# CAT 2022 QA Slot 1 Question Paper with Solutions

## General Instructions

(i) This booklet contains 22 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 13 single-correct multiple-choice questions and 9 numerical / type-in-the-answer questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For numerical questions, report the answer rounded exactly as asked.

**1.** Pinky is standing in a queue at a ticket counter. Suppose the ratio of the number of persons standing ahead of Pinky to the number of persons standing behind her in the queue is 3:5. If the total number of persons in the queue is less than 300, then the maximum possible number of persons standing ahead of Pinky is

**Correct Answer:** —

**Solution:**

# 1. Let the Variables Represent

Let:

- Number of persons standing ahead of Pinky $= 3x$
- Number of persons standing behind Pinky $= 5x$
- Here, $x$ is a positive integer

The ratio is:

$$\frac{3x}{5x} = \frac{3}{5}$$

## 2. Total Persons in the Queue

Total persons = Persons ahead + Pinky herself + Persons behind

$$3x + 1 + 5x = 8x + 1$$

Given: Total is less than 300

$$8x + 1 < 300$$

Subtract 1:

$$8x < 299$$

Divide by 8:

$$x < \frac{299}{8} \Rightarrow x < 37.375$$

Maximum possible integer value of $x$ is 37

## 3. Calculate Maximum Number Ahead of Pinky

$$\text{Maximum } 3x = 3 \times 37 = 111$$

**Therefore, the maximum number of persons ahead of Pinky is: 111**

**2.** The largest real value of a for which the equation$|x + a| + |x - 1| = 2$ has an infinite number of solutions for x is

(A) -1

(B) 0

(C) 1

(D) 2

**Correct Answer:** (C) 1

**Solution:**

To find the largest real value of $a$ for which the equation $|x + a| + |x - 1| = 2$ has an infinite number of solutions for $x$, we need to analyze the possible cases when the absolute value expressions change their signs.

**Case 1:** When both expressions are positive:

$x + a + x - 1 = 2$

$2x + a - 1 = 2$

$2x + a = 3$

**Case 2:** When the first expression is positive and the second is negative:

$x + a - (x - 1) = 2$

$x + a - x + 1 = 2$

$a + 1 = 2$

$a = 1$

**Case 3:** When both expressions are negative:

$-(x + a) - (x - 1) = 2$

$-x - a - x + 1 = 2$

$-2x - a + 1 = 2$

$-2x - a = 1$

**Case4:** When the first expression is negative and the second is positive:

$-(x+a)+(x-1)=2$

$-x-a+x-1=2$

$-a-1=2$

$a=-3$

Now, let's analyze the critical points:

1. When $a>1$, we have the solution $a=3$, but it does not satisfy the condition that both expressions should be positive.
2. When $a=1$, we have the solution $a=1$, which satisfies the condition for both expressions to be positive.
3. When $a<1$, we have the solution $a=-3$, but it does not satisfy the condition that both expressions should be positive.

Hence, the largest real value of "a" for which the equation has an infinite number of solutions is $a=1$.

So, the correct option is (C): 1

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**3.** The average of three integers is 13. When a natural number n is included, the average of these four integers remains an odd integer. The minimum possible value of n is

(A) 3

(B) 4

(C) 5

(D) 1

**Correct Answer:** (C) 5

**Solution:**

## 1. Given

Let the three integers be $A, B, C$, and let the natural number to be added be $n$.

Given average of the three numbers:

$$\frac{A+B+C}{3} = 13 \Rightarrow A+B+C = 39$$

## 2. Condition After Adding n

New average becomes:

$$\frac{A+B+C+n}{4}$$

which must be an odd integer.

Let an odd integer be represented as $2k+1$. Then:

$$\frac{39+n}{4} = 2k+1$$

Multiply both sides by 4:

$$39+n = 4(2k+1) = 8k+4 \Rightarrow n = 8k+4-39 = 8k-35$$

## 3. Find Minimum Natural Number n

We want $n > 0$. Try increasing integer values of $k$:

- For $k = 0$: $n = -35$ ❌
- For $k = 1$: $n = -27$ ❌
- For $k = 2$: $n = -19$ ❌
- For $k = 3$: $n = -11$ ❌
- For $k = 4$: $n = -3$ ❌

- For $k = 5$: $n = 5$ ✅

So, the smallest natural number $n$ such that the new average is odd is:

$$\boxed{5}$$

## 4. Final Answer

**Correct Option: C. 5**

**4.** Let $A$ be the largest positive integer that divides all the numbers of form $3^k + 4^k + 5^k$, and $B$ be the largest positive integer that divides all the numbers of the form $4^k + 3(4^k) + 4^{k+2}$, where k is any positive integer. Then $(A + B)$ equals

**Correct Answer:** —

**Solution:**

### Step 1: Values of the form $3^k + 4^k + 5^k$

Let's compute for different values of $k$:

- For $k = 1$: $3^1 + 4^1 + 5^1 = 3 + 4 + 5 = 12$
- For $k = 2$: $3^2 + 4^2 + 5^2 = 9 + 16 + 25 = 50$
- For $k = 3$: $3^3 + 4^3 + 5^3 = 27 + 64 + 125 = 216$

Now compute the HCF of these results:

$$\gcd(12, 50, 216) = 2$$

So, $A = 2$

### Step 2: Values of the form $4^k + 3(4^k) + 4^{k+2}$

We simplify:

$4^k + 3(4^k) + 4^{k+2} = 4^k(1+3) + 4^{k+2} = 4^{k+1} + 4^{k+2} = 4^{k+1}(1+4) = 5 \cdot 4^{k+1}$

Evaluate for different values of $k$:

- For $k = 1$: $B = 5 \cdot 4^2 = 5 \cdot 16 = 80$
- For $k = 2$: $B = 5 \cdot 4^3 = 5 \cdot 64 = 320$
- For $k = 3$: $B = 5 \cdot 4^4 = 5 \cdot 256 = 1280$

Now compute the HCF:

$$\gcd(80, 320, 1280) = 80$$

So, $B = 80$

### Final Step: Sum of A and B

$$A + B = 2 + 80 = \boxed{82}$$

### Answer:

$\boxed{82}$

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**5.** In a village, the ratio of number of males to females is 5:4. The ratio of number of literate males to literate females is 2:3. The ratio of the number of illiterate males to illiterate females is 4:3. If 3600 males in the village are literate, then the total number of females in the village is

**Correct Answer:** —

**Solution:**

**Given:**

- Ratio of males to females in the village is **5:4**
- Ratio of literate males to literate females is **2:3**
- Ratio of illiterate males to illiterate females is **4:3**
- Number of literate males = **3600**

**Step 1:** Let the number of males and females in the village be:

- Males $= 5x$
- Females $= 4x$

**Step 2:** Literate males and females are in the ratio $2:3$.

Let the common multiple be $y$, then:

- Literate males $= 2y$
- Literate females $= 3y$

**Step 3:** Given that literate males = 3600

So, $2y = 3600 \Rightarrow y = 1800$

Thus:

- Literate females $= 3 \times 1800 = 5400$

**Step 4:** Now consider illiterate males and females in the ratio $4:3$.

Let the common multiple be $z$, then:

- Illiterate males $= 4z$
- Illiterate females $= 3z$

**Step 5:** Total males = literate males + illiterate males

$5x = 2y + 4z = 3600 + 4z$

**Step 6:** Total females = literate females + illiterate females

$4x = 3y + 3z = 5400 + 3z$

**Now, solve for $x$ using either equation:**

From Step 5: $5x = 3600 + 4z \Rightarrow x = \frac{3600+4z}{5}$

From Step 6: $4x = 5400 + 3z \Rightarrow x = \frac{5400+3z}{4}$

Equating both expressions for $x$:

$\frac{3600+4z}{5} = \frac{5400+3z}{4}$

Cross-multiply:

$4(3600 + 4z) = 5(5400 + 3z)$

$14400 + 16z = 27000 + 15z$

$16z - 15z = 27000 - 14400$

$z = 12600$

**Now find $x$:**

$x = \frac{3600+4\times12600}{5} = \frac{3600+50400}{5} = \frac{54000}{5} = 10800$

**Total number of females** $= 4x = 4 \times 10800 = \boxed{43200}$

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**6.** Let ABCD be a parallelogram such that the coordinates of its three vertices A, B, C are (1, 1), (3, 4), and (−2, 8), respectively. Then, the coordinates of the vertex D are

(A) (−4, 5)

(B) (4, 5)

(C) (−3, 4)

(D) (0, 11)

**Correct Answer:** (A) (−4, 5)

**Solution:**

# Given

Parallelogram ABCD with vertices:

- $A = (1, 1)$
- $B = (3, 4)$
- $C = (-2, 8)$
- $D = (x, y)$ — to be found

In a parallelogram, diagonals bisect each other.

# Step 1: Use Midpoint of Diagonal AC

Find the midpoint of diagonal AC:

$$\text{Midpoint}_{AC} = \left(\frac{1 + (-2)}{2}, \frac{1 + 8}{2}\right) = \left(\frac{-1}{2}, \frac{9}{2}\right)$$

# Step 2: Let Point D Be $(x, y)$

Midpoint of diagonal BD is:

$$\text{Midpoint}_{BD} = \left(\frac{3 + x}{2}, \frac{4 + y}{2}\right)$$

Since diagonals bisect each other:

$$\frac{3 + x}{2} = \frac{-1}{2} \quad \text{and} \quad \frac{4 + y}{2} = \frac{9}{2}$$

## Step 3: Solve the Equations

First equation:

$$\frac{3+x}{2} = \frac{-1}{2} \Rightarrow 3+x=-1 \Rightarrow x=-4$$

Second equation:

$$\frac{4+y}{2} = \frac{9}{2} \Rightarrow 4+y=9 \Rightarrow y=5$$

**Final Answer**

Therefore, the coordinates of point $D$ are:

$$\boxed{(-4, 5)}$$

**7.** Alex invested his savings in two parts. The simple interest earned on the first part at 15% per annum for 4 years is the same as the simple interest earned on the second part at 12% per annum for 3 years. Then, the percentage of his savings invested in the first part is

(A) 62.50%

(B) 37.50%

(C) 60%

(D) 40%

**Correct Answer:** (B) 37.50%

**Solution:**

The correct answer is B: 37.50%

Alex divided his savings into two parts: one invested at a 15% annual interest rate for 4 years, and the other invested at a 12% annual interest rate for 3 years.

Let's denote the amount invested in the first part as ₹x and in the second part as ₹y.

The interest earned from the first part is calculated as $0.15 \times x \times 4$, and the interest earned from the second part is calculated as $0.12 \times y \times 3$.

Equating these two interests:

$0.15 \times x \times 4 = 0.12 \times y \times 3$

Solving for x and y:

20x=12y

This implies the ratio of x to y is 3:5.

Therefore, the percentage of savings invested in the first part is $\frac{3}{(3+5)} = \frac{3}{8} = 0.375$.

Converting this to a percentage gives us 37.5%.

So, Alex invested 37.5% of his savings in the first part.

**8.** The average weight of students in a class increases by 600 gm when some new students join the class. If the average weight of the new students is 3 kg more than the average weight of the original students, then the ratio of the number of original students to the number of new students is

(A) 1:2

(B) 3:1

(C) 1:4

(D) 4:1

**Correct Answer:** (D) 4:1

**Solution:**

Let's denote the following variables:

- $n$: number of original students
- $m$: number of new students
- $W$: average weight of the original students (in kg)
- $W + 3$: average weight of the new students (in kg)

Initially, the total weight of the original students is $nW$. When new students join, the new average weight becomes $W + 0.6$ kg. The total weight equation for both original and new students is:

$(n + m)(W + 0.6) = nW + m(W + 3)$

Expanding and simplifying:

$nW + mW + 0.6n + 0.6m = nW + mW + 3m$

Cancel out the common terms $nW + mW$:

$0.6n + 0.6m = 3m$

Simplify to find a relationship between $n$ and $m$:

$0.6n = 3m - 0.6m$

$0.6n = 2.4m$

$n = \frac{2.4m}{0.6}$

$n = 4m$

This shows that the ratio of the number of original students to the number of new students is $\frac{n}{m} = \frac{4}{1}$.

| Original Students | New Students | Ratio |
|---|---|---|
| 4 | 1 | 4:1 |

**9.** A mixture contains lemon juice and sugar syrup in equal proportion. If a new mixture is created by adding this mixture and sugar syrup in the ratio 1 : 3, then the ratio of lemon juice and sugar syrup in the new mixture is

(A) 1:6

(B) 1:4

(C) 1:5

(D) 1:7

**Correct Answer:** (D) 1:7

**Solution:**

An initial mixture contains lemon juice and sugar syrup in equal proportion. A new mixture is formed by mixing the initial mixture with pure sugar syrup in the ratio 1:3. Find the final ratio of lemon juice to sugar syrup in the new mixture.

## Step 1: Initial Mixture Composition

Since the mixture contains lemon juice and sugar syrup in equal proportions:

$$\text{Lemon Juice} = \frac{1}{2}, \quad \text{Sugar Syrup} = \frac{1}{2}$$

## Step 2: Creating the New Mixture

The new mixture combines:

- 1 part of initial mixture
- 3 parts of pure sugar syrup

Total parts $= 1 + 3 = 4$

## Step 3: Sugar Syrup in the New Mixture

Sugar syrup from initial mixture:

$$\frac{1}{2} \times 1 = \frac{1}{2}$$

Sugar syrup from 3 parts of pure syrup:

$$1 \times 3 = 3$$

Total sugar syrup:

$$\frac{1}{2} + 3 = \frac{7}{2}$$

## Step 4: Lemon Juice in the New Mixture

Lemon juice comes only from the initial mixture:

$$\frac{1}{2} \times 1 = \frac{1}{2}$$

## Step 5: Final Ratio

Lemon Juice : Sugar Syrup =

$$\frac{1}{2} : \frac{7}{2} \Rightarrow 1 : 7 \quad \text{(after multiplying both terms by 2)}$$

## Final Answer:

$$\boxed{1 : 7}$$

**Correct Option: (D)**

**10.** Amal buys 110kg of syrup and 120kg of juice, syrup being 20% less costly than juice, per kg. He sells 10kg of syrup at 10% profit and 20kg of juice at 20% profit. Mixing the remaining juice and syrup, Amal sells the mixture at ₹ 308.32 per kg and makes an overall profit of 64%. Then, Amal's cost price for syrup, in rupees per kg, is

**Correct Answer:** —

**Solution:**

**The correct answer is ₹160:**

**Step 1:** Given Information

Amal purchases 110 kg of syrup and 120 kg of juice.

The cost price of syrup is 20% less than the cost price of juice.

**Step 2:** Cost Price of Syrup and Juice

Let the cost price of 1 kg of juice be $10\,\text{CP}$.

Therefore, the cost price of 1 kg of syrup is $8\,\text{CP}$ (20% less than juice).

**Step 3:** Selling of Syrup and Juice

Amal sells 10 kg of syrup at 10% profit:

Selling price of 10 kg syrup $= 1.1 \times 8\,\text{CP} = 8.8\,\text{CP}$

Amal sells 20 kg of juice at 20% profit:

Selling price of 20 kg juice $= 1.2 \times 10\,\text{CP} = 12\,\text{CP}$

**Step 4:** Selling the Mixture

Amal combines the remaining syrup and juice and sells the mixture at ₹308.32 per kg.

Total selling price of the mixture =

$308.32 \times (110 + 120) = 308.32 \times 230 = 70,912$

**Step 5:** Calculating Total Cost and Profit

Total cost price = Cost of syrup + Cost of juice

$= 110 \times 8\,\text{CP} + 120 \times 10\,\text{CP} = 880\,\text{CP} + 1200\,\text{CP} = 2080\,\text{CP}$

Overall profit = 64%

So, total selling price = $1.64 \times 2080\,\text{CP} = 3411.2\,\text{CP}$

Already sold 10 kg syrup and 20 kg juice for:

$10 \times 8.8 + 20 \times 12 = 88 + 240 = 328$

So, mixture sold for = $3411.2 - 328 = 3083.2$

Amount of mixture = $230 - 10 - 20 = 200$ kg

So, price per kg = $\frac{3083.2}{200} = 15.416 =$₹ 308.32 (scaling CP to ₹)

**Step 6:** Finding the Cost Price of 1 CP

We equate ₹308.32 with 15.416 CP, so ₹1 = $\frac{308.32}{15.416} = 20$

Thus, **1 CP = ₹20**

**Step 7:** Cost Price of Syrup

Cost price of syrup per kg = $8\times$ ₹ 20 =₹ 160

**Final Answer: ₹160 per kg**

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**11.** A trapezium ABCD has side AD parallel to BC, ∠BAD=90°, BC=3 cm, and AD=8 cm. If the perimeter of this trapezium is 36 cm, then its area, in sq. cm, is

**Correct Answer:** —

**Solution:**

The correct answer is **66**:

Given the information:
- Side BC = 3 cm
- Side AD = 8 cm
- Perimeter of the trapezium = 36 cm

Let the other two sides of the trapezium be AB and CD. Since AD is parallel to BC, we have AB = CD.

The perimeter of a trapezium is given by the sum of all its four sides:
Perimeter = BC + CD + DA + AB
Given: BC = 3 cm and AD = 8 cm

Substituting:
$36 = 3 + CD + 8 + AB$
$AB + CD = 36 - 11 = 25$
Since $AB = CD$:
$2AB = 25 \Rightarrow AB = CD = \frac{25}{2} = 12.5\,\text{cm}$

Now, use Pythagoras theorem (since $\angle BAD = 90°$) to find the height:
$BD^2 = AB^2 - AD^2 = (12.5)^2 - (8)^2 = 156.25 - 64 = 92.25$
$BD = \sqrt{92.25} \approx 9.61\,\text{cm}$

Area of the trapezium:
$\text{Area} = \frac{1}{2} \times (\text{AB} + \text{CD}) \times \text{height}$
$\text{Area} = \frac{1}{2} \times (12.5 + 12.5) \times 9.61 = \frac{1}{2} \times 25 \times 9.61 \approx 66\,\text{cm}^2$

**Hence, the area of the trapezium is approximately 66 cm$^2$.**

**12.** All the vertices of a rectangle lie on a circle of radius R. If the perimeter of the rectangle is P, then the area of the rectangle is

(A) $\frac{P^2}{2} - 2PR$

(B) $\frac{P^2}{8} - 2R^2$

(C) $\frac{P^2}{16} - R^2$

(D) $\frac{P^2}{8} - \frac{R^2}{2}$

**Correct Answer:** (B) $\frac{P^2}{8} - 2R^2$

**Solution:**

To find the area of a rectangle inscribed in a circle (i.e., with all vertices on the circle), we need to consider the properties of such a rectangle. If we denote the rectangle's side lengths as $a$ and $b$, the diagonals are equal to the circle's diameter, which is $2R$. Thus, using the Pythagorean theorem: $a^2 + b^2 = (2R)^2 = 4R^2$. Additionally, the perimeter $P$ of the rectangle is given by: $P = 2(a + b)$.

Using the perimeter expression, we have:

$$a + b = \frac{P}{2}$$

We are asked to find the area $A$ of the rectangle, which can be expressed as:

$$A = a \times b$$

We can square the sum expression $a + b$ and use the identity:

$$(a + b)^2 = a^2 + b^2 + 2ab$$

Substituting the known values:

$$\left(\frac{P}{2}\right)^2 = 4R^2 + 2ab$$

Solve for $ab$:

$$2ab = \frac{P^2}{4} - 4R^2$$

$$ab = \frac{P^2}{8} - 2R^2$$

Thus, the area of the rectangle is:

$$A = \frac{P^2}{8} - 2R^2$$

This confirms the correct answer is: $\frac{P^2}{8} - 2R^2$

**13.** Let a,b,c be non-zero real numbers such that $b^2 < 4ac$, and $f(x) = ax^2 + bx + c$. If the set S consists of all integers m such that $f(m) < 0$, then the set S must necessarily be

(A) the set of all integers

(B) either the empty set or the set of all integers

(C) the empty set the set of all positive integers

(D) the set of all positive integers

**Correct Answer:** (B) either the empty set or the set of all integers

**Solution:**

Given the quadratic function

$$f(x) = ax^2 + bx + c$$

where

$$a, b,$$

and

$$c$$

are non-zero real numbers such that

$$b^2 < 4ac$$

, we want to find the set

$$S$$

of all integers

$$m$$

such that

$$f(m) < 0$$

.

The quadratic function

$$f(x)$$

represents a parabola. The discriminant

$$\Delta$$

of the quadratic equation

$$ax^2 + bx + c = 0$$

is given by

$$\Delta = b^2 - 4ac$$

.
Since

$$b^2 < 4ac$$

, the discriminant is negative (

$$\Delta < 0$$

), which means that the quadratic equation has two distinct complex roots.
The vertex of the parabola is given by the point

$$(h, k)$$

where

$$h = -\frac{b}{2a}$$

and

$$k = f(h)$$

.
In this case, since

$$a$$

is non-zero, the parabola opens upwards if

$$a > 0$$

and downwards if

$$a < 0$$

.
Since the parabola opens upwards or downwards and the discriminant is negative, the parabola does not intersect the x-axis. This implies that the function

$$f(x)$$

is either entirely above the x-axis (if

$$a > 0$$

) or entirely below the x-axis (if

$$a < 0$$

).
Now, we want to find the set

$$S$$

of all integers

$$m$$

such that

$$f(m) < 0$$

.
Depending on the sign of

$$a$$

, the parabola is either above or below the x-axis.
If the parabola is above the x-axis, there will be no integer

$$m$$

for which

$$f(m) < 0\,(f(m)$$

will always be positive or zero).
If the parabola is below the x-axis, then

$$f(m) < 0$$

for all integers

$$m$$

.
Therefore, the set

$$S$$

must be either the empty set (if

$$a > 0$$

and the parabola opens upwards) or the set of all integers (if

$$a < 0$$

and the parabola opens downwards), depending on the sign of

$$a$$

.
Hence, the correct answer is: either the empty set or the set of all integers.

**14.** Let a and b be natural numbers. If $a^2 + ab + a = 14$ and $b^2 + ab + b = 28$, then (2a+b) equals

(A) 7

(B) 10

(C) 9

(D) 8

**Correct Answer:** (D) 8

**Solution:**

To solve the problem, we need to find the values of $a$ and $b$ that satisfy the given equations and then compute $2a + b$.

We have the following two equations:

$a^2 + ab + a = 14$ (Equation 1)

$b^2 + ab + b = 28$ (Equation 2)

Let's rewrite Equation 1:

$$a(a + b + 1) = 14$$

Since $a$ is a natural number and the factors of 14 are 1, 2, 7, and 14, we will test these values:

1. If $a = 1$, then the equation becomes $1(b + 2) = 14$ which gives $b + 2 = 14$, so $b = 12$.

2. If $a = 2$, then the equation becomes $2(b + 3) = 14$ which gives $b + 3 = 7$, so $b = 4$.

3. If $a = 7$, then the equation becomes $7(b + 8) = 14$, which is not possible since $b$ must be a natural number.

4. If $a = 14$, then the equation becomes $14(b + 15) = 14$, which is not possible as $b + 15$ cannot be 1.

Now, let's check these values by substituting into Equation 2:

Substitute $a = 2, b = 4$ into Equation 2:

$$b^2 + ab + b = 28$$

$$4^2 + 2 \times 4 + 4 = 16 + 8 + 4 = 28$$

This solution satisfies both equations, so $a = 2$ and $b = 4$.

Finally, calculate $2a + b$:

$$2a + b = 2(2) + 4 = 4 + 4 = 8$$

The answer is 8.

**15.** In a class of 100 students, 73 like coffee, 80 like tea, and 52 like lemonade. It may be possible that some students do not like any of these three drinks. Then the difference between the maximum and minimum possible number of students who like all the three drinks is

(A) 48

(B) 53

(C) 47

(D) 52

**Correct Answer:** (C) 47

**Solution:**

**Step 1**: Define the variables

Let (n),(s),(d),and (t) represent the number of students who like none, exactly one, exactly two, and all three drinks respectively.

**Step 2:** Write the equations

We have two equations based on the information given:

Equation $1:(n+s+d+t=100)$

Equation $2:(s+2d+3t=205)$

**Step 3**: Express (d) in terms of (n) and (t)

Subtract Equation 1 from Equation 2 to eliminate (s):

$[s+2d+3t-(n+s+d+t)=205-100]$

Simplify:

$[d+2t-n=105]$

This equation relates the number of students who like exactly two drinks (d) with the number of students who like none (n) and all three (t)

**Step 4**: Find maximum and minimum values of (t)

We know that (0 leq n,s,d,t leq 100) and $(n+s+d+t=100)$.

To find the maximum and minimum possible values of (t), we consider extreme cases:

**a) Maximum:** Let $(t=52)$ (all students like all three drinks).

Solve Equation 3 for (d):

$[d+2(52)-n=105 \text{ implies } d-n=1]$

Since (d) and (n) should be non-negative, the only possible solution is $(d=1)$ and $(n=0)$.

**b) Minimum:** Let $(t=5)$ (only a few students like all three drinks).

Solve Equation 3 for (d):

$[d+2(5)-n=105 \text{ implies } d-n=95]$

Again, the only possible solution is $(d=95)\text{and}(n=0)$.

**Step 5**: Calculate the difference

The difference between the maximum and minimum possible values of (t) is:

$[47=52-5]$

Therefore, the difference between the maximum and minimum number of students who like all three drinks is **47.**

**16.** Trains A and B start traveling at the same time towards each other with constant speeds from stations X and Y, respectively. Train A reaches station Y in 10 minutes while train B takes 9 minutes to reach station X after meeting train A. Then the total time taken, in minutes, by train B to travel from station Y to station X is

(A) 15

(B) 12

(C) 6

(D) 10

**Correct Answer:** (A) 15

**Solution:**

# 1. Given:

- Train A reaches station Y in 10 minutes after meeting Train B
- Train B takes 9 minutes to reach station X after meeting Train A
- Let the meeting point be M

# 2. Let t be the time (in minutes) each train took to reach point M

From the diagram:

- Train A: X $\rightarrow$ M in $t$ minutes, then M $\rightarrow$ Y in $10 - t$ minutes
- Train B: Y $\rightarrow$ M in $t$ minutes, then M $\rightarrow$ X in 9 minutes

Since the distances XM and MY are same for both trains (they meet at the same point), the ratio of times should be inverse of speeds:

$$\frac{t}{9} = \frac{10 - t}{t}$$

## 3. Solving the Equation

Cross-multiplying:

$$t^2 = 9(10 - t)$$

$$t^2 = 90 - 9t \Rightarrow t^2 + 9t - 90 = 0$$

Solving the quadratic:

$$(t + 15)(t - 6) = 0 \Rightarrow t = -15 \text{ or } t = 6$$

Since time cannot be negative, we take:

$$t = 6$$

## 4. Final Answer

Train B took:

$$\text{From Y to M: } t = 6 \text{ minutes}$$

$$\text{From M to X: 9 minutes}$$

$$\Rightarrow \text{Total time from Y to X} = 6 + 9 = \boxed{15} \text{ minutes}$$

**Correct Answer: 15 minutes**

**17.** Ankita buys 4kg cashews, 14kg peanuts, and 6kg almonds when the cost of 7kg cashews is the same as that of 30kg peanuts or 9kg almonds. She mixes all three nuts and marks a price for the mixture in order to make a profit of ₹1752. She sells 4kg of the mixture at this marked price and the remaining at a 20% discount on the marked price, thus making a total profit of ₹744. Then the amount, in rupees, that she had spent buying almonds is

(A) 1440

(B) 1176

(C) 1680

(D) 2520

**Correct Answer:** (C) 1680

**Solution:**

# 1. Given Quantities

Ankita has:

- 4 kg of cashews
- 14 kg of peanuts
- 6 kg of almonds

Total quantity $= 4 + 14 + 6 = 24$ kg

# 2. Selling Plan

She plans to earn a total profit of ₹1752.

Assume the average cost price of the mixture is $x$ rupees/kg.

So, marked price $= x + 73$ rupees/kg

## 3. Selling Distribution

Ankita sells:

- 4 kg at full marked price
- 20 kg at a 20% discount

Total Selling Price =

$$4(x + 73) + 0.8 \times 20(x + 73) = 4(x + 73) + 16(x + 73)$$

$$= 20(x + 73)$$

## 4. Profit Equation

Given total profit is ₹1752:

$$\text{Profit} = \text{Selling Price} - \text{Cost Price}$$

$$1752 = 20(x + 73) - \text{Cost Price}$$

$$\text{Cost Price} = 20x + 1460 - 1752 = 20x - 292$$

## 5. Given Price Ratio

We know:

$$7C = 30P = 9A$$

Let all equal ₹630k (a constant multiple), then:

$$C = \frac{630k}{7} = 90k, \quad P = \frac{630k}{30} = 21k, \quad A = \frac{630k}{9} = 70k$$

# 6. Total Cost Price in Terms of k

Total cost =

$$4C + 14P + 6A = 4(90k) + 14(21k) + 6(70k)$$

$$= 360k + 294k + 420k = 1074k$$

Given: Total cost = ₹4296

$$1074k = 4296 \Rightarrow k = \frac{4296}{1074} = 4$$

# 7. Amount Spent on Almonds

Since $A = 70k = 70 \times 4 =$₹ 280, and Ankita had 6 kg of almonds:

$$\text{Total almond cost} = 6 \times 280 = ₹\ 1680$$

**Answer:** ₹1680

**18.** For natural numbers x,y, and z, if xy+yz=19 and yz+xz=51, then the minimum possible value of xyz is

**Correct Answer:** —

**Solution:**

# 1. Given Equations

We are given:

$$y(x+z) = 19 \tag{1}$$

$$z(x+y) = 51 \tag{2}$$

# 2. Deduction from Equation (1)

From equation (1):

$$y(x+z) = 19$$

Since 19 is a prime number, and $y$ is a positive integer, the only integer values possible for $y$ are 1 and 19.
If $y = 1$, then:

$$x + z = 19$$

Try this value in equation (2):

$$z(x+1) = 51 \tag{3}$$

From equation (1), we now explore pairs $(x, z)$ such that $x + z = 19$

# 3. Exploring Cases

**Case 1:** $z = 3 \Rightarrow x = 16$

Plug into equation (3):

$$z(x+1) = 3 \times (16+1) = 3 \times 17 = 51 \quad \checkmark \text{ Satisfies}$$

So,

$$xyz = x \cdot y \cdot z = 16 \cdot 1 \cdot 3 = 48$$

**Case 2:** $z = 17 \Rightarrow x = 2$

Check equation (3):

$$z(x+1) = 17 \cdot (2+1) = 17 \cdot 3 = 51 \quad \checkmark \text{ Satisfies}$$

So,

$$xyz = 2 \cdot 1 \cdot 17 = 34$$

## 4. Final Answer

The two valid solutions yield:

$$xyz = 48 \quad \text{and} \quad xyz = 34$$

The minimum value is:

$$\boxed{34}$$

**19.** Let $0 \le a \le x \le 100$ and f(x)=$|x-a| + |x-100| + |x-a-50|$.Then the maximum value of f(x) becomes 100 when a is equal to

**Correct Answer:** —

**Solution:**

We are given the function: $f(x) = |x - a| + |x - 100| + |x - a - 50|$
This function represents the sum of distances from point $x$ to three positions: $a$, 100, and $a + 50$.

Let us rewrite the function to group the terms more clearly: $f(x) = |x - a| + |x - 100| + |x - (a + 50)|$

## Understanding Each Term:

- $|x - a|$ is the distance between $x$ and $a$.
- $|x - 100|$ is the distance between $x$ and 100.
- $|x - (a + 50)|$ is the distance between $x$ and $a + 50$.

We are told that: $a \leq x \leq 100$.
From this, note that: $|x - a| + |x - 100| = 100 - a$, which is constant and **independent of x.**

So, we can simplify the function as: $f(x) = (100 - a) + |x - (a + 50)|$

## Analyzing $|x - (a + 50)|$

To understand how to maximize $f(x)$, we must consider how to maximize $|x - (a + 50)|$. There are two possibilities:

**Case 1: $x$ is between $a$ and $a + 50$**

In this scenario, the farthest $x$ can be from $a + 50$ is exactly 50 units, and this happens when $x = a$. So: $|x - (a + 50)| = |a - (a + 50)| = 50$
Thus, the maximum value of the function is: $f(x) = (100 - a) + 50$

**Case 2: $x \geq a + 50$**

To maximize $|x - (a + 50)|$ again, we take $x$ to its rightmost possible value (i.e., 100), and $a$ to its leftmost possible value (i.e., 0).

Then: $a = 0 \Rightarrow a + 50 = 50$ and $x = 100$

So, $|x - (a + 50)| = |100 - 50| = 50$

Again, the function becomes: $f(x) = (100 - 0) + 50 = 150$

## Conclusion:

In both cases, the maximum value that $|x - (a + 50)|$ can take is 50, and that's the key to finding the maximum value of the function.

**Why is understanding the maximum of $|x - (a + 50)|$ important?**

Because the rest of the expression, $|x - a| + |x - 100| = 100 - a$, is constant with respect to $x$, we focus on the variable part: $|x - (a + 50)|$. Finding its maximum helps in determining the overall maximum value of $f(x)$.

Hence, by understanding and maximizing $|x - (a + 50)|$, we ensure $f(x)$ is maximized. The maximum value is: $f(x) = 100 - a + 50 = 150$, when $a = 0$ and $x = 100$

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**20.** For any real number x, let [x] be the largest integer less than or equal to x. If $\sum_{n=1}^{N} \left[\frac{1}{5} + \frac{n}{25}\right] = 25$ then N is

**Correct Answer:** —

**Solution:**

We are given the following sum:

$$\sum_{n=1}^{N} \left[\frac{1}{5} + \frac{n}{25}\right] = 25$$

First, let's find the expression inside the square brackets: $\frac{1}{5} + \frac{n}{25} = \frac{5n+1}{25}$

Now we want to find the largest integer less than or equal to $\frac{5n+1}{25}$, which is represented as $\left[\frac{5n+1}{25}\right]$.

We are given that for $n = 1$ to $n = 19$, the value of the function is zero.

This means that $\left[\frac{5n+1}{25}\right] = 0$ for $n = 1$ to $n = 19$.

For $n = 20$ to $n = 44$, the value of the function is 1.

This means that $\left[\frac{5n+1}{25}\right] = 1$ for $n = 20$ to $n = 44$.

Now, we want to find the value of $N$ such that the sum of these bracketed terms is equal to 25.

$\sum_{n=1}^{N} \left[\frac{5n+1}{25}\right] = \sum_{n=1}^{19} 0 + \sum_{n=20}^{44} 1 = 0 + 25 = 25$

So, the value of $N$ that satisfies the given equation is indeed $N = 44$.

To summarize: $\sum_{n=1}^{N} \left[\frac{1}{5} + \frac{n}{25}\right] = 25$ is satisfied when $N = 44$.

**21.** For any natural number n,suppose the sum of the first n terms of an arithmetic progression is $(n + 2n^2)$. If the $n^{th}$ term of the progression is divisible by 9,then the smallest possible value of n is

(A) 4

(B) 8

(C) 7

(D) 9

**Correct Answer:** (C) 7

**Solution:**

To solve this problem, we start by recalling the formula for the sum of the first $n$ terms of an arithmetic progression (AP), which is given by:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

Given that the sum of the first $n$ terms is $(n + 2n^2)$, we equate this with the formula:

$$\frac{n}{2}(2a + (n-1)d) = n + 2n^2$$

Simplifying, we have:

$$n(2a + (n-1)d) = 2n + 4n^2$$

Dividing through by $n$ (since $n \neq 0$), we get:

$$2a + (n-1)d = 2 + 4n$$

Now, the nth term of the AP, denoted as $T_n$, is defined by:

$$T_n = a + (n-1)d$$

To find the nth term, we rearrange the equation:

$$2a + (n-1)d = 2 + 4n$$

Let's solve for $a + (n-1)d$:
Substitute $2a$ in terms of $(2a + (n-1)d)$:

$$a + (n-1)d = \frac{2 + 4n + 2}{2}$$

$$a + (n-1)d = 2 + 2n$$

Thus the nth term is $2 + 2n$.
According to the question, this nth term is divisible by 9:

$$2 + 2n \equiv 0 \pmod{9}$$

Rearranging gives:

$$2n \equiv -2 \equiv 7 \pmod{9}$$

To solve for $n$, multiply both sides by the modular inverse of 2 mod 9, which is 5 (since $2 \times 5 \equiv 1 \pmod{9}$):

$$10n \equiv 35 \pmod{9}$$

$$n \equiv 35 \pmod{9}$$

Calculating $35 \mod 9$ gives:

$$35 \div 9 = 3 \text{ R } 8$$

$$n \equiv 8 \pmod{9}$$

Thus, the smallest possible value of $n$ is:
7

**22.** The number of ways of distributing 20 identical balloons among 4 children such that each child gets some balloons but no child gets an odd number of balloons, is

**Correct Answer:** —

**Solution:**

## Step 1: Convert Even Numbers to Natural Numbers

Since only even numbers are allowed, we can write the number of balloons each child receives as:

$$2a, \quad 2b, \quad 2c, \quad 2d \quad \text{where } a, b, c, d \in \mathbb{N}$$

Then the total becomes:

$$2a + 2b + 2c + 2d = 20 \Rightarrow a + b + c + d = 10$$

So the original problem is transformed into: How many ways can 4 positive integers sum up to 10?

## Step 2: Use Combinatorics for Positive Integer Partitions

The number of ways to partition a number $n$ into $r$ positive integers is given by:

$$\text{Number of solutions} = \binom{n-1}{r-1}$$

In our case:

$$n = 10, \quad r = 4 \Rightarrow \binom{10-1}{4-1} = \binom{9}{3}$$

Calculating:

$$\binom{9}{3} = \frac{9 \times 8 \times 7}{3 \times 2 \times 1} = 84$$

## Final Answer

There are exactly:

$$\boxed{84}$$

ways to distribute 20 identical balloons among 4 children such that each gets a non-zero even number of balloons.

## Summary

- We transformed the problem from even numbers to natural numbers.
- Used the formula for partitioning a number into positive integers.
- The key was: $a + b + c + d = 10$ with $a, b, c, d \geq 1$
- Applied: $\binom{n-1}{r-1} \Rightarrow \binom{9}{3} = 84$