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NCERT Exemplar Solutions

Solved NCERT Exemplar Class 10 Maths Chapter 5, Arithmetic Progressions: every question, step by step (2026-27)

Chapter 5: Arithmetic Progressions

About this Chapter

Full worked solutions to every **NCERT Exemplar** problem in Class 10 Maths Chapter 5 Arithmetic Progressions Exercise 5.1. An **AP** adds a fixed **common difference** d to each term. After this set you can find any term with $a_n = a + (n-1)d$, sum n terms, and solve word problems on savings and instalments.

Topics covered: Meaning of an AP and common difference • nth term $a_n = a + (n-1)d$ • Sum of n terms $S_n = \frac{n}{2}[2a+(n-1)d]$ • Sum using first and last term $S_n = \frac{n}{2}(a+l)$ • Middle terms • Term from the end • Word problems on savings, instalments and distances

Quick Formula Sheet

General term (nth term):
$a_n = a + (n-1)d$

Sum of first n terms:
$S_n = \frac{n}{2}[2a + (n-1)d]$

Sum with last term l:
$S_n = \frac{n}{2}(a + l)$

Common difference:
$d = a_n - a_{n-1}$

nth term from the sum:
$a_n = S_n - S_{n-1}$

Also see for this chapter on Collegedunia: NCERT Solutions, Revision Notes, Formula Sheet and the Exemplar Book PDF for Class 10 Maths Chapter 5, Arithmetic Progressions.

I. Multiple Choice Questions (Exercise 5.1)

Q 5.1 **In an AP, if $d = -4$, $n = 7$, $a_n = 4$, then a is**

(A) 6 **(B)** 7 **(C)** 20 **(D)** 28

SOLUTION

Correct option: (D) 28.

Concept used. The nth term of an AP is $a_n = a + (n-1)d$. We are given a_n, d and n, so we rearrange to find the first term a.

Step 1. Write the formula: $a_n = a + (n-1)d$.

Step 2. Substitute $a_n = 4$, $n = 7$, $d = -4$:

$$4 = a + (7-1)(-4)$$

$$4 = a + 6 \times (-4)$$

$$4 = a - 24.$$

Step 3. Solve for a: $a = 4 + 24 = 28$.

> **Final Answer:** The first term is $a = 28$; option **(D)**.

☞ Rearrange, don't guess

When three of the four quantities a, d, n, a_n are given, the nth-term formula has exactly one unknown. Substitute first, then isolate it.

EXPERT'S SOLUTION *: Aarav Mehta, M.Sc Mathematics, IIT Kanpur*

Walk backwards from the seventh term.

- **Count the steps:** going from the first term to the seventh adds the common difference six times, so the total shift is $6 \times (-4) = -24$.
- **Undo the shift:** since the seventh term is 4, the start must have been 24 higher, that is $4 + 24 = 28$.
- **Check forward:** starting at 28 and stepping down by 4 gives $28, 24, 20, 16, 12, 8, 4$, and the seventh entry is indeed 4, so the first term is 28.

> **Final Answer:** Option **(D)**, $a = 28$.

Q 5.2 **In an AP, if $a = 3.5$, $d = 0$, $n = 101$, then a_n will be**

(A) 0 **(B)** 3.5 **(C)** 103.5 **(D)** 104.5

SOLUTION

Correct option: (B) 3.5.
Concept used. When the common difference $d = 0$, every term of the AP equals the first term, because nothing is added as you move along.

Step 1. Write the formula: $a_n = a + (n-1)d$.

Step 2. Substitute $a = 3.5$, $d = 0$, $n = 101$:

$$a_{101} = 3.5 + (101 - 1) \times 0$$

$$a_{101} = 3.5 + 0.$$

Step 3. So $a_{101} = 3.5$.

Final Answer: With $d = 0$ the term stays at 3.5; option **(B)**.

☞ **A constant list is still an AP**

A list like $3.5, 3.5, 3.5, \ldots$ has common difference 0, so it counts as an AP. Every term is the same number.

EXPERT'S SOLUTION *: Priya Nair, M.Sc Mathematics, University of Delhi*

Zero step means no change.

- **What d does:** the common difference is the size of each jump from one term to the next, and here that jump is 0.
- **Effect on the list:** with a zero jump the list never moves off its starting value of 3.5, so the 2nd term and the 101st term are the same 3.5.
- **Spot the trap:** the large value of n is only a distractor, because multiplying it by $d = 0$ wipes it out completely.

Final Answer: Option **(B)**, 3.5.

Q 5.3 **The list of numbers $-10, -6, -2, 2, \ldots$ is**
(A) an AP with $d = -16$ **(B) an AP with $d = 4$**
(C) an AP with $d = -4$ **(D) not an AP**

SOLUTION

Correct option: (B) an AP with $d = 4$.
Concept used. A list is an AP when the difference between every pair of consecutive

terms is the same. That fixed difference is d.

Step 1. Find each consecutive difference:

$$a_2 - a_1 = -6 - (-10) = 4$$

$$a_3 - a_2 = -2 - (-6) = 4$$

$$a_4 - a_3 = 2 - (-2) = 4.$$

Step 2. All differences equal 4, so the list is an AP with $d = 4$.

Final Answer: The list is an AP with common difference $d = 4$; option **(B)**.

✗ Mind the signs

Subtracting a negative number adds it: $-6 - (-10) = -6 + 10 = 4$. Dropping the double-negative is the most common slip and would wrongly suggest $d = -16$.

EXPERT'S SOLUTION *: Rohan Verma, M.Sc Mathematics, IIT Bombay*

Test the gaps, not the terms.

- **The deciding rule:** an AP is decided purely by whether the gaps between neighbours stay constant.
- **Apply it:** stepping from -10 to -6 is a rise of 4, and the same rise of 4 repeats from -6 to -2 and from -2 to 2, so the list is an AP with common difference 4.
- **The trap options:** option (A) comes from a sign error and option (C) from reversing the order of subtraction.

Final Answer: Option **(B)**, $d = 4$.

Q 5.4 **The 11th term of the AP:** $-5,\ -\frac{5}{2},\ 0,\ \frac{5}{2},\ \ldots$ **is**

(A) -20 **(B)** 20 **(C)** -30 **(D)** 30

SOLUTION

Correct option: (B) 20.

Concept used. Find the common difference first, then apply $a_n = a + (n-1)d$ with $n = 11$.

Step 1. First term $a = -5$. Common difference $d = -\frac{5}{2} - (-5) = -\frac{5}{2} + 5 = \frac{5}{2}$.

Step 2. Write the formula: $a_{11} = a + (11-1)d$.

Step 3. Substitute $a = -5$, $d = \dfrac{5}{2}$:

$$a_{11} = -5 + 10 \times \frac{5}{2}$$
$$a_{11} = -5 + 25.$$

Step 4. So $a_{11} = 20$.

Final Answer: The 11th term is 20; option **(B)**.

☞ **Fractions in d are fine**

A common difference can be a fraction. Keep it as $\frac{5}{2}$ through the working, multiply by $(n-1)$, and the fractions clear at the end.

EXPERT'S SOLUTION *: Sneha Kulkarni, M.Sc Mathematics, IISc Bangalore*

Ten steps of half-fives.

- **Size of each step:** every move up the list adds $\frac{5}{2}$, and reaching the 11th term takes 10 such moves, a total of $10 \times \frac{5}{2} = 25$.
- **Add to the start:** putting that on the first term -5 gives $-5 + 25 = 20$.
- **Pattern check:** every two steps add a whole 5, so ten steps add 25, which lands the eleventh term at 20 as well.

Final Answer: Option **(B)**, 20.

Q 5.5 **The first four terms of an AP, whose first term is -2 and the common difference is -2, are**

(A) $-2, 0, 2, 4$ **(B)** $-2, 4, -8, 16$

(C) $-2, -4, -6, -8$ **(D)** $-2, -4, -8, -16$

SOLUTION

Correct option: (C) $-2, -4, -6, -8$.

Concept used. Build an AP by repeatedly adding the common difference d to the previous term, starting from the first term a.

Step 1. First term $a = -2$.

Step 2. Second term $= a + d = -2 + (-2) = -4$.

Step 3. Third term $= -4 + (-2) = -6$.

Step 4. Fourth term $= -6 + (-2) = -8$.

Step 5. So the first four terms are $-2, -4, -6, -8$.

> **Final Answer:** The first four terms are $-2, -4, -6, -8$; option **(C)**.

✗ Add, do not multiply

An AP grows by *adding* d each time, not by multiplying. The doubling lists in (B) and (D) are geometric, not arithmetic.

EXPERT'S SOLUTION *: Ananya Iyer, M.Sc Mathematics, ISI Kolkata*

Keep stepping down by two.

- **Build the list:** from -2, each new term is two less than the one before, giving $-2, -4, -6, -8$.
- **Why the traps fail:** options (B) and (D) multiply instead of add, jumping $-2 \to 4 \to -8$ or doubling to -16, which breaks the constant-difference rule of an AP.
- **The clean route:** only the steady step of -2 keeps the gap fixed and produces the correct list.

> **Final Answer:** Option **(C)**, $-2, -4, -6, -8$.

Q 5.6 **The 21st term of the AP whose first two terms are -3 and 4 is**

(A) 17 **(B)** 137 **(C)** 143 **(D)** -143

SOLUTION

Correct option: (B) 137.

Concept used. The common difference is the second term minus the first; then use $a_n = a + (n-1)d$ with $n = 21$.

Step 1. First term $a = -3$; second term $= 4$, so $d = 4 - (-3) = 7$.

Step 2. Write the formula: $a_{21} = a + (21 - 1)d$.

Step 3. Substitute $a = -3$, $d = 7$:

$$a_{21} = -3 + 20 \times 7$$

$$a_{21} = -3 + 140.$$

Step 4. So $a_{21} = 137$.

Final Answer: The 21st term is 137; option **(B)**.

☞ Two terms fix the whole AP

Knowing the first two terms gives both a and d. From there any term is one formula away, so you never need to list out all of them.

EXPERT'S SOLUTION *: Karthik Reddy, M.Sc Mathematics, IIT Madras*

One subtraction, then one formula.

- **Find the step:** the jump from -3 to 4 is a rise of 7, so $d = 7$.
- **Reach term 21:** the twenty-first term sits 20 jumps past the start, adding $20 \times 7 = 140$ to -3 for a result of 137.
- **Rule out the trap:** the negative option -143 tries to trick you into subtracting, but the terms clearly increase once $d = 7$ is positive.

Final Answer: Option **(B)**, 137.

Q 5.7 **If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term?**

(A) 30 **(B)** 33 **(C)** 37 **(D)** 38

SOLUTION

Correct option: (B) 33.

Concept used. Two known terms give two equations in a and d. Subtracting them finds d, after which the required term follows.

Step 1. $a_2 = a + d = 13$ and $a_5 = a + 4d = 25$.

Step 2. Subtract the first from the second:

$$(a + 4d) - (a + d) = 25 - 13$$

$$3d = 12, \text{ so } d = 4.$$

Step 3. Then $a = 13 - d = 13 - 4 = 9$.

Step 4. Required term: $a_7 = a + 6d = 9 + 6 \times 4 = 9 + 24 = 33$.

Final Answer: The 7th term is 33; option **(B)**.

☞ **Subtract to kill a**

When two term-equations share the same a, subtracting them removes a straight away and leaves a single equation in d.

EXPERT'S SOLUTION *: Meera Joshi, M.Sc Applied Mathematics, IIT Kharagpur*

Use the gap between known terms.

- **Find the step:** going from the 2nd term to the 5th covers 3 steps and a rise of $25 - 13 = 12$, so each step is $d = 12/3 = 4$.
- **Extend to term 7:** the seventh term is two steps beyond the fifth, adding $2 \times 4 = 8$ to 25 to give 33.
- **Why it is faster:** this term-gap shortcut avoids solving for the first term a at all, yet lands on the same answer.

Final Answer: Option **(B)**, 33.

Q 5.8 **Which term of the AP: $21, 42, 63, 84, \ldots$ is 210?**

(A) 9th (B) 10th (C) 11th (D) 12th

SOLUTION

Correct option: (B) 10th.

Concept used. Set $a_n = 210$ in the formula $a_n = a + (n-1)d$ and solve for the position n.

Step 1. Here $a = 21$ and $d = 42 - 21 = 21$.

Step 2. Write the equation: $a_n = a + (n-1)d = 210$.

Step 3. Substitute:

$$210 = 21 + (n-1) \times 21$$

$$210 - 21 = (n-1) \times 21$$

$$189 = (n-1) \times 21.$$

Step 4. Solve: $n - 1 = \dfrac{189}{21} = 9$, so $n = 10$.

Final Answer: 210 is the 10th term; option **(B)**.

☞ Solve for the position

"Which term is…" means the value is known and n is unknown. Put the value into $a_n = a + (n-1)d$ and solve the resulting linear equation for n.

EXPERT'S SOLUTION *: Vikram Singh, M.Sc Mathematics, Jadavpur University*

The list is just the 21 times table.

- **Spot the pattern:** every term is a multiple of 21, the first being 21×1 and the second 21×2, so the nth term is simply $21n$.
- **Solve directly:** setting $21n = 210$ gives $n = 10$ at once.
- **Why it works:** the common difference happens to equal the first term, which turns the AP into a clean multiplication table and beats the full formula here.

Final Answer: Option **(B)**, the 10th term.

Q 5.9 **If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?**

(A) 5 **(B)** 20 **(C)** 25 **(D)** 30

SOLUTION

Correct option: (C) 25.

Concept used. The difference between two terms of an AP equals the number of steps between them times the common difference: $a_m - a_n = (m-n)d$.

Step 1. Write both terms: $a_{18} = a + 17d$ and $a_{13} = a + 12d$.

Step 2. Subtract:

$$a_{18} - a_{13} = (a + 17d) - (a + 12d)$$
$$= (17 - 12)d = 5d.$$

Step 3. Substitute $d = 5$: $5d = 5 \times 5 = 25$.

Final Answer: $a_{18} - a_{13} = 5d = 25$; option **(C)**.

☞ Term gap rule

$a_m - a_n = (m-n)d$. The first term a always cancels, so only the step count and d matter.

EXPERT'S SOLUTION *: Divya Menon, M.Sc Mathematics, University of Hyderabad*

Count the steps between the two terms.

- **Count the steps:** from the 13th term to the 18th term there are $18 - 13 = 5$ steps, each worth the common difference 5.
- **Total the rise:** so the climb is $5 \times 5 = 25$, regardless of where the AP actually starts.
- **Why no a is given:** the first term never enters the answer, which is exactly why the question supplies no value for it.

Final Answer: Option **(C)**, 25.

Q 5.10 **What is the common difference of an AP in which $a_{18} - a_{14} = 32$?**

(A) 8 **(B)** -8 **(C)** -4 **(D)** 4

SOLUTION

Correct option: (A) 8.

Concept used. Use the term-gap rule $a_m - a_n = (m - n)d$ and solve for d.

Step 1. The gap is $18 - 14 = 4$ steps, so $a_{18} - a_{14} = 4d$.

Step 2. Set it equal to the given value:

$$4d = 32.$$

Step 3. Solve: $d = \dfrac{32}{4} = 8$.

Final Answer: The common difference is $d = 8$; option **(A)**.

☞ **Reverse of Q9**

Q9 gave d and asked for the gap; this one gives the gap and asks for d. Same rule $a_m - a_n = (m - n)d$, just solved for the other unknown.

EXPERT'S SOLUTION *: Arjun Pillai, M.Sc Mathematics, IIT Delhi*

Divide the rise by the steps.

- **Per-step size:** over 4 steps the terms climb by 32, so a single step is $32/4 = 8$.
- **Read off d:** that single-step size is exactly the common difference, giving $d = 8$.
- **Reject negatives:** the negative choices would only fit a decreasing AP, but a positive total rise of 32 rules them out.

Final Answer: Option **(A)**, $d = 8$.

Q 5.11 **Two APs have the same common difference. The first term of one of these is -1 and that of the other is -8. Then the difference between their 4th terms is**

(A) -1 **(B)** -8 **(C)** 7 **(D)** -9

SOLUTION

Correct option: (C) 7.

Concept used. When two APs share the same d, the difference between matching terms equals the difference between their first terms, because the $(n-1)d$ part is identical and cancels.

Step 1. Fourth terms: first AP $a_4 = -1 + 3d$; second AP $a_4' = -8 + 3d$.

Step 2. Subtract:

$$a_4 - a_4' = (-1 + 3d) - (-8 + 3d)$$
$$= -1 + 3d + 8 - 3d$$
$$= -1 + 8 = 7.$$

Final Answer: The difference between the 4th terms is 7; option **(C)**.

☞ **Same d keeps gaps fixed**

If two APs step by the same amount, the gap between their matching terms never changes; it stays equal to the gap between their starts.

EXPERT'S SOLUTION *: Nisha Agarwal, M.Sc Mathematics, BHU Varanasi*

The shared d does nothing to the gap.

- **What cancels:** both fourth terms carry the same $3d$, so subtracting one from the other makes the $3d$ vanish.
- **What survives:** only the starting gap is left, and that gap is $-1 - (-8) = 7$.
- **Why d is hidden:** the step is the same in both APs and cancels for every matching pair of terms, not just the fourth, so its value is never asked for.

Final Answer: Option **(C)**, 7.

Q 5.12 **If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be**

(A) 7 **(B)** 11 **(C)** 18 **(D)** 0

SOLUTION

Correct option: (D) 0.

Concept used. Turn the word condition $7a_7 = 11a_{11}$ into an equation in a and d, simplify, and read off a_{18}.

Step 1. Write the condition: $7a_7 = 11a_{11}$, i.e. $7(a + 6d) = 11(a + 10d)$.

Step 2. Expand:

$$7a + 42d = 11a + 110d.$$

Step 3. Collect terms:

$$7a - 11a = 110d - 42d$$

$$-4a = 68d, \text{ so } a = -17d.$$

Step 4. Now $a_{18} = a + 17d = -17d + 17d = 0$.

Final Answer: The 18th term is 0; option **(D)**.

☞ **Watch for a hidden zero term**

A relation like $p\,a_p = q\,a_q$ often forces the $(p + q)$th term to be 0. Here 7 and 11 give the 18th term, which works out to zero.

EXPERT'S SOLUTION *: Siddharth Rao, M.Sc Applied Mathematics, IIT Roorkee*

Solve the relation, then jump to term 18.

- **Simplify:** expanding $7(a + 6d) = 11(a + 10d)$ leads to $-4a = 68d$, so $a = -17d$.
- **Evaluate term 18:** the eighteenth term is $a + 17d$, and replacing a gives $-17d + 17d = 0$.
- **No accident:** the condition $7a_7 = 11a_{11}$ is built so the position $7 + 11 = 18$ is forced to zero whatever the value of d.

Final Answer: Option **(D)**, 0.

Q 5.13 **The 4th term from the end of the AP:** $-11, -8, -5, \ldots, 49$ **is**

(A) 37 **(B)** 40 **(C)** 43 **(D)** 58

SOLUTION

Correct option: (B) 40.

Concept used. The kth term from the end of an AP with last term l and common difference d is $l-(k-1)d$. This treats the AP backwards, where the step size is still d but counted from the end.

Step 1. Here $d=-8-(-11)=3$, last term $l=49$, and $k=4$.

Step 2. Write the formula: kth term from end $=l-(k-1)d$.

Step 3. Substitute $l=49$, $k=4$, $d=3$:

$$=49-(4-1)\times 3$$

$$=49-3\times 3$$

$$=49-9.$$

Step 4. So the 4th term from the end is 40.

> **Final Answer:** The 4th term from the end is 40; option **(B)**.

☞ **Count back from the last term**

From the end, the last term is the "1st from end", so the 4th from end is only 3 steps back: subtract $3d$ from l.

EXPERT'S SOLUTION *: Kavya Desai, M.Sc Mathematics, Savitribai Phule Pune University*

Read the AP from right to left.

- **Step from the end:** looking back from 49, each step down is the same $d=3$, and the first-from-end is 49 itself.
- **Count back three:** the fourth-from-end is three steps back, $49-3\times 3=40$.
- **Reverse and read:** listing the AP backwards as $49, 46, 43, 40, \ldots$ shows the fourth entry as 40 too, without ever finding how many terms the AP has.

> **Final Answer:** Option **(B)**, 40.

Q 5.14 **The famous mathematician associated with finding the sum of the first 100 natural numbers is**

(A) Pythagoras (B) Newton (C) Gauss (D) Euclid

SOLUTION

Correct option: (C) Gauss.

Concept used. This is a history fact tied to the AP sum formula. Carl Friedrich Gauss, as a schoolboy, paired the numbers 1 to 100 to add them quickly, which is the idea behind $S_n = \frac{n}{2}(a + l)$.

Step 1. Gauss paired $1 + 100 = 101$, $2 + 99 = 101$, and so on, making 50 pairs each summing to 101.

Step 2. Total $= 50 \times 101 = 5050$, matching $S_{100} = \frac{100}{2}(1 + 100) = 50 \times 101 = 5050$.

Step 3. The pairing trick is exactly the "first plus last" sum formula, so the name attached to it is Gauss.

Final Answer: The mathematician is Gauss; option **(C)**.

☞ **The pairing story is the formula**

$S_n = \frac{n}{2}(a + l)$ is Gauss's pairing written as algebra: number of pairs $\times$ the common pair-sum. Remember the story to recall the formula.

EXPERT'S SOLUTION *: Manish Gupta, M.Sc Mathematics, IIT Guwahati*

Match the name to the method.

- **The famous trick:** the clever pairing of 1 with 100, 2 with 99 and so on into fifty 101s is the schoolboy story of Carl Friedrich Gauss, giving $50 \times 101 = 5050$.
- **Rule out the rest:** Pythagoras and Euclid are linked with geometry and Newton with calculus, none of them with this sum trick.
- **Tie to the formula:** the same pairing is the reason the first-plus-last sum formula works, so the correct name is Gauss.

Final Answer: Option **(C)**, Gauss.

Q 5.15 **If the first term of an AP is -5 and the common difference is 2, then the sum of the first 6 terms is**

(A) 0 **(B)** 5 **(C)** 6 **(D)** 15

SOLUTION

Correct option: (A) 0.
Concept used. The sum of the first n terms is $S_n = \frac{n}{2}[2a + (n-1)d]$. Substitute a, d and n directly.

Step 1. Write the formula: $S_6 = \frac{6}{2}[2a + (6-1)d]$.

Step 2. Substitute $a = -5$, $d = 2$:

$$S_6 = 3\,[2 \times (-5) + 5 \times 2]$$

$$S_6 = 3\,[-10 + 10]$$

$$S_6 = 3 \times 0.$$

Step 3. So $S_6 = 0$.

Final Answer: The sum of the first 6 terms is 0; option **(A)**.

☞ **A sum can be zero**

If the negative early terms balance the positive later terms, the running sum can hit 0. Here $-5, -3, -1, 1, 3, 5$ cancel in pairs.

EXPERT'S SOLUTION *: Shreya Banerjee, M.Sc Applied Mathematics, IIT Hyderabad*

The terms cancel in pairs.

- **List them:** the first six terms are $-5, -3, -1, 1, 3, 5$.
- **Pair the outers:** $-5 + 5 = 0$, $-3 + 3 = 0$ and $-1 + 1 = 0$, so the whole sum is 0.
- **Formula agrees:** the sum formula gives $3[2(-5) + 5(2)] = 3[-10 + 10] = 0$, and the symmetry around zero is what makes the total vanish exactly.

Final Answer: Option **(A)**, 0.

Q 5.16 **The sum of first 16 terms of the AP: $10, 6, 2, \ldots$ is**

(A) -320 **(B)** 320 **(C)** -352 **(D)** -400

SOLUTION

Correct option: (A) -320.
Concept used. Use $S_n = \frac{n}{2}[2a + (n-1)d]$ with $a = 10$, $d = 6 - 10 = -4$ and $n = 16$.

Step 1. Write the formula: $S_{16} = \frac{16}{2}[2a + (16-1)d]$.

Step 2. Substitute $a = 10$, $d = -4$:

$$S_{16} = 8\,[2 \times 10 + 15 \times (-4)]$$

$$S_{16} = 8\,[20 - 60]$$

$$S_{16} = 8 \times (-40).$$

Step 3. So $S_{16} = -320$.

Final Answer: The sum of the first 16 terms is -320; option **(A)**.

✗ Keep the sign of d

With $d = -4$, the term $15d = -60$. Forgetting the minus gives $+60$ and the wrong sum 320 (option B). Track the negative all the way through.

EXPERT'S SOLUTION *: Tanvi Shah, M.Sc Mathematics, St. Stephen's College Delhi*

A falling AP drags the sum negative.

- **Direction first:** the terms start at 10 and drop by 4 each time, so they soon go negative and pull the total below zero.
- **Plug in:** the sum formula gives $8[20 - 60] = 8(-40) = -320$.
- **Sift the options:** the size 320 with a minus sign rules out every positive choice, while the wrong value -352 comes from miscounting the steps as 16 instead of 15.

Final Answer: Option **(A)**, -320.

Q 5.17 **In an AP if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is**

(A) 19 **(B)** 21 **(C)** 38 **(D)** 42

SOLUTION

Correct option: (C) 38.

Concept used. When the first term a and the last term $a_n = l$ are known, the sum is $S_n = \frac{n}{2}(a + l)$. Solve it for n.

Step 1. Write the formula: $S_n = \frac{n}{2}(a + a_n)$.

Step 2. Substitute $a = 1$, $a_n = 20$, $S_n = 399$:

$$399 = \frac{n}{2}(1 + 20)$$

$$399 = \frac{n}{2} \times 21.$$

Step 3. Solve for n:

$$n = \frac{399 \times 2}{21} = \frac{798}{21} = 38.$$

Final Answer: The number of terms is $n = 38$; option **(C)**.

☞ Pick the right sum formula

When the last term is given, $S_n = \frac{n}{2}(a + l)$ is shorter than the $2a + (n-1)d$ version because you skip finding d.

EXPERT'S SOLUTION *: Harish Kumar, M.Sc Mathematics, Anna University*

Use first-plus-last directly.

- **Average the ends:** both ends of the AP are known, 1 and 20, so each pair averages $\frac{1+20}{2} = 10.5$.
- **Divide to count:** the sum 399 divided by this average gives the number of terms, $399/10.5 = 38$, the same as $n = 2 \times 399/21 = 38$.
- **Skip d:** choosing this formula avoids solving for the common difference, which the question never actually needs.

Final Answer: Option **(C)**, 38.

Q 5.18 **The sum of first five multiples of 3 is**

(A) 45 **(B)** 55 **(C)** 65 **(D)** 75

SOLUTION

Correct option: (A) 45.

Concept used. The first five multiples of 3 form an AP $3, 6, 9, 12, 15$ with $a = 3$, $d = 3$, $n = 5$. Use $S_n = \frac{n}{2}[2a + (n-1)d]$.

Step 1. Write the formula: $S_5 = \frac{5}{2}[2a + (5-1)d]$.

Step 2. Substitute $a = 3$, $d = 3$:

$$S_5 = \frac{5}{2}[2 \times 3 + 4 \times 3]$$

$$S_5 = \frac{5}{2}[6 + 12]$$

$$S_5 = \frac{5}{2} \times 18.$$

Step 3. So $S_5 = 5 \times 9 = 45$.

> **Final Answer:** The sum of the first five multiples of 3 is 45; option **(A)**.

☞ **Factor out the 3**

First five multiples of $3 = 3(1 + 2 + 3 + 4 + 5) = 3 \times 15 = 45$. Pulling out the common 3 turns it into the familiar sum $1 + 2 + \cdots + 5$.

EXPERT'S SOLUTION *: Aishwarya Rao, M.Sc Mathematics, Christ University Bengaluru*

Add the small list straight.

- **Direct addition:** the five numbers are $3, 6, 9, 12, 15$, and adding them gives $3 + 6 + 9 + 12 + 15 = 45$.
- **Factor instead:** pulling the 3 out gives $3(1 + 2 + 3 + 4 + 5) = 3 \times 15 = 45$, the same total.
- **Formula check:** the sum formula confirms $\frac{5}{2}(6 + 12) = 45$, though with only five terms direct addition is the quickest safe route.

> **Final Answer:** Option **(A)**, 45.

Download the Full chapter resource: the NCERT Solutions for Class 10 Maths Chapter 5 Arithmetic Progressions to drill the regular exercises.
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NCERT Exemplar Solutions

Solved NCERT Exemplar Class 10 Maths Chapter 5, Arithmetic Progressions: every question, step by step (2026-27)

Chapter 5: Arithmetic Progressions

About this Chapter

Full worked solutions to every **NCERT Exemplar** problem in Class 10 Maths Chapter 5 Arithmetic Progressions Exercise 5.2. An **AP** adds a fixed **common difference** d to each term. After this set you can find any term with $a_n = a + (n-1)d$, sum n terms, and solve word problems on savings and instalments.

Topics covered: Meaning of an AP and common difference • nth term $a_n = a + (n-1)d$ • Sum of n terms $S_n = \frac{n}{2}[2a+(n-1)d]$ • Sum using first and last term $S_n = \frac{n}{2}(a+l)$ • Middle terms • Term from the end • Word problems on savings, instalments and distances

Quick Formula Sheet

General term (nth term):
$a_n = a + (n-1)d$

Sum of first n terms:
$S_n = \frac{n}{2}[2a + (n-1)d]$

Sum with last term l:
$S_n = \frac{n}{2}(a + l)$

Common difference:
$d = a_n - a_{n-1}$

nth term from the sum:
$a_n = S_n - S_{n-1}$

Also see for this chapter on Collegedunia: NCERT Solutions, Revision Notes, Formula Sheet and the Exemplar Book PDF for Class 10 Maths Chapter 5, Arithmetic Progressions.

II. Short Answer Questions with Reasoning, True / False (Exercise 5.2)

Q 5.1 **Which of the following form an AP? Justify your answer.**

(i) $-1, -1, -1, -1, \ldots$ **(ii)** $0, 2, 0, 2, \ldots$ **(iii)** $1, 1, 2, 2, 3, 3, \ldots$

(iv) $11, 22, 33, \ldots$ **(v)** $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ **(vi)** $2, 2^2, 2^3, 2^4, \ldots$

(vii) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

SOLUTION

Verdict: (i), (iv) and (vii) form an AP; the rest do not.

Concept used. A list is an AP only when $a_{k+1} - a_k$ is the *same* for every k. Test each list by checking consecutive differences.

Step 1. (i) $-1, -1, -1, \ldots$: every difference is 0, constant, so it is an AP with $d = 0$.

Step 2. (ii) $0, 2, 0, 2, \ldots$: differences are $2, -2, 2, \ldots$, not constant, so not an AP.

Step 3. (iii) $1, 1, 2, 2, 3, 3, \ldots$: differences are $0, 1, 0, 1, \ldots$, not constant, so not an AP.

Step 4. (iv) $11, 22, 33, \ldots$: every difference is 11, constant, so it is an AP with $d = 11$.

Step 5. (v) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$: differences are $-\frac{1}{6}, -\frac{1}{12}, \ldots$, not constant, so not an AP.

Step 6. (vi) $2, 4, 8, 16, \ldots$: differences are $2, 4, 8, \ldots$, not constant (this is a GP), so not an AP.

Step 7. (vii) simplify the surds: $\sqrt{3},\ 2\sqrt{3},\ 3\sqrt{3},\ 4\sqrt{3}$; every difference is $\sqrt{3}$, constant, so it is an AP with $d = \sqrt{3}$.

> **Final Answer:** (i), (iv) and (vii) are APs (with $d = 0, 11, \sqrt{3}$); (ii), (iii), (v), (vi) are not.

☞ Simplify surds before testing

$\sqrt{12} = 2\sqrt{3}$ and $\sqrt{27} = 3\sqrt{3}$. Reducing surds to a common form makes the constant gap $\sqrt{3}$ obvious, so list (vii) is clearly an AP.

✗ A GP is not an AP

List (vi) doubles each time, so the gaps grow. Equal *ratios* make a GP; an AP needs equal *differences*. Do not confuse the two.

EXPERT'S SOLUTION *: Pooja Chatterjee, M.Sc Mathematics, Presidency University Kolkata*

One test, applied seven times.

- **The single rule:** the only question to ask of each list is whether the gaps between neighbours stay equal all the way along, and nothing else matters.
- **Clear passes:** lists (i) and (iv) have flat, unchanging gaps of 0 and 11, so they are arithmetic progressions straight away with no extra work.
- **Hidden pass:** list (vii) buries its AP behind surds, but once it is rewritten as $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, 4\sqrt{3}$ the gap is a steady $\sqrt{3}$, so it also qualifies.
- **The failures:** the rest each break the rule in their own way, since (ii) and (iii) bounce up and down, list (v) shrinks towards zero, and list (vi) keeps doubling, so none of

them holds a constant gap.

- **Verdict:** only constant gaps count, which leaves (i), (iv) and (vii) as the genuine arithmetic progressions here.

Final Answer: APs: (i), (iv), (vii). Not APs: (ii), (iii), (v), (vi).

Q 5.2 **Justify whether it is true to say that $-1, -\frac{3}{2}, -2, \frac{5}{2}, \ldots$ forms an AP as $a_2 - a_1 = a_3 - a_2$.**

SOLUTION

Verdict: False. It is not an AP, so the reasoning is wrong.
Concept used. For an AP, *every* consecutive difference must be equal, not just the first two. Checking only $a_2 - a_1 = a_3 - a_2$ is not enough.

Step 1. Compute the early differences:

$$a_2 - a_1 = -\frac{3}{2} - (-1) = -\frac{1}{2}$$

$$a_3 - a_2 = -2 - \left(-\frac{3}{2}\right) = -\frac{1}{2}.$$

Step 2. These two match, but now check the next one:

$$a_4 - a_3 = \frac{5}{2} - (-2) = \frac{5}{2} + 2 = \frac{9}{2}.$$

Step 3. Since $a_4 - a_3 = \frac{9}{2} \neq -\frac{1}{2}$, the differences are not all equal, so the list is not an AP.

Final Answer: False, because $a_4 - a_3 = \frac{9}{2} \neq a_3 - a_2 = -\frac{1}{2}$; one matching pair does not make an AP.

✗ Two equal gaps prove nothing

A list can start with two equal gaps and still break later, as here. You must check the gaps run consistently, not just the first pair.

EXPERT'S SOLUTION *: Nikhil Pandey, M.Sc Applied Mathematics, IIT Indore*

The fourth term betrays the list.

- **The bait:** the first two gaps are both $-\frac{1}{2}$, which is exactly what the claim leans on.

- **The giveaway:** the fourth term $\frac{5}{2}$ jumps up by $\frac{9}{2}$ from -2, nothing like $-\frac{1}{2}$, so the gap changes and the list fails the AP test.
- **The lesson:** matching only $a_2 - a_1 = a_3 - a_2$ is too weak, since a genuine AP needs the same gap running all the way through.

Final Answer: False; the gap is not constant ($a_4 - a_3 = \frac{9}{2}$).

Q 5.3 **For the AP: $-3, -7, -11, \ldots$, can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}? Give reasons for your answer.**

SOLUTION

Verdict: Yes. We can find it directly as $a_{30} - a_{20} = -40$.
Concept used. The difference between two terms of an AP is $a_m - a_n = (m - n)d$. The first term a cancels, so only the step count and d are needed.

Step 1. Common difference $d = -7 - (-3) = -4$.

Step 2. Use the term-gap rule:

$$a_{30} - a_{20} = (30 - 20)d$$
$$= 10 \times (-4).$$

Step 3. So $a_{30} - a_{20} = -40$, found without computing either term.

Final Answer: Yes; $a_{30} - a_{20} = 10d = 10 \times (-4) = -40$.

☞ **Skip the heavy terms**

You never need a_{30} or a_{20} separately. Their difference is just the number of steps between them, 10, times d.

EXPERT'S SOLUTION *: Ishita Roy, M.Sc Mathematics, Calcutta University*

The starting point cancels out.

- **Write both terms:** $a_{30} = a + 29d$ and $a_{20} = a + 19d$ both carry the same first term a.
- **Subtract:** the a disappears and leaves $(29 - 19)d = 10d$, which with $d = -4$ is -40.
- **Cross-check:** computing the two large terms, -119 and -79, and subtracting gives the same -40, but the gap rule reaches it in a single line.

Final Answer: Yes; the answer is $10d = -40$.

Q 5.4 **Two APs have the same common difference. The first term of one AP is 2 and that of the other is 7. The difference between their 10th terms is the same as the difference between their 21st terms, which is the same as the difference between any two corresponding terms. Why?**

SOLUTION

Verdict: True. All these differences equal 5, the difference of the first terms.
Concept used. For two APs with the *same* d, the difference between corresponding terms is fixed, because the common $(n-1)d$ part cancels and only the first-term gap is left.

Step 1. Let the two APs be $a_n = 2 + (n-1)d$ and $b_n = 7 + (n-1)d$.

Step 2. Their corresponding-term difference:

$$b_n - a_n = [7 + (n-1)d] - [2 + (n-1)d]$$
$$= 7 - 2 = 5.$$

Step 3. This 5 does not depend on n, so the 10th, 21st and every corresponding pair differ by the same 5.

Final Answer: Because the common $(n-1)d$ cancels, every corresponding-term difference equals $7 - 2 = 5$, independent of the term number.

☞ **Parallel staircases**

Two APs with equal d are like two staircases of the same step height. They stay a constant vertical distance apart, that distance being the gap between their starting steps.

EXPERT'S SOLUTION *: Aditya Bhat, M.Sc Mathematics, NIT Trichy*

Equal steps keep the distance fixed.

- **Lockstep:** both APs add the same d at each move, so they rise together at exactly the same rate.
- **What cancels:** whatever term number you pick, the $(n-1)d$ contributions are identical and subtract away.
- **What is left:** only the unchanging head start of $7 - 2 = 5$ survives, so the 10th-term gap, the 21st-term gap and every corresponding-term gap are all the same 5.

Final Answer: All differences equal 5, the gap between the first terms.

Q 5.5 **Is 0 a term of the AP: $31, 28, 25, \ldots$? Justify your answer.**

SOLUTION

Verdict: No. 0 is not a term of this AP.

Concept used. A value is a term only if $a_n = a + (n-1)d$ gives a *positive integer* n. Set $a_n = 0$ and check whether n is a whole number.

Step 1. Here $a = 31$, $d = 28 - 31 = -3$.

Step 2. Set $a_n = 0$ in the formula:

$$0 = 31 + (n-1)(-3)$$

$$3(n-1) = 31$$

$$n - 1 = \frac{31}{3}.$$

Step 3. Then $n = 1 + \dfrac{31}{3} = \dfrac{34}{3}$, which is not a whole number.

Step 4. Since n must be a positive integer, 0 cannot be a term.

Final Answer: No; solving $a_n = 0$ gives $n = \dfrac{34}{3}$, not a whole number.

☞ **n must be a counting number**

A number is a term of an AP only if the position n comes out as a positive whole number. A fractional n means the value is skipped over.

EXPERT'S SOLUTION : *Rahul Saxena, M.Sc Mathematics, University of Mumbai*

The terms step over zero.

- **List it out:** starting at 31 and dropping by 3 gives $31, 28, 25, 22, \ldots, 4, 1, -2, \ldots$, which lands on 1 and then -2, jumping straight past 0.
- **Confirm by algebra:** demanding $a_n = 0$ forces $n = \frac{34}{3}$, a non-integer position, so the value is never actually reached.
- **Verdict:** since n must be a whole number, 0 is not a term of this AP.

Final Answer: No; the terms go $\ldots, 4, 1, -2, \ldots$, skipping 0.

Q 5.6 **The taxi fare after each km, when the fare is Rs 15 for the first km and Rs 8 for each additional km, does not form an AP as the total fare (in Rs) after each km is $15, 8, 8, 8, \ldots$. Is the statement true? Give reasons.**

SOLUTION

Verdict: False. The reasoning is wrong; the total fares actually form an AP.
Concept used. "Total fare after each km" is the running total, not the cost of each separate km. Build the cumulative list and test it.

Step 1. Total after 1 km $= 15$.

Step 2. Total after 2 km $= 15 + 8 = 23$.

Step 3. Total after 3 km $= 23 + 8 = 31$; after 4 km $= 31 + 8 = 39$.

Step 4. So the totals are $15, 23, 31, 39, \ldots$, with constant difference 8, which is an AP.

Step 5. The list $15, 8, 8, 8, \ldots$ in the statement is the per-km cost, not the total, so the statement misreads "total".

Final Answer: False; the totals $15, 23, 31, 39, \ldots$ form an AP with $d = 8$.

✗ Total means running sum

"Total fare after each km" adds up as you travel further. Listing $15, 8, 8, 8$ records each km's cost, not the accumulated fare, which is the error in the statement.

EXPERT'S SOLUTION *: Snigdha Das, M.Sc Mathematics, Visva-Bharati University*

Add as the meter ticks.

- **Running total:** a real taxi meter keeps a running total of 15, then 23, then 31, then 39, each 8 more than the last, which is a textbook AP.
- **The misread:** the statement's list $15, 8, 8, 8$ is the charge for each individual km and never accumulates, so it confuses "per km" with "total".
- **Verdict:** read correctly, the cumulative fares do form an AP, so the statement is false.

Final Answer: False; running totals $15, 23, 31, 39, \ldots$ are an AP, $d = 8$.

Q 5.7 **In which of the following situations, do the lists of numbers involved form an AP? Give reasons for your answers.**

(i) The fee charged from a student every month by a school for the whole session, when the monthly fee is Rs 400.

(ii) The fee charged every month by a school from Classes I to XII, when the monthly fee for Class I is Rs 250, and it increases by Rs 50 for the next higher class.

(iii) The amount of money in the account of Varun at the end of every year when Rs 1000 is deposited at simple interest of 10% per annum.

(iv) The number of bacteria in a certain food item after each second, when they double every second.

SOLUTION

Verdict: (i), (ii) and (iii) form an AP; (iv) does not.

Concept used. A situation gives an AP only if a *fixed amount* is added each step. Check what is added (or multiplied) in each case.

Step 1. (i) Fee each month is $400, 400, 400, \ldots$; the same number repeats, so $d = 0$, which is an AP.

Step 2. (ii) Fees are $250, 300, 350, \ldots$, rising by a fixed Rs 50 each class, so $d = 50$, an AP.

Step 3. (iii) Simple interest adds a fixed Rs 100 (10% of 1000) each year: $1100, 1200, 1300, \ldots$, so $d = 100$, an AP.

Step 4. (iv) Bacteria double: $a, 2a, 4a, 8a, \ldots$ This multiplies by 2 each second (a GP), so the gaps grow and it is not an AP.

> **Final Answer:** (i), (ii), (iii) are APs ($d = 0, 50, 100$); (iv) is not (it is a doubling, a GP).

☞ Added vs multiplied

Fixed amount added each step $\Rightarrow$ AP. Fixed factor multiplied each step $\Rightarrow$ GP. Simple interest adds; doubling multiplies.

♥ Simple interest is linear by design

Simple interest pays a fixed amount each period, so balances climb in an AP. This is exactly why simple-interest growth is straight-line, unlike compound interest, which multiplies and curves upward.

EXPERT'S SOLUTION *: Mohit Arora, M.Sc Applied Mathematics, IIT (ISM) Dhanbad*

Look for a steady add-on.

- **Flat fee:** a repeating Rs 400 fee has gap 0, which still counts as an AP.
- **Fixed climbs:** class fees climb by a fixed Rs 50 and a simple-interest balance climbs by a fixed Rs 100, both clean arithmetic progressions.
- **The odd one out:** bacteria double, so the increase itself grows from a to $2a$ to $4a$, a multiplying pattern that is a GP and not an AP, leaving only the first three as APs.

> **Final Answer:** APs: (i), (ii), (iii). Not an AP: (iv).

Q 5.8 **Justify whether it is true to say that the following are the nth terms of an AP.**
(i) $2n - 3$ **(ii)** $3n^2 + 5$ **(iii)** $1 + n + n^2$

SOLUTION

Verdict: only (i) is the nth term of an AP; (ii) and (iii) are not.
Concept used. The nth term of an AP is $a_n = a + (n-1)d$, which is a *linear* expression in n. If a_n is linear, the list is an AP; if it contains n^2, it is not.

Step 1. (i) $a_n = 2n - 3$ is linear in n. Check the gap:
$a_n - a_{n-1} = (2n-3) - (2(n-1)-3) = 2$, a constant, so it is an AP with $d = 2$.

Step 2. (ii) $a_n = 3n^2 + 5$ has an n^2 term. Gap: $a_n - a_{n-1} = 3n^2 - 3(n-1)^2 = 6n - 3$, which depends on n, so not an AP.

Step 3. (iii) $a_n = 1 + n + n^2$ also has an n^2 term. Gap: $a_n - a_{n-1} = 2n$, which depends on n, so not an AP.

> **Final Answer:** Only (i) $2n - 3$ gives an AP ($d = 2$); the n^2 forms (ii) and (iii) do not.

☞ Linear in n is the signature

$a_n = a + (n-1)d = dn + (a - d)$ is always degree 1 in n. Any n^2 in the formula means the common difference would change, so it cannot be an AP.

EXPERT'S SOLUTION *: Lakshmi Pillai, M.Sc Mathematics, University of Madras*

Degree of n decides it.

- **The signature:** an AP's nth term is a straight-line function of the form $dn + (a - d)$, so a valid formula must be degree one in n and nothing higher.

- **The clean fit:** the expression $2n - 3$ matches that shape exactly, with slope $d = 2$, so the first list is a genuine AP.
- **Why the squares fail:** the other two carry an n^2 term, and working out their gaps gives $6n - 3$ and $2n$, both of which keep growing as n grows.
- **Verdict:** a changing gap is forbidden in an AP, so only the linear form (i) qualifies and the two quadratic formulas do not.

> **Final Answer:** Only (i) is an AP nth term ($d = 2$).

Related Collegedunia resources: pair these NCERT Exemplar Class 10 Maths solutions for Arithmetic Progressions with the chapter Notes, the NCERT Solutions, and the all-formulas sheet to revise the whole unit before your board exam.

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NCERT Exemplar Solutions

Solved NCERT Exemplar Class 10 Maths Chapter 5, Arithmetic Progressions: every question, step by step (2026-27)

Chapter 5: Arithmetic Progressions

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Topics covered: Meaning of an AP and common difference • nth term $a_n = a + (n-1)d$ • Sum of n terms $S_n = \frac{n}{2}[2a+(n-1)d]$ • Sum using first and last term $S_n = \frac{n}{2}(a+l)$ • Middle terms • Term from the end • Word problems on savings, instalments and distances

Quick Formula Sheet

General term (nth term):
$a_n = a + (n-1)d$

Sum of first n terms:
$S_n = \frac{n}{2}[2a + (n-1)d]$

Sum with last term l:
$S_n = \frac{n}{2}(a + l)$

Common difference:
$d = a_n - a_{n-1}$

nth term from the sum:
$a_n = S_n - S_{n-1}$

Also see for this chapter on Collegedunia: NCERT Solutions, Revision Notes, Formula Sheet and the Exemplar Book PDF for Class 10 Maths Chapter 5, Arithmetic Progressions.

III. Short Answer Questions (Exercise 5.3)

Q 5.1 **Match the APs given in column A with suitable common differences given in column B.**

Column A: **(A$_1$)** $2, -2, -6, -10, \ldots$; **(A$_2$)** $a = -18,\ n = 10,\ a_n = 0$; **(A$_3$)** $a = 0,\ a_{10} = 6$; **(A$_4$)** $a_2 = 13,\ a_4 = 3$.

Column B: **(B$_1$)** $\frac{2}{3}$; **(B$_2$)** -5; **(B$_3$)** 4; **(B$_4$)** -4; **(B$_5$)** 2; **(B$_6$)** $\frac{1}{2}$; **(B$_7$)** 5.

SOLUTION

Matches: $(A_1)\to(B_4)$, $(A_2)\to(B_5)$, $(A_3)\to(B_1)$, $(A_4)\to(B_2)$.

Concept used. Find d in each row, using $d = a_2 - a_1$ when terms are listed, or $d = \dfrac{a_n - a}{n-1}$ when first and later terms are given.

Step 1. (A_1) $d = a_2 - a_1 = -2 - 2 = -4$, so it matches (B_4).

Step 2. (A_2) $a_n = a + (n-1)d \Rightarrow 0 = -18 + 9d \Rightarrow d = \dfrac{18}{9} = 2$, so it matches (B_5).

Step 3. (A_3) $a_{10} = a + 9d \Rightarrow 6 = 0 + 9d \Rightarrow d = \dfrac{6}{9} = \dfrac{2}{3}$, so it matches (B_1).

Step 4. (A_4) $a_4 - a_2 = 2d \Rightarrow 3 - 13 = 2d \Rightarrow d = \dfrac{-10}{2} = -5$, so it matches (B_2).

Final Answer: $(A_1)\to(B_4) = -4$, $(A_2)\to(B_5) = 2$, $(A_3)\to(B_1) = \frac{2}{3}$, $(A_4)\to(B_2) = -5$.

☞ Pick the right d route

If consecutive terms are given, subtract them. If a far-off term is given, divide the term gap by the step count: $d = \dfrac{a_n - a}{n-1}$.

EXPERT'S SOLUTION *: Varun Kapoor, M.Sc Mathematics, NIT Surathkal*

Four short calculations, one per row.

- **Listed terms:** row A_1 shows terms, so d is a direct subtraction giving -4.
- **Distant terms:** rows A_2 and A_3 give a far-off term, so divide its gap from the first term by the step count, $18/9 = 2$ and $6/9 = \frac{2}{3}$.
- **Two apart:** row A_4 gives two terms two steps apart, so halve their gap, $(3 - 13)/2 = -5$.
- **Match up:** lining these against Column B gives B_4, B_5, B_1 and B_2 in turn.

Final Answer: A_1B_4, A_2B_5, A_3B_1, A_4B_2.

Q 5.2 **Verify that each of the following is an AP, and then write its next three terms.**

(i) $0, \dfrac{1}{4}, \dfrac{1}{2}, \dfrac{3}{4}, \ldots$ **(ii)** $5, \dfrac{14}{3}, \dfrac{13}{3}, 4, \ldots$ **(iii)** $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$

(iv) $a + b, (a+1) + b, (a+1) + (b+1), \ldots$ **(v)** $a, 2a + 1, 3a + 2, 4a + 3, \ldots$

SOLUTION

Concept used. Confirm a constant common difference, then add it repeatedly to the last given term to get the next three terms.

Step 1. (i) $d = \frac{1}{4} - 0 = \frac{1}{4}$ (constant). Next three: $1, \frac{5}{4}, \frac{3}{2}$.

Step 2. (ii) $d = \frac{14}{3} - 5 = -\frac{1}{3}$ (constant). Next three: $\frac{11}{3}, \frac{10}{3}, 3$.

Step 3. (iii) $d = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$ (constant). Next three: $4\sqrt{3}, 5\sqrt{3}, 6\sqrt{3}$.

Step 4. (iv) $d = (a + 1 + b) - (a + b) = 1$ (constant). Next three: $(a + 2) + (b + 1)$, $(a + 2) + (b + 2)$, $(a + 3) + (b + 2)$.

Step 5. (v) $d = (2a + 1) - a = a + 1$ (constant). Next three: $5a + 4$, $6a + 5$, $7a + 6$.

> **Final Answer:** (i) $1, \frac{5}{4}, \frac{3}{2}$; (ii) $\frac{11}{3}, \frac{10}{3}, 3$; (iii) $4\sqrt{3}, 5\sqrt{3}, 6\sqrt{3}$; (iv) $(a+2)+(b+1), (a+2)+(b+2), (a+3)+(b+2)$; (v) $5a + 4, 6a + 5, 7a + 6$.

☞ Verify before you extend

First confirm the gap is constant; only then keep adding it. If the gap were not constant, "next terms" would be meaningless.

EXPERT'S SOLUTION *: Deepika Menon, M.Sc Mathematics, Cochin University of Science and Technology*

Find d, then keep adding it.

- **Verify first:** each list passes the constant-gap test with $d = \frac{1}{4}, -\frac{1}{3}, \sqrt{3}, 1, a + 1$ in turn.
- **Extend mechanically:** once d is known, the next three terms come from adding it three more times.
- **Letters are harmless:** the symbolic lists (iv) and (v) look fearsome but behave like any AP, since the gaps 1 and $a + 1$ stay fixed regardless of the symbols, so the same add-on rule finishes them.

> **Final Answer:** Next three terms found by adding the verified d thrice in each part (see boxed list above).

Q 5.3 **Write the first three terms of the APs when a and d are as given below:**

(i) $a = \frac{1}{2},\ d = -\frac{1}{6}$ **(ii)** $a = -5,\ d = -3$ **(iii)** $a = \sqrt{2},\ d = \frac{1}{\sqrt{2}}$

SOLUTION

Concept used. The first three terms of an AP are a, $a+d$ and $a+2d$. Substitute the given a and d.

Step 1. (i) $a=\frac{1}{2}$; $a+d=\frac{1}{2}-\frac{1}{6}=\frac{3-1}{6}=\frac{1}{3}$; $a+2d=\frac{1}{2}-\frac{2}{6}=\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$. Terms: $\frac{1}{2},\frac{1}{3},\frac{1}{6}$.

Step 2. (ii) $a=-5$; $a+d=-5-3=-8$; $a+2d=-5-6=-11$. Terms: $-5,-8,-11$.

Step 3. (iii) $a=\sqrt{2}$; $a+d=\sqrt{2}+\frac{1}{\sqrt{2}}=\frac{2+1}{\sqrt{2}}=\frac{3}{\sqrt{2}}$; $a+2d=\sqrt{2}+\frac{2}{\sqrt{2}}=\frac{2+2}{\sqrt{2}}=\frac{4}{\sqrt{2}}$. Terms: $\sqrt{2},\frac{3}{\sqrt{2}},\frac{4}{\sqrt{2}}$.

Final Answer: (i) $\frac{1}{2},\frac{1}{3},\frac{1}{6}$; (ii) $-5,-8,-11$; (iii) $\sqrt{2},\frac{3}{\sqrt{2}},\frac{4}{\sqrt{2}}$.

☞ Common denominator helps

For fraction or surd d, write each new term over a common denominator (6 in part (i), $\sqrt{2}$ in part (iii)) so the addition is clean.

EXPERT'S SOLUTION *: Sameer Khan, M.Sc Mathematics, Aligarh Muslim University*

Three plug-ins, nothing more.

- **The recipe:** the first three terms are simply a, then $a+d$, then $a+2d$.
- **Part by part:** the sixths in (i) combine to $\frac{1}{2},\frac{1}{3},\frac{1}{6}$, and the steps of -3 in (ii) give $-5,-8,-11$.
- **Surd part:** in (iii), writing everything over $\sqrt{2}$ turns $\sqrt{2}+\frac{1}{\sqrt{2}}$ into $\frac{3}{\sqrt{2}}$ and the next into $\frac{4}{\sqrt{2}}$, with no rule beyond adding d each time.

Final Answer: First three terms as listed in the box above.

Q 5.4 **Find a, b and c such that the following numbers are in AP: $a,\ 7,\ b,\ 23,\ c$.**

SOLUTION

Concept used. In an AP, the gap d between neighbours is constant. The terms 7 and 23 are two steps apart, which fixes d, and then the others follow.

Step 1. 7 and 23 are separated by two steps, so

$$2d=23-7=16$$

$$d=8.$$

Step 2. a is one step before 7: $a=7-d=7-8=-1$.

Step 3. b is one step after 7: $b = 7 + d = 7 + 8 = 15$.

Step 4. c is one step after 23: $c = 23 + d = 23 + 8 = 31$.

> **Final Answer:** $a = -1,\ b = 15,\ c = 31$ (with $d = 8$).

☞ **Equally spaced positions**

In $a, 7, b, 23, c$ the positions are $1, 2, 3, 4, 5$, so each adjacent pair differs by the same d. Two known terms two apart pin d down.

EXPERT'S SOLUTION *: Ritika Malhotra, M.Sc Applied Mathematics, Delhi Technological University*

Anchor on the two known terms.

- **Fix the step:** the terms 7 at position 2 and 23 at position 4 sit two steps apart, so $2d = 16$ and $d = 8$.
- **Fill the gaps:** a is one step below 7 giving -1, b is one step above 7 giving 15, and c is one step above 23 giving 31.
- **Check:** the full list $-1, 7, 15, 23, 31$ rises by a steady 8, as it should.

> **Final Answer:** $a = -1,\ b = 15,\ c = 31$.

Q 5.5 Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.

SOLUTION

Concept used. Turn each clue into an equation in a and d: $a_5 = 19$ and $a_{13} - a_8 = 20$. Solve the pair.

Step 1. Clue 2: $a_{13} - a_8 = (a + 12d) - (a + 7d) = 5d = 20$, so $d = 4$.

Step 2. Clue 1: $a_5 = a + 4d = 19$, so $a + 4 \times 4 = 19$.

$$a + 16 = 19$$

$$a = 3.$$

Step 3. The AP is $a, a + d, a + 2d, \cdots = 3, 7, 11, 15, \ldots$

> **Final Answer:** The AP is $3, 7, 11, 15, \ldots$ (with $a = 3,\ d = 4$).

☞ Use the gap clue first

A clue like "$a_{13} - a_8$" gives d on its own, since a cancels. Find d first, then use the other clue to get a.

EXPERT'S SOLUTION *: Anil Kumawat, M.Sc Mathematics, University of Rajasthan*

The difference clue isolates d.

- **Get d free:** subtracting the 8th term from the 13th leaves $5d = 20$, so $d = 4$ at once with no a involved.
- **Then get a:** feeding $d = 4$ into $a_5 = a + 16 = 19$ gives $a = 3$, so the AP starts at 3 and climbs by 4 as $3, 7, 11, 15, \ldots$
- **Order matters:** starting from the gap clue keeps the algebra cleaner than tackling a_5 first.

Final Answer: $3, 7, 11, 15, \ldots$

Q 5.6 **The 26th, 11th and the last term of an AP are 0, 3 and $-\frac{1}{5}$, respectively. Find the common difference and the number of terms.**

SOLUTION

Concept used. Use $a_{26} = 0$ and $a_{11} = 3$ to get a and d, then set the last term equal to $-\frac{1}{5}$ to find the number of terms n.

Step 1. $a_{26} - a_{11} = (a + 25d) - (a + 10d) = 15d$, and this equals $0 - 3 = -3$.

$$15d = -3$$
$$d = -\frac{1}{5}.$$

Step 2. From $a_{11} = a + 10d = 3$: $a + 10\left(-\frac{1}{5}\right) = 3$, so $a - 2 = 3$ and $a = 5$.

Step 3. Last term $a_n = a + (n-1)d = -\frac{1}{5}$:

$$5 + (n-1)\left(-\tfrac{1}{5}\right) = -\tfrac{1}{5}$$
$$(n-1)\left(-\tfrac{1}{5}\right) = -\tfrac{1}{5} - 5 = -\tfrac{26}{5}$$
$$n - 1 = 26, \text{ so } n = 27.$$

Final Answer: Common difference $d = -\frac{1}{5}$ and number of terms $n = 27$.

☞ **Two terms give a and d**

The 26th and 11th terms together fix both a and d. Only after that does the last-term value tell you how many terms there are.

EXPERT'S SOLUTION *: Gauri Sathe, M.Sc Mathematics, Fergusson College Pune*

Build the AP, then count to the end.

- **Shape the AP:** the gap between the 26th and 11th terms is $15d = 0 - 3 = -3$, so $d = -\frac{1}{5}$, and then $a_{11} = a - 2 = 3$ gives $a = 5$.
- **Count the length:** the last term $-\frac{1}{5}$ must satisfy $5 + (n-1)(-\frac{1}{5}) = -\frac{1}{5}$, which unwinds to $n - 1 = 26$, so there are 27 terms.
- **One job each:** two clues shape the AP and the third counts how far it runs.

Final Answer: $d = -\dfrac{1}{5}$, $n = 27$.

Q 5.7 **The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the AP.**

SOLUTION

Concept used. Convert the two clues into equations in a and d and solve the pair.

Step 1. Clue 1: $a_5 + a_7 = (a + 4d) + (a + 6d) = 2a + 10d = 52$, so $a + 5d = 26$.

Step 2. Clue 2: $a_{10} = a + 9d = 46$.

Step 3. Subtract Clue 1 from Clue 2:

$$(a + 9d) - (a + 5d) = 46 - 26$$

$4d = 20$, so $d = 5$.

Step 4. Then $a + 5 \times 5 = 26$, i.e. $a = 26 - 25 = 1$.

Step 5. The AP is $1, 6, 11, 16, \ldots$

Final Answer: The AP is $1, 6, 11, 16, \ldots$ (with $a = 1,\ d = 5$).

☞ **Average two symmetric terms**

$a_5 + a_7 = 2a_6$, so the first clue is really $2a_6 = 52$, i.e. $a_6 = 26$. Spotting this saves a line of algebra.

EXPERT'S SOLUTION *: Pranav Joshi, M.Sc Mathematics, NIT Warangal*

Two linear clues, one subtraction.

- **Use symmetry:** the fifth and seventh terms straddle the sixth, so their sum 52 means $a_6 = 26$.
- **Find the step:** with $a_{10} = 46$, the four-step gap from the 6th to the 10th term is $46 - 26 = 20$, giving $d = 5$.
- **Back to the start:** then $a = a_6 - 5d = 26 - 25 = 1$, so the AP is $1, 6, 11, 16, \ldots$ and the symmetry trick made the first clue almost free.

Final Answer: $1, 6, 11, 16, \ldots$

Q 5.8 **Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12.**

SOLUTION

Concept used. "a_7 is 24 less than a_{11}" means $a_{11} - a_7 = 24$. This gives d; then $a_{20} = a + 19d$.

Step 1. $a_{11} - a_7 = (a + 10d) - (a + 6d) = 4d = 24$, so $d = 6$.

Step 2. First term is given: $a = 12$.

Step 3. Required term: $a_{20} = a + 19d$.

$= 12 + 19 \times 6$

$= 12 + 114$.

Step 4. So $a_{20} = 126$.

Final Answer: The 20th term is $a_{20} = 126$.

✗ Translate "less than" carefully

"a_7 is 24 less than a_{11}" means $a_7 = a_{11} - 24$, i.e. $a_{11} - a_7 = 24$. Reversing it would flip the sign of d.

EXPERT'S SOLUTION *: Sunita Yadav, M.Sc Mathematics, Banasthali Vidyapith*

Decode the wording, then march to term 20.

- **Read the phrase:** "24 less" fixes the four-step gap $a_{11} - a_7 = 4d = 24$, so $d = 6$.

- **March forward:** starting from the given $a = 12$ and adding 19 steps of 6 gives $12 + 114 = 126$.
- **The only trap:** the wording is the hard part, but once it is read as a positive gap of 24 over four steps the arithmetic is straightforward.

Final Answer: $a_{20} = 126$.

Q 5.9 If the 9th term of an AP is zero, prove that its 29th term is twice its 19th term.

SOLUTION

Concept used. Use $a_9 = 0$ to express a in terms of d, then compute a_{29} and a_{19} and compare.

Step 1. Given $a_9 = a + 8d = 0$, so $a = -8d$.

Step 2. $a_{29} = a + 28d = -8d + 28d = 20d$.

Step 3. $a_{19} = a + 18d = -8d + 18d = 10d$.

Step 4. Compare: $a_{29} = 20d = 2 \times 10d = 2\,a_{19}$.

Final Answer: $a_{29} = 20d = 2(10d) = 2\,a_{19}$, as required.

☞ **Anchor at the zero term**

A term equal to 0 is the most useful anchor: it lets you replace a by $-8d$ and turn every other term into a pure multiple of d.

EXPERT'S SOLUTION *: Tarun Bose, M.Sc Mathematics, Jadavpur University Salt Lake*

Measure both terms from the zero.

- **Step from zero:** since the 9th term is 0, counting forward 20 steps reaches the 29th term at $20d$ and 10 steps reaches the 19th term at $10d$.
- **The doubling:** because $20d$ is exactly twice $10d$, the 29th term is twice the 19th.
- **Why it is clean:** the zero term acts like a fresh origin, making the doubling obvious from the step counts alone.

Final Answer: $a_{29} = 2\,a_{19}$.

Q 5.10 **Find whether 55 is a term of the AP: $7, 10, 13, \ldots$ or not. If yes, find which term it is.**

SOLUTION

Concept used. Set $a_n = 55$ in $a_n = a + (n-1)d$ and check whether n is a positive whole number.

Step 1. Here $a = 7$, $d = 10 - 7 = 3$.

Step 2. Set $a_n = 55$:

$$55 = 7 + (n-1) \times 3$$

$$55 - 7 = 3(n-1)$$

$$48 = 3(n-1).$$

Step 3. Solve: $n - 1 = \dfrac{48}{3} = 16$, so $n = 17$, a whole number.

> **Final Answer:** Yes; 55 is the 17th term of the AP.

☞ **Whole n means it is a term**

If solving for n gives a positive integer, the value is a genuine term at that position. A fraction would mean it is not in the list.

EXPERT'S SOLUTION *: Naveen Chandra, M.Sc Mathematics, NIT Calicut*

Test membership through the position.

- **Term rule:** the AP climbs by 3 from 7, so any term is $7 + 3(n-1)$.
- **Solve for n:** demanding this equals 55 gives $3(n-1) = 48$ and $n = 17$, a clean whole number, so 55 is genuinely the 17th term.
- **The test:** had n come out fractional, the value would have fallen between two terms and not belonged to the AP at all.

> **Final Answer:** Yes; the 17th term.

Q 5.11 **Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP.**

SOLUTION

Concept used. Three terms t_1, t_2, t_3 are consecutive AP terms exactly when $t_2 - t_1 = t_3 - t_2$ (the middle is the average of the outer two).

Step 1. Left gap: $t_2 - t_1 = (2k^2 + 3k + 6) - (k^2 + 4k + 8) = k^2 - k - 2$.

Step 2. Right gap: $t_3 - t_2 = (3k^2 + 4k + 4) - (2k^2 + 3k + 6) = k^2 + k - 2$.

Step 3. Set the gaps equal:

$$k^2 - k - 2 = k^2 + k - 2$$

$$-k = k$$

$2k = 0$, so $k = 0$.

Final Answer: $k = 0$ makes the three expressions consecutive AP terms.

☞ **Middle term is the average**

For three AP terms, $2t_2 = t_1 + t_3$. This is the same condition as "equal gaps" and is sometimes faster to apply.

EXPERT'S SOLUTION *: Bhavna Sharma, M.Sc Mathematics, Miranda House Delhi*

Force the two gaps to agree.

- **Find the gaps:** subtracting in pairs gives the left gap as $k^2 - k - 2$ and the right gap as $k^2 + k - 2$.
- **Set them equal:** matching the gaps removes the k^2 and the constant, leaving $-k = k$, so $k = 0$.
- **Why it collapses:** the quadratic clutter looks intimidating but falls apart because the squared terms are identical on both sides.

Final Answer: $k = 0$.

Q 5.12 **Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.**

SOLUTION

Concept used. Represent three AP numbers symmetrically as $a - d$, a, $a + d$ so their sum is $3a$. Then use the product clue to find d.

Step 1. Sum: $(a - d) + a + (a + d) = 3a = 207$, so $a = 69$.

Step 2. The two smaller parts are $a-d$ and a (taking $d>0$). Their product:

$$(a-d)\,a = 4623$$

$$69\,(69-d) = 4623$$

$$69-d = \frac{4623}{69} = 67.$$

Step 3. So $d = 69-67 = 2$, and the parts are $67, 69, 71$.

> **Final Answer:** The three parts are $67,\ 69,\ 71$.

☞ **Symmetric form simplifies sums**

Writing three AP numbers as $a-d, a, a+d$ makes the sum $3a$ instantly, so the middle term drops out of the first equation.

EXPERT'S SOLUTION *: Yogesh Patil, M.Sc Applied Mathematics, Institute of Chemical Technology Mumbai*

Centre on the middle part.

- **Symmetric form:** calling the parts $a-d, a, a+d$ turns the sum 207 into $3a$, so the middle part is $a = 69$ outright.
- **Use the product:** the two smaller parts are $69-d$ and 69, and their product 4623 gives $69-d = 67$, hence $d = 2$.
- **Confirm:** the three numbers are $67, 69, 71$, which add to 207 and have $67 \times 69 = 4623$ as required.

> **Final Answer:** $67,\ 69,\ 71$.

Q 5.13 **The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.**

SOLUTION

Concept used. Write the three angles symmetrically as $a-d,\ a,\ a+d$. Use the angle-sum of a triangle ($180°$) and the "greatest = twice least" condition.

Step 1. Angle sum: $(a-d)+a+(a+d) = 180°$, so $3a = 180°$ and $a = 60°$.

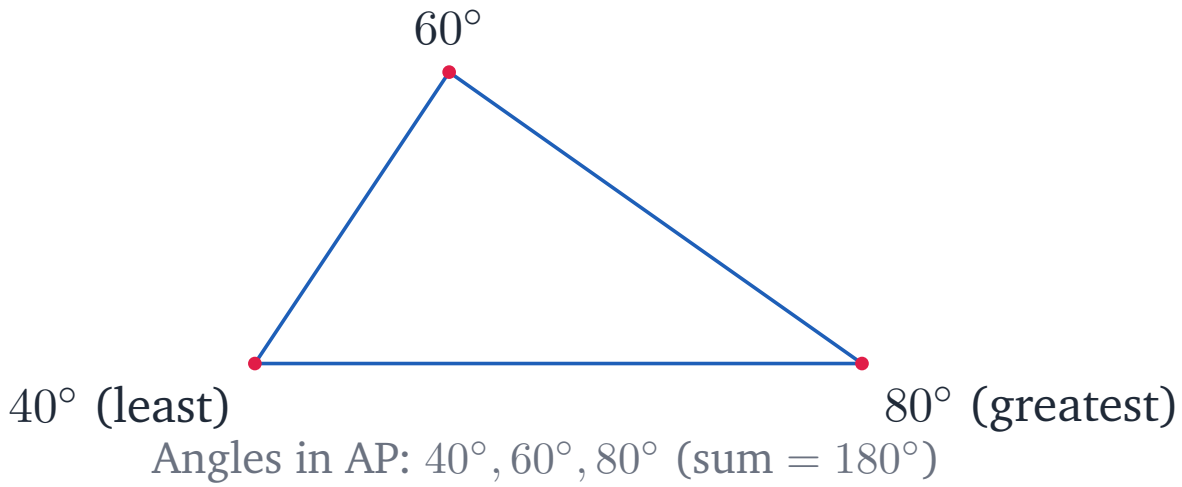
Step 2. Greatest is $a+d$, least is $a-d$. Condition: $a+d = 2(a-d)$.

$$60+d = 2(60-d)$$

$60 + d = 120 - 2d$

$3d = 60$, so $d = 20°$.

Step 3. Angles: $a - d = 40°$, $a = 60°$, $a + d = 80°$.



Final Answer: The angles are $40°,\ 60°,\ 80°$.

☞ **Middle angle is always $60°$**

Whenever a triangle's angles are in AP, the middle angle is $\frac{180°}{3} = 60°$, no matter what d is.

♥ **Why the middle angle is fixed**

The angle sum $180°$ forces $3a = 180°$, so the middle angle is locked at $60°$ for every AP triangle. Only the spread d changes with the extra condition, here pushing the others to $40°$ and $80°$.

EXPERT'S SOLUTION *: Radhika Krishnan, M.Sc Mathematics, Stella Maris College Chennai*

Symmetry pins the middle, the ratio sets the spread.

- **Lock the middle:** three angles in AP sum to $3a = 180°$, so the middle angle is fixed at $60°$ straight away, before the second condition is even used.
- **Set the spread:** the condition that the greatest angle is twice the least becomes $60 + d = 2(60 - d)$, which solves to give the common difference $d = 20°$.
- **Read the angles:** the three angles then spread out to $40°$, $60°$ and $80°$, sitting evenly around the locked middle of $60°$.
- **Both checks pass:** the three angles add back to $180°$ and the largest is indeed twice the smallest since $80 = 2 \times 40$, so the answer is consistent on both counts.

Final Answer: $40°,\ 60°,\ 80°$.

Q 5.14 **If the nth terms of the two APs: $9, 7, 5, \ldots$ and $24, 21, 18, \ldots$ are the same, find the value of n. Also find that term.**

SOLUTION

Concept used. Write the nth term of each AP with $a_n = a + (n-1)d$, set them equal, and solve for n.

Step 1. First AP: $a = 9$, $d = -2$, so $a_n = 9 + (n-1)(-2) = 11 - 2n$.

Step 2. Second AP: $a = 24$, $d = -3$, so $a'_n = 24 + (n-1)(-3) = 27 - 3n$.

Step 3. Set equal:

$$11 - 2n = 27 - 3n$$

$$3n - 2n = 27 - 11$$

$$n = 16.$$

Step 4. The common term: $11 - 2(16) = 11 - 32 = -21$.

Final Answer: $n = 16$, and that common term is -21.

☞ **Make each nth term linear in n**

Reduce both general terms to the form (slope) n+(constant). Equating two linear expressions in n leaves a single easy equation.

EXPERT'S SOLUTION *: Imran Sheikh, M.Sc Mathematics, Jamia Millia Islamia*

Equate two formulas, solve once.

- **Two term rules:** the first list shrinks by 2 giving $11 - 2n$, and the second shrinks by 3 giving $27 - 3n$.
- **Set equal:** they agree when $11 - 2n = 27 - 3n$, which rearranges to $n = 16$.
- **Find the value:** plugging back gives the shared value $11 - 32 = -21$, so the two lists, though starting far apart at 9 and 24, cross at their 16th terms.

Final Answer: $n = 16$, common term $= -21$.

Q 5.15 **If sum of the 3rd and the 8th terms of an AP is 7 and the sum of the 7th and the 14th terms is -3, find the 10th term.**

SOLUTION

Concept used. Turn each clue into an equation in a and d, solve for a and d, then compute $a_{10} = a + 9d$.

Step 1. Clue 1: $a_3 + a_8 = (a + 2d) + (a + 7d) = 2a + 9d = 7$.

Step 2. Clue 2: $a_7 + a_{14} = (a + 6d) + (a + 13d) = 2a + 19d = -3$.

Step 3. Subtract Clue 1 from Clue 2:

$$(2a + 19d) - (2a + 9d) = -3 - 7$$

$10d = -10$, so $d = -1$.

Step 4. From Clue 1: $2a + 9(-1) = 7$, so $2a = 16$ and $a = 8$.

Step 5. Required: $a_{10} = a + 9d = 8 + 9(-1) = 8 - 9 = -1$.

Final Answer: The 10th term is $a_{10} = -1$.

☞ **Subtract paired-sum equations**

Both clues are sums of two terms, giving $2a + \ldots$ each. Subtracting removes $2a$ at once and leaves a single equation in d.

EXPERT'S SOLUTION *: Komal Jain, M.Sc Mathematics, Hansraj College Delhi*

Two paired sums, one clean subtraction.

- **Same head:** each clue doubles up the first term, so both read $2a$ plus a multiple of d.
- **Eliminate:** subtracting them wipes out the $2a$ and gives $10d = -10$, hence $d = -1$ and then $a = 8$.
- **Finish:** the tenth term is $8 + 9(-1) = -1$, and the two-term structure of the clues is exactly what makes the elimination so quick.

Final Answer: $a_{10} = -1$.

Q 5.16 **Find the 12th term from the end of the AP: $-2, -4, -6, \ldots, -100$.**

SOLUTION

Concept used. The kth term from the end is $l-(k-1)d$, where l is the last term and d the common difference.

Step 1. Here $a=-2$, $d=-4-(-2)=-2$, last term $l=-100$, and $k=12$.

Step 2. Write the formula: kth term from end $=l-(k-1)d$.

Step 3. Substitute:

$$=-100-(12-1)(-2)$$

$$=-100-11\times(-2)$$

$$=-100+22.$$

Step 4. So the 12th term from the end is -78.

> **Final Answer:** The 12th term from the end is -78.

☞ **Stepping back flips the sign**

From the end you move against d, so you subtract $(k-1)d$. With a negative d, that subtraction adds a positive amount.

EXPERT'S SOLUTION *: Suresh Babu, M.Sc Mathematics, Osmania University*

Count back from the last term.

- **Reverse the step:** the AP ends at -100 and steps by -2, so reading it backwards means adding 2 each step.
- **Eleven steps back:** the 12th-from-end is 11 steps back from -100, that is $-100+11\times2=-78$.
- **Or just flip it:** reversing the AP into $-100,-98,-96,\ldots$ and taking the 12th entry again gives -78.

> **Final Answer:** -78.

Q 5.17 **Which term of the AP: $53,48,43,\ldots$ is the first negative term?**

SOLUTION

Concept used. Find the smallest n for which $a_n<0$, using $a_n=a+(n-1)d$.

Step 1. Here $a=53$, $d=48-53=-5$. The nth term is

$$a_n = 53 + (n-1)(-5) = 58 - 5n.$$

Step 2. Require $a_n < 0$:

$$58 - 5n < 0$$

$$5n > 58$$

$$n > 11.6.$$

Step 3. The smallest whole number greater than 11.6 is 12.

Step 4. Check: $a_{12} = 58 - 5(12) = 58 - 60 = -2 < 0$, while $a_{11} = 58 - 55 = 3 > 0$.

Final Answer: The 12th term ($a_{12} = -2$) is the first negative term.

☞ **Round up after the inequality**

Solving $a_n < 0$ gives a decimal cutoff. The first negative term is at the next whole number above it (here $11.6 \to 12$), so always round up.

EXPERT'S SOLUTION *: Anjali Deshmukh, M.Sc Mathematics, University of Pune*

Cross from positive to negative.

- **Stay positive:** the terms fall by 5 from 53, so they remain positive while $58 - 5n > 0$, that is up to $n = 11$ with value 3.
- **First dip:** the very next term at $n = 12$ drops to -2, the first one below zero.
- **Inequality route:** solving $n > 11.6$ and rounding up to 12 gives the same answer without listing every term.

Final Answer: The 12th term, $a_{12} = -2$.

Q 5.18 How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?

SOLUTION

Concept used. Numbers leaving remainder 3 on division by 4 form an AP with $d = 4$. Find the first and last such numbers in the range, then count the terms with $n = \dfrac{l - a}{d} + 1$.

Step 1. Smallest number > 10 of the form $4q + 3$: $11 = 4(2) + 3$. So $a = 11$.

Step 2. Largest such number < 300: $299 = 4(74) + 3$. So $l = 299$.

Step 3. These form the AP $11, 15, 19, \ldots, 299$ with $d = 4$.

Step 4. Count the terms:

$$n = \frac{l - a}{d} + 1 = \frac{299 - 11}{4} + 1$$
$$= \frac{288}{4} + 1 = 72 + 1 = 73.$$

Final Answer: There are 73 such numbers.

☞ **Term-count formula**

For an AP, the number of terms is $\dfrac{\text{last} - \text{first}}{d} + 1$. The "+1" counts the first term itself; forgetting it is a common off-by-one error.

EXPERT'S SOLUTION *: Rajat Kulthe, M.Sc Mathematics, NIT Rourkela*

List the endpoints, then count.

- **The pattern:** remainder-3 numbers run $\ldots, 11, 15, 19, \ldots, 299, \ldots$, stepping by 4.
- **Pin the ends:** the first one inside the range is 11 and the last is 299.
- **Count the AP:** from 11 to 299 in steps of 4 there are $\frac{288}{4} + 1 = 73$ numbers, with the endpoints doing the heavy lifting and the $+1$ finishing the count.

Final Answer: 73 numbers.

Q 5.19 **Find the sum of the two middle most terms of the AP:** $-\frac{4}{3}, -1, -\frac{2}{3}, \ldots, 4\frac{1}{3}$.

SOLUTION

Concept used. First find how many terms the AP has. If n is even there are two middle terms, at positions $\frac{n}{2}$ and $\frac{n}{2} + 1$.

Step 1. Here $a = -\frac{4}{3}$, $d = -1 - \left(-\frac{4}{3}\right) = \frac{1}{3}$, and the last term $l = 4\frac{1}{3} = \frac{13}{3}$.

Step 2. Find n from $l = a + (n - 1)d$:

$$\tfrac{13}{3} = -\tfrac{4}{3} + (n - 1)\tfrac{1}{3}$$
$$\tfrac{13}{3} + \tfrac{4}{3} = (n - 1)\tfrac{1}{3}$$
$$\tfrac{17}{3} = (n - 1)\tfrac{1}{3}, \text{ so } n - 1 = 17 \text{ and } n = 18.$$

Step 3. $n = 18$ is even; the middle terms are the 9th and 10th.

$a_9 = a + 8d = -\frac{4}{3} + \frac{8}{3} = \frac{4}{3}$

$a_{10} = a + 9d = -\frac{4}{3} + \frac{9}{3} = \frac{5}{3}$.

Step 4. Sum: $a_9 + a_{10} = \frac{4}{3} + \frac{5}{3} = \frac{9}{3} = 3$.

Final Answer: The two middle terms are $\frac{4}{3}$ and $\frac{5}{3}$; their sum is 3.

☞ **Even n has two middles**

For an even number of terms n, the two middle terms sit at $\frac{n}{2}$ and $\frac{n}{2}+1$. For odd n there is a single middle at $\frac{n+1}{2}$.

EXPERT'S SOLUTION *: Vidya Hegde, M.Sc Mathematics, Mangalore University*

Count first, then grab the centre pair.

- **Find the count:** solving $\frac{13}{3} = -\frac{4}{3} + (n-1)\frac{1}{3}$ gives $n = 18$, an even number of terms.
- **Grab the centre:** the two middle terms are the 9th and 10th, $\frac{4}{3}$ and $\frac{5}{3}$, whose sum is 3.
- **Neat check:** in any AP the two middle terms average the first and last, and $\frac{1}{2}(-\frac{4}{3} + \frac{13}{3}) = \frac{3}{2}$, so their sum is 3, matching.

Final Answer: Sum $= 3$.

Q 5.20 **The first term of an AP is -5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.**

SOLUTION

Concept used. With first term a and last term l known, use $S_n = \frac{n}{2}(a+l)$ to find n, then $l = a + (n-1)d$ to find d.

Step 1. Sum formula: $S_n = \frac{n}{2}(a+l) = 120$.

$120 = \frac{n}{2}(-5 + 45)$

$120 = \frac{n}{2} \times 40 = 20n$.

Step 2. Solve: $n = \frac{120}{20} = 6$.

Step 3. Last term: $l = a + (n-1)d$, so $45 = -5 + (6-1)d$.

$50 = 5d$, hence $d = 10$.

Final Answer: Number of terms $n = 6$ and common difference $d = 10$.

☞ First and last term sum

When both ends and the sum are known, start with $S_n = \frac{n}{2}(a + l)$ to get n. Save the d calculation for after n is fixed.

EXPERT'S SOLUTION *: Faizan Ahmed, M.Sc Applied Mathematics, NIT Srinagar*

Get the count, then the step.

- **Count from the average:** the first-plus-last sum is $-5 + 45 = 40$, so each term averages 20, and dividing the total 120 by that average gives $n = 6$ terms.
- **Then the step:** with six terms spanning -5 to 45 there are five gaps, so $d = \frac{45-(-5)}{5} = 10$.
- **Two formulas:** the sum formula hands over n and the last-term formula then hands over d.

Final Answer: $n = 6$, $d = 10$.

Q 5.21 **Find the sum:**

(i) $1 + (-2) + (-5) + (-8) + \cdots + (-236)$ **(ii)** $\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \cdots$ **upto** n **terms**

(iii) $\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \cdots$ **to** 11 **terms.**

SOLUTION

Concept used. Identify a and d for each AP, find the term count where needed, and apply $S_n = \frac{n}{2}[2a + (n-1)d]$ (or $\frac{n}{2}(a + l)$).

Step 1. (i) $a = 1$, $d = -3$, last term $l = -236$. Find n:

$$-236 = 1 + (n-1)(-3) \Rightarrow -237 = -3(n-1) \Rightarrow n = 80.$$

$$S_{80} = \frac{80}{2}(1 + (-236)) = 40 \times (-235) = -9400.$$

Step 2. (ii) Term k is $4 - \frac{k}{n}$; summing n terms,

$$\sum_{k=1}^{n}\left(4 - \frac{k}{n}\right) = 4n - \frac{1}{n}(1 + 2 + \cdots + n) = 4n - \frac{1}{n} \cdot \frac{n(n+1)}{2}$$

$$= 4n - \frac{n+1}{2} = \frac{8n - n - 1}{2} = \frac{7n - 1}{2}.$$

Step 3. (iii) Each term is $\dfrac{\text{(numerator)}}{a+b}$ with numerators $a-b,\ 3a-2b,\ 5a-3b, \ldots$, an AP of first term $a-b$ and $d = 2a-b$. For 11 terms,

$$\text{numerator sum} = \frac{11}{2}\big[2(a-b) + 10(2a-b)\big] = \frac{11}{2}(22a - 12b) = 11(11a - 6b).$$

So the sum $= \dfrac{11(11a-6b)}{a+b}$.

Final Answer: (i) -9400; (ii) $\dfrac{7n-1}{2}$; (iii) $\dfrac{11(11a-6b)}{a+b}$.

☞ **Pull common factors out of sums**

In (ii) and (iii) the constant part (4 or the fixed denominator $a+b$) factors out, leaving a plain AP sum to handle. Separate the parts before summing.

EXPERT'S SOLUTION *: Megha Tiwari, M.Sc Mathematics, University of Lucknow*

Three sums, three tactics.

- **Numeric AP:** part (i) finds $n = 80$ from the last term -236, then $\frac{80}{2}(1 - 236) = -9400$.
- **Split the sum:** part (ii) breaks as $4n$ minus $\frac{1}{n}$ times the triangular number $\frac{n(n+1)}{2}$, simplifying to $\frac{7n-1}{2}$.
- **Factor the denominator:** part (iii) keeps the fixed denominator $a+b$ aside and sums the numerator AP, first term $a-b$ and $d = 2a-b$, over 11 terms to $11(11a-6b)$.
- **The key:** each part hides a plain AP inside a disguise, and spotting it is what makes all three sums routine.

Final Answer: (i) -9400; (ii) $\frac{7n-1}{2}$; (iii) $\frac{11(11a-6b)}{a+b}$.

Q 5.22 **Which term of the AP: $-2, -7, -12, \ldots$ will be -77? Find the sum of this AP upto the term -77.**

SOLUTION

Concept used. First locate -77 with $a_n = a + (n-1)d$, then sum up to that term using $S_n = \dfrac{n}{2}(a + l)$.

Step 1. Here $a = -2$, $d = -7 - (-2) = -5$. Set $a_n = -77$:

$$-77 = -2 + (n-1)(-5)$$

$$-75 = -5(n-1)$$

$n - 1 = 15$, so $n = 16$.

Step 2. Sum up to the 16th term, with $l = -77$:

$$S_{16} = \frac{16}{2}(-2 + (-77))$$

$$= 8 \times (-79)$$

$$= -632.$$

> **Final Answer:** -77 is the 16th term, and the sum up to it is -632.

☞ Find n before summing

A "sum upto" problem needs the position of the final term first. Once n is known, the first-plus-last sum formula finishes the job.

EXPERT'S SOLUTION *: Karan Bhatia, M.Sc Mathematics, Panjab University Chandigarh*

Two stages: locate, then add.

- **Locate the term:** the AP drops by 5 from -2, so -77 sits where $-5(n-1) = -75$, that is $n = 16$.
- **Add the terms:** averaging the ends gives $\frac{-2+(-77)}{2} = -39.5$, and times 16 that is -632, the same as $\frac{16}{2}(-79) = -632$.
- **Order matters:** pinpointing $n = 16$ first is what makes the sum straightforward.

> **Final Answer:** 16th term; sum $= -632$.

Q 5.23 **If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, \ldots$ form an AP. Also find S_{20}.**

SOLUTION

Concept used. A sequence is an AP if $a_n - a_{n-1}$ is a constant. Then use $S_n = \frac{n}{2}[2a + (n-1)d]$.

Step 1. Compute the common difference:

$$a_n - a_{n-1} = (3 - 4n) - (3 - 4(n-1))$$

$$= (3 - 4n) - (7 - 4n) = -4 \text{ (constant)}.$$

Step 2. Since the difference is the constant -4, the sequence is an AP with $a_1 = 3 - 4 = -1$ and $d = -4$.

Step 3. Sum of 20 terms:

$$S_{20} = \frac{20}{2}\left[2(-1) + (20-1)(-4)\right]$$
$$= 10\left[-2 - 76\right]$$
$$= 10 \times (-78) = -780.$$

Final Answer: The sequence is an AP ($a = -1,\ d = -4$) and $S_{20} = -780$.

☞ **Linear a_n means AP**

$a_n = 3 - 4n$ is linear in n, so its consecutive differences are constant. Any first-degree formula in n defines an AP.

EXPERT'S SOLUTION *: Sarita Nayak, M.Sc Mathematics, Utkal University*

Constant gap proves it, then sum.

- **Prove the AP:** subtracting consecutive terms of $3 - 4n$ always gives -4, so the list is an AP starting at -1 with step -4.
- **Sum twenty:** the bracket is $2(-1) + 19(-4) = -78$, and half of 20 times that is -780.
- **The shortcut:** the linear shape of a_n guarantees the AP, and the standard sum formula does the rest.

Final Answer: AP with $d = -4$; $S_{20} = -780$.

Q 5.24 In an AP, if $S_n = n(4n + 1)$, find the AP.

SOLUTION

Concept used. The nth term comes from the sums by $a_n = S_n - S_{n-1}$, and $a_1 = S_1$. Build the first few terms.

Step 1. $a_1 = S_1 = 1(4 \cdot 1 + 1) = 5$.

Step 2. $S_2 = 2(4 \cdot 2 + 1) = 2 \times 9 = 18$, so $a_2 = S_2 - S_1 = 18 - 5 = 13$.

Step 3. $S_3 = 3(4 \cdot 3 + 1) = 3 \times 13 = 39$, so $a_3 = S_3 - S_2 = 39 - 18 = 21$.

Step 4. Common difference $d = a_2 - a_1 = 13 - 5 = 8$. The AP is $5, 13, 21, \ldots$

Final Answer: The AP is $5, 13, 21, \ldots$ (with $a = 5,\ d = 8$).

☞ Recover terms from sums

Given a sum formula S_n, the term rule is $a_n = S_n - S_{n-1}$, with $a_1 = S_1$ as the starting case. Two or three terms reveal a and d.

EXPERT'S SOLUTION *: Devendra Rathi, M.Sc Mathematics, IIT (BHU) Varanasi*

Differences of sums give the terms.

- **First term:** the first sum is $S_1 = 5$, so $a_1 = 5$.
- **Peel off more:** the next sums 18 and 39 give $a_2 = 18 - 5 = 13$ and $a_3 = 39 - 18 = 21$, a steady step of 8, so the AP is $5, 13, 21, \ldots$
- **Formula check:** the direct route $a_n = S_n - S_{n-1} = 8n - 3$ gives the same terms, confirming $a = 5$ and $d = 8$.

Final Answer: $5, 13, 21, \ldots$

Q 5.25 **In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.**

SOLUTION

Concept used. Find the nth-term rule from $a_n = S_n - S_{n-1}$, then set $a_k = 164$ and solve for k.

Step 1. Compute $a_n = S_n - S_{n-1}$:

$S_n = 3n^2 + 5n$ and $S_{n-1} = 3(n-1)^2 + 5(n-1)$.

$a_n = (3n^2 + 5n) - \left[3(n-1)^2 + 5(n-1)\right]$

$= 3(2n - 1) + 5 = 6n - 3 + 5 = 6n + 2.$

Step 2. Set $a_k = 164$: $6k + 2 = 164$.

$6k = 162$, so $k = 27$.

Final Answer: $k = 27$ (since $a_n = 6n + 2$ and $6k + 2 = 164$).

☞ **a_n is linear from a quadratic S_n**

A quadratic $S_n = An^2 + Bn$ always gives a linear term rule $a_n = 2An - A + B$. Here $a_n = 6n + 2$, so solving $a_k = 164$ is one step.

EXPERT'S SOLUTION *: Tanya Sengupta, M.Sc Mathematics, Lady Shri Ram College Delhi*

Turn the sum into a term rule.

- **Build the rule:** subtracting S_{n-1} from $S_n = 3n^2 + 5n$ collapses the squares to give the clean linear term $a_n = 6n + 2$.
- **Solve for k:** demanding $6k + 2 = 164$ yields $6k = 162$ and $k = 27$.
- **Why this tool:** the quadratic sum hides a simple linear term, which is exactly why $a_n = S_n - S_{n-1}$ is the right approach.

Final Answer: $k = 27$.

Q 5.26 **If S_n denotes the sum of first n terms of an AP, prove that $S_{12} = 3(S_8 - S_4)$.**

SOLUTION

Concept used. Write each sum with $S_n = \frac{n}{2}[2a + (n-1)d]$, then simplify both sides and compare.

Step 1. $S_{12} = \frac{12}{2}[2a + 11d] = 6(2a + 11d) = 12a + 66d.$

Step 2. $S_8 = \frac{8}{2}[2a + 7d] = 4(2a + 7d) = 8a + 28d.$

Step 3. $S_4 = \frac{4}{2}[2a + 3d] = 2(2a + 3d) = 4a + 6d.$

Step 4. Right side: $3(S_8 - S_4) = 3[(8a + 28d) - (4a + 6d)] = 3(4a + 22d) = 12a + 66d.$

Step 5. Left side equals right side: $S_{12} = 12a + 66d = 3(S_8 - S_4)$.

Final Answer: $S_{12} = 12a + 66d = 3(S_8 - S_4)$, so the identity holds.

☞ **Expand all sums to a and d**

Identity proofs become routine once every S_n is written as a multiple of a and d. Then it is just algebra to match the two sides.

EXPERT'S SOLUTION *: Hemant Pawar, M.Sc Mathematics, Shivaji University Kolhapur*

Reduce both sides to a and d.

- **Write the sums:** the three sums expand to $S_{12} = 12a + 66d$, $S_8 = 8a + 28d$ and $S_4 = 4a + 6d$.
- **Form the right side:** the difference $S_8 - S_4 = 4a + 22d$, tripled, is $12a + 66d$, which is exactly S_{12}.
- **The conclusion:** both sides reduce to the same expression in a and d, so the identity holds for every AP whatever its first term and step.

Final Answer: $S_{12} = 3(S_8 - S_4)$.

Q 5.27 **Find the sum of first 17 terms of an AP whose 4th and 9th terms are -15 and -30 respectively.**

SOLUTION

Concept used. Use the two given terms to find a and d, then apply $S_{17} = \frac{17}{2}[2a + 16d]$.

Step 1. $a_9 - a_4 = (a + 8d) - (a + 3d) = 5d$, and this equals $-30 - (-15) = -15$.

$5d = -15$, so $d = -3$.

Step 2. From $a_4 = a + 3d = -15$: $a + 3(-3) = -15$, so $a - 9 = -15$ and $a = -6$.

Step 3. Sum of 17 terms:

$$S_{17} = \frac{17}{2}\left[2(-6) + (17 - 1)(-3)\right]$$

$$= \frac{17}{2}[-12 - 48]$$

$$= \frac{17}{2} \times (-60) = 17 \times (-30) = -510.$$

Final Answer: The sum of the first 17 terms is $S_{17} = -510$.

☞ **Gap of given terms gives d**

$a_9 - a_4 = 5d$, so the difference of two known terms divided by the step count is the common difference. Here $-15/5 = -3$.

EXPERT'S SOLUTION *: Neha Kulshrestha, M.Sc Applied Mathematics, MNNIT Allahabad*

Two terms set up the whole sum.

- **Get the step:** the drop from the 4th to the 9th term, -15 over five steps, gives $d = -3$.
- **Get the start:** then $a_4 = -15$ yields $a = -6$.
- **Sum it:** the bracket for seventeen terms is $2(-6) + 16(-3) = -60$, and half of 17 times that is -510, just one substitution once a and d are known.

Final Answer: $S_{17} = -510$.

Q 5.28 **If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.**

SOLUTION

Concept used. Form two equations from S_6 and S_{16}, solve for a and d, then compute S_{10}.

Step 1. $S_6 = \frac{6}{2}[2a + 5d] = 3(2a + 5d) = 36$, so $2a + 5d = 12$.

Step 2. $S_{16} = \frac{16}{2}[2a + 15d] = 8(2a + 15d) = 256$, so $2a + 15d = 32$.

Step 3. Subtract the first from the second:

$$(2a + 15d) - (2a + 5d) = 32 - 12$$

$10d = 20$, so $d = 2$.

Step 4. Then $2a + 5(2) = 12$, giving $2a = 2$ and $a = 1$.

Step 5. $S_{10} = \frac{10}{2}[2(1) + (10 - 1)(2)] = 5[2 + 18] = 5 \times 20 = 100$.

Final Answer: The sum of the first 10 terms is $S_{10} = 100$.

☞ **Two sums give two equations**

Each given S_n becomes one equation in a and d. With two of them, eliminate a by subtraction to find d first.

EXPERT'S SOLUTION *: Joseph Thomas, M.Sc Mathematics, Loyola College Chennai*

Solve the simultaneous pair.

- **Strip constants:** S_6 and S_{16} reduce to $2a + 5d = 12$ and $2a + 15d = 32$.

- **Eliminate:** subtracting kills the $2a$ and leaves $10d = 20$, so $d = 2$ and $a = 1$.
- **Finish:** the ten-term sum is then $5(2 + 18) = 100$, and forcing each sum into the same $2a$-plus form is what makes the elimination immediate.

> **Final Answer:** $S_{10} = 100$.

Q 5.29 **Find the sum of all the 11 terms of an AP whose middle most term is 30.**

SOLUTION

Concept used. For an odd number of terms, the sum equals (number of terms) $\times$ (middle term), because the middle term is the average of the whole AP.

Step 1. With 11 terms, the middle is the 6th term, so $a_6 = 30$.

Step 2. The sum can be written $S_{11} = \frac{11}{2}(a_1 + a_{11})$.

Step 3. In an AP, $a_1 + a_{11} = 2a_6$ (the ends average to the middle), so

$$S_{11} = \frac{11}{2} \times 2a_6 = 11\, a_6.$$

Step 4. Substitute $a_6 = 30$: $S_{11} = 11 \times 30 = 330$.

> **Final Answer:** The sum of all 11 terms is $S_{11} = 330$.

☞ **Odd terms: sum $= n\times$ middle**

For an odd count n, $S_n = n \times$ (middle term). Here $11 \times 30 = 330$ with no need for a or d.

EXPERT'S SOLUTION *: Pallavi Rane, M.Sc Mathematics, University of Goa*

The middle term carries the average.

- **Symmetry:** in any AP the terms are symmetric about the centre, so the middle term is the mean of them all.
- **Multiply:** with eleven terms and a middle of 30, the total is simply $11 \times 30 = 330$.
- **Why it fits:** this shortcut sidesteps finding a and d entirely, which the question never supplies.

> **Final Answer:** $S_{11} = 330$.

Q 5.30 **Find the sum of last ten terms of the AP:** $8, 10, 12, \ldots, 126$.

SOLUTION

Concept used. Read the AP from the end. The last ten terms form their own AP starting at 126 with common difference -2. Sum those ten.

Step 1. The original AP has $a = 8$, $d = 2$, last term $l = 126$.

Step 2. Reading backward, the last ten terms are $126, 124, 122, \ldots$, an AP with first term 126 and $d = -2$.

Step 3. Sum of these ten terms:

$$S = \frac{10}{2}\left[2(126) + (10-1)(-2)\right]$$
$$= 5\left[252 - 18\right]$$
$$= 5 \times 234 = 1170.$$

Final Answer: The sum of the last ten terms is 1170.

☞ **Turn "last terms" around**

The simplest way to add the last few terms is to reverse the AP, so the last term becomes the new first term and the step flips sign.

EXPERT'S SOLUTION *: Aman Chauhan, M.Sc Mathematics, NIT Kurukshetra*

Reverse, then sum ten.

- **Flip the AP:** the last ten terms read $126, 124, \ldots$, a fresh AP with start 126 and step -2.
- **Sum them:** its sum is $5(252 - 18) = 1170$.
- **Cross-check:** the 10th-from-last term is $126 - 9 \times 2 = 108$, and averaging the ends gives $\frac{126+108}{2} \times 10 = 1170$, the same total.

Final Answer: 1170.

Q 5.31 **Find the sum of first seven numbers which are multiples of 2 as well as of 9. [Hint: Take the LCM of 2 and 9.]**

SOLUTION

Concept used. A number that is a multiple of both 2 and 9 is a multiple of their LCM, 18. The first seven such numbers form an AP with $a = 18$, $d = 18$.

Step 1. $\text{LCM}(2, 9) = 18$, so the numbers are $18, 36, 54, \ldots$ with $a = 18$, $d = 18$, $n = 7$.

Step 2. Sum of the first 7 terms:

$$S_7 = \frac{7}{2}\left[2(18) + (7-1)(18)\right]$$

$$= \frac{7}{2}[36 + 108]$$

$$= \frac{7}{2} \times 144 = 7 \times 72 = 504.$$

Final Answer: The sum of the first seven such numbers is 504.

☞ **Common multiple means LCM multiple**

A number divisible by both 2 and 9 must be divisible by $\text{LCM}(2, 9) = 18$. So the list is simply the multiples of 18.

EXPERT'S SOLUTION *: Swati Pradhan, M.Sc Mathematics, Sambalpur University*

Reduce to multiples of 18**.**

- **Use the LCM:** being a multiple of 2 and of 9 at once is the same as being a multiple of 18, so the seven numbers are $18, 36, \ldots, 126$.
- **Sum directly:** their sum is $\frac{7}{2}(36 + 108) = 504$.
- **Factor instead:** pulling 18 out gives $18(1 + 2 + \cdots + 7) = 18 \times 28 = 504$, the same total.

Final Answer: 504.

Q 5.32 **How many terms of the AP: $-15, -13, -11, \ldots$ are needed to make the sum -55? Explain the reason for double answer.**

SOLUTION

Concept used. Set $S_n = -55$ in $S_n = \frac{n}{2}[2a + (n-1)d]$. This gives a quadratic in n, which can have two valid positive roots.

Step 1. Here $a = -15$, $d = 2$. Write the sum:

$$-55 = \frac{n}{2}[2(-15) + (n-1)(2)]$$
$$-55 = \frac{n}{2}[-30 + 2n - 2]$$
$$-55 = \frac{n}{2}(2n - 32) = n(n - 16).$$

Step 2. Rearrange: $n^2 - 16n + 55 = 0$.

Step 3. Factorise: $(n-5)(n-11) = 0$, so $n = 5$ or $n = 11$.

Step 4. Both are positive whole numbers, so both are valid.

Reason for two answers. The terms from position 6 to 11 are $-5, -3, -1, 1, 3, 5$, which add to 0. So the running sum at $n = 11$ equals the running sum at $n = 5$, both -55.

> **Final Answer:** $n = 5$ or $n = 11$; the extra six terms (positions 6 to 11) sum to 0, leaving the total unchanged.

♥ Why a sum can repeat

When later terms cancel out (here they sum to 0), the running total returns to an earlier value. That is why a single target sum can be reached at two different term counts.

EXPERT'S SOLUTION *: Girish Naidu, M.Sc Mathematics, Andhra University*

A quadratic with two honest roots.

- **Form the quadratic:** the sum condition becomes $n^2 - 16n + 55 = 0$, factoring to $(n-5)(n-11) = 0$, so $n = 5$ or 11.
- **Why both hold:** the terms from the 6th to the 11th, namely $-5, -3, -1, 1, 3, 5$, cancel to 0.
- **The reason:** adding those six extra terms leaves the total of -55 unchanged, which is exactly why two answers appear.

> **Final Answer:** $n = 5$ or 11; terms 6 to 11 sum to zero.

Q 5.33 **The sum of the first n terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first $2n$ terms of another AP whose first term is -30 and the common difference is 8. Find n.**

SOLUTION

Concept used. Write each sum with $S = \frac{(\text{count})}{2}[2a + (\text{count} - 1)d]$, set them equal, and solve for n.

Step 1. First AP, n terms ($a = 8$, $d = 20$):

$$S = \frac{n}{2}[2(8) + (n-1)(20)] = \frac{n}{2}[16 + 20n - 20] = \frac{n}{2}(20n - 4) = n(10n - 2) = 10n^2 - 2n.$$

Step 2. Second AP, $2n$ terms ($a = -30$, $d = 8$):

$$S' = \frac{2n}{2}[2(-30) + (2n-1)(8)] = n[-60 + 16n - 8] = n(16n - 68) = 16n^2 - 68n.$$

Step 3. Set $S = S'$:

$$10n^2 - 2n = 16n^2 - 68n$$

$$0 = 6n^2 - 66n = 6n(n - 11).$$

Step 4. Since $n \neq 0$, we get $n = 11$.

Final Answer: $n = 11$.

☞ Mind the different counts

One AP runs to n terms, the other to $2n$. Substitute the right count into each sum formula before equating, or the algebra will be off.

EXPERT'S SOLUTION *: Ruchi Mittal, M.Sc Mathematics, Kirori Mal College Delhi*

Two sum expressions, set equal.

- **Write both sums:** the first AP's n-term sum is $10n^2 - 2n$, while the second AP's $2n$-term sum is $16n^2 - 68n$.
- **Equate:** tidying gives $6n^2 - 66n = 0$, that is $6n(n - 11) = 0$, so discarding the useless $n = 0$ leaves $n = 11$.
- **The one trap:** the second sum runs to $2n$ terms, not n, and using that count carefully is the only subtlety.

Final Answer: $n = 11$.

Q 5.34 **Kanika was given her pocket money on Jan 1st, 2008. She puts Re 1 on Day 1, Rs 2 on Day 2, Rs 3 on Day 3, and continued doing so till the end of the month, into her piggy bank. She also spent Rs 204 of her pocket money, and found that at the end**

of the month she still had Rs 100 with her. How much was her pocket money for the month?

SOLUTION

Concept used. The daily deposits $1, 2, 3, \ldots, 31$ form an AP; their sum is what she saved. Then pocket money = saved + spent + left over.

Step 1. January has 31 days, so she saved $1 + 2 + 3 + \cdots + 31$, an AP with $a = 1$, $d = 1$, $n = 31$.

Step 2. Sum saved:

$$S_{31} = \frac{31}{2}(1 + 31) = \frac{31}{2} \times 32 = 31 \times 16 = 496.$$

Step 3. Her pocket money is split into saved, spent and remaining:

$$\text{pocket money} = 496 + 204 + 100.$$

$$= 800.$$

Final Answer: Kanika's pocket money for the month was Rs 800.

☞ **Sum 1 to n quickly**

$1 + 2 + \cdots + n = \dfrac{n(n+1)}{2}$. For January, $\dfrac{31 \times 32}{2} = 496$ without listing all the days.

EXPERT'S SOLUTION *: Abhishek Dubey, M.Sc Applied Mathematics, IIIT Hyderabad*

Account for every rupee.

- **Sum the savings:** the piggy-bank deposits add to $1 + 2 + \cdots + 31 = 496$ using $\frac{31 \times 32}{2}$.
- **List the parts:** the pocket money had to cover three things, the 496 saved, the 204 spent and the 100 still in hand.
- **Add up:** summing them gives $496 + 204 + 100 = 800$, with the AP handling only the savings and the rest being plain bookkeeping.

Final Answer: Rs 800.

Q 5.35 **Yasmeen saves Rs 32 during the first month, Rs 36 in the second month and Rs 40 in the third month. If she continues to save in this manner, in how many months will she save Rs 2000?**

SOLUTION

Concept used. Monthly savings form an AP ($a = 32$, $d = 4$). Set the total $S_n = 2000$ and solve the resulting quadratic for n.

Step 1. Write the sum:

$$2000 = \frac{n}{2}[2(32) + (n-1)(4)]$$

$$2000 = \frac{n}{2}[64 + 4n - 4]$$

$$2000 = \frac{n}{2}(4n + 60) = n(2n + 30).$$

Step 2. Expand and rearrange:

$$2000 = 2n^2 + 30n$$

$$n^2 + 15n - 1000 = 0.$$

Step 3. Factorise: $(n - 25)(n + 40) = 0$, so $n = 25$ or $n = -40$.

Step 4. A month count cannot be negative, so $n = 25$.

> **Final Answer:** Yasmeen will save Rs 2000 in 25 months.

✗ Reject the negative root

A quadratic for n often gives one negative root. Since n counts months, discard $n = -40$ and keep only the positive $n = 25$.

EXPERT'S SOLUTION *: Sphurti Kale, M.Sc Mathematics, SP College Pune*

Set total savings to the target.

- **Model the savings:** saving $32, 36, 40, \ldots$ is an AP with step 4, and its n-month total is $n(2n + 30)$.
- **Form the quadratic:** setting this to 2000 gives $n^2 + 15n - 1000 = 0$, which factors as $(n - 25)(n + 40) = 0$.
- **Pick the root:** only the positive root $n = 25$ makes sense for a month count, so it takes 25 months to reach Rs 2000.

> **Final Answer:** 25 months.

Related Collegedunia resources: pair these NCERT Exemplar Class 10 Maths solutions for Arithmetic Progressions with the chapter Notes, the NCERT Solutions, and the all-formulas sheet to revise the whole unit before your board exam.

NCERT Exemplar Solutions

Solved NCERT Exemplar Class 10 Maths Chapter 5, Arithmetic Progressions: every question, step by step (2026-27)

Chapter 5: Arithmetic Progressions

About this Chapter

Full worked solutions to every **NCERT Exemplar** problem in Class 10 Maths Chapter 5 Arithmetic Progressions Exercise 5.4. An **AP** adds a fixed **common difference** d to each term. After this set you can find any term with $a_n = a + (n-1)d$, sum n terms, and solve word problems on savings and instalments.

Topics covered: Meaning of an AP and common difference • nth term $a_n = a + (n-1)d$ • Sum of n terms $S_n = \frac{n}{2}[2a+(n-1)d]$ • Sum using first and last term $S_n = \frac{n}{2}(a+l)$ • Middle terms • Term from the end • Word problems on savings, instalments and distances

Quick Formula Sheet

General term (nth term):
$a_n = a + (n-1)d$

Sum of first n terms:
$S_n = \frac{n}{2}[2a + (n-1)d]$

Sum with last term l:
$S_n = \frac{n}{2}(a + l)$

Common difference:
$d = a_n - a_{n-1}$

nth term from the sum:
$a_n = S_n - S_{n-1}$

Also see for this chapter on Collegedunia: NCERT Solutions, Revision Notes, Formula Sheet and the Exemplar Book PDF for Class 10 Maths Chapter 5, Arithmetic Progressions.

IV. Long Answer Questions (Exercise 5.4)

Q 5.1 **The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.**

SOLUTION

Concept used. Express S_5, S_7 and S_{10} with $S_n = \frac{n}{2}[2a + (n-1)d]$. Two clues give two equations in a and d; solve, then compute S_{20}.

Step 1. Clue 1: $S_5 + S_7 = 167$.

$$S_5 = \frac{5}{2}(2a + 4d) = 5(a + 2d) \text{ and } S_7 = \frac{7}{2}(2a + 6d) = 7(a + 3d).$$

$$5(a + 2d) + 7(a + 3d) = 12a + 31d = 167.$$

Step 2. Clue 2: $S_{10} = \frac{10}{2}(2a + 9d) = 5(2a + 9d) = 235$, so $2a + 9d = 47$.

Step 3. Solve the pair $12a + 31d = 167$ and $2a + 9d = 47$. From the second, $a = \frac{47 - 9d}{2}$; substitute:

$$6(47 - 9d) + 31d = 167$$

$$282 - 54d + 31d = 167$$

$$-23d = -115, \text{ so } d = 5 \text{ and } a = \frac{47 - 45}{2} = 1.$$

Step 4. Required: $S_{20} = \frac{20}{2}[2(1) + (20-1)(5)] = 10[2 + 95] = 10 \times 97 = 970$.

> **Final Answer:** The sum of the first twenty terms is $S_{20} = 970$.

☞ **Reduce each sum before combining**

Simplify $S_5 = 5(a + 2d)$ and $S_7 = 7(a + 3d)$ before adding, so Clue 1 lands neatly as $12a + 31d = 167$ rather than a messy fraction.

♥ **Two clues lock the AP**

Each independent sum condition pins one combination of a and d. Two of them fix the whole AP, after which any sum, including S_{20}, follows from one substitution.

EXPERT'S SOLUTION *: Vivek Menon, M.Sc Mathematics, NIT Karnataka*

Set up two equations, solve, then extend.

- **Two clues:** writing the sums out, the first clue becomes $12a + 31d = 167$ and the ten-term clue becomes $2a + 9d = 47$.
- **Solve:** eliminating a gives $d = 5$, then $a = 1$.
- **Extend:** with the AP fixed, the twenty-term sum is $10(2 + 95) = 970$, so the work is two linear equations followed by one clean plug-in.

Final Answer: $S_{20} = 970$.

Q 5.2 **Find the (i) sum of those integers between 1 and 500 which are multiples of 2 as well as of 5; (ii) sum of those integers from 1 to 500 which are multiples of 2 as well as of 5; (iii) sum of those integers from 1 to 500 which are multiples of 2 or 5.**

SOLUTION

Concept used. "Multiple of 2 and 5" means multiple of $\mathrm{LCM}(2,5) = 10$. Sum each AP with $S_n = \frac{n}{2}(a+l)$, and for part (iii) use multiples of 2 plus multiples of 5 minus multiples of 10.

Step 1. (i) Between 1 and 500 (endpoints excluded): $10, 20, \ldots, 490$, with $a = 10$, $d = 10$, $l = 490$. Number of terms $n = \dfrac{490 - 10}{10} + 1 = 49$.

$$S = \frac{49}{2}(10 + 490) = \frac{49}{2} \times 500 = 49 \times 250 = 12250.$$

Step 2. (ii) From 1 to 500 (now 500 is included): $10, 20, \ldots, 500$, so $n = 50$.

$$S = \frac{50}{2}(10 + 500) = 25 \times 510 = 12750.$$

Step 3. (iii) Multiples of 2 from 1 to 500: $2, \ldots, 500$, $n = 250$, sum $= \dfrac{250}{2}(2 + 500) = 125 \times 502 = 62750$.

Multiples of 5: $5, \ldots, 500$, $n = 100$, sum $= \dfrac{100}{2}(5 + 500) = 50 \times 505 = 25250$.

Multiples of 10 (counted twice) $= 12750$ from part (ii).

So multiples of 2 or $5 = 62750 + 25250 - 12750 = 75250$.

Final Answer: (i) 12250; (ii) 12750; (iii) 75250.

✗ "And" means LCM, "or" means add then subtract

"Multiple of 2 and 5" is multiples of 10. "Multiple of 2 or 5" needs inclusion-exclusion: add the two lists, then subtract the overlap (multiples of 10) once.

♥ Why subtract the overlap

Multiples of 10 are counted in both the "multiples of 2" list and the "multiples of 5" list, so adding the two lists double-counts them. Subtracting them once corrects the total in part (iii).

EXPERT'S SOLUTION *: Damini Rawat, M.Sc Mathematics, Doon University Dehradun*

Three sums tied together by LCM.

- **Two near-twins:** parts (i) and (ii) are both multiples of 10, differing only in whether the last term 500 is counted, so 49 terms sum to 12250 and 50 terms sum to 12750.
- **Inclusion-exclusion:** part (iii) wants the "or" count, so add the multiples of 2 giving 62750 and the multiples of 5 giving 25250, then remove the doubly counted multiples of 10.
- **Finish:** subtracting that overlap of 12750 once leaves 75250, and this overlap step is the heart of the third part.

Final Answer: (i) 12250; (ii) 12750; (iii) 75250.

Q 5.3 **The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15th term.**

SOLUTION

Concept used. Turn each sentence into an equation in a and d, solve the pair, then evaluate $a_{15} = a + 14d$.

Step 1. "a_8 is half a_2": $a + 7d = \frac{1}{2}(a + d)$. Multiply by 2:

$2a + 14d = a + d$, so $a = -13d$.

Step 2. "a_{11} exceeds one third of a_4 by 1": $a + 10d = \frac{1}{3}(a + 3d) + 1$. Multiply by 3:

$3a + 30d = a + 3d + 3$

$2a + 27d = 3$.

Step 3. Substitute $a = -13d$: $2(-13d) + 27d = 3$, i.e. $-26d + 27d = 3$, so $d = 3$ and $a = -13 \times 3 = -39$.

Step 4. Required: $a_{15} = a + 14d = -39 + 14 \times 3 = -39 + 42 = 3$.

Final Answer: The 15th term is $a_{15} = 3$.

☞ Clear fractions early

Multiply each condition by its denominator (2 then 3) at the start, so you work with whole-number coefficients of a and d throughout.

♥ Negative a is fine

The first term comes out as -39, which is allowed; an AP can start negative and rise. The 15th term then lands exactly on 3, showing the list has climbed back through zero.

EXPERT'S SOLUTION *: Prakash Iyengar, M.Sc Mathematics, Bangalore University*

Translate, clear fractions, solve.

- **First clue:** doubling the "eighth is half the second" sentence clears the half and gives the relation $a = -13d$.
- **Second clue:** tripling the "exceeds one third by one" sentence clears the third and gives $2a + 27d = 3$.
- **Combine:** substituting the first relation into the second leaves $d = 3$ and then $a = -39$, so the fifteenth term is $-39 + 14(3) = 3$, with each English clause turning into one tidy equation once the denominators are gone.

Final Answer: $a_{15} = 3$.

Q 5.4 **An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.**

SOLUTION

Concept used. For 37 terms, the middle is the 19th term. Three consecutive terms sum to $3\times$(their middle term). Use both clues to find a and d.

Step 1. The three middle terms are the 18th, 19th, 20th, summing to $3a_{19} = 225$, so $a_{19} = 75$, i.e. $a + 18d = 75$.

Step 2. The last three terms are the 35th, 36th, 37th, summing to $3a_{36} = 429$, so $a_{36} = 143$, i.e. $a + 35d = 143$.

Step 3. Subtract the first from the second:

$$(a + 35d) - (a + 18d) = 143 - 75$$

$17d = 68$, so $d = 4$.

Step 4. Then $a + 18(4) = 75$, giving $a = 75 - 72 = 3$.

Step 5. The AP is $3, 7, 11, 15, \ldots$

Final Answer: The AP is $3, 7, 11, 15, \ldots$ (with $a = 3,\ d = 4$).

☞ **Three terms average to the middle**

Any three consecutive AP terms sum to three times the central one. So $225 = 3a_{19}$ and $429 = 3a_{36}$, turning each clue into a single term.

♥ **Why pick the 19th as middle**

With an odd 37 terms, the exact centre is term $\frac{37+1}{2} = 19$. Knowing the middle position lets you replace the "three middle terms" sum with one term, a_{19}.

EXPERT'S SOLUTION *: Shalini Verghese, M.Sc Mathematics, Mount Carmel College Bengaluru*

Collapse each triple to its centre.

- **Middle triple:** the three central terms average to the 19th, so the sum $225 = 3a_{19}$ gives $a_{19} = 75$ at once.
- **Last triple:** the same trick on the final three gives $a_{36} = 143$, two clean single-term facts instead of six.
- **Solve:** the gap $a_{36} - a_{19} = 17d = 68$ yields $d = 4$, and back-substitution gives $a = 3$, so the AP is $3, 7, 11, 15, \ldots$ with the triple-to-centre shortcut clearing all the clutter.

Final Answer: $3, 7, 11, 15, \ldots$

Q 5.5 **Find the sum of the integers between 100 and 200 that are (i) divisible by 9; (ii) not divisible by 9.**

SOLUTION

Concept used. The integers divisible by 9 form an AP. For part (ii), subtract that sum from the sum of all integers from 101 to 199.

Step 1. (i) Multiples of 9 between 100 and 200: $108, 117, \ldots, 198$, with $a = 108$, $d = 9$, $l = 198$. Number of terms $n = \dfrac{198 - 108}{9} + 1 = 11$.

$$S_9 = \frac{11}{2}(108 + 198) = \frac{11}{2} \times 306 = 11 \times 153 = 1683.$$

Step 2. (ii) Sum of all integers from 101 to 199: $a = 101$, $l = 199$, $n = 99$.

$$S_{\text{all}} = \frac{99}{2}(101 + 199) = \frac{99}{2} \times 300 = 99 \times 150 = 14850.$$

Step 3. Not divisible by $9 = S_{\text{all}} - S_9 = 14850 - 1683 = 13167$.

Final Answer: (i) 1683; (ii) 13167.

☞ Total minus the wanted part

"Not divisible by 9" is easiest as (sum of everything) minus (sum of the multiples of 9). Counting the non-multiples directly is far slower.

♥ Complement counting saves work

There are 89 integers not divisible by 9 in this range, an awkward AP to sum head-on. Subtracting the 11 multiples of 9 from the full total sidesteps that, a complement trick worth keeping.

EXPERT'S SOLUTION *: Nitin Bansal, M.Sc Mathematics, Thapar University Patiala*

Sum the easy set, then subtract.

- **Wanted set:** the multiples of 9 between 100 and 200 are eleven terms from 108 to 198, summing to 1683.
- **Whole range:** every integer from 101 to 199 adds up to 14850, an easy first-plus-last sum.
- **Complement:** removing the multiples of 9 from that total leaves $14850 - 1683 = 13167$ for the non-multiples, which is far quicker than summing the 89 scattered non-multiples directly.

Final Answer: (i) 1683; (ii) 13167.

Q 5.6 **The ratio of the 11th term to the 18th term of an AP is $2:3$. Find the ratio of the 5th term to the 21st term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.**

SOLUTION

Concept used. Convert the given ratio into a relation between a and d, then express the required term-ratio and sum-ratio in terms of d.

Step 1. Given $\dfrac{a_{11}}{a_{18}} = \dfrac{a+10d}{a+17d} = \dfrac{2}{3}$. Cross-multiply:

$$3(a+10d) = 2(a+17d)$$

$3a + 30d = 2a + 34d$, so $a = 4d$.

Step 2. Ratio of 5th to 21st term:

$$\frac{a_5}{a_{21}} = \frac{a+4d}{a+20d} = \frac{4d+4d}{4d+20d} = \frac{8d}{24d} = \frac{1}{3}.$$

Step 3. Sums: $S_5 = \frac{5}{2}(2a + 4d) = 5(a + 2d) = 5(6d) = 30d$;
$S_{21} = \frac{21}{2}(2a + 20d) = 21(a + 10d) = 21(14d) = 294d$.

Step 4. Ratio of sums: $\frac{S_5}{S_{21}} = \frac{30d}{294d} = \frac{30}{294} = \frac{5}{49}$.

Final Answer: $\frac{a_5}{a_{21}} = 1 : 3$ and $\frac{S_5}{S_{21}} = 5 : 49$.

☞ **Ratios make a proportional to d**

A term ratio gives one equation, here $a = 4d$. After that, every term and every sum becomes a pure multiple of d, and the d cancels in any ratio.

♥ **Why d disappears**

Both numerator and denominator of each required ratio end up as a number times d, so d cancels. That is why a single proportional relation $a = 4d$ is enough to pin down both ratios.

EXPERT'S SOLUTION *: Asmita Ghosh, M.Sc Mathematics, Bethune College Kolkata*

One relation powers both ratios.

- **The key relation:** the given $2 : 3$ term ratio cross-multiplies and rearranges down to the single fact $a = 4d$.
- **Term ratio:** substituting makes the fifth and twenty-first terms $8d$ and $24d$, which reduces to the ratio $1 : 3$.
- **Sum ratio:** likewise the sums become $30d$ and $294d$, a ratio of $5 : 49$, and because d cancels each time the answers come out as pure numbers needing no value of d.

Final Answer: $1 : 3$ and $5 : 49$.

Q 5.7 **Show that the sum of an AP whose first term is a, the second term b and the last term c, is equal to $\frac{(a + c)(b + c - 2a)}{2(b - a)}$.**

SOLUTION

Concept used. The common difference is $d = b - a$. Find the number of terms n from the last term c, then use $S_n = \frac{n}{2}(a + c)$ and simplify.

Step 1. Common difference: $d = b - a$.

Step 2. Last term: $c = a + (n-1)d = a + (n-1)(b-a)$, so

$$c - a = (n-1)(b-a)$$

$$n - 1 = \frac{c-a}{b-a}, \text{ hence } n = \frac{c-a}{b-a} + 1 = \frac{c-a+b-a}{b-a} = \frac{b+c-2a}{b-a}.$$

Step 3. Sum with first term a and last term c:

$$S_n = \frac{n}{2}(a+c) = \frac{1}{2} \cdot \frac{b+c-2a}{b-a} \cdot (a+c).$$

Step 4. So $S_n = \dfrac{(a+c)(b+c-2a)}{2(b-a)}$, as required.

Final Answer: $S_n = \dfrac{(a+c)(b+c-2a)}{2(b-a)}$.

☞ Letters behave like numbers

Treat a, b, c exactly as you would numeric values: $d = b - a$, find n, then sum. The algebra is identical, just symbolic.

♥ A reusable closed form

This formula gives the sum straight from the first, second and last terms, with no need to count n separately. It is handy whenever a problem hands you exactly those three values.

EXPERT'S SOLUTION *: Rakesh Meena, M.Sc Mathematics, Central University of Rajasthan*

Build n, then substitute into the sum.

- **Step size:** the second term being b fixes the common difference as $d = b - a$, the only step the proof needs.
- **Count the terms:** the last-term equation $c = a + (n-1)(b-a)$ unwinds to $n = \frac{b+c-2a}{b-a}$.
- **Substitute:** feeding this into $S_n = \frac{n}{2}(a+c)$ produces the target form straight away, so the proof is just two standard AP formulas chained together with symbols in place of numbers.

Final Answer: $S_n = \dfrac{(a+c)(b+c-2a)}{2(b-a)}$.

Q 5.8 **Solve the equation $-4 + (-1) + 2 + \cdots + x = 437$.**

SOLUTION

Concept used. The left side is an AP with $a = -4$, $d = 3$, last term x. Use $S_n = \frac{n}{2}[2a + (n-1)d] = 437$ to find n, then $x = a + (n-1)d$.

Step 1. Here $a = -4$, $d = -1 - (-4) = 3$. Write the sum:

$$437 = \frac{n}{2}[2(-4) + (n-1)(3)]$$

$$437 = \frac{n}{2}[-8 + 3n - 3]$$

$$437 = \frac{n}{2}(3n - 11).$$

Step 2. Clear the fraction: $874 = n(3n - 11)$, i.e. $3n^2 - 11n - 874 = 0$.

Step 3. Solve by the quadratic formula:

$$n = \frac{11 \pm \sqrt{121 + 4 \cdot 3 \cdot 874}}{6} = \frac{11 \pm \sqrt{10609}}{6} = \frac{11 \pm 103}{6}.$$

Step 4. This gives $n = 19$ or $n = -\frac{92}{6}$; reject the negative, so $n = 19$.

Step 5. Last term: $x = a + (n-1)d = -4 + (19-1)(3) = -4 + 54 = 50$.

> **Final Answer:** $n = 19$ and the last term is $x = 50$.

✗ Keep only the whole-number root

The quadratic gives two roots, but n counts terms, so the negative (and non-integer) root is rejected. Only $n = 19$ is valid, giving $x = 50$.

♥ The unknown is the last term

Here x is not a stray variable but the final term of the AP. Finding how many terms reach the sum 437 tells you which term x is, and then its value follows.

EXPERT'S SOLUTION *: Pooja Hegde, M.Sc Applied Mathematics, VNIT Nagpur*

Sum to 437, then read off the last term.

- **Set up the sum:** the series steps by 3 from -4, so its n-term sum is $\frac{n}{2}(3n - 11)$.
- **Solve the quadratic:** setting this equal to 437 gives $3n^2 - 11n - 874 = 0$, whose only positive root is $n = 19$.
- **Read off x:** the nineteenth term is $-4 + 18 \times 3 = 50$, which is the unknown x, so the equation holds exactly when the running sum reaches its 19th term.

Final Answer: $x = 50$ (at $n = 19$).

Q 5.9 **Jaspal Singh repays his total loan of Rs 118000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month, what amount will be paid by him in the 30th instalment? What amount of loan does he still have to pay after the 30th instalment?**

SOLUTION

Concept used. The instalments form an AP with $a = 1000$, $d = 100$. The 30th instalment is a_{30}, and the loan repaid in 30 months is S_{30}; subtract that from the total loan for the balance.

Step 1. 30th instalment: $a_{30} = a + (30 - 1)d = 1000 + 29 \times 100$.

$= 1000 + 2900 = 3900$.

Step 2. Total repaid in 30 months:

$$S_{30} = \frac{30}{2}[2(1000) + (30 - 1)(100)]$$

$$= 15\,[2000 + 2900]$$

$$= 15 \times 4900 = 73500.$$

Step 3. Balance still to pay $= 118000 - 73500 = 44500$.

Final Answer: The 30th instalment is Rs 3900, and Rs 44500 of the loan remains after it.

☞ Two answers, two formulas

"Amount in the 30th instalment" needs the nth-term formula; "loan still to pay" needs the sum formula. Match each question to the right tool.

♥ Rising instalments clear loans faster

Because each instalment grows by Rs 100, the monthly payments speed up repayment compared with fixed instalments. After just 30 months, Rs 73500 of the Rs 118000 loan is already cleared.

EXPERT'S SOLUTION *: Sandeep Yadav, M.Sc Mathematics, Kurukshetra University*

One term, one sum, one subtraction.

- **The 30th payment:** the instalments climb by Rs 100 each month, so the thirtieth is $1000 + 29 \times 100 = 3900$.
- **Total paid:** the sum of all thirty instalments is $S_{30} = 15 \times 4900 = 73500$, the money handed over so far.
- **Balance left:** taking that off the Rs 118000 loan leaves Rs 44500 outstanding, so the term formula answers the first part and the sum formula, minus the total, answers the second.

Final Answer: 30th instalment Rs 3900; balance Rs 44500.

Q 5.10 **The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?**

SOLUTION

Concept used. With 27 flags, the middle is the 14th, leaving 13 flags on each side at $2, 4, \ldots, 26$ m. For each side, the round-trip distances form an AP; sum them, double for both sides, and the longest single carry is to the farthest flag.

Step 1. The 14th flag sits at the store; on each side there are 13 flags at distances $2, 4, 6, \ldots, 26$ m.

Step 2. Carrying one flag to a flag at distance x and walking back costs $2x$ m. For one side the total is $2(2 + 4 + 6 + \cdots + 26)$.

Step 3. The bracket is an AP with $a = 2$, $d = 2$, $n = 13$:

$$2 + 4 + \cdots + 26 = \frac{13}{2}(2 + 26) = \frac{13}{2} \times 28 = 13 \times 14 = 182.$$

Step 4. One side total $= 2 \times 182 = 364$ m. Both sides $= 2 \times 364 = 728$ m.

Step 5. The longest single carry is to the farthest flag, 26 m away (the 13th flag from the centre).

Final Answer: Total distance covered $= 728$ m; maximum distance carrying a flag $= 26$ m.

☞ Round trip doubles the distance

Each delivery is "go and come back", so a flag at distance x costs $2x$ m. That is why the per-side total is $2(2 + 4 + \cdots + 26)$, not just the one-way sum.

♥ Why both sides are equal

The store is at the exact middle, so the layout is symmetric: 13 flags to the left mirror 13 to the right. Computing one side and doubling is valid only because of that symmetry.

EXPERT'S SOLUTION *: Trisha Mukherjee, M.Sc Mathematics, Scottish Church College Kolkata*

Symmetry, round trips, and one AP.

- **The layout:** the middle 14th flag holds the store, so 13 flags lie on each side at distances $2, 4, \ldots, 26$ m.
- **Round trips:** each flag needs a there-and-back walk of $2x$, so one side totals $2(2 + 4 + \cdots + 26) = 2 \times 182 = 364$ m and both sides give 728 m.
- **Longest carry:** the single hardest trip is to the farthest flag at 26 m, with the AP sum $2 + 4 + \cdots + 26 = 182$ doing the main work and the symmetry and doubling wrapped around it.

Final Answer: Total 728 m; longest single carry 26 m.

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