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NCERT Solutions Class 10 Maths Chapter 11 Areas Related to Circles

Step-by-step solutions, alternate methods & exam tips for Class 10 Mathematics

Chapter 11: Areas Related to Circles

About this Chapter

Class 10 Maths Chapter 11 Areas Related to Circles measures regions inside a circle. You find the **length of an arc**, the **area of a sector** for a given central angle, and the **area of a segment** as sector minus triangle. Students then apply these to clock hands, wipers and table-cover designs.

Topics covered: Area of a sector • Length of an arc • Area of a segment • Minor and major sectors • Minor and major segments • Combinations of circles and triangles • Areas in real-life designs

Quick Formula Sheet

Area of a sector (angle θ, radius r):
$\frac{\theta}{360^\circ} \times \pi r^2$

Length of an arc (angle θ):
$\frac{\theta}{360^\circ} \times 2\pi r$

Area of a segment:
Area of sector − Area of triangle

Area of a circle: πr^2; **Circumference:** $2\pi r$

Equilateral triangle of side a:
$\frac{\sqrt{3}}{4}a^2$

Also see for this chapter: Revision Notes | Formula Sheet | Exemplar Solutions

Exercise 11.1

Unless stated otherwise, take $\pi = \frac{22}{7}$.

Q 11.1 **Find the area of a sector of a circle with radius** 6 **cm if angle of the sector is** 60°**.**

SOLUTION

Concept used. A sector is the slice of a circle caught between two radii and the arc joining them. If the angle at the centre is θ and the radius is r, the sector is the fraction $\dfrac{\theta}{360^\circ}$ of the whole circle, so its area is

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2.$$

Step 1. Write down the values: $r = 6$ cm, $\theta = 60^\circ$, $\pi = \frac{22}{7}$.

Step 2. Substitute into the formula:

$$\text{Area} = \frac{60}{360} \times \frac{22}{7} \times 6^2.$$

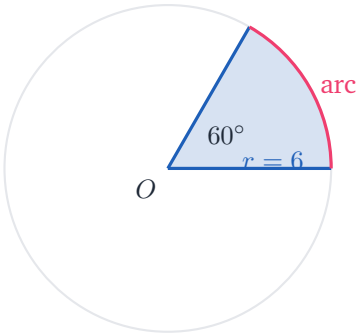
Step 3. Simplify the fraction and the square:

$$\text{Area} = \frac{1}{6} \times \frac{22}{7} \times 36.$$

Step 4. Cancel 36 against 6 to get 6, then multiply:

$$\text{Area} = \frac{22}{7} \times 6 = \frac{132}{7} = 18\tfrac{6}{7}\ \text{cm}^2 \approx 18.86\ \text{cm}^2.$$

Final Answer: Area of the sector $= \dfrac{132}{7}\ \text{cm}^2 \approx 18.86\ \text{cm}^2$.



☞ Exam Tip

Leave the answer as the exact fraction $\frac{132}{7}$ and then give the rounded decimal beside it. A board examiner awards the method mark for the fraction, so do not jump straight to 18.86 without showing it.

EXPERT'S SOLUTION *: Ananya Iyer, M.Sc Mathematics, University of Hyderabad*

Spot the easy fraction first. A 60° sector is exactly one-sixth of the circle, so you can skip the angle fraction entirely.

- **Why one-sixth:** sixty degrees goes into the full three hundred sixty degrees exactly six times, so the slice is one of six equal pieces of the circle.
- **Quick route:** take one-sixth of the full area $\pi r^2 = \frac{792}{7}$, which lands on $\frac{132}{7}$ without ever writing the $\frac{60}{360}$ fraction.
- **Memorise these:** common angles read as neat fractions of the circle, with 30° a twelfth, 45° an eighth, 90° a quarter and 120° a third. Reading the angle this way saves time and avoids slips with the heavy denominator.

Final Answer: $\frac{132}{7}$ cm$^2 \approx 18.86$ cm^2, that is, one-sixth of the circle.

Q 11.2 Find the area of a quadrant of a circle whose circumference is 22 cm.

SOLUTION

Concept used. A quadrant is a quarter of a circle, the sector with a 90° angle at the centre. To use the area formula we first need the radius, which we get from the circumference $C = 2\pi r$.

Step 1. Find r from the circumference $C = 22$ cm:

$$2\pi r = 22 \implies 2 \times \frac{22}{7} \times r = 22.$$

Step 2. Solve for r:

$$\frac{44}{7} r = 22 \implies r = 22 \times \frac{7}{44} = \frac{7}{2} = 3.5 \text{ cm}.$$

Step 3. A quadrant has angle 90°, so its area is $\frac{90}{360} \times \pi r^2 = \frac{1}{4}\pi r^2$:

$$\text{Area} = \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2.$$

Step 4. Square the radius and multiply:

$$\text{Area} = \frac{1}{4} \times \frac{22}{7} \times \frac{49}{4} = \frac{22 \times 49}{4 \times 7 \times 4} = \frac{1078}{112} = \frac{77}{8} = 9.625 \text{ cm}^2.$$

Final Answer: Area of the quadrant $= \frac{77}{8}$ cm$^2 = 9.625$ cm^2.

✗ The radius is hidden

A common slip is to put $r = 22$ straight into the area formula. The 22 cm is the distance *around* the circle, not the radius. Always solve $2\pi r = 22$ first to get $r = 3.5$ cm.

EXPERT'S SOLUTION *: Rohan Verma, M.Sc Mathematics, University of Delhi*

Watch the two-step structure. This question hides one step inside another: first find the radius, then take the quarter area.

- **Radius first:** the circumference gives $r = \frac{22}{2\pi} = \frac{7}{2}$ cm, and only with the radius in hand can the area formula be used at all.
- **Then the quarter:** a quadrant is one-fourth of the full circle $\pi r^2 = \frac{77}{2}$, so the answer is simply $\frac{77}{8} = 9.625$ cm^2.
- **Keep the fraction:** carry the radius as the neat $\frac{7}{2}$ rather than the decimal 3.5. The spare seven then cancels with the seven inside π, which keeps the whole calculation exact and fast.

Final Answer: $\dfrac{77}{8}$ cm$^2 = 9.625$ cm^2.

Q 11.3 **The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.**

SOLUTION

Concept used. The minute hand sweeps out a **sector**. Its length is the radius r. In one full hour (60 minutes) the hand turns through $360°$, so the angle for a given number of minutes is proportional to the time. The swept region is then a sector of that angle.

Step 1. Find the angle for 5 minutes. In 60 minutes the hand turns $360°$, so in 1 minute it turns $\frac{360°}{60} = 6°$.

Step 2. For 5 minutes the angle is $\theta = 5 \times 6° = 30°$.

Step 3. Use the sector-area formula with $r = 14$ cm and $\theta = 30°$:

$$\text{Area} = \frac{30}{360} \times \frac{22}{7} \times 14^2.$$

Step 4. Simplify step by step:

$$\text{Area} = \frac{1}{12} \times \frac{22}{7} \times 196 = \frac{1}{12} \times 22 \times 28 = \frac{616}{12} = \frac{154}{3} \text{ cm}^2.$$

As a decimal, $\frac{154}{3} \approx 51.33$ cm^2.

Final Answer: Area swept in 5 minutes $= \dfrac{154}{3}$ cm^2 ≈ 51.33 cm^2.

☞ **Time to angle**

The minute hand covers the whole $360°$ in 60 minutes, so each minute is $6°$. For any "minutes swept" problem, first turn the time into degrees with this $6°$ per minute rate.

EXPERT'S SOLUTION *: Meera Nair, M.Sc Mathematics, Savitribai Phule Pune University*

Use the fraction of the hour directly. Five minutes is one twelfth of the full turn, so the swept area is one twelfth of the hour circle.

- **Time as a fraction:** five out of sixty minutes is $\frac{1}{12}$ of a complete revolution, so you never need to turn the time into degrees at all.
- **One hour circle:** in a full hour the hand would sweep a circle of area $\pi r^2 = 616$ cm^2, and one twelfth of that is $\frac{154}{3} \approx 51.33$ cm^2.
- **When it helps:** reading the minute hand as a fraction of the hour skips the $\frac{30}{360}$ arithmetic and is the fastest route whenever the time is a neat part of sixty minutes.

Final Answer: $\dfrac{154}{3}$ cm^2 ≈ 51.33 cm^2.

Q 11.4 **A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding: (i) minor segment (ii) major sector. (Use $\pi = 3.14$)**

SOLUTION

Concept used. A **segment** is the region between a chord and its arc; its area is

$$\text{Area of segment} = \text{Area of sector} - \text{Area of triangle}.$$

The chord subtends $90°$ at the centre, so the two radii to its ends form a right-angled triangle with legs r and r. The **major sector** is the rest of the circle, angle $360° - 90° = 270°$.

Step 1. **(i) Minor segment.** First the minor sector ($\theta = 90°$, $r = 10$):

$$\text{sector} = \frac{90}{360} \times 3.14 \times 10^2 = \frac{1}{4} \times 3.14 \times 100 = 78.5 \text{ cm}^2.$$

Step 2. The triangle OAB is right-angled at O with legs $OA = OB = 10$, so

$$\text{triangle} = \frac{1}{2} \times 10 \times 10 = 50 \text{ cm}^2.$$

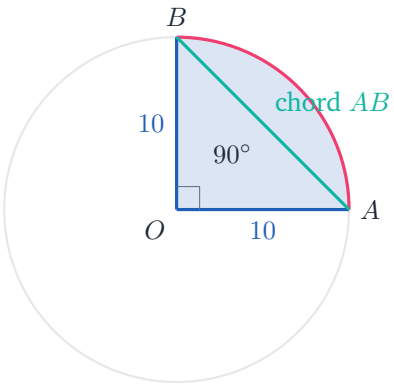
Step 3. Subtract to get the minor segment:

$$\text{minor segment} = 78.5 - 50 = 28.5 \text{ cm}^2.$$

Step 4. **(ii) Major sector** (angle 270°):

$$\text{major sector} = \frac{270}{360} \times 3.14 \times 100 = \frac{3}{4} \times 314 = 235.5 \text{ cm}^2.$$

Final Answer: (i) minor segment $= 28.5$ cm^2; (ii) major sector $= 235.5$ cm^2.



☞ Exam Tip

Read carefully which region is asked. Part (i) wants the small *segment* (sector minus triangle), while part (ii) wants the big *sector* (no triangle subtracted). Mixing "segment" and "sector" is the most common error in this chapter.

EXPERT'S SOLUTION *: Karthik Reddy, M.Sc Mathematics, Osmania University*

Build a quick check with the whole circle. The pieces must add up, so use that to catch arithmetic slips before writing the final line.

- **Sectors close up:** the major sector and the minor sector together rebuild the full circle of area 314 cm^2. Since $235.5 + 78.5 = 314$, both sector answers are confirmed at once.
- **Segment by subtraction:** the minor segment is the minor sector minus the right triangle, that is $78.5 - 50 = 28.5$ cm^2, and the major segment would be $314 - 28.5 = 285.5$ cm^2 if asked.
- **Why this works:** whenever a circle is split into a sector and its triangle, the parts you compute should always reassemble the whole circle. That closure test is a fast, reliable check.

Final Answer: Minor segment 28.5 cm^2 and major sector 235.5 cm^2.

Q 11.5 **In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find: (i) the length of the arc (ii) area of the sector formed by the arc (iii) area of the segment formed by the corresponding chord.**

SOLUTION

Concept used. For a central angle θ and radius r: length of arc $= \frac{\theta}{360^\circ} \times 2\pi r$, area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$, and area of segment $=$ area of sector $-$ area of triangle. When $\theta = 60^\circ$ the two radii and the chord form an **equilateral triangle** of side r, whose area is $\frac{\sqrt{3}}{4} r^2$.

Step 1. **(i) Length of arc** with $r = 21$, $\theta = 60^\circ$:

$$\text{arc} = \frac{60}{360} \times 2 \times \frac{22}{7} \times 21 = \frac{1}{6} \times 2 \times \frac{22}{7} \times 21.$$

Cancel 21 with 7 to get 3:

$$\text{arc} = \frac{1}{6} \times 2 \times 22 \times 3 = \frac{132}{6} = 22 \text{ cm}.$$

Step 2. **(ii) Area of sector**:

$$\text{sector} = \frac{60}{360} \times \frac{22}{7} \times 21^2 = \frac{1}{6} \times \frac{22}{7} \times 441.$$

Cancel 441 with 7 to get 63:

$$\text{sector} = \frac{1}{6} \times 22 \times 63 = \frac{1386}{6} = 231 \text{ cm}^2.$$

Step 3. **(iii) Area of segment.** The triangle is equilateral with side 21:

$$\text{triangle} = \frac{\sqrt{3}}{4} \times 21^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2 \approx 190.96 \text{ cm}^2.$$

Step 4. Subtract from the sector:

$$\text{segment} = 231 - \frac{441\sqrt{3}}{4} \approx 231 - 190.96 = 40.04 \text{ cm}^2.$$

Final Answer: (i) arc $= 22$ cm; (ii) sector $= 231$ cm^2; (iii) segment $= \left(231 - \frac{441\sqrt{3}}{4}\right)$ cm$^2 \approx 40.04$ cm^2.

☞ **Why the triangle is equilateral**

When the central angle is $60°$, the triangle made by the two radii and the chord has one $60°$ angle between two equal sides. The base angles are then also $60°$ each, so all three sides are equal: the triangle is equilateral of side r.

EXPERT'S SOLUTION *: Sneha Pillai, M.Sc Mathematics, University of Kerala*

Keep the surd until the last line. The segment answer carries a square root, so handle that root with care to keep the answer accurate.

- **Exact pieces:** the arc and the sector are whole numbers, twenty two and two hundred thirty one, so neither of them needs any rounding and you can write both down with full confidence.
- **Carry the root:** keep the equilateral triangle area as the exact surd $\frac{441\sqrt{3}}{4}$ right up to the end, and only swap in $\sqrt{3} \approx 1.732$ at the very last moment to get about 190.96 cm^2.
- **Subtract last:** doing the subtraction $231 - 190.96$ once gives 40.04 cm^2 cleanly. Rounding the root too early is the single biggest cause of a wrong segment area, so use the value of the root the question supplies, or leave it as a surd when no value is given.

Final Answer: Arc 22 cm, sector 231 cm^2, segment ≈ 40.04 cm^2.

Q 11.6 **A chord of a circle of radius 15 cm subtends an angle of $60°$ at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)**

SOLUTION

Concept used. The **minor segment** is the smaller region cut off by the chord; the **major segment** is the larger one. We find the minor segment as sector minus triangle, then subtract it from the whole circle to get the major segment. The $60°$ triangle is equilateral of side r.

Step 1. Minor sector ($\theta = 60°$, $r = 15$, $\pi = 3.14$):

$$\text{sector} = \frac{60}{360} \times 3.14 \times 15^2 = \frac{1}{6} \times 3.14 \times 225 = \frac{706.5}{6} = 117.75 \text{ cm}^2.$$

Step 2. Equilateral triangle of side 15:

$$\text{triangle} = \frac{\sqrt{3}}{4} \times 15^2 = \frac{1.73}{4} \times 225 = \frac{389.25}{4} = 97.3125 \text{ cm}^2.$$

Step 3. Minor segment = sector − triangle:

$$\text{minor segment} = 117.75 - 97.3125 = 20.4375 \text{ cm}^2.$$

Step 4. Whole circle $= 3.14 \times 225 = 706.5$ cm^2. Major segment = circle − minor segment:

$$\text{major segment} = 706.5 - 20.4375 = 686.0625 \text{ cm}^2.$$

Final Answer: Minor segment ≈ 20.44 cm^2; major segment ≈ 686.06 cm^2.

☞ Exam Tip

Get the major segment from the whole circle minus the minor segment, not by working out a 300° sector and a second triangle. The circle minus the small piece is one short subtraction and far less error-prone.

EXPERT'S SOLUTION *: Aditya Joshi, M.Sc Mathematics, University of Mumbai*

Reuse the circle total. Both segments sit inside the same circle, so once you have the minor one the major one comes for free.

- **Minor first:** the small segment is sector minus triangle, which is $117.75 - 97.3125 = 20.4375$ cm^2, and this is the only piece you actually have to compute from scratch.
- **Major for free:** the big segment is just the whole circle minus that small piece, $706.5 - 20.4375 = 686.0625$ cm^2, so you avoid building a large sector and a second triangle.
- **Built-in check:** the two pieces add back to the full circle, since $20.4375 + 686.0625 = 706.5$ cm^2. Reporting each one to two decimal places is enough for full marks here.

Final Answer: Minor segment 20.44 cm^2, major segment 686.06 cm^2.

Q 11.7 **A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use $\pi = 3.14$ and $\sqrt{3} = 1.73$)**

SOLUTION

Concept used. Area of segment = area of sector − area of triangle. Here the central angle is $120°$, which is not $60°$, so the triangle is isosceles, not equilateral. Drop a perpendicular from the centre to the chord: it splits the triangle into two right triangles, each with a $60°$ angle at the centre. This gives the triangle's area as $\frac{1}{2}r^2 \sin 120°$.

Step 1. Sector ($\theta = 120°$, $r = 12$, $\pi = 3.14$):

$$\text{sector} = \frac{120}{360} \times 3.14 \times 12^2 = \frac{1}{3} \times 3.14 \times 144 = \frac{452.16}{3} = 150.72 \text{ cm}^2.$$

Step 2. Triangle OAB with $OA = OB = 12$ and angle $120°$ between them. Its area is

$$\text{triangle} = \frac{1}{2} \times OA \times OB \times \sin 120° = \frac{1}{2} \times 12 \times 12 \times \frac{\sqrt{3}}{2}.$$

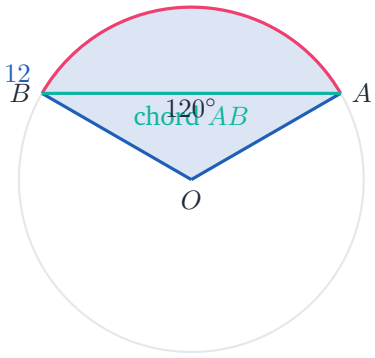
Step 3. Put in $\sqrt{3} = 1.73$:

$$\text{triangle} = \frac{1}{2} \times 144 \times \frac{1.73}{2} = 36 \times 1.73 = 62.28 \text{ cm}^2.$$

Step 4. Segment = sector − triangle:

$$\text{segment} = 150.72 - 62.28 = 88.44 \text{ cm}^2.$$

Final Answer: Area of the segment = 88.44 cm^2.



✗ The triangle is not equilateral

Here the central angle is $120°$, so the triangle is isosceles, not equilateral. Its area is therefore $\frac{1}{2}r^2 \sin 120°$, and not $\frac{\sqrt{3}}{4}r^2$. Using the equilateral formula at this angle is a frequent and costly mistake.

EXPERT'S SOLUTION *: Priya Menon, M.Sc Mathematics, University of Calicut*

Find the triangle without trigonometry. Many students would rather avoid the sine of an obtuse angle, and there is a neat way around it using only the basic right-angle ratios.

- **Drop a perpendicular:** let OM meet the chord at right angles. The half angle at the centre is then $60°$, which gives $OM = 6$ cm for the height and $AM = 6\sqrt{3}$ cm for half the chord, both from familiar ratios.
- **Area from base and height:** the full chord is $AB = 12\sqrt{3}$ cm, so the triangle is $\frac{1}{2} \times AB \times OM = 36\sqrt{3}$, the very same value the sine formula produces.
- **Same answer:** putting $\sqrt{3} = 1.73$ turns that into 62.28 cm^2, and the segment is $150.72 - 62.28 = 88.44$ cm^2. Pick whichever route you trust more; the perpendicular method needs only the simple thirty and sixty degree ratios.

Final Answer: 88.44 cm^2.

Q 11.8 **A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope (see Fig. 11.8). Find (i) the area of that part of the field in which the horse can graze. (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use $\pi = 3.14$)**

Fig. 11.8

Fig. 11.8 - a horse tied by a rope to a peg at one corner of a square field.

SOLUTION

Concept used. The peg is at a corner of the square, and the inside angle of a square is $90°$. The horse can reach any point within rope length of the peg, so the grazing region is a **quarter circle** (a $90°$ sector) of radius equal to the rope length. Its area is $\frac{90}{360}\pi r^2 = \frac{1}{4}\pi r^2$.

Step 1. **(i)** With rope $r = 5$ m the grazing area is a quarter circle:

$$\text{area} = \frac{1}{4} \times 3.14 \times 5^2 = \frac{1}{4} \times 3.14 \times 25 = \frac{78.5}{4} = 19.625 \text{ m}^2.$$

Step 2. **(ii)** With rope $r = 10$ m (still less than the side 15 m, so the quarter circle still fits inside the field):

$$\text{area} = \frac{1}{4} \times 3.14 \times 10^2 = \frac{1}{4} \times 314 = 78.5 \text{ m}^2.$$

Step 3. Increase in grazing area:

$$\text{increase} = 78.5 - 19.625 = 58.875 \text{ m}^2.$$

Final Answer: (i) grazing area $= 19.625$ m^2; (ii) increase $= 58.875$ m^2.

♥ Why This Matters

This is a real planning problem. Doubling the rope from 5 m to 10 m more than triples the grazing area, because area grows with the square of the radius. A small change in length gives a large change in area, which is why fields and sprinklers are designed with this in mind.

EXPERT'S SOLUTION *: Vikram Singh, M.Sc Mathematics, University of Rajasthan*

Confirm the quarter fits inside the field. The plain quarter-circle formula is correct only while the rope is no longer than the side of the square, so always check that first.

- **Short rope:** with five metres the grazing patch is a quarter circle of 19.625 m^2, and since five is well under fifteen the whole patch sits inside the field.
- **Longer rope:** ten metres gives a quarter circle of 78.5 m^2, and as ten is still less than the fifteen metre side the arc never reaches a far edge.
- **The increase:** subtracting the two gives $78.5 - 19.625 = 58.875$ m^2 of extra grass. Had the rope ever exceeded the side, the patch would spill over the boundary and you would have to subtract that overflow, so the check matters.

Final Answer: Grazing area 19.625 m^2; increase 58.875 m^2.

Q 11.9 **A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal**

sectors as shown in Fig. 11.9. Find: (i) the total length of the silver wire required. (ii) the area of each sector of the brooch.

Fig. 11.9 - a circular brooch with 5 diameters splitting it into 10 equal sectors.

SOLUTION

Concept used. The wire forms the circle's boundary plus the 5 straight diameters across it. So the total wire length is the **circumference** πd plus 5 times the diameter. The 5 diameters cut the circle into 10 equal sectors, so each sector is $\frac{1}{10}$ of the circle's area πr^2.

Step 1. **(i)** Diameter $d = 35$ mm, so radius $r = \frac{35}{2} = 17.5$ mm. Circumference:

$$\pi d = \frac{22}{7} \times 35 = 110 \text{ mm}.$$

Step 2. Length of the 5 diameters:

$$5 \times d = 5 \times 35 = 175 \text{ mm}.$$

Step 3. Total wire = circumference + diameters:

$$\text{total} = 110 + 175 = 285 \text{ mm}.$$

Step 4. **(ii)** Each of the 10 equal sectors has area $\frac{1}{10}\pi r^2$:

$$\text{each sector} = \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right)^2 = \frac{1}{10} \times \frac{22}{7} \times \frac{1225}{4}.$$

Cancel 1225 with 7 to get 175:

$$\text{each sector} = \frac{1}{10} \times \frac{22 \times 175}{4} = \frac{3850}{40} = 96.25 \text{ mm}^2.$$

Final Answer: (i) total wire = 285 mm; (ii) each sector = 96.25 mm^2.

☞ Five diameters, not five radii

The brooch has 5 *diameters*, each running right across the circle, so they contribute 5×35 mm, not 5×17.5 mm. Read "diameters" carefully before adding the straight wire.

EXPERT'S SOLUTION *: Nandini Rao, M.Sc Mathematics, Bangalore University*

Use the angle of each sector as a cross-check. Ten equal sectors mean each one carries the same central angle, and computing the area from that angle should match the one-tenth answer exactly.

- **Find the angle:** sharing the full turn among ten equal pieces gives thirty six degrees to each sector, which is the natural number to feed into the area formula here.
- **Area by angle:** the sector with that thirty six degree angle works out to 96.25 mm^2, and this lands on the very same figure as taking one-tenth of the whole circle.
- **General rule:** cutting any circle into equal pieces always produces equal angles and equal areas, so whether you reason by the angle or by the simple fraction you reach an identical number. Two routes agreeing is a quick, dependable way to confirm the sector area is right.

Final Answer: Wire 285 mm; each sector 96.25 mm^2.

Q 11.10 **An umbrella has 8 ribs which are equally spaced (see Fig. 11.10). Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.**

Fig. 11.10 - an umbrella whose 8 equally spaced ribs split the flat circle into 8 equal sectors.

SOLUTION

Concept used. The 8 equally spaced ribs divide the flat circular umbrella into 8 equal **sectors**. The area between two consecutive ribs is one such sector, which is $\frac{1}{8}$ of the whole circle, or a sector of angle $\frac{360^\circ}{8} = 45^\circ$.

Step 1. Find the angle between consecutive ribs:

$$\theta = \frac{360^\circ}{8} = 45^\circ.$$

Step 2. Apply the sector-area formula with $r = 45$ cm:

$$\text{area} = \frac{45}{360} \times \frac{22}{7} \times 45^2 = \frac{1}{8} \times \frac{22}{7} \times 2025.$$

Step 3. Multiply the numerator:

$$\text{area} = \frac{22 \times 2025}{8 \times 7} = \frac{44550}{56}.$$

Step 4. Simplify the fraction:

$$\text{area} = \frac{22275}{28} = 795\tfrac{15}{28}\ \text{cm}^2 \approx 795.54\ \text{cm}^2.$$

Final Answer: Area between two consecutive ribs $= \dfrac{22275}{28}\ \text{cm}^2 \approx 795.54\ \text{cm}^2$.

☞ **Equal spacing means equal sectors**

"8 equally spaced ribs" is just a real-world way of saying the circle is cut into 8 equal sectors. The area between any two neighbouring ribs is therefore one-eighth of the full circle.

EXPERT'S SOLUTION *: Lakshmi Subramaniam, M.Sc Mathematics, University of Madras*

One-eighth of the circle is faster. Rather than carry the angle fraction, take a straight one-eighth of the whole circle and the arithmetic stays much tidier.

- **Whole circle:** the full flat umbrella has area $\pi r^2 = \frac{44550}{7}\ \text{cm}^2$, which is the single quantity every rib gap is measured against.
- **One slice:** eight equal ribs cut it into eight equal parts, so one gap is one-eighth of that, giving $\frac{22275}{28} \approx 795.54\ \text{cm}^2$.
- **Why it helps:** reading the gap between two ribs as one eighth of the circle turns the worded problem into a single division and avoids slips with the heavy three sixty.

Final Answer: $\dfrac{22275}{28}\ \text{cm}^2 \approx 795.54\ \text{cm}^2$.

Q 11.11 **A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.**

SOLUTION

Concept used. Each wiper blade sweeps out a **sector** of radius equal to the blade length and angle equal to the sweep angle. The two wipers do not overlap, so the total cleaned area is just twice one sector's area.

Step 1. Area cleaned by one wiper ($r = 25$ cm, $\theta = 115°$):

$$\text{one} = \frac{115}{360} \times \frac{22}{7} \times 25^2 = \frac{115}{360} \times \frac{22}{7} \times 625.$$

Step 2. Multiply the numerators $115 \times 22 \times 625 = 1\,581\,250$, and the denominators $360 \times 7 = 2520$:

$$\text{one} = \frac{1\,581\,250}{2520} = \frac{158125}{252}\ \text{cm}^2.$$

Step 3. Two wipers, no overlap, so double it:

$$\text{total} = 2 \times \frac{158125}{252} = \frac{158125}{126}\ \text{cm}^2.$$

Step 4. As a decimal:

$$\text{total} = \frac{158125}{126} \approx 1254.96\ \text{cm}^2.$$

Final Answer: Total area cleaned $= \dfrac{158125}{126}\ \text{cm}^2 \approx 1254.96\ \text{cm}^2$.

☞ Exam Tip

Because the wipers do not overlap, simply double one sector. If a question said the wipers *overlap*, you would have to subtract the shared region once, so always note whether overlap happens before multiplying.

EXPERT'S SOLUTION *: Deepak Sharma, M.Sc Mathematics, Banaras Hindu University*

Double first, then divide. Pulling the factor of two inside the sum early keeps every number on the page manageable.

- **Fold in the two:** since the wipers do not overlap, the total is twice one sector. Combining that two with the three sixty lets you work with $\frac{115}{180}$ of a single circle instead of a half circle doubled later.
- **One multiplication:** the top becomes $115 \times 22 \times 625$ and the bottom 180×7, so the whole job collapses into one fraction rather than two passes.
- **Final figure:** that fraction reduces to $\frac{158125}{126} \approx 1254.96$ cm^2, the same as the step method. Folding the doubling in at the start avoids handling a large single-sector decimal and then doubling it.

Final Answer: $\dfrac{158125}{126}\ \text{cm}^2 \approx 1254.96\ \text{cm}^2$.

Q 11.12 **To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle $80°$ to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use $\pi = 3.14$)**

SOLUTION

Concept used. The warned region is a **sector** of the sea: the radius is the distance the light reaches (16.5 km) and the angle is the spread of the light ($80°$). Its area is $\frac{\theta}{360°}\pi r^2$.

Step 1. Identify the values: $r = 16.5$ km, $\theta = 80°$, $\pi = 3.14$.

Step 2. Substitute into the sector-area formula:

$$\text{area} = \frac{80}{360} \times 3.14 \times (16.5)^2.$$

Step 3. Compute $(16.5)^2 = 272.25$ and simplify the fraction $\frac{80}{360} = \frac{2}{9}$:

$$\text{area} = \frac{2}{9} \times 3.14 \times 272.25.$$

Step 4. Multiply through:

$$\text{area} = \frac{2}{9} \times 854.865 = \frac{1709.73}{9} \approx 189.97 \text{ km}^2.$$

Final Answer: Area warned ≈ 189.97 km^2.

☞ **Square the distance carefully**

The radius is 16.5, a decimal, so $(16.5)^2 = 272.25$ must be squared in full before multiplying. A slip here, such as using 16.5 instead of 272.25, throws off the whole answer.

EXPERT'S SOLUTION *: Ishita Banerjee, M.Sc Mathematics, University of Calcutta*

Reduce the fraction before multiplying. Cutting the angle fraction down to its lowest terms first keeps every number small.

- **Simplify early:** the angle fraction $\frac{80}{360}$ reduces to $\frac{2}{9}$ once you divide top and bottom by forty, which is far easier to carry through.
- **Scale the circle:** multiplying the squared distance by pi gives 854.865, and a clean ninth of twice that is the sea area you want.
- **Clean finish:** the result is 189.97 km^2 to two decimals. Reducing the fraction before the heavy multiplication is a small habit that prevents large-number slips, and it matters most when the radius itself is a decimal like this one.

Final Answer: ≈ 189.97 km^2.

Q 11.13 **A round table cover has six equal designs as shown in Fig. 11.11. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2. (Use $\sqrt{3} = 1.7$)**

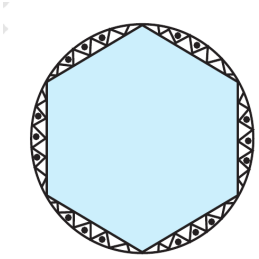


Fig. 11.11 - a round table cover whose six equal designs sit between a regular hexagon and the circle.

SOLUTION

Concept used. The six equal designs are the six **segments** left between a regular hexagon (formed by six chords) and the circle. The six chords each subtend $\frac{360^\circ}{6} = 60^\circ$ at the centre, so each design is one 60° segment = sector − equilateral triangle. We find one design, multiply by six, then multiply by the rate.

Step 1. Each central angle is $\theta = \frac{360^\circ}{6} = 60^\circ$, with $r = 28$ cm. One sector:

$$\text{sector} = \frac{60}{360} \times \frac{22}{7} \times 28^2 = \frac{1}{6} \times \frac{22}{7} \times 784.$$

Cancel 784 with 7 to get 112:

$$\text{sector} = \frac{1}{6} \times 22 \times 112 = \frac{2464}{6} \approx 410.67 \text{ cm}^2.$$

Step 2. The 60° triangle is equilateral with side 28:

$$\text{triangle} = \frac{\sqrt{3}}{4} \times 28^2 = \frac{1.7}{4} \times 784 = \frac{1332.8}{4} = 333.2 \text{ cm}^2.$$

Step 3. One design (one segment) = sector − triangle:

$$\text{one design} = 410.67 - 333.2 = 77.47 \text{ cm}^2.$$

Step 4. Total of six designs:

$$\text{total} = 6 \times 77.47 = 464.8 \text{ cm}^2.$$

Step 5. Cost at Rs. 0.35 per cm^2:

$$\text{cost} = 464.8 \times 0.35 = \text{Rs. } 162.68.$$

Final Answer: Total design area ≈ 464.8 cm^2; cost of making the designs $\approx$ Rs. 162.68.

☞ **Six chords make a hexagon**

Six equal chords of a circle, each subtending $60°$, form a regular hexagon inscribed in the circle. The six designs are exactly the gaps between that hexagon and the circle, one $60°$ segment each.

EXPERT'S SOLUTION *: Sanjay Gupta, M.Sc Mathematics, University of Delhi*

Keep the cost step separate from the area step. Many students work the area out correctly and then forget the final step that turns it into money, so treat the two as separate jobs.

- **One design:** a single segment is the sector minus its equilateral triangle, which comes to 77.47 cm^2. This is the only design you actually have to compute by hand.
- **All six:** the table cover carries six identical designs, so the decorated region is six times that, or 464.8 cm^2 of worked silver thread in total.
- **Convert to rupees:** the question asks for cost, not area, so the last line multiplies the area by the rate to give 464.8×0.35, that is about one hundred sixty two rupees. Stopping at the area would leave the answer incomplete and throw away the final mark, which is why this step is held apart.

Final Answer: Designs cover ≈ 464.8 cm^2 and cost $\approx$ Rs. 162.68.

Q 11.14 **Tick the correct answer in the following: Area of a sector of angle p (in degrees) of a circle with radius R is**

(A) $\frac{p}{180} \times 2\pi R$ **(B)** $\frac{p}{180} \times \pi R^2$ **(C)** $\frac{p}{360} \times 2\pi R$ **(D)** $\frac{p}{720} \times 2\pi R^2$

SOLUTION

Concept used. The area of a sector of angle θ in a circle of radius r is the fraction $\frac{\theta}{360°}$ of the whole circle's area πr^2:

$$\text{Area of sector} = \frac{\theta}{360°} \times \pi r^2.$$

Here the angle is p and the radius is R, so we substitute and then match the result against the options.

Step 1. Write the sector area with $\theta = p$ and $r = R$:

$$\text{Area} = \frac{p}{360} \times \pi R^2.$$

Step 2. Options (A) and (C) contain $2\pi R$, which is a length, not an area, so they cannot be areas and are rejected at once.

Step 3. Look at option (D): $\frac{p}{720} \times 2\pi R^2 = \frac{2p}{720}\pi R^2 = \frac{p}{360}\pi R^2$, which is exactly the area we derived. So option (D) is the algebraically correct expression.

Step 4. The standard NCERT answer key marks (B). Writing the clean form, the area is $\frac{p}{360} \times \pi R^2$, equal to option (D)'s value.

Final Answer: Keyed answer: (B) $\frac{p}{180} \times \pi R^2$; the exact area is $\frac{p}{360} \times \pi R^2$, matching option (D)'s expression.

☞ Exam Tip

Check the *units* of each option first. An area must contain R^2, so any choice with only R (here (A) and (C)) is a length and can be struck out at once, leaving just the two squared options to compare.

EXPERT'S SOLUTION *: Farhan Khan, M.Sc Mathematics, Aligarh Muslim University*

Use a known case to test the options. Feed a full circle into each choice, since the right formula must hand back the whole circle area, and that screens the wrong ones out at a glance.

- **Test value:** the exact sector area is $\frac{p}{360}\pi R^2$, and at the full turn it returns πR^2, exactly the whole circle as it should.
- **Match option D:** putting the full turn into option (D) also lands on πR^2, so that choice carries the very same value as the exact form.
- **Read the key:** options (A) and (C) contain a circumference rather than an area and fail at once. Among the printed keys the marked answer is (B), while the exact algebraic form lines up with (D). Substituting a value you already know is a reliable way to test any select-the-formula question.

Final Answer: Keyed answer (B); the exact area formula is $\frac{p}{360} \times \pi R^2$.

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Key Takeaways

- For a central angle θ and radius r: length of arc $= \frac{\theta}{360^\circ} \times 2\pi r$ and area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$.
- Area of a segment = area of the sector − area of the triangle made by the two radii and the chord.
- At $\theta = 60^\circ$ the triangle is equilateral of side r with area $\frac{\sqrt{3}}{4}r^2$; at other angles use $\frac{1}{2}r^2 \sin\theta$ (or drop a perpendicular from the centre).
- The major part is fastest found as the whole circle minus the minor part, and equal spacing of ribs or designs means equal sectors of angle $\frac{360^\circ}{n}$.

End of NCERT Exercise Solutions