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NCERT Solutions Class 10 Maths Chapter 6 Triangles Exercise 6.1

Step-by-step solutions, alternate methods & exam tips for Class 10 Mathematics

Chapter 6: Triangles

About this Chapter

Class 10 Maths Chapter 6 Triangles is about **similar figures**, shapes with the same form but different sizes. You learn the **Basic Proportionality Theorem** and its converse, plus the **AA, SSS and SAS** similarity tests. After this chapter you can prove triangles similar, set up side ratios, and find heights using shadows.

Topics covered: Similar figures • Similar triangles • Basic Proportionality Theorem • Converse of BPT • AAA and AA similarity • SSS similarity • SAS similarity • Corresponding sides in proportion • Heights and shadows by similarity

Quick Formula Sheet

Basic Proportionality Theorem (BPT):
In $\triangle ABC$, if $DE \parallel BC$ with D on AB and E on AC, then $\frac{AD}{DB} = \frac{AE}{EC}$.

Converse of BPT:
If $\frac{AD}{DB} = \frac{AE}{EC}$ in $\triangle ABC$, then $DE \parallel BC$.

Similarity tests:
AA: two angles equal. SSS: all three side ratios equal. SAS: one angle equal and the two sides around it in the same ratio.

Corresponding sides of similar triangles:
If $\triangle ABC \sim \triangle DEF$ then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$.

Also see for this chapter: Revision Notes | Formula Sheet | Exemplar Solutions

Exercise 6.1

Q 6.1 **Fill in the blanks using the correct word given in brackets:**

(i) All circles are _____. (congruent, similar)

(ii) All squares are _____. (similar, congruent)

(iii) All _____ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _____ and (b) their corresponding sides are _____. (equal, proportional)

SOLUTION

Concept used. Two figures are **similar** when they have the same shape but not necessarily the same size. The full rule for polygons is that two polygons with the same number of sides are similar only when both conditions hold together: their corresponding angles are equal and their corresponding sides are in the same ratio (proportional). We test each blank against this definition.

Step 1. (i) Any two circles have the same round shape; only their radius can differ. Same shape, possibly different size, means *similar*.

Step 2. (ii) Any two squares have four right angles and four equal sides, so they share the same shape; the side length may differ. Hence *similar*.

Step 3. (iii) An *equilateral* triangle always has all angles 60°, so any two equilateral triangles are equiangular and share shape. Isosceles triangles can have different apex angles, so they need not be similar.

Step 4. (iv) The polygon definition supplies the two missing words directly: corresponding angles *equal*, corresponding sides *proportional*.

Final Answer: (i) similar (ii) similar (iii) equilateral (iv) (a) equal, (b) proportional

☞ **Useful aside**

For (iii) the trap word is "isosceles". Two isosceles triangles can look very different (a tall thin one and a short wide one), so equal shape is not guaranteed. Equilateral triangles are forced to be equiangular, which is why they are always similar.

EXPERT'S SOLUTION *: Anjali Verma, M.Sc Mathematics, University of Delhi*

How to lock these one-word answers in a board exam. Each blank is marked on the exact definitional word, so think "same shape always?" before writing each one.

- **Shape-fixed figures:** a circle is set by its radius and a square by its side, so changing the size never changes the shape. Both are therefore always similar, whatever their

size.

- **Triangles:** the shape can change unless the angles are pinned down. Only the equilateral triangle fixes all three angles at $60°$, which is why it alone is always similar.
- **Both conditions:** the polygon blank wants equal angles and proportional sides together. Writing only one of the two would lose half the mark.
- **Common slip:** students write "congruent" where "similar" is meant. Congruent demands equal size as well as equal shape, so reading the definition first prevents the mix-up.

Final Answer: (i) similar, (ii) similar, (iii) equilateral, (iv) equal and proportional.

Q 6.2 Give two different examples of pair of

(i) similar figures. (ii) non-similar figures.

SOLUTION

Concept used. A pair of figures is **similar** when one is a scaled copy of the other, that is, same shape and corresponding sides in the same ratio. A pair is **non-similar** when the shapes differ, so no single scale factor maps one onto the other. We pick everyday pairs that clearly satisfy or break this rule.

Step 1. Similar example 1: two photographs of the same scene printed at passport size and at postcard size. One is an exact enlargement of the other, so all lengths share one ratio.

Step 2. Similar example 2: any two equilateral triangles, or two circles of different radii. Their shape is fixed, only the size changes.

Step 3. Non-similar example 1: a square and a rectangle that is not a square. The angles match ($90°$ each) but the side ratios do not, so they are not similar.

Step 4. Non-similar example 2: a circle and a square. Their shapes are completely different, so no scaling turns one into the other.

Final Answer: Similar: (a) two photos of the same picture in different print sizes, (b) two circles. Non-similar: (a) a square and a non-square rectangle, (b) a circle and a square.

☞ **Recall**

Similarity needs *both* equal corresponding angles *and* proportional corresponding sides. A square and a rectangle fail the side test; a circle and a square fail both. Either failure is enough to make a pair non-similar.

EXPERT'S SOLUTION *: Rohan Deshmukh, M.Sc Mathematics, Savitribai Phule Pune University*

Why simple, well-chosen examples score full marks here. An open question like this rewards a clear pair plus a one-line reason, not a long list.

- **Best similar pair:** two photographs of the same image at different print sizes. Every length scales by the same factor, so the reason writes itself; two circles work equally well.
- **Best non-similar pair:** a square and a longer rectangle keep equal angles but break the side ratio, while a circle and a square break the very idea of a common shape.
- **Name the failing test:** stating which condition fails turns a guess into a justified answer, and that justification is what the examiner actually rewards.
- **Avoid vague pairs:** do not write "two triangles" without detail, since two general triangles may or may not be similar at all.

The examiner is checking that you can connect a chosen pair back to the definition, so one good pair with a clear reason beats four vague pairs with none. A safe routine is to pick the pair first and then ask which similarity condition it passes or fails before writing anything down.

> **Final Answer:** Pick a scaled photo pair or two circles for similar; a square with a non-square rectangle, or a circle with a square, for non-similar, always naming which condition fails.

Q 6.3 State whether the following quadrilaterals are similar or not:

Fig. 6.8 - square $ABCD$ of side 3 cm and quadrilateral $PQRS$ with all sides 1.5 cm.

SOLUTION

Concept used. Two polygons are **similar** only when both tests pass at the same time: corresponding angles equal *and* corresponding sides in the same ratio. If either test fails, the polygons are not similar. From the figure $ABCD$ is a square (all sides 3 cm, all angles $90°$) and $PQRS$ has all four sides 1.5 cm, which makes it a rhombus whose angles are not right angles.

Step 1. Check the side ratio. Every side of $ABCD$ is 3 cm and every side of $PQRS$ is 1.5 cm, so

$$\frac{AB}{PQ} = \frac{3}{1.5} = 2.$$

All corresponding sides share the ratio $2:1$, so the side test passes.

Step 2. Check the angles. In the square each angle is $90°$. In the rhombus $PQRS$ the angles are not $90°$ (a slanted rhombus has two acute and two obtuse angles).

Step 3. Compare. The corresponding angles are *not* equal, so the angle test fails even though the side test passed.

> **Final Answer:** The two quadrilaterals are not similar, because their corresponding sides are proportional but their corresponding angles are not equal.

✗ Common Mistake

Seeing equal side ratios ($3 : 1.5 = 2$) tempts students to call the figures similar. For polygons one matching condition is never enough. A square and a rhombus share proportional

sides yet differ in angles, so they fail similarity.

EXPERT'S SOLUTION *: Karthik Iyer, M.Sc Mathematics, Anna University*

The square-versus-rhombus trap, stated cleanly. This is the classic counter-example showing why *both* similarity conditions are needed at the same time.

- **Read the figure:** $PQRS$ has equal sides but tilted corners, which makes it a rhombus rather than a square. Note this before testing anything.
- **Sides pass:** all ratios reduce to $2:1$, so the proportional-sides condition holds and might fool a quick reader into calling them similar.
- **Angles fail:** a square is locked at four 90° corners while a non-square rhombus is not, so the corresponding angles disagree and similarity breaks.
- **Why both matter:** this single example is the reason the definition asks for equal angles and equal side ratios together, not either one on its own.

Writing the side ratio first and then pointing to the angle mismatch gives a complete, examiner-friendly justification. Many students stop at the equal sides and wrongly call the pair similar, so the angle line is the one that actually earns the mark here.

> **Final Answer:** Not similar: sides are in ratio $2:1$ but the square's 90° angles do not match the rhombus angles.

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NCERT Solutions Class 10 Maths Chapter 6 Triangles Exercise 6.2

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Exercise 6.2

Q 6.1 **In Fig. 6.17, (i) and (ii), $DE \parallel BC$. Find EC in (i) and AD in (ii).**

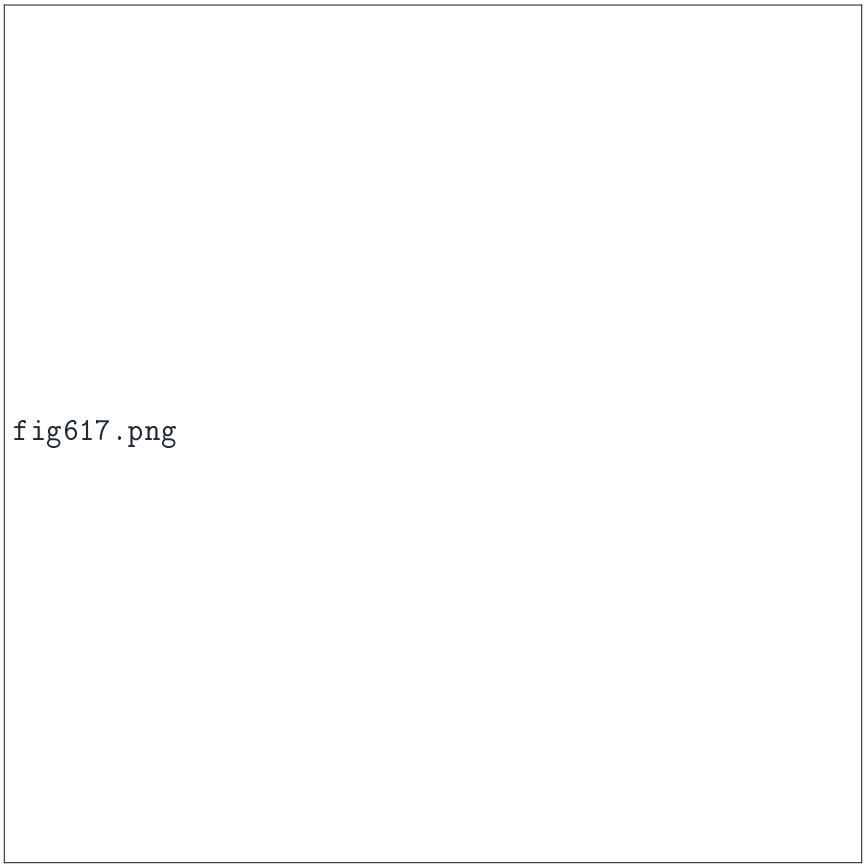


Fig. 6.17 - (i) $AD = 1.5$ *cm,* $DB = 3$ *cm,* $AE = 1$ *cm; (ii)* $DB = 7.2$ *cm,* $AE = 1.8$ *cm,* $EC = 5.4$ *cm.*

SOLUTION

Concept used. The **Basic Proportionality Theorem (BPT)** says that if a line is drawn parallel to one side of a triangle and cuts the other two sides, it divides those two sides in the same ratio. So in $\triangle ABC$ with $DE \parallel BC$,

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Part (i).

Step 1. Write BPT for the figure:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Step 2. Substitute $AD = 1.5$, $DB = 3$, $AE = 1$:

$$\frac{1.5}{3} = \frac{1}{EC}.$$

Step 3. Cross-multiply:

$$1.5 \times EC = 1 \times 3.$$

Step 4. Solve for EC:

$$EC = \frac{3}{1.5} = 2 \text{ cm}.$$

Part (ii).

Step 1. Here D is on AB and E on AC with $DB = 7.2$, $AE = 1.8$, $EC = 5.4$. BPT gives

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Step 2. Substitute the known values:

$$\frac{AD}{7.2} = \frac{1.8}{5.4}.$$

Step 3. Simplify the right side:

$$\frac{1.8}{5.4} = \frac{1}{3}.$$

Step 4. Solve for AD:

$$AD = 7.2 \times \frac{1}{3} = 2.4 \text{ cm}.$$

Final Answer: (i) $EC = 2$ cm (ii) $AD = 2.4$ cm.

☞ **Useful aside**

Always match each numerator with the segment on the *same* side of the triangle. AD pairs with AE (both touch vertex A), and DB pairs with EC (both touch the base). Mixing these up flips the ratio.

EXPERT'S SOLUTION *: Sneha Patel, M.Sc Mathematics, Sardar Patel University*

Set the ratio up by position, not by name order. The reliable habit for any BPT problem is to put the upper pieces (touching the apex) on top and the lower pieces (touching the base) underneath, on both sides of the equation.

- **Part (i):** that layout fixes $\frac{AD}{DB} = \frac{AE}{EC}$, and a single cross-multiplication gives $EC = 2$ cm with no fuss.
- **Part (ii):** the same layout applies; simplifying $\frac{1.8}{5.4}$ to $\frac{1}{3}$ first keeps the arithmetic clean and leads straight to $AD = 2.4$ cm.
- **Reduce early:** cutting the known ratio to lowest terms before multiplying avoids clumsy decimals and the slips they cause in a timed paper.
- **Sanity check:** each answer must be positive and smaller than the whole side it sits on, which both clearly are here.

Final Answer: By BPT, $EC = 2$ cm in (i) and $AD = 2.4$ cm in (ii).

Q 6.2 **E and F are points on the sides PQ and PR respectively of a $\triangle PQR$. For each of the following cases, state whether $EF \parallel QR$:**

(i) $PE = 3.9$ **cm,** $EQ = 3$ **cm,** $PF = 3.6$ **cm and** $FR = 2.4$ **cm**

(ii) $PE = 4$ **cm,** $QE = 4.5$ **cm,** $PF = 8$ **cm and** $RF = 9$ **cm**

(iii) $PQ = 1.28$ **cm,** $PR = 2.56$ **cm,** $PE = 0.18$ **cm and** $PF = 0.36$ **cm**

SOLUTION

Concept used. The converse of the Basic Proportionality Theorem says that if a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side. So $EF \parallel QR$ exactly when

$$\frac{PE}{EQ} = \frac{PF}{FR}.$$

In part (iii) we first turn the whole-side data into the two segment pieces by subtraction.

Part (i).

Step 1. Compute each ratio:

$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3, \qquad \frac{PF}{FR} = \frac{3.6}{2.4} = 1.5.$$

Step 2. Compare: $1.3 \neq 1.5$, so the two ratios are unequal.

So EF is *not* parallel to QR.

Part (ii).

Step 1. Compute each ratio:

$$\frac{PE}{QE} = \frac{4}{4.5} = \frac{8}{9}, \qquad \frac{PF}{RF} = \frac{8}{9}.$$

Step 2. Compare: both equal $\frac{8}{9}$, so the ratios match.

So $EF \parallel QR$.

Part (iii).

Step 1. Find EQ and FR by subtraction:

$$EQ = PQ - PE = 1.28 - 0.18 = 1.10 \text{ cm},$$

$$FR = PR - PF = 2.56 - 0.36 = 2.20 \text{ cm}.$$

Step 2. Compute each ratio:

$$\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{9}{55}, \qquad \frac{PF}{FR} = \frac{0.36}{2.20} = \frac{9}{55}.$$

Step 3. Compare: both equal $\frac{9}{55}$, so the ratios match.

So $EF \parallel QR$.

Final Answer: (i) Not parallel (ii) $EF \parallel QR$ (iii) $EF \parallel QR$.

✗ Common Mistake

In part (iii) the data gives whole sides PQ and PR, not the lower pieces. Forgetting to subtract ($EQ = PQ - PE$, $FR = PR - PF$) is the usual slip. Always split a full side into its two parts before forming the converse-BPT ratio.

EXPERT'S SOLUTION *: Vikram Menon, M.Sc Mathematics, University of Calicut*

Decide parallelism by a clean ratio test, not by the picture. The converse of BPT turns the parallel question into a single equality of ratios, which is far safer than eyeballing a sketch.

- **Part (i):** the two ratios come out as 1.3 and 1.5. Since they differ, EF cannot be parallel to QR, so the answer is not parallel.
- **Part (ii):** both ratios reduce to $\frac{8}{9}$, so the equality holds and the line is parallel to the third side.
- **Part (iii):** the question gives whole sides, so subtract first to get $EQ = 1.10$ and $FR = 2.20$; both ratios then become $\frac{9}{55}$ and parallelism follows.
- **Takeaway:** reduce each fraction to lowest terms before comparing. The same reduced form makes the equality obvious and kills the urge to round decimals.

Final Answer: Using converse of BPT: (i) not parallel, (ii) parallel, (iii) parallel.

Q 6.3 **In Fig. 6.18, if $LM \parallel CB$ and $LN \parallel CD$, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.**

fig618.png

Fig. 6.18 - point L on AC; $LM \parallel CB$ (M on AB) and $LN \parallel CD$ (N on AD).

SOLUTION

Concept used. The **Basic Proportionality Theorem** says a line parallel to one side of a triangle splits the other two sides in the same ratio. We use it once in $\triangle ABC$ and once in $\triangle ACD$, then link the two results through the side AC that they share.

Step 1. In $\triangle ABC$, $LM \parallel CB$ with L on AC and M on AB. By BPT,

$$\frac{AM}{MB} = \frac{AL}{LC}. \tag{1}$$

Step 2. In $\triangle ACD$, $LN \parallel CD$ with L on AC and N on AD. By BPT,

$$\frac{AN}{ND} = \frac{AL}{LC}. \tag{2}$$

Step 3. The right sides of (1) and (2) are identical, so

$$\frac{AM}{MB} = \frac{AN}{ND}. \tag{3}$$

Step 4. Add 1 to both sides of (3). Using $\frac{AM}{MB} + 1 = \frac{AM+MB}{MB} = \frac{AB}{MB}$ and likewise for the other side,

$$\frac{AB}{MB} = \frac{AD}{ND}.$$

Step 5. Take reciprocals:

$$\frac{MB}{AB} = \frac{ND}{AD}.$$

Step 6. Subtract each side from 1, since $1 - \frac{MB}{AB} = \frac{AB-MB}{AB} = \frac{AM}{AB}$:

$$\frac{AM}{AB} = \frac{AN}{AD}.$$

Final Answer: $\dfrac{AM}{AB} = \dfrac{AN}{AD}$, proved by applying BPT in $\triangle ABC$ and $\triangle ACD$ about the common ratio $\dfrac{AL}{LC}$.

☞ **Recall**

The trick that links two triangles is a *common* segment. Here $\frac{AL}{LC}$ appears in both BPT statements, so the two ratios equal each other. Hunting for the shared ratio is the standard first move in these two-triangle proofs.

EXPERT'S SOLUTION *: Meera Krishnan, M.Sc Mathematics, University of Kerala*

Bridge the two triangles through the shared ratio. Both parallel lines start from the same point L on AC, which is the key that links the two triangles together.

- **Shared fraction:** applying BPT in each triangle produces the same right-hand side $\frac{AL}{LC}$, so the two left sides must be equal: $\frac{AM}{MB} = \frac{AN}{ND}$.
- **Add one:** adding 1 to each side converts a part-to-part ratio into a part-to-whole ratio, turning MB into AB and ND into AD in a single move.
- **Clean write-up:** state both BPT lines, equate the right sides, then do the $+1$ step. The result $\frac{AM}{AB} = \frac{AN}{AD}$ is exactly the part-to-whole form asked for.

Finishing with the whole-side denominators is what earns the final mark. The add-one move is worth practising on its own, because it appears in several proofs in this chapter whenever a part-to-part ratio has to become a part-to-whole one. If you forget it, you are left with MB and ND in the denominators and the answer never matches the form the question asked for, so the conversion step is not optional here.

Final Answer: Both parallels give the ratio $\frac{AL}{LC}$, so $\frac{AM}{AB} = \frac{AN}{AD}$.

Q 6.4 **In Fig. 6.19, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.**

fig619.png

Fig. 6.19 - D on AB; $DE \parallel AC$ (E on BC) and $DF \parallel AE$ (F on BE).

SOLUTION

Concept used. The **Basic Proportionality Theorem** is applied twice, in two triangles that share the vertex B. We use the common ratio $\frac{BD}{DA}$ to connect the two results.

Step 1. In $\triangle ABC$, $DE \parallel AC$ with D on AB and E on BC. By BPT,

$$\frac{BD}{DA} = \frac{BE}{EC}. \tag{1}$$

Step 2. In $\triangle ABE$, $DF \parallel AE$ with D on AB and F on BE. By BPT,

$$\frac{BD}{DA} = \frac{BF}{FE}. \tag{2}$$

Step 3. The left sides of (1) and (2) are the same fraction $\frac{BD}{DA}$, so their right sides are equal:

$$\frac{BE}{EC} = \frac{BF}{FE}.$$

Step 4. Rewriting gives the required result

$$\frac{BF}{FE} = \frac{BE}{EC}.$$

Final Answer: $\frac{BF}{FE} = \frac{BE}{EC}$, since both equal the common ratio $\frac{BD}{DA}$ by BPT.

☞ Exam Tip

Name the triangle before each BPT line ($\triangle ABC$ for $DE \parallel AC$, $\triangle ABE$ for $DF \parallel AE$). Examiners award a mark for clearly stating the triangle and the parallel side used, so write them out rather than jumping straight to the ratios.

EXPERT'S SOLUTION *: Aditya Rao, M.Sc Mathematics, Osmania University*

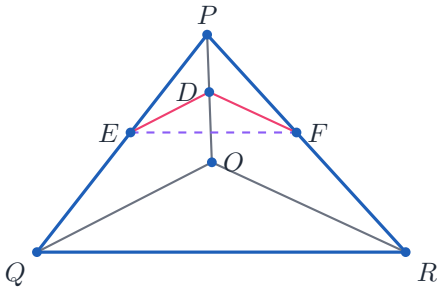
Use the same dividing point in two triangles. The point D on AB is the hinge of this proof, because both parallel lines sit in configurations that contain AB.

- **First triangle:** BPT in $\triangle ABC$ turns $DE \parallel AC$ into $\frac{BD}{DA} = \frac{BE}{EC}$, which fixes one half of the link.
- **Second triangle:** BPT in $\triangle ABE$ turns $DF \parallel AE$ into $\frac{BD}{DA} = \frac{BF}{FE}$, the same left-hand fraction as before.
- **Conclude:** since the left sides match, the right sides are equal, which is the required result $\frac{BF}{FE} = \frac{BE}{EC}$.

The one habit that matters is naming which triangle each parallel line belongs to; choosing $\triangle ABE$ for the second line is what makes BF and FE appear at all.

Final Answer: $\frac{BF}{FE} = \frac{BD}{DA} = \frac{BE}{EC}$, so $\frac{BF}{FE} = \frac{BE}{EC}$.

Q 6.5 **In Fig. 6.20, $DE \parallel OQ$ and $DF \parallel OR$. Show that $EF \parallel QR$.**



SOLUTION

Concept used. We use the **Basic Proportionality Theorem** in two triangles that share the segment PO, and then the **converse of BPT** in the big triangle PQR. BPT: a line parallel to a side cuts the other two sides in the same ratio. Converse: if a line cuts two sides in the same ratio, it is parallel to the third.

Step 1. In $\triangle POQ$, $DE \parallel OQ$ with D on PO and E on PQ. By BPT,

$$\frac{PD}{DO} = \frac{PE}{EQ}. \tag{1}$$

Step 2. In $\triangle POR$, $DF \parallel OR$ with D on PO and F on PR. By BPT,

$$\frac{PD}{DO} = \frac{PF}{FR}. \tag{2}$$

Step 3. The left sides of (1) and (2) are the same fraction, so

$$\frac{PE}{EQ} = \frac{PF}{FR}. \tag{3}$$

Step 4. Equation (3) says the line EF divides the two sides PQ and PR of $\triangle PQR$ in the same ratio. By the converse of BPT, $EF \parallel QR$.

Final Answer: $EF \parallel QR$, because $\dfrac{PE}{EQ} = \dfrac{PF}{FR}$ (both equal $\dfrac{PD}{DO}$), and the converse of BPT then forces EF parallel to QR.

☞ **Useful aside**

This is a two-step pattern: BPT (forward) turns parallel lines into equal ratios, and BPT (converse) turns equal ratios back into a parallel line. Spotting the shared segment PO is what lets the two ratios meet.

EXPERT'S SOLUTION *: Priya Nambiar, M.Sc Mathematics, Cochin University of Science and Technology*

Run BPT forward twice, then once in reverse. The structure here is worth memorising because it appears again and again in this exercise.

- **Forward twice:** both lines come from D on PO, so BPT in $\triangle POQ$ and $\triangle POR$ gives the same left-hand fraction $\frac{PD}{DO}$ each time.
- **Equate:** setting the right sides equal yields $\frac{PE}{EQ} = \frac{PF}{FR}$, which is exactly the condition the converse of BPT needs in the outer triangle PQR.
- **Reverse once:** invoking that converse then delivers $EF \parallel QR$, closing the proof in one clean step.

Marks split between the two forward applications and the final converse, so name each triangle and state the converse explicitly to collect all three. The pattern of two forward uses followed by one reverse use is the template for every "show this line is parallel" question in the exercise, so once you spot it here you can reuse it almost without thinking on the very next problem.

Final Answer: Forward BPT twice gives $\frac{PE}{EQ} = \frac{PF}{FR}$; the converse of BPT then gives $EF \parallel QR$.

Q 6.6 **In Fig. 6.21, A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.**

fig621.png

Fig. 6.21 - A on OP, B on OQ, C on OR with $AB \parallel PQ$ and $AC \parallel PR$.

SOLUTION

Concept used. As before we use the **Basic Proportionality Theorem** forward in two triangles sharing the segment OP, then its **converse** in the triangle OQR.

Step 1. In $\triangle OPQ$, $AB \parallel PQ$ with A on OP and B on OQ. By BPT,

$$\frac{OA}{AP} = \frac{OB}{BQ}. \tag{1}$$

Step 2. In $\triangle OPR$, $AC \parallel PR$ with A on OP and C on OR. By BPT,

$$\frac{OA}{AP} = \frac{OC}{CR}. \tag{2}$$

Step 3. The left sides of (1) and (2) are equal, so

$$\frac{OB}{BQ} = \frac{OC}{CR}. \tag{3}$$

Step 4. Equation (3) says the line BC divides the sides OQ and OR of $\triangle OQR$ in the same ratio. By the converse of BPT, $BC \parallel QR$.

Final Answer: $BC \parallel QR$, since $\frac{OB}{BQ} = \frac{OC}{CR}$ (both equal $\frac{OA}{AP}$) and the converse of BPT applies in $\triangle OQR$.

♥ Why This Matters

This "parallel cascade" is exactly how shadows and scale models work. Cutting across an angle with one parallel line forces a fixed ratio, and a second parallel from the same point carries that ratio to a new side, so the far line ends up parallel too. The same chain of similar triangles lets surveyors copy a shape at a different size.

EXPERT'S SOLUTION *: Sanjay Gupta, M.Sc Mathematics, Banaras Hindu University*

Identical structure to the previous problem, with O as apex. Recognising this as the same proof pattern saves time in a timed paper.

- **Hinge point:** A on OP anchors both parallel lines, so BPT in $\triangle OPQ$ and $\triangle OPR$ yields the same left-hand fraction $\frac{OA}{AP}$ each time.
- **Equate:** the right sides give $\frac{OB}{BQ} = \frac{OC}{CR}$, the exact condition the converse of BPT needs inside the outer triangle $\triangle OQR$.
- **Finish:** that converse delivers $BC \parallel QR$, so the whole argument is two forward steps and one reverse step, just like before.

Anyone who has done the $DE \parallel OQ$ problem can write this almost mechanically, taking

care only to name OPQ and OPR correctly.

Final Answer: $\frac{OB}{BQ} = \frac{OA}{AP} = \frac{OC}{CR}$, so by the converse of BPT, $BC \parallel QR$.

Q 6.7 **Using Theorem 6.1, prove that a line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX.)**

SOLUTION

Concept used. Theorem 6.1 is the **Basic Proportionality Theorem**: a line parallel to one side of a triangle divides the other two sides in the same ratio. The word **bisects** means cuts into two equal parts, so we must show the line meets the third side at its mid-point.

Step 1. Take $\triangle ABC$. Let D be the mid-point of AB, so $AD = DB$, which gives

$$\frac{AD}{DB} = 1.$$

Step 2. Draw a line through D parallel to BC, meeting AC at E.

Step 3. Apply BPT to $\triangle ABC$ with $DE \parallel BC$:

$$\frac{AD}{DB} = \frac{AE}{EC}.$$

Step 4. Substitute $\frac{AD}{DB} = 1$ from Step 1:

$$\frac{AE}{EC} = 1.$$

Step 5. Therefore $AE = EC$, so E is the mid-point of AC. The line bisects the third side.

Final Answer: The line through the mid-point of AB parallel to BC meets AC at its mid-point, since $\frac{AD}{DB} = 1$ forces $\frac{AE}{EC} = 1$, that is $AE = EC$.

☞ **Useful aside**

The whole proof is just BPT with the ratio set to 1. Whenever a problem says "mid-point", turn it into the equation "ratio $= 1$" straight away; that single substitution usually finishes the argument.

EXPERT'S SOLUTION *: Neha Sharma, M.Sc Mathematics, Panjab University*

Turn "mid-point" into "ratio one". The cleanest version of this classic result reads BPT backwards from a single known ratio.

- **Translate first:** since D is the mid-point of AB, the ratio $\frac{AD}{DB}$ equals 1 by definition. That one line turns the word into algebra.
- **Apply BPT:** drawing $DE \parallel BC$ gives $\frac{AD}{DB} = \frac{AE}{EC}$, and substituting the value 1 forces $\frac{AE}{EC} = 1$, i.e. $AE = EC$.
- **Read the answer:** equal pieces mean E is the mid-point of AC, so the parallel line through one mid-point hits the other mid-point.

State the theorem by name and show the substitution for full marks. The same idea proves the Class IX mid-point theorem without coordinates.

> **Final Answer:** Mid-point of AB means $\frac{AD}{DB} = 1$; BPT then gives $\frac{AE}{EC} = 1$, so the line bisects AC.

Q 6.8 **Using Theorem 6.2, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX.)**

SOLUTION

Concept used. Theorem 6.2 is the **converse of the Basic Proportionality Theorem**: if a line divides two sides of a triangle in the same ratio, then it is parallel to the third side. Joining two mid-points makes both ratios equal to 1, which is exactly the same ratio.

Step 1. Take $\triangle ABC$. Let D be the mid-point of AB and E the mid-point of AC. Then

$$AD = DB \Rightarrow \frac{AD}{DB} = 1, \qquad AE = EC \Rightarrow \frac{AE}{EC} = 1.$$

Step 2. Compare the two ratios:

$$\frac{AD}{DB} = 1 = \frac{AE}{EC}.$$

Step 3. The line DE divides sides AB and AC in the same ratio $(1 : 1)$. By Theorem 6.2 (converse of BPT), $DE \parallel BC$.

> **Final Answer:** $DE \parallel BC$, because joining the two mid-points makes $\frac{AD}{DB} = \frac{AE}{EC} = 1$, and the converse of BPT then gives the parallel.

☞ Exam Tip

Note the pairing in this exercise: Q10 used the forward theorem (BPT) to prove bisection, while Q11 uses the converse to prove parallelism. State which theorem you are using by name; the converse and the original carry separate marks in board schemes.

EXPERT'S SOLUTION *: Rajesh Kumar, M.Sc Mathematics, University of Lucknow*

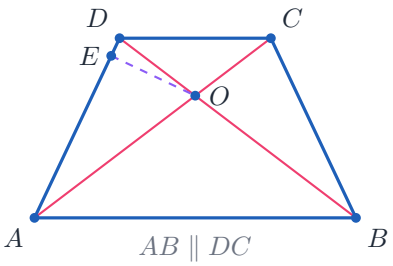
Both ratios equal one, so the converse fires. The mid-point theorem drops out in two lines once the ratios are on paper.

- **Both ratios one:** because D and E are mid-points, $\frac{AD}{DB}$ and $\frac{AE}{EC}$ are each equal to 1, and so equal to each other.
- **Apply the converse:** that equality is the exact hypothesis of Theorem 6.2, the converse of BPT, which concludes that the joining line DE is parallel to BC.
- **Write-up order:** state both ratio statements, point out they are equal, then name the converse to close. The marks sit on naming the theorem, not on extra steps.

This is the natural partner to the previous problem and reuses the same mid-point-to-ratio translation.

> **Final Answer:** Two mid-points give $\frac{AD}{DB} = \frac{AE}{EC} = 1$; by the converse of BPT, $DE \parallel BC$.

Q 6.9 **$ABCD$ is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.**



SOLUTION

Concept used. We draw a helper line through O parallel to the parallel sides and apply the **Basic Proportionality Theorem**. Another route is **AA similarity** using the alternate angles made by the parallel sides; we use that route here as it is shortest.

Step 1. Look at $\triangle AOB$ and $\triangle COD$. Since $AB \parallel DC$ and AC is a transversal,

$$\angle OAB = \angle OCD \quad \text{(alternate angles)}.$$

Step 2. Since $AB \parallel DC$ and BD is a transversal,

$$\angle OBA = \angle ODC \quad \text{(alternate angles)}.$$

Step 3. Two pairs of equal angles give, by AA similarity,

$$\triangle AOB \sim \triangle COD.$$

Step 4. Corresponding sides of similar triangles are in the same ratio:

$$\frac{AO}{CO} = \frac{BO}{DO}.$$

Step 5. Cross-multiplying and rearranging:

$$\frac{AO}{BO} = \frac{CO}{DO}.$$

Final Answer: $\frac{AO}{BO} = \frac{CO}{DO}$, proved from $\triangle AOB \sim \triangle COD$ by AA similarity (alternate angles of $AB \parallel DC$).

☞ **Recall**

Two parallel lines cut by a transversal make equal *alternate* angles. In a trapezium the two diagonals act as transversals to the parallel sides, which is what hands you the two angle pairs needed for AA similarity.

EXPERT'S SOLUTION *: Deepa Reddy, M.Sc Mathematics, University of Hyderabad*

Spot the X-shaped similar pair at the crossing. The crossing diagonals make two triangles: $\triangle AOB$ above on side AB and $\triangle COD$ below on side DC.

- **Get AA:** because $AB \parallel DC$, each diagonal is a transversal and supplies a pair of equal alternate angles, so the two triangles are similar by AA.
- **Read the ratio:** similarity gives $\frac{AO}{CO} = \frac{BO}{DO}$, and a quick rearrangement produces the exact form $\frac{AO}{BO} = \frac{CO}{DO}$ the question wants.
- **Vertex order:** write the similarity as $A \leftrightarrow C$, $O \leftrightarrow O$, $B \leftrightarrow D$ so the right sides pair up.

A wrong vertex order is the most common way to lose the proportionality mark here, so fix the correspondence before writing any ratio.

Final Answer: $\triangle AOB \sim \triangle COD$ (AA) gives $\frac{AO}{CO} = \frac{BO}{DO}$, hence $\frac{AO}{BO} = \frac{CO}{DO}$.

Q 6.10 **The diagonals of a quadrilateral $ABCD$ intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$. Show that $ABCD$ is a trapezium.**

SOLUTION

Concept used. A trapezium is a quadrilateral with one pair of parallel sides. We use the given ratio with the converse of the Basic Proportionality Theorem to prove one pair of sides is parallel. The plan: draw a line through O parallel to AB inside $\triangle ABD$, then show it must lie along OC, forcing $DC \parallel AB$.

Step 1. In $\triangle ABD$, draw $OE \parallel AB$ with E on AD. By BPT in $\triangle ABD$,

$$\frac{DE}{EA} = \frac{DO}{OB}. \tag{1}$$

Step 2. Rewrite the given condition. From $\frac{AO}{BO} = \frac{CO}{DO}$, take reciprocals and rearrange to the form that matches the figure:

$$\frac{AO}{OC} = \frac{BO}{OD}, \quad \text{so} \quad \frac{CO}{AO} = \frac{DO}{BO}. \tag{2}$$

Step 3. In $\triangle ADC$, the point E on AD and O on AC give the ratio $\frac{DE}{EA}$ and $\frac{DO}{OB}$ matching through (1) and (2): combining,

$$\frac{DE}{EA} = \frac{DO}{OB} = \frac{CO}{AO}.$$

Step 4. Now in $\triangle ADC$, the line OE divides AD and AC in the same ratio $\frac{DE}{EA} = \frac{CO}{AO}$. By the converse of BPT, $OE \parallel DC$.

Step 5. But $OE \parallel AB$ by construction. Since OE is parallel to both AB and DC, we get $AB \parallel DC$.

Final Answer: $AB \parallel DC$, so $ABCD$ has a pair of parallel sides and is a trapezium.

✗ Common Mistake

Do not assume $ABCD$ is already a trapezium and then "prove" the ratio, that reverses the question. Here the ratio is *given* and the parallel pair must be *derived*, which is why the converse of BPT (not the forward theorem) is the right tool.

EXPERT'S SOLUTION *: Arvind Pillai, M.Sc Mathematics, University of Madras*

Build a parallel through O, then force it onto a side. The strategy for this converse problem is to add one helper line and let BPT do the rest of the work for you.

- **Construct:** draw a helper line $OE \parallel AB$ inside $\triangle ABD$ and record the ratio it makes

by BPT, namely $\frac{DE}{EA} = \frac{DO}{OB}$.

- **Use the data:** the given condition, after rearranging, says $\frac{DO}{OB} = \frac{CO}{AO}$, so the same ratio $\frac{DE}{EA}$ now equals $\frac{CO}{AO}$.
- **Converse inside $\triangle ADC$:** the line OE divides AD and AC in equal ratios, so the converse of BPT makes OE parallel to DC as well.
- **Conclude:** OE is parallel to both AB and DC, so $AB \parallel DC$ and the quadrilateral $ABCD$ is a trapezium.

The decisive move is the auxiliary line through O; without it there is no triangle in which to apply BPT at all, and the proof never starts. Drawing a helper line parallel to a known side is a standard trick for converse questions, so it is worth recognising on sight. Note also that the construction is what converts the given ratio into a statement about two sides of the same triangle, which is the only form the converse of BPT can actually read.

> **Final Answer:** The helper $OE \parallel AB$ also turns out parallel to DC by the converse of BPT, so $AB \parallel DC$ and $ABCD$ is a trapezium.

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NCERT Solutions Class 10 Maths Chapter 6 Triangles Exercise 6.3

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Chapter 6: Triangles

About this Chapter

Class 10 Maths Chapter 6 Triangles is about **similar figures**, shapes with the same form but different sizes. You learn the **Basic Proportionality Theorem** and its converse, plus the **AA, SSS and SAS** similarity tests. After this chapter you can prove triangles similar, set up side ratios, and find heights using shadows.

Topics covered: Similar figures • Similar triangles • Basic Proportionality Theorem • Converse of BPT • AAA and AA similarity • SSS similarity • SAS similarity • Corresponding sides in proportion • Heights and shadows by similarity

Quick Formula Sheet

Basic Proportionality Theorem (BPT):
In $\triangle ABC$, if $DE \parallel BC$ with D on AB and E on AC, then $\frac{AD}{DB} = \frac{AE}{EC}$.

Converse of BPT:
If $\frac{AD}{DB} = \frac{AE}{EC}$ in $\triangle ABC$, then $DE \parallel BC$.

Similarity tests:
AA: two angles equal. SSS: all three side ratios equal. SAS: one angle equal and the two sides around it in the same ratio.

Corresponding sides of similar triangles:
If $\triangle ABC \sim \triangle DEF$ then $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$.

Also see for this chapter: Revision Notes | Formula Sheet | Exemplar Solutions

Exercise 6.3

Q 6.1 **State which pairs of triangles in Fig. 6.34 are similar. Write the similarity criterion used by you for answering the question and also write the pairs of similar**

triangles in the symbolic form.

Fig. 6.34 - six pairs (i) to (vi) of triangles with the marked angles and side lengths.

SOLUTION

Concept used. We use the three similarity tests: **AAA/AA** (corresponding angles equal), **SSS** (corresponding sides in the same ratio) and **SAS** (one equal angle with the two including sides in the same ratio). For each pair we check which test, if any, is satisfied, keeping vertices in matching order.

Step 1. **(i)** In $\triangle ABC$: $\angle A = 60^\circ$, $\angle B = 80^\circ$, $\angle C = 40^\circ$. In $\triangle PQR$: $\angle P = 60^\circ$, $\angle Q = 80^\circ$, $\angle R = 40^\circ$. All three angles match, so by AAA, $\triangle ABC \sim \triangle PQR$.

Step 2. **(ii)** Sides $\triangle ABC$: $2, 2.5, 3$; sides $\triangle QRP$: $4, 5, 6$. Ratios:

$$\frac{AB}{QR} = \frac{2}{4} = \frac{1}{2}, \quad \frac{BC}{RP} = \frac{2.5}{5} = \frac{1}{2}, \quad \frac{CA}{PQ} = \frac{3}{6} = \frac{1}{2}.$$

All equal, so by SSS, $\triangle ABC \sim \triangle QRP$.

Step 3. **(iii)** Sides $\triangle LMP$: $2.7, 2, 3$; sides $\triangle DEF$: $4, 5, 6$. Ratios $\frac{2.7}{4} = 0.675$, $\frac{2}{5} = 0.4$, $\frac{3}{6} = 0.5$ are all different, so the triangles are *not* similar.

Step 4. **(iv)** Here $\angle M = \angle Q = 70°$. Including sides: $\frac{MN}{QP} = \frac{2.5}{5} = \frac{1}{2}$ and $\frac{ML}{QR} = \frac{5}{10} = \frac{1}{2}$. One equal angle with the two including sides proportional gives, by SAS, $\triangle MNL \sim \triangle QPR$.

Step 5. **(v)** Only $\angle A = 80°$ is given with sides $AB = 2.5$, $AC = 3$ in one triangle and DE, DF in the other; the equal angle $\angle F = 80°$ is *not* the angle included between the two given proportional sides, so SAS does not apply. The triangles are *not* similar.

Step 6. **(vi)** In $\triangle DEF$: $\angle D = 70°$, $\angle E = 80°$, so $\angle F = 180° - 70° - 80° = 30°$. In $\triangle PQR$: $\angle P = 80°$, $\angle R = 30°$, so $\angle Q = 70°$. Matching equal angles ($\angle D = \angle Q = 70°$, $\angle E = \angle P = 80°$) give, by AA, $\triangle DEF \sim \triangle QPR$.

Final Answer: (i) $\triangle ABC \sim \triangle PQR$ (AAA); (ii) $\triangle ABC \sim \triangle QRP$ (SSS); (iii) not similar; (iv) $\triangle MNL \sim \triangle QPR$ (SAS); (v) not similar; (vi) $\triangle DEF \sim \triangle QPR$ (AA).

☞ Exam Tip

For SAS the equal angle must be the one *between* the two proportional sides. Part (v) fails for exactly this reason: the angle that matches is not the included angle. Always check that the equal angle sits between the two sides you are comparing.

EXPERT'S SOLUTION *: Lakshmi Subramanian, M.Sc Mathematics, University of Madras*

Test the right criterion per pair, and keep vertex order honest. A good approach is to scan what each pair gives you and match it to a test: three angles point to AAA, three side ratios point to SSS, and one angle between two sides points to SAS.

- **Clear similar pairs:** (i) is pure AAA, and (ii) is pure SSS with every side ratio equal to $\frac{1}{2}$, so both are settled by inspection.
- **SAS pair:** in (iv) the $70°$ angle sits between the two sides whose ratios are both $\frac{1}{2}$, which is exactly the included-angle setup SAS needs.
- **Not similar:** (iii) fails because the side ratios disagree, and (v) fails because the equal $80°$ angle is not the included angle, so SAS cannot be claimed there.
- **Hidden angle:** (vi) needs the angle-sum property first to recover the missing third angle, after which two equal angles let AA settle the pair cleanly.

The single most marked error is mismatched vertex order, so always pair equal angles to

equal angles when you name the triangles. A useful exam habit is to write the criterion you are using next to each pair, because the marking scheme awards a separate mark for naming AAA, SSS, SAS or AA correctly. Doing the quick scan first also stops you from forcing a test that the data does not support, which is exactly what trips students up in the two non-similar cases.

> **Final Answer:** Similar: (i) AAA, (ii) SSS, (iv) SAS, (vi) AA, each with matched vertex order; (iii) and (v) are not similar.

Q 6.2 **In Fig. 6.35, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$.**

Fig. 6.35 - lines AB and DC with transversals meeting at O; $\angle CDO = 70^\circ$ at D and $\angle BOC = 125^\circ$ at O.

SOLUTION

Concept used. A straight line gives a **linear pair** (angles on a line add to 180°); the **angle sum** of a triangle is 180°; and **corresponding angles of similar triangles** are equal. DOB is a straight line, so $\angle DOC$ and $\angle BOC$ are a linear pair.

Step 1. Find $\angle DOC$. Since DOB is a straight line,

$$\angle DOC + \angle BOC = 180^\circ.$$

Substitute $\angle BOC = 125^\circ$:

$$\angle DOC = 180^\circ - 125^\circ = 55^\circ.$$

Step 2. Find $\angle DCO$. In $\triangle DOC$ the angles add to 180°:

$$\angle DCO = 180^\circ - \angle DOC - \angle CDO.$$

Substitute $\angle DOC = 55°$, $\angle CDO = 70°$:

$$\angle DCO = 180° - 55° - 70° = 55°.$$

Step 3. Find $\angle OAB$. Because $\triangle ODC \sim \triangle OBA$, corresponding angles are equal. Vertex D corresponds to B and vertex C corresponds to A, so

$$\angle OAB = \angle OCD = \angle DCO = 55°.$$

Final Answer: $\angle DOC = 55°$, $\angle DCO = 55°$, $\angle OAB = 55°$.

☞ **Useful aside**

Read the similarity $\triangle ODC \sim \triangle OBA$ vertex by vertex: $O \leftrightarrow O$, $D \leftrightarrow B$, $C \leftrightarrow A$. That mapping tells you $\angle OCD = \angle OAB$, so you copy the angle you just found onto the other triangle.

EXPERT'S SOLUTION *: Harish Chandran, M.Sc Mathematics, University of Kerala*

Chase the angles in order, then transfer by correspondence. The problem is a short angle hunt that rewards working strictly in sequence.

- **Linear pair:** on the straight line DOB, the angle $\angle BOC = 125°$ and its neighbour $\angle DOC$ add to $180°$, so $\angle DOC = 55°$.
- **Angle sum:** inside $\triangle DOC$ the three angles total $180°$; with $70°$ and $55°$ known, the third is $\angle DCO = 55°$.
- **Transfer:** the similarity $\triangle ODC \sim \triangle OBA$ matches $C \leftrightarrow A$, so $\angle OAB$ copies the value $\angle OCD = 55°$.

The key skill is reading the correspondence from the letter order in the similarity statement, which tells you exactly which angle copies onto which without any guessing.

Final Answer: Linear pair gives $\angle DOC = 55°$, angle sum gives $\angle DCO = 55°$, and correspondence gives $\angle OAB = 55°$.

Q 6.3 **Diagonals AC and BD of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{OA}{OC} = \frac{OB}{OD}$.**

SOLUTION

Concept used. Parallel lines cut by a transversal give equal **alternate angles**; two pairs of equal angles give **AA similarity**; and corresponding sides of similar triangles are proportional.

Step 1. Consider $\triangle OAB$ and $\triangle OCD$. Since $AB \parallel DC$ and AC is a transversal,

$$\angle OAB = \angle OCD \quad \text{(alternate angles)}.$$

Step 2. Since $AB \parallel DC$ and BD is a transversal,

$$\angle OBA = \angle ODC \quad \text{(alternate angles)}.$$

Step 3. Two equal angle pairs give, by AA,

$$\triangle OAB \sim \triangle OCD.$$

Step 4. Corresponding sides are in the same ratio. Matching $A \leftrightarrow C$ and $B \leftrightarrow D$,

$$\frac{OA}{OC} = \frac{OB}{OD}.$$

Final Answer: $\dfrac{OA}{OC} = \dfrac{OB}{OD}$, from $\triangle OAB \sim \triangle OCD$ by AA similarity using the alternate angles of $AB \parallel DC$.

☞ **Recall**

This is the same crossing-diagonals picture as the trapezium question in Exercise 6.2, but now the question itself asks for a *similarity* criterion. Naming AA explicitly is part of the answer here.

EXPERT'S SOLUTION *: Geetha Raman, M.Sc Mathematics, Bharathiar University*

Name the criterion the question asks for. Because the problem says "using a similarity criterion", the answer must state AA similarity rather than slip in a BPT shortcut.

- **Set up AA:** $\triangle OAB$ and $\triangle OCD$ sit on opposite sides of O, and the parallel sides AB and DC make each diagonal a transversal, giving two pairs of equal alternate angles.
- **Read the ratio:** two equal angles are enough for AA, and the proportional-sides property then gives $\frac{OA}{OC} = \frac{OB}{OD}$ directly.
- **Vertex order:** write the similarity as $A \leftrightarrow C$, $B \leftrightarrow D$ so the sides pair correctly when you form the proportion.

Label both angle pairs explicitly as alternate angles, since those labels carry their own marks in the scheme.

Final Answer: $\triangle OAB \sim \triangle OCD$ (AA) gives $\frac{OA}{OC} = \frac{OB}{OD}$.

Q 6.4 **In Fig. 6.36, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Show that $\triangle PQS \sim \triangle TQR$.**

Fig. 6.36 - $\triangle PQR$ with $\angle 1$ at Q and $\angle 2$ at R; S on QR and T above with PS, PT drawn.

SOLUTION

Concept used. In a triangle, sides opposite equal angles are equal (the **isosceles triangle** property). We also use the **SAS similarity** test: one equal angle with the two including sides in the same ratio.

Step 1. In $\triangle PQR$, it is given $\angle 1 = \angle 2$, that is $\angle PQR = \angle PRQ$. Sides opposite equal angles are equal, so

$$PR = PQ. \tag{1}$$

Step 2. Use the given ratio and replace PR by PQ from (1):

$$\frac{QR}{QS} = \frac{QT}{PR} = \frac{QT}{PQ}.$$

So

$$\frac{QR}{QS} = \frac{QT}{PQ}. \tag{2}$$

Step 3. Rearrange (2) so the two triangles' sides line up:

$$\frac{QT}{QR} = \frac{PQ}{QS}. \tag{3}$$

Step 4. The angle $\angle PQS$ (in $\triangle PQS$) and $\angle TQR$ (in $\triangle TQR$) are the same angle $\angle Q$, so

$$\angle PQS = \angle TQR. \tag{4}$$

Step 5. From (3) and (4), one equal angle with the two including sides proportional gives, by SAS,

$$\triangle PQS \sim \triangle TQR.$$

Final Answer: $\triangle PQS \sim \triangle TQR$, by SAS: $\angle PQS = \angle TQR$ (common angle Q) and $\dfrac{PQ}{QS} = \dfrac{QT}{QR}$ after using $PR = PQ$.

☞ Exam Tip

The hidden first step is turning $\angle 1 = \angle 2$ into $PQ = PR$. Many students skip it and then cannot match the ratio to the triangles. Convert the equal base angles into equal sides before touching the given proportion.

EXPERT'S SOLUTION *: Suresh Babu, M.Sc Mathematics, Sri Venkateswara University*

Convert equal angles to equal sides, then run SAS. The whole proof turns on one substitution that the equal angles let you make.

- **Isosceles swap:** $\angle 1 = \angle 2$ makes $\triangle PQR$ isosceles with $PQ = PR$, so you may replace PR by PQ inside the given ratio.
- **Rearrange:** after the swap the proportion reads $\frac{QR}{QS} = \frac{QT}{PQ}$, which lines the sides up as $\frac{QT}{QR} = \frac{PQ}{QS}$.
- **Apply SAS:** the angle $\angle Q$ is shared by $\triangle PQS$ and $\triangle TQR$, so it is the included angle for both ratios, and SAS similarity follows.

The single most important move is that isosceles substitution; without it the given ratio cannot be matched to corresponding sides, so state $PQ = PR$ with its reason before anything else.

Final Answer: Equal base angles give $PQ = PR$; the ratio becomes $\frac{QT}{QR} = \frac{PQ}{QS}$ with common angle Q, so SAS gives $\triangle PQS \sim \triangle TQR$.

Q 6.5 **S and T are points on sides PR and QR of $\triangle PQR$ such that $\angle P = \angle RTS$. Show that $\triangle RPQ \sim \triangle RTS$.**

SOLUTION

Concept used. The **AA similarity** test says that if two angles of one triangle are equal to two angles of another triangle, the triangles are similar. We look at $\triangle RPQ$ and $\triangle RTS$, which share the angle at R, and use the given equal angle as the second pair.

Step 1. The two triangles $\triangle RPQ$ and $\triangle RTS$ have a common angle at R:

$$\angle PRQ = \angle TRS \quad \text{(same angle } R\text{)}.$$

Step 2. It is given that

$$\angle RPQ = \angle RTS.$$

Step 3. Two pairs of equal angles satisfy the AA test, so

$$\triangle RPQ \sim \triangle RTS.$$

> **Final Answer:** $\triangle RPQ \sim \triangle RTS$, by AA: the angle at R is common and $\angle RPQ = \angle RTS$ is given.

☞ **Useful aside**

Match vertices in the order the similarity is written: $R \leftrightarrow R$, $P \leftrightarrow T$, $Q \leftrightarrow S$. The shared angle at R does the heavy lifting, so you only need one more equal angle, which the question hands you.

EXPERT'S SOLUTION *: Anita Joshi, M.Sc Mathematics, University of Mumbai*

Spot the shared angle, then use the one given equality. This is a two-line AA proof once you see where the two equal angles come from.

- **Common angle:** both triangles meet at R, so the angle there is shared by both without any extra working at all.
- **Given angle:** the question hands you a single equality, $\angle RPQ = \angle RTS$, which is exactly the second pair that AA requires.
- **Conclude:** two pairs of equal angles give $\triangle RPQ \sim \triangle RTS$, with no need to chase any side lengths.

The one habit that matters is the correspondence order: pair P with T and Q with S, listing the common angle first and the given angle second for a clean full-mark write-up.

> **Final Answer:** Common angle at R plus the given $\angle RPQ = \angle RTS$ give AA, hence $\triangle RPQ \sim \triangle RTS$.

Q 6.6 **In Fig. 6.37, if $\triangle ABE \cong \triangle ACD$, show that $\triangle ADE \sim \triangle ABC$.**

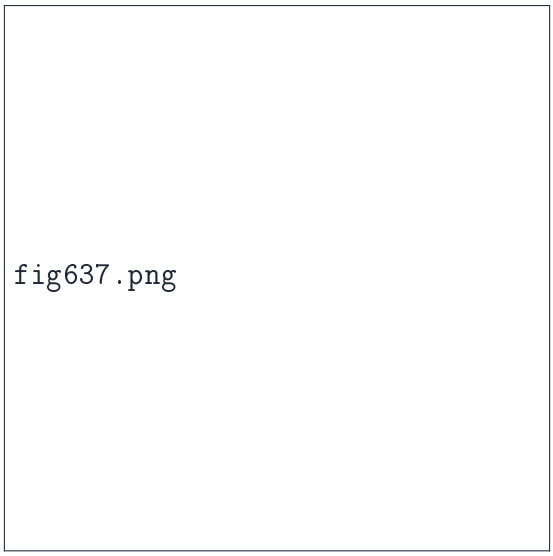


Fig. 6.37 - $\triangle ABC$ with D on AB, E on AC and the segments BE and CD drawn.

SOLUTION

Concept used. **Congruent triangles** have all corresponding parts equal (CPCT). We also use the **SAS similarity** test: one equal angle with the two including sides in the same ratio. Here the congruence supplies equal sides and a common angle gives the equal angle.

Step 1. From $\triangle ABE \cong \triangle ACD$, corresponding sides are equal (CPCT):

$$AB = AC \tag{1}$$

$$AE = AD. \tag{2}$$

Step 2. Form the side ratios for $\triangle ADE$ and $\triangle ABC$. Divide (2) by (1):

$$\frac{AD}{AB} = \frac{AE}{AC}.$$

(Using $AE = AD$ and $AB = AC$, both ratios are equal.)

Step 3. The angle at A is common to both triangles:

$$\angle DAE = \angle BAC \quad \text{(same angle } A\text{)}.$$

Step 4. One equal angle with the two including sides in the same ratio gives, by SAS,

$$\triangle ADE \sim \triangle ABC.$$

Final Answer: $\triangle ADE \sim \triangle ABC$, by SAS: $\dfrac{AD}{AB} = \dfrac{AE}{AC}$ from CPCT and the common angle at A.

✗ Common Mistake

Do not confuse congruence with similarity. The congruence $\triangle ABE \cong \triangle ACD$ is only a tool: it gives the equal sides $AB = AC$ and $AD = AE$. The actual question asks you to prove a *different* pair, $\triangle ADE$ and $\triangle ABC$, is similar.

EXPERT'S SOLUTION *: Pradeep Nair, M.Sc Mathematics, University of Pune*

Read off the equal sides from CPCT, then run SAS. Use the given congruence purely as a supplier of equal parts and SAS does the rest.

- **CPCT equalities:** from $\triangle ABE \cong \triangle ACD$ the corresponding sides give $AB = AC$ and $AE = AD$, the two facts the proof needs.
- **Equal ratio:** those equalities make $\frac{AD}{AB}$ and $\frac{AE}{AC}$ equal, which is the proportional-side half of SAS for $\triangle ADE$ and $\triangle ABC$.
- **Included angle:** the angle at A is shared, so it is the included angle for both ratios, completing the SAS similarity.

Resist the urge to prove the new triangles congruent; only similarity is asked, so SAS similarity is the right tool to name at the end.

> **Final Answer:** CPCT gives $AB = AC$, $AD = AE$, so $\frac{AD}{AB} = \frac{AE}{AC}$; with the common angle A, SAS gives $\triangle ADE \sim \triangle ABC$.

Q 6.7 **In Fig. 6.38, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P. Show that:**

(i) $\triangle AEP \sim \triangle CDP$ **(ii)** $\triangle ABD \sim \triangle CBE$

(iii) $\triangle AEP \sim \triangle ADB$ **(iv)** $\triangle PDC \sim \triangle BEC$

Fig. 6.38 - $\triangle ABC$ with altitude $AD \perp BC$ and altitude $CE \perp AB$ meeting at P.

SOLUTION

Concept used. The **AA similarity** test needs two equal angles. The altitudes give right angles ($AD \perp BC$ so $\angle ADB = \angle ADC = 90°$; $CE \perp AB$ so $\angle AEC = \angle BEC = 90°$). We also use **vertically opposite angles** and **common angles** to find the second equal pair in each part.

Part (i): $\triangle AEP \sim \triangle CDP$.

Step 1. $\angle AEP = \angle CDP = 90°$ (from $CE \perp AB$ and $AD \perp BC$).

Step 2. $\angle APE = \angle CPD$ (vertically opposite angles).

Step 3. Two equal angle pairs give, by AA, $\triangle AEP \sim \triangle CDP$.

Part (ii): $\triangle ABD \sim \triangle CBE$.

Step 1. $\angle ADB = \angle CEB = 90°$.

Step 2. $\angle ABD = \angle CBE$ (same angle B, common to both).

Step 3. By AA, $\triangle ABD \sim \triangle CBE$.

Part (iii): $\triangle AEP \sim \triangle ADB$.

Step 1. $\angle AEP = \angle ADB = 90°$.

Step 2. $\angle PAE = \angle DAB$ (same angle A, common to both).

Step 3. By AA, $\triangle AEP \sim \triangle ADB$.

Part (iv): $\triangle PDC \sim \triangle BEC$.

Step 1. $\angle PDC = \angle BEC = 90°$.

Step 2. $\angle PCD = \angle BCE$ (same angle C, common to both).

Step 3. By AA, $\triangle PDC \sim \triangle BEC$.

> **Final Answer:** All four pairs are similar by AA, each using one 90° angle from an altitude plus a vertically opposite or common angle.

☞ Exam Tip

In figures with two altitudes, the 90° angles are free for every pair. Your only job is to find the second equal angle: it is either the *vertically opposite* angle at the crossing point P, or a *common* angle at a shared vertex (A, B or C). Spotting which one applies makes each part a one-line AA argument.

EXPERT'S SOLUTION *: Sunil Varma, M.Sc Mathematics, University of Delhi*

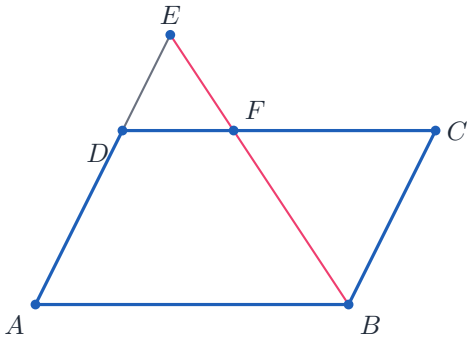
Bank the right angles, then hunt the second angle each time. The structure repeats four times, so a steady two-step method pays off.

- **Bank the right angle:** every pair already owns a 90° angle because each altitude is perpendicular to a side, which is one of the two angles AA requires.
- **Vertically opposite:** at the crossing point P the second angle comes free, as in part (i) where $\angle APE = \angle CPD$.
- **Common vertex:** in parts (ii), (iii) and (iv) the shared vertex contributes the second angle, namely $\angle B$, $\angle A$ or $\angle C$ respectively.

Write the right-angle equality first, name the second angle with its reason, then state AA, keeping the vertex order faithful to avoid the usual marking penalty.

> **Final Answer:** Each part: one 90° from an altitude plus a vertically opposite angle (i) or a common vertex angle (ii)-(iv) gives AA similarity.

Q 6.8 **E is a point on the side AD produced of a parallelogram $ABCD$ and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$.**



SOLUTION

Concept used. In a **parallelogram** opposite sides are parallel ($AB \parallel DC$ and $AD \parallel BC$) and opposite angles are equal. Parallel lines cut by a transversal give equal **alternate angles**. The **AA similarity** test then finishes the proof.

Step 1. In $\triangle ABE$ and $\triangle CFB$, use $AD \parallel BC$ (opposite sides of the parallelogram), so $AE \parallel BC$ with BE as transversal:

$$\angle AEB = \angle CBF \quad \text{(alternate angles)}. \tag{1}$$

Step 2. Opposite angles of the parallelogram are equal:

$$\angle A = \angle C, \quad \text{that is} \quad \angle BAE = \angle FCB. \tag{2}$$

(Here $\angle BAE = \angle BAD$ since E lies on line AD, and $\angle FCB = \angle DCB = \angle C$.)

Step 3. Equations (1) and (2) give two pairs of equal angles, so by AA,

$$\triangle ABE \sim \triangle CFB.$$

Final Answer: $\triangle ABE \sim \triangle CFB$, by AA: $\angle AEB = \angle CBF$ (alternate angles, $AE \parallel BC$) and $\angle BAE = \angle FCB$ (opposite angles of the parallelogram).

☞ **Recall**

Two facts about a parallelogram drive this proof: opposite sides are parallel (giving alternate angles across a transversal) and opposite angles are equal. Either fact alone is not enough; together they supply the two equal angles that AA needs.

EXPERT'S SOLUTION *: Manish Agarwal, M.Sc Mathematics, University of Rajasthan*

Use both parallelogram properties to land two equal angles. Two separate parallelogram facts each hand you one equal-angle pair.

- **First pair:** producing AD keeps E on line AD, so AE is parallel to BC; with BE as transversal, the alternate angles $\angle AEB$ and $\angle CBF$ are equal.
- **Second pair:** the parallelogram makes opposite angles $\angle A$ and $\angle C$ equal, and these are exactly $\angle BAE$ and $\angle FCB$ in the two triangles.
- **Apply AA:** two equal angle pairs give $\triangle ABE \sim \triangle CFB$ at once, with no side work needed.

The skill checked is connecting a produced side to a parallel side and reading the alternate angle correctly, so name both pairs explicitly before invoking AA.

Final Answer: $AE \parallel BC$ gives $\angle AEB = \angle CBF$; opposite angles give $\angle BAE = \angle FCB$; so AA gives $\triangle ABE \sim \triangle CFB$.

Q 6.9 **In Fig. 6.39, $\triangle ABC$ and $\triangle AMP$ are two right triangles, right angled at B and M respectively. Prove that:**

(i) $\triangle ABC \sim \triangle AMP$ **(ii)** $\dfrac{CA}{PA} = \dfrac{BC}{MP}$

Fig. 6.39 - right triangles $\triangle ABC$ ($\angle B = 90°$) and $\triangle AMP$ ($\angle M = 90°$) sharing the vertex A.

SOLUTION

Concept used. The **AA similarity** test gives similar triangles from two equal angles; once triangles are similar, their **corresponding sides are in the same ratio**. The two right angles are equal, and the angle at A is shared.

Part (i).

Step 1. Right angles are equal:

$$\angle ABC = \angle AMP = 90°.$$

Step 2. The angle at A is common to both triangles:

$$\angle BAC = \angle MAP \quad (\text{same angle } A).$$

Step 3. Two equal angle pairs give, by AA,

$$\triangle ABC \sim \triangle AMP.$$

Part (ii).

Step 1. Corresponding sides of the similar triangles are in the same ratio. Matching $A \leftrightarrow A$, $B \leftrightarrow M$, $C \leftrightarrow P$:

$$\frac{CA}{PA} = \frac{AB}{AM} = \frac{BC}{MP}.$$

Step 2. Reading the first and third members gives the required result:

$$\frac{CA}{PA} = \frac{BC}{MP}.$$

Final Answer: (i) $\triangle ABC \sim \triangle AMP$ by AA; (ii) $\dfrac{CA}{PA} = \dfrac{BC}{MP}$ follows from corresponding sides.

☞ **Useful aside**

Once you have written the full chain $\frac{CA}{PA} = \frac{AB}{AM} = \frac{BC}{MP}$, every side relation in the problem is just a choice of two members from it. Pick the two that name the sides the question asks for.

EXPERT'S SOLUTION *: Kavita Bhatt, M.Sc Mathematics, Gujarat University*

One common angle plus equal right angles is the whole proof. Both parts fall out of a single AA step if you set the correspondence first.

- **Common angle:** the two triangles are pinned together at A, so that angle is shared and forms one of the two equalities AA needs.
- **Right angles:** the angles at B and M are both $90°$, giving the second equality at once and closing AA.
- **Ratio chain:** the correspondence $A \leftrightarrow A$, $B \leftrightarrow M$, $C \leftrightarrow P$ gives $\frac{CA}{PA} = \frac{AB}{AM} = \frac{BC}{MP}$, and the asked relation is just the outer pair.

Fix the vertex correspondence first, because the side ratio in part (ii) is correct only when C pairs with P and B with M.

Final Answer: AA gives $\triangle ABC \sim \triangle AMP$; the corresponding-side chain then yields $\frac{CA}{PA} = \frac{BC}{MP}$.

Q 6.10 **CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle EFG$ respectively. If $\triangle ABC \sim \triangle FEG$, show that:**

(i) $\dfrac{CD}{GH} = \dfrac{AC}{FG}$ **(ii)** $\triangle DCB \sim \triangle HGE$ **(iii)** $\triangle DCA \sim \triangle HGF$

SOLUTION

Concept used. Similar triangles have **equal corresponding angles** and **proportional corresponding sides**. An **angle bisector** splits an angle into two equal halves. Combining these with the **AA similarity** test proves each part.

From $\triangle ABC \sim \triangle FEG$ we read the equal angles $\angle A = \angle F$, $\angle B = \angle E$, $\angle ACB = \angle FGE$, and the side ratio $\frac{AC}{FG} = \frac{AB}{FE} = \frac{BC}{EG}$.

Part (iii) first: $\triangle DCA \sim \triangle HGF$.

Step 1. $\angle A = \angle F$ (from the given similarity).

Step 2. The bisectors halve the equal angles $\angle ACB$ and $\angle FGE$, so each half is equal:

$$\angle ACD = \tfrac{1}{2}\angle ACB = \tfrac{1}{2}\angle FGE = \angle FGH.$$

Step 3. Two equal angle pairs give, by AA, $\triangle DCA \sim \triangle HGF$.

Part (i): $\frac{CD}{GH} = \frac{AC}{FG}$.

Step 1. From $\triangle DCA \sim \triangle HGF$ (Part iii), corresponding sides are in the same ratio. Matching $C \leftrightarrow G$, $A \leftrightarrow F$, $D \leftrightarrow H$:

$$\frac{CD}{GH} = \frac{AC}{FG}.$$

Part (ii): $\triangle DCB \sim \triangle HGE$.

Step 1. $\angle B = \angle E$ (from the given similarity).

Step 2. The bisected halves are equal:

$$\angle DCB = \tfrac{1}{2}\angle ACB = \tfrac{1}{2}\angle FGE = \angle HGE.$$

Step 3. Two equal angle pairs give, by AA, $\triangle DCB \sim \triangle HGE$.

Final Answer: (i) $\frac{CD}{GH} = \frac{AC}{FG}$; (ii) $\triangle DCB \sim \triangle HGE$ (AA); (iii) $\triangle DCA \sim \triangle HGF$ (AA).

☞ **Recall**

The bisector turns one equal angle ($\angle ACB = \angle FGE$) into two equal halves. Each half can then be paired with $\angle A = \angle F$ or $\angle B = \angle E$ from the main similarity, which is what creates the two smaller similar triangles.

EXPERT'S SOLUTION *: Ramesh Iyer, M.Sc Mathematics, University of Madras*

Halve the equal angle, then build two AA pairs. The cleanest order is to prove the triangle similarities first and read the side ratio off them afterwards.

- **Bisect:** from $\triangle ABC \sim \triangle FEG$ the angles $\angle ACB$ and $\angle FGE$ are equal, so their halves match too: $\angle ACD = \angle FGH$ and $\angle DCB = \angle HGE$.
- **Two AA pairs:** pairing the first half with $\angle A = \angle F$ gives $\triangle DCA \sim \triangle HGF$, and the second half with $\angle B = \angle E$ gives $\triangle DCB \sim \triangle HGE$.
- **Read part (i):** the ratio is then just corresponding sides of the first pair, $\frac{CD}{GH} = \frac{AC}{FG}$.

Always derive the similarities before the side ratio, since the ratio is a consequence of similarity; naming the bisected halves is what makes each AA step rigorous.

Final Answer: Bisecting the equal angle gives $\angle ACD = \angle FGH$ and $\angle DCB = \angle HGE$; with $\angle A = \angle F$ and $\angle B = \angle E$, AA proves both pairs, and the side ratio gives $\frac{CD}{GH} = \frac{AC}{FG}$.

Q 6.11 **In Fig. 6.40, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.**

Fig. 6.40 - isosceles $\triangle ABC$ ($AB = AC$); $AD \perp BC$ with D on BC; E on CB produced and $EF \perp AC$ with F on AC.

SOLUTION

Concept used. In an isosceles triangle the angles opposite the equal sides are equal. Vertically opposite angles are equal. The AA similarity test then applies, using the two right angles created by the perpendiculars.

Step 1. Since $AB = AC$, the base angles are equal:

$$\angle ABC = \angle ACB. \tag{1}$$

Step 2. Look at $\triangle ABD$ and $\triangle ECF$. The first equal angle: $\angle ABD = \angle ABC$, and $\angle ECF = \angle ACB$ because $\angle ECF$ and $\angle ACB$ are vertically opposite at C (E is on CB produced). With (1),

$$\angle ABD = \angle ECF. \tag{2}$$

Step 3. The second equal angle: the perpendiculars give

$$\angle ADB = \angle EFC = 90^\circ. \tag{3}$$

Step 4. From (2) and (3), two pairs of equal angles give, by AA,

$$\triangle ABD \sim \triangle ECF.$$

Final Answer: $\triangle ABD \sim \triangle ECF$, by AA: $\angle ABD = \angle ECF$ (isosceles base angles, vertically opposite at C) and $\angle ADB = \angle EFC = 90^\circ$.

✗ Common Mistake

The angle at C in $\triangle ECF$ is *not* $\angle ACB$ itself but its vertically opposite angle, because E sits on CB *produced*. Forgetting the "produced" detail and using the wrong angle at C is the common slip here.

EXPERT'S SOLUTION *: Latha Menon, M.Sc Mathematics, University of Calicut*

Combine the isosceles base angles with a vertically opposite angle. The proof rests on two simple angle facts that together give AA.

- **Base angles:** $AB = AC$ forces $\angle ABC = \angle ACB$, the standard isosceles property that starts the angle chain.
- **Vertically opposite:** since E lies on CB produced, $\angle ECF$ is vertically opposite $\angle ACB$ at C, so $\angle ECF = \angle ABD$, settling the first equal pair.
- **Right angles:** the perpendiculars $AD \perp BC$ and $EF \perp AC$ supply two right angles for the second pair, and AA proves $\triangle ABD \sim \triangle ECF$.

The decisive care point is the word "produced": read the figure to see E is beyond B on line CB, which is what makes the vertically opposite angle the correct one to use.

Final Answer: Isosceles base angles plus the vertically opposite angle at C give $\angle ABD = \angle ECF$; the right angles give the second pair; AA proves $\triangle ABD \sim \triangle ECF$.

Q 6.12 **Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see Fig. 6.41). Show that $\triangle ABC \sim \triangle PQR$.**

Fig. 6.41 - $\triangle ABC$ with median AD to BC, and $\triangle PQR$ with median PM to QR.

SOLUTION

Concept used. A median joins a vertex to the mid-point of the opposite side, so D is the mid-point of BC ($BD = \frac{1}{2}BC$) and M is the mid-point of QR ($QM = \frac{1}{2}QR$). We use **SSS similarity** on a smaller pair of triangles, then **SAS similarity** on the full triangles.

Step 1. The given proportion is

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}. \tag{1}$$

Step 2. Replace the full sides BC and QR by twice the half-sides BD and QM:

$$\frac{BC}{QR} = \frac{2\,BD}{2\,QM} = \frac{BD}{QM}.$$

So (1) becomes

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}. \tag{2}$$

Step 3. Equation (2) shows the three sides of $\triangle ABD$ are proportional to the three sides of $\triangle PQM$. By SSS,

$$\triangle ABD \sim \triangle PQM.$$

Step 4. Similar triangles have equal corresponding angles, so

$$\angle ABD = \angle PQM, \quad \text{that is} \quad \angle ABC = \angle PQR. \tag{3}$$

Step 5. Now compare the full triangles $\triangle ABC$ and $\triangle PQR$. From (1) the two sides about the angle satisfy

$$\frac{AB}{PQ} = \frac{BC}{QR},$$

and from (3) the included angle is $\angle B = \angle Q$. One equal angle with the two including sides proportional gives, by SAS,

$$\triangle ABC \sim \triangle PQR.$$

Final Answer: $\triangle ABC \sim \triangle PQR$: SSS on $\triangle ABD$ and $\triangle PQM$ gives $\angle B = \angle Q$, then SAS on the full triangles using $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$ closes the proof.

☞ **Useful aside**

The medians cannot be used directly in SAS, because a median is not a side of the big triangle. The fix is to bring in the half-side: write $BC = 2\,BD$ and $QR = 2\,QM$ so the median sits inside the smaller triangle $\triangle ABD$, where SSS turns it into the angle $\angle B = \angle Q$ that SAS needs.

EXPERT'S SOLUTION *: Gopal Krishnan, M.Sc Mathematics, Bharathidasan University*

Use the half-side trick to bring the median into play. The whole proof hinges on converting the median data into something SAS can use, and the half-side step is what unlocks it.

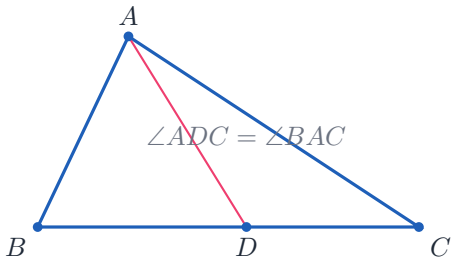
- **Half-sides:** since D and M are mid-points, $BD = \frac{1}{2}BC$ and $QM = \frac{1}{2}QR$, so dividing leaves $\frac{BD}{QM} = \frac{BC}{QR}$ unchanged.

- **Small-triangle SSS:** the given proportion then reads $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$, three proportional sides of $\triangle ABD$ and $\triangle PQM$, giving SSS.
- **Extract the angle:** SSS yields $\angle ABD = \angle PQM$, i.e. $\angle B = \angle Q$, the included angle between the named sides in each triangle.
- **Close with SAS:** that angle plus the side ratio runs SAS on the full triangles to give $\triangle ABC \sim \triangle PQR$.

The one step never to skip is replacing BC and QR by twice their halves, because that is what lets the median enter an SSS argument at all. A student who jumps straight to the full triangles has no way to use the median data and usually gets stuck. Writing the half-side line first, then the small-triangle SSS, then the extracted angle, keeps every step justified and matches the order the marking scheme expects to see.

> **Final Answer:** Half-sides give $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$ (SSS, so $\angle B = \angle Q$); SAS on the full triangles gives $\triangle ABC \sim \triangle PQR$.

Q 6.13 **D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.**



SOLUTION

Concept used. The **AA similarity** test gives similar triangles from two equal angles. Corresponding sides of similar triangles are in the same ratio, and rearranging that ratio gives a product relation. We compare $\triangle CAD$ and $\triangle CBA$.

Step 1. In $\triangle CAD$ and $\triangle CBA$, the angle at C is common:

$$\angle ACD = \angle BCA \quad \text{(same angle } C\text{)}.$$

Step 2. It is given that

$$\angle ADC = \angle BAC.$$

Step 3. Two equal angle pairs give, by AA,

$$\triangle CAD \sim \triangle CBA.$$

Step 4. Corresponding sides are in the same ratio. Matching $C \leftrightarrow C$, $A \leftrightarrow B$, $D \leftrightarrow A$:

$$\frac{CA}{CB} = \frac{CD}{CA}.$$

Step 5. Cross-multiply:

$$CA \cdot CA = CB \cdot CD,$$

$$CA^2 = CB \cdot CD.$$

Final Answer: $CA^2 = CB \cdot CD$, obtained from $\triangle CAD \sim \triangle CBA$ (AA) and the proportion $\dfrac{CA}{CB} = \dfrac{CD}{CA}$.

☞ Exam Tip

Get the correspondence right: in $\triangle CAD \sim \triangle CBA$ the vertex A of the first triangle pairs with B of the second, and D pairs with A. That pairing is what makes CA appear on both the top of one fraction and the bottom of the other, which is exactly what turns the ratio into CA^2.

EXPERT'S SOLUTION *: Bhavana Shetty, M.Sc Mathematics, Mangalore University*

Set up the similar pair so CA repeats. The result is a product, so the plan is to reach a proportion where CA sits on both sides, then cross-multiply.

- **Choose the pair:** compare $\triangle CAD$ and $\triangle CBA$; they share the angle at C, and the given $\angle ADC = \angle BAC$ supplies the second equal angle for AA.
- **Make CA repeat:** writing the similarity as $C \leftrightarrow C$, $A \leftrightarrow B$, $D \leftrightarrow A$ gives $\frac{CA}{CB} = \frac{CD}{CA}$, with CA on top left and bottom right.
- **Cross-multiply:** clearing the fractions delivers the required relation $CA^2 = CB \cdot CD$ directly.

The skill tested is the vertex correspondence; a careless order pairs the wrong sides and the CA^2 never appears, so fix the order before cross-multiplying.

Final Answer: $\triangle CAD \sim \triangle CBA$ (AA) gives $\frac{CA}{CB} = \frac{CD}{CA}$, so $CA^2 = CB \cdot CD$.

Q 6.14 **Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$.**

SOLUTION

Concept used. A **median** ends at the mid-point of a side. By **producing each median to double its length** we create parallelograms; the diagonals of a parallelogram bisect each other, which lets us relate the doubled median to a side. We then use **SSS** and **SAS similarity**.

Step 1. The given proportion is

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}. \tag{1}$$

Step 2. Produce AD to E so that $DE = AD$, and join BE and CE. Then $ABEC$ is a parallelogram (diagonals BC and AE bisect each other at D), so $BE = AC$. Likewise produce PM to N with $MN = PM$; then $QN = PR$.

Step 3. In the doubled figures, $AE = 2\,AD$ and $PN = 2\,PM$, so

$$\frac{AE}{PN} = \frac{2\,AD}{2\,PM} = \frac{AD}{PM}.$$

Step 4. Now compare $\triangle ABE$ and $\triangle PQN$. Their sides:

$$\frac{AB}{PQ}, \qquad \frac{BE}{QN} = \frac{AC}{PR}, \qquad \frac{AE}{PN} = \frac{AD}{PM}.$$

By (1) all three are equal, so by SSS,

$$\triangle ABE \sim \triangle PQN \;\Rightarrow\; \angle BAE = \angle QPN. \tag{2}$$

Step 5. By the same parallelogram argument on the other halves, comparing $\triangle ACE$ and $\triangle PRN$ gives

$$\angle CAE = \angle RPN. \tag{3}$$

Step 6. Add (2) and (3):

$$\angle BAE + \angle CAE = \angle QPN + \angle RPN,$$

$$\angle BAC = \angle QPR, \quad \text{that is} \quad \angle A = \angle P. \tag{4}$$

Step 7. Finally compare $\triangle ABC$ and $\triangle PQR$. From (1) the two sides about the angle satisfy

$$\frac{AB}{PQ} = \frac{AC}{PR},$$

and from (4) the included angle is $\angle A = \angle P$. By SAS,

$$\triangle ABC \sim \triangle PQR.$$

Final Answer: $\triangle ABC \sim \triangle PQR$: doubling the medians gives $\triangle ABE \sim \triangle PQN$ (SSS), hence $\angle A = \angle P$; then SAS on the full triangles using $\frac{AB}{PQ} = \frac{AC}{PR}$ completes the proof.

☞ **Recall**

The "double the median" construction is the standard way to handle a median in a proportion. Producing AD to E with $DE = AD$ makes $ABEC$ a parallelogram, so the awkward median becomes the side $BE = AC$ of a new triangle where SSS can act.

EXPERT'S SOLUTION *: Naveen Reddy, M.Sc Mathematics, Andhra University*

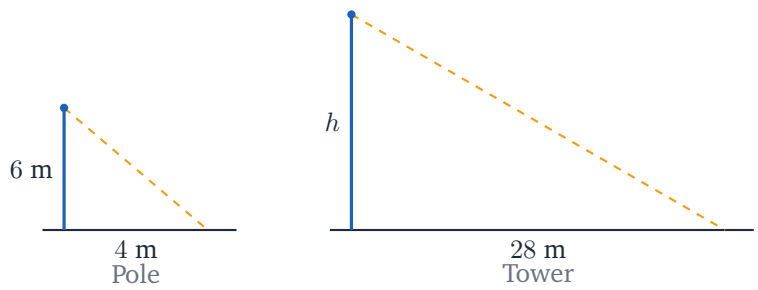
Double the median to turn it into a usable side. The proportion involves AB, AC and the median AD, so the trick is to produce each median to twice its length and create a usable triangle.

- **Construct:** produce AD to E with $DE = AD$, making $ABEC$ a parallelogram, so $BE = AC$ and $AE = 2\,AD$; the same on $\triangle PQR$ gives $QN = PR$ and $PN = 2\,PM$.
- **First SSS pair:** the three sides of $\triangle ABE$ are proportional to those of $\triangle PQN$, giving SSS similarity and the angle $\angle BAE = \angle QPN$.
- **Recover the apex:** repeating on the other side yields $\angle CAE = \angle RPN$, and adding the two recovers the full angle $\angle A = \angle P$.
- **Close with SAS:** that included angle with the proportion $\frac{AB}{PQ} = \frac{AC}{PR}$ runs SAS on the full triangles to finish.

The one step to do carefully is the parallelogram claim $BE = AC$, which depends on the diagonals of $ABEC$ bisecting each other at D. Spell that reason out, because an unjustified $BE = AC$ is the most common place to lose a mark in this proof. Notice also that the median doubling is done on both triangles in the same way, so the two constructions mirror each other and the SSS comparison lines up side for side. Once the apex angles are recovered by addition, the final SAS step is short, so most of the marks here sit in the careful construction and the SSS stage rather than in the closing line.

Final Answer: Doubling the medians gives $\triangle ABE \sim \triangle PQN$ (SSS), so $\angle A = \angle P$; SAS then gives $\triangle ABC \sim \triangle PQR$.

Q 6.15 **A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.**



SOLUTION

Concept used. At the same time of day the sun's rays fall at the same angle, so the pole, the tower and their shadows form two **similar triangles** (each has a right angle at the ground and the same sun-angle). In similar triangles, **corresponding sides are in the same ratio**. So

$$\frac{\text{height of pole}}{\text{shadow of pole}} = \frac{\text{height of tower}}{\text{shadow of tower}}.$$

Step 1. Let the tower height be h metres. Write the ratio of height to shadow for both:

$$\frac{6}{4} = \frac{h}{28}.$$

Step 2. Cross-multiply:

$$4 \times h = 6 \times 28.$$

Step 3. Compute the right side:

$$4h = 168.$$

Step 4. Solve for h:

$$h = \frac{168}{4} = 42 \text{ m}.$$

> **Final Answer:** The height of the tower is 42 m.

♥ Why This Matters

This is exactly how the heights of tall objects, trees, towers, even the Great Pyramid, are measured without climbing them. As long as two objects are measured at the same moment, the ratio of height to shadow is the same for both, so one known height calibrates all the others.

EXPERT'S SOLUTION *: Shalini Desai, M.Sc Mathematics, University of Mumbai*

Match height-to-shadow ratios, not heights to heights. The safe setup keeps each object's own height over its own shadow, then equates the two ratios.

- **Set up ratios:** the pole gives $\frac{6}{4}$ and the tower gives $\frac{h}{28}$; equal sun angles at the same time make the triangles similar, so these ratios are equal.
- **Solve:** one cross-multiplication, $4h = 6 \times 28 = 168$, leads straight to $h = 42$ m.
- **Avoid the slip:** the common error is pairing the pole's shadow with the tower's height, so write each fraction as height over shadow for the same object before equating.

A quick reasonableness check helps: the tower's shadow is 7 times the pole's, so the tower should be 7 times as tall, and $7 \times 6 = 42$ m confirms it.

Final Answer: Equal height-to-shadow ratios give $\frac{6}{4} = \frac{h}{28}$, so $h = 42$ m.

Q 6.16 **If AD and PM are medians of triangles ABC and PQR, respectively where $\triangle ABC \sim \triangle PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.**

SOLUTION

Concept used. Similar triangles have **equal corresponding angles** and **proportional corresponding sides**. A **median** ends at the mid-point of a side, so D and M are mid-points of BC and QR. We prove a smaller pair similar by SAS and read off the side ratio.

Step 1. From $\triangle ABC \sim \triangle PQR$, corresponding sides give

$$\frac{AB}{PQ} = \frac{BC}{QR}, \tag{1}$$

and corresponding angles give $\angle B = \angle Q$.

Step 2. D and M are mid-points, so $BD = \frac{1}{2}BC$ and $QM = \frac{1}{2}QR$. Hence

$$\frac{BD}{QM} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BC}{QR}.$$

Combining with (1),

$$\frac{AB}{PQ} = \frac{BD}{QM}. \tag{3}$$

Step 3. Compare $\triangle ABD$ and $\triangle PQM$. From (3) the two sides about the angle are proportional, and from (2) the included angles are equal ($\angle B = \angle Q$). By SAS,

$$\triangle ABD \sim \triangle PQM.$$

Step 4. Corresponding sides of these similar triangles are in the same ratio. Matching $A \leftrightarrow P$, $B \leftrightarrow Q$, $D \leftrightarrow M$:

$$\frac{AB}{PQ} = \frac{AD}{PM}.$$

Final Answer: $\frac{AB}{PQ} = \frac{AD}{PM}$, proved from $\triangle ABD \sim \triangle PQM$ (SAS, using $\frac{AB}{PQ} = \frac{BD}{QM}$ and $\angle B = \angle Q$).

☞ **Useful aside**

The median ratio equals the side ratio in similar triangles. The same argument shows every matching "cevian" (median, altitude or angle bisector) of two similar triangles is in the same ratio as the sides, which is the scale factor of the similarity.

EXPERT'S SOLUTION *: Faisal Khan, M.Sc Mathematics, Aligarh Muslim University*

Shrink to a half-side triangle, then run SAS. The result says a median scales by the same factor as the sides, and the clean way to see it is through the smaller triangles $\triangle ABD$ and $\triangle PQM$.

- **Start from the main pair:** the given similarity supplies $\frac{AB}{PQ} = \frac{BC}{QR}$ and the equal included angle $\angle B = \angle Q$.
- **Halve the sides:** since D and M are mid-points, halving BC and QR keeps the ratio, so $\frac{BD}{QM} = \frac{BC}{QR} = \frac{AB}{PQ}$.
- **Run SAS:** $\triangle ABD$ and $\triangle PQM$ now have two proportional sides about the equal angle, so SAS gives their similarity and hence $\frac{AB}{PQ} = \frac{AD}{PM}$.

The single key move is recognising that the half-sides keep the same ratio as the full sides, which is exactly what lets the median enter as a corresponding side.

> **Final Answer:** $\triangle ABD \sim \triangle PQM$ (SAS) gives $\frac{AB}{PQ} = \frac{AD}{PM}$.

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