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Step-by-step solutions, alternate methods & exam tips for Class 10 Mathematics

Chapter 3: Pair of Linear Equations in Two Variables

About this Chapter

Class 10 Maths Chapter 3 Pair of Linear Equations in Two Variables Exercise 3.1 works with two equations in x and y together. These solutions solve every exercise by the **graphical method**, **substitution** and **elimination**. Students also learn to tell a **consistent** pair from an inconsistent one and to crack word problems on ages, money and digits.

Topics covered: Pair of linear equations • Graphical method • Consistent and inconsistent pairs • Comparing coefficient ratios • Substitution method • Elimination method • Word problems

Quick Formula Sheet

General pair:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Unique solution (intersecting):

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Infinitely many (coincident):

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

No solution (parallel):

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Also see for this chapter: Revision Notes | Formula Sheet | Exemplar Solutions

NCERT Exercise Solutions

Exercise 3.1

Q 3.1 **Form the pair of linear equations in the following problems, and find their solutions graphically. (i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz. (ii) 5 pencils and 7 pens together cost Rs. 50, whereas 7 pencils and 5 pens together cost Rs. 46. Find the cost of one pencil and that of one pen.**

SOLUTION

Concept used. To solve a **pair of linear equations** graphically, we plot each equation as a straight line by finding two solution points for it. The point where the two lines **intersect** gives the common solution, that is the values of x and y that satisfy both equations at once.

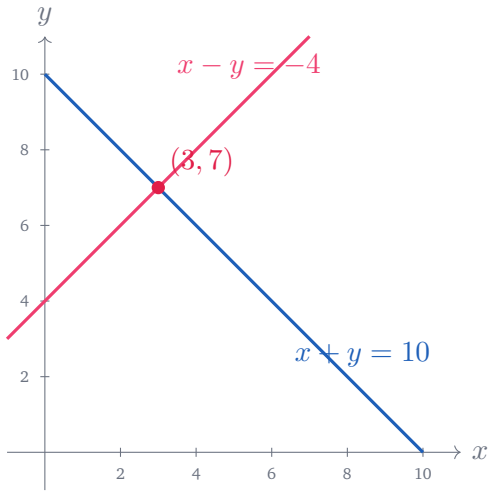
Step 1. **(i) Forming the equations.** Let the number of boys be x and the number of girls be y. The total is 10, so $x + y = 10$. The girls are 4 more than the boys, so $y = x + 4$, that is $x - y = -4$. The pair is

$$x + y = 10, \qquad x - y = -4.$$

Step 2. **Two points on each line.** For $x + y = 10$: if $x = 0$ then $y = 10$; if $x = 10$ then $y = 0$. Points $(0, 10)$ and $(10, 0)$. For $x - y = -4$ (that is $y = x + 4$): if $x = 0$ then $y = 4$; if $x = 2$ then $y = 6$. Points $(0, 4)$ and $(2, 6)$.

Step 3. **Reading the graph.** The two lines meet at $(3, 7)$.

A sketch of the two lines and their meeting point:



Step 1. So $x = 3$ and $y = 7$: there are 3 boys and 7 girls.

Step 2. **(ii) Forming the equations.** Let the cost of one pencil be Rs. x and one pen be Rs. y. Then

$$5x + 7y = 50, \qquad 7x + 5y = 46.$$

Step 3. **Two points on each line.** For $5x + 7y = 50$: if $x = 3$ then $y = 5$; if $x = 10$ then $y = 0$. Points $(3, 5)$ and $(10, 0)$. For $7x + 5y = 46$: if $x = 3$ then $y = 5$; if $x = 8$ then $y = -2$. Points $(3, 5)$ and $(8, -2)$.

Step 4. Both lines pass through $(3, 5)$, so they intersect there. Hence $x = 3$ and $y = 5$: one pencil costs Rs. 3 and one pen costs Rs. 5.

Final Answer: (i) Boys $= 3$, girls $= 7$. (ii) Pencil $=$ Rs. 3, pen $=$ Rs. 5.

☞ Pick easy points for the graph

For each line, choose values of x (often 0 and one more) that give whole number y. Two clean points are enough to draw a straight line, and tidy points make the intersection easy to read.

EXPERT'S SOLUTION *: Anjali Rao, M.Sc Mathematics, University of Delhi*

Translate each sentence into one equation, then let the graph confirm the meeting point. The first job in both parts is to name the two unknowns and write exactly two equations, because a graphical solution needs a pair of lines and nothing more.

- **Word to equation:** keep the wording in the same order it is written. In part (i) "girls are 4 more than boys" becomes $y = x + 4$ and "ten students in all" becomes $x + y = 10$, which saves a student from the common sign slip of writing the difference the wrong way round as $y - x = -4$.
- **Read the crossing:** plot each line through two neat points and read where they cross. In part (i) the lines meet at $(3, 7)$, giving 3 boys and 7 girls. In part (ii) the two cost lines both pass through $(3, 5)$, so the pencil is Rs. 3 and the pen is Rs. 5.
- **Where the marks sit:** the examiner rewards three separate things, the correctly formed pair of equations, a small table of two points for each line, and a clearly marked meeting point, so a student should label that crossing on the graph itself and not merely state it in words below the diagram where it is easy to miss.
- **Defend the answer:** put the read-off point back into both of the original equations as a final step. A graph that is drawn slightly off the true crossing can still be defended by that one line of checking, since here the totals $3 + 7 = 10$ and the difference $7 - 3 = 4$ both come out exactly right and confirm the reading.

Final Answer: Boys 3, girls 7; pencil Rs. 3, pen Rs. 5.

Q 3.2 **On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:**

(i) $5x-4y+8=0$, $7x+6y-9=0$ **(ii)** $9x+3y+12=0$, $18x+6y+24=0$ **(iii)** $6x-3y+10=0$, $2x-y+9=0$

SOLUTION

Concept used. For two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, the three coefficient ratios decide their geometry: if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ the lines **intersect** at one point; if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ they are **coincident**; and if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ they are **parallel**.

Step 1. **(i)** $5x - 4y + 8 = 0$ **and** $7x + 6y - 9 = 0$**.** Here $\frac{a_1}{a_2} = \frac{5}{7}$ and $\frac{b_1}{b_2} = \frac{-4}{6} = \frac{-2}{3}$. Since $\frac{5}{7} \neq \frac{-2}{3}$, the lines **intersect at a point**.

Step 2. **(ii)** $9x + 3y + 12 = 0$ **and** $18x + 6y + 24 = 0$**.** Here $\frac{a_1}{a_2} = \frac{9}{18} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{3}{6} = \frac{1}{2}$ and $\frac{c_1}{c_2} = \frac{12}{24} = \frac{1}{2}$. All three ratios are equal, so the lines are **coincident**.

Step 3. **(iii)** $6x - 3y + 10 = 0$ **and** $2x - y + 9 = 0$**.** Here $\frac{a_1}{a_2} = \frac{6}{2} = 3$, $\frac{b_1}{b_2} = \frac{-3}{-1} = 3$ and $\frac{c_1}{c_2} = \frac{10}{9}$. Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = 3$ but $\frac{c_1}{c_2} = \frac{10}{9}$ is different, the lines are **parallel**.

Final Answer: (i) Intersecting; (ii) Coincident; (iii) Parallel.

☞ Keep the signs with the coefficients

When you read off a, b, c, carry the sign in front of each term. Here $b_1 = -4$ and $b_2 = 6$, so $\frac{b_1}{b_2} = \frac{-4}{6}$, not $\frac{4}{6}$. A dropped sign flips the conclusion.

EXPERT'S SOLUTION *: Vikram Iyer, M.Sc Mathematics, Savitribai Phule Pune University*

Compare the ratios in a fixed order and stop at the first that differs. The whole question is one rule applied three times, so the work is fast once the order of checking is fixed.

- **Fixed order:** write all three lines in the form $ax + by + c = 0$, then compute the first two ratios a_1/a_2 and b_1/b_2. If those two differ, as in part (i), the lines intersect and there is no need to look at the constant ratio at all.
- **Then the constant:** only when the first two match do you check the third ratio. Equal everywhere means coincident as in part (ii), while the constant breaking the pattern means parallel as in part (iii).
- **Sign trap:** the most common error is dropping a minus, because $b_1 = -4$ and $b_2 = -1$ must keep their signs, and rounding the constant ratio $\frac{10}{9}$ in part (iii) to a careless 1 would wrongly turn parallel lines into coincident ones.
- **Earning the marks:** show each ratio reduced to lowest terms and then write the one word conclusion, since the reasoning marks sit in the comparison rather than in the verdict on its own.

Final Answer: Intersecting; coincident; parallel.

Q 3.3 **On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pairs of linear equations are consistent, or inconsistent. (i)** $3x + 2y = 5$; $2x - 3y = 7$ **(ii)** $2x - 3y = 8$; $4x - 6y = 9$ **(iii)** $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$ **(iv)** $5x - 3y = 11$; $-10x + 6y = -22$ **(v)** $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$

SOLUTION

Concept used. A pair of linear equations is **consistent** when it has at least one solution, and **inconsistent** when it has none. Comparing the ratios tells us which: if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ the pair is consistent (a unique solution); if all three ratios are equal the pair is consistent (infinitely many solutions); and if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ the pair is inconsistent.

Step 1. **(i)** $3x + 2y - 5 = 0$ **and** $2x - 3y - 7 = 0$. $\frac{a_1}{a_2} = \frac{3}{2}$, $\frac{b_1}{b_2} = \frac{2}{-3}$. These differ, so the pair is **consistent** (unique solution).

Step 2. **(ii)** $2x - 3y - 8 = 0$ **and** $4x - 6y - 9 = 0$. $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-8}{-9} = \frac{8}{9}$. Here $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, so the pair is **inconsistent**.

Step 3. **(iii)** $\frac{3}{2}x + \frac{5}{3}y - 7 = 0$ **and** $9x - 10y - 14 = 0$. $\frac{a_1}{a_2} = \frac{3/2}{9} = \frac{1}{6}$, $\frac{b_1}{b_2} = \frac{5/3}{-10} = \frac{-1}{6}$. These differ, so the pair is **consistent** (unique solution).

Step 4. **(iv)** $5x - 3y - 11 = 0$ **and** $-10x + 6y + 22 = 0$. $\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}$, $\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$, $\frac{c_1}{c_2} = \frac{-11}{22} = \frac{-1}{2}$. All three are equal, so the pair is **consistent** (infinitely many solutions).

Step 5. **(v)** $\frac{4}{3}x + 2y - 8 = 0$ **and** $2x + 3y - 12 = 0$. $\frac{a_1}{a_2} = \frac{4/3}{2} = \frac{2}{3}$, $\frac{b_1}{b_2} = \frac{2}{3}$, $\frac{c_1}{c_2} = \frac{-8}{-12} = \frac{2}{3}$. All three are equal, so the pair is **consistent** (infinitely many solutions).

Final Answer: (i) Consistent; (ii) Inconsistent; (iii) Consistent; (iv) Consistent; (v) Consistent.

✗ Move every term to one side first

Each equation must be in the form $ax + by + c = 0$ before you read off c. Writing $3x + 2y = 5$ as $3x + 2y - 5 = 0$ gives $c_1 = -5$. Comparing c values without shifting the constant is the usual slip.

EXPERT'S SOLUTION *: Meera Krishnan, M.Sc Mathematics, University of Madras*

Consistent or inconsistent is decided by the ratio test, so classify, never solve. This question never asks for the actual values of x and y, only whether a solution exists, so a student should resist solving and read off the three ratios instead.

- **Clean routine:** write each equation as $ax + by + c = 0$, simplify any fractions, and compare a_1/a_2 with b_1/b_2 before touching the constant ratio.
- **The verdicts:** parts (i) and (iii) have unequal first two ratios, so they are consistent with a unique solution; part (ii) matches in the first two but not the third, the exact signature of an inconsistent pair; parts (iv) and (v) match in all three, so they are consistent with infinitely many solutions.
- **Fraction care:** the fractional coefficients in (iii) and (v) are where errors creep in, so dividing $\frac{3}{2}$ by 9 to get $\frac{1}{6}$ should be done carefully, and in (iv) keeping the minus on -10 and -22 is what makes all three ratios equal.
- **Full marks:** state the three ratios, the comparison and the one word verdict, and add whether a consistent pair has one solution or infinitely many, since that distinction often carries a mark of its own.

Final Answer: Consistent: (i), (iii), (iv), (v). Inconsistent: (ii).

Q 3.4 **Which of the following pairs of linear equations are consistent/inconsistent? If consistent, obtain the solution graphically: (i)** $x + y = 5$, $2x + 2y = 10$ **(ii)** $x - y = 8$, $3x - 3y = 16$ **(iii)** $2x + y - 6 = 0$, $4x - 2y - 4 = 0$ **(iv)** $2x - 2y - 2 = 0$, $4x - 4y - 5 = 0$

SOLUTION

Concept used. We first use the ratio test to decide whether each pair is **consistent** or **inconsistent**. For a consistent pair with a unique solution, we then draw both lines and read off the point where they cross. For a coincident pair the two lines lie on top of each other, giving infinitely many solutions.

Step 1. **(i)** $x + y = 5$ **and** $2x + 2y = 10$. $\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{2}, \frac{c_1}{c_2} = \frac{-5}{-10} = \frac{1}{2}$. All equal, so the lines are coincident: the pair is **consistent** with infinitely many solutions. (The second equation is just twice the first.)

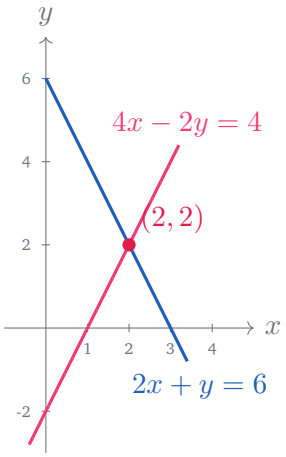
Step 2. **(ii)** $x - y = 8$ **and** $3x - 3y = 16$. $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$. Since $\frac{1}{3} = \frac{1}{3} \neq \frac{1}{2}$, the lines are parallel: the pair is **inconsistent**, no solution.

Step 3. **(iii)** $2x + y - 6 = 0$ **and** $4x - 2y - 4 = 0$. $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{1}{-2}$. These differ, so the pair is **consistent** with a unique solution. To find it, take two points on each

line: for $2x + y = 6$, points $(0, 6)$ and $(3, 0)$; for $4x - 2y = 4$ (that is $y = 2x - 2$), points $(0, -2)$ and $(1, 0)$. The lines meet at $(2, 2)$.

Step 4. **(iv)** $2x - 2y - 2 = 0$ **and** $4x - 4y - 5 = 0$**.** Simplify the first to $x - y - 1 = 0$ and the second to $4x - 4y - 5 = 0$. $\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}$, $\frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}$, $\frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$. Since $\frac{1}{2} = \frac{1}{2} \neq \frac{2}{5}$, the lines are parallel: the pair is **inconsistent**, no solution.

The graph for part (iii) shows the unique meeting point:



Final Answer: (i) Consistent (infinitely many solutions); (ii) Inconsistent; (iii) Consistent, $x = 2,\ y = 2$; (iv) Inconsistent.

☞ Exam Tip

Run the ratio test before drawing anything. It tells you in one line whether a solution even exists, so you only spend time graphing the parts that are consistent with a unique solution, here just part (iii).

EXPERT'S SOLUTION *: Rohan Desai, M.Sc Mathematics, University of Mumbai*

Classify with the ratios, then graph only the one part that needs it. The smart order here is to apply the ratio test to all four pairs first, because three of them are settled without a single plotted point.

- **Settle by ratio:** part (i) has all three ratios equal, so the lines coincide and the pair is consistent with infinitely many solutions; parts (ii) and (iv) match in the first two ratios but not the third, the fingerprint of parallel lines, so both are inconsistent.
- **Graph only one:** only part (iii) has unequal first two ratios, which guarantees a single crossing point, so that is the only pair worth drawing, and plotting two clean points on each line gives the meeting at $(2, 2)$.
- **Check the point:** put that point into both equations, since $2(2) + 2 = 6$ and

$4(2) - 2(2) = 4$ are both true, which confirms the read-off before any marks are claimed.

- **Standard form first:** in parts (ii) and (iv), write each equation as $ax + by + c = 0$ before reading the constant, because a term left on the wrong side flips the third ratio and could turn a parallel pair into a coincident one. Simplifying also helps, since dividing $2x - 2y - 2 = 0$ by 2 gives $x - y - 1 = 0$ and makes the comparison obvious.
- **Marks rule:** state every verdict with its ratio reasoning, and supply a graph only where a unique solution exists, because graphing the inconsistent pairs earns nothing while skipping the one unique pair loses the plotting marks.

Final Answer: Consistent: (i) infinite, (iii) $x = 2, y = 2$. Inconsistent: (ii), (iv).

Q 3.5 **Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.**

SOLUTION

Concept used. For a rectangle, the perimeter is $2(\text{length} + \text{width})$, so half the perimeter is simply length + width. We name the two dimensions, write one equation from "half the perimeter is 36" and another from "length is 4 m more than width", and solve the pair.

Step 1. Let the length be x m and the width be y m. Half the perimeter is length + width, so

$$x + y = 36.$$

Step 2. The length is 4 m more than the width:

$$x = y + 4 \;\Rightarrow\; x - y = 4.$$

Step 3. Add the two equations $x + y = 36$ and $x - y = 4$:

$$(x + y) + (x - y) = 36 + 4 \;\Rightarrow\; 2x = 40 \;\Rightarrow\; x = 20.$$

Step 4. Substitute $x = 20$ into $x + y = 36$:

$$20 + y = 36 \;\Rightarrow\; y = 16.$$

Final Answer: The length is 20 m and the width is 16 m.

☞ **Use half the perimeter directly**

The problem hands you "half the perimeter $= 36$", which equals length + width. There is no need to write the full perimeter $2(l + w) = 72$ and divide; using the half straight away keeps the numbers small.

EXPERT'S SOLUTION *: Priya Menon, M.Sc Mathematics, Anna University*

Read "half the perimeter" as length plus width to avoid an extra step. The one idea that simplifies this problem is recognising that half of $2(l + w)$ is just $l + w$.

- **Clean first equation:** half the perimeter gives the tidy $x + y = 36$ straight away, rather than the longer $2(x + y) = 72$ that you would then have to divide down.
- **Add, do not substitute:** the second condition gives $x - y = 4$, and adding the two equations cancels y at once to give $x = 20$, after which $y = 16$ follows. Adding is faster here because the y terms are already $+y$ and $-y$, so they vanish with no multiplication.
- **Protect the marks:** label which letter is length and which is width, state both equations, show the elimination, and finish with a check that $20 + 16 = 36$ and $20 - 16 = 4$. Forgetting both dimensions or mixing up length and width are the two slips that cost marks, so naming the variables clearly at the start guards the answer.

Final Answer: Length $= 20$ m, width $= 16$ m.

Q 3.6 **Given the linear equation $2x + 3y - 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is: (i) intersecting lines (ii) parallel lines (iii) coincident lines**

SOLUTION

Concept used. Starting from $2x + 3y - 8 = 0$ (so $a_1 = 2$, $b_1 = 3$, $c_1 = -8$), we choose the second equation $a_2x + b_2y + c_2 = 0$ to match the required geometry, using the ratio conditions: **intersecting** needs $a_1/a_2 \neq b_1/b_2$, **parallel** needs $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, and **coincident** needs $a_1/a_2 = b_1/b_2 = c_1/c_2$. Many correct answers are possible; one valid choice is given for each case.

Step 1. **(i) Intersecting lines.** We need $a_1/a_2 \neq b_1/b_2$. Choose, for example, $a_2 = 3$, $b_2 = 2$, since $\frac{2}{3} \neq \frac{3}{2}$. A valid equation is

$$3x + 2y - 7 = 0.$$

Step 2. **(ii) Parallel lines.** We need $a_1/a_2 = b_1/b_2 \neq c_1/c_2$. Keep the same x, y ratios but change the constant: multiply a_1, b_1 by 2 to get $a_2 = 4$, $b_2 = 6$, and pick a

constant that breaks the third ratio, say $c_2 = -9$. A valid equation is

$$4x + 6y - 9 = 0,$$

because $\frac{2}{4} = \frac{3}{6} = \frac{1}{2}$ but $\frac{-8}{-9} = \frac{8}{9} \neq \frac{1}{2}$.

Step 3. **(iii) Coincident lines.** We need all three ratios equal, so simply multiply the whole given equation by any non-zero constant, say 2:

$$4x + 6y - 16 = 0,$$

because $\frac{2}{4} = \frac{3}{6} = \frac{-8}{-16} = \frac{1}{2}$.

Final Answer: (i) $3x + 2y - 7 = 0$; (ii) $4x + 6y - 9 = 0$; (iii) $4x + 6y - 16 = 0$ (one valid answer each; others exist).

♥ Why This Matters

This question reverses the usual task. Instead of testing a given pair, you build a second equation to order. Understanding the three ratio conditions lets you design lines that intersect, run parallel, or coincide on demand.

EXPERT'S SOLUTION *: Arjun Nair, M.Sc Mathematics, University of Kerala*

Engineer the second equation to fit the ratio rule for each case. The question is open ended, so the goal is not a unique answer but a second equation that provably meets each geometric condition, and the cleanest way to guarantee that is to work backwards from the ratio test.

- **Intersecting:** any choice with unequal first two ratios works, so simply swapping the coefficients to give $3x + 2y - 7 = 0$ is a safe pick that is easy to justify.
- **Parallel:** keep a and b in the same ratio as the original, easiest by multiplying $2x + 3y$ by a constant, and then choose a constant term that breaks the third ratio, which $4x + 6y - 9 = 0$ does.
- **Coincident:** the surest move is to multiply the entire given equation by a non-zero number, here 2, giving $4x + 6y - 16 = 0$, because every ratio then equals that same number automatically.
- **Show the why:** state the ratio condition being targeted before writing each equation and confirm the ratios afterwards, since the marks are for the justification rather than the particular numbers chosen, and an examiner accepts any equation that satisfies the stated condition with working shown.

Final Answer: Intersecting $3x + 2y - 7 = 0$; parallel $4x + 6y - 9 = 0$; coincident $4x + 6y - 16 = 0$.

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Q 3.7 **Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.**

SOLUTION

Concept used. Each linear equation is a straight line. Two such lines together with the x-axis (the line $y = 0$) enclose a triangle. The vertices of that triangle are the three crossing points: where the two lines meet each other, and where each line meets the x-axis.

Step 1. **Points on each line.** For $x - y + 1 = 0$ (that is $y = x + 1$): if $x = 0$, $y = 1$; if $x = 2$, $y = 3$. For $3x + 2y - 12 = 0$ (that is $y = \dfrac{12 - 3x}{2}$): if $x = 0$, $y = 6$; if $x = 4$, $y = 0$.

Step 2. **Where the two lines meet.** Solve $y = x + 1$ and $3x + 2y = 12$ together. Substitute:

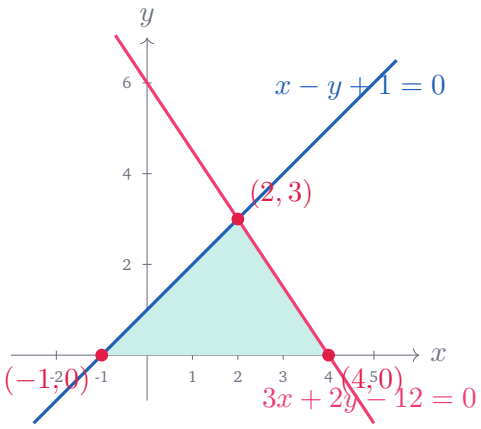
$$3x + 2(x + 1) = 12 \;\Rightarrow\; 5x + 2 = 12 \;\Rightarrow\; x = 2,$$

then $y = 2 + 1 = 3$. The lines meet at $(2, 3)$.

Step 3. **Where each line meets the x-axis ($y = 0$).** For $x - y + 1 = 0$: $x - 0 + 1 = 0$, so $x = -1$, giving $(-1, 0)$. For $3x + 2y - 12 = 0$: $3x - 12 = 0$, so $x = 4$, giving $(4, 0)$.

Step 4. The triangle has vertices $(2, 3)$, $(-1, 0)$ and $(4, 0)$.

The shaded triangle formed by the two lines and the x-axis:



Final Answer: The vertices of the triangle are $(2, 3)$, $(-1, 0)$ and $(4, 0)$.

☞ The x-axis is the line y = 0

To find where a line cuts the x-axis, set $y = 0$ and solve for x. The third side of this triangle is the x-axis itself, so two of its vertices lie on it.

EXPERT'S SOLUTION *: Sneha Patel, M.Sc Mathematics, The Maharaja Sayajirao University of Baroda*

Find the three corners as three separate intersections, then shade. The triangle here is bounded by three lines, the two given equations and the x-axis, so its three vertices are exactly the three pairwise crossings.

- **The apex:** the top corner is where the two given lines meet, and solving them together gives $(2, 3)$, the single point that satisfies both equations at once.
- **The base corners:** the base lies along the x-axis, so the other two vertices come from setting $y = 0$ in each equation, which gives $(-1, 0)$ from the first line and $(4, 0)$ from the second.
- **Draw and shade:** a student who draws both lines accurately through two plotted points each will see the same three corners, and shading the enclosed region answers the final part of the question.
- **Where marks live:** the graph must show both lines labelled, the three vertices marked with their coordinates, and the region clearly shaded, because stating the vertices without the diagram, or the diagram without the coordinates, leaves easy marks on the table.
- **Sanity check:** the base from $x = -1$ to $x = 4$ has length 5 and the height to the apex is 3, so the area is $\frac{1}{2} \times 5 \times 3 = 7.5$ square units, a tidy confirmation that the three corners are right.

Final Answer: Vertices $(2, 3)$, $(-1, 0)$, $(4, 0)$.

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Chapter 3: Pair of Linear Equations in Two Variables

About this Chapter

Class 10 Maths Chapter 3 Pair of Linear Equations in Two Variables Exercise 3.2 works with two equations in x and y together. These solutions solve every exercise by the **graphical method**, **substitution** and **elimination**. Students also learn to tell a **consistent** pair from an inconsistent one and to crack word problems on ages, money and digits.

Topics covered: Pair of linear equations • Graphical method • Consistent and inconsistent pairs • Comparing coefficient ratios • Substitution method • Elimination method • Word problems

Quick Formula Sheet

General pair:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Unique solution (intersecting):

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Infinitely many (coincident):

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

No solution (parallel):

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Also see for this chapter: Revision Notes | Formula Sheet | Exemplar Solutions

NCERT Exercise Solutions

Exercise 3.2

Q 3.1 **Solve the following pair of linear equations by the substitution method.**

(i) $x + y = 14,\ x - y = 4$ **(ii)** $s - t = 3,\ \frac{s}{3} + \frac{t}{2} = 6$ **(iii)** $3x - y = 3,\ 9x - 3y = 9$ **(iv)** $0.2x + 0.3y = 1.3,\ 0.4x + 0.5y = 2.3$ **(v)** $\sqrt{2}\,x + \sqrt{3}\,y = 0,\ \sqrt{3}\,x - \sqrt{8}\,y = 0$ **(vi)** $\frac{3x}{2} - \frac{5y}{3} = -2,\ \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$

SOLUTION

Concept used. In the **substitution method** we take one equation, express one variable in terms of the other, and substitute that expression into the second equation. This leaves a single equation in one variable, which we solve, and then back-substitute to find the other variable.

Step 1. **(i)** $x + y = 14$, $x - y = 4$. From the second, $x = 4 + y$. Substitute into the first:

$$(4 + y) + y = 14 \;\Rightarrow\; 4 + 2y = 14 \;\Rightarrow\; 2y = 10 \;\Rightarrow\; y = 5.$$

Then $x = 4 + 5 = 9$. So $x = 9$, $y = 5$.

Step 2. **(ii)** $s - t = 3$, $\dfrac{s}{3} + \dfrac{t}{2} = 6$. From the first, $s = 3 + t$. Substitute:

$$\frac{3 + t}{3} + \frac{t}{2} = 6.$$

Multiply through by 6: $2(3 + t) + 3t = 36$, so $6 + 2t + 3t = 36$, that is $5t = 30$, giving $t = 6$. Then $s = 3 + 6 = 9$. So $s = 9$, $t = 6$.

Step 3. **(iii)** $3x - y = 3$, $9x - 3y = 9$. From the first, $y = 3x - 3$. Substitute into the second:

$$9x - 3(3x - 3) = 9 \;\Rightarrow\; 9x - 9x + 9 = 9 \;\Rightarrow\; 9 = 9.$$

This is always true, so the pair has **infinitely many solutions** (the second equation is three times the first).

Step 4. **(iv)** $0.2x + 0.3y = 1.3$, $0.4x + 0.5y = 2.3$. Multiply both by 10 to clear decimals: $2x + 3y = 13$ and $4x + 5y = 23$. From the first, $x = \dfrac{13 - 3y}{2}$. Substitute into $4x + 5y = 23$:

$$4 \cdot \frac{13 - 3y}{2} + 5y = 23 \;\Rightarrow\; 2(13 - 3y) + 5y = 23.$$

So $26 - 6y + 5y = 23$, that is $26 - y = 23$, giving $y = 3$. Then $x = \dfrac{13 - 9}{2} = 2$. So $x = 2$, $y = 3$.

Step 5. **(v)** $\sqrt{2}\,x + \sqrt{3}\,y = 0$, $\sqrt{3}\,x - \sqrt{8}\,y = 0$. From the first, $x = -\dfrac{\sqrt{3}}{\sqrt{2}}\,y$. Substitute into the second:

$$\sqrt{3}\left(-\frac{\sqrt{3}}{\sqrt{2}}\,y\right) - \sqrt{8}\,y = 0 \;\Rightarrow\; -\frac{3}{\sqrt{2}}\,y - 2\sqrt{2}\,y = 0.$$

Since $-\dfrac{3}{\sqrt{2}} - 2\sqrt{2} = -\dfrac{3 + 4}{\sqrt{2}} = -\dfrac{7}{\sqrt{2}} \neq 0$, the only possibility is $y = 0$, and then $x = 0$. So $x = 0$, $y = 0$.

Step 6. **(vi)** $\dfrac{3x}{2} - \dfrac{5y}{3} = -2$, $\dfrac{x}{3} + \dfrac{y}{2} = \dfrac{13}{6}$. Clear fractions. Multiply the first by 6: $9x - 10y = -12$. Multiply the second by 6: $2x + 3y = 13$, so $x = \dfrac{13 - 3y}{2}$.

Substitute into $9x - 10y = -12$:

$$9 \cdot \frac{13 - 3y}{2} - 10y = -12 \;\Rightarrow\; \frac{117 - 27y}{2} - 10y = -12.$$

Multiply by 2: $117 - 27y - 20y = -24$, so $117 - 47y = -24$, that is $47y = 141$, giving $y = 3$. Then $x = \frac{13 - 9}{2} = 2$. So $x = 2$, $y = 3$.

Final Answer: (i) $x = 9, y = 5$; (ii) $s = 9, t = 6$; (iii) infinitely many solutions; (iv) $x = 2, y = 3$; (v) $x = 0, y = 0$; (vi) $x = 2, y = 3$.

☞ **Clear fractions and decimals first**

Parts (ii), (iv) and (vi) look messy only because of fractions and decimals. Multiplying each equation by a suitable number to make every coefficient a whole number turns them into routine substitution problems.

EXPERT'S SOLUTION *: Kavya Sharma, M.Sc Mathematics, Banaras Hindu University*

Tidy the coefficients before substituting, and read a vanishing variable correctly. Substitution is reliable across all six parts, but the practical skill is preparing each pair first.

- **Clear the mess:** multiply by 10 to remove the decimals in part (iv), and by 6 to remove the fractions in parts (ii) and (vi), so the substitution then runs over whole numbers and the chance of an arithmetic slip drops sharply.
- **Read an identity:** part (iii) collapses to $9 = 9$, true for every value, which means the two equations are the same line and there are infinitely many solutions, not "no answer". A student must state that conclusion rather than leave the problem blank, because the mark is for recognising the dependent pair.
- **The zero case:** part (v) is the mirror, since both constants are zero, so the lines pass through the origin and substitution forces $y = 0$ and then $x = 0$, the single solution $(0, 0)$. The surds look frightening but never need to be evaluated, because the combined coefficient is simply non-zero.
- **Isolate wisely:** pick the equation where a coefficient is one or easy, as with $x = 4 + y$ in part (i) and $s = 3 + t$ in part (ii), since that keeps the algebra clean and the substitution short.
- **Board habit:** clear the fractions on a visible line, isolate one variable, substitute, solve, and back-substitute, finishing each part with both values stated together and a one-line check that puts the pair back into the original equations.

Final Answer: $x = 9, y = 5$; $s = 9, t = 6$; infinite; $x = 2, y = 3$; $x = 0, y = 0$; $x = 2, y = 3$.

Q 3.2 **Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.**

SOLUTION

Concept used. We first solve the pair of linear equations for x and y. Once both values are known, we put them into the relation $y = mx + 3$ and solve for the constant m. The same point (x, y) must satisfy this relation, so it fixes a single value of m.

Step 1. Subtract the second equation from the first to eliminate x:

$$(2x + 3y) - (2x - 4y) = 11 - (-24) \Rightarrow 7y = 35 \Rightarrow y = 5.$$

Step 2. Substitute $y = 5$ into $2x + 3y = 11$:

$$2x + 15 = 11 \Rightarrow 2x = -4 \Rightarrow x = -2.$$

So $x = -2$, $y = 5$.

Step 3. Put $x = -2$, $y = 5$ into $y = mx + 3$:

$$5 = m(-2) + 3 \Rightarrow 5 - 3 = -2m \Rightarrow 2 = -2m \Rightarrow m = -1.$$

Final Answer: $x = -2$, $y = 5$, and $m = -1$.

☞ Solve for the point, then fit the line

The relation $y = mx + 3$ is a line through $(0, 3)$ with unknown slope m. Once you know the solution point $(-2, 5)$, demand that the line passes through it; that single condition gives m.

EXPERT'S SOLUTION *: Imran Khan, M.Sc Mathematics, Aligarh Muslim University*

Get the solution point first, because m is decided only after x and y are known. The question has two stages, and the order matters: solve the pair, then use the point to find m.

- **Eliminate x:** since both equations carry $2x$, subtracting one from the other removes x in a single step and gives $y = 5$, after which $x = -2$ follows from either original equation.

- **Fit the slope:** the relation $y = mx + 3$ is just one more condition the same point must obey, so substituting $x = -2$ and $y = 5$ turns it into $5 = -2m + 3$, which solves to $m = -1$.
- **Avoid the trap:** a frequent error is to chase m before solving for the point, or to drop the values into the wrong slots, so writing the relation with $y = 5$ and $x = -2$ clearly labelled prevents that, and the check $5 = (-1)(-2) + 3 = 2 + 3 = 5$ confirms the slope. Present the elimination, the solved point, and the substitution for m as three clear stages.

Final Answer: $x = -2$, $y = 5$, $m = -1$.

Q 3.3 **Form the pair of linear equations for the following problems and find their solution by substitution method. (i) The difference between two numbers is 26 and one number is three times the other. Find them. (ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them. (iii) The coach of a cricket team buys 7 bats and 6 balls for Rs. 3800. Later, she buys 3 bats and 5 balls for Rs. 1750. Find the cost of each bat and each ball. (iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs. 105 and for a journey of 15 km, the charge paid is Rs. 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km? (v) A fraction becomes $\frac{9}{11}$ if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$. Find the fraction. (vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?**

SOLUTION

Concept used. Each word problem is turned into a pair of linear equations by naming two unknowns and writing two relations from the conditions. We then solve each pair by substitution, expressing one variable in terms of the other and substituting.

Step 1. **(i) Two numbers.** Let the numbers be x (larger) and y. Then $x - y = 26$ and $x = 3y$. Substitute $x = 3y$ into $x - y = 26$:

$$3y - y = 26 \;\Rightarrow\; 2y = 26 \;\Rightarrow\; y = 13, \quad x = 3(13) = 39.$$

The numbers are 39 and 13.

Step 2. **(ii) Supplementary angles.** Two angles are supplementary when they add to $180°$. Let the larger be x and the smaller y. Then $x + y = 180$ and $x - y = 18$.

From the second, $x = 18 + y$. Substitute:

$$(18 + y) + y = 180 \;\Rightarrow\; 18 + 2y = 180 \;\Rightarrow\; 2y = 162 \;\Rightarrow\; y = 81,$$

so $x = 18 + 81 = 99$. The angles are 99° and 81°.

Step 3. **(iii) Bats and balls.** Let a bat cost Rs. x and a ball Rs. y. Then $7x + 6y = 3800$ and $3x + 5y = 1750$. From the second, $x = \dfrac{1750 - 5y}{3}$. Substitute into $7x + 6y = 3800$:

$$7 \cdot \frac{1750 - 5y}{3} + 6y = 3800.$$

Multiply by 3: $7(1750 - 5y) + 18y = 11400$, so $12250 - 35y + 18y = 11400$, that is $12250 - 17y = 11400$, giving $17y = 850$, so $y = 50$. Then $x = \dfrac{1750 - 250}{3} = \dfrac{1500}{3} = 500$. A bat costs Rs. 500 and a ball Rs. 50.

Step 4. **(iv) Taxi charges.** Let the fixed charge be Rs. x and the charge per km be Rs. y. Then $x + 10y = 105$ and $x + 15y = 155$. From the first, $x = 105 - 10y$. Substitute into the second:

$$(105 - 10y) + 15y = 155 \;\Rightarrow\; 105 + 5y = 155 \;\Rightarrow\; 5y = 50 \;\Rightarrow\; y = 10,$$

so $x = 105 - 100 = 5$. Fixed charge Rs. 5, per km Rs. 10. For 25 km the charge is $x + 25y = 5 + 25(10) =$ Rs. 255.

Step 5. **(v) The fraction.** Let the fraction be $\dfrac{x}{y}$. Adding 2 to each: $\dfrac{x+2}{y+2} = \dfrac{9}{11}$, so $11(x + 2) = 9(y + 2)$, that is $11x - 9y = -4$. Adding 3 to each: $\dfrac{x+3}{y+3} = \dfrac{5}{6}$, so $6(x + 3) = 5(y + 3)$, that is $6x - 5y = -3$. From $6x - 5y = -3$, $x = \dfrac{5y - 3}{6}$. Substitute into $11x - 9y = -4$:

$$11 \cdot \frac{5y - 3}{6} - 9y = -4.$$

Multiply by 6: $11(5y - 3) - 54y = -24$, so $55y - 33 - 54y = -24$, that is $y - 33 = -24$, giving $y = 9$. Then $x = \dfrac{45 - 3}{6} = \dfrac{42}{6} = 7$. The fraction is $\dfrac{7}{9}$.

Step 6. **(vi) Jacob and son.** Let Jacob's present age be x years and his son's y years. Five years hence: $x + 5 = 3(y + 5)$, that is $x - 3y = 10$. Five years ago: $x - 5 = 7(y - 5)$, that is $x - 7y = -30$. From the first, $x = 10 + 3y$. Substitute into the second:

$$(10 + 3y) - 7y = -30 \;\Rightarrow\; 10 - 4y = -30 \;\Rightarrow\; -4y = -40 \;\Rightarrow\; y = 10,$$

so $x = 10 + 30 = 40$. Jacob is 40 years and his son 10 years.

Final Answer: (i) 39 and 13; (ii) 99° and 81°; (iii) bat Rs. 500, ball Rs. 50; (iv) fixed Rs. 5, per km Rs. 10, and Rs. 255 for 25 km; (v) $\dfrac{7}{9}$; (vi) Jacob 40 yr, son 10 yr.

✗ Supplementary means a sum of 180 degrees

In part (ii) the key fact is that supplementary angles add to $180°$, not $90°$ (that is complementary). Using the wrong total changes both angles and loses the question.

EXPERT'S SOLUTION *: Deepa Reddy, M.Sc Mathematics, University of Hyderabad*

Translate each clue into an equation in the order it is written, then substitute the simplest variable. Every part of this question rewards careful reading far more than heavy algebra, so the time is best spent on the set-up.

- **Isolate what is given:** "difference is 26" and "one is three times the other" give $x - y = 26$ and $x = 3y$, and because x is already isolated in the second relation, the substitution into the first is instant and needs no rearranging.
- **Recall the fact:** the supplementary angles in part (ii) hinge on the angles summing to $180°$, the one piece of recalled knowledge the question tests, and a student who confuses supplementary with complementary uses $90°$ and gets both angles wrong.
- **Tame the big numbers:** the bats and balls pair in part (iii) has no modelling trick, but the numbers are large, so isolating x from the simpler equation $3x + 5y = 1750$ keeps the substitution manageable and avoids unwieldy fractions partway through.
- **Rearrange first:** the fraction problem (v) and the age problem (vi) each need a small rearrangement, cross-multiplying $\frac{x+2}{y+2} = \frac{9}{11}$ into $11x - 9y = -4$, and turning "three times" and "seven times" into linear forms, after which the substitution is routine.
- **Shift both ages:** a common slip in age problems is to apply the "five years ago" and "five years hence" change to only one person, so each age must shift by the same amount before the multiple is taken.
- **Answer everything:** the taxi problem (iv) has a final part students often skip, the charge for 25 km, so after finding the fixed charge 5 and rate 10 the answer must include $5 + 25 \times 10 = 255$. Reading the question to the very end is what secures the last mark in parts (iv) and (vi).
- **Verify each part:** name both unknowns clearly, write the two equations, substitute, solve, and finish each part with a quick check such as $7 + 2$ over $9 + 2$ giving $\frac{9}{11}$, since one verifying line per part catches an arithmetic slip before it ever costs the accuracy mark on that part of the answer.

Final Answer: $39, 13$; $99°, 81°$; bat 500, ball 50; fixed 5, rate 10, 25 km $= 255$; $\frac{7}{9}$; Jacob 40, son 10.

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Chapter 3: Pair of Linear Equations in Two Variables

About this Chapter

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Topics covered: Pair of linear equations • Graphical method • Consistent and inconsistent pairs • Comparing coefficient ratios • Substitution method • Elimination method • Word problems

Quick Formula Sheet

General pair:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Unique solution (intersecting):

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Infinitely many (coincident):

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

No solution (parallel):

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Also see for this chapter: Revision Notes | Formula Sheet | Exemplar Solutions

NCERT Exercise Solutions

Exercise 3.3

Q 3.1 **Solve the following pair of linear equations by the elimination method and the substitution method: (i)** $x + y = 5$ **and** $2x - 3y = 4$ **(ii)** $3x + 4y = 10$ **and** $2x - 2y = 2$ **(iii)** $3x - 5y - 4 = 0$ **and** $9x = 2y + 7$ **(iv)** $\frac{x}{2} + \frac{2y}{3} = -1$ **and** $x - \frac{y}{3} = 3$

SOLUTION

Concept used. In the **elimination method** we multiply the equations by suitable numbers so that one variable has equal coefficients, then add or subtract to remove it. In the **substitution method** we express one variable from one equation and put it into the other. Both methods must give the same solution. The working below uses elimination; the substitution route is noted briefly for each.

Step 1. **(i)** $x + y = 5$, $2x - 3y = 4$. Multiply the first by 3: $3x + 3y = 15$. Add to $2x - 3y = 4$ to eliminate y:

$$(3x + 3y) + (2x - 3y) = 15 + 4 \;\Rightarrow\; 5x = 19 \;\Rightarrow\; x = \frac{19}{5}.$$

From $x + y = 5$, $y = 5 - \dfrac{19}{5} = \dfrac{6}{5}$. (By substitution: $y = 5 - x$, then $2x - 3(5 - x) = 4$ gives the same x.)

Step 2. **(ii)** $3x + 4y = 10$, $2x - 2y = 2$. Multiply the second by 2: $4x - 4y = 4$. Add to $3x + 4y = 10$ to eliminate y:

$$(3x + 4y) + (4x - 4y) = 10 + 4 \;\Rightarrow\; 7x = 14 \;\Rightarrow\; x = 2.$$

From $2x - 2y = 2$, $4 - 2y = 2$, so $y = 1$.

Step 3. **(iii)** $3x - 5y - 4 = 0$, $9x = 2y + 7$. Write both in standard form: $3x - 5y = 4$ and $9x - 2y = 7$. Multiply the first by 3: $9x - 15y = 12$. Subtract $9x - 2y = 7$:

$$(9x - 15y) - (9x - 2y) = 12 - 7 \;\Rightarrow\; -13y = 5 \;\Rightarrow\; y = -\frac{5}{13}.$$

From $3x - 5y = 4$, $3x = 4 + 5\left(-\dfrac{5}{13}\right) = 4 - \dfrac{25}{13} = \dfrac{27}{13}$, so $x = \dfrac{9}{13}$.

Step 4. **(iv)** $\dfrac{x}{2} + \dfrac{2y}{3} = -1$, $x - \dfrac{y}{3} = 3$. Clear fractions. Multiply the first by 6: $3x + 4y = -6$. Multiply the second by 3: $3x - y = 9$. Subtract the second from the first to remove x:

$$(3x + 4y) - (3x - y) = -6 - 9 \;\Rightarrow\; 5y = -15 \;\Rightarrow\; y = -3.$$

From $3x - y = 9$, $3x + 3 = 9$, so $x = 2$.

Final Answer: (i) $x = \dfrac{19}{5},\ y = \dfrac{6}{5}$; (ii) $x = 2,\ y = 1$; (iii) $x = \dfrac{9}{13},\ y = -\dfrac{5}{13}$; (iv) $x = 2,\ y = -3$.

☞ **Match coefficients with the smallest multiplier**

Before eliminating, look for the variable whose coefficients line up with the least effort. In part (ii), doubling $2x - 2y = 2$ matches the $4y$ in the other equation, so y cancels in one step.

EXPERT'S SOLUTION *: Sanjay Gupta, M.Sc Mathematics, Panjab University*

Pick the variable that is cheapest to eliminate, and always tidy fractions first. Both methods are required by the question, but elimination is usually the quicker route when the coefficients can be matched with a small multiplier, and the real skill is spotting that match before any arithmetic.

- **Find the match:** in part (i), tripling the first equation lines up the terms in y; in part (ii), doubling the second matches the term in y; in part (iii), tripling the first matches the term in x, so the right multiplier removes a variable in one clean step every time.
- **Clear before eliminating:** in part (iv), clearing the fractions first to $3x + 4y = -6$ and $3x - y = 9$ makes the terms in x cancel directly by subtraction, which is far tidier than wrestling with fractional coefficients while you eliminate.
- **Mind the signs:** keep signs straight when subtracting, since $-13y = 5$ in part (iii) gives a negative value, not a positive one, and a single sign slip there cascades into a wrong value for the other variable as well.
- **Check and present:** after finding both values, substituting them back into either original equation is a fast check, such as $3(2) + 4(1) = 10$ in part (ii). The marks reward a clear elimination step, the solved variable, the back-substitution and the final pair, so write each stage on its own line to keep the method transparent.

Final Answer: $\frac{19}{5}, \frac{6}{5}$; $2, 1$; $\frac{9}{13}, -\frac{5}{13}$; $2, -3$.

Q 3.2 **Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method: (i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction? (ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu? (iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number. (iv) Meena went to a bank to withdraw Rs. 2000. She asked the cashier to give her Rs. 50 and Rs. 100 notes only. Meena got 25 notes in all. Find how many notes of Rs. 50 and Rs. 100 she received. (v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs. 27 for a book kept for seven days, while Susy paid Rs. 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.**

SOLUTION

Concept used. Each situation is written as a **pair of linear equations** in two unknowns. We then solve by the **elimination method**, making the coefficient of one variable equal in both equations and adding or subtracting to remove it.

Step 1. **(i) The fraction.** Let it be $\frac{x}{y}$. "$+1$ to numerator, -1 from denominator gives 1": $\frac{x+1}{y-1}=1$, so $x+1=y-1$, that is $x-y=-2$. "$+1$ to denominator gives $\frac{1}{2}$": $\frac{x}{y+1}=\frac{1}{2}$, so $2x=y+1$, that is $2x-y=1$. Subtract $x-y=-2$ from $2x-y=1$:

$$(2x-y)-(x-y)=1-(-2) \;\Rightarrow\; x=3.$$

Then from $x-y=-2$, $y=5$. The fraction is $\frac{3}{5}$.

Step 2. **(ii) Nuri and Sonu.** Let Nuri be x years and Sonu y years now. Five years ago: $x-5=3(y-5)$, that is $x-3y=-10$. Ten years later: $x+10=2(y+10)$, that is $x-2y=10$. Subtract the first from the second:

$$(x-2y)-(x-3y)=10-(-10) \;\Rightarrow\; y=20.$$

Then from $x-2y=10$, $x=10+40=50$. Nuri is 50 years and Sonu 20 years.

Step 3. **(iii) Two-digit number.** Let the ten's digit be x and the unit's digit y, so the number is $10x+y$ and its reverse is $10y+x$. Digit sum: $x+y=9$. "Nine times the number is twice the reverse": $9(10x+y)=2(10y+x)$, so $90x+9y=20y+2x$, that is $88x-11y=0$, which simplifies to $8x-y=0$. Add $x+y=9$ and $8x-y=0$:

$$(x+y)+(8x-y)=9+0 \;\Rightarrow\; 9x=9 \;\Rightarrow\; x=1,$$

then $y=9-1=8$. The number is $10(1)+8=18$.

Step 4. **(iv) The notes.** Let the number of Rs. 50 notes be x and Rs. 100 notes be y. Total notes: $x+y=25$. Total value: $50x+100y=2000$, which simplifies to $x+2y=40$. Subtract $x+y=25$ from $x+2y=40$:

$$(x+2y)-(x+y)=40-25 \;\Rightarrow\; y=15,$$

then $x=25-15=10$. So 10 notes of Rs. 50 and 15 notes of Rs. 100.

Step 5. **(v) Library charges.** Let the fixed charge (for the first three days) be Rs. x and the charge for each extra day be Rs. y. Saritha kept the book 7 days, so 4 extra days: $x+4y=27$. Susy kept it 5 days, so 2 extra days: $x+2y=21$. Subtract the second from the first:

$$(x+4y)-(x+2y)=27-21 \;\Rightarrow\; 2y=6 \;\Rightarrow\; y=3,$$

then $x+2(3)=21$, so $x=15$. Fixed charge Rs. 15, extra day Rs. 3.

> **Final Answer:** (i) $\frac{3}{5}$; (ii) Nuri 50 yr, Sonu 20 yr; (iii) 18; (iv) 10 notes of Rs. 50, 15 notes of Rs. 100; (v) fixed Rs. 15, extra day Rs. 3.

♥ Why This Matters

Digit, age, money and fraction problems all become two linear equations once you name the right unknowns. The same elimination routine then cracks every one of them, which is why mastering the set-up matters more than memorising each type.

EXPERT'S SOLUTION *: Lakshmi Pillai, M.Sc Mathematics, University of Calicut*

Name the unknowns to match the question, then simplify before eliminating. The hard part of each problem is the translation, not the arithmetic, so the unknowns should be chosen to fit exactly what the question asks.

- **Fraction care:** in part (i) the two conditions become $x - y = -2$ and $2x - y = 1$, and a student must take care that "denominator subtract 1" lands on the denominator, not the numerator, since swapping the two changes the whole answer.
- **Shift both ages:** in part (ii) each person's age must shift by the same five or ten years before the multiple "thrice" or "twice" is applied, so the relation is $x - 5 = 3(y - 5)$ rather than $x - 5 = 3y$, and getting that shift onto both ages is the most common stumbling point.
- **Use place value:** in part (iii) the number is $10x + y$ and its reverse $10y + x$, and writing them this way turns "nine times the number is twice the reverse" into $88x - 11y = 0$, which divides down to the tidy $8x - y = 0$. Treating the number as just $x + y$ is the error that wrecks this part.
- **Simplify and read clauses:** parts (iv) and (v) both simplify before elimination, since $50x + 100y = 2000$ reduces to $x + 2y = 40$, while "seven days" means four extra days beyond the first three and "five days" means two extra, not the full counts.
- **Eliminate then check:** once each pair is in clean form, subtracting to remove the matching variable is one quick step, and back-substitution gives the second value. Verify it, for instance that ten fifties and fifteen hundreds total 25 notes and Rs. 2000, or that the digits of 18 add to 9 and nine times 18 equals twice 81.
- **The recurring trap:** across all five parts the real danger is mis-modelling the words, especially the "first three days" clause, the equal shift in ages, and the place value of a two-digit number, so a careful set-up wins the marks and a quick verification secures them.

> **Final Answer:** $\frac{3}{5}$; Nuri 50, Sonu 20; 18; ten Rs. 50 and fifteen Rs. 100; fixed 15, per extra day 3.

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