them to Kanu at a 20% profit.

Step 1: Minu's Sale to Kanu

Minu paid Rs. 1000 for the sunglasses and sold them to Kanu for Rs. 1200. This means Minu made a profit of Rs. 200 on this transaction.

Step 2: Kanu's Sale to Minu

At a 20% loss, Kanu sells the sunglasses back to Minu. This means Kanu sold the glasses to Minu for:

$80\% \times 1200 = 960$

So, Minu buys back the sunglasses for Rs. 960 from Kanu.

Step 3: Minu's Sale to Tanu

Minu needs to make a profit of Rs. 500 in total. Since she has already made Rs. 200 from the first transaction, she needs an additional Rs. 300 profit.

Therefore, Minu sells the sunglasses to Tanu for:

$960 + 300 = 1260$

Step 4: Profit Percentage

The profit made on the final sale is Rs. 300. To calculate the profit percentage, we use the formula:

$\frac{300}{960} \times 100 = 31.25\%$

Conclusion

The profit percentage on the final sale is **31.25%**.

Therefore, the correct option is (D): **31.25%**.

12. The price of a precious stone is directly proportional to the square of its weight. Sita has a precious stone weighing 18 units. If she breaks it into four pieces with each piece having distinct integer weight, then the difference between the highest and lowest possible values of the total price of the four pieces will be 288000. Then, the price of the original precious stone is

(A) 1620000

(B) 1296000

(C) 1944000

(D) 972000

Correct Answer: (B) 1296000

Solution:

"A precious stone's price is directly correlated with the square of its weight."

$P = k \times W^2$, where W and P are the stone's weight and price, respectively.

The total cost of the intact stone will be $18^2 \times k = 324k$.

"The difference between the highest and lowest possible values of the total price of the four pieces will be 288000 if she breaks it into four pieces, each with a distinct integer weight."

When the shattered stone weights are close to one another, that is, when the weights are 3, 4, 5, and 6 units, the minimum profit is made.

In this instance, the four stones' combined worth $=(3^2 + 4^2 + 5^2 + 6^2)k$ $= 86k$

When the broken stone weights are separated by a large amount, i.e., 1, 2, 3, and 12 units, the largest benefit is realized.

The combined value of the four stones in this instance is equal to $(1^2 + 2^2 + 3^2 + 12^2)k = 158k$.

The overall value difference is 2,88,000.

1,88,000 – 86 000 = 72 000 = 4,000

Thus, the original stone cost 324 k, or 12,96,000.

The correct option is (B): 1296000

13. In a company, 20% of the employees work in the manufacturing department. If the total salary obtained by all the manufacturing employees is one-sixth of the total salary obtained by all the employees in the company, then the ratio of the average salary obtained by the manufacturing employees to the average salary obtained by the non-manufacturing employees is

(A) 6 : 5

(B) 4 : 5

(C) 5 : 4

(D) 5 : 6

Correct Answer: (B) 4 : 5

Solution:

Average Salary Calculation in Manufacturing and Non-Manufacturing Departments

We are given that the total number of employees in the organization is $100x$, and each employee's wage is $100y$. Let's analyze the situation step by step:

Step 1: Manufacturing Department

The manufacturing department employs 20% of the total workforce. Therefore, the number of employees in the manufacturing department is $20x$. Additionally, the combined salary of all manufacturing staff members is one-sixth of the total salary received by all employees.

The combined salary of the manufacturing department is:

$\frac{1}{6} \times 100x \times 100y = \frac{10000xy}{6}$

The average salary in the manufacturing department is then:

$\frac{\frac{10000xy}{6}}{20x} = \frac{500y}{6x}$

Step 2: Non-Manufacturing Department

The non-manufacturing department employs the remaining 80% of the workforce, meaning there are $80x$ employees in the non-manufacturing department. The combined salary of the non-manufacturing department is:

$$\frac{50000xy}{6}$$

The average salary in the non-manufacturing department is:

$$\frac{\frac{50000xy}{6}}{80x} = \frac{500y}{24x}$$

Step 3: Ratio of Average Salaries

We now calculate the ratio of the average salary in the manufacturing department to the average salary in the non-manufacturing department:

$$\frac{\frac{500y}{6x}}{\frac{500y}{24x}} = \frac{500y}{6x} \times \frac{24x}{500y} = \frac{24}{6} = 4 : 15$$

Conclusion

Thus, the ratio of the average salary in the manufacturing department to the average salary in the non-manufacturing department is **4:15**.

Therefore, the correct choice is **(B): 4:15**.

14. A container has 40 liters of milk. Then, 4 liters are removed from the container and replaced with 4 liters of water. This process of replacing 4 liters of the liquid in the container with an equal volume of water is continued repeatedly. The smallest number of times of doing this process, after which the volume of milk in the container becomes less than that of water, is

(A) 5

(B) 7

(C) 4

(D) None of Above

Correct Answer: (B) 7

Solution:

Step 1: Understanding the Substitution Process

We are told that each time 10% of the mixture is substituted with pure adulterant (water), the concentration of the mixture decreases to 90% of its initial concentration.

In general, when a proportion p of a mixture is substituted with pure adulterant, the concentration of the resulting mixture becomes $(1 - p)$ times the previous concentration. Here, $p = \frac{4}{40} = 0.1$.

Initially, the mixture contains pure milk, so the concentration or strength is 100% or 1 (in terms of proportion). After undergoing the substitution process n times, the concentration of milk in the mixture will be represented by the expression:

$1 \times (0.9)^n$.

Step 2: Finding the Smallest n for Milk Concentration Less Than 50%

We are required to find the smallest number of substitutions, n, for which the concentration of milk becomes less than 50%. This condition is represented as:

$1 \times (0.9)^n < 0.5$.

Step 3: Solving the Inequality

To solve this, take the logarithm of both sides:

$\log\left((0.9)^n\right) < \log(0.5)$

Using the properties of logarithms, this simplifies to:

$n\log(0.9) < \log(0.5)$

Step 4: Calculating the Logarithms

The logarithms are approximately:

$\log(0.9) \approx -0.045757 \text{ and } \log(0.5) \approx -0.3010$

Now, substitute these values into the inequality:

$n \times (-0.045757) < -0.3010$

Solving for n

$n > \frac{-0.3010}{-0.045757} \approx 6.58$

Step 5: Conclusion

The smallest integer greater than 6.58 is $n = 7$.

Final Answer:

The smallest number of substitutions, n, that satisfies this condition is 7.

The correct option is (B): 7.

15. If a certain amount of money is divided equally among n persons, each one receives Rs 352 . However, if two persons receive Rs 506 each and the remaining amount is divided equally among the other persons, each of them receive less than or equal to Rs 330 . Then, the maximum possible value of n is

(A) 15

(B) 17

(C) 16

(D) None of Above

Correct Answer: (C) 16

Solution:

The problem states that:

> *"If a certain amount of money is divided equally among n persons, each one receives Rs 352."*

Thus, the total amount of money is:

$$\text{Total amount} = 352 \times n = 352n$$

Next, we are told that:

> *"If two persons receive Rs 506 each and the remaining amount is divided equally among the other persons, each of them receives less than or equal to Rs 330."*

From this, we can calculate the total amount of money in two parts:

- First, two persons receive Rs 506 each, so the total amount for these two persons is $506 \times 2 = 1012$.
- The remaining amount is $352n - 1012$, and this remaining amount is divided equally among the remaining $n - 2$ persons, each receiving Rs 330 or less. Therefore, the total amount for the remaining persons is:

$$\text{Remaining amount} = (n - 2) \times 330$$

The total amount of money can thus be written as:

Total money $= 1012 + (n - 2) \times 330$

Expanding this expression:

$1012 + 330n - 660 = 352 + 330n$

Now, the total money is also equal to $352n$, so we equate the two expressions:

$352 + 330n \geq 352n$

Next, simplify the inequality:

$330n \geq 352n - 352$

Rearranging the inequality gives:

$22n \leq 352$

Now, solving for n:

$n \leq \frac{352}{22}$

$n \leq 16$

Thus, the maximum value that n can take is 16.

The correct option is (C): 16.

16. Jayant bought a certain number of white shirts at the rate of Rs 1000 per piece and a certain number of blue shirts at the rate of Rs 1125 per piece. For each shirt, he then set a fixed market price which was 25% higher than the average cost of all the shirts. He sold all the shirts at a discount of 10% and made a total profit of Rs 51000. If he bought both colors of shirts, then the maximum possible total number of shirts that he could have bought is

(A) 395

(B) 407

(C) 413

(D) None of Above

Correct Answer: (B) 407

Solution:

Let's assume the following:

- **Number of blue shirts:** n
- **Number of white shirts:** m
- **Total number of shirts:** $m + n$

Therefore, the total cost of the shirts is:

$$1000m + 1125n$$

The average cost of a shirt is:

$$\frac{1000m+1125n}{m+n}$$

It is mentioned that the market price is set to be 25% higher than the average cost of all the shirts. Therefore, the selling price is:

$$\frac{1000m+1125n}{m+n} \times \frac{5}{4} \times \frac{9}{10}$$

After simplifying, the average selling price of the shirts becomes:

$$\frac{9}{8} \times \frac{1000m+1125n}{m+n}$$

The average profit per shirt is:

$$\frac{1}{8} \times \frac{1000m+1125n}{m+n}$$

The total profit for all the shirts is:

$$\frac{1}{8} \times \frac{1000m+1125n}{m+n} \times (m+n)$$

Which simplifies to:

$\frac{1}{8}(1000m + 1125n)$

The total profit is given as 51,000, so:

$\frac{1}{8}(1000m + 1125n) = 51000$

Multiplying both sides by 8:

$1000m + 1125n = 408000$

To maximize the total number of shirts, we need to minimize the value of n, which can't be zero. Therefore, we need to maximize m.

Rearranging the equation for m:

$m = \frac{408000 - 1125n}{1000}$

Now, we maximize m by minimizing n, and after inspection, we find that the maximum value for m is 399 and n is 8.

The total number of shirts is:

$m + n = 399 + 8 = 407$

Conclusion:

The maximum number of shirts is $\boxed{407}$.

The correct option is (B): 407.

17. A triangle is drawn with its vertices on the circle C such that one of its sides is a diameter of C and the other two sides have their lengths in the ratio a:b. If the radius of the circle is r, then the area of the triangle is

(A) $\frac{2abr^2}{a^2+b^2}$

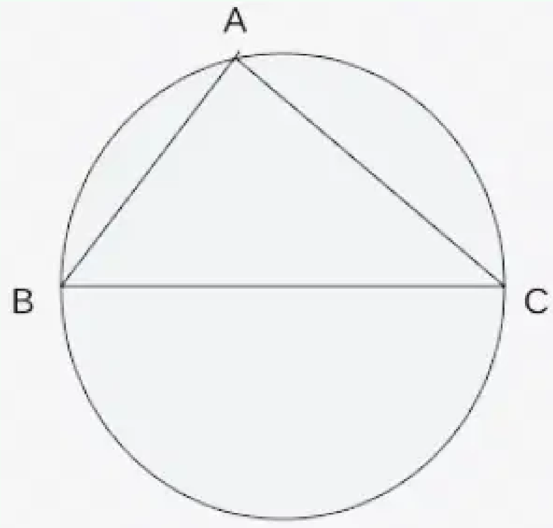
(B) $\frac{abr^2}{a^2+b^2}$

(C) $\frac{abr^2}{2(a^2+b^2)}$

(D) $\frac{4abr^2}{a^2+b^2}$

Correct Answer: (A) $\frac{2abr^2}{a^2+b^2}$

Solution:



Given: $\angle BAC = 90°$ because BC is the diameter of the circle.

Let $AB = a$ cm and $AC = b$ cm.

Since $\angle BAC = 90°$, triangle ABC is a right-angled triangle, and we can apply the Pythagoras Theorem:

$$BC = \sqrt{a^2 + b^2}$$

Since BC is the diameter of the circle, its length is $2r$, where r is the radius of the circle. So,

$$2r = \sqrt{a^2 + b^2} \Rightarrow 4r^2 = a^2 + b^2$$

Area of triangle ABC is:

$$\text{Area} = \frac{1}{2} \times a \times b$$

Now, we multiply and divide this expression by $a^2 + b^2$:

$$\text{Area} = \frac{ab}{2(a^2 + b^2)} \times (a^2 + b^2)$$

Substitute $a^2 + b^2 = 4r^2$:

$$= \frac{ab}{2(a^2 + b^2)} \times 4r^2 = \frac{ab}{(a^2 + b^2)} \times 2r^2$$

Therefore,

$$\text{Area} = \frac{2abr^2}{a^2 + b^2}$$

So, the correct option is (A): $\boxed{\frac{2abr^2}{a^2 + b^2}}$

18. In a rectangle ABCD, AB = 9 cm and BC = 6 cm. P and Q are two points on BC such that the areas of the figures ABP, APQ, and AQCD are in geometric progression. If the area of the figure AQCD is four times the area of triangle ABP, then BP : PQ : QC is

(A) 1 : 1 : 2

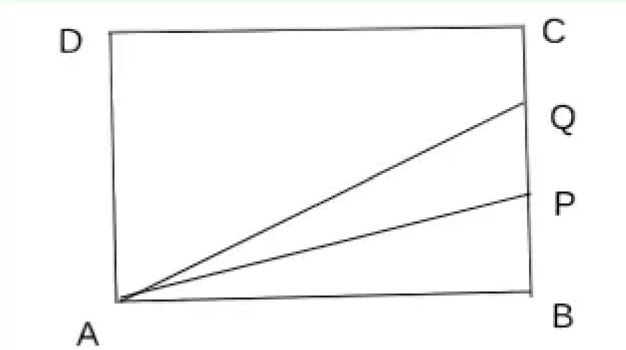
(B) 1 : 2 : 1

(C) 1 : 2 : 4

(D) 2 : 4 : 1

Correct Answer: (D) 2 : 4 : 1

Solution:



Given:

AB = 9cm, BC = 6cm

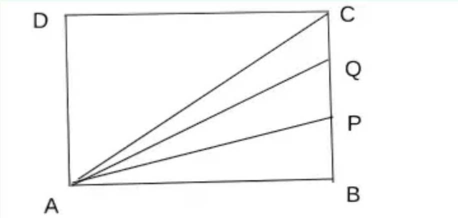
We are also given that the areas of the ABP, APQ and AQCD are in geometric progression.

Therefore, it can be assumed as:

Area of ABP, APQ and AQCD as k, 2k and 4k respectively.

As per the question, the ratio of BP, PQ and QC will be the ratio of the respective triangles.

So, we can draw a line from Point A to C.



Let the area of $\triangle AQC$ be x. The area of $\triangle ADC$ is given by:

$\triangle ADC = ADQC = AQC = 4k - x$

This is equal to the sum of the areas of triangles $\triangle APB$, $\triangle AQP$, and $\triangle ACQ$, which can be expressed as:

$4k - x = 3k + x$

Simplifying this equation:

$4k - x = 3k + x \Rightarrow x = \frac{k}{2}$

Now, let's calculate the ratio of $BP : PQ : CQ$:

$BP : PQ : CQ = k : 2k : \frac{k}{2}$

By simplifying the ratio:

$= 2 : 4 : 1$

Therefore, the correct option is:

(D): 2 : 4 : 1

19. The area of the quadrilateral bounded by the Y -axis, the line $x = 5$, and the lines $|x - y| - |x - 5| = 2$, is

(A) 45

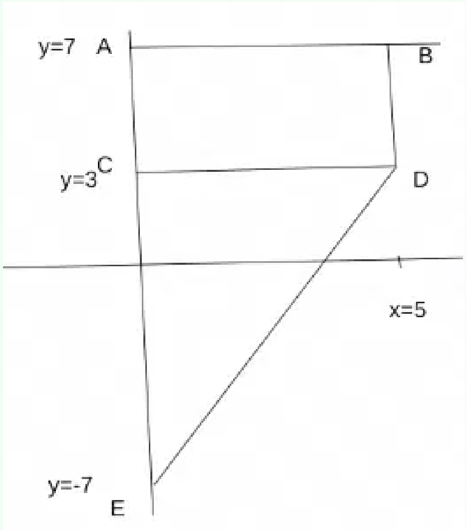
(B) 55

(C) 60

(D) None of Above

Correct Answer: (A) 45

Solution:



To find: Area of quadrilateral $ABDE$

Approach: The quadrilateral $ABDE$ is made up of:

- Rectangle $ABCD$
- Triangle $\triangle CDE$

So, Area of $ABDE$ = Area of rectangle $ABCD$ + Area of triangle $\triangle CDE$

Step 1: Area of rectangle *ABCD*

Length $= 7 - 3 = 4$ units

Breadth $=$ 5 units

Area $=$ Length $\times$ Breadth $= 4 \times 5 = 20$ square units

Step 2: Area of triangle $\triangle CDE$

Given: Base $=$ 10 units, Height $=$ 5 units

Area $= \frac{1}{2} \times$ Base $\times$ Height $= \frac{1}{2} \times 10 \times 5 = 25$ square units

Step 3: Total Area of Quadrilateral *ABDE*

$= 20 + 25 =$ **45** square units

Final Answer: (A) 45

20. If $p^2 + q^2 - 29 = 2pq - 20 = 52 - 2pq$, then the difference between the maximum and minimum possible value of $(p^3 - q^3)$is

(A) 486

(B) 378

(C) 243

(D) 189

Correct Answer: (B) 378

Solution:

Given:

$2pq - 20 = 52 - 2pq$

$\Rightarrow 2pq + 2pq = 52 + 20$

$\Rightarrow 4pq = 72$

$\Rightarrow pq = 18$ ...... **(1)**

Now consider:

$$p^2 + q^2 - 29 = 2pq - 20$$
$$\Rightarrow p^2 + q^2 - 2pq = 9$$
$$\Rightarrow (p - q)^2 = 9$$
$$\Rightarrow p - q = \pm 3$$

From (1): $pq = 18$

Again, using the same identity:
$$p^2 + q^2 = 2pq + 9$$
$$\Rightarrow p^2 + q^2 = 2(18) + 9 = 36 + 9 = 45$$

Now, using the identity:
$$p^3 - q^3 = (p - q)(p^2 + pq + q^2)$$

From above:
$p^2 + q^2 = 45$, and $pq = 18$
So,
$$p^2 + pq + q^2 = 45 + 18 = 63$$

Case 1: $p - q = 3$

$$p^3 - q^3 = 3 \cdot 63 = 189$$

Case 2: $p - q = -3$

$$p^3 - q^3 = (-3) \cdot 63 = -189$$

Difference between the two values:

$189 - (-189) = 189 + 189 = \mathbf{378}$

Final Answer: (B) 378

21. Let both the series $a_1, a_2, a_3, ...$ and $b_1, b_2, b_3, ...$ be in arithmetic progression such that the common differences of both the series are prime numbers. If $a_5 = b_9, a_{19} = b_{19}$ and $b_2 = 0$, then a_{11} equals

(A) 79

(B) 83

(C) 84

(D) 86

Correct Answer: (A) 79

Solution:

Let the first term of both series be a1 and b1, respectively, and the common differences be d1 and d2, respectively.

It is given that $a_5 = b_9$, which implies

$a_1 + 4d_1 = b_1 + 8d_2$:

$a_1 - b_1 = 8d_2 - 4d_1$--------(1)

Similarly, it is known that $a_{19} = b_{19}$, which implies

$a_1 + 18d_1 = b_1 + 18d_2$:

$a_1 - b_1 = 18d_2 - 18d_1$----------(2)

Equating (1) and (2), we get:

$18d_2 - 18d_1 = 8d_2 - 4d_1$

$10d_2 = 14d_1$

$5d_2 = 7d_1$

Since d_1 and d_2 are prime numbers, this implies $d_1 = 5$ and $d_2 = 7$.

It is also known that $b_2 = 0$, which implies $b_1 + d_2 = 0 \Rightarrow b_1 = -d_2 = -7$.

Putting the values of b_1, d_1, and d_2 in Eq(1), we get:

$a_1 = 8d_2 - 4d_1 + b_1 = 56 - 20 - 7 = 29$

Hence,

$a_{11} = a_1 + 10d_1 = 29 + 10 \times 5 = 29 + 50 = 79$

Therefore, $a_{11} = 79$.

22. Let a_n and b_n be two sequences such that $a_n = 13 + 6(n-1)$ and $b_n = 15 + 7(n-1)$ for all natural numbers n . Then, the largest three digit integer that is common to both these sequences, is

(A) 937

(B) 1037

(C) 967

(D) None of Above

Correct Answer: (C) 967

Solution:

Given:

$a_n = 13 + 6(n-1)$

$\Rightarrow a_n = 13 + 6n - 6 = 7 + 6n$

Similarly, $b_n = 15 + 7(n-1)$

$\Rightarrow b_n = 15 + 7n - 7 = 8 + 7n$

The common differences are 6 and 7 respectively.

LCM of 6 and 7 is $\text{LCM}(6,7) = 42$

The first common term is found by inspection to be 43.

So, we form a new AP starting at 43 with common difference 42:

$t_m = 43 + (m-1) \cdot 42$

We need the largest term less than 1000:

$43 + (m-1) \cdot 42 < 1000$

$\Rightarrow (m-1) \cdot 42 < 957$

$\Rightarrow m - 1 < \frac{957}{42} \approx 22.78$

$\Rightarrow m = 23$

Therefore, the 23rd term is:

$t_{23} = 43 + (23 - 1) \cdot 42 = 43 + 22 \cdot 42 = 967$

Correct option: (C) 967