Collegedunia NCERT Notes

The Ultimate NCERT Revision Guide for Class 10 Maths (Mathematics)

Chapter 4: Quadratic Equations

What this chapter covers: a quadratic equation is any equation of the form $ax^2 + bx + c = 0$ with $a \neq 0$. This chapter teaches three things students must know cold for the board exam: how to spot and write a quadratic equation in standard form, how to find its roots by *factorisation* and by the *quadratic formula*, and how the *discriminant* $b^2 - 4ac$ tells you the nature of the roots before you even solve. We finish with the word problems that make this one of the highest-scoring chapters in Class 10 Mathematics. These notes follow the rationalised NCERT syllabus for the session **2026-27**.

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Also see for this chapter: NCERT Solutions | Formula Sheet | Exemplar Solutions

1 Introduction and Standard Form

In Chapter 2 you studied polynomials. A polynomial of degree 2, like $2x^2+x-300$, is called a **quadratic polynomial**. When we set such a polynomial equal to zero, we get a **quadratic equation**. These equations turn up everywhere: areas of plots, speeds of trains, ages, and product-of-numbers puzzles all lead to a quadratic. This chapter shows you how to write them and how to solve them.

The chapter is short and the methods are fixed. Once you can factorise confidently and remember one formula, you can solve almost any board question here. So learn the standard form first, then practise until the steps feel automatic.

1.1 What is a quadratic equation?

A **quadratic equation** in the variable x is an equation that can be written in the form

$$ax^2 + bx + c = 0, \qquad a \neq 0,$$

where a, b and c are real numbers. This is called the **standard form**. The condition $a \neq 0$ is essential: if $a = 0$ the x^2 term vanishes and the equation is only linear, not quadratic.

Standard Form of a Quadratic Equation

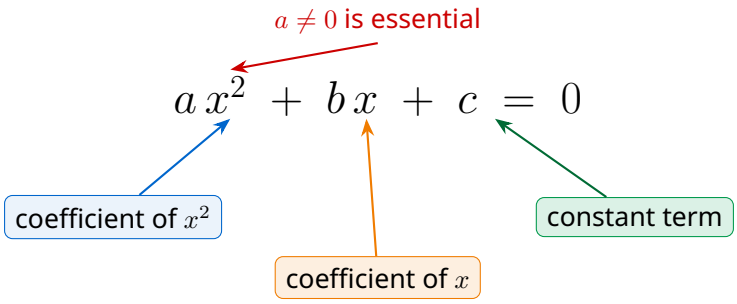
$$ax^2 + bx + c = 0, \qquad a \neq 0.$$

- a is the **coefficient of** x^2 (the leading coefficient, must be non-zero).
- b is the **coefficient of** x (can be zero).
- c is the **constant term** (can be zero).

Any equation of the form $p(x) = 0$, where $p(x)$ is a degree-2 polynomial, is a quadratic equation.

Examples of quadratic equations: $2x^2+x-300=0$, $x^2-3x+1=0$, $4x-3x^2+2=0$, and $1 - x^2 + 300 = 0$.

The diagram below names the three parts of the standard form so you can read off a, b and c at a glance.



Read the equation left to right

Going left to right, the coefficients are always a (with x^2), then b (with x), then c (alone). If a term is missing, its coefficient is zero: in $x^2 - 9 = 0$ we have $a = 1$, $b = 0$, $c = -9$.

1.2 Putting an equation into standard form

Many equations do not look quadratic at first. You must simplify both sides, bring everything to one side, and write the terms in descending order of degree before you can read off a, b and c.

Worked example. Check whether $(x-2)^2+1=2x-3$ is a quadratic equation.

Expand the left-hand side:

$$\begin{aligned}(x-2)^2+1 &= x^2-4x+4+1\\ &= x^2-4x+5.\end{aligned}$$

So the equation becomes

$$\begin{aligned}x^2-4x+5 &= 2x-3\\ x^2-4x+5-2x+3 &= 0\\ x^2-6x+8 &= 0.\end{aligned}$$

This is of the form $ax^2+bx+c=0$ with $a=1$, $b=-6$, $c=8$. So yes, it is a quadratic equation.

Always simplify before you decide

An equation can *look* quadratic and turn out linear, or *look* cubic and turn out quadratic. For example $x(x+1)+8=(x+2)(x-2)$ simplifies to $x+12=0$, which is *linear*. And $(x+2)^3=x^3-4$ simplifies to $6x^2+12x+12=0$, which *is* quadratic. Never judge by the first look; simplify, then check the degree.

1.3 Roots of a quadratic equation

A real number α is called a **root** of the quadratic equation $ax^2+bx+c=0$ if it satisfies the equation, that is, if $a\alpha^2+b\alpha+c=0$. We also say $x=\alpha$ is a **solution** of the equation.

Roots and zeroes are the same thing

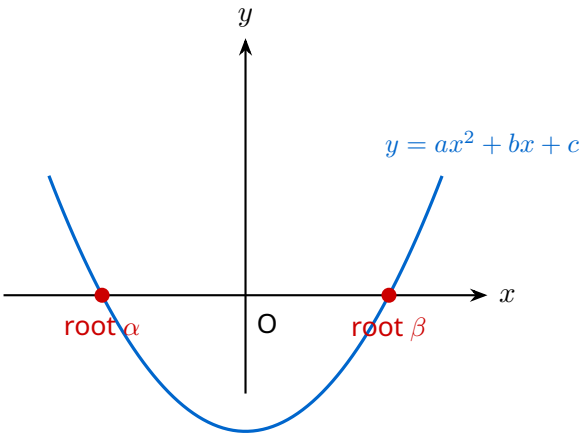
The roots of the quadratic equation $ax^2+bx+c=0$ are exactly the **zeroes** of the quadratic polynomial ax^2+bx+c. Since a quadratic polynomial has at most two zeroes, a quadratic equation has **at most two roots**.

Quick check. Is $x=1$ a root of $2x^2-3x+1=0$? Substitute $x=1$:

$$2(1)^2-3(1)+1=2-3+1=0.$$

The left side equals zero, so $x=1$ is indeed a root.

A quadratic always traces a U-shaped curve called a **parabola** (it opens upward when $a>0$ and downward when $a<0$). The roots are the x-values where this parabola crosses the x-axis.



The picture above shows the typical case: the parabola meets the x-axis at two points, giving two distinct real roots α and β. Later we will see that the parabola can also just touch the axis (equal roots) or miss it entirely (no real roots).

Where quadratics come from

The very first example in the NCERT chapter is a prayer hall whose length is one metre more than twice its breadth, with floor area 300 square metres. If the breadth is x metres, the length is $(2x + 1)$ metres, and the area gives $x(2x + 1) = 300$, i.e. $2x^2 + x - 300 = 0$. Plot dimensions, speed-distance-time problems, and number puzzles all turn into quadratics in just this way.

2 Solution by Factorisation

The first method for solving a quadratic equation is **factorisation**. The idea rests on one simple fact about real numbers: if a product of two factors is zero, then at least one of the factors must be zero. We split the middle term, write the quadratic as a product of two linear factors, and set each factor equal to zero.

2.1 The zero-product rule

Zero-Product Property

For any two real numbers P and Q:

$$P \cdot Q = 0 \iff P = 0 \text{ or } Q = 0.$$

So if $(x - \alpha)(x - \beta) = 0$, then either $x - \alpha = 0$ or $x - \beta = 0$, giving the roots $x = \alpha$ and $x = \beta$.

This is why factorisation works. Once the quadratic is written as a product of two brackets equal to zero, each bracket on its own gives a root.

2.2 Splitting the middle term

To factorise $ax^2 + bx + c$, we split the middle coefficient b into two numbers whose *sum* is b and whose *product* is $a \times c$. This lets us group the four terms into two pairs and pull out a common bracket.

Splitting the Middle Term

To factorise $ax^2 + bx + c$, find two numbers p and q such that

$$p + q = b \qquad \text{and} \qquad p \cdot q = a \cdot c.$$

Then $ax^2 + bx + c = ax^2 + px + qx + c$, and group to factor.

Worked example. Find the roots of $2x^2 - 5x + 3 = 0$ by factorisation.

Here $a = 2$, $b = -5$, $c = 3$, so we need two numbers with sum -5 and product $a \cdot c = 2 \times 3 = 6$. The numbers -2 and -3 work, since $(-2) + (-3) = -5$ and $(-2)(-3) = 6$.

$$\begin{aligned} 2x^2 - 5x + 3 &= 2x^2 - 2x - 3x + 3 \\ &= 2x(x - 1) - 3(x - 1) \\ &= (2x - 3)(x - 1). \end{aligned}$$

Setting each factor to zero:

$$\begin{aligned} 2x - 3 = 0 &\implies x = \tfrac{3}{2}, \\ x - 1 = 0 &\implies x = 1. \end{aligned}$$

So the roots are $x = \dfrac{3}{2}$ and $x = 1$.

Find the two numbers fast

List the factor pairs of $a \cdot c$, then look for the pair whose *difference* or *sum* matches b. Fix the signs at the end: if the product $a \cdot c$ is positive, both numbers share the sign of b; if it is negative, the numbers have opposite signs.

2.3 A worked example with surds

Factorisation also handles equations with surd coefficients. The method is identical; only the arithmetic looks heavier.

Worked example. Find the roots of $3x^2 - 2\sqrt{6}\,x + 2 = 0$.

We need two terms with product $a \cdot c = 3 \times 2 = 6$ and sum $-2\sqrt{6}$. Splitting the middle term as $-\sqrt{6}\,x - \sqrt{6}\,x$:

$$\begin{aligned} 3x^2 - 2\sqrt{6}\,x + 2 &= 3x^2 - \sqrt{6}\,x - \sqrt{6}\,x + 2 \\ &= \sqrt{3}\,x\left(\sqrt{3}\,x - \sqrt{2}\right) - \sqrt{2}\left(\sqrt{3}\,x - \sqrt{2}\right) \\ &= \left(\sqrt{3}\,x - \sqrt{2}\right)\left(\sqrt{3}\,x - \sqrt{2}\right). \end{aligned}$$

So $\left(\sqrt{3}\,x - \sqrt{2}\right)^2 = 0$, which gives the single repeated root

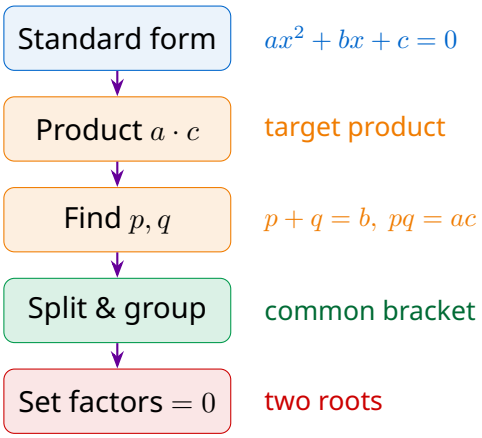
$$\sqrt{3}\,x - \sqrt{2} = 0 \implies x = \frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}.$$

Here both factors are the same, so the root $x = \sqrt{\frac{2}{3}}$ is repeated twice. This is our first example of **equal roots**, a case we study fully in the discriminant section.

2.4 Step-by-step factorisation method

Factorisation in five steps

1. Write the equation in standard form $ax^2 + bx + c = 0$.
2. Compute the product $a \cdot c$.
3. Find two numbers with sum b and product $a \cdot c$.
4. Split the middle term and group into two brackets; pull out the common factor.
5. Set each linear factor to zero and solve for x.



Equation must be set to zero first

You can only use the zero-product rule when one side is *exactly zero*. Writing $(2x - 3)(x - 1) = 5$ and then saying $2x - 3 = 5$ is wrong. Always rearrange so the right-hand side is 0 before factorising.

2.5 Quick practice: factorisation at a glance

The table collects standard NCERT-style equations with their split, factors and roots. Cover the last two columns and try each one yourself first.

Equation	Split of b	Factors	Roots
$x^2 - 3x - 10 = 0$	$-5,\ 2$	$(x-5)(x+2)$	$5,\ -2$
$2x^2 + x - 6 = 0$	$4,\ -3$	$(2x-3)(x+2)$	$\frac{3}{2},\ -2$
$6x^2 - x - 2 = 0$	$-4,\ 3$	$(3x-2)(2x+1)$	$\frac{2}{3},\ -\frac{1}{2}$
$100x^2 - 20x + 1 = 0$	$-10,\ -10$	$(10x-1)(10x-1)$	$\frac{1}{10}$ (equal)
$x^2 - 45x + 324 = 0$	$-36,\ -9$	$(x-36)(x-9)$	$36,\ 9$

Notice the third column always multiplies back to the original equation, and the roots come straight from setting each factor to zero. The fourth row gives two equal roots, the case we meet again with the discriminant.

3 The Quadratic Formula

Not every quadratic factorises with nice whole numbers. The **quadratic formula** solves *any* quadratic equation directly from its coefficients. It comes from completing the square on the general form $ax^2 + bx + c = 0$, and it is the single most useful formula in this chapter.

3.1 The formula

Quadratic Formula (Sridharacharya's formula)

The roots of $ax^2 + bx + c = 0$ (with $a \neq 0$) are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad \text{provided } b^2 - 4ac \geq 0.$$

- a, b, c are the coefficients read off the standard form.
- The quantity $b^2 - 4ac$ under the root is the **discriminant**, written D.
- The "$\pm$" gives the two roots: one with $+$, one with $-$.

The formula is often credited to the Indian mathematician Sridharacharya. To use it, just substitute the values of a, b and c carefully, watching the signs.

3.2 Worked example using the formula

Worked example. Solve $x^2 + 7x - 60 = 0$ using the quadratic formula.

Step 1: read off the coefficients. Here $a = 1$, $b = 7$, $c = -60$.

Step 2: compute the discriminant.

$$\begin{aligned} b^2 - 4ac &= (7)^2 - 4(1)(-60) \\ &= 49 + 240 \\ &= 289. \end{aligned}$$

Step 3: substitute into the formula. Start from the general formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Putting $a = 1$, $b = 7$ and $\sqrt{D} = \sqrt{289} = 17$:

$$x = \frac{-7 \pm \sqrt{289}}{2(1)} = \frac{-7 \pm 17}{2}.$$

Step 4: split the $\pm$ into two roots.

$$x = \frac{-7 + 17}{2} = \frac{10}{2} = 5,$$
$$x = \frac{-7 - 17}{2} = \frac{-24}{2} = -12.$$

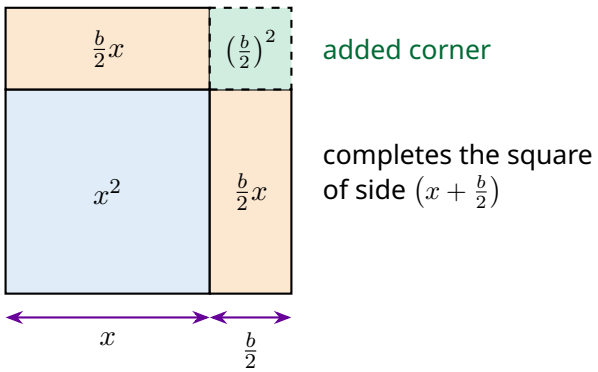
So the roots are $x = 5$ and $x = -12$.

When to use the formula over factorisation

Try factorisation first when $a \cdot c$ is small and easy to split. Switch to the quadratic formula whenever the numbers are awkward, the discriminant is not a perfect square, or you simply cannot spot the factors quickly. The formula never fails on a quadratic with real roots.

3.3 The idea behind the formula: completing the square

The formula is not magic; it comes from **completing the square**. Starting from $ax^2 + bx + c = 0$, we make the left side a perfect square plus a constant, then take square roots. The diagram below shows the geometric idea: we turn $x^2 + bx$ into a square of side $\left(x + \frac{b}{2}\right)$ by adding a small corner square of area $\left(\frac{b}{2}\right)^2$.



Algebraically the same idea runs like this. Divide through by a, move the constant

across, and add $\left(\frac{b}{2a}\right)^2$ to both sides:

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}.$$

Taking square roots and rearranging gives exactly $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Mind the sign of b and of c

The commonest formula error is sign carelessness. In $2x^2 + 14x - 120 = 0$ we have $c = -120$, so $-4ac = -4(2)(-120) = +960$, a *plus*. Writing -960 here wrecks the answer. Put each value in brackets before squaring or multiplying.

4 Nature of Roots and the Discriminant

The expression $b^2 - 4ac$ under the square root is so useful that it has its own name: the **discriminant**, written D. Its *sign* alone tells you how many real roots a quadratic has, without solving the equation. This is a favourite one-mark and two-mark board question.

4.1 The discriminant

Discriminant

For the quadratic equation $ax^2 + bx + c = 0$,

$$D = b^2 - 4ac.$$

The sign of D decides the nature of the roots:

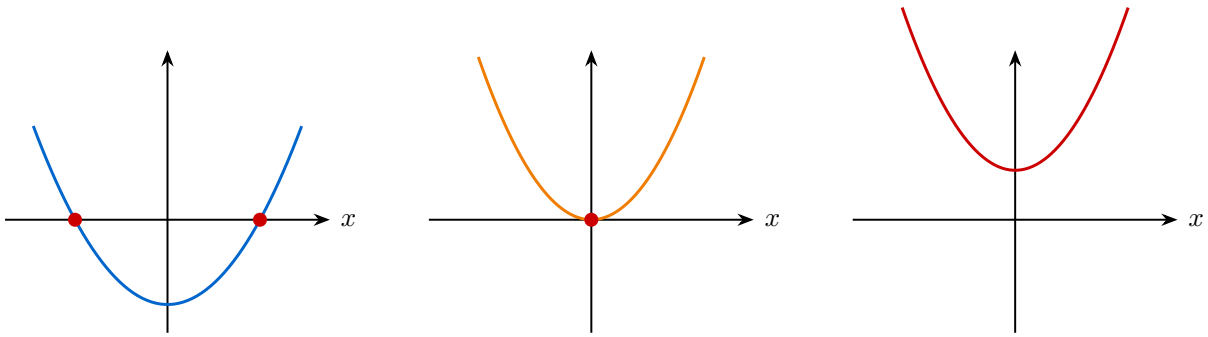
- $D > 0$: two distinct real roots.
- $D = 0$: two equal real roots (one repeated root).
- $D < 0$: no real roots.

Why the sign of D matters

The roots are $x = \dfrac{-b \pm \sqrt{D}}{2a}$. If $D > 0$ the square root is a positive number, so the two roots differ. If $D = 0$ the root term vanishes and both roots collapse to $x = -\frac{b}{2a}$. If $D < 0$ we would need the square root of a negative number, which is not real, so there are no real roots.

4.2 The three cases as parabolas

Each case has a clear picture. The parabola either cuts the x-axis at two points, just touches it, or floats above (or below) it.



$D > 0$ two distinct roots | $D = 0$ two equal roots | $D < 0$ no real roots

When the parabola crosses the axis twice ($D > 0$) we get two roots. When it just touches at the vertex ($D = 0$) the two roots merge into one repeated value. When it stays entirely on one side ($D < 0$) it never meets the axis, so there are no real roots.

4.3 Worked examples on nature of roots

Example 1. Find the discriminant of $2x^2 - 4x + 3 = 0$ and state the nature of its roots.

Here $a = 2$, $b = -4$, $c = 3$.

$$\begin{aligned} D = b^2 - 4ac &= (-4)^2 - 4(2)(3) \\ &= 16 - 24 \\ &= -8. \end{aligned}$$

Since $D = -8 < 0$, the equation has **no real roots**.

Example 2. Find the discriminant of $3x^2 - 2x + \frac{1}{3} = 0$ and find the roots if they are real.

Here $a = 3$, $b = -2$, $c = \frac{1}{3}$.

$$\begin{aligned} D = b^2 - 4ac &= (-2)^2 - 4(3)\left(\tfrac{1}{3}\right) \\ &= 4 - 4 \\ &= 0. \end{aligned}$$

Since $D = 0$, the equation has **two equal real roots**, each equal to

$$x = \frac{-b}{2a} = \frac{-(-2)}{2(3)} = \frac{2}{6} = \frac{1}{3}.$$

So both roots are $x = \dfrac{1}{3}$.

Equal roots short-cut

For a repeated root ($D = 0$) you do not need the full formula. The single root is always $x = -\dfrac{b}{2a}$, the x-coordinate of the vertex of the parabola. This saves time in the exam.

4.4 Finding an unknown for equal roots

A standard board question gives a quadratic with an unknown coefficient k and asks for the value of k that makes the roots equal. The condition is simply $D = 0$.

Worked example. Find the values of k for which $2x^2 + kx + 3 = 0$ has two equal roots.

For equal roots, $D = 0$:

$$\begin{aligned} b^2 - 4ac &= 0 \\ k^2 - 4(2)(3) &= 0 \\ k^2 - 24 &= 0 \\ k^2 &= 24 \\ k &= \pm 2\sqrt{6}. \end{aligned}$$

So the roots are equal when $k = 2\sqrt{6}$ or $k = -2\sqrt{6}$.

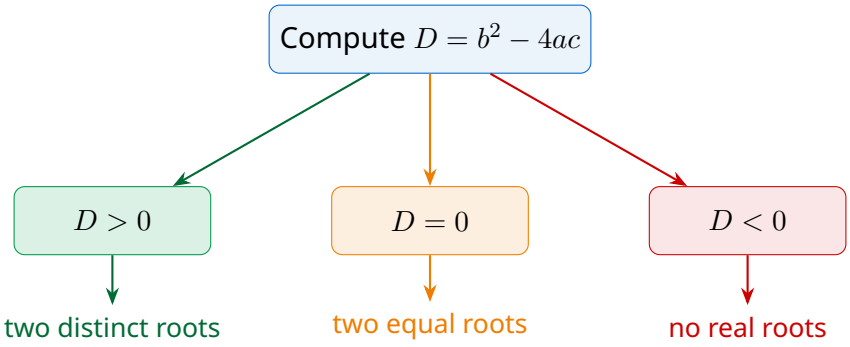
"No real roots" is a complete answer

If $D < 0$, do *not* try to push on and write a square root of a negative number in a Class 10 paper. The correct, full-mark answer is simply: "Since $D < 0$, the equation has no real roots." Stop there.

4.5 Summary table of the three cases

Discriminant D	Nature of roots	Roots
$D > 0$	Two distinct real roots	$x = \dfrac{-b \pm \sqrt{D}}{2a}$
$D = 0$	Two equal real roots	$x = -\dfrac{b}{2a}$ (repeated)
$D < 0$	No real roots	none (in real numbers)

The same logic fits neatly into a decision tree. Compute D once, then branch on its sign.



The discriminant in design problems

Many "Is it possible to design ...?" board questions are really discriminant questions. Asking whether a rectangular park with a given perimeter and area can exist leads to a quadratic; if its discriminant is negative, the park simply cannot be built. The sign of D becomes a yes-or-no answer about whether a real design exists.

5 Word Problems and Applications

The real marks in this chapter come from **word problems**. The pattern is always the same: name the unknown, translate the sentence into an equation, simplify to standard form, solve, and *reject any root that makes no physical sense* (such as a negative length or a fractional number of articles).

5.1 A four-step method for word problems

Solving word problems in four steps

1. **Let** the unknown quantity be x (state its units).
2. **Form** an equation from the conditions; simplify to $ax^2 + bx + c = 0$.
3. **Solve** by factorisation or the quadratic formula.
4. **Reject** any root that is impossible in context; state the valid answer.

5.2 Number problems

Worked example. Find two consecutive positive integers, the sum of whose squares is 365.

Let the integers be x and $x+1$. Then

$$\begin{aligned} x^2 + (x+1)^2 &= 365 \\ x^2 + x^2 + 2x + 1 &= 365 \\ 2x^2 + 2x - 364 &= 0 \\ x^2 + x - 182 &= 0. \end{aligned}$$

Factorising (two numbers with sum 1 and product -182 are 14 and -13):

$$\begin{aligned} x^2 + 14x - 13x - 182 &= 0 \\ x(x+14) - 13(x+14) &= 0 \\ (x-13)(x+14) &= 0. \end{aligned}$$

So $x = 13$ or $x = -14$. Since the integers must be *positive*, we reject $x = -14$. Thus $x = 13$, and the two integers are **13** and **14**.

Check: $13^2 + 14^2 = 169 + 196 = 365$. (verified)

5.3 Geometry and area problems

Worked example. The area of a rectangular plot is 528 m^2. The length is one more than twice the breadth. Find the dimensions.

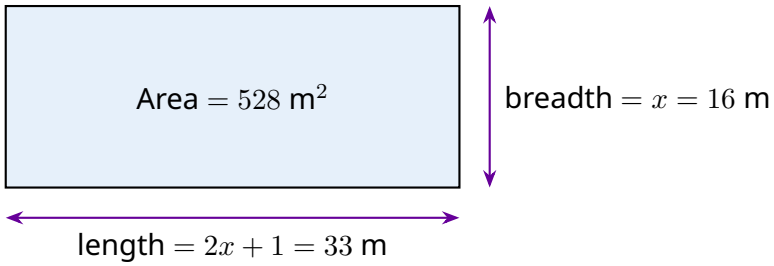
Let the breadth be x metres. Then the length is $(2x+1)$ metres, and

$$\begin{aligned} x(2x+1) &= 528 \\ 2x^2 + x - 528 &= 0. \end{aligned}$$

Using the quadratic formula with $a = 2$, $b = 1$, $c = -528$:

$$\begin{aligned} D &= 1^2 - 4(2)(-528) = 1 + 4224 = 4225, \\ \sqrt{D} &= 65, \\ x &= \frac{-1 \pm 65}{4}. \end{aligned}$$

So $x = \dfrac{64}{4} = 16$ or $x = \dfrac{-66}{4} = -16.5$. A breadth cannot be negative, so $x = 16$. Then the length is $2(16) + 1 = 33$. The plot is **16** m by **33** m.



5.4 Speed, distance and time problems

Worked example. A train travels 480 km at a uniform speed. If its speed had been 8 km/h less, it would have taken 3 hours more for the same journey. Find the speed of the train.

Let the speed be x km/h. Time at speed x is $\dfrac{480}{x}$ hours; at speed $(x-8)$ it is $\dfrac{480}{x-8}$ hours. The slower trip takes 3 hours longer:

$$\begin{aligned} \frac{480}{x-8} - \frac{480}{x} &= 3 \\ 480\left(\frac{x-(x-8)}{x(x-8)}\right) &= 3 \\ 480 \cdot \frac{8}{x^2-8x} &= 3 \\ 3840 &= 3(x^2-8x) \\ x^2 - 8x - 1280 &= 0. \end{aligned}$$

Factorising (sum -8, product -1280 gives -40 and 32):

$$(x-40)(x+32) = 0 \implies x = 40 \text{ or } x = -32.$$

Speed cannot be negative, so $x = 40$. The train's speed is **40** km/h.

> **Why we reject roots**
>
> Quadratics naturally throw up two answers, but the physical world often allows only one. A length, a speed, a count of marbles, or an age cannot be negative. Always finish a word problem by checking each root against the context and discarding the impossible one. Forgetting this loses a mark even when the algebra is perfect.

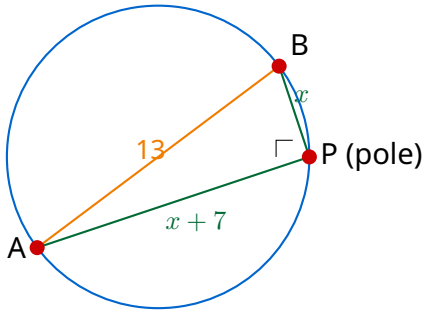
5.5 The pole-on-a-circle problem

Worked example. A pole is to be erected on the boundary of a circular park of diameter 13 m so that the difference of its distances from two diametrically opposite gates A and B is 7 m. At what distances from the two gates should the pole stand?

Let the distance from gate B be x m, so the distance from gate A is $(x+7)$ m. Since AB is a diameter, the angle at the pole P is a right angle (angle in a semicircle). By the Pythagoras theorem:

$$\begin{aligned} \text{AP}^2 + \text{BP}^2 &= \text{AB}^2 \\ (x+7)^2 + x^2 &= 13^2 \\ x^2 + 14x + 49 + x^2 &= 169 \\ 2x^2 + 14x - 120 &= 0 \\ x^2 + 7x - 60 &= 0. \end{aligned}$$

We solved this exact equation with the formula earlier and got $x = 5$ or $x = -12$. Distance cannot be negative, so $x = 5$. The pole stands **5** m from gate B and $5 + 7 =$ **12** m from gate A.



L-F-S-R for word problems

Remember **L-F-S-R**: **L**et the unknown, **F**orm the equation, **S**olve it, **R**eject the impossible root. Four letters, four steps, full marks.

5.6 Age problems

Age problems combine two unknowns into one quadratic using the "years ago" or "years from now" shift.

Worked example. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48. Find whether this situation is possible, and if so, their present ages.

Let one friend's present age be x years; then the other is $(20-x)$ years. Four years ago their ages were $(x-4)$ and $(16-x)$. The product condition gives

$$\begin{aligned}(x-4)(16-x) &= 48\\ 16x - x^2 - 64 + 4x &= 48\\ -x^2 + 20x - 64 &= 48\\ x^2 - 20x + 112 &= 0.\end{aligned}$$

Check the discriminant before solving:

$$D = (-20)^2 - 4(1)(112) = 400 - 448 = -48.$$

Since $D = -48 < 0$, the equation has no real roots. So the given situation is **not possible**: no pair of ages fits both conditions.

Why we check D in "is it possible?" questions

When a problem asks whether a situation can exist, you do not always need the roots. Form the quadratic and test the discriminant. If $D < 0$, the answer is "not possible" and you are done. If $D \geq 0$, solve and then reject any impossible root. This saves time and earns the full mark.

5.7 Design-possibility problems

Worked example. Is it possible to design a rectangular mango grove whose length is twice its breadth and whose area is 800 m^2? If so, find its dimensions.

Let the breadth be x metres; the length is then $2x$ metres, and

$$2x \cdot x = 800$$
$$2x^2 = 800$$
$$x^2 = 400.$$

Here $D = 0^2 - 4(1)(-400) = 1600 > 0$, so real roots exist and the design *is* possible. Solving, $x = \pm 20$. A breadth cannot be negative, so $x = 20$. The grove is **20** m broad and $2(20) =$ **40** m long.

6 Quick Reference Summary

Everything in this chapter, on one card, for last-minute revision before the board exam.

6.1 Key facts and formulae

Memorise these

Standard form: $ax^2 + bx + c = 0,\ a \neq 0$

Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant: $D = b^2 - 4ac$

Equal-root value: $x = -\dfrac{b}{2a}$ (when $D = 0$)

6.2 Methods at a glance

Method	When to use	Key step
Factorisation	$a \cdot c$ splits into easy factors	Split middle term; zero-product rule
Quadratic formula	Any quadratic with real roots	Substitute a, b, c into the formula
Completing the square	To derive the formula / special cases	Add $\left(\frac{b}{2a}\right)^2$ to both sides
Discriminant test	To find the nature of roots only	Compute D and read its sign

6.3 Nature-of-roots checklist

When asked about the nature of the roots, work through this:

1. Write the equation in standard form and read off a, b, c.

2. Compute $D = b^2 - 4ac$, keeping signs in brackets.
3. State the case: $D > 0$ (two distinct), $D = 0$ (two equal), $D < 0$ (no real roots).
4. If asked for the roots and $D \geq 0$, use the formula; if $D = 0$, use $x = -\frac{b}{2a}$.

6.4 Common errors to avoid

Five traps that cost marks

- Forgetting the condition $a \neq 0$ when claiming an equation is quadratic.
- Sign slips in $b^2 - 4ac$, especially when c is negative.
- Using the zero-product rule when one side is not yet zero.
- Keeping a negative or fractional answer where the context forbids it.
- Writing $\sqrt{D}$ when $D < 0$ instead of stating "no real roots".

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6.5 Final word

Quadratic equations are one of the most dependable mark-earners in Class 10 Mathematics. The methods are fixed and the patterns repeat, so practice pays off directly here.

Bank these marks

Practise five mixed word problems a day for a week before the exam. Mix in an area problem, a speed problem, a number problem, a nature-of-roots question, and one equal-roots "find k" question. That single habit makes this whole chapter feel routine on exam day.

All the best for your board exam!

Master the formula, practise the word problems, and these marks are yours.

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