collegedunia

Collegedunia NCERT Solutions

Step-by-step solutions, alternate methods & exam tips for Class 10 Mathematics

Chapter 4: Quadratic Equations

About this Chapter

These solutions for Class 10 Maths Chapter 4 Quadratic Equations Exercise 4.2 work out every exercise question in full. A **quadratic equation** has the form $ax^2 + bx + c = 0$ with $a \neq 0$. Students learn to find **roots** by factorisation, judge the nature of roots from the **discriminant** $b^2 - 4ac$, and set up word problems on ages, areas and speeds.

Topics covered: Standard form $ax^2 + bx + c = 0$ • Roots and zeroes • Solving by factorisation • Splitting the middle term • Discriminant $b^2 - 4ac$ • Nature of roots • Word problems

Quick Formula Sheet

Standard form:
$ax^2 + bx + c = 0,\ a \neq 0$

Quadratic formula:
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant:
$D = b^2 - 4ac$

Nature of roots:
$D > 0$: two distinct real roots
$D = 0$: two equal real roots
$D < 0$: no real roots

Also see for this chapter: Revision Notes | Formula Sheet | Exemplar Solutions

NCERT Exercise Solutions

Exercise 4.2

Q 4.1 **Find the roots of the following quadratic equations by factorisation: (i)** $x^2 - 3x - 10 = 0$ **(ii)** $2x^2 + x - 6 = 0$ **(iii)** $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$ **(iv)** $2x^2 - x + \dfrac{1}{8} = 0$ **(v)** $100x^2 - 20x + 1 = 0$

SOLUTION

Concept used. To solve $ax^2 + bx + c = 0$ by **factorisation**, we **split the middle term**: find two numbers whose product is $a \times c$ and whose sum is b. We rewrite the middle

term using these two numbers, take out common factors in pairs, and write the expression as a product of two linear factors. Setting each factor equal to zero gives the **roots**, because if a product is zero then at least one factor is zero.

Step 1. **(i)** $x^2 - 3x - 10 = 0$. Here $a \times c = 1 \times (-10) = -10$ and we need two numbers with product -10 and sum -3: these are -5 and $+2$. Split:

$$x^2 - 5x + 2x - 10 = 0 \;\Rightarrow\; x(x-5) + 2(x-5) = 0 \;\Rightarrow\; (x-5)(x+2) = 0.$$

So $x - 5 = 0$ or $x + 2 = 0$, giving $x = 5$ or $x = -2$.

Step 2. **(ii)** $2x^2 + x - 6 = 0$. Here $a \times c = 2 \times (-6) = -12$, and two numbers with product -12 and sum $+1$ are $+4$ and -3. Split:

$$2x^2 + 4x - 3x - 6 = 0 \;\Rightarrow\; 2x(x+2) - 3(x+2) = 0 \;\Rightarrow\; (x+2)(2x-3) = 0.$$

So $x + 2 = 0$ or $2x - 3 = 0$, giving $x = -2$ or $x = \frac{3}{2}$.

Step 3. **(iii)** $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$. Here $a \times c = \sqrt{2} \times 5\sqrt{2} = 5 \times 2 = 10$, and two numbers with product 10 and sum 7 are 5 and 2. Split:

$$\sqrt{2}\,x^2 + 5x + 2x + 5\sqrt{2} = 0.$$

Group: $x(\sqrt{2}\,x + 5) + \sqrt{2}(\sqrt{2}\,x + 5) = 0$, that is $(\sqrt{2}\,x + 5)(x + \sqrt{2}) = 0$. So $\sqrt{2}\,x + 5 = 0$ or $x + \sqrt{2} = 0$, giving $x = -\frac{5}{\sqrt{2}} = -\frac{5\sqrt{2}}{2}$ or $x = -\sqrt{2}$.

Step 4. **(iv)** $2x^2 - x + \frac{1}{8} = 0$. Multiply throughout by 8 to clear the fraction:

$$16x^2 - 8x + 1 = 0.$$

Now $a \times c = 16 \times 1 = 16$ and two numbers with product 16 and sum -8 are -4 and -4. Split:

$$16x^2 - 4x - 4x + 1 = 0 \;\Rightarrow\; 4x(4x-1) - 1(4x-1) = 0 \;\Rightarrow\; (4x-1)(4x-1) = 0.$$

So $4x - 1 = 0$ twice, giving the repeated root $x = \frac{1}{4}$.

Step 5. **(v)** $100x^2 - 20x + 1 = 0$. Here $a \times c = 100 \times 1 = 100$ and two numbers with product 100 and sum -20 are -10 and -10. Split:

$$100x^2 - 10x - 10x + 1 = 0 \;\Rightarrow\; 10x(10x-1) - 1(10x-1) = 0 \;\Rightarrow\; (10x-1)(10x-1) = 0.$$

So $10x - 1 = 0$ twice, giving the repeated root $x = \frac{1}{10}$.

Final Answer: (i) $x = 5,\ -2$; (ii) $x = \dfrac{3}{2},\ -2$; (iii) $x = -\sqrt{2},\ -\dfrac{5\sqrt{2}}{2}$; (iv) $x = \dfrac{1}{4}$ (repeated); (v) $x = \dfrac{1}{10}$ (repeated).

☞ **Product and sum at a glance**

Splitting the middle term needs two numbers whose **product** is $a \times c$ and whose **sum** is the middle coefficient b. List the factor pairs of $a \times c$ first, then pick the pair that adds to b.

EXPERT'S SOLUTION *: Meera Krishnan, M.Sc Mathematics, University of Madras*

Clear fractions and surds before splitting the middle term. The two trickier parts here become routine once the equation is tidied, so always smooth the coefficients before you go hunting for the split.

- **Kill the fraction:** in part (iv) the eighth makes the product awkward, so multiply the whole equation by eight to get $16x^2 - 8x + 1 = 0$, where every number is now whole and the split is obvious; multiplying by a constant never changes the roots, so the step is completely safe.
- **Tame the surd:** in part (iii) the root of two sits in both the leading and the constant term, which looks intimidating, but the product comes out to the clean integer ten, so the ordinary product-and-sum method still works once the grouping keeps each surd with the right factor.
- **Spot the perfect square:** parts (iv) and (v) each give a repeated root, the tell-tale sign that the quadratic is a perfect square, so recognising that pattern lets you both find and confirm the answer at once without any splitting; report a repeated root once, with a short note, never twice as if it were two different values.

A useful board habit across all five parts is to substitute each root back into the original equation as a final line, since the two-factor form makes any sign or arithmetic slip show up immediately, and that one quick check very often rescues an otherwise careless answer and protects the accuracy mark.

> **Final Answer:** $x = 5, -2$; $x = \frac{3}{2}, -2$; $x = -\sqrt{2}, -\frac{5\sqrt{2}}{2}$; $x = \frac{1}{4}$; $x = \frac{1}{10}$.

Q 4.2 **Solve the problems given in Example 1. (i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find how many marbles they had to start with. (ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs. 750. Find the number of toys produced on that day.**

SOLUTION

Concept used. Example 1 already reduced each situation to a quadratic equation. We now **solve** each one by **factorisation** (splitting the middle term) and then pick the values that make sense in the real-life context (for instance, a count of marbles or toys must be a positive whole number). From Example 1, the marble equation is $x^2 - 45x + 324 = 0$ and the toy equation is $x^2 - 55x + 750 = 0$.

Step 1. **(i) Marbles.** The equation is $x^2 - 45x + 324 = 0$, where x is the number of marbles John had. Split the middle term using two numbers with product 324 and sum -45, namely -9 and -36:

$$x^2 - 9x - 36x + 324 = 0 \;\Rightarrow\; x(x-9) - 36(x-9) = 0 \;\Rightarrow\; (x-9)(x-36) = 0.$$

So $x = 9$ or $x = 36$. If John had 9, Jivanti had $45 - 9 = 36$; if John had 36, Jivanti had $45 - 36 = 9$. Both cases describe the same pair.

Step 2. **(ii) Toys.** The equation is $x^2 - 55x + 750 = 0$, where x is the number of toys produced. Two numbers with product 750 and sum -55 are -25 and -30:

$$x^2 - 25x - 30x + 750 = 0 \;\Rightarrow\; x(x-25) - 30(x-25) = 0 \;\Rightarrow\; (x-25)(x-30) = 0.$$

So $x = 25$ or $x = 30$. Both are positive whole numbers, so each is a valid number of toys.

> **Final Answer:** (i) The two friends started with 36 and 9 marbles. (ii) The number of toys produced is 25 or 30.

☞ Read the context for valid roots

A quadratic can give two roots, but a count of objects must be a whole positive number. Always test both roots against the real situation and reject any that make no sense.

EXPERT'S SOLUTION *: Rohan Desai, M.Sc Mathematics, University of Mumbai*

Both roots can be acceptable, so do not discard one by habit. Many word problems force you to throw away a negative or fractional root, but these two are different and worth a second look before you reject anything.

- **Marbles, swapped:** the two roots are simply the two friends exchanged, so they describe a single physical answer, namely one friend with thirty-six marbles and the other with nine, not two rival situations.
- **Toys, both valid:** here both roots are positive whole numbers and each one satisfies the cost rule given in the problem, so the day could genuinely have produced either count and the question honestly allows two answers.

- **Interpret, do not just list:** factorise, state both roots, then write one short sentence reading each root back in the words of the problem, because that interpretation line is where the reasoning mark sits and it stops you wrongly rejecting a valid root.

A quick safety check is to confirm the cost per toy stays positive: at one count it works out to thirty rupees and at the other to twenty-five rupees, both perfectly sensible figures.

> **Final Answer:** Marbles: 36 and 9. Toys: 25 or 30.

Q 4.3 **Find two numbers whose sum is 27 and product is 182.**

SOLUTION

Concept used. When two numbers have a known sum and a known product, we name one number x, write the other in terms of x using the sum, and form a quadratic equation from the product condition. Solving by **factorisation** then gives both numbers.

Step 1. Let the first number be x. Since the sum is 27, the second number is $27 - x$.

Step 2. Their product is 182, so write the equation:

$$x(27 - x) = 182.$$

Step 3. Expand and bring to standard form:

$$27x - x^2 = 182 \;\Rightarrow\; x^2 - 27x + 182 = 0.$$

Step 4. Split the middle term using two numbers with product 182 and sum -27, namely -13 and -14:

$$x^2 - 13x - 14x + 182 = 0 \;\Rightarrow\; x(x-13) - 14(x-13) = 0 \;\Rightarrow\; (x-13)(x-14) = 0.$$

Step 5. So $x = 13$ or $x = 14$. If $x = 13$, the other number is $27 - 13 = 14$; if $x = 14$, the other is $27 - 14 = 13$. Either way the pair is 13 and 14.

> **Final Answer:** The two numbers are 13 and 14.

☞ Exam Tip

A quick mental check: two numbers near half of 27 (about 13.5) with product 182 point straight to 13 and 14, since $13 \times 14 = 182$. The factorising confirms the guess and earns the method marks.

EXPERT'S SOLUTION *: Priya Menon, M.Sc Mathematics, Anna University*

Sum-and-product problems are quadratics in disguise. Whenever a question fixes both the sum and the product of two numbers, those numbers are exactly the roots of $x^2 - sx + p = 0$, so the template removes any need to guess.

- **Set up cleanly:** let the second number be $27 - x$, form the product equation, expand it to standard form, and split the middle term with the pair that multiplies to the constant and adds to the middle coefficient.
- **Keep both roots:** the two numbers are interchangeable, so the answer is the unordered pair rather than a single value, and a student should resist stopping at one root when the question clearly asks for two numbers.
- **Verify both facts:** write one line checking that the pair adds to twenty-seven and multiplies to one hundred eighty-two at the same time, which confirms the answer without any need to solve again.

Final Answer: The numbers are 13 and 14 (since $13 + 14 = 27$ and $13 \times 14 = 182$).

Q 4.4 Find two consecutive positive integers, sum of whose squares is 365.

SOLUTION

Concept used. Two consecutive integers differ by 1, so if one is x the next is $x + 1$. We translate "sum of their squares is 365" into an equation, reduce it to standard form, solve by factorisation, and keep only the root that is a positive integer.

Step 1. Let the smaller integer be x. The next consecutive integer is $x + 1$.

Step 2. Sum of their squares is 365:

$$x^2 + (x + 1)^2 = 365.$$

Step 3. Expand $(x + 1)^2 = x^2 + 2x + 1$:

$$x^2 + x^2 + 2x + 1 = 365 \;\Rightarrow\; 2x^2 + 2x + 1 = 365.$$

Step 4. Bring to one side: $2x^2 + 2x - 364 = 0$. Divide every term by 2:

$$x^2 + x - 182 = 0.$$

Step 5. Split the middle term with two numbers of product -182 and sum $+1$, namely $+14$ and -13:

$$x^2 + 14x - 13x - 182 = 0 \;\Rightarrow\; x(x+14) - 13(x+14) = 0 \;\Rightarrow\; (x+14)(x-13) = 0.$$

So $x = -14$ or $x = 13$. The integers must be positive, so $x = 13$ and the next integer is 14.

Final Answer: The two consecutive positive integers are 13 and 14.

✗ Keep only the positive root

The equation gives $x = -14$ and $x = 13$. The problem says **positive** integers, so $x = -14$ must be rejected. Dropping the wrong root costs the final accuracy mark.

EXPERT'S SOLUTION *: Arjun Nair, M.Sc Mathematics, University of Kerala*

Divide out the common factor early to keep numbers small. After expanding, the equation carries a common factor of two, and dividing through by it gives a friendlier equation with the same roots but smaller coefficients.

- **Smaller is faster:** once the factor of two is removed, splitting the middle term needs a pair that multiplies to the constant and adds to one, and noticing that thirteen times fourteen makes one hundred eighty-two lands that pair immediately.
- **Reject in writing:** the equation offers a positive and a negative root, but the word "positive" in the question rules the negative one out, and the examiner wants that rejection stated in a line, not made silently in your head.
- **Square and add to check:** the fastest verification is to square each integer and add, which reproduces the given total and confirms the pair without re-solving the equation at all.

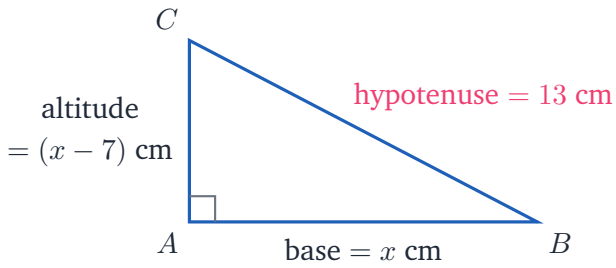
Final Answer: The integers are 13 and 14, since $13^2 + 14^2 = 365$.

Q 4.5 **The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.**

SOLUTION

Concept used. In a **right triangle** the base and the altitude are the two legs that meet at the right angle, and the **hypotenuse** is the longest side opposite that angle. By the **Pythagoras theorem**, $(\text{base})^2 + (\text{altitude})^2 = (\text{hypotenuse})^2$. We set the base as x, write the altitude in terms of x, form the equation, and solve.

A labelled sketch of the right triangle makes the set-up clear:



Step 1. Let the base be x cm. The altitude is 7 cm less, so altitude $= (x - 7)$ cm. The hypotenuse is 13 cm.

Step 2. Apply the Pythagoras theorem:

$$x^2 + (x - 7)^2 = 13^2.$$

Step 3. Expand $(x - 7)^2 = x^2 - 14x + 49$ and $13^2 = 169$:

$$x^2 + x^2 - 14x + 49 = 169 \;\Rightarrow\; 2x^2 - 14x + 49 = 169.$$

Step 4. Bring to one side: $2x^2 - 14x - 120 = 0$. Divide by 2:

$$x^2 - 7x - 60 = 0.$$

Step 5. Split the middle term with product -60 and sum -7, namely -12 and $+5$:

$$x^2 - 12x + 5x - 60 = 0 \;\Rightarrow\; x(x - 12) + 5(x - 12) = 0 \;\Rightarrow\; (x - 12)(x + 5) = 0.$$

So $x = 12$ or $x = -5$. A length cannot be negative, so $x = 12$ cm is the base and the altitude is $12 - 7 = 5$ cm.

Final Answer: The base is 12 cm and the altitude is 5 cm (check: $12^2 + 5^2 = 144 + 25 = 169 = 13^2$).

♥ Why This Matters

Right-triangle side problems turn straight into quadratics through the Pythagoras theorem. Spotting that the three sides satisfy $\text{leg}^2 + \text{leg}^2 = \text{hyp}^2$ is the key step that converts a geometry question into an equation you can solve.

EXPERT'S SOLUTION *: Sneha Patel, M.Sc Mathematics, The Maharaja Sayajirao University of Baroda*

Set the larger leg as the unknown so the other comes out positive. The base is described as seven more than the altitude, so naming the base x keeps the altitude $x - 7$ and avoids a second variable, holding the algebra in one unknown.

- **Build the equation:** the Pythagoras relation follows directly because the base and altitude are the two legs and the thirteen is the hypotenuse, and after expanding, dividing the whole equation by two keeps the numbers small and factorising easy.
- **Reject the negative:** one root is negative and so impossible for a length, and it should be discarded with a one-line reason, leaving a base of twelve and an altitude of five rather than a silent drop.
- **Label the hypotenuse:** students sometimes mark the wrong side as the hypotenuse, which wrecks the equation from the start, so remember it is always the longest side and sits opposite the right angle, which fixes the thirteen as the hypotenuse and the two unknowns as the legs.

Several small habits earn full marks here: draw a quick labelled triangle so the right angle is clearly marked, name the Pythagoras theorem before using it, and finish with a square-and-add check that reproduces the hypotenuse squared, confirming both the right angle and the arithmetic; finally, give both sides in the answer, since the question asks for the other two sides, not just the base.

Final Answer: Base $= 12$ cm, altitude $= 5$ cm.

Q 4.6 **A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs. 90, find the number of articles produced and the cost of each article.**

SOLUTION

Concept used. The total cost equals the number of articles multiplied by the cost of each article. We name the number of articles x, write the cost per article in terms of x from the condition given, form the **quadratic equation** from the total-cost relation, solve by **factorisation**, and keep the root that is a positive whole number of articles.

Step 1. Let the number of articles produced be x. The cost of each article is "3 more than twice the number of articles", so cost per article $= (2x + 3)$ rupees.

Step 2. Total cost $=$ number of articles $\times$ cost of each article:

$$x(2x + 3) = 90.$$

Step 3. Expand and bring to standard form:

$$2x^2 + 3x = 90 \;\Rightarrow\; 2x^2 + 3x - 90 = 0.$$

Step 4. Split the middle term with product $2 \times (-90) = -180$ and sum $+3$, namely $+15$ and -12:

$$2x^2+15x-12x-90=0 \;\Rightarrow\; x(2x+15)-6(2x+15)=0 \;\Rightarrow\; (2x+15)(x-6)=0.$$

So $2x + 15 = 0$ or $x - 6 = 0$, giving $x = -\frac{15}{2}$ or $x = 6$.

Step 5. The number of articles must be a positive whole number, so $x = -\frac{15}{2}$ is rejected. Hence $x = 6$ articles, and the cost of each article is $2(6) + 3 = 15$ rupees.

Final Answer: 6 articles were produced, and the cost of each article was Rs. 15.

☞ **Translate the cost phrase carefully**

"3 more than twice the number" is $2x + 3$, not $2(x + 3)$ or $3x + 2$. Reading the phrase in the right order is what keeps the equation correct from the start.

EXPERT'S SOLUTION *: Kavya Sharma, M.Sc Mathematics, Banaras Hindu University*

Reject the impossible root and report both required quantities. The equation gives one positive whole root and one negative fraction, but only a positive whole number can count pottery articles, so the fraction is discarded with a one-line reason.

- **Answer both parts:** a frequent oversight is to stop at the number of articles and forget that the question also asks for the cost of each article, so substitute the valid root into the cost expression and put both figures in the final answer.
- **Pick the split:** the product to factorise is two times minus ninety, and the pair fifteen and minus twelve is the only one that adds to the middle coefficient of three, which is what makes the grouping come out cleanly.
- **Sanity-check the total:** six articles at fifteen rupees each give exactly the stated ninety rupees, and the cost rule holds at the same value, so writing that check secures the verification mark and guards against an arithmetic slip.

Final Answer: Articles $= 6$, cost per article $=$ Rs. 15 (since $6 \times 15 = 90$).

Download the Full Revision Notes PDF →

Related Collegedunia Resources

Same chapter, other resources:

- Revision Notes
- Formula Sheet
- NCERT Book PDF
- Exemplar Book PDF
- Exemplar Solutions
- Handwritten Notes

Continue learning:

- Ch 3: Pair of Linear Equations in Two Variables
- Ch 5: Arithmetic Progressions
- Class 10 Maths - All Chapters