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NCERT Solutions for Class 10 Maths (Mathematics): step-by-step answers, alternate methods & board-exam tips

Chapter 5: Arithmetic Progressions

About this Chapter

Class 10 Maths Chapter 5 Arithmetic Progressions Exercise 5.3 builds an **AP**: a list where each term adds a fixed **common difference** d. You learn to test a list, find any term with $a_n = a + (n-1)d$, and add the first n terms using $S_n = \frac{n}{2}[2a + (n-1)d]$. Then you solve word problems on salaries, savings and stacked logs.

Topics covered: What is an AP • First term & common difference • Checking if a list is an AP • nth term $a_n = a + (n-1)d$ • Middle and missing terms • Sum of n terms S_n • Word problems

Quick Formula Sheet

General form of an AP:
$a,\ a+d,\ a+2d,\ a+3d,\ \ldots$

Common difference:
$d = a_2 - a_1 = a_{k+1} - a_k$

nth (general) term:
$a_n = a + (n-1)d$
a = first term, d = common difference, n = term number

Sum of first n terms:
$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Sum when last term l is known:
$S_n = \frac{n}{2}(a + l)$

Link between S_n and a_n:
$a_n = S_n - S_{n-1}$

Arithmetic mean of a and c:
$b = \frac{a+c}{2}$

Also see for this chapter: Revision Notes | Formula Sheet | Exemplar Solutions

NCERT Exercise Solutions

Exercise 5.3

Q 5.1 **Find the sum of the following APs:**

(i) $2,\ 7,\ 12,\ \ldots$**, to 10 terms.** **(ii)** $-37,\ -33,\ -29,\ \ldots$**, to 12 terms.**
(iii) $0.6,\ 1.7,\ 2.8,\ \ldots$**, to 100 terms.** **(iv)** $\frac{1}{15},\ \frac{1}{12},\ \frac{1}{10},\ \ldots$**, to 11 terms.**

SOLUTION

Concept used. The sum of the first n terms of an AP is $S_n = \frac{n}{2}\big[\,2a + (n-1)d\,\big]$, where a is the first term and $d = a_2 - a_1$.

Step 1. **(i)** $a = 2$, $d = 5$, $n = 10$:

$$S_{10} = \tfrac{10}{2}\big[2(2) + (10-1)(5)\big] = 5\,[4 + 45] = 5(49) = 245.$$

Step 2. **(ii)** $a = -37$, $d = -33 - (-37) = 4$, $n = 12$:

$$S_{12} = \tfrac{12}{2}\big[2(-37) + (12-1)(4)\big] = 6\,[-74 + 44] = 6(-30) = -180.$$

Step 3. **(iii)** $a = 0.6$, $d = 1.1$, $n = 100$:

$$S_{100} = \tfrac{100}{2}\big[2(0.6) + (100-1)(1.1)\big] = 50\,[1.2 + 108.9] = 50(110.1) = 5505.$$

Step 4. **(iv)** $a = \frac{1}{15}$, $d = \frac{1}{12} - \frac{1}{15} = \frac{5-4}{60} = \frac{1}{60}$, $n = 11$:

$$S_{11} = \tfrac{11}{2}\left[2 \cdot \tfrac{1}{15} + (11-1) \cdot \tfrac{1}{60}\right] = \tfrac{11}{2}\left[\tfrac{2}{15} + \tfrac{10}{60}\right].$$

Now $\frac{2}{15} = \frac{8}{60}$, so $\frac{8}{60} + \frac{10}{60} = \frac{18}{60} = \frac{3}{10}$.

$$S_{11} = \tfrac{11}{2} \cdot \tfrac{3}{10} = \tfrac{33}{20}.$$

Final Answer: (i) 245 (ii) -180 (iii) 5505 (iv) $\frac{33}{20}$

☞ Bracket then halve

Compute $2a + (n-1)d$ fully inside the bracket first, then multiply by $\frac{n}{2}$. Doing the bracket before the half keeps the steps tidy.

EXPERT'S SOLUTION *: Gaurav Malhotra, B.Tech Production Engineering, NIT Jamshedpur*

Match the denominators in the fraction AP.

- **Spot the hard part:** the first three sums are routine, and only the fraction AP in part (iv) needs care, so choose a single common denominator of sixty up front and the rest becomes plain arithmetic.
- **Find the step:** subtracting the first two terms over the common denominator gives a difference of one over sixty.

- **Add inside the bracket:** rewrite the leading fraction over sixty so it adds cleanly to the rest, which simplifies to three over ten.
- **Finish the sum:** multiplying by the half-count factor gives the final value of thirty-three over twenty.
- **Why it matters:** sum problems with fractions are really a test of denominator handling, and converting everything to a single denominator before adding is the safest route to the exact answer the board expects.

Final Answer: (i) 245; (ii) -180; (iii) 5505; (iv) $\frac{33}{20}$.

Q 5.2 Find the sums given below:

(i) $7 + 10\frac{1}{2} + 14 + \ldots + 84$

(ii) $34 + 32 + 30 + \ldots + 10$

(iii) $-5 + (-8) + (-11) + \ldots + (-230)$

SOLUTION

Concept used. When the last term l is given, first find the number of terms n from $l = a + (n-1)d$, then use $S_n = \frac{n}{2}(a + l)$.

Step 1. **(i)** $a = 7$, $d = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$, $l = 84$. Find n:
$84 = 7 + (n-1)\frac{7}{2} \Rightarrow 77 = \frac{7}{2}(n-1) \Rightarrow n - 1 = \frac{77 \times 2}{7} = 22 \Rightarrow n = 23$. Then
$S_{23} = \frac{23}{2}(7 + 84) = \frac{23}{2}(91) = \frac{2093}{2} = 1046\frac{1}{2}$.

Step 2. **(ii)** $a = 34$, $d = 32 - 34 = -2$, $l = 10$. Find n:
$10 = 34 + (n-1)(-2) \Rightarrow -24 = -2(n-1) \Rightarrow n - 1 = 12 \Rightarrow n = 13$. Then
$S_{13} = \frac{13}{2}(34 + 10) = \frac{13}{2}(44) = 13 \times 22 = 286$.

Step 3. **(iii)** $a = -5$, $d = -8 - (-5) = -3$, $l = -230$. Find n:
$-230 = -5 + (n-1)(-3) \Rightarrow -225 = -3(n-1) \Rightarrow n - 1 = 75 \Rightarrow n = 76$. Then
$S_{76} = \frac{76}{2}(-5 + (-230)) = 38 \times (-235) = -8930$.

Final Answer: (i) $1046\frac{1}{2}$ (ii) 286 (iii) -8930

☞ **Last term given? Use $\frac{n}{2}(a + l)$**

When the final term is printed, the form $S_n = \frac{n}{2}(a + l)$ is shortest, but you still need n first from $l = a + (n-1)d$.

EXPERT'S SOLUTION *: Shruti Kulkarni, M.Sc Mathematics, Pune University*

Two-stage routine: count, then average.

- **Same shape:** every "up to a last term" sum follows the same two stages, so handle all three parts with one routine.
- **Stage one, count:** use the last-term equation to find the number of terms, which gives twenty-three, thirteen and seventy-six.
- **Stage two, average:** average the two ends and scale by the count, $S_n = n \times \frac{a+l}{2}$; for part (iii) the average term is $\frac{-5+(-230)}{2}$, and times seventy-six gives minus eight thousand nine hundred thirty.
- **Read the form:** this half-count times sum-of-ends is simply the number of terms times the average term.
- **Why it matters:** reading the sum as count times average term demystifies the formula and gives a built-in sanity check, since the sum should sit near the count times the middle value.

Final Answer: (i) $1046\frac{1}{2}$; (ii) 286; (iii) -8930.

Q 5.3 **In an AP:**

(i) given $a = 5,\ d = 3,\ a_n = 50$**, find** n **and** S_n**.** **(ii) given** $a = 7,\ a_{13} = 35$**, find** d **and** S_{13}**.** **(iii) given** $a_{12} = 37,\ d = 3$**, find** a **and** S_{12}**.** **(iv) given** $a_3 = 15,\ S_{10} = 125$**, find** d **and** a_{10}**.** **(v) given** $d = 5,\ S_9 = 75$**, find** a **and** a_9**.** **(vi) given** $a = 2,\ d = 8,\ S_n = 90$**, find** n **and** a_n**.** **(vii) given** $a = 8,\ a_n = 62,\ S_n = 210$**, find** n **and** d**.** **(viii) given** $a_n = 4,\ d = 2,\ S_n = -14$**, find** n **and** a**.**

(ix) given $a = 3,\ n = 8,\ S = 192$**, find** d**.** **(x) given** $l = 28,\ S = 144$**, and there are total 9 terms. Find** a**.**

SOLUTION

Concept used. We use the three relations $a_n = a + (n-1)d$, $S_n = \frac{n}{2}[2a + (n-1)d]$ and $S_n = \frac{n}{2}(a + l)$, picking the one that fits the known quantities, and solve for the unknowns.

Step 1. **(i)** $50 = 5 + (n-1)(3) \Rightarrow 45 = 3(n-1) \Rightarrow n = 16.$
$S_{16} = \frac{16}{2}(5 + 50) = 8(55) = 440.$

Step 2. **(ii)** $a_{13} = 7 + 12d = 35 \Rightarrow 12d = 28 \Rightarrow d = \frac{7}{3}.$
$S_{13} = \frac{13}{2}(a + a_{13}) = \frac{13}{2}(7 + 35) = \frac{13}{2}(42) = 273.$

Step 3. **(iii)** $a_{12} = a + 11d = 37 \Rightarrow a + 33 = 37 \Rightarrow a = 4.$
$S_{12} = \frac{12}{2}(a + a_{12}) = 6(4 + 37) = 6(41) = 246.$

Step 4. **(iv)** $a_3 = a + 2d = 15 \ldots (1)$.
$S_{10} = \frac{10}{2}[2a + 9d] = 125 \Rightarrow 5(2a + 9d) = 125 \Rightarrow 2a + 9d = 25 \ldots (2)$. From (1), $a = 15 - 2d$; put in (2):
$2(15 - 2d) + 9d = 25 \Rightarrow 30 + 5d = 25 \Rightarrow 5d = -5 \Rightarrow d = -1$, so $a = 17$.
$a_{10} = a + 9d = 17 + 9(-1) = 8$.

Step 5. **(v)** $S_9 = \frac{9}{2}[2a + 8d] = 75$. With $d = 5$:
$\frac{9}{2}(2a + 40) = 75 \Rightarrow 2a + 40 = \frac{150}{9} = \frac{50}{3} \Rightarrow 2a = \frac{50}{3} - 40 = -\frac{70}{3} \Rightarrow a = -\frac{35}{3}$.
$a_9 = a + 8d = -\frac{35}{3} + 40 = \frac{-35+120}{3} = \frac{85}{3}$.

Step 6. **(vi)** $S_n = \frac{n}{2}[2(2) + (n-1)(8)] = 90 \Rightarrow \frac{n}{2}[4 + 8n - 8] = 90 \Rightarrow \frac{n}{2}(8n - 4) = 90 \Rightarrow n(4n - 2) = 90 \Rightarrow 4n^2 - 2n - 90 = 0$. Divide by 2:
$2n^2 - n - 45 = 0 \Rightarrow (2n + 9)(n - 5) = 0 \Rightarrow n = 5$ (positive).
$a_n = a_5 = 2 + 4(8) = 34$.

Step 7. **(vii)** $S_n = \frac{n}{2}(a + a_n) = \frac{n}{2}(8 + 62) = 210 \Rightarrow 35n = 210 \Rightarrow n = 6$. Then
$a_n = a + (n-1)d \Rightarrow 62 = 8 + 5d \Rightarrow 5d = 54 \Rightarrow d = \frac{54}{5}$.

Step 8. **(viii)** $S_n = \frac{n}{2}(a + a_n) = \frac{n}{2}(a + 4) = -14$. Also
$a_n = a + (n-1)d \Rightarrow 4 = a + 2(n-1) \Rightarrow a = 4 - 2(n-1) = 6 - 2n$. Put into the sum: $\frac{n}{2}\big((6 - 2n) + 4\big) = -14 \Rightarrow \frac{n}{2}(10 - 2n) = -14 \Rightarrow n(5 - n) = -14 \Rightarrow n^2 - 5n - 14 = 0 \Rightarrow (n - 7)(n + 2) = 0 \Rightarrow n = 7$. Then $a = 6 - 2(7) = -8$.

Step 9. **(ix)**
$S = \frac{8}{2}[2(3) + 7d] = 192 \Rightarrow 4(6 + 7d) = 192 \Rightarrow 6 + 7d = 48 \Rightarrow 7d = 42 \Rightarrow d = 6$.

Step 10. **(x)** $S = \frac{9}{2}(a + l) = 144 \Rightarrow \frac{9}{2}(a + 28) = 144 \Rightarrow a + 28 = \frac{288}{9} = 32 \Rightarrow a = 4$.

Final Answer: (i) $n = 16,\ S_{16} = 440$ (ii) $d = \frac{7}{3},\ S_{13} = 273$ (iii) $a = 4,\ S_{12} = 246$ (iv) $d = -1,\ a_{10} = 8$ (v) $a = -\frac{35}{3},\ a_9 = \frac{85}{3}$ (vi) $n = 5,\ a_n = 34$ (vii) $n = 6,\ d = \frac{54}{5}$ (viii) $n = 7,\ a = -8$ (ix) $d = 6$ (x) $a = 4$

☞ Pick the formula that uses your knowns

If you know a and l, use $S_n = \frac{n}{2}(a + l)$. If you know a and d, use $S_n = \frac{n}{2}[2a + (n-1)d]$. Choosing the right form saves whole steps.

EXPERT'S SOLUTION *: Venkatesh Murthy, M.Sc Mathematics, Sri Venkateswara University*

Reject any negative or fractional term count.

- **Watch the quadratics:** parts (vi) and (viii) turn into quadratics in the term count, and only a positive whole number is a valid count.
- **Part (vi):** the quadratic factors cleanly, so reject the negative root, keep five, and the fifth term is thirty-four.

- **Part (viii):** again the quadratic factors, so reject the negative root, keep seven, and the first term is minus eight.
- **The rest:** everywhere else just match the known quantities to the right formula and solve linearly, taking extra care with the fractions in parts (ii) and (v).
- **Why it matters:** the sum formula can produce a quadratic in the count, and knowing the count must be a positive integer lets you discard the extra root instantly, a step examiners specifically check.

> **Final Answer:** Solve the matching formula for the unknowns; discard any non-positive or non-integer n.

Q 5.4 **How many terms of the AP: $9,\ 17,\ 25,\ \ldots$ must be taken to give a sum of 636?**

SOLUTION

Concept used. Set $S_n = \frac{n}{2}[2a + (n-1)d]$ equal to the target sum. This gives a quadratic in n; solve and keep the positive integer root.

Step 1. $a = 9$, $d = 17 - 9 = 8$, $S_n = 636$.

Step 2. $636 = \frac{n}{2}\big[2(9) + (n-1)(8)\big] = \frac{n}{2}\big[18 + 8n - 8\big] = \frac{n}{2}(8n + 10)$.

Step 3. $636 = n(4n + 5) = 4n^2 + 5n$, so $4n^2 + 5n - 636 = 0$.

Step 4. Quadratic formula:

$$n = \frac{-5 \pm \sqrt{5^2 - 4(4)(-636)}}{2(4)} = \frac{-5 \pm \sqrt{25 + 10176}}{8} = \frac{-5 \pm \sqrt{10201}}{8}.$$

Step 5. $\sqrt{10201} = 101$, so $n = \dfrac{-5 + 101}{8} = \dfrac{96}{8} = 12$ (rejecting the negative root).

> **Final Answer:** 12 terms of the AP add up to 636.

✗ Reject the negative root

The quadratic gives $n = 12$ and $n = -\frac{106}{8}$. A term count cannot be negative or fractional, so only $n = 12$ is accepted.

EXPERT'S SOLUTION *: Nisha Thomas, M.Sc Mathematics, Mahatma Gandhi University*

Tidy the quadratic before you solve it.

- **The crux:** the whole question turns on reducing the quadratic carefully and then

recognising that its discriminant is a perfect square.

- **Form the equation:** the sum formula gives the tidy quadratic $4n^2 + 5n - 636 = 0$.
- **Check the discriminant:** the discriminant works out to ten thousand two hundred one, which is exactly the square of one hundred one.
- **Take the valid root:** this gives a count of twelve once the negative root is rejected, $n = \frac{-5+101}{8} = 12$.
- **Why it matters:** a clean perfect-square discriminant signals you are on track, and spotting it confirms the count without a calculator, exactly the kind of number boards design.

Final Answer: $n = 12$ terms.

Q 5.5 **The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.**

SOLUTION

Concept used. With first term a, last term l and sum S_n known, use $S_n = \frac{n}{2}(a + l)$ to get n, then $l = a + (n - 1)d$ to get d.

Step 1. $a = 5$, $l = 45$, $S_n = 400$. From $S_n = \frac{n}{2}(a + l)$: $400 = \frac{n}{2}(5 + 45) = \frac{n}{2}(50) = 25n$.

Step 2. So $n = \frac{400}{25} = 16$.

Step 3. Now use $l = a + (n - 1)d$: $45 = 5 + (16 - 1)d = 5 + 15d$.

Step 4. $15d = 40 \Rightarrow d = \frac{40}{15} = \frac{8}{3}$.

Final Answer: Number of terms $n = 16$; common difference $d = \frac{8}{3}$.

☞ **Use the sum to get n first**

With a, l and S_n all known, the form $S_n = \frac{n}{2}(a + l)$ gives n in one step. Only then bring in $l = a + (n - 1)d$ for d.

EXPERT'S SOLUTION *: Abhishek Jain, M.Sc Mathematics, University of Delhi*

Read the sum as count times average term.

- **The shortcut:** the sum equals the count times the average of the two ends, so the count appears at sight once you know that average.
- **Average term:** the average of the first and last terms is twenty-five.

- **Count the terms:** dividing the total of four hundred by that average of twenty-five gives sixteen terms.
- **Find the step:** the last-term equation then gives the common difference as eight-thirds.
- **Why it matters:** treating the sum as count times average term makes the count obvious and gives a quick check, since sixteen terms averaging twenty-five indeed add to four hundred.

Final Answer: $n = 16$, $d = \frac{8}{3}$.

Q 5.6 **The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?**

SOLUTION

Concept used. Find the number of terms from $l = a + (n-1)d$, then the sum from $S_n = \frac{n}{2}(a + l)$.

Step 1. $a = 17$, $l = 350$, $d = 9$. From $l = a + (n-1)d$: $350 = 17 + (n-1)(9)$.

Step 2. $350 - 17 = 9(n-1) \Rightarrow 333 = 9(n-1) \Rightarrow n - 1 = 37 \Rightarrow n = 38$.

Step 3. Sum: $S_{38} = \frac{38}{2}(a + l) = 19(17 + 350) = 19(367) = 6973$.

Final Answer: There are 38 terms and their sum is 6973.

☞ **Order of operations**

First get n (needs a, l, d), then get S_n (needs n, a, l). Trying the sum before the count leaves you with two unknowns.

EXPERT'S SOLUTION *: Pallavi Rane, M.Sc Mathematics, University of Mumbai*

Span over step for the count, then average.

- **Find the count:** the total rise from seventeen to three hundred fifty is three hundred thirty-three, and dividing by the step of nine gives thirty-seven steps, so there are thirty-eight terms.
- **Average the ends:** the average of the first and last terms is one hundred eighty-three point five.
- **Scale to the sum:** multiplying that average by the thirty-eight terms gives six

thousand nine hundred seventy-three, matching the formula.

- **Why it matters:** splitting the work into counting the terms and then averaging them gives two simple computations and a natural cross-check on the final sum.

Final Answer: 38 terms; sum = 6973.

Q 5.7 Find the sum of first 22 terms of an AP in which $d = 7$ and 22nd term is 149.

SOLUTION

Concept used. The 22nd term is the last term here, $l = a_{22} = 149$. First find a from $a_{22} = a + 21d$, then use $S_{22} = \frac{22}{2}(a + a_{22})$.

Step 1. $a_{22} = a + 21d = 149$ with $d = 7$: $a + 21(7) = 149 \Rightarrow a + 147 = 149 \Rightarrow a = 2$.

Step 2. $S_{22} = \frac{22}{2}(a + a_{22}) = 11(2 + 149) = 11(151) = 1661$.

Final Answer: The sum of the first 22 terms is 1661.

☞ **The nth term can be the last term**

"22nd term is 149" means $l = 149$ for a 22-term sum, so $S_{22} = \frac{22}{2}(a + l)$ applies directly once a is known.

EXPERT'S SOLUTION *: Sanjay Patil, M.Sc Mathematics, North Maharashtra University*

Two clean steps: find the start, then average.

- **Find the first term:** the twenty-second-term equation collapses to a first term of two.
- **Average the ends:** the average of the first and last terms is seventy-five point five.
- **Scale to the sum:** multiplying that average by the twenty-two terms gives one thousand six hundred sixty-one.
- **Why it matters:** once the last term is identified as the twenty-second term, the sum needs only the first term, which one equation supplies, keeping a seemingly long problem to two lines.

Final Answer: $S_{22} = 1661$.

Q 5.8 Find the sum of first 51 terms of an AP whose second and third terms are 14

and 18 respectively.

SOLUTION

Concept used. From two consecutive terms, $d = a_3 - a_2$ and $a = a_2 - d$. Then use $S_{51} = \frac{51}{2}[2a + 50d]$.

Step 1. $a_2 = 14$, $a_3 = 18$, so $d = a_3 - a_2 = 18 - 14 = 4$.

Step 2. First term $a = a_2 - d = 14 - 4 = 10$.

Step 3. $S_{51} = \frac{51}{2}\big[2(10) + (51-1)(4)\big] = \frac{51}{2}\big[20 + 200\big] = \frac{51}{2}(220)$.

Step 4. $= 51 \times 110 = 5610$.

> **Final Answer:** The sum of the first 51 terms is 5610.

☞ **Recover a from a_2**

Given the second term, step back one place: $a = a_2 - d$. Many students wrongly use a_2 as the first term, shifting the whole AP.

EXPERT'S SOLUTION *: Aarti Saxena, M.Sc Mathematics, Aligarh Muslim University*

Build the first term and step, then plug in.

- **The pivot:** the whole problem rests on getting the first term right from the given second term.
- **Find the step:** the two consecutive terms give a common difference of four.
- **Step back:** moving one place back from the second term gives a first term of ten.
- **Plug into the sum:** the sum formula for fifty-one terms then gives five thousand six hundred ten.
- **Why it matters:** a single misread, using the second term as the first, would throw off the entire sum, so stepping back to the true first term is the safeguard examiners reward.

> **Final Answer:** $S_{51} = 5610$.

Q 5.9 **If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.**

SOLUTION

Concept used. Write S_7 and S_{17} using $S_n = \frac{n}{2}[2a + (n-1)d]$, form two equations in a and d, solve, then substitute back to get the general S_n.

Step 1. $S_7 = \frac{7}{2}[2a + 6d] = 49 \Rightarrow 7(a + 3d) = 49 \Rightarrow a + 3d = 7$ (1)

Step 2. $S_{17} = \frac{17}{2}[2a + 16d] = 289 \Rightarrow 17(a + 8d) = 289 \Rightarrow a + 8d = 17$ (2)

Step 3. Subtract (1) from (2): $5d = 10 \Rightarrow d = 2$.

Step 4. From (1): $a + 3(2) = 7 \Rightarrow a = 1$.

Step 5. General sum: $S_n = \frac{n}{2}[2(1) + (n-1)(2)] = \frac{n}{2}[2 + 2n - 2] = \frac{n}{2}(2n) = n^2$.

Final Answer: $S_n = n^2$.

☞ **Simplify before solving**

Dividing $S_7 = 49$ by 7 and $S_{17} = 289$ by 17 early gives the tidy pair $a + 3d = 7$ and $a + 8d = 17$, which are far easier to handle.

EXPERT'S SOLUTION *: Ramesh Babu, M.Sc Mathematics, Andhra University*

Notice the perfect squares in the data.

- **Early hint:** the two given sums are the squares of seven and seventeen, which already hints that the sum of the first count of terms may equal the count squared.
- **Reduce the conditions:** dividing each condition by its count gives the tidy pair that yields a step of two and a first term of one.
- **Get the closed form:** substituting these back collapses the sum formula to the count squared.
- **Check it:** the closed form reproduces both given sums exactly, matching the perfect squares.
- **Why it matters:** finding a clean closed form lets you write any partial sum instantly and is a satisfying confirmation that the given data were consistent.

Final Answer: $S_n = n^2$.

Q 5.10 **Show that $a_1, a_2, \ldots, a_n, \ldots$ form an AP where a_n is defined as below. Also find the sum of the first 15 terms in each case.**

(i) $a_n = 3 + 4n$ **(ii)** $a_n = 9 - 5n$

SOLUTION

Concept used. A list is an AP if $a_{n+1} - a_n$ is the same constant for all n; that constant is d. Then a_1 is the first term, and $S_{15} = \frac{15}{2}[2a + 14d]$.

Step 1. **(i)** $a_n = 3 + 4n$. Then $a_{n+1} = 3 + 4(n+1) = 7 + 4n$.
$a_{n+1} - a_n = (7 + 4n) - (3 + 4n) = 4$, a constant, so it is an AP with $d = 4$.
First term $a_1 = 3 + 4(1) = 7$.
$S_{15} = \frac{15}{2}[2(7) + 14(4)] = \frac{15}{2}[14 + 56] = \frac{15}{2}(70) = 525$.

Step 2. **(ii)** $a_n = 9 - 5n$. Then $a_{n+1} = 9 - 5(n+1) = 4 - 5n$.
$a_{n+1} - a_n = (4 - 5n) - (9 - 5n) = -5$, a constant, so it is an AP with $d = -5$.
First term $a_1 = 9 - 5(1) = 4$.
$S_{15} = \frac{15}{2}[2(4) + 14(-5)] = \frac{15}{2}[8 - 70] = \frac{15}{2}(-62) = -465$.

Final Answer: (i) $d = 4$, AP confirmed, $S_{15} = 525$. (ii) $d = -5$, AP confirmed, $S_{15} = -465$.

☞ Proving "it is an AP"

To show a defined list is an AP, compute $a_{n+1} - a_n$ in general. If it simplifies to a number free of n, you have proved it is an AP and named d.

EXPERT'S SOLUTION *: Sunita Rao, M.Sc Mathematics, Gujarat University*

The coefficient of the term number is the step.

- **Key shortcut:** when the term formula is linear in the term number, the coefficient of that number is automatically the common difference.
- **Part (i):** the term coefficient is four, so the difference is four and the first term is seven.
- **Part (ii):** the term coefficient is minus five, so the difference is minus five and the first term is four.
- **The sums:** the fifteen-term sums then come out as five hundred twenty-five and minus four hundred sixty-five.
- **Why it matters:** recognising that any linear term formula is always an AP with that coefficient as the difference lets you read the step off instantly and focus your working on the sum.

Final Answer: (i) AP with $d = 4$, $S_{15} = 525$; (ii) AP with $d = -5$, $S_{15} = -465$.

Q 5.11 **If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that**

is S_1)? What is the sum of first two terms? What is the second term? Similarly, find the 3rd, the 10th and the nth terms.

SOLUTION

Concept used. Given S_n, the first term is S_1, and any term is the gap between successive sums: $a_n = S_n - S_{n-1}$.

Step 1. $S_n = 4n - n^2$. First term: $S_1 = 4(1) - 1^2 = 4 - 1 = 3$. So $a_1 = 3$.

Step 2. Sum of first two terms: $S_2 = 4(2) - 2^2 = 8 - 4 = 4$.

Step 3. Second term: $a_2 = S_2 - S_1 = 4 - 3 = 1$.

Step 4. Common difference $d = a_2 - a_1 = 1 - 3 = -2$.

Step 5. Third term: $a_3 = a_2 + d = 1 + (-2) = -1$. (Check: $S_3 = 12 - 9 = 3$, $a_3 = S_3 - S_2 = 3 - 4 = -1$.)

Step 6. Tenth term: $a_{10} = a + 9d = 3 + 9(-2) = 3 - 18 = -15$.

Step 7. General nth term: $a_n = S_n - S_{n-1} = (4n - n^2) - \left[4(n-1) - (n-1)^2\right]$.
$= 4n - n^2 - \left[4n - 4 - (n^2 - 2n + 1)\right] = 4n - n^2 - 4n + 4 + n^2 - 2n + 1 = 5 - 2n$.

Final Answer: $a_1 = 3$; $S_2 = 4$; $a_2 = 1$; $a_3 = -1$; $a_{10} = -15$; $a_n = 5 - 2n$.

☞ $a_n = S_n - S_{n-1}$

Each term equals the sum to n terms minus the sum to $(n-1)$ terms. This is the bridge that turns a sum formula into the individual terms.

EXPERT'S SOLUTION *: Mohit Bansal, B.Tech Chemical Engineering, NIT Rourkela*

Difference the sum to get the term formula.

- **Clean route:** the tidy way to the general term is the difference between successive sums, which collapses the quadratic neatly.
- **Shift the sum:** compute the sum to one fewer term, which expands to a simple quadratic in the term number.
- **Subtract:** the difference of the two sums cancels the square term and leaves the linear term formula five minus twice the count.
- **Check it:** that formula reproduces the first, second and tenth terms exactly, all matching the direct values.
- **Why it matters:** once you have the term formula, every individual term and the difference of minus two follow at once, and deriving the term formula from the sum is a core skill the board tests through this exact question type.

Final Answer: $a_n = 5 - 2n$, giving $a_1 = 3,\ a_2 = 1,\ a_3 = -1,\ a_{10} = -15$, and $S_2 = 4$.

Q 5.12 Find the sum of the first 40 positive integers divisible by 6.

SOLUTION

Concept used. The positive integers divisible by 6 are $6, 12, 18, \ldots$, an AP with $a = 6$ and $d = 6$. Sum the first 40 using $S_n = \frac{n}{2}[2a + (n-1)d]$.

Step 1. $a = 6$, $d = 6$, $n = 40$.

Step 2. $S_{40} = \frac{40}{2}\big[2(6) + (40-1)(6)\big] = 20\big[12 + 234\big] = 20(246)$.

Step 3. $= 4920$.

Final Answer: The sum of the first 40 positive integers divisible by 6 is 4920.

☞ **Factor out the 6**

The first 40 multiples of 6 are $6(1 + 2 + \ldots + 40)$. Since $1 + \ldots + 40 = \frac{40 \cdot 41}{2} = 820$, the sum is $6 \times 820 = 4920$, a neat check.

EXPERT'S SOLUTION *: Geetha Subramanian, M.Sc Mathematics, Bharathiar University*

Pull the common factor out of the sum.

- **The idea:** the multiples of six are just six times the plain counting numbers, so factor the six out and add the counting numbers.
- **Sum the counts:** the first forty counting numbers add to eight hundred twenty by the well-known formula.
- **Multiply back:** six times eight hundred twenty is four thousand nine hundred twenty, agreeing with the direct AP formula.
- **Why it matters:** factoring out the common multiplier turns the problem into the familiar sum of the first forty natural numbers, a fast and reliable cross-check.

Final Answer: 4920.

Q 5.13 Find the sum of the first 15 multiples of 8.

SOLUTION

Concept used. The multiples of 8 are $8, 16, 24, \ldots$, an AP with $a = 8$ and $d = 8$. Use $S_n = \frac{n}{2}[2a + (n-1)d]$ for $n = 15$.

Step 1. $a = 8$, $d = 8$, $n = 15$.

Step 2. $S_{15} = \frac{15}{2}\big[2(8) + (15-1)(8)\big] = \frac{15}{2}\big[16 + 112\big] = \frac{15}{2}(128)$.

Step 3. $= 15 \times 64 = 960$.

Final Answer: The sum of the first 15 multiples of 8 is 960.

☞ **Factor-out check**

$8(1 + 2 + \ldots + 15) = 8 \times \frac{15 \cdot 16}{2} = 8 \times 120 = 960$, confirming the answer.

EXPERT'S SOLUTION *: Tarun Chauhan, M.Sc Mathematics, Himachal Pradesh University*

Use the same factor-out idea here.

- **The idea:** the first fifteen multiples of eight are just eight times the first fifteen counting numbers, so pull the eight out.
- **Sum the counts:** the first fifteen counting numbers add to one hundred twenty.
- **Multiply back:** eight times one hundred twenty is nine hundred sixty.
- **Why it matters:** recognising the first few multiples of a number as that number times the triangular sum is a shortcut that works for any divisor and saves plugging into the long AP formula.

Final Answer: 960.

Q 5.14 Find the sum of the odd numbers between 0 and 50.

SOLUTION

Concept used. The odd numbers between 0 and 50 are $1, 3, 5, \ldots, 49$, an AP with $a = 1$, $d = 2$, last term $l = 49$. Find n, then sum with $S_n = \frac{n}{2}(a + l)$.

Step 1. $a = 1$, $d = 2$, $l = 49$. Find n:

$$49 = 1 + (n-1)(2) \Rightarrow 48 = 2(n-1) \Rightarrow n - 1 = 24 \Rightarrow n = 25.$$

Step 2. $S_{25} = \frac{25}{2}(a + l) = \frac{25}{2}(1 + 49) = \frac{25}{2}(50) = 25 \times 25 = 625$.

Final Answer: The sum of the odd numbers between 0 and 50 is 625.

☞ **Sum of first n odd numbers is n^2**

The first 25 odd numbers add to $25^2 = 625$. This well-known result is an instant check on the answer.

EXPERT'S SOLUTION *: Rekha Nambiar, M.Sc Mathematics, University of Calicut*

Use the odd-number square law.

- **The law:** the sum of the first count of odd numbers equals that count squared, which short-circuits the whole problem.
- **Count them:** the odd numbers from one to forty-nine number twenty-five in all.
- **Apply it:** the sum of the first twenty-five odd numbers is therefore twenty-five squared, six hundred twenty-five.
- **Cross-check:** this matches the full AP-sum computation exactly.
- **Why it matters:** the identity is worth remembering, since it answers this whole question in one line and reinforces the link between APs and perfect squares.

Final Answer: 625.

Q 5.15 **A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?**

SOLUTION

Concept used. Daily penalties form an AP: $a = 200$, $d = 50$. The total penalty for 30 days is $S_{30} = \frac{30}{2}[2a + (30-1)d]$.

Step 1. $a = 200$, $d = 50$, $n = 30$.

Step 2. $S_{30} = \frac{30}{2}\big[2(200) + (30-1)(50)\big] = 15\big[400 + 1450\big] = 15(1850)$.

Step 3. $= 27750$.

Final Answer: The contractor has to pay Rs 27750 as penalty for the 30-day delay.

☞ **Total, not last-day, penalty**

The question asks for the *total* over 30 days, so use S_{30}, not the 30th day's penalty alone. Read whether the problem wants a single term or a sum.

EXPERT'S SOLUTION *: Pankaj Tiwari, M.Sc Mathematics, University of Allahabad*

Average daily penalty times the number of days.

- **Last-day penalty:** the thirtieth day's penalty is the first day plus twenty-nine raises of fifty, which is one thousand six hundred fifty.
- **Average it:** the average of the first and last day penalties is nine hundred twenty-five.
- **Scale to total:** multiplying that average by the thirty days gives twenty-seven thousand seven hundred fifty, matching the sum formula.
- **Why it matters:** computing the last term and averaging is a sturdy way to cross-check a penalty or instalment total, making sure you have not confused the final-day amount with the running total.

Final Answer: Rs 27750.

Q 5.16 **A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.**

SOLUTION

Concept used. The prizes form an AP with $n = 7$ terms, common difference $d = -20$ (each is Rs 20 less), and total $S_7 = 700$. Use $S_n = \frac{n}{2}[2a + (n-1)d]$ to find the first (largest) prize a, then list all seven.

Step 1. $n = 7$, $d = -20$, $S_7 = 700$.

Step 2. $700 = \frac{7}{2}\big[2a + (7-1)(-20)\big] = \frac{7}{2}\big[2a - 120\big] = 7(a - 60)$.

Step 3. $a - 60 = \frac{700}{7} = 100 \Rightarrow a = 160$.

Step 4. The seven prizes (subtracting Rs 20 each time): $160,\ 140,\ 120,\ 100,\ 80,\ 60,\ 40$ (in rupees).

Final Answer: The prizes are Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60 and Rs 40.

☞ **Check the prizes add to the total**

$160 + 140 + 120 + 100 + 80 + 60 + 40 = 700$. Always confirm the listed prizes sum back to the given amount.

EXPERT'S SOLUTION *: Sangeeta Roy, M.Sc Mathematics, Jadavpur University*

Anchor on the middle prize.

- **Find the centre:** for an odd number of terms the average prize is the middle one, and the total divided by seven makes that middle prize one hundred rupees.
- **Build outward:** step up by twenty to the higher prizes and down by twenty to the lower ones around that centre to get the seven values.
- **Sanity check:** the seven prizes are symmetric about one hundred rupees and add back to the given total of seven hundred.
- **Why it matters:** for an odd-length AP the middle term equals the average, so spotting that the fourth prize is one hundred rupees lets you build the list outward without solving for the first term first.

Final Answer: Rs 160, 140, 120, 100, 80, 60, 40.

Q 5.17 **In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?**

SOLUTION

Concept used. One section of class k plants k trees, and there are 3 sections per class. The total per class is $3k$, for $k = 1$ to 12. These totals $3, 6, 9, \ldots, 36$ form an AP; sum them.

Step 1. Trees by all sections of class k: $3 \times k$. For $k = 1$ to 12: $3, 6, 9, \ldots, 36$.

Step 2. This is an AP with $a = 3$, $d = 3$, $n = 12$.

Step 3. $S_{12} = \frac{12}{2}\big[2(3) + (12-1)(3)\big] = 6\big[6 + 33\big] = 6(39) = 234$.

Final Answer: The students will plant 234 trees in all.

☞ Factor the section count

Total trees $= 3 \times (1 + 2 + \cdots + 12)$. Since $1 + \cdots + 12 = 78$, the total is $3 \times 78 = 234$, a fast check.

EXPERT'S SOLUTION *: Imran Khan, M.Sc Mathematics, Jamia Millia Islamia*

Sum the classes first, then triple.

- **The plan:** it is cleaner to add the per-section counts first and multiply by the three sections only at the end.
- **One section:** a single section across the twelve classes plants the sum of one to twelve, which is seventy-eight trees.
- **All sections:** with three sections per class the total is three times seventy-eight, which is two hundred thirty-four.
- **Cross-check:** this matches the direct AP sum of three, six and so on up to thirty-six.
- **Why it matters:** pulling the section count outside the sum reduces the problem to adding the first twelve natural numbers, a quicker and less error-prone path.

Final Answer: 234 trees.

Q 5.18 **A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... as shown in Fig. 5.4. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take $\pi = \frac{22}{7}$)**

Fig. 5.4: the spiral of thirteen successive semicircles with centres alternately at A and B.

SOLUTION

Concept used. The length of a semicircle of radius r is half the circumference, πr. The thirteen semicircle lengths form an AP because the radii $0.5, 1.0, 1.5, \ldots$ increase by a fixed step. Sum the thirteen lengths.

Step 1. Length of the kth semicircle $= \pi r_k$, where the radii are $r_1 = 0.5,\ r_2 = 1.0,\ r_3 = 1.5, \ldots$ cm (each 0.5 cm more).

Step 2. So the lengths are $\pi(0.5),\ \pi(1.0),\ \pi(1.5),\ \ldots$, an AP with $a = 0.5\pi$ and $d = 0.5\pi$, for $n = 13$ terms.

Step 3. Total length $L = S_{13} = \pi\,(0.5 + 1.0 + 1.5 + \cdots + 13 \text{ terms})$. The radii sum is $0.5(1 + 2 + \cdots + 13) = 0.5 \times \frac{13\times14}{2} = 0.5 \times 91 = 45.5$ cm.

Step 4. $L = \pi \times 45.5 = \frac{22}{7} \times 45.5 = \frac{22\times45.5}{7} = \frac{1001}{7} = 143$ cm.

Final Answer: The total length of the spiral is 143 cm.

☞ **Semicircle, not full circle**

Each piece is a *semi*circle, so its length is πr, not $2\pi r$. Using the full circumference would double the answer.

EXPERT'S SOLUTION *: Bhavna Shah, M.Sc Mathematics, Sardar Patel University*

Sum the radii, then multiply by pi.

- **Factor pi out:** every semicircle length is pi times its radius, so pull the pi outside all thirteen lengths and add only the radii.
- **Add the radii:** the thirteen radii form their own AP and sum to forty-five point five centimetres.
- **Multiply back:** the total length is pi times forty-five point five, using the given value of twenty-two over seven.
- **Clean cancellation:** the numerator becomes one thousand and one, which divides by seven to a clean one hundred forty-three centimetres.
- **Why it matters:** taking the common pi outside turns a geometry problem into a plain AP sum of radii, and the clean cancellation confirms the chosen value of pi was meant to give a whole number.

Final Answer: 143 cm.

Q 5.19 **200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on (see Fig. 5.5). In how many rows are the 200 logs placed and how many logs are in the top row?**

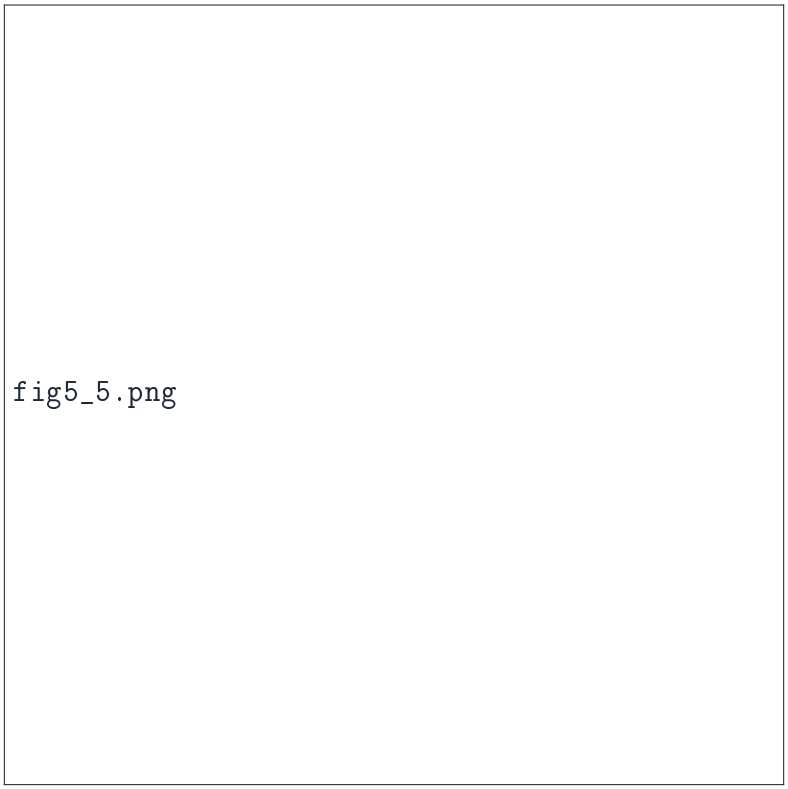

Fig. 5.5: logs stacked with 20 in the bottom row, then 19, 18, and so on upward.

SOLUTION

Concept used. Logs per row form an AP: $a = 20$, $d = -1$. The total is $S_n = 200$. Solving $S_n = \frac{n}{2}[2a + (n-1)d] = 200$ gives n (number of rows), and $a_n = a + (n-1)d$ gives the logs in the top row.

Step 1. $a = 20$, $d = -1$, $S_n = 200$.

Step 2. $200 = \frac{n}{2}\big[2(20) + (n-1)(-1)\big] = \frac{n}{2}\big[40 - n + 1\big] = \frac{n}{2}(41 - n)$.

Step 3. $400 = n(41 - n) = 41n - n^2$, so $n^2 - 41n + 400 = 0$.

Step 4. Factor: $(n - 16)(n - 25) = 0 \Rightarrow n = 16$ or $n = 25$.

Step 5. Test $n = 25$: top row $a_{25} = 20 + 24(-1) = -4$, which is impossible (a row cannot have negative logs). So reject $n = 25$.

Step 6. Take $n = 16$: top row $a_{16} = 20 + 15(-1) = 20 - 15 = 5$ logs.

Final Answer: The logs are placed in 16 rows, with 5 logs in the top row.

✗ Reject the impossible root

Both $n = 16$ and $n = 25$ solve the quadratic, but $n = 25$ forces a negative top row (-4 logs). Always test each root against the physical meaning.

EXPERT'S SOLUTION *: Dinesh Gowda, M.Sc Mathematics, Tumkur University*

Reality-check both roots of the quadratic.

- **Two candidates:** the quadratic for the number of rows gives two values, sixteen and twenty-five, but only one can be physically real.
- **Reject the bad root:** for twenty-five rows the top row would hold minus four logs, which is meaningless.
- **Keep the good root:** for sixteen rows the top row holds five logs, a sensible positive count.
- **Why it matters:** word problems often yield two algebraic roots, but physical limits such as no negative logs decide the real answer, and checking the top-row count is the deciding step examiners look for.

Final Answer: 16 rows; 5 logs in the top row.

Q 5.20 **In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see Fig. 5.6). A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run? [Hint: To pick up the first potato and the second potato, the total distance (in metres) run by a competitor is** $2 \times 5 + 2 \times (5 + 3)$**.]**

Fig. 5.6: the bucket, then ten potatoes placed 5 m, then 3 m apart along a straight line.

SOLUTION

Concept used. For each potato the competitor runs to it and back, so the distance is twice the potato's distance from the bucket. These round-trip distances form an AP; sum the ten of them.

Step 1. Distances of potatoes from the bucket: $5,\ 5+3,\ 5+6, \ldots = 5, 8, 11, \ldots$ m.

Step 2. Each potato needs a there-and-back run, so the run distances are $2\times 5 = 10,\ 2\times 8 = 16,\ 2\times 11 = 22, \ldots$ m.

Step 3. These form an AP with $a = 10$, $d = 6$, $n = 10$.

Step 4. $S_{10} = \frac{10}{2}\big[2(10) + (10-1)(6)\big] = 5\big[20 + 54\big] = 5(74) = 370.$

Final Answer: The total distance the competitor has to run is 370 m.

☞ Double for the return run

Each trip is to the potato and back, so multiply every one-way distance by 2 before summing. Forgetting the return trip halves the answer.

EXPERT'S SOLUTION *: Anita Pradhan, M.Sc Mathematics, Utkal University*

Factor the doubling out of the sum.

- **The idea:** the total run is twice the sum of the one-way distances, so add the one-way distances first and double at the end.
- **One-way AP:** the one-way distances form a simple AP starting at five with a step of three over ten terms.
- **One-way sum:** that AP sums to one hundred eighty-five metres.
- **Double it:** the total run is twice one hundred eighty-five, which is three hundred seventy metres.
- **Why it matters:** pulling the doubling outside lets you sum the simpler one-way distances first, keeping the numbers small and the doubling explicit so the return trip is never missed.

> **Final Answer:** 370 m.

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