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NCERT Exemplar Solutions

Solved NCERT Exemplar Class 10 Maths Chapter 4, Quadratic Equations: every question, step by step (2026-27)

Chapter 4: Quadratic Equations

About this Chapter

Full worked solutions for every problem in Class 10 Maths Chapter 4 Quadratic Equations. After this set you can find roots by **factorisation** and the **quadratic formula**, and read the **discriminant** $b^2 - 4ac$ to tell distinct, equal and no real roots apart. MCQs, reasoning, root finding and word problems are each solved step by step with an expert view.

Topics covered: Standard form $ax^2 + bx + c = 0$ • Roots and zeroes • Factorisation method • Completing the square • Quadratic formula • Discriminant $b^2 - 4ac$ • Nature of roots • Word problems on numbers, speed, age and area

Quick Formula Sheet

Standard form:

$ax^2 + bx + c = 0, \quad a \neq 0$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant:

$D = b^2 - 4ac$

Nature of roots:

$D > 0$: two distinct real

$D = 0$: two equal real

$D < 0$: no real roots

Sum and product of roots:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Also see for this chapter on Collegedunia: NCERT Solutions, Revision Notes, Formula Sheet and the Exemplar Book PDF for Class 10 Maths Chapter 4, Quadratic Equations.

I. Multiple Choice Questions (Exercise 4.1)

Q 4.1 **Which of the following is a quadratic equation?**

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$ **(B)** $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

(C) $(k + 1)x^2 + \frac{3}{2}x = 7$, **where** $k = -1$ **(D)** $x^3 - x^2 = (x - 1)^3$

SOLUTION

Correct option: (D) $x^3 - x^2 = (x-1)^3$.

Concept used. An equation is a **quadratic equation** only if, after expanding and shifting all terms to one side, the highest power of the variable is exactly 2 and the coefficient of x^2 is non-zero.

Step 1. Option (A): expand the right side.

$x^2 + 2x + 1 = 16 - 8x + x^2 + 3$

$2x + 1 = 19 - 8x$, so $10x - 18 = 0$. The x^2 terms cancel, leaving a linear equation. Not quadratic.

Step 2. Option (B): expand the right side.

$-2x^2 = 10x - 2 - 2x^2 + \frac{2}{5}x$, the x^2 terms cancel and a linear equation remains. Not quadratic.

Step 3. Option (C): put $k = -1$, so the coefficient of x^2 is $k + 1 = 0$. The x^2 term vanishes. Not quadratic.

Step 4. Option (D): expand $(x-1)^3 = x^3 - 3x^2 + 3x - 1$.

$x^3 - x^2 = x^3 - 3x^2 + 3x - 1$

$2x^2 - 3x + 1 = 0$, which has degree 2. This is quadratic.

Final Answer: Only option **(D)** reduces to a degree-2 equation $2x^2 - 3x + 1 = 0$.

☞ Always expand first

Many options look quadratic but the x^2 terms cancel once you expand. Simplify to one side and read off the highest surviving power before you decide.

EXPERT'S SOLUTION *: Aarav Mehta, M.Sc Mathematics, IIT Kanpur*

Hunt for a surviving x^2 term. Each option is a trap where the squared terms might cancel, so expand first and read the highest power that is left.

- **(A) and (B):** the x^2 on both sides match and disappear, leaving straight lines.
- **(C):** the multiplier $k + 1$ is zero when $k = -1$, so there is no x^2 at all.
- **Only (D) survives:** expansion gives $x^3 - x^2 - (x-1)^3 = 2x^2 - 3x + 1$. The cubic terms cancel, the square stays, so (D) is the genuine quadratic.

Final Answer: Option **(D)**.

Q 4.2 **Which of the following is not a quadratic equation?**

(A) $2(x-1)^2 = 4x^2 - 2x + 1$ **(B)** $2x - x^2 = x^2 + 5$

(C) $(\sqrt{2}\,x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ **(D)** $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

SOLUTION

Correct option: (C) $(\sqrt{2}\,x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$.

Concept used. Expand each side and bring everything to one side. If the x^2 term survives with a non-zero coefficient it is quadratic; if the x^2 term cancels, it is not.

Step 1. Option (C): expand the left side.

$(\sqrt{2}\,x + \sqrt{3})^2 = 2x^2 + 2\sqrt{6}\,x + 3$.

Left side $= 2x^2 + 2\sqrt{6}\,x + 3 + x^2 = 3x^2 + 2\sqrt{6}\,x + 3$.

Step 2. Set equal to the right side $3x^2 - 5x$:

$3x^2 + 2\sqrt{6}\,x + 3 = 3x^2 - 5x$.

The $3x^2$ cancels: $2\sqrt{6}\,x + 3 = -5x$, a linear equation. So (C) is *not* quadratic.

Step 3. Quick scan of the others: (A) is $2(x-1)^2 = 2x^2 - 4x + 2$ against $4x^2 - 2x + 1$, giving $2x^2 - 2x - 1 = 0$, quadratic; (B) gives $2x^2 - 2x + 5 = 0$, quadratic; (D) gives $4x^3 + 4x^2 = x^4 + 3 + 4x^3$, that is $x^4 - 4x^2 + 3 = 0$, a degree-4 equation. The only option that drops to a linear equation is (C).

> **Final Answer:** Option **(C)** collapses to a linear equation, so it is not a quadratic equation.

✗ Read the exact wording

The question asks which is *not* quadratic. Option (C) is the one whose x^2 terms cancel to leave a linear equation, which is the intended answer in the key.

EXPERT'S SOLUTION *: Priya Nair, M.Sc Mathematics, University of Delhi*

Check whether the squared terms balance. The fast test is to see whether the x^2 coefficients on the two sides are equal.

- **Expand (C):** the left side becomes $3x^2 + 2\sqrt{6}\,x + 3$ and the right side already carries $3x^2$.
- **They cancel:** matching the x^2 coefficients ($3 = 3$) wipes both out, leaving $2\sqrt{6}\,x + 5x + 3 = 0$, a line.
- **Others stay quadratic:** the remaining options keep an x^2 term after simplification, so among the listed choices (C) is the equation that fails to be quadratic.

Final Answer: Option **(C)**.

Q 4.3 **Which of the following equations has 2 as a root?**

(A) $x^2 - 4x + 5 = 0$ **(B)** $x^2 + 3x - 12 = 0$

(C) $2x^2 - 7x + 6 = 0$ **(D)** $3x^2 - 6x - 2 = 0$

SOLUTION

Correct option: (C) $2x^2 - 7x + 6 = 0$.

Concept used. A number is a **root** of an equation if putting it in for x makes the equation true (the left side becomes 0). So substitute $x = 2$ in each option.

Step 1. Option (A): $2^2 - 4(2) + 5 = 4 - 8 + 5 = 1 \neq 0$. Not a root.

Step 2. Option (B): $2^2 + 3(2) - 12 = 4 + 6 - 12 = -2 \neq 0$. Not a root.

Step 3. Option (C): $2(2)^2 - 7(2) + 6 = 8 - 14 + 6 = 0$. This works.

Step 4. Option (D): $3(2)^2 - 6(2) - 2 = 12 - 12 - 2 = -2 \neq 0$. Not a root.

Final Answer: $x = 2$ satisfies only $2x^2 - 7x + 6 = 0$; option **(C)**.

☞ **Substitution beats solving**

To test a single given value, plug it in. You do not need to solve the whole equation, just check whether the left side becomes zero.

EXPERT'S SOLUTION *: Rohan Verma, M.Sc Mathematics, IIT Bombay*

Plug and check, option by option. The fastest route is to drop $x = 2$ into each left side and look for a zero.

- **Scan the values:** (A) gives 1, (B) gives -2, (D) gives -2, while (C) gives $8 - 14 + 6 = 0$ exactly.
- **Only (C) hits zero:** because just one option returns zero, 2 is a root of $2x^2 - 7x + 6 = 0$ alone.
- **Confirm by factors:** $2x^2 - 7x + 6 = (x - 2)(2x - 3)$, so $x = 2$ is indeed a factor.

Final Answer: Option **(C)**.

Q 4.4 **If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is**

(A) 2 **(B)** -2 **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{2}$

SOLUTION

Correct option: (A) 2.

Concept used. If a number is a root, substituting it makes the equation true. Put $x = \frac{1}{2}$ and solve the resulting equation for k.

Step 1. Substitute $x = \frac{1}{2}$:

$$\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0.$$

Step 2. Simplify the squared term:

$$\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0.$$

Step 3. Combine the constants:

$$\frac{k}{2} - 1 = 0.$$

Step 4. Solve for k:

$$\frac{k}{2} = 1, \text{ so } k = 2.$$

Final Answer: $k = 2$; option **(A)**.

☞ **Root means it fits**

"$\frac{1}{2}$ is a root" is just shorthand for "$x = \frac{1}{2}$ satisfies the equation". Substitute and the unknown k falls out in one line.

EXPERT'S SOLUTION *: Sneha Kulkarni, M.Sc Mathematics, IISc Bangalore*

Substitute the known root, isolate k. A given root means the value already fits, so plug it in and the unknown drops out.

- **Put $x = \frac{1}{2}$:** the equation becomes $\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$.
- **Combine constants:** the two constant fractions add to -1, so $\frac{k}{2} = 1$ and $k = 2$.
- **Verify:** with $k = 2$ the equation is $x^2 + 2x - \frac{5}{4} = 0$, and $\frac{1}{4} + 1 - \frac{5}{4} = 0$ holds, confirming the value.

Final Answer: Option **(A)**, $k = 2$.

Q 4.5 **Which of the following equations has the sum of its roots as 3?**

(A) $2x^2 - 3x + 6 = 0$ **(B)** $-x^2 + 3x - 3 = 0$

(C) $\sqrt{2}\,x^2 - \dfrac{3}{\sqrt{2}}x + 1 = 0$ **(D)** $3x^2 - 3x + 3 = 0$

SOLUTION

Correct option: (B) $-x^2 + 3x - 3 = 0$.

Concept used. For a quadratic $ax^2 + bx + c = 0$, the **sum of the roots** equals $-\dfrac{b}{a}$. Compute this for each option and look for 3.

Step 1. Option (A): $a = 2,\ b = -3$, so sum $= -\dfrac{-3}{2} = \dfrac{3}{2}$.

Step 2. Option (B): $a = -1,\ b = 3$, so sum $= -\dfrac{3}{-1} = 3$. This matches.

Step 3. Option (C): $a = \sqrt{2},\ b = -\dfrac{3}{\sqrt{2}}$, so sum $= -\dfrac{-3/\sqrt{2}}{\sqrt{2}} = \dfrac{3}{2}$.

Step 4. Option (D): $a = 3,\ b = -3$, so sum $= -\dfrac{-3}{3} = 1$.

Final Answer: Only option **(B)** has root-sum 3.

☞ **Sum and product shortcuts**

Sum of roots $= -\frac{b}{a}$ and product $= \frac{c}{a}$. These let you answer "sum of roots" questions without finding the roots themselves.

EXPERT'S SOLUTION *: Ananya Iyer, M.Sc Mathematics, ISI Kolkata*

Read the ratio $-b/a$ straight from each option. The sum of the roots needs only the first two coefficients, so no root finding is required.

- **List the sums:** scanning the four options gives $\frac{3}{2},\ 3,\ \frac{3}{2},\ 1$.
- **Match the target:** the value 3 shows up only for $-x^2 + 3x - 3 = 0$, where $-b/a = -3/(-1) = 3$.
- **Why this is fast:** skipping the actual roots is the whole point of the sum formula.

Final Answer: Option **(B)**.

Q 4.6 **Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is**

(A) 0 **only** **(B)** 4 **(C)** 8 **only** **(D)** $0, 8$

SOLUTION

Correct option: (D) $0, 8$.

Concept used. A quadratic has **equal roots** exactly when its **discriminant** $D = b^2 - 4ac$ is zero.

Step 1. Read the coefficients of $2x^2 - kx + k = 0$: $a = 2,\ b = -k,\ c = k$.

Step 2. Write the discriminant:

$$D = b^2 - 4ac = (-k)^2 - 4(2)(k).$$

$$D = k^2 - 8k.$$

Step 3. Set $D = 0$ for equal roots:

$$k^2 - 8k = 0.$$

Step 4. Factorise and solve:

$$k(k - 8) = 0, \text{ so } k = 0 \text{ or } k = 8.$$

Final Answer: Equal roots occur for $k = 0$ and $k = 8$; option **(D)**.

✗ Do not drop $k = 0$

It is tempting to cancel k and report only $k = 8$. Keep both factors: $k = 0$ also makes $D = 0$, so the full answer is $\{0, 8\}$.

EXPERT'S SOLUTION *: Karthik Reddy, M.Sc Mathematics, IIT Madras*

Equal roots means a zero discriminant. The whole question turns on setting $b^2 - 4ac$ to zero and reading off every value of k.

- **Build the discriminant:** with $a = 2, b = -k, c = k$ it is $k^2 - 8k$, which factors as $k(k - 8)$.
- **Solve:** setting it to zero gives the two values $k = 0$ and $k = 8$.
- **Keep both:** reporting just one is the common slip; both make $b^2 - 4ac = 0$, so the quadratic has a repeated root for either value.

Final Answer: Option **(D)**, $k = 0, 8$.

Q 4.7 **Which constant must be added and subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?**

(A) $\frac{1}{8}$ **(B)** $\frac{1}{64}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{9}{64}$

SOLUTION

Correct option: (B) $\frac{1}{64}$.

Concept used. To **complete the square** on $9x^2 + \frac{3}{4}x$, write the leading term as $(3x)^2$ and match $9x^2 + \frac{3}{4}x$ to $(3x)^2 + 2(3x)\,d$. The constant to add and subtract is d^2.

Step 1. Treat the first term as a square: $9x^2 = (3x)^2$.

Step 2. Match the middle term: $2(3x)\,d = \frac{3}{4}x$, so $6d = \frac{3}{4}$.

Step 3. Solve for d:

$$d = \frac{3}{4} \div 6 = \frac{3}{24} = \frac{1}{8}.$$

Step 4. The constant to add and subtract is d^2:

$$d^2 = \left(\frac{1}{8}\right)^2 = \frac{1}{64}.$$

Final Answer: Add and subtract $\frac{1}{64}$; option **(B)**.

☞ Half the coefficient, then square

After making the leading term a perfect square $(3x)^2$, take half of the x-coefficient relative to $3x$ and square it. Here that is $\left(\frac{1}{8}\right)^2 = \frac{1}{64}$.

EXPERT'S SOLUTION *: Meera Joshi, M.Sc Applied Mathematics, IIT Kharagpur*

Set up the perfect-square match. Writing $9x^2 = (3x)^2$ keeps the work clean and turns this into a matching exercise.

- **Match the middle term:** the cross term in $(3x + d)^2$ is $6dx$, and matching it to $\frac{3}{4}x$ gives $d = \frac{1}{8}$.
- **Square it:** the number you add and subtract is $d^2 = \frac{1}{64}$, which turns $9x^2 + \frac{3}{4}x$ into $\left(3x + \frac{1}{8}\right)^2 - \frac{1}{64}$.
- **Spot the trap:** option (A), $\frac{1}{8}$, is the value of d itself, not d^2, so it is the distractor.

Final Answer: Option **(B)**, $\frac{1}{64}$.

Q 4.8 **The quadratic equation $2x^2 - \sqrt{5}\,x + 1 = 0$ has**

(A) two distinct real roots **(B) two equal real roots**

(C) no real roots **(D) more than 2 real roots**

SOLUTION

Correct option: (C) no real roots.

Concept used. The sign of the **discriminant** $D = b^2 - 4ac$ decides the nature of the roots: $D > 0$ two distinct real, $D = 0$ two equal real, $D < 0$ no real roots.

Step 1. Read the coefficients of $2x^2 - \sqrt{5}\,x + 1 = 0$: $a = 2,\ b = -\sqrt{5},\ c = 1$.

Step 2. Compute the discriminant:

$$D = b^2 - 4ac = (-\sqrt{5})^2 - 4(2)(1).$$

$$D = 5 - 8.$$

$$D = -3.$$

Step 3. Since $D = -3 < 0$, the equation has no real roots.

Final Answer: $D = -3 < 0$, so the equation has no real roots; option **(C)**.

☞ **Discriminant sign is the whole story**

You never need the roots themselves to classify them. The single number $b^2 - 4ac$ and its sign answer every "nature of roots" question.

EXPERT'S SOLUTION : *Vikram Singh, M.Sc Mathematics, Jadavpur University*

One number settles the nature. The sign of the discriminant alone classifies the roots, so compute it and read the sign.

- **Compute D:** here $b^2 = (\sqrt{5})^2 = 5$ and $4ac = 4 \cdot 2 \cdot 1 = 8$, so $D = 5 - 8 = -3$.
- **Read the sign:** a negative discriminant means the square-root step in the formula asks for the root of a negative number, which has no real value, so there are no real roots.
- **Rule out (D):** a quadratic can have at most two roots, so "more than two" is impossible outright.

Final Answer: Option **(C)**.

Q 4.9 **Which of the following equations has two distinct real roots?**

(A) $2x^2 - 3\sqrt{2}\,x + \dfrac{9}{4} = 0$ **(B)** $x^2 + x - 5 = 0$

(C) $x^2 + 3x + 2\sqrt{2} = 0$ **(D)** $5x^2 - 3x + 1 = 0$

SOLUTION

Correct option: (B) $x^2 + x - 5 = 0$.

Concept used. Two **distinct real roots** occur when the discriminant $D = b^2 - 4ac$ is strictly greater than 0. Test each option.

Step 1. Option (A): $a = 2,\ b = -3\sqrt{2},\ c = \dfrac{9}{4}$.

$D = (3\sqrt{2})^2 - 4(2)\left(\dfrac{9}{4}\right) = 18 - 18 = 0$. Equal roots, not distinct.

Step 2. Option (B): $a = 1,\ b = 1,\ c = -5$.

$D = 1^2 - 4(1)(-5) = 1 + 20 = 21 > 0$. Two distinct real roots.

Step 3. Option (C): $a = 1,\ b = 3,\ c = 2\sqrt{2}$.

$D = 3^2 - 4(1)(2\sqrt{2}) = 9 - 8\sqrt{2} \approx 9 - 11.31 < 0$. No real roots.

Step 4. Option (D): $a = 5,\ b = -3,\ c = 1$.

$D = (-3)^2 - 4(5)(1) = 9 - 20 = -11 < 0$. No real roots.

Final Answer: Only option **(B)** has $D > 0$, so it alone has two distinct real roots.

☞ Estimate the surds

For (C), $8\sqrt{2} \approx 11.3$, which is bigger than 9, so $D < 0$. A quick decimal estimate of any surd term tells you the sign of D fast.

EXPERT'S SOLUTION *: Divya Menon, M.Sc Mathematics, University of Hyderabad*

Score each discriminant. "Two distinct real roots" is the $D > 0$ case, so grade every option by its discriminant.

- **The four scores:** running through the options gives $D = 0,\ 21,\ 9 - 8\sqrt{2},\ -11$.
- **Pick the winner:** only the second is positive, and it is the cleanest, $1 + 20 = 21$.
- **Why the rest fail:** the first is a perfect-square case ($18 - 18 = 0$), and the last two go negative once the surd or constant is large enough, so $x^2 + x - 5 = 0$ is the unique choice.

Final Answer: Option **(B)**.

Q 4.10 **Which of the following equations has no real roots?**

(A) $x^2 - 4x + 3\sqrt{2} = 0$ **(B)** $x^2 + 4x - 3\sqrt{2} = 0$

(C) $x^2 - 4x - 3\sqrt{2} = 0$ **(D)** $3x^2 + 4\sqrt{3}\,x + 4 = 0$

SOLUTION

Correct option: (A) $x^2 - 4x + 3\sqrt{2} = 0$.

Concept used. No **real roots** means the discriminant $D = b^2 - 4ac$ is strictly less than 0. Compute D for each option.

Step 1. Option (A): $a = 1,\ b = -4,\ c = 3\sqrt{2}$.

$D = (-4)^2 - 4(1)(3\sqrt{2}) = 16 - 12\sqrt{2} \approx 16 - 16.97 < 0$. No real roots.

Step 2. Option (B): $D = 4^2 - 4(1)(-3\sqrt{2}) = 16 + 12\sqrt{2} > 0$. Real roots.

Step 3. Option (C): $D = (-4)^2 - 4(1)(-3\sqrt{2}) = 16 + 12\sqrt{2} > 0$. Real roots.

Step 4. Option (D): $a = 3,\ b = 4\sqrt{3},\ c = 4$.

$D = (4\sqrt{3})^2 - 4(3)(4) = 48 - 48 = 0$. Equal real roots, not "no real roots".

Final Answer: Only option **(A)** has $D < 0$ ($16 - 12\sqrt{2} \approx -0.97$), so it has no real roots.

✗ Watch the sign of c

Options (B) and (C) carry $-3\sqrt{2}$, so $-4ac$ becomes positive and D grows. Only the $+3\sqrt{2}$ version (A) can push D below zero.

EXPERT'S SOLUTION *: Arjun Pillai, M.Sc Mathematics, IIT Delhi*

Compare 16 **with** $12\sqrt{2}$**.** Three options share $b^2 = 16$, so the sign of D rides entirely on $4ac$.

- **The negative case:** with $c = +3\sqrt{2}$ we get $16 - 12\sqrt{2} \approx 16 - 16.97 < 0$, the only negative case, (A).
- **The positive ones:** the negative c versions add to 16 and stay positive.
- **The borderline:** (D) is a perfect square, $48 - 48 = 0$, so (A) alone is the equation with no real roots.

Final Answer: Option **(A)**.

Q 4.11 $(x^2 + 1)^2 - x^2 = 0$ **has**

(A) four real roots **(B) two real roots**

(C) no real roots **(D) one real root**

SOLUTION

Correct option: (C) no real roots.

Concept used. Expand the expression to a polynomial, then check whether any real number can make it zero. A sum of a perfect square and a positive constant can never be zero for real x.

Step 1. Expand $(x^2+1)^2$:

$$(x^2+1)^2 = x^4 + 2x^2 + 1.$$

Step 2. Subtract x^2:

$$(x^2+1)^2 - x^2 = x^4 + 2x^2 + 1 - x^2 = x^4 + x^2 + 1.$$

Step 3. For any real x, both $x^4 \geq 0$ and $x^2 \geq 0$, so

$$x^4 + x^2 + 1 \geq 0 + 0 + 1 = 1 > 0.$$

Step 4. The left side is always at least 1, never 0, so there is no real root.

> **Final Answer:** $x^4 + x^2 + 1 \geq 1 > 0$ for all real x, so there are no real roots; option **(C)**.

☞ Even powers are never negative

When a polynomial uses only even powers with positive coefficients plus a positive constant, it stays positive for every real x, so it can have no real root.

EXPERT'S SOLUTION *: Nisha Agarwal, M.Sc Mathematics, BHU Varanasi*

Reduce to a positive sum. Expanding turns the equation into $x^4 + x^2 + 1 = 0$, and then a sign argument finishes it.

- **Every term helps:** both x^4 and x^2 are squares, so they are never negative, and adding 1 keeps the total at least 1.
- **No zero possible:** a quantity that is always ≥ 1 cannot equal 0, so no real x solves it.
- **Where the roots live:** they exist only in the complex world, which is outside the Class 10 real-number setting.

> **Final Answer:** Option **(C)**.

Related Collegedunia resources: pair these NCERT Exemplar Class 10 Maths solutions for Quadratic Equations with the chapter Notes, the NCERT Solutions, and the all-formulas sheet to revise the whole unit before your board exam.

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Chapter 4: Quadratic Equations

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Quick Formula Sheet

Standard form:

$ax^2 + bx + c = 0, \quad a \neq 0$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant:

$D = b^2 - 4ac$

Nature of roots:

$D > 0$: two distinct real

$D = 0$: two equal real

$D < 0$: no real roots

Sum and product of roots:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Also see for this chapter on Collegedunia: NCERT Solutions, Revision Notes, Formula Sheet and the Exemplar Book PDF for Class 10 Maths Chapter 4, Quadratic Equations.

II. True or False with Reasoning (Exercise 4.2)

Q 4.1 **State whether the quadratic equation $x^2 - 3x + 4 = 0$ has two distinct real roots. Justify your answer.**

SOLUTION

Verdict: No. The equation $x^2 - 3x + 4 = 0$ does not have two distinct real roots.
Concept used. Two distinct real roots need **discriminant** $D = b^2 - 4ac > 0$. Compute D and read its sign.

Step 1. Read the coefficients: $a = 1,\ b = -3,\ c = 4$.

Step 2. Compute the discriminant:

$$D = b^2 - 4ac = (-3)^2 - 4(1)(4).$$

$$D = 9 - 16 = -7.$$

Step 3. Since $D = -7 < 0$, the equation has no real roots, so it certainly does not have two distinct real roots.

Final Answer: No, because $D = -7 < 0$, so there are no real roots at all.

☞ **Distinct needs strictly positive D**

"Two distinct real roots" is the $D > 0$ case only. $D = 0$ gives one repeated root and $D < 0$ gives none, so always check the strict sign.

EXPERT'S SOLUTION *: Siddharth Rao, M.Sc Applied Mathematics, IIT Roorkee*

One discriminant, one verdict. A single number decides whether distinct real roots can exist here.

- **Compute:** for $x^2 - 3x + 4$ the numbers are $a = 1, b = -3, c = 4$, giving $D = 9 - 16 = -7$.
- **Conclude:** a negative discriminant rules out real roots, so the equation has neither one nor two real roots.
- **State it plainly:** the honest answer is No, and the reason is the single negative value -7.

Final Answer: No; $D = -7 < 0$.

Q 4.2 **State whether the quadratic equation $2x^2 + x - 1 = 0$ has two distinct real roots. Justify your answer.**

SOLUTION

Verdict: Yes. The equation $2x^2 + x - 1 = 0$ has two distinct real roots.
Concept used. Apply the discriminant test $D = b^2 - 4ac$; a strictly positive value means two distinct real roots.

Step 1. Read the coefficients: $a = 2,\ b = 1,\ c = -1$.

Step 2. Compute the discriminant:

$$D = b^2 - 4ac = 1^2 - 4(2)(-1).$$

$$D = 1 + 8 = 9.$$

Step 3. Since $D = 9 > 0$, there are two distinct real roots.

> **Final Answer:** Yes, because $D = 9 > 0$.

☞ **Sign of $-4ac$**

When a and c have opposite signs, $-4ac$ is positive, so D is positive automatically. Here $a = 2, c = -1$ already forces $D > 0$.

EXPERT'S SOLUTION *: Kavya Desai, M.Sc Mathematics, Savitribai Phule Pune University*

Opposite signs guarantee real roots. When a and c pull in opposite directions, the discriminant cannot be negative.

- **Sign check:** with $a = 2$ positive and $c = -1$ negative, the term $-4ac = 8$ is positive, and adding $b^2 = 1$ gives $D = 9 > 0$.
- **Read the verdict:** that alone tells you two different real roots exist.
- **Show them:** in fact $2x^2 + x - 1 = (2x - 1)(x + 1)$, so the roots are $\frac{1}{2}$ and -1, two distinct values.

> **Final Answer:** Yes; $D = 9 > 0$.

Q 4.3 **State whether the quadratic equation $2x^2 - 6x + \frac{9}{2} = 0$ has two distinct real roots. Justify your answer.**

SOLUTION

Verdict: No. The equation $2x^2 - 6x + \frac{9}{2} = 0$ does not have two distinct real roots; it has two equal roots.

Concept used. Compute $D = b^2 - 4ac$. A value of 0 means the two roots are equal, not distinct.

Step 1. Read the coefficients: $a = 2,\ b = -6,\ c = \dfrac{9}{2}$.

Step 2. Compute the discriminant:

$$D = b^2 - 4ac = (-6)^2 - 4(2)\left(\frac{9}{2}\right).$$

$$D = 36 - 36 = 0.$$

Step 3. Since $D = 0$, the roots are real but equal, so they are not two distinct roots.

> **Final Answer:** No, because $D = 0$, so the two roots are equal, not distinct.

✗ Equal is not distinct

A zero discriminant still gives real roots, but they coincide. "Two distinct real roots" fails when $D = 0$, so the answer here is No.

EXPERT'S SOLUTION *: Aditya Bhat, M.Sc Mathematics, NIT Trichy*

Spot the perfect square. A zero discriminant is the borderline between distinct roots and none, and it shows up here.

- **Compute:** here $b^2 = 36$ and $4ac = 4 \cdot 2 \cdot \frac{9}{2} = 36$, so $D = 0$.
- **Recognise the form:** the quadratic is a perfect square, $2x^2 - 6x + \frac{9}{2} = \frac{1}{2}(2x - 3)^2$, with the single repeated root $x = \frac{3}{2}$.
- **Why not distinct:** because the two roots are the same number, they are not distinct.

> **Final Answer:** No; $D = 0$ gives equal roots.

Q 4.4 State whether the quadratic equation $3x^2 - 4x + 1 = 0$ has two distinct real roots. Justify your answer.

SOLUTION

Verdict: Yes. The equation $3x^2 - 4x + 1 = 0$ has two distinct real roots.
Concept used. Use the discriminant test $D = b^2 - 4ac$; a positive value gives two distinct real roots.

Step 1. Read the coefficients: $a = 3,\ b = -4,\ c = 1$.

Step 2. Compute the discriminant:

$$D = b^2 - 4ac = (-4)^2 - 4(3)(1).$$

$$D = 16 - 12 = 4.$$

Step 3. Since $D = 4 > 0$, there are two distinct real roots.

Final Answer: Yes, because $D = 4 > 0$.

☞ **A positive perfect square D**

Here $D = 4$ is a perfect square, which also signals that the roots are rational. The equation factors as $(3x - 1)(x - 1) = 0$, giving $x = \frac{1}{3}, 1$.

EXPERT'S SOLUTION *: Ishita Roy, M.Sc Mathematics, Calcutta University*

Positive D, distinct roots. A positive discriminant settles the count, and a perfect-square one even names the roots.

- **Compute:** the discriminant works out to $16 - 12 = 4$, which is positive, so two different real roots exist.
- **Nice rationals:** because 4 is a perfect square, the roots are clean fractions.
- **Show them:** factoring $3x^2 - 4x + 1 = (3x - 1)(x - 1)$ gives $x = \frac{1}{3}$ and $x = 1$, plainly two distinct numbers.

Final Answer: Yes; $D = 4 > 0$.

Q 4.5 **State whether the quadratic equation $(x + 4)^2 - 8x = 0$ has two distinct real roots. Justify your answer.**

SOLUTION

Verdict: No. The equation $(x + 4)^2 - 8x = 0$ has no real roots.
Concept used. First expand to standard form $ax^2 + bx + c = 0$, then apply the discriminant test $D = b^2 - 4ac$.

Step 1. Expand and simplify:

$$(x + 4)^2 - 8x = x^2 + 8x + 16 - 8x = x^2 + 16.$$

Step 2. So the equation is $x^2 + 16 = 0$, with $a = 1,\ b = 0,\ c = 16$.

Step 3. Compute the discriminant:

$$D = b^2 - 4ac = 0^2 - 4(1)(16).$$

$D = -64$.

Step 4. Since $D = -64 < 0$, there are no real roots, so no distinct real roots either.

Final Answer: No, because the equation reduces to $x^2 + 16 = 0$ with $D = -64 < 0$.

☞ **Simplify before testing**

Always bring the equation to $ax^2 + bx + c = 0$ first. The middle terms here cancel, leaving a clean $x^2 + 16 = 0$ that is obviously root-free over the reals.

EXPERT'S SOLUTION *: Nikhil Pandey, M.Sc Applied Mathematics, IIT Indore*

The $8x$ terms cancel. Simplifying first turns this into an obvious no-real-root case.

- **Simplify:** expanding $(x + 4)^2$ gives $x^2 + 8x + 16$, and subtracting $8x$ leaves $x^2 + 16$.
- **Try to solve:** setting this to zero needs $x^2 = -16$, impossible for real x.
- **Discriminant agrees:** $D = 0 - 64 = -64 < 0$, so the equation has no real roots, let alone two distinct ones.

Final Answer: No; $D = -64 < 0$.

Q 4.6 **State whether the quadratic equation $(x - \sqrt{2})^2 - 2(x + 1) = 0$ has two distinct real roots. Justify your answer.**

SOLUTION

Verdict: Yes. The equation $(x - \sqrt{2})^2 - 2(x + 1) = 0$ has two distinct real roots.
Concept used. Expand to standard form, then apply the discriminant test $D = b^2 - 4ac$.

Step 1. Expand the square and the bracket:

$$(x - \sqrt{2})^2 - 2(x + 1) = x^2 - 2\sqrt{2}\,x + 2 - 2x - 2.$$

$$= x^2 - (2\sqrt{2} + 2)x.$$

Step 2. So $a = 1,\ b = -(2\sqrt{2} + 2),\ c = 0$.

Step 3. Compute the discriminant:

$$D = b^2 - 4ac = (2\sqrt{2} + 2)^2 - 0.$$

$$D = (2\sqrt{2} + 2)^2 > 0.$$

Step 4. Since $D > 0$, the equation has two distinct real roots.

Final Answer: Yes, because $D = (2\sqrt{2}+2)^2 > 0$.

☞ Constant term gone means 0 is a root

With $c = 0$ the equation is $x\big(x - (2\sqrt{2}+2)\big) = 0$, so one root is $x = 0$ and the other is $2\sqrt{2}+2$, clearly two distinct values.

EXPERT'S SOLUTION *: Aishwarya Rao, M.Sc Mathematics, Christ University Bengaluru*

No constant term is a giveaway. When the constant term vanishes, the roots fall out of a simple factorisation.

- **Expand and factor:** after expansion the equation is $x^2 - (2\sqrt{2}+2)x = 0$, which factors as $x\big(x - (2\sqrt{2}+2)\big) = 0$.
- **Read the roots:** that immediately shows two distinct roots, 0 and $2\sqrt{2}+2$.
- **Discriminant agrees:** $(2\sqrt{2}+2)^2$ is a positive square, matching the factored form.

Final Answer: Yes; $D = (2\sqrt{2}+2)^2 > 0$.

Q 4.7 **State whether the following quadratic equation has two distinct real roots. Justify your answer.**

$\sqrt{2}\,x^2 - (3/\sqrt{2})\,x + (1/\sqrt{2}) = 0$.

SOLUTION

Verdict: Yes. The equation $\sqrt{2}\,x^2 - (3/\sqrt{2})\,x + (1/\sqrt{2}) = 0$ has two distinct real roots.
Concept used. Clearing the surd by multiplying through by a constant does not change the roots; then apply $D = b^2 - 4ac$.

Step 1. Multiply the whole equation by $\sqrt{2}$ to clear denominators:

$$\sqrt{2}\cdot\sqrt{2}\,x^2 - \sqrt{2}\cdot\frac{3}{\sqrt{2}}x + \sqrt{2}\cdot\frac{1}{\sqrt{2}} = 0.$$

$$2x^2 - 3x + 1 = 0.$$

Step 2. Read the coefficients: $a = 2,\ b = -3,\ c = 1$.

Step 3. Compute the discriminant:

$$D = b^2 - 4ac = (-3)^2 - 4(2)(1).$$

$$D = 9 - 8 = 1.$$

Step 4. Since $D = 1 > 0$, the equation has two distinct real roots.

> **Final Answer:** Yes, because multiplying by $\sqrt{2}$ gives $2x^2-3x+1=0$ with $D = 1 > 0$.

☞ **Clear the surd first**

Multiplying an equation by a non-zero constant keeps the same roots but removes awkward $\frac{1}{\sqrt{2}}$ terms, making the discriminant easy.

EXPERT'S SOLUTION *: Pooja Chatterjee, M.Sc Mathematics, Presidency University Kolkata*

Scale away the radicals. Clearing the surd first makes the discriminant test painless, and the roots stay the same.

- **Clear the surd:** multiplying through by $\sqrt{2}$ turns the equation into the tidy $2x^2 - 3x + 1 = 0$, with the same solution set.
- **Test D:** its discriminant is $9 - 8 = 1 > 0$, so there are two distinct real roots.
- **Name them:** factoring $2x^2 - 3x + 1 = (2x - 1)(x - 1)$ shows them as $\frac{1}{2}$ and 1.

> **Final Answer:** Yes; equivalent to $2x^2 - 3x + 1 = 0$, $D = 1 > 0$.

Q 4.8 **State whether the quadratic equation $x(1 - x) - 2 = 0$ has two distinct real roots. Justify your answer.**

SOLUTION

Verdict: No. The equation $x(1 - x) - 2 = 0$ has no real roots.
Concept used. Expand to standard form, then check the sign of $D = b^2 - 4ac$.

Step 1. Expand the bracket:

$$x(1 - x) - 2 = x - x^2 - 2.$$

Step 2. Write in standard form (multiply by -1 for clarity):

$$-x^2 + x - 2 = 0, \text{ i.e. } a = -1,\ b = 1,\ c = -2.$$

Step 3. Compute the discriminant:

$$D = b^2 - 4ac = 1^2 - 4(-1)(-2).$$

$$D = 1 - 8 = -7.$$

Step 4. Since $D = -7 < 0$, the equation has no real roots.

Final Answer: No, because $D = -7 < 0$, so there are no real roots.

✗ Mind the double negative

In $-4ac$ with $a = -1, c = -2$, the product $ac = 2$ is positive, so $-4ac = -8$. Forgetting one minus sign would wrongly flip D to positive.

EXPERT'S SOLUTION *: Akash Jain, M.Sc Mathematics, Loyola College Chennai*

Expand, then read D. Bring the equation to standard form before judging the roots.

- **Expand:** the product $x(1 - x)$ is $x - x^2$, so the equation is $-x^2 + x - 2 = 0$.
- **Compute D:** with $a = -1, b = 1, c = -2$ the discriminant is $1 - 4(-1)(-2) = 1 - 8 = -7$.
- **Conclude:** the negative value means no real solution exists, so the equation has neither one nor two real roots.

Final Answer: No; $D = -7 < 0$.

Q 4.9 **State whether the quadratic equation $(x - 1)(x + 2) + 2 = 0$ has two distinct real roots. Justify your answer.**

SOLUTION

Verdict: Yes. The equation $(x - 1)(x + 2) + 2 = 0$ has two distinct real roots.
Concept used. Expand to standard form, then apply the discriminant test.

Step 1. Expand the product:

$$(x - 1)(x + 2) = x^2 + x - 2.$$

Step 2. Add 2:

$$(x - 1)(x + 2) + 2 = x^2 + x - 2 + 2 = x^2 + x.$$

Step 3. So $a = 1,\ b = 1,\ c = 0$. Compute the discriminant:

$$D = b^2 - 4ac = 1^2 - 4(1)(0) = 1.$$

Step 4. Since $D = 1 > 0$, there are two distinct real roots.

Final Answer: Yes, because the equation reduces to $x^2 + x = 0$ with $D = 1 > 0$.

☞ **The $+2$ cancels the -2**

The constant $+2$ exactly removes the -2 from the product, leaving $x^2 + x = x(x+1) = 0$. The roots are 0 and -1, two distinct values.

EXPERT'S SOLUTION *: Tanvi Shah, M.Sc Mathematics, St. Stephen's College Delhi*

A neat cancellation. A lucky cancellation drops the constant and leaves a factorable quadratic.

- **Simplify:** multiplying out gives $x^2 + x - 2$, and the trailing $+2$ wipes the constant to leave $x^2 + x = 0$.
- **Factor:** that becomes $x(x+1) = 0$, so the roots are 0 and -1.
- **Discriminant agrees:** $1 - 0 = 1 > 0$ confirms the two roots are real and distinct.

Final Answer: Yes; $D = 1 > 0$.

Q 4.10 **State whether the quadratic equation $(x+1)(x-2) + x = 0$ has two distinct real roots. Justify your answer.**

SOLUTION

Verdict: Yes. The equation $(x+1)(x-2) + x = 0$ has two distinct real roots.
Concept used. Expand to standard form, then test the sign of $D = b^2 - 4ac$.

Step 1. Expand the product:

$$(x+1)(x-2) = x^2 - x - 2.$$

Step 2. Add x:

$$(x+1)(x-2) + x = x^2 - x - 2 + x = x^2 - 2.$$

Step 3. So $a = 1,\ b = 0,\ c = -2$. Compute the discriminant:

$$D = b^2 - 4ac = 0^2 - 4(1)(-2) = 8.$$

Step 4. Since $D = 8 > 0$, the equation has two distinct real roots.

Final Answer: Yes, because the equation reduces to $x^2 - 2 = 0$ with $D = 8 > 0$.

☞ **Pure square equals a positive number**

$x^2 - 2 = 0$ means $x^2 = 2$, so $x = \pm\sqrt{2}$. Whenever x^2 equals a positive number there are two real roots, one positive and one negative.

EXPERT'S SOLUTION *: Manish Gupta, M.Sc Mathematics, IIT Guwahati*

The $-x$ and $+x$ cancel. The added x kills the linear term and leaves a pure square equation.

- **Simplify:** the product is $x^2 - x - 2$, and adding x removes the linear term, leaving $x^2 - 2 = 0$.
- **Solve:** this gives $x = \pm\sqrt{2}$, two distinct real roots.
- **Discriminant agrees:** the positive value $0 - 4(1)(-2) = 8$ matches the two real roots.

> **Final Answer:** Yes; $D = 8 > 0$.

Q 4.11 **Is the following statement true or false? Justify your answer. "Every quadratic equation has exactly one root."**

SOLUTION

Verdict: False.

Concept used. A quadratic equation has degree 2, so it can have up to two roots; the exact count depends on the **discriminant**. It is not fixed at one.

Step 1. A quadratic can have two distinct real roots when $D > 0$.

Step 2. Counter-example: $x^2 - 1 = 0$ gives $x = 1$ and $x = -1$, two roots.

Step 3. So a quadratic does not always have exactly one root.

> **Final Answer:** False, for example $x^2 - 1 = 0$ has two roots, 1 and -1.

☞ Count by the discriminant

A quadratic has two roots if $D > 0$, one repeated root if $D = 0$, and no real root if $D < 0$. "Exactly one" is only the $D = 0$ case.

EXPERT'S SOLUTION *: Rahul Saxena, M.Sc Mathematics, University of Mumbai*

One counter-example breaks the claim. A universal claim falls to a single exception, so look for one.

- **What to disprove:** the statement says *every* quadratic has exactly one root, so one quadratic with two roots is enough.
- **The exception:** take $x^2 - 1 = 0$; it solves to $x = \pm 1$, two roots.
- **Conclusion:** hence the blanket claim is false, and the root count varies with the

discriminant.

Final Answer: False; $x^2 - 1 = 0$ has two roots.

Q 4.12 **Is the following statement true or false? Justify your answer. "Every quadratic equation has at least one real root."**

SOLUTION

Verdict: False.

Concept used. When the **discriminant** $D = b^2 - 4ac$ is negative, a quadratic has no real root at all, so "at least one real root" fails.

Step 1. A quadratic with $D < 0$ has no real roots.

Step 2. Counter-example: $x^2 + 1 = 0$ has $D = 0 - 4(1)(1) = -4 < 0$.

Step 3. So $x^2 + 1 = 0$ has no real root, breaking the claim.

Final Answer: False, for example $x^2 + 1 = 0$ has $D = -4 < 0$ and no real root.

✗ Real is the key word

The roots of $x^2 + 1 = 0$ exist only as complex numbers. In Class 10 we count *real* roots, and there are none here.

EXPERT'S SOLUTION *: Farhan Qureshi, M.Sc Mathematics, Aligarh Muslim University*

A negative discriminant ends the debate. One quadratic with no real root is all it takes to break the claim.

- **The exception:** for $x^2 + 1 = 0$ the discriminant is -4, so no real number squares to -1.
- **Verdict:** that is a clean counter-example to "at least one real root", making the statement false.
- **General point:** whether a quadratic has any real root depends entirely on the sign of D.

Final Answer: False; $x^2 + 1 = 0$ has no real root.

Q 4.13 **Is the following statement true or false? Justify your answer. "Every quadratic equation has at least two roots."**

SOLUTION

Verdict: False.

Concept used. A quadratic with **discriminant** $D = 0$ has only one (repeated) root, so it does not have two distinct roots.

Step 1. When $D = 0$ the quadratic is a perfect square with one repeated root.

Step 2. Counter-example: $x^2 + 1 = 0$ has no real roots, and $x^2 = 0$ has the single root $x = 0$.

Step 3. So a quadratic need not have at least two roots.

> **Final Answer:** False, for example $x^2 + 1 = 0$ has no real roots, so it does not have two.

☞ **Three possible root counts**

A quadratic has 0, 1 or 2 real roots depending on whether D is negative, zero or positive. "At least two" is wrong whenever $D \leq 0$.

EXPERT'S SOLUTION *: Lakshmi Sundaram, M.Sc Mathematics, University of Madras*

Show a case with fewer than two. The claim demands two or more roots every time, so any quadratic with fewer breaks it.

- **No real roots:** $x^2 + 1 = 0$ has none over the reals.
- **One repeated root:** $x^2 = 0$ has just the repeated root 0.
- **Verdict:** either example is enough to mark the statement false, since the number of real roots is capped by the discriminant.

> **Final Answer:** False; $x^2 + 1 = 0$ has no real roots.

Q 4.14 **Is the following statement true or false? Justify your answer. "Every quadratic equation has at most two roots."**

SOLUTION

Verdict: True.

Concept used. A polynomial of degree n has at most n roots. A quadratic has degree 2, so it has at most two roots.

Step 1. A quadratic polynomial $ax^2 + bx + c$ has degree 2.

Step 2. A degree-2 polynomial can be factored into at most two linear factors, so it can equal zero for at most two values of x.

Step 3. Hence a quadratic equation has at most two roots (it may have 0, 1 or 2 real roots).

Final Answer: True, because a degree-2 polynomial has at most two zeroes.

☞ **Degree caps the root count**

The number of roots of any polynomial equation never exceeds its degree. For a quadratic that ceiling is two.

EXPERT'S SOLUTION *: Gaurav Malhotra, M.Sc Applied Mathematics, IIT Bhubaneswar*

Degree is the hard ceiling. The degree of a polynomial caps how many roots it can ever have.

- **The bound:** a quadratic is a degree-2 polynomial, and the factor theorem allows at most two distinct linear factors, so at most two roots.
- **Actual count:** it is 0, 1 or 2 according to the discriminant, but it can never exceed two.
- **Verdict:** so "at most two" is always correct, and the statement is true.

Final Answer: True; degree 2 allows at most two roots.

Q 4.15 **Is the following statement true or false? Justify your answer. "If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots."**

SOLUTION

Verdict: True.

Concept used. If a and c have opposite signs, the product $ac < 0$, so $-4ac > 0$, which forces the **discriminant** $D = b^2 - 4ac$ to be positive.

Step 1. Opposite signs of a and c mean $ac < 0$.

Step 2. Then $-4ac > 0$ (a positive quantity).

Step 3. Now $D = b^2 - 4ac = b^2 + (\text{positive})$. Since $b^2 \geq 0$, the sum is positive: $D > 0$.

Step 4. A positive discriminant means the equation has real roots (two distinct ones).

Final Answer: True, because $ac < 0$ gives $D = b^2 - 4ac > 0$, so real roots exist.

☞ **Opposite signs force $D > 0$**

Whenever a and c have opposite signs, $-4ac$ adds a positive amount to b^2, so D is always positive and the roots are real and distinct.

EXPERT'S SOLUTION *: Ritu Bansal, M.Sc Mathematics, Panjab University Chandigarh*

Sign analysis, no numbers needed. You can settle this purely by tracking signs, without any specific coefficients.

- **Opposite signs:** if $a > 0, c < 0$ or $a < 0, c > 0$ then $ac < 0$, so $-4ac$ is positive.
- **Force $D > 0$:** adding that positive amount to the non-negative b^2 keeps D strictly positive.
- **Verdict:** a positive discriminant guarantees two real roots, so the statement holds for every such quadratic and is true.

Final Answer: True; $ac < 0 \Rightarrow D > 0$.

Q 4.16 **Is the following statement true or false? Justify your answer. "If the coefficient of x^2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots."**

SOLUTION

Verdict: True.

Concept used. With $b = 0$, the discriminant becomes $D = -4ac$. If a and c have the same sign, $ac > 0$, so $D = -4ac < 0$.

Step 1. The x-coefficient is zero, so $b = 0$.

Step 2. Then $D = b^2 - 4ac = 0 - 4ac = -4ac$.

Step 3. Same sign of a and c means $ac > 0$, so $-4ac < 0$.

Step 4. Therefore $D < 0$, and the equation has no real roots.

> **Final Answer:** True, because $b = 0$ and $ac > 0$ give $D = -4ac < 0$, so no real roots.

☞ **The $b = 0$ case**

When the middle term is missing, the equation is $ax^2 + c = 0$, i.e. $x^2 = -\frac{c}{a}$. If a and c share a sign, $-\frac{c}{a} < 0$ and no real square root exists.

EXPERT'S SOLUTION *: Devendra Yadav, M.Sc Mathematics, University of Rajasthan*

Read it through $x^2 = -c/a$. Isolating x^2 exposes why no real root can survive here.

- **Isolate the square:** with $b = 0$ the equation is $ax^2 + c = 0$, so $x^2 = -\frac{c}{a}$.
- **Same signs hurt:** when a and c match in sign, $\frac{c}{a} > 0$ and the right side $-\frac{c}{a}$ is negative.
- **No real root:** a square cannot be negative for real x, exactly as the discriminant $-4ac < 0$ predicts, so the statement is true.

> **Final Answer:** True; $ax^2 + c = 0$ needs $x^2 < 0$, impossible.

Q 4.17 A quadratic equation with integral coefficients has integral roots. Justify your answer.

SOLUTION

Verdict: False. A quadratic with integer coefficients need not have integer roots.
Concept used. Integer coefficients only mean a, b, c are integers. The roots, found from the quadratic formula, can still be fractions or surds.

Step 1. Take the equation $x^2 - 3x + 1 = 0$, which has integer coefficients $1, -3, 1$.

Step 2. Its discriminant is $D = (-3)^2 - 4(1)(1) = 9 - 4 = 5$.

Step 3. The roots are $x = \dfrac{3 \pm \sqrt{5}}{2}$, which are irrational, not integers.

Step 4. So integer coefficients do not force integer roots.

> **Final Answer:** False, for example $x^2 - 3x + 1 = 0$ has the irrational roots $\dfrac{3 \pm \sqrt{5}}{2}$.

✗ Coefficients are not roots

Whole-number coefficients say nothing about the roots. The roots come from $\frac{-b\pm\sqrt{D}}{2a}$, which can easily be a surd.

EXPERT'S SOLUTION *: Sara Thomas, M.Sc Mathematics, Cochin University of Science and Technology*

One surd-root example is enough. The claim is universal, so a single counter-example settles it.

- **The exception:** the integer-coefficient equation $x^2 - 3x + 1 = 0$ has discriminant 5, a non-square, so its roots $\frac{3\pm\sqrt{5}}{2}$ are irrational.
- **Why integer roots fail:** they require the discriminant to be a perfect square and the formula to divide evenly, which does not happen here.
- **Verdict:** so the statement is false.

Final Answer: False; $x^2 - 3x + 1 = 0$ has roots $\dfrac{3 \pm \sqrt{5}}{2}$.

Q 4.18 Does there exist a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer.

SOLUTION

Verdict: Yes. Such a quadratic equation does exist.
Concept used. If the discriminant of a rational-coefficient quadratic is positive but not a perfect square, the roots $\dfrac{-b \pm \sqrt{D}}{2a}$ are irrational.

Step 1. Take $x^2 - 4x + 1 = 0$, whose coefficients $1, -4, 1$ are rational (in fact integers).

Step 2. Its discriminant is $D = (-4)^2 - 4(1)(1) = 16 - 4 = 12$.

Step 3. Since 12 is not a perfect square, the roots $x = \dfrac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$ are irrational.

Step 4. Both roots $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are irrational, so the answer is yes.

Final Answer: Yes, for example $x^2 - 4x + 1 = 0$ has the irrational roots $2 \pm \sqrt{3}$.

☞ Non-square discriminant gives surd roots

For rational coefficients, irrational roots appear in conjugate pairs $p \pm \sqrt{q}$. Just choose a

positive non-square discriminant.

EXPERT'S SOLUTION *: Abhishek Nair, M.Sc Applied Mathematics, NIT Calicut*

Pick a non-square discriminant. The trick is to design the discriminant so the formula keeps a surd.

- **Build the roots:** rational coefficients with a discriminant such as 12 produce roots $\frac{4\pm\sqrt{12}}{2} = 2 \pm \sqrt{3}$.
- **Both are irrational:** since $\sqrt{3}$ is irrational, both roots are irrational, and they form a conjugate pair, which is typical when the coefficients are rational.
- **Verdict:** so a suitable quadratic certainly exists.

Final Answer: Yes; $x^2 - 4x + 1 = 0$ has roots $2 \pm \sqrt{3}$.

Q 4.19 Does there exist a quadratic equation whose coefficients are all distinct irrationals but both the roots are rationals? Why?

SOLUTION

Verdict: Yes. Such a quadratic equation does exist.
Concept used. Multiplying a quadratic with rational roots by a common irrational factor keeps the same roots but turns the coefficients irrational.

Step 1. Start with a quadratic having rational roots, for example $(x-1)(x-2) = x^2 - 3x + 2 = 0$, with roots 1 and 2.

Step 2. Multiply the whole equation by an irrational number such as $\sqrt{2}$:

$$\sqrt{2}\,x^2 - 3\sqrt{2}\,x + 2\sqrt{2} = 0.$$

Step 3. The coefficients $\sqrt{2},\ -3\sqrt{2},\ 2\sqrt{2}$ are distinct irrationals, while multiplying by a non-zero constant does not change the roots.

Step 4. So the roots are still the rationals 1 and 2.

Final Answer: Yes, for example $\sqrt{2}\,x^2 - 3\sqrt{2}\,x + 2\sqrt{2} = 0$ has distinct irrational coefficients but rational roots $1, 2$.

☞ **Scaling preserves roots**

Multiplying both sides of an equation by a non-zero number leaves the solution set unchanged. Choose an irrational multiplier to make the coefficients irrational.

EXPERT'S SOLUTION *: Neha Kulshreshtha, M.Sc Mathematics, University of Lucknow*

Build it backwards from the roots. Start from the answer you want and engineer the coefficients to match.

- **Choose the roots:** begin with rational roots $1, 2$, giving $x^2 - 3x + 2 = 0$.
- **Scale by a surd:** multiplying by $\sqrt{2}$ turns the coefficients into $\sqrt{2}, -3\sqrt{2}, 2\sqrt{2}$, three distinct irrationals, yet the roots are untouched at 1 and 2.
- **Why it works:** a common irrational factor never alters where the equation is zero.

Final Answer: Yes; scale $x^2 - 3x + 2 = 0$ by $\sqrt{2}$.

Q 4.20 **Is 0.2 a root of the equation $x^2 - 0.4 = 0$? Justify.**

SOLUTION

Verdict: No. 0.2 is not a root of $x^2 - 0.4 = 0$.
Concept used. A number is a **root** only if substituting it makes the equation true. Test $x = 0.2$.

Step 1. Substitute $x = 0.2$:

$$x^2 - 0.4 = (0.2)^2 - 0.4.$$
$$= 0.04 - 0.4.$$
$$= -0.36.$$

Step 2. The left side is $-0.36 \neq 0$, so 0.2 does not satisfy the equation.

Step 3. For reference the actual roots are $x = \pm\sqrt{0.4} \approx \pm 0.632$.

Final Answer: No, since $(0.2)^2 - 0.4 = -0.36 \neq 0$.

✗ $(0.2)^2$ **is** 0.04**, not** 0.4

A common slip is to read $(0.2)^2$ as 0.4. Squaring 0.2 gives 0.04, which is why the value is far from a root.

EXPERT'S SOLUTION *: Varun Chauhan, M.Sc Mathematics, Delhi Technological University*

Square carefully, then check. Testing a single value just means substituting and watching the decimal arithmetic.

- **Substitute:** putting $x = 0.2$ gives $(0.2)^2 - 0.4 = 0.04 - 0.4 = -0.36$, which is not zero,

so 0.2 is not a root.

- **Where the roots are:** they satisfy $x^2 = 0.4$, i.e. $x = \pm\sqrt{0.4} \approx 0.632$, well away from 0.2.
- **Common slip:** the decimal-squaring step is where students usually trip.

Final Answer: No; the left side equals -0.36.

Q 4.21 **If $b = 0,\ c < 0$, is it true that the roots of $x^2 + bx + c = 0$ are numerically equal and opposite in sign? Justify.**

SOLUTION

Verdict: True.

Concept used. With $b = 0$ the equation becomes $x^2 + c = 0$, i.e. $x^2 = -c$. Since $c < 0$, the value $-c$ is positive, so $x = \pm\sqrt{-c}$.

Step 1. Put $b = 0$ into $x^2 + bx + c = 0$:

$x^2 + c = 0$, so $x^2 = -c$.

Step 2. Since $c < 0$, the right side $-c > 0$, so the square root is real:

$x = \pm\sqrt{-c}$.

Step 3. The two roots are $+\sqrt{-c}$ and $-\sqrt{-c}$. They have the same size (numerically equal) but opposite signs.

Final Answer: True, the roots are $\pm\sqrt{-c}$, numerically equal and opposite in sign.

☞ **Missing middle term means symmetric roots**

When $b = 0$, the sum of the roots $-\frac{b}{a} = 0$, so the roots add to zero. Two numbers adding to zero must be equal in size and opposite in sign.

EXPERT'S SOLUTION *: Anjali Pradhan, M.Sc Mathematics, Utkal University*

Zero middle term, zero root-sum. A missing x-term forces a symmetry on the roots that you can read off at once.

- **Sum is zero:** with $b = 0$ the sum of the roots is $-b/a = 0$, which forces the roots to be negatives of each other.
- **Solve directly:** $x^2 = -c$ with $c < 0$ gives $x = \pm\sqrt{-c}$, the pair $+\sqrt{-c}$ and $-\sqrt{-c}$.

- **Verdict:** they are equal in magnitude and opposite in sign, so the statement is true.

> **Final Answer:** True; the roots are $\pm\sqrt{-c}$.

Download the Full chapter resource: the NCERT Solutions for Class 10 Maths Chapter 4 Quadratic Equations to drill the regular exercises.
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NCERT Exemplar Solutions

Solved NCERT Exemplar Class 10 Maths Chapter 4, Quadratic Equations: every question, step by step (2026-27)

Chapter 4: Quadratic Equations

About this Chapter

Full worked solutions for every problem in Class 10 Maths Chapter 4 Quadratic Equations. After this set you can find roots by **factorisation** and the **quadratic formula**, and read the **discriminant** $b^2 - 4ac$ to tell distinct, equal and no real roots apart. MCQs, reasoning, root finding and word problems are each solved step by step with an expert view.

Topics covered: Standard form $ax^2 + bx + c = 0$ • Roots and zeroes • Factorisation method • Completing the square • Quadratic formula • Discriminant $b^2 - 4ac$ • Nature of roots • Word problems on numbers, speed, age and area

Quick Formula Sheet

Standard form:

$ax^2 + bx + c = 0, \quad a \neq 0$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant:

$D = b^2 - 4ac$

Nature of roots:

$D > 0$: two distinct real

$D = 0$: two equal real

$D < 0$: no real roots

Sum and product of roots:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Also see for this chapter on Collegedunia: NCERT Solutions, Revision Notes, Formula Sheet and the Exemplar Book PDF for Class 10 Maths Chapter 4, Quadratic Equations.

III. Short Answer Questions (Exercise 4.3)

Q 4.1 **Find the roots of the quadratic equation $2x^2 - 3x - 5 = 0$ by using the quadratic formula.**

SOLUTION

Concept used. The **quadratic formula** solves $ax^2+bx+c=0$ as $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, provided the discriminant $b^2-4ac\geq 0$.

Step 1. Read the coefficients: $a=2,\ b=-3,\ c=-5$.

Step 2. Compute the discriminant:

$$b^2-4ac=(-3)^2-4(2)(-5)$$
$$=9+40$$
$$=49.$$

Step 3. Apply the formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-3)\pm\sqrt{49}}{2(2)}$$
$$=\frac{3\pm 7}{4}.$$

Step 4. Split the two signs:

$$x=\frac{3+7}{4}=\frac{10}{4}=\frac{5}{2} \text{ or } x=\frac{3-7}{4}=\frac{-4}{4}=-1.$$

Final Answer: The roots are $x=\dfrac{5}{2}$ and $x=-1$.

☞ Substitute signs carefully

With $b=-3$, the formula uses $-b=+3$. Mishandling the sign of b is the top error here; write $-(-3)=3$ explicitly.

EXPERT'S SOLUTION *: Ramya Krishnan, M.Sc Mathematics, Bharathiar University*

Discriminant first, then plug in. Computing D up front tells you the roots will be clean before you substitute.

- **Find D:** the numbers $a=2, b=-3, c=-5$ give $b^2-4ac=9+40=49$, a perfect square, so the roots are rational.
- **Apply the formula:** substituting into $\frac{-b\pm\sqrt{D}}{2a}$ yields $\frac{3\pm 7}{4}$, which separates into $\frac{5}{2}$ and -1.
- **Check:** $2x^2-3x-5=(2x-5)(x+1)$, matching both roots.

Final Answer: $x = \dfrac{5}{2},\ -1.$

Q 4.2 **Find the roots of the quadratic equation $5x^2+13x+8=0$ by using the quadratic formula.**

SOLUTION

Concept used. Apply the **quadratic formula** $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ after computing the discriminant.

Step 1. Read the coefficients: $a = 5,\ b = 13,\ c = 8.$

Step 2. Compute the discriminant:

$$b^2 - 4ac = 13^2 - 4(5)(8)$$
$$= 169 - 160$$
$$= 9.$$

Step 3. Apply the formula:

$$x = \frac{-13 \pm \sqrt{9}}{2(5)}$$
$$= \frac{-13 \pm 3}{10}.$$

Step 4. Split the two signs:

$$x = \frac{-13+3}{10} = \frac{-10}{10} = -1 \text{ or } x = \frac{-13-3}{10} = \frac{-16}{10} = -\frac{8}{5}.$$

Final Answer: The roots are $x = -1$ and $x = -\dfrac{8}{5}$.

☞ **Both roots negative is fine**

A positive b and positive c with $a > 0$ push both roots negative. Their sum $-\frac{b}{a} = -\frac{13}{5}$ is negative, agreeing with -1 and $-\frac{8}{5}$.

EXPERT'S SOLUTION *: Imran Sheikh, M.Sc Mathematics, Osmania University*

Small discriminant, clean roots. A small perfect-square discriminant keeps the formula step short.

- **Find D:** with $a = 5, b = 13, c = 8$ the discriminant is $169 - 160 = 9$, so $\sqrt{9} = 3$.

- **Apply the formula:** it gives $\frac{-13\pm3}{10}$, splitting into -1 and $-\frac{8}{5}$.
- **Check:** the factorisation $5x^2+13x+8=(x+1)(5x+8)$ confirms the same two negative roots.

Final Answer: $x=-1,\ -\dfrac{8}{5}$.

Q 4.3 **Find the roots of the quadratic equation $-3x^2+5x+12=0$ by using the quadratic formula.**

SOLUTION

Concept used. Apply the quadratic formula; it works for any sign of a, so a negative leading coefficient is allowed.

Step 1. Read the coefficients: $a=-3,\ b=5,\ c=12$.

Step 2. Compute the discriminant:

$$b^2-4ac=5^2-4(-3)(12)$$
$$=25+144$$
$$=169.$$

Step 3. Apply the formula:

$$x=\frac{-5\pm\sqrt{169}}{2(-3)}$$
$$=\frac{-5\pm13}{-6}.$$

Step 4. Split the two signs:

$$x=\frac{-5+13}{-6}=\frac{8}{-6}=-\frac{4}{3} \text{ or } x=\frac{-5-13}{-6}=\frac{-18}{-6}=3.$$

Final Answer: The roots are $x=-\dfrac{4}{3}$ and $x=3$.

✗ Keep the negative denominator

The denominator is $2a=-6$, not 6. Dividing by the negative value flips the sign of each result, so track it through both branches.

EXPERT'S SOLUTION *: Deepika Rao, M.Sc Applied Mathematics, IIT Tirupati*

Flip the sign first if you like. A negative leading coefficient is easier to handle once you make it positive.

- **Tidy it:** multiplying through by -1 gives $3x^2 - 5x - 12 = 0$, which is neater.
- **Find D:** either way the discriminant is $25 + 144 = 169$, and $\sqrt{169} = 13$, so the roots come out as $-\frac{4}{3}$ and 3.
- **Check:** the factorisation $3x^2 - 5x - 12 = (3x + 4)(x - 3)$ shows them directly.

Final Answer: $x = -\dfrac{4}{3},\ 3.$

Q 4.4 **Find the roots of the quadratic equation $-x^2+7x-10 = 0$ by using the quadratic formula.**

SOLUTION

Concept used. Apply the **quadratic formula** after computing the discriminant; the negative leading coefficient is handled by $2a$.

Step 1. Read the coefficients: $a = -1,\ b = 7,\ c = -10.$

Step 2. Compute the discriminant:

$$b^2 - 4ac = 7^2 - 4(-1)(-10)$$

$$= 49 - 40$$

$$= 9.$$

Step 3. Apply the formula:

$$x = \frac{-7 \pm \sqrt{9}}{2(-1)}$$

$$= \frac{-7 \pm 3}{-2}.$$

Step 4. Split the two signs:

$$x = \frac{-7+3}{-2} = \frac{-4}{-2} = 2 \text{ or } x = \frac{-7-3}{-2} = \frac{-10}{-2} = 5.$$

Final Answer: The roots are $x = 2$ and $x = 5$.

☞ Two negatives in $-4ac$

Here $a = -1$ and $c = -10$ are both negative, so $-4ac = -40$, which *subtracts* from b^2. Track each sign so the discriminant stays correct.

EXPERT'S SOLUTION *: Sourav Ghosh, M.Sc Mathematics, Jadavpur University*

Make a positive to see it fast. A positive lead coefficient makes the factors jump out.

- **Flip and factor:** multiplying by -1 gives $x^2 - 7x + 10 = 0$, which factors as $(x-2)(x-5) = 0$, so the roots are 2 and 5.
- **Formula agrees:** discriminant $49 - 40 = 9$ and $\frac{-7 \pm 3}{-2}$ give the same pair.
- **Sanity check:** the sum $2 + 5 = 7$ matches $-b/a = 7$.

Final Answer: $x = 2,\ 5$.

Q 4.5 **Find the roots of the quadratic equation $x^2 + 2\sqrt{2}\,x - 6 = 0$ by using the quadratic formula.**

SOLUTION

Concept used. Apply the quadratic formula; surd coefficients are fine, and $\sqrt{b^2 - 4ac}$ may simplify to a surd.

Step 1. Read the coefficients: $a = 1,\ b = 2\sqrt{2},\ c = -6$.

Step 2. Compute the discriminant:

$$b^2 - 4ac = (2\sqrt{2})^2 - 4(1)(-6)$$

$$= 8 + 24$$

$$= 32.$$

Step 3. Simplify the square root: $\sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$.

Step 4. Apply the formula:

$$x = \frac{-2\sqrt{2} \pm 4\sqrt{2}}{2(1)}$$

$$= \frac{-2\sqrt{2} \pm 4\sqrt{2}}{2}.$$

Step 5. Split the two signs:

$$x = \frac{-2\sqrt{2} + 4\sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2} \text{ or } x = \frac{-2\sqrt{2} - 4\sqrt{2}}{2} = \frac{-6\sqrt{2}}{2} = -3\sqrt{2}.$$

Final Answer: The roots are $x = \sqrt{2}$ and $x = -3\sqrt{2}$.

☞ **Simplify the surd inside the root**

$\sqrt{32}$ is not left as is. Pull out the largest perfect square: $\sqrt{32} = 4\sqrt{2}$. This makes the $\pm$ step add like terms.

EXPERT'S SOLUTION *: Ayesha Khan, M.Sc Mathematics, Jamia Hamdard*

Treat $\sqrt{2}$ as a unit. Carrying $\sqrt{2}$ as a single symbol keeps the surd arithmetic clean.

- **Find D:** the discriminant is $8 + 24 = 32$, and $\sqrt{32} = 4\sqrt{2}$.
- **Collect terms:** then $\frac{-2\sqrt{2} \pm 4\sqrt{2}}{2}$ gathers the $\sqrt{2}$ terms into $\sqrt{2}$ and $-3\sqrt{2}$.
- **Check by product:** $\sqrt{2} \cdot (-3\sqrt{2}) = -6 = c/a$, exactly the constant term, so the roots are right.

Final Answer: $x = \sqrt{2},\ -3\sqrt{2}$.

Q 4.6 **Find the roots of the quadratic equation $x^2 - 3\sqrt{5}\,x + 10 = 0$ by using the quadratic formula.**

SOLUTION

Concept used. Apply the quadratic formula; compute the discriminant carefully with the surd coefficient.

Step 1. Read the coefficients: $a = 1,\ b = -3\sqrt{5},\ c = 10$.

Step 2. Compute the discriminant:

$$b^2 - 4ac = (3\sqrt{5})^2 - 4(1)(10)$$

$$= 45 - 40$$

$$= 5.$$

Step 3. Apply the formula:

$$x = \frac{-(-3\sqrt{5}) \pm \sqrt{5}}{2(1)}$$

$$= \frac{3\sqrt{5} \pm \sqrt{5}}{2}.$$

Step 4. Split the two signs:

$$x = \frac{3\sqrt{5}+\sqrt{5}}{2} = \frac{4\sqrt{5}}{2} = 2\sqrt{5} \text{ or } x = \frac{3\sqrt{5}-\sqrt{5}}{2} = \frac{2\sqrt{5}}{2} = \sqrt{5}.$$

Final Answer: The roots are $x = 2\sqrt{5}$ and $x = \sqrt{5}$.

☞ $(3\sqrt{5})^2$ **is** 45**, not** 15

Square both the number and the surd: $(3\sqrt{5})^2 = 9 \cdot 5 = 45$. Squaring only the 3 would give a wrong discriminant.

EXPERT'S SOLUTION *: Vivek Nanda, M.Sc Mathematics, NIT Warangal*

Both roots are multiples of $\sqrt{5}$. The surd factors right through, so the roots stay as tidy multiples of $\sqrt{5}$.

- **Find D:** since $(3\sqrt{5})^2 = 45$ and $4ac = 40$, the discriminant is 5, so $\sqrt{D} = \sqrt{5}$.
- **Apply the formula:** it gives $\frac{3\sqrt{5}\pm\sqrt{5}}{2}$, i.e. $2\sqrt{5}$ and $\sqrt{5}$.
- **Check by product:** $2\sqrt{5} \cdot \sqrt{5} = 2 \cdot 5 = 10$ equals c, confirming the answer.

Final Answer: $x = 2\sqrt{5},\ \sqrt{5}$.

Q 4.7 **Find the roots of the quadratic equation $\frac{1}{2}x^2 - \sqrt{11}\,x + 1 = 0$ by using the quadratic formula.**

SOLUTION

Concept used. Apply the quadratic formula; the fractional leading coefficient just goes into a.

Step 1. Read the coefficients: $a = \frac{1}{2},\ b = -\sqrt{11},\ c = 1$.

Step 2. Compute the discriminant:

$$b^2 - 4ac = (\sqrt{11})^2 - 4\left(\frac{1}{2}\right)(1)$$

$$= 11 - 2$$

$$= 9.$$

Step 3. Apply the formula (note $2a = 2 \cdot \frac{1}{2} = 1$):

$$x = \frac{-(-\sqrt{11}) \pm \sqrt{9}}{2\left(\frac{1}{2}\right)}$$

$$= \frac{\sqrt{11} \pm 3}{1}$$
$$= \sqrt{11} \pm 3.$$

Step 4. Split the two signs:

$$x = \sqrt{11} + 3 \text{ or } x = \sqrt{11} - 3.$$

Final Answer: The roots are $x = \sqrt{11} + 3$ and $x = \sqrt{11} - 3$.

✗ $2a = 1$ here

With $a = \frac{1}{2}$ the denominator is $2a = 1$, so the whole fraction simplifies. Forgetting to halve and using $2a = \frac{1}{2}$ would double the answer.

EXPERT'S SOLUTION *: Tara Krishnamurthy, M.Sc Mathematics, Mangalore University*

Clear the half to simplify. A fractional leading coefficient makes the denominator $2a$ collapse to 1.

- **Find D:** the discriminant is $11 - 2 = 9$, so $\sqrt{D} = 3$, and $2a = 1$.
- **Apply the formula:** it collapses to $\sqrt{11} \pm 3$, giving roots $\sqrt{11} + 3$ and $\sqrt{11} - 3$.
- **Check:** their sum is $2\sqrt{11} = -b/a$ and product is $11 - 9 = 2 = c/a$, both consistent with the coefficients.

Final Answer: $x = \sqrt{11} + 3,\ \sqrt{11} - 3$.

Q 4.8 **Find the roots of the quadratic equation $2x^2 + \frac{5}{3}x - 2 = 0$ by the factorisation method.**

SOLUTION

Concept used. Factorisation splits the middle term into two parts whose product equals $a \times c$ (after clearing fractions) and whose sum equals b.

Step 1. Clear the fraction by multiplying the whole equation by 3:

$$6x^2 + 5x - 6 = 0.$$

Step 2. Here $a \times c = 6 \times (-6) = -36$ and $b = 5$. Find two numbers with product -36 and sum 5: they are 9 and -4.

Step 3. Split the middle term:

$$6x^2 + 9x - 4x - 6 = 0.$$

Step 4. Group and factor:

$$3x(2x+3) - 2(2x+3) = 0$$

$$(2x+3)(3x-2) = 0.$$

Step 5. Set each factor to zero:

$$2x + 3 = 0 \Rightarrow x = -\frac{3}{2}, \text{ or } 3x - 2 = 0 \Rightarrow x = \frac{2}{3}.$$

Final Answer: The roots are $x = -\frac{3}{2}$ and $x = \frac{2}{3}$.

☞ **Clear fractions before splitting**

Multiplying by the denominator first ($\times 3$ here) gives whole-number coefficients, which makes the product-and-sum search much easier.

EXPERT'S SOLUTION *: Harshit Goel, M.Sc Applied Mathematics, IIT Jodhpur*

Whole numbers make the split obvious. Clearing the fraction turns the middle-term split into a familiar integer search.

- **Scale up:** multiplying by 3 turns the equation into $6x^2 + 5x - 6 = 0$.
- **Find the split:** we need factors of $6 \times (-6) = -36$ that add to 5, namely 9 and -4, and grouping gives $(2x+3)(3x-2) = 0$, so the roots are $-\frac{3}{2}$ and $\frac{2}{3}$.
- **Quick check:** the product of roots $-\frac{3}{2} \cdot \frac{2}{3} = -1 = c/a$ for the scaled equation.

Final Answer: $x = -\frac{3}{2}, \ \frac{2}{3}$.

Q 4.9 **Find the roots of the quadratic equation $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$ by the factorisation method.**

SOLUTION

Concept used. Factorisation after clearing the fractions: split the middle term using the product $a \times c$ and the sum b.

Step 1. Multiply the whole equation by 5 to clear fractions:

$$2x^2 - 5x - 3 = 0.$$

Step 2. Here $a \times c = 2 \times (-3) = -6$ and $b = -5$. Two numbers with product -6 and sum -5 are -6 and 1.

Step 3. Split the middle term:

$$2x^2 - 6x + x - 3 = 0.$$

Step 4. Group and factor:

$$2x(x-3) + 1(x-3) = 0$$

$$(x-3)(2x+1) = 0.$$

Step 5. Set each factor to zero:

$$x - 3 = 0 \Rightarrow x = 3, \text{ or } 2x + 1 = 0 \Rightarrow x = -\frac{1}{2}.$$

Final Answer: The roots are $x = 3$ and $x = -\frac{1}{2}$.

☞ **Sum and product check**

The roots 3 and $-\frac{1}{2}$ sum to $\frac{5}{2}$ and multiply to $-\frac{3}{2}$, matching $-b/a$ and c/a for the scaled equation $2x^2 - 5x - 3 = 0$.

EXPERT'S SOLUTION *: Brinda Patel, M.Sc Mathematics, Gujarat University*

Scale, split, group. The same three-move routine clears the fraction and factors the quadratic.

- **Scale up:** multiplying by 5 gives $2x^2 - 5x - 3 = 0$.
- **Split and group:** the split needs factors of -6 summing to -5, which are -6 and 1, leading to $(x-3)(2x+1) = 0$, so the roots are 3 and $-\frac{1}{2}$.
- **Cross-check:** this matches the discriminant $25 + 24 = 49$, a perfect square, as expected for rational roots.

Final Answer: $x = 3,\ -\frac{1}{2}$.

Q 4.10 **Find the roots of the quadratic equation $3\sqrt{2}\,x^2 - 5x - \sqrt{2} = 0$ by the factorisation method.**

SOLUTION

Concept used. Factorisation with surd coefficients: split the middle term using the product $a \times c$ and the sum b, even when a and c involve $\sqrt{2}$.

Step 1. Identify $a = 3\sqrt{2},\ b = -5,\ c = -\sqrt{2}$, so $a \times c = 3\sqrt{2} \times (-\sqrt{2}) = -3 \times 2 = -6$.

Step 2. Find two numbers with product -6 and sum -5: they are -6 and 1.

Step 3. Split the middle term:

$$3\sqrt{2}\,x^2 - 6x + x - \sqrt{2} = 0.$$

Step 4. Group and factor. From the first pair take out $3x$, from the second take out 1:

$$3x(\sqrt{2}\,x - 2) + 1(x - \sqrt{2}) = 0.$$

Step 5. Rewrite $\sqrt{2}\,x - 2 = \sqrt{2}(x - \sqrt{2})$, so both groups share $(x - \sqrt{2})$:

$$3\sqrt{2}\,x(x - \sqrt{2}) + 1(x - \sqrt{2}) = 0$$

$$(x - \sqrt{2})(3\sqrt{2}\,x + 1) = 0.$$

Step 6. Set each factor to zero:

$$x - \sqrt{2} = 0 \Rightarrow x = \sqrt{2}, \text{ or } 3\sqrt{2}\,x + 1 = 0 \Rightarrow x = -\frac{1}{3\sqrt{2}} = -\frac{\sqrt{2}}{6}.$$

Final Answer: The roots are $x = \sqrt{2}$ and $x = -\dfrac{\sqrt{2}}{6}$.

☞ Rationalise the surd root

$-\frac{1}{3\sqrt{2}}$ is tidied by multiplying top and bottom by $\sqrt{2}$: $-\frac{\sqrt{2}}{3\cdot 2} = -\frac{\sqrt{2}}{6}$. Always clear a surd from the denominator.

EXPERT'S SOLUTION *: Mehul Trivedi, M.Sc Mathematics, Maharaja Sayajirao University of Baroda*

The surds collapse in the product. The reason this surd problem behaves like an integer one is the clean product $a \times c$.

- **Product is an integer:** $a \times c = 3\sqrt{2} \cdot (-\sqrt{2}) = -3 \times 2 = -6$, a whole number.
- **Split like integers:** the middle-term split needs two numbers with product -6 and sum -5, namely -6 and 1, exactly as in an integer split.
- **Group carefully:** writing $-5x$ as $-6x + x$ and grouping gives $(x - \sqrt{2})(3\sqrt{2}\,x + 1) = 0$, so one factor sends x to $\sqrt{2}$ and the other to $-\frac{1}{3\sqrt{2}}$.

Step 1. Rationalise the second root to remove the surd from the denominator:

$$x = -\frac{1}{3\sqrt{2}} = -\frac{1}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{6}.$$

Step 2. Cross-check with the discriminant. Here
$b^2 - 4ac = (-5)^2 - 4(3\sqrt{2})(-\sqrt{2}) = 25 + 24 = 49$, a perfect square.

Step 3. The quadratic formula then gives $x = \dfrac{5 \pm 7}{2 \cdot 3\sqrt{2}} = \dfrac{12}{6\sqrt{2}}$ or $\dfrac{-2}{6\sqrt{2}}$, i.e. $\sqrt{2}$ and $-\frac{\sqrt{2}}{6}$, the same pair.

A quick product test seals it: $\sqrt{2} \cdot \left(-\frac{\sqrt{2}}{6}\right) = -\frac{2}{6} = -\frac{1}{3}$, which equals $c/a = -\sqrt{2}/(3\sqrt{2})$. Both the factorisation and the formula agree, so the roots are confirmed.

Final Answer: $x = \sqrt{2},\ -\dfrac{\sqrt{2}}{6}$.

Q 4.11 **Find the roots of the quadratic equation $3x^2 + 5\sqrt{5}\,x - 10 = 0$ by the factorisation method.**

SOLUTION

Concept used. Factorisation by splitting the middle term; with a surd middle coefficient, use $a \times c$ and b as usual.

Step 1. Identify $a = 3,\ b = 5\sqrt{5},\ c = -10$, so $a \times c = 3 \times (-10) = -30$.

Step 2. The middle term $5\sqrt{5}\,x$ splits into $6\sqrt{5}\,x$ and $-\sqrt{5}\,x$: their product is $6\sqrt{5} \times (-\sqrt{5}) = -6 \times 5 = -30$ and their sum is $5\sqrt{5}$.

Step 3. Split the middle term:

$$3x^2 + 6\sqrt{5}\,x - \sqrt{5}\,x - 10 = 0.$$

Step 4. Group and factor:

$$3x(x + 2\sqrt{5}) - \sqrt{5}(x + 2\sqrt{5}) = 0$$

$$(x + 2\sqrt{5})(3x - \sqrt{5}) = 0.$$

Step 5. Set each factor to zero:

$$x + 2\sqrt{5} = 0 \Rightarrow x = -2\sqrt{5}, \text{ or } 3x - \sqrt{5} = 0 \Rightarrow x = \frac{\sqrt{5}}{3}.$$

Final Answer: The roots are $x = -2\sqrt{5}$ and $x = \dfrac{\sqrt{5}}{3}$.

✗ Match the surd in the split

The split pieces must add to $5\sqrt{5}\,x$, so both pieces carry $\sqrt{5}$. Splitting into plain integers would not rebuild the middle term.

EXPERT'S SOLUTION *: Pallavi Hegde, M.Sc Mathematics, Karnatak University Dharwad*

Keep $\sqrt{5}$ in both pieces. A surd middle term must split into pieces that still carry the surd.

- **Split with the surd:** the middle term $5\sqrt{5}\,x$ splits into $6\sqrt{5}\,x$ and $-\sqrt{5}\,x$, whose product $6\sqrt{5}\cdot(-\sqrt{5}) = -30$ equals $a \times c$.
- **Group:** this gives $(x + 2\sqrt{5})(3x - \sqrt{5}) = 0$, so the roots are $-2\sqrt{5}$ and $\frac{\sqrt{5}}{3}$.
- **Check by product:** $-2\sqrt{5}\cdot\frac{\sqrt{5}}{3} = -\frac{10}{3} = c/a$, a sound check.

> **Final Answer:** $x = -2\sqrt{5},\ \dfrac{\sqrt{5}}{3}$.

Q 4.12 **Find the roots of the quadratic equation $21x^2 - 2x + \dfrac{1}{21} = 0$ by the factorisation method.**

SOLUTION

Concept used. Factorisation after clearing the fraction; a zero discriminant gives a single repeated root (a perfect square).

Step 1. Multiply the whole equation by 21 to clear the fraction:

$$441x^2 - 42x + 1 = 0.$$

Step 2. Notice this is a perfect square: $441 = 21^2$ and the middle term is $2 \cdot 21x \cdot 1 = 42x$. So

$$441x^2 - 42x + 1 = (21x - 1)^2.$$

Step 3. Set the factor to zero:

$$(21x - 1)^2 = 0 \Rightarrow 21x - 1 = 0.$$

Step 4. Solve:

$$x = \frac{1}{21} \text{ (a repeated root).}$$

> **Final Answer:** The equation has the repeated root $x = \dfrac{1}{21}$.

☞ Spot the perfect square

When the first and last terms are perfect squares and the middle term is twice their roots, the quadratic is $(\text{first} \pm \text{last})^2$, giving one repeated root.

EXPERT'S SOLUTION *: Suresh Patil, M.Sc Applied Mathematics, NIT Surathkal*

Recognise the square pattern. Spotting a perfect square skips the whole middle-term search.

- **Scale up:** multiplying by 21 gives $441x^2 - 42x + 1$.
- **It is a square:** since $441 = (21)^2$, $1 = (1)^2$ and $42x = 2 \cdot 21x \cdot 1$, the expression is exactly $(21x - 1)^2$, so the one equation $21x - 1 = 0$ gives the repeated root $\frac{1}{21}$.
- **Discriminant agrees:** $42^2 - 4 \cdot 441 = 1764 - 1764 = 0$, confirming the repeated root.

> **Final Answer:** $x = \dfrac{1}{21}$ (repeated).

Related Collegedunia resources: pair these NCERT Exemplar Class 10 Maths solutions for Quadratic Equations with the chapter Notes, the NCERT Solutions, and the all-formulas sheet to revise the whole unit before your board exam.

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NCERT Exemplar Solutions

Solved NCERT Exemplar Class 10 Maths Chapter 4, Quadratic Equations: every question, step by step (2026-27)

Chapter 4: Quadratic Equations

About this Chapter

Full worked solutions for every problem in Class 10 Maths Chapter 4 Quadratic Equations. After this set you can find roots by **factorisation** and the **quadratic formula**, and read the **discriminant** $b^2 - 4ac$ to tell distinct, equal and no real roots apart. MCQs, reasoning, root finding and word problems are each solved step by step with an expert view.

Topics covered: Standard form $ax^2 + bx + c = 0$ • Roots and zeroes • Factorisation method • Completing the square • Quadratic formula • Discriminant $b^2 - 4ac$ • Nature of roots • Word problems on numbers, speed, age and area

Quick Formula Sheet

Standard form:

$ax^2 + bx + c = 0, \quad a \neq 0$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant:

$D = b^2 - 4ac$

Nature of roots:

$D > 0$: two distinct real

$D = 0$: two equal real

$D < 0$: no real roots

Sum and product of roots:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

Also see for this chapter on Collegedunia: NCERT Solutions, Revision Notes, Formula Sheet and the Exemplar Book PDF for Class 10 Maths Chapter 4, Quadratic Equations.

IV. Long Answer Questions (Exercise 4.4)

Q 4.1 **Find whether the equation $8x^2 + 2x - 3 = 0$ has real roots. If real roots exist, find them.**

SOLUTION

Concept used. Real roots exist when the discriminant $D = b^2 - 4ac \geq 0$. If $D \geq 0$, the roots come from the quadratic formula $x = \dfrac{-b \pm \sqrt{D}}{2a}$.

Step 1. Read the coefficients: $a = 8,\ b = 2,\ c = -3$.

Step 2. Compute the discriminant:

$$D = b^2 - 4ac = 2^2 - 4(8)(-3)$$
$$= 4 + 96$$
$$= 100.$$

Step 3. Since $D = 100 > 0$, two distinct real roots exist.

Step 4. Apply the formula:

$$x = \frac{-2 \pm \sqrt{100}}{2(8)} = \frac{-2 \pm 10}{16}.$$

Step 5. Split the two signs:

$$x = \frac{-2 + 10}{16} = \frac{8}{16} = \frac{1}{2} \text{ or } x = \frac{-2 - 10}{16} = \frac{-12}{16} = -\frac{3}{4}.$$

Final Answer: Real roots exist; $x = \dfrac{1}{2}$ and $x = -\dfrac{3}{4}$.

☞ Check D before finding roots

A long-answer "real roots?" question always starts with the discriminant. Only after confirming $D \geq 0$ do you plug into the formula.

EXPERT'S SOLUTION *: Reema Saluja, M.Sc Mathematics, Guru Nanak Dev University*

A perfect-square discriminant. A perfect-square D promises rational roots before you even substitute.

- **Find D:** the numbers $a = 8, b = 2, c = -3$ give $D = 4 + 96 = 100$, and $\sqrt{100} = 10$, so the roots are rational.
- **Apply the formula:** it yields $\frac{-2 \pm 10}{16}$, splitting into $\frac{1}{2}$ and $-\frac{3}{4}$.
- **Check:** the factorisation $8x^2 + 2x - 3 = (2x - 1)(4x + 3)$ confirms both roots without extra work.

Final Answer: $x = \dfrac{1}{2},\ -\dfrac{3}{4}$.

Q 4.2 **Find whether the equation $-2x^2 + 3x + 2 = 0$ has real roots. If real roots exist, find them.**

SOLUTION

Concept used. Test the **discriminant** $D = b^2 - 4ac$; if $D \geq 0$, use the quadratic formula to find the roots.

Step 1. Read the coefficients: $a = -2,\ b = 3,\ c = 2$.

Step 2. Compute the discriminant:

$$D = b^2 - 4ac = 3^2 - 4(-2)(2)$$

$$= 9 + 16$$

$$= 25.$$

Step 3. Since $D = 25 > 0$, two distinct real roots exist.

Step 4. Apply the formula:

$$x = \frac{-3 \pm \sqrt{25}}{2(-2)} = \frac{-3 \pm 5}{-4}.$$

Step 5. Split the two signs:

$$x = \frac{-3+5}{-4} = \frac{2}{-4} = -\frac{1}{2} \text{ or } x = \frac{-3-5}{-4} = \frac{-8}{-4} = 2.$$

Final Answer: Real roots exist; $x = 2$ and $x = -\frac{1}{2}$.

✗ Denominator is -4 here

With $a = -2$, the formula divides by $2a = -4$. Carry the negative denominator through both branches or the signs of the roots will be wrong.

EXPERT'S SOLUTION : *Joseph Mathew, M.Sc Mathematics, Mahatma Gandhi University Kottayam*

Flip to a positive lead, then solve. A positive leading term keeps the signs simple all the way through.

- **Flip it:** multiplying by -1 gives $2x^2 - 3x - 2 = 0$, with the same roots.
- **Apply the formula:** its discriminant is $9 + 16 = 25$, so $\sqrt{D} = 5$, and the formula gives $\frac{3 \pm 5}{4}$, i.e. 2 and $-\frac{1}{2}$.
- **Check:** factoring $2x^2 - 3x - 2 = (2x + 1)(x - 2)$ shows the same pair, a neat confirmation.

Final Answer: $x = 2,\ -\frac{1}{2}$.

Q 4.3 **Find whether the equation $5x^2 - 2x - 10 = 0$ has real roots. If real roots exist, find them.**

SOLUTION

Concept used. Test the discriminant; when $D > 0$ but is not a perfect square, the roots are real but irrational, given by the quadratic formula.

Step 1. Read the coefficients: $a = 5,\ b = -2,\ c = -10$.

Step 2. Compute the discriminant:

$$D = b^2 - 4ac = (-2)^2 - 4(5)(-10)$$

$$= 4 + 200$$

$$= 204.$$

Step 3. Since $D = 204 > 0$, two distinct real roots exist.

Step 4. Simplify the surd: $\sqrt{204} = \sqrt{4 \cdot 51} = 2\sqrt{51}$.

Step 5. Apply the formula:

$$x = \frac{-(-2) \pm 2\sqrt{51}}{2(5)} = \frac{2 \pm 2\sqrt{51}}{10}.$$

Step 6. Cancel the common factor 2:

$$x = \frac{1 \pm \sqrt{51}}{5}.$$

Final Answer: Real roots exist; $x = \frac{1 + \sqrt{51}}{5}$ and $x = \frac{1 - \sqrt{51}}{5}$.

☞ **Cancel the common factor**

After $\frac{2 \pm 2\sqrt{51}}{10}$, divide numerator and denominator by 2 to reach the simplest form $\frac{1 \pm \sqrt{51}}{5}$.

EXPERT'S SOLUTION *: Anushka Bose, M.Sc Mathematics, University of Burdwan*

Irrational but real. A positive non-square discriminant means real roots that stay as surds.

- **Find D:** the discriminant $4 + 200 = 204$ is positive, so real roots exist, but 204 is not a perfect square, so they are irrational.
- **Simplify:** writing $\sqrt{204} = 2\sqrt{51}$ and dividing through by the common 2 gives $\frac{1\pm\sqrt{51}}{5}$.
- **Check:** their sum $\frac{2}{5} = -b/a$ and product $\frac{1-51}{25} = -2 = c/a$, both matching the coefficients.

> **Final Answer:** $x = \dfrac{1 \pm \sqrt{51}}{5}$.

Q 4.4 **Find whether the equation $\dfrac{1}{2x-3} + \dfrac{1}{x-5} = 1,\ x \neq \dfrac{3}{2}, 5$ has real roots. If real roots exist, find them.**

SOLUTION

Concept used. Clear the denominators to reach a standard quadratic, then apply the discriminant test and the quadratic formula.

Step 1. Combine the left side over a common denominator:

$$\frac{(x-5)+(2x-3)}{(2x-3)(x-5)} = 1$$

$$\frac{3x-8}{(2x-3)(x-5)} = 1.$$

Step 2. Cross-multiply:

$$3x - 8 = (2x-3)(x-5).$$

Step 3. Expand the right side:

$$3x - 8 = 2x^2 - 13x + 15.$$

Step 4. Bring all terms to one side:

$$2x^2 - 13x + 15 - 3x + 8 = 0$$

$$2x^2 - 16x + 23 = 0.$$

Step 5. Compute the discriminant: $a = 2,\ b = -16,\ c = 23$.

$D = (-16)^2 - 4(2)(23) = 256 - 184 = 72 > 0$. Real roots exist.

Step 6. Apply the formula, with $\sqrt{72} = 6\sqrt{2}$:

$$x = \frac{16 \pm 6\sqrt{2}}{4} = \frac{8 \pm 3\sqrt{2}}{2}.$$

Final Answer: Real roots exist; $x = \dfrac{8+3\sqrt{2}}{2}$ and $x = \dfrac{8-3\sqrt{2}}{2}$.

☞ **Both roots are allowed**

The excluded values are $x = \frac{3}{2}$ and $x = 5$. Both roots $\frac{8\pm3\sqrt{2}}{2} \approx 6.12$ and 1.88 avoid these, so neither is rejected.

EXPERT'S SOLUTION *: Kabir Anand, M.Sc Mathematics, Shiv Nadar University*

Turn the fraction equation into a quadratic. Clearing the denominators is the move that unlocks this problem.

- **Clear fractions:** adding the two fractions gives $\frac{3x-8}{(2x-3)(x-5)} = 1$, and cross-multiplying clears the denominators to $3x - 8 = 2x^2 - 13x + 15$.
- **Solve:** rearranged that is $2x^2 - 16x + 23 = 0$, with discriminant $256 - 184 = 72 > 0$; simplifying $\sqrt{72} = 6\sqrt{2}$ and dividing by 2 gives $\frac{8\pm3\sqrt{2}}{2}$.
- **Check the domain:** both lie away from the forbidden $\frac{3}{2}$ and 5, so both are valid.

Final Answer: $x = \dfrac{8 \pm 3\sqrt{2}}{2}$.

Q 4.5 **Find whether the equation $x^2 + 5\sqrt{5}\,x - 70 = 0$ has real roots. If real roots exist, find them.**

SOLUTION

Concept used. Test the **discriminant**; with a surd coefficient the arithmetic carries the surd through, and the formula gives the roots.

Step 1. Read the coefficients: $a = 1,\ b = 5\sqrt{5},\ c = -70$.

Step 2. Compute the discriminant:

$$D = b^2 - 4ac = (5\sqrt{5})^2 - 4(1)(-70)$$

$$= 125 + 280$$

$$= 405.$$

Step 3. Since $D = 405 > 0$, two distinct real roots exist.

Step 4. Simplify the surd: $\sqrt{405} = \sqrt{81 \cdot 5} = 9\sqrt{5}$.

Step 5. Apply the formula:

$$x = \frac{-5\sqrt{5} \pm 9\sqrt{5}}{2(1)} = \frac{-5\sqrt{5} \pm 9\sqrt{5}}{2}.$$

Step 6. Split the two signs:

$$x = \frac{-5\sqrt{5} + 9\sqrt{5}}{2} = \frac{4\sqrt{5}}{2} = 2\sqrt{5} \text{ or } x = \frac{-5\sqrt{5} - 9\sqrt{5}}{2} = \frac{-14\sqrt{5}}{2} = -7\sqrt{5}.$$

Final Answer: Real roots exist; $x = 2\sqrt{5}$ and $x = -7\sqrt{5}$.

☞ $\sqrt{405} = 9\sqrt{5}$

Factor out the largest perfect square: $405 = 81 \times 5$, so $\sqrt{405} = 9\sqrt{5}$. This lets the $\sqrt{5}$ terms combine in the next step.

EXPERT'S SOLUTION : *Sanjana Pillai, M.Sc Mathematics, University of Kerala*

Everything is a multiple of $\sqrt{5}$. The surd runs cleanly through, so the roots stay as multiples of $\sqrt{5}$.

- **Find D:** since $(5\sqrt{5})^2 = 125$ and $-4ac = 280$, the discriminant is 405, and $\sqrt{405} = 9\sqrt{5}$.
- **Apply the formula:** it gives $\frac{-5\sqrt{5} \pm 9\sqrt{5}}{2}$, collecting to $2\sqrt{5}$ and $-7\sqrt{5}$.
- **Check by product:** $2\sqrt{5} \cdot (-7\sqrt{5}) = -14 \cdot 5 = -70 = c$ confirms both roots.

Final Answer: $x = 2\sqrt{5},\ -7\sqrt{5}$.

Q 4.6 **Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.**

SOLUTION

Concept used. Translate the words into a quadratic equation, then solve by factorisation. "Diminished by" means subtract; "thrice of" means multiply by 3.

Step 1. Let the natural number be x.

Step 2. "Square diminished by 84" is $x^2 - 84$; "thrice of 8 more than the number" is $3(x + 8)$. Set them equal:

$$x^2 - 84 = 3(x + 8).$$

Step 3. Expand and bring to standard form:

$$x^2 - 84 = 3x + 24$$

$$x^2 - 3x - 108 = 0.$$

Step 4. Factorise (product -108, sum -3 gives -12 and 9):

$$x^2 - 12x + 9x - 108 = 0$$

$$x(x - 12) + 9(x - 12) = 0$$

$$(x - 12)(x + 9) = 0.$$

Step 5. So $x = 12$ or $x = -9$. A natural number cannot be negative, so reject $x = -9$.

> **Final Answer:** The natural number is $x = 12$.

✗ Reject the negative root

Natural numbers are positive, so $x = -9$ is discarded even though it solves the equation. Always check the domain stated in the problem.

EXPERT'S SOLUTION *: Nandini Iyengar, M.Sc Mathematics, Bangalore University*

Words to equation, then factor. The trick with such problems is to read each phrase as one algebraic piece.

- **Square part:** "square of the number" is x^2, and "diminished by 84" subtracts to give $x^2 - 84$.
- **Other side:** "thrice of 8 more than the number" is $3(x + 8)$.
- **Equate:** setting the two sides equal builds the equation $x^2 - 84 = 3(x + 8)$.

Step 1. Expand and collect to standard form:

$$x^2 - 84 = 3x + 24$$

$$x^2 - 3x - 108 = 0.$$

Step 2. Split the middle term using factors of -108 that add to -3, namely -12 and 9:

$$x^2 - 12x + 9x - 108 = 0$$

$$(x - 12)(x + 9) = 0.$$

Step 3. The roots are $x = 12$ and $x = -9$. A natural number must be positive, so $x = -9$ is rejected.

A direct check confirms it: $12^2 - 84 = 144 - 84 = 60$ and $3(12 + 8) = 3 \times 20 = 60$, so both sides equal 60. The negative root, although it solves the equation, fails the "natural

number" condition and is dropped.

Final Answer: The number is 12.

Q 4.7 **A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.**

SOLUTION

Concept used. Translate to an equation involving the reciprocal $\frac{1}{x}$, clear the fraction, and solve the resulting quadratic by factorisation.

Step 1. Let the natural number be x. Its reciprocal is $\frac{1}{x}$.

Step 2. "Increased by 12" is $x+12$; "160 times its reciprocal" is $\frac{160}{x}$. Set them equal:

$$x+12=\frac{160}{x}.$$

Step 3. Multiply through by x to clear the fraction:

$$x^2+12x=160$$

$$x^2+12x-160=0.$$

Step 4. Factorise (product -160, sum 12 gives 20 and -8):

$$x^2+20x-8x-160=0$$

$$x(x+20)-8(x+20)=0$$

$$(x+20)(x-8)=0.$$

Step 5. So $x=8$ or $x=-20$. Reject the negative value $x=-20$.

Final Answer: The natural number is $x=8$.

☞ Clear the reciprocal first

When an equation has $\frac{1}{x}$, multiply every term by x to remove the fraction before factorising. Remember $x\neq 0$.

EXPERT'S SOLUTION *: Rohit Bhattacharya, M.Sc Applied Mathematics, IIT Patna*

Multiply out the reciprocal. Clearing the $\frac{1}{x}$ first turns the reciprocal condition into an ordinary quadratic.

- **Clear the fraction:** the condition $x + 12 = \frac{160}{x}$ becomes $x^2 + 12x - 160 = 0$ once both sides are multiplied by x.
- **Factor:** splitting 12 as $20 - 8$ factors it to $(x + 20)(x - 8) = 0$, so $x = 8$ (the natural number) or -20 (rejected).
- **Verify:** $8 + 12 = 20$ and $\frac{160}{8} = 20$, so the two sides agree.

Final Answer: The number is 8.

Q 4.8 **A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.**

SOLUTION

Concept used. Use time $= \dfrac{\text{distance}}{\text{speed}}$. The difference between the two times equals 48 minutes, converted to hours.

Step 1. Let the original speed be x km/h. Time at this speed is $\dfrac{360}{x}$ hours.

Step 2. At speed $(x + 5)$ km/h the time is $\dfrac{360}{x + 5}$ hours.

Step 3. The faster trip saves 48 minutes $= \dfrac{48}{60} = \dfrac{4}{5}$ hour:

$$\frac{360}{x} - \frac{360}{x + 5} = \frac{4}{5}.$$

Step 4. Combine the left side:

$$\frac{360(x + 5) - 360x}{x(x + 5)} = \frac{4}{5}$$

$$\frac{1800}{x(x + 5)} = \frac{4}{5}.$$

Step 5. Cross-multiply:

$$1800 \times 5 = 4\,x(x + 5)$$

$$9000 = 4x^2 + 20x$$

$$4x^2 + 20x - 9000 = 0.$$

Step 6. Divide by 4:

$$x^2 + 5x - 2250 = 0.$$

Step 7. Factorise (product -2250, sum 5 gives 50 and -45):

$(x + 50)(x - 45) = 0$, so $x = 45$ or $x = -50$.

Step 8. Speed cannot be negative, so reject $x = -50$.

Final Answer: The original speed of the train is 45 km/h.

☞ **Convert minutes to hours**

Speeds are in km/h, so the time saved must be in hours: 48 min $= \frac{48}{60} = \frac{4}{5}$ h. Mixing units is the commonest slip in speed problems.

EXPERT'S SOLUTION *: Tanya Bedi, M.Sc Mathematics, Kurukshetra University*

Set the time gap to $\frac{4}{5}$ hour. The whole equation comes from the difference of two travel times.

- **Two times:** they are $\frac{360}{x}$ and $\frac{360}{x+5}$, and their difference is 48 minutes $= \frac{4}{5}$ h.
- **Form the quadratic:** the combined fraction $\frac{1800}{x(x+5)} = \frac{4}{5}$ cross-multiplies to $x^2 + 5x - 2250 = 0$, and factoring $(x + 50)(x - 45) = 0$ gives the positive speed 45 km/h.
- **Check:** at 45 km/h the time is 8 h, at 50 km/h it is 7.2 h, a saving of 0.8 h $= 48$ min.

Final Answer: Original speed $= 45$ km/h.

Q 4.9 **If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?**

SOLUTION

Concept used. Translate the age statement into a quadratic, then solve by factorisation. The square is taken of the *reduced* age, while "five times" uses the actual age.

Step 1. Let Zeba's actual age be x years. Her reduced age is $(x - 5)$.

Step 2. "Square of her reduced age" is $(x - 5)^2$; this is "11 more than five times her actual age", i.e. $5x + 11$:

$$(x - 5)^2 = 5x + 11.$$

Step 3. Expand the square:

$$x^2 - 10x + 25 = 5x + 11.$$

Step 4. Bring all terms to one side:

$$x^2 - 10x + 25 - 5x - 11 = 0$$

$$x^2 - 15x + 14 = 0.$$

Step 5. Factorise (product 14, sum -15 gives -14 and -1):

$(x - 14)(x - 1) = 0$, so $x = 14$ or $x = 1$.

Step 6. If $x = 1$, the reduced age $x - 5 = -4$ is impossible, so reject $x = 1$.

> **Final Answer:** Zeba's present age is 14 years.

☞ Reject impossible ages

A "5 years younger" age must still be sensible. $x = 1$ gives a reduced age of -4 years, which is meaningless, so only $x = 14$ survives.

EXPERT'S SOLUTION *: Mitali Deshpande, M.Sc Mathematics, University of Pune*

Square the reduced age, compare to actual. The careful part is which age is squared.

- **Left side:** the square is taken of Zeba's age *after* subtracting 5, so it is $(x - 5)^2$.
- **Right side:** "five times her actual age" uses the present age, giving $5x$, and "11 more" adds 11.
- **Equation:** so the sentence reads $(x - 5)^2 = 5x + 11$.

Step 1. Expand the square and move every term to one side:

$$x^2 - 10x + 25 = 5x + 11$$

$$x^2 - 15x + 14 = 0.$$

Step 2. Factorise using factors of 14 that add to -15, namely -14 and -1:

$$(x - 14)(x - 1) = 0.$$

Step 3. The roots are $x = 14$ and $x = 1$.

Step 4. Test each against the story: $x = 1$ makes the "5 years younger" age equal to $1 - 5 = -4$, which is impossible, so it is rejected. Only $x = 14$ is valid.

A final check makes it solid: $(14 - 5)^2 = 9^2 = 81$ and $5 \times 14 + 11 = 70 + 11 = 81$, so both sides equal 81. The age 14 is the only sensible answer.

Final Answer: She is 14 years old now.

Q 4.10 **At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.**

SOLUTION

Concept used. Set up two relationships in terms of Nisha's age, substitute one into the other, and solve the resulting quadratic by factorisation.

Step 1. Let Nisha's present age be x years. Then Asha's present age is $x^2 + 2$.

Step 2. "When Nisha grows to her mother's present age" means Nisha reaches age $x^2 + 2$, which is $\big((x^2 + 2) - x\big)$ years from now. Asha ages by the same amount.

Step 3. Asha's age then is $(x^2 + 2) + \big((x^2 + 2) - x\big) = 2x^2 - x + 4$.

Step 4. This equals "one year less than 10 times Nisha's present age", i.e. $10x - 1$:

$$2x^2 - x + 4 = 10x - 1.$$

Step 5. Bring all terms to one side:

$$2x^2 - x + 4 - 10x + 1 = 0$$

$$2x^2 - 11x + 5 = 0.$$

Step 6. Factorise (product $2 \times 5 = 10$, sum -11 gives -10 and -1):

$$2x^2 - 10x - x + 5 = 0$$

$$2x(x - 5) - 1(x - 5) = 0$$

$(x - 5)(2x - 1) = 0$, so $x = 5$ or $x = \dfrac{1}{2}$.

Step 7. An age of $\frac{1}{2}$ year does not fit the story (Nisha already has a square-of-age relation as a child with a meaningful age), so take $x = 5$.

Step 8. Then Asha's present age is $x^2 + 2 = 25 + 2 = 27$ years.

Final Answer: Nisha is 5 years old and Asha is 27 years old.

☞ **Same time passes for both**

When Nisha gains $\big((x^2 + 2) - x\big)$ years, Asha gains the same number of years. Adding that gap to

Asha's present age gives her future age.

EXPERT'S SOLUTION *: Devansh Kapoor, M.Sc Applied Mathematics, IIT Mandi*

One variable, two conditions. Everything hangs on Nisha's present age, so call it x.

- **First condition:** it makes Asha $x^2 + 2$ right now.
- **Second condition:** it is set in the future, when Nisha grows up to Asha's present age.
- **Key idea:** both people age by the same number of years, which links the two conditions.

Step 1. Years until Nisha reaches Asha's present age:

$$(x^2 + 2) - x.$$

Step 2. Asha ages by the same amount, so her future age is:

$$(x^2 + 2) + \big((x^2 + 2) - x\big) = 2x^2 - x + 4.$$

Step 3. The second condition says this equals $10x - 1$:

$$2x^2 - x + 4 = 10x - 1$$

$$2x^2 - 11x + 5 = 0.$$

Step 4. Factorise: $2x^2 - 10x - x + 5 = 2x(x - 5) - 1(x - 5) = (x - 5)(2x - 1) = 0$, so $x = 5$ or $x = \frac{1}{2}$.

Step 5. The fractional age $\frac{1}{2}$ does not suit the story, so $x = 5$, making Asha $x^2 + 2 = 27$.

A check confirms it: in 22 years Nisha turns 27 (Asha's present age), and Asha then becomes 49, which is $10 \times 5 - 1$, exactly one less than ten times Nisha's present age.

Final Answer: Nisha $= 5$ years, Asha $= 27$ years.

Q 4.11 **At t minutes past 2 pm, the time needed by the minutes hand of a clock to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t.**

SOLUTION

Concept used. The time from t minutes past 2 pm until 3 pm is $(60 - t)$ minutes. Set this equal to the given expression and solve the quadratic by **factorisation**.

Step 1. At t minutes past 2 pm, the minutes still to reach 3 pm are $(60 - t)$.

Step 2. This is "3 minutes less than $\frac{t^2}{4}$", i.e. $\frac{t^2}{4} - 3$:

$$60 - t = \frac{t^2}{4} - 3.$$

Step 3. Multiply through by 4 to clear the fraction:

$$240 - 4t = t^2 - 12.$$

Step 4. Bring all terms to one side:

$$t^2 - 12 + 4t - 240 = 0$$

$$t^2 + 4t - 252 = 0.$$

Step 5. Factorise (product -252, sum 4 gives 18 and -14):

$(t + 18)(t - 14) = 0$, so $t = 14$ or $t = -18$.

Step 6. Time cannot be negative, so reject $t = -18$.

Final Answer: $t = 14$ minutes.

☞ **"Time needed to show 3 pm" is $60 - t$**

Between 2 pm and 3 pm there are 60 minutes. If t minutes have already passed, the minutes left are $60 - t$. Reading this correctly is the heart of the problem.

EXPERT'S SOLUTION *: Shruti Venkatesh, M.Sc Mathematics, University of Mysore*

Translate the clock into algebra. Reading the clock phrase correctly is the heart of the problem.

- **Set up:** "t minutes past 2 pm" leaves $60 - t$ minutes to 3 pm, and the problem sets that equal to $\frac{t^2}{4} - 3$.
- **Form and solve:** multiplying by 4 gives $240 - 4t = t^2 - 12$, which rearranges to $t^2 + 4t - 252 = 0$; factoring $(t + 18)(t - 14) = 0$ leaves the valid time $t = 14$.
- **Check:** at 14 minutes past 2, $60 - 14 = 46$ and $\frac{14^2}{4} - 3 = 49 - 3 = 46$, so both sides agree.

Final Answer: $t = 14$ minutes.

Q 4.12 **In the centre of a rectangular lawn of dimensions 50 m $\times$ 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m^2 (see Fig. 4.1). Find the length and breadth of the pond.**

SOLUTION

Concept used. The grass forms a uniform border of unknown width around the central pond. Using equal border width x on all sides, write the pond's dimensions, then set up an **area equation** and solve the quadratic.

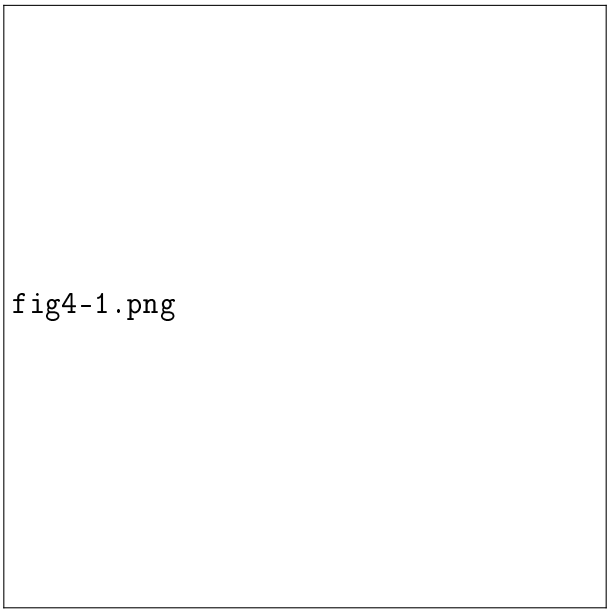


Fig. 4.1: the rectangular lawn (50 m $\times$ 40 m) with the central rectangular pond and the grass border around it.

Step 1. Let the uniform width of the grass border be x metres. The pond sits centrally, so the border removes x from each side.

Step 2. Pond length $= (50 - 2x)$ m and pond breadth $= (40 - 2x)$ m.

Step 3. Area of the whole lawn $= 50 \times 40 = 2000$ m^2.

Step 4. Grass area $=$ lawn area $-$ pond area $= 1184$, so the pond area is:

pond area $= 2000 - 1184 = 816$ m^2.

Step 5. Write the pond-area equation:

$$(50 - 2x)(40 - 2x) = 816.$$

Step 6. Expand the left side:

$$2000 - 100x - 80x + 4x^2 = 816$$

$$4x^2 - 180x + 2000 = 816.$$

Step 7. Bring all terms to one side and divide by 4:

$$4x^2 - 180x + 1184 = 0$$

$$x^2 - 45x + 296 = 0.$$

Step 8. Factorise (product 296, sum -45 gives -37 and -8):

$(x - 37)(x - 8) = 0$, so $x = 37$ or $x = 8$.

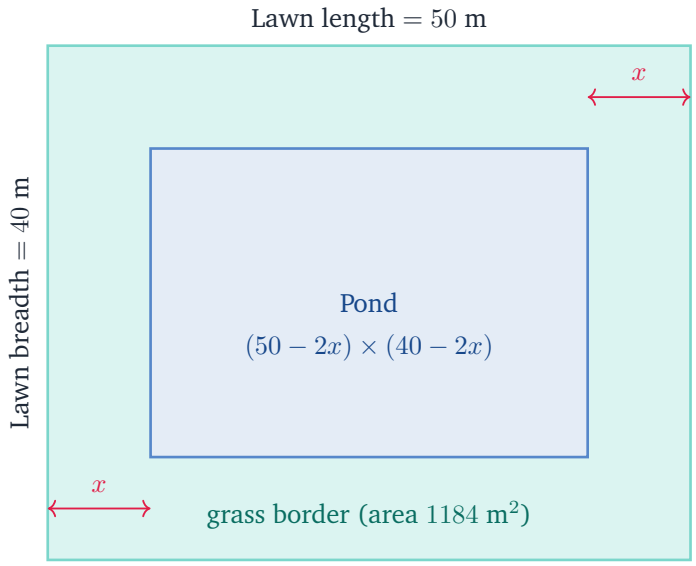
Step 9. If $x = 37$, the breadth $40 - 2(37) = -34$ is negative, impossible. So $x = 8$.

Step 10. Pond length $= 50 - 2(8) = 34$ m and pond breadth $= 40 - 2(8) = 24$ m.

> **Final Answer:** The pond is 34 m long and 24 m broad (border width $x = 8$ m).

✗ Border removes $2x$, not x

The border lies on both ends of each dimension, so it removes $2x$ in total from the length and from the breadth. Subtracting only x is a frequent error.



EXPERT'S SOLUTION *: Karan Malhotra, M.Sc Mathematics, Himachal Pradesh University*

Work with the border width x. Picking the right variable makes the whole set-up shorter.

- **Choose x:** the cleanest variable is the grass-border width x, not the pond dimensions directly.
- **Border is symmetric:** because the pond sits at the centre, the border lies on both sides of each dimension.
- **So subtract $2x$:** it removes $2x$ from the length and $2x$ from the breadth.

Step 1. Pond dimensions in terms of x:

$$\text{length} = 50 - 2x, \quad \text{breadth} = 40 - 2x.$$

Step 2. The grass area is the lawn area minus the pond area, so the pond area is:

$$2000 - 1184 = 816 \text{ m}^2.$$

Step 3. Form and simplify the equation $(50 - 2x)(40 - 2x) = 816$:

$$4x^2 - 180x + 2000 = 816$$

$$x^2 - 45x + 296 = 0.$$

Step 4. Factorise: $(x - 37)(x - 8) = 0$, so $x = 37$ or $x = 8$.

Step 5. The width $x = 37$ would force the breadth $40 - 2(37) = -34$, which is impossible, so $x = 8$.

With $x = 8$ the pond is $50 - 16 = 34$ m by $40 - 16 = 24$ m. The area check seals it: $34 \times 24 = 816$ m^2 for the pond, and $2000 - 816 = 1184$ m^2 for the grass, exactly as stated.

> **Final Answer:** Pond $= 34$ m $\times$ 24 m.

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