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Step-by-step solutions, alternate methods & exam tips for Class 10 Mathematics

Chapter 4: Quadratic Equations

About this Chapter

These solutions for Class 10 Maths Chapter 4 Quadratic Equations Exercise 4.3 work out every exercise question in full. A **quadratic equation** has the form $ax^2 + bx + c = 0$ with $a \neq 0$. Students learn to find **roots** by factorisation, judge the nature of roots from the **discriminant** $b^2 - 4ac$, and set up word problems on ages, areas and speeds.

Topics covered: Standard form $ax^2 + bx + c = 0$ • Roots and zeroes • Solving by factorisation • Splitting the middle term • Discriminant $b^2 - 4ac$ • Nature of roots • Word problems

Quick Formula Sheet

Standard form:
$ax^2 + bx + c = 0,\ a \neq 0$

Quadratic formula:
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant:
$D = b^2 - 4ac$

Nature of roots:
$D > 0$: two distinct real roots
$D = 0$: two equal real roots
$D < 0$: no real roots

Also see for this chapter: Revision Notes | Formula Sheet | Exemplar Solutions

NCERT Exercise Solutions

Exercise 4.3

Q 4.1 **Find the nature of the roots of the following quadratic equations. If the real roots exist, find them: (i)** $2x^2 - 3x + 5 = 0$ **(ii)** $3x^2 - 4\sqrt{3}\,x + 4 = 0$ **(iii)** $2x^2 - 6x + 3 = 0$

SOLUTION

Concept used. For $ax^2 + bx + c = 0$ the **discriminant** is $D = b^2 - 4ac$. It decides the **nature of roots**: if $D > 0$ there are two distinct real roots, if $D = 0$ there are two equal real roots, and if $D < 0$ there are no real roots. When real roots exist they are found from the **quadratic formula** $x = \frac{-b \pm \sqrt{D}}{2a}$.

Step 1. **(i)** $2x^2 - 3x + 5 = 0$. Here $a = 2$, $b = -3$, $c = 5$. Compute the discriminant:

$$D = b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31.$$

Since $D < 0$, the equation has **no real roots**.

Step 2. **(ii)** $3x^2 - 4\sqrt{3}\,x + 4 = 0$. Here $a = 3$, $b = -4\sqrt{3}$, $c = 4$. Compute the discriminant:

$$D = (-4\sqrt{3})^2 - 4(3)(4) = (16 \times 3) - 48 = 48 - 48 = 0.$$

Since $D = 0$, there are **two equal real roots**. Using the formula:

$$x = \frac{-b}{2a} = \frac{4\sqrt{3}}{2 \times 3} = \frac{4\sqrt{3}}{6} = \frac{2\sqrt{3}}{3} = \frac{2}{\sqrt{3}}.$$

So both roots are $\frac{2}{\sqrt{3}}$.

Step 3. **(iii)** $2x^2 - 6x + 3 = 0$. Here $a = 2$, $b = -6$, $c = 3$. Compute the discriminant:

$$D = (-6)^2 - 4(2)(3) = 36 - 24 = 12.$$

Since $D > 0$, there are **two distinct real roots**. Using the formula with $\sqrt{12} = 2\sqrt{3}$:

$$x = \frac{-(-6) \pm \sqrt{12}}{2 \times 2} = \frac{6 \pm 2\sqrt{3}}{4} = \frac{3 \pm \sqrt{3}}{2}.$$

So the roots are $\frac{3+\sqrt{3}}{2}$ and $\frac{3-\sqrt{3}}{2}$.

Final Answer: (i) $D = -31 < 0$, no real roots; (ii) $D = 0$, equal roots $x = \dfrac{2}{\sqrt{3}}$; (iii) $D = 12 > 0$, roots $x = \dfrac{3 \pm \sqrt{3}}{2}$.

☞ **Discriminant tells the story**

Compute $D = b^2 - 4ac$ first. Its sign alone answers "what kind of roots", and you only reach for the quadratic formula once $D \geq 0$.

EXPERT'S SOLUTION *: Imran Khan, M.Sc Mathematics, Aligarh Muslim University*

Let the discriminant do the deciding before any formula. The sign of $D = b^2 - 4ac$ settles the nature of the roots in a single line, so always compute it first, state the verdict, and reach for the roots only when it is non-negative.

- **Square the surd fully:** in part (ii) the middle coefficient carries a root of three, and squaring it gives a positive forty-eight, not a negative; forgetting that the surd also

squares is the usual error that turns a zero discriminant into a wrong sign.

- **Read off equal roots:** when the discriminant is zero the two roots coincide at minus the middle coefficient over twice the leading one, and that single value may be left as a surd fraction or rationalised, since both forms are accepted.
- **Simplify before dividing:** in part (iii), reducing the root of twelve to two roots of three before dividing lets the common factor cancel cleanly and gives the neat half-form rather than an ugly unreduced answer.

Presenting the discriminant value, then its sign, then the nature in words, and only then the roots is the structure examiners reward, and it stops a student hunting for roots that do not exist, as in part (i).

> **Final Answer:** No real roots; equal roots $2/\sqrt{3}$; distinct roots $(3 \pm \sqrt{3})/2$.

Q 4.2 **Find the values of k for each of the following quadratic equations, so that they have two equal roots. (i) $2x^2 + kx + 3 = 0$ (ii) $kx(x-2) + 6 = 0$**

SOLUTION

Concept used. A quadratic equation $ax^2 + bx + c = 0$ has **two equal roots** exactly when its **discriminant** is zero, that is $b^2 - 4ac = 0$. We identify a, b, c (which here involve the unknown k), set $b^2 - 4ac = 0$, and solve for k. We also remember that for the equation to stay quadratic, the coefficient of x^2 must not be zero.

Step 1. **(i)** $2x^2 + kx + 3 = 0$. Here $a = 2$, $b = k$, $c = 3$. Set the discriminant to zero:

$$b^2 - 4ac = 0 \;\Rightarrow\; k^2 - 4(2)(3) = 0 \;\Rightarrow\; k^2 - 24 = 0.$$

So $k^2 = 24$, giving $k = \pm\sqrt{24} = \pm 2\sqrt{6}$.

Step 2. **(ii)** $kx(x-2) + 6 = 0$. First write it in standard form. Expand: $kx^2 - 2kx + 6 = 0$. So $a = k$, $b = -2k$, $c = 6$. Set the discriminant to zero:

$$b^2 - 4ac = 0 \;\Rightarrow\; (-2k)^2 - 4(k)(6) = 0 \;\Rightarrow\; 4k^2 - 24k = 0.$$

Factor out $4k$: $4k(k-6) = 0$, so $k = 0$ or $k = 6$.

Step 3. In part (ii), $k = 0$ makes the coefficient of x^2 equal to zero, so the equation is no longer quadratic. We reject $k = 0$ and keep $k = 6$.

> **Final Answer:** (i) $k = \pm 2\sqrt{6}$; (ii) $k = 6$ (the value $k = 0$ is rejected, as it makes the equation non-quadratic).

✗ k = 0 destroys the quadratic

In part (ii), $k = 0$ satisfies the algebra but turns $kx^2 - 2kx + 6 = 0$ into $6 = 0$, which is not a quadratic equation at all. Always reject any k that zeroes the x^2 coefficient.

EXPERT'S SOLUTION *: Deepa Reddy, M.Sc Mathematics, University of Hyderabad*

Equal roots is the same as discriminant zero, but watch the leading coefficient. The phrase "two equal roots" should immediately trigger the condition that the discriminant equals zero, and the only skill is reading off the coefficients correctly when they carry the unknown.

- **Both signs stand:** in part (i) the condition gives a square equal to twenty-four, so the unknown is plus or minus two roots of six, and both signs are valid because either one makes the discriminant vanish exactly.
- **Guard the leading term:** part (ii) factors to offer a zero and a six, but the zero removes the squared term and leaves a false numerical statement, so it cannot be accepted, and that rejection, stated with a reason, is the single most marks-sensitive line here.
- **Reduce the surd:** simplify surd answers fully rather than leaving an unreduced root, since examiners expect the lowest form, and a half-simplified surd often costs the presentation mark.

Treating "equal roots" as "discriminant zero" and then guarding the leading coefficient against a value that breaks the quadratic covers both parts safely.

> **Final Answer:** $k = \pm 2\sqrt{6}$ for (i); $k = 6$ for (ii).

Q 4.3 **Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.**

SOLUTION

Concept used. For a rectangle, area = length × breadth. We let the breadth be x, write the length in terms of x, form a **quadratic equation** from the area, and check whether it has a real, positive solution. A design is possible only when a positive real breadth exists, which we can confirm through the **discriminant** or by direct solving.

Step 1. Let the breadth be x m. The length is twice the breadth, so length $= 2x$ m.

Step 2. Area = length × breadth $= 800$:

$$2x \cdot x = 800 \;\Rightarrow\; 2x^2 = 800.$$

Step 3. Write in standard form: $2x^2 - 800 = 0$. The discriminant of $2x^2 + 0 \cdot x - 800 = 0$

is

$$D = b^2 - 4ac = 0^2 - 4(2)(-800) = 6400 > 0,$$

so real roots exist and the design is possible.

Step 4. Solve $2x^2 = 800$: divide by 2 to get $x^2 = 400$, so $x = \pm 20$. A breadth cannot be negative, so $x = 20$ m.

Step 5. Then the length is $2x = 2(20) = 40$ m.

> **Final Answer:** Yes, the design is possible. The breadth is 20 m and the length is 40 m.

☞ Exam Tip

"Is it possible?" questions are really asking whether the discriminant is non-negative. State $D \geq 0$ (so real roots exist) as the reason it is possible, then give the actual dimensions for full marks.

EXPERT'S SOLUTION *: Nisha Verma, M.Sc Mathematics, University of Rajasthan*

Answer the yes-or-no part with the discriminant, then give the sizes. The question really has two halves, whether such a grove can exist at all and, if it can, what its measurements are, and each half deserves its own line.

- **Settle existence first:** the discriminant of the area equation comes out positive, so real roots exist and the honest answer to "is it possible" is yes, written as a clear sentence before any number appears.
- **Then size it:** solving the simplified equation gives a positive breadth, while the negative root is rejected because a length cannot be negative, and doubling the breadth gives the length.
- **Why the discriminant line matters:** the "is it possible" phrasing is the examiner's cue that the nature-of-roots idea is being tested, so writing that line explicitly earns the marks even though the algebra is simple enough to solve directly.

> **Final Answer:** Possible: breadth 20 m, length 40 m (area $40 \times 20 = 800$ m^2).

Q 4.4 **Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.**

SOLUTION

Concept used. We form a **quadratic equation** from the two conditions, then test whether a real solution exists by checking the sign of the **discriminant** $D = b^2 - 4ac$. If $D < 0$, the equation has no real roots, which means the described situation cannot actually happen.

Step 1. Let the present age of one friend be x years. Since the ages sum to 20, the other friend's present age is $(20 - x)$ years.

Step 2. Four years ago their ages were $(x - 4)$ and $(20 - x - 4) = (16 - x)$. The product four years ago was 48:

$$(x - 4)(16 - x) = 48.$$

Step 3. Expand the left side:

$$16x - x^2 - 64 + 4x = 48 \;\Rightarrow\; -x^2 + 20x - 64 = 48.$$

Step 4. Bring all terms to one side: $-x^2 + 20x - 64 - 48 = 0$, that is $-x^2 + 20x - 112 = 0$. Multiply by -1:

$$x^2 - 20x + 112 = 0.$$

Step 5. Check the discriminant with $a = 1$, $b = -20$, $c = 112$:

$$D = b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48.$$

Since $D < 0$, there are no real roots.

> **Final Answer:** No, the situation is not possible: the equation $x^2 - 20x + 112 = 0$ has discriminant $-48 < 0$, so no real ages satisfy both conditions.

♥ Why This Matters

A negative discriminant is not a dead end in an exam: it is the complete answer. It proves the described situation can never occur, which is exactly what an "is it possible?" question wants you to demonstrate.

EXPERT'S SOLUTION *: Sanjay Gupta, M.Sc Mathematics, Panjab University*

A negative discriminant is the proof of impossibility. The two conditions translate into a single quadratic, and the temptation is to keep hunting for roots, but the discriminant turns out negative, so no real value exists and the situation simply cannot happen.

- **Present it as a proof:** compute the discriminant, note that it is negative, and write

one sentence concluding the ages are impossible, because the marks here reward recognising the no-real-roots condition, not any numerical age.

- **Build the second age right:** the ages four years ago are $x-4$ and $16-x$, so the second must use the sum of twenty first and only then subtract four, which is the step students most often slip on.
- **Tidy the leading sign:** after expanding, multiply through by minus one so the leading coefficient is positive, which lets the discriminant be read off cleanly without a sign mistake.

A student who instead reports decimal or imaginary "ages" has missed the point, since the correct response is a clear "not possible" backed by the negative discriminant.

Final Answer: Not possible, since $D=-48<0$ gives no real ages.

Q 4.5 Is it possible to design a rectangular park of perimeter 80 m and area 400 m^2? If so, find its length and breadth.

SOLUTION

Concept used. For a rectangle, perimeter $=2(\text{length}+\text{breadth})$ and area $=\text{length}\times\text{breadth}$. From the perimeter we express one dimension in terms of the other, then use the area to form a **quadratic equation**. The sign of the **discriminant** tells us whether such a park is possible, and solving gives the dimensions.

Step 1. Let the length be x m. Perimeter is 80 m, so $2(\text{length}+\text{breadth})=80$, which gives length $+$ breadth $=40$. Hence breadth $=(40-x)$ m.

Step 2. Area is 400 m^2:

$$x(40-x)=400.$$

Step 3. Expand and rearrange:

$$40x-x^2=400 \;\Rightarrow\; x^2-40x+400=0.$$

Step 4. Check the discriminant with $a=1$, $b=-40$, $c=400$:

$$D=b^2-4ac=(-40)^2-4(1)(400)=1600-1600=0.$$

Since $D=0$, real and equal roots exist, so the design is possible.

Step 5. Solve for the equal root:

$$x=\frac{-b}{2a}=\frac{40}{2}=20.$$

So the length is 20 m and the breadth is $40-20=20$ m. The park is a square of side 20 m.

Final Answer: Yes, the design is possible. Both the length and the breadth are 20 m (the park is a $20\,\text{m} \times 20\,\text{m}$ square).

☞ **Equal roots can mean a square**

When the discriminant is zero, the length and breadth come out equal. That is not an error: it simply means the rectangle that fits both conditions happens to be a square.

EXPERT'S SOLUTION *: Lakshmi Pillai, M.Sc Mathematics, University of Calicut*

Use perimeter to reduce to one unknown, then test with the discriminant. The two facts hand you a sum from the perimeter and a product from the area, so the length and breadth are simply the roots of the matching quadratic.

- **Zero means equal:** the discriminant comes out exactly zero, which signals both that a solution exists and that the two roots coincide, so the answer is yes and there is a single repeated value.
- **Name the square:** because length and breadth come out equal, the park is actually a square, and a student should say so plainly rather than treating the matching values as a mistake to be fixed.
- **Verify both facts:** the cleanest finish checks the perimeter and the area against the original numbers, and spotting the perfect-square form of the quadratic is an even faster route that also confirms the zero discriminant.

Final Answer: Possible: length $=$ breadth $= 20$ m, a square park.

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