GATE 2026 Aerospace Engineering Question Paper with Solutions

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General Instructions

(i) This booklet contains 63 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 29 single-correct multiple-choice questions, 13 multiple-correct questions and 21 numerical / integer-type questions.

(iii) These questions follow the GATE Aerospace Engineering examination pattern.

(iv) Questions carry 1 or 2 marks. MCQs have negative marking (1/3 or 2/3 of the marks); MSQ and numerical-answer (NAT) questions carry no negative marking.

(v) GATE is a 3-hour paper of 65 questions (100 marks).

(vi) A virtual scientific calculator is provided; physical calculators are not allowed.

(vii) Attempt each question on your own before reviewing the given solution.

(viii) For multiple-correct questions, all correct options must be selected to earn full marks.

(ix) For numerical questions, report the answer rounded exactly as asked.

1. "He often _____ the numbers. False claims are not going to help. Honesty _____ trust", said the manager.

Choose the option with the correct order of words to fill the blanks.

(A) exaggerates; engenders

(B) excels; encourages

(C) aggravates; alleviates

(D) diminishes; eliminates

Correct Answer: (A) exaggerates; engenders

Solution:

This question checks vocabulary in context. Read the three sentences as one connected thought: the manager is criticizing someone for inflating numbers, then stating a general truth about honesty and trust. Each option pair must fit both blanks and keep the logic of the passage intact.

1. **exaggerates; engenders**: "He often exaggerates the numbers" means he often makes the numbers sound bigger or more impressive than they really are. This directly matches the next line, "False claims are not going to help", since exaggeration is a form of a false or inflated claim. "Honesty engenders trust" is also correct: engender means to give rise to or produce, and honesty is exactly what produces trust. Both blanks fit.
2. **excels; encourages**: "Excels" needs a preposition ("excels at" or "excels in"); "He often excels the numbers" is not correct English, so the first blank already fails. "Honesty encourages trust" is at least sensible, but the first blank rules this option out.
3. **aggravates; alleviates**: Aggravate means to make a bad situation worse, and it does not pair naturally with "the numbers" here. Alleviate means to ease pain or a burden, so "honesty alleviates trust" makes no sense since trust is not a burden to be eased.
4. **diminishes; eliminates**: "He often diminishes the numbers" would mean he plays the numbers down, which does not connect with the warning that "false claims are not going to help". Also "honesty eliminates trust" reverses the intended meaning: honesty is supposed to build trust, not remove it.

Only "exaggerates; engenders" keeps both blanks logically consistent with the surrounding sentences, so option (A) is correct.

Let's summarize:

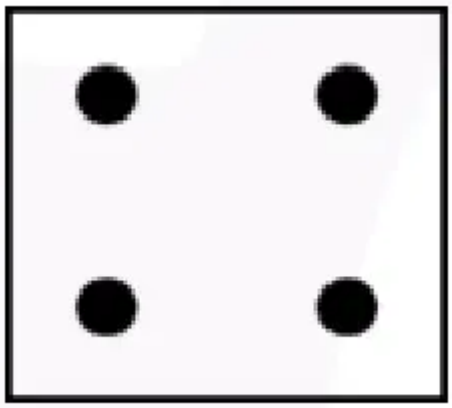
- The first blank needs a word describing inflating or overstating numbers, which is exactly what "false claims" refers to.
- The second blank needs a word meaning "creates" or "gives rise to", since honesty is the cause and trust is the effect.

The correct order of words is "exaggerates; engenders", which is option (A).

Quick Tip: Read the two blanks as linked to "false claims" and to the idea that honesty creates trust; find words that mean "inflating" and "creating".

2. In the sequence of tiles shown below, the missing tile indicated by the question mark should be

(A)



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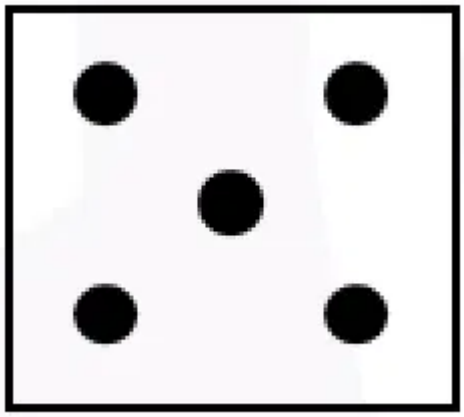
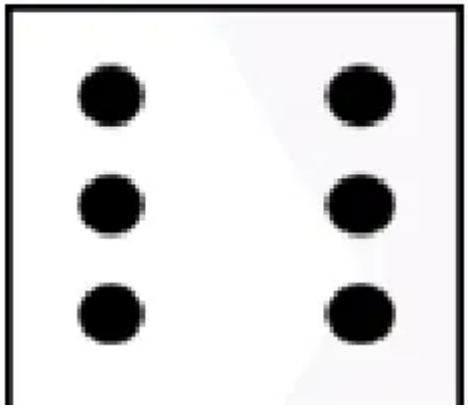
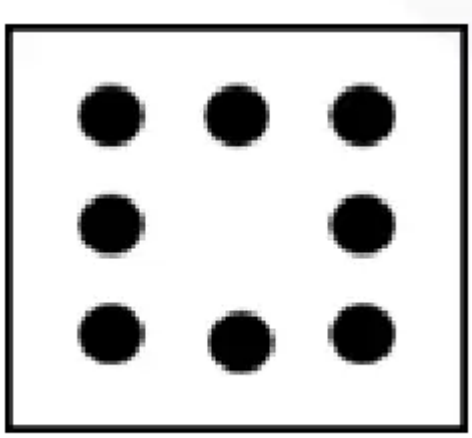


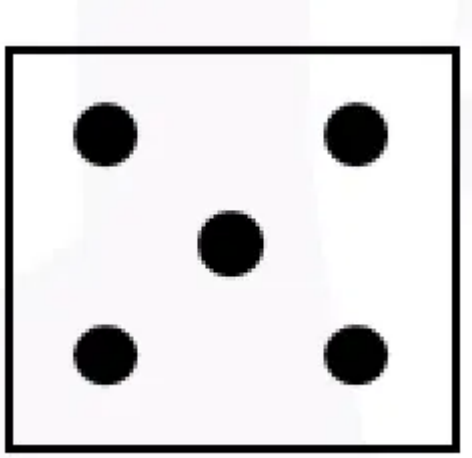
(C)



(D)



Correct Answer: (B)



Solution:

Step 1: Count the dots in each given tile.

The first tile has 2 dots, the second tile has 3 dots, the fourth tile (just after the missing one) has clearly more dots than the second tile, and the fifth tile has even more dots than the fourth. Since each tile is bigger than the one before it, and the jump keeps getting larger, a simple "add 1 each time" rule will not explain the picture; we need a rule that grows faster.

Step 2: Test a Fibonacci-style rule.

A natural way for a growing sequence to speed up the way this one does is for each term to equal the sum of the two terms before it, instead of a fixed common difference. Starting from the first two given tiles, 2 and 3, add them:

$$2 + 3 = 5$$

This predicts the missing third tile should have 5 dots.

Step 3: Check the rule against the remaining tiles.
Continue the same rule forward using the predicted value 5:

$$3 + 5 = 8$$

$$5 + 8 = 13$$

These predicted values, 8 and 13, match the larger and denser dot counts visible in the fourth and fifth tiles of the sequence. Since the rule correctly predicts both tiles that are already given, and not just the missing one, it confirms the underlying pattern really is "each tile's dot count equals the sum of the two tiles before it."

Step 4: Match the predicted count to an option.
The missing tile must show exactly 5 dots. Checking the four answer choices: option (A) shows 4 dots in a 2 by 2 grid, option (B) shows 5 dots arranged as four corner dots with one dot in the centre, the standard five-pattern seen on a die, option (C) shows 6 dots, and option (D) shows 7 dots. Only option (B) has the required count of 5 dots.

Final Answer:
The missing tile has 5 dots, arranged as four corner dots with one centre dot.

Option (B)

Quick Tip: Check whether each tile's dot count equals the sum of the two tiles before it (a Fibonacci-style rule): 2, 3, 5, 8, 13.

3. A school has 100 students distributed among 1^{st} to 10^{th} standards. Based on this, which one of the following statements is always correct?

(A) There are at least 10 students who belong to the same standard.

(B) There is at least one student in each standard.

(C) There are at most 10 students in 10^{th} standard.

(D) The total number of students from 1^{st} to 5^{th} standards is at least 50.

Correct Answer: (A) There are at least 10 students who belong to the same standard.

Solution:

Step 1: Understand what "always correct" means.
The 100 students can be split among the 10 standards in any way at all, some standards could have very few students and others could have a lot, as long as the total is 100. A statement is "always correct" only if it stays true for every single one of these possible splits, not just for one convenient example.

Step 2: Apply the pigeonhole principle to test option (A).
Suppose, for the sake of argument, that no standard has 10 or more students, so every standard has at most 9 students. There are 10 standards, so the largest possible total under this assumption is

$$9 \times 10 = 90$$

This is less than 100, which contradicts the fact that there are 100 students in total. So the assumption "every standard has at most 9 students" cannot be true. This means at least one standard must have 10 or more students, which is exactly what option (A) says. This is the pigeonhole principle: 100 students placed into 10 groups force at least one group to hold at least $\lceil 100/10 \rceil = 10$ students.

Step 3: Show the other options can fail, using specific distributions.
For option (B), put all 100 students in the 1st standard and 0 in every other standard. This is a valid distribution of 100 students among 10 standards, but standards 2nd to 10th have zero students, so "at least one student in each standard" is false here. Option (B) is not always true.
For option (C), put all 100 students in the 10th standard. Then the 10th standard has 100 students, far more than 10, so "at most 10 students in 10th standard" is false. Option (C) is not always true.
For option (D), put all 100 students in standards 6th to 10th (for example, 20 in each of those five standards) and 0 in standards 1st to 5th. Then the total from 1st to 5th standards is 0, which is less than 50, so "at least 50 students from 1st to 5th" is false. Option (D) is not always true.

Step 4: Final Answer.

Only option (A) holds for every possible distribution of the 100 students, because it follows directly from the pigeonhole principle.

Option (A)

Quick Tip: Use the pigeonhole principle: 100 students placed into 10 standards forces at least one standard to have 100/10 = 10 or more students.

4. How many 3-digit numbers can be formed using three distinct single digit prime numbers?

(A) 64

(B) 24

(C) 12

(D) 4

Correct Answer: (B) 24

Solution:

Step 1: List the single digit prime numbers.
A single digit number runs from 0 to 9. The prime numbers among these are 2, 3, 5, and 7, so there are exactly 4 single digit primes to choose from. None of them is 0, so there is no risk of a leading zero in any 3-digit number we build.

Step 2: Understand the condition "three distinct" primes.
The 3-digit number must use three different primes from this list of 4, with no digit repeated. So we are really asking: in how many ways can we pick 3 of these 4 primes and place them in order to form a

number, where changing the order gives a different number (for example, 235 and 253 are different 3-digit numbers).

Step 3: Count using the multiplication principle.
Fill the hundreds place first: any of the 4 primes can go there, giving 4 choices. Once the hundreds digit is fixed, the tens place must be a different prime, so only 3 primes remain, giving 3 choices. Once both of those digits are fixed, the units place must be different from both, so only 2 primes remain, giving 2 choices. Multiplying the choices for each position:

$$4 \times 3 \times 2 = 24$$

Step 4: Rule out the other options.
64 equals 4^3, which would be the count if digits were allowed to repeat, but the question requires distinct digits, so 64 is wrong. 12 and 4 are both smaller than the correct count of arrangements and undercount the actual number of orderings possible.

Final Answer:
The number of 3-digit numbers using three distinct single digit primes is 24.

$$\boxed{24}$$

Quick Tip: The single digit primes are 2, 3, 5, 7. Since the three digits must be distinct, count ordered selections: 4 x 3 x 2.

5. In a group of students, 10 students like Mathematics, 12 students like English, 4 students like both Mathematics and English, and 6 students like neither Mathematics nor English. The number of students in the group is

(A) 18

(B) 20

(C) 24

(D) 32

Correct Answer: (C) 24

Solution:

Step 1: Set up the sets.

Let M be the set of students who like Mathematics and E be the set of students who like English. We are given $|M| = 10$, $|E| = 12$, $|M \cap E| = 4$ (students who like both), and 6 students who like neither Mathematics nor English at all.

Step 2: Find the number of students who like at least one of the two subjects.

Use the inclusion-exclusion rule for two sets, which avoids double-counting the students who like both subjects:

$$|M \cup E| = |M| + |E| - |M \cap E|$$

$$|M \cup E| = 10 + 12 - 4 = 18$$

So 18 students like Mathematics or English (or both).

Step 3: Add the students who like neither subject.

The 6 students who like neither Mathematics nor English are outside the set $M \cup E$ entirely, so they must be added separately to get the full group size:

$$\text{Total} = |M \cup E| + \text{neither} = 18 + 6 = 24$$

Step 4: Final Answer.

The total number of students in the group is 24, which matches option (C).

$$\boxed{24}$$

Quick Tip: Use inclusion-exclusion for the two subjects, then add the students who like neither: (10 + 12 - 4) + 6.

6. Charity : P :: Retaliation : Q

Choose the appropriate pair of words P and Q that fit the analogy.

(A) P = Parsimonious; Q = Vengeful

(B) P = Altruistic; Q = Amicable

(C) P = Resentful; Q = Spiteful

(D) P = Magnanimous; Q = Vindictive

Correct Answer: (D) P = Magnanimous; Q = Vindictive

Solution:

This is a noun to matching adjective analogy. Charity is the noun for the quality of giving generously without expecting anything back, and

Retaliation is the noun for the act of paying back harm with harm. We need P to be the adjective that describes a person who shows charity, and Q to be the adjective that describes a person who is inclined to retaliate, so that both pairs follow the exact same noun to personality-adjective relationship.

1. **P = Parsimonious; Q = Vengeful**: Parsimonious means stingy or unwilling to spend, which is close to the opposite of charitable, not a match for "Charity". Q, "Vengeful", would fit Retaliation, but since P already fails, the pair as a whole is wrong.
2. **P = Altruistic; Q = Amicable**: Altruistic, meaning unselfish and caring for others, does describe someone charitable, so P works. But Amicable means friendly and cooperative, which describes the opposite temperament of someone who retaliates, not someone prone to revenge. Q fails, so the pair is wrong.
3. **P = Resentful; Q = Spiteful**: Resentful means holding a grudge, which has nothing to do with charity, so P fails immediately.
4. **P = Magnanimous; Q = Vindictive**: Magnanimous means generous and big-hearted, exactly the personality trait behind an act of charity, so P is a correct match. Vindictive means having a strong desire for revenge, exactly the personality trait behind an act of retaliation, so Q is also a correct match. Both halves of the analogy line up the same way.

Only option (D) keeps the same noun to matching personality-adjective relationship on both sides of the analogy, so P = Magnanimous and Q = Vindictive is correct.

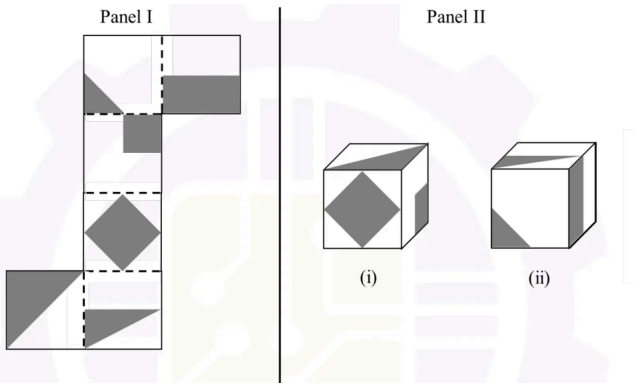
Let's summarize:

- Charity is to Magnanimous as Retaliation is to Vindictive: in both pairs the second word is the adjective describing a person prone to the first word's action.
- The other options break this pattern on at least one side, either P does not match Charity, or Q does not match Retaliation.

The appropriate pair is P = Magnanimous, Q = Vindictive, option (D).

Quick Tip: Find the adjective describing a person's character for each noun: a generous character for Charity, a revenge-seeking character for Retaliation.

7. A paper shown in Panel I is folded along the dashed lines (- - -) to construct a cube. The shaded regions shown in Panel I appear on the outer surface of the cube. Referring to cubes shown in Panel II, which one of the options is correct?



(A) Only (i) can correspond to the unfolded cube in Panel I.

(B) Only (ii) can correspond to the unfolded cube in Panel I.

(C) Both (i) and (ii) can correspond to the unfolded cube in Panel I.

(D) Neither (i) nor (ii) can correspond to the unfolded cube in Panel I.

Correct Answer: (A) Only (i) can correspond to the unfolded cube in Panel I.

Solution:

Step 1: Label every face of the net in Panel I.

The net is a chain of six squares joined edge to edge along the marked dashed fold lines, one flap attached near the top of the chain and one flap attached near the bottom, with four squares forming the main column in between. Each square carries its own shaded mark: a diagonal triangle, a solid small square, a rotated shaded diamond, and further diagonal triangles on the two end flaps. The clearest, most identifiable face is the one carrying the shaded diamond in the middle of the strip, since a diamond (a square rotated 45 degrees) cannot be confused with any triangular mark, and this same diamond appears on the front face of both candidate cubes (i) and (ii) in Panel II.

Step 2: Use the diamond face as a fixed reference.

Since both cube (i) and cube (ii) place the diamond on the same visible (front) face, the diamond face alone cannot distinguish between them. The two candidate cubes must instead be told apart by the faces adjacent to the diamond, that is, the top face and the right side face visible in the drawing, and by how each neighboring face's own diagonal shading is oriented once the net is actually folded.

Step 3: Track which net-faces fold up against the diamond face, and in what orientation.

Folding a strip of squares along their shared edges only ever rotates a face about that shared hinge, it never mirrors or flips the pattern printed on the paper. So each neighboring face keeps a fixed, predictable orientation relative to the diamond face once folded,

determined entirely by which edge of the net it was hinged from. Walking through the net from the diamond square outward to its immediate neighbors on the strip shows that the adjacent triangular shading lands in the upper-right area of the top face and the lower portion of the right face, exactly the placement drawn in cube (i).

Step 4: Check cube (ii) against the same folding.
Cube (ii) shows the same diamond front face, but its neighboring triangular shading sits in the mirror-image position compared to what the physical fold produces, the kind of left-right reversal that would only occur if the paper's shaded side were flipped over before folding. Since the problem states the shaded regions must appear on the outer surface after folding this specific net (not a mirrored version of it), cube (ii) cannot be produced.

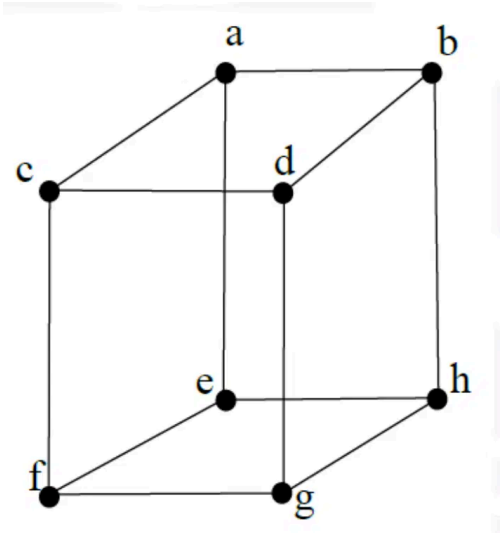
Final Answer:
Only cube (i) is consistent with folding the given net, so option (A) is correct.

Option (A)

Quick Tip: Use the shaded diamond face as a fixed reference point, then check whether the neighboring faces' shading lands in the same rotational position (never mirrored) as a genuine fold of the net would produce.

8. Consider the cube shown below with its 8 corners labelled a, b, c, d, e, f, g, and h. The figure is representative. All corners are to be colored such that any two corners that are connected by an edge must be of different colors. The minimum number of colors required to achieve this is

_____



(A) 8

(B) 4

(C) 3

(D) 2

Correct Answer: (D) 2

Solution:

Step 1: Model the problem as a graph coloring problem.
Each corner of the cube is a vertex, and each edge of the cube (the line joining two corners that are physically connected) is a graph edge. Two vertices joined by an edge must get different colors. This is exactly the standard proper vertex coloring problem, and we want the smallest number of colors, called the chromatic number of the graph.

Step 2: Check whether the cube graph has any odd length cycle.

A graph needs only 2 colors if and only if it has no cycle of odd length, which is the definition of a bipartite graph. Every face of a cube is a square, so every face boundary is a cycle of length 4, which is even. Any other cycle you can trace on a cube, for example going around two faces, is built out of these even length face boundaries and also comes out even in length. So the cube graph has no odd cycle anywhere, which means it is bipartite.

Step 3: Build an explicit 2-coloring to show 2 colors are enough.
Give each corner of the cube a coordinate triple (x, y, z) with $x, y, z \in \{0, 1\}$, one coordinate per direction of the cube. Two corners are joined by a cube edge exactly when their coordinates differ in one place only. Color a corner red if $x + y + z$ is even, and blue if $x + y + z$ is odd. Moving along any single edge changes exactly one coordinate, so it always flips the parity of $x + y + z$. This means every edge always joins a red corner to a blue corner, never two corners of the same color, so this 2-coloring is valid for all 8 labelled corners a to h.

Step 4: Rule out 1 color.
The cube clearly has edges, for example a is joined to its neighbouring corners, so at least two corners are adjacent. Using only 1 color would force two adjacent corners to share a color, which breaks the rule. So 1 color is never enough, and 2 is the smallest number that can work.

Why the other options are wrong:
8 colors works but wastes 6 colors that are not needed, since Step 3 already gives a valid coloring with only 2. 4 colors and 3 colors both work too, since any coloring that already needs only 2 colors also succeeds with more colors, but the question asks for the minimum, and Steps 3 and 4 together show that number is exactly 2, not 3 or 4.

Final Answer:

The cube graph is bipartite, since it has no odd cycles, so the minimum number of colors needed is 2.

$$\boxed{2}$$

Quick Tip: A cube graph has no odd cycles, since every face is a 4-sided square, so it is bipartite. A bipartite graph with at least one edge always needs exactly 2 colors.

9. Four hills H1, H2, H3, and H4 are present in an area. The following observations are made about them:

i. Neither H2 nor H3 is the easternmost hill.

ii. Neither H2 nor H3 is the westernmost hill.

iii. Neither the easternmost hill nor the westernmost hill is the southernmost hill.

iv. Two hills are located to the west of H2.

v. The southernmost hill has at least two hills to its east.

The southernmost hill is _____.

(A) H1

(B) H2

(C) H3

(D) H4

Correct Answer: (C) H3

Solution:

Step 1: Use clues (i) and (ii) to place H2 and H3.
Clue (i) says H2 and H3 are not the easternmost hill. Clue (ii) says H2 and H3 are not the westernmost hill either. So neither H2 nor H3 sits at either end of the west to east line of hills. With 4 hills in a row, that leaves only the two middle positions for H2 and H3, and forces H1 and H4 to occupy the two end positions, though we do not yet know which of H1, H4 is on which end.

Step 2: Use clue (iv) to fix H2's exact position.
Clue (iv) says two hills lie to the west of H2. Counting positions from the west as 1st, 2nd, 3rd, 4th, having exactly two hills to the west of H2 means H2 itself is in the 3rd position. Since Step 1 already restricted H2 to one of the two middle positions, this confirms H2 is 3rd from the west, and therefore H3, the only other middle hill, takes the 2nd position.

Step 3: Write out the confirmed order.
From west to east, the order is now: position 1 (H1 or H4), position 2 = H3, position 3 = H2, position 4 (whichever of H1, H4 is left). The easternmost hill is position 4, and the westernmost hill is position 1.

Step 4: Use clue (iii) to shortlist the southernmost hill.
Clue (iii) says the southernmost hill is neither the easternmost hill nor the westernmost hill. From Step 3 those are position 1 and position 4, that is, H1 and H4 in some order. So the southernmost hill must be one of the two middle hills, H2 or H3.

Step 5: Use clue (v) to choose between H2 and H3.
Clue (v) says the southernmost hill has at least two hills to its east. If H2, position 3, were the southernmost hill, only position 4 lies to its east, just 1 hill, not enough. So H2 fails clue (v). If H3, position 2, is

the southernmost hill, then positions 3 and 4 lie to its east, 2 hills, and that satisfies at least two.

Final Answer:

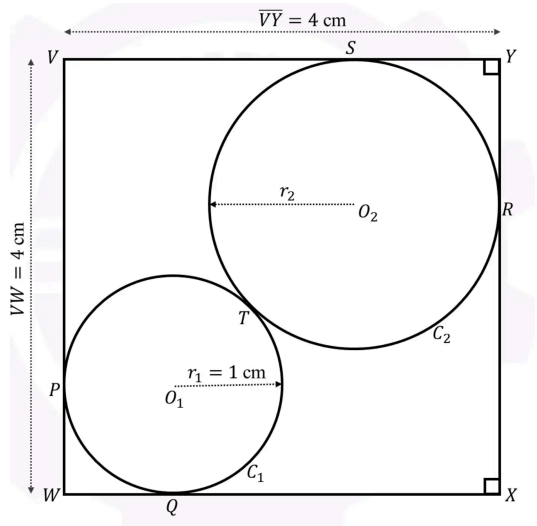
Only H3 satisfies every clue, so the southernmost hill is H3.

$$\boxed{\text{H3}}$$

Quick Tip: First fix the west to east order using clues i, ii and iv, then use clues iii and v to pin down which hill is southernmost.

10. As shown in the figure, circle C_1 with center O_1 and radius r_1 touches the square $VWXY$ at points P and Q while circle C_2 with center O_2 and radius r_2 touches the square $VWXY$ at points R and S. The two circles touch each other at T.

Given $r_1 = 1$ cm and $\overline{VY} = \overline{VW} = 4$ cm, $r_2 =$ _____ cm.



(A) $4-3\sqrt{2}$

(B) $1+2\sqrt{2}$

(C) $7-4\sqrt{2}$

(D) $5+3\sqrt{2}$

Correct Answer: (C) $7-4\sqrt{2}$

Solution:

Step 1: Set up coordinates for the square.

Place corner W at the origin $(0,0)$, with side WX along the positive x-axis and side WV along the positive y-axis. Since $\overline{VW} = \overline{VY} = 4$ cm, the square has side length 4, so the four corners are $W(0,0)$, $X(4,0)$, $Y(4,4)$, $V(0,4)$.

Step 2: Locate the center of the smaller circle C_1.

Circle C_1 touches the square at P, on side WV, and Q, on side WX, so it sits in the corner near W, tangent to both the left side and the bottom side. A circle tangent to both these sides has its center at a distance r_1 from each, so $O_1 = (r_1, r_1) = (1,1)$.

Step 3: Locate the center of the larger circle C_2.

Circle C_2 touches the square at S, on side VY, the top, and R, on side YX, the right side, so it sits in the corner near Y, tangent to the top and right sides. Its center is a distance r_2 in from each of these sides, so $O_2 = (4-r_2,\ 4-r_2)$.

Step 4: Use the tangency condition between the two circles.

The two circles touch each other externally at T, so the distance between their centers must equal the sum of their radii:

$$O_1O_2 = r_1 + r_2$$

Using the coordinates from Steps 2 and 3, and $r_1 = 1$:

$$O_1O_2 = \sqrt{(3 - r_2)^2 + (3 - r_2)^2} = \sqrt{2}(3 - r_2)$$

Step 5: Solve for r_2.

Setting the two expressions for O_1O_2 equal:

$$\sqrt{2}(3 - r_2) = 1 + r_2$$

$$3\sqrt{2} - \sqrt{2}r_2 = 1 + r_2$$

$$3\sqrt{2} - 1 = r_2(1 + \sqrt{2})$$

$$r_2 = \frac{3\sqrt{2} - 1}{1 + \sqrt{2}}$$

Rationalize by multiplying top and bottom by $(\sqrt{2} - 1)$:

$$r_2 = \frac{(3\sqrt{2} - 1)(\sqrt{2} - 1)}{(1 + \sqrt{2})(\sqrt{2} - 1)} = \frac{6 - 3\sqrt{2} - \sqrt{2} + 1}{1} = 7 - 4\sqrt{2}$$

Step 6: Sanity check the value.

Numerically, $\sqrt{2} \approx 1.4142$, so $r_2 \approx 7 - 5.657 = 1.343$ cm, a positive number smaller than the square's side, which fits a circle sitting

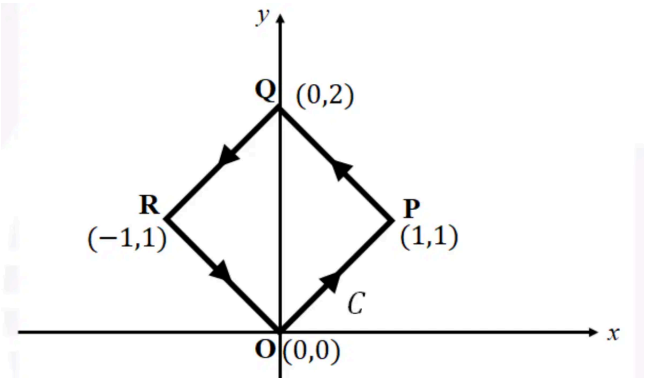
snugly in the opposite corner.

Final Answer:

$$\boxed{r_2 = 7 - 4\sqrt{2}\ \text{cm}}$$

Quick Tip: Place the square in a coordinate system with W at the origin. Both circle centers then lie on the diagonal WY, at perpendicular distances r1 and r2 from the two nearby sides. Set the center to center distance equal to r1 plus r2.

11. Consider the contour C shown in the figure below. For the vector $\vec{F} = (x + 2y)\hat{e}_x + (2x + 4y)\hat{e}_y$, the integral $\oint_C \vec{F} \cdot d\vec{l}$ =_____.
Here $d\vec{l}$ represents an infinitesimal length along the contour C.



(A) 0

(B) 2

(C) 4

(D) 6

Correct Answer: (A) 0

Solution:

Step 1: Identify the components of the vector field.

Write $\vec{F} = P\,\hat{e}_x + Q\,\hat{e}_y$, so here $P(x, y) = x + 2y$ and $Q(x, y) = 2x + 4y$. The contour C is the closed diamond shaped path through $O(0, 0)$, $P(1, 1)$, $Q(0, 2)$, $R(-1, 1)$, and back to O.

Step 2: Recall Green's theorem for a closed contour in the plane.

For a closed curve C enclosing a region A in the xy-plane,

$$\oint_C \vec{F} \cdot d\vec{l} = \oint_C (P\,dx + Q\,dy) = \pm \iint_A \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

where the sign depends on whether the contour is traversed counterclockwise or clockwise. This turns a line integral around the boundary into an area integral of the field's curl over the enclosed region.

Step 3: Compute the two partial derivatives.

$$\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x}(2x + 4y) = 2$$

$$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}(x + 2y) = 2$$

So the curl term is

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2 - 2 = 0$$

Step 4: Apply Green's theorem.

Since the curl term is 0 at every single point in the plane, not just on average, the area integral over any region A is $\iint_A 0\,dA = 0$. This holds regardless of the size or shape of the diamond region enclosed by C, and regardless of whether C is traced clockwise or counterclockwise, since a zero integrand gives zero area integral either way.

Step 5: Conclude the value of the line integral.
By Green's theorem, the closed line integral equals this area integral.

$$\oint_C \vec{F} \cdot d\vec{l} = 0$$

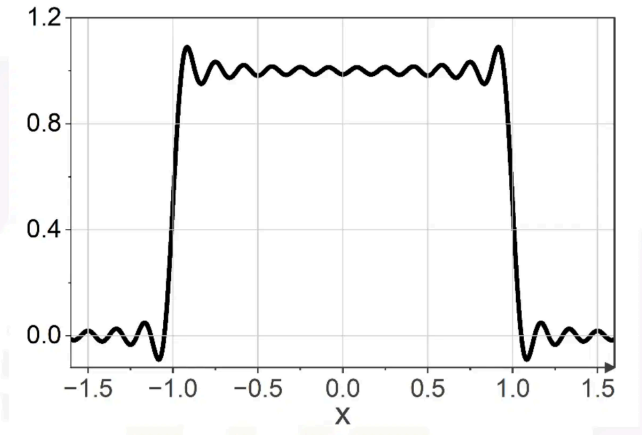
Why the other options are wrong:
2, 4, and 6 would only appear if the curl of $\vec{F}$ were nonzero, so the enclosed area of the diamond would matter in the answer. Since the curl here is exactly zero everywhere, the enclosed area never enters the answer, and those nonzero values do not arise.

Final Answer:

$$\boxed{0}$$

Quick Tip: Check the curl of F first. If it is zero everywhere, the closed loop integral is zero no matter what shape or orientation the contour has.

12. The Fourier series representation of a square wave is shown in the figure below. The fluctuations seen near $x = \pm 1$ are named after which one of the following scientists?



(A) Cauchy

(B) Fourier

(C) Gibbs

(D) Laplace

Correct Answer: (C) Gibbs

Solution:

The plot shows a truncated Fourier series trying to reproduce a square wave, which has sharp jump discontinuities at $x = \pm 1$. Near each jump, the reconstructed curve overshoots the true value, dips, and rings before settling down. This ringing pattern is a well known named effect in Fourier analysis, and matching it to the right name is what this question tests.

1. **Cauchy**: Augustin-Louis Cauchy's name is attached to results in complex analysis and to the Cauchy sequence and Cauchy integral theorem, not to the overshoot near a jump in a Fourier series. This does not match.

2. **Fourier**: Joseph Fourier gave his name to the series itself, the representation of a periodic function as a sum of sines and cosines. The series is called Fourier series throughout, but the specific ringing artifact near a discontinuity is credited to a different mathematician, not to Fourier.
3. **Gibbs**: Josiah Willard Gibbs analyzed exactly this behavior. When a Fourier series for a function with a jump discontinuity is truncated to a finite number of terms, the partial sum overshoots the jump by a fixed fraction, about 9 percent of the jump size, and this overshoot does not vanish even as more terms are added, it only squeezes closer to the discontinuity. This is precisely the ringing seen near $x = \pm 1$ in the figure, and it is called the Gibbs phenomenon.
4. **Laplace**: Pierre-Simon Laplace is known for the Laplace transform and Laplace's equation, both different tools used elsewhere in engineering mathematics, and unrelated to this specific Fourier series overshoot behavior.

The correct option is Gibbs, since the fixed size overshoot and ringing near a jump discontinuity of a truncated Fourier series is specifically named the Gibbs phenomenon.

Let's sum up:

- A square wave has a jump discontinuity at $x = \pm 1$, and no finite Fourier sum can reproduce a jump exactly.
- Near the jump, the partial sum overshoots by roughly 9 percent of the jump height, no matter how many terms are used.
- This persistent overshoot and ringing pattern is called the Gibbs phenomenon, named after J. Willard Gibbs.

So the fluctuations near $x = \pm 1$ are named after Gibbs.

Quick Tip: These overshoot ripples near a jump discontinuity never shrink in height as more Fourier terms are added, only in width. This named effect is a classic property of truncated Fourier series.

13. The following equation with respect to $\varphi(x, t)$, where a is a non-zero constant, represents _____.

$$\frac{\partial \varphi}{\partial t} + a\frac{\partial \varphi}{\partial x} = 0$$

(A) linear wave propagation

(B) transient heat conduction

(C) Newton's law of cooling

(D) radiative transfer

Correct Answer: (A) linear wave propagation

Solution:

Step 1: Identify the type and order of the equation.

The equation $\frac{\partial \varphi}{\partial t} + a\frac{\partial \varphi}{\partial x} = 0$ is a first order, linear, homogeneous partial differential equation in the two independent variables x and t, with constant coefficient a. This particular form is called the linear advection equation, or the one way wave equation.

Step 2: Solve it using the method of characteristics.

Along a curve in the x-t plane where x and t change together such that $\frac{dx}{dt} = a$, the chain rule gives

$$\frac{d\varphi}{dt} = \frac{\partial \varphi}{\partial t} + \frac{dx}{dt}\frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial t} + a\frac{\partial \varphi}{\partial x}$$

which is exactly the left side of the given PDE. Since the PDE says this equals zero, φ does not change at all along any curve satisfying $\frac{dx}{dt} = a$, meaning φ is constant along the straight lines $x - at = \text{constant}$. These lines are called the characteristics of the equation.

Step 3: Write the general solution.
Since φ only depends on which characteristic line it sits on, the general solution has the form

$$\varphi(x, t) = f(x - at)$$

for an arbitrary function f, fixed by whatever initial shape $\varphi(x, 0) = f(x)$ is given.

Step 4: Interpret this solution physically.
At time $t = 0$, the profile is $f(x)$. At a later time t, the profile is $f(x - at)$, the same shape f simply shifted to the right by a distance at if $a > 0$, or to the left if $a < 0$. The waveform keeps its exact shape and just translates rigidly at constant speed a, with no spreading, no decay, and no change in profile. This rigid, undistorted translation of a fixed shape at constant speed is exactly what wave propagation means, and because the governing equation is linear, it is specifically linear wave propagation.

Why the other options are wrong:
Transient heat conduction is governed by the heat, or diffusion, equation $\varphi_t = \kappa\varphi_{xx}$, which has a second order spatial derivative and describes smoothing and spreading of a temperature profile over time, not translation, so it does not match this first order equation.

Newton's law of cooling is an ordinary differential equation in time alone, $dT/dt = -k(T - T_\infty)$, with no spatial derivative or dependence on x at all, so it cannot match a PDE involving $\partial\varphi/\partial x$. Radiative transfer equations involve integral terms over direction and frequency and usually a fourth power temperature term from the Stefan-Boltzmann law, nothing like this simple linear first order balance.

Final Answer:
The equation describes a fixed profile translating at constant speed a, which is linear wave propagation.

linear wave propagation

Quick Tip: Solve this first order PDE by the method of characteristics. The solution turns out to be any function of (x minus at), which is the mathematical signature of a wave moving at constant speed without changing shape.

14. Which one of the following makes an ideal air-standard Stirling cycle?

(A) Two reversible isobars, and two reversible adiabatics

(B) Two reversible isotherms, and two reversible isobars

(C) Two reversible isotherms, and two reversible isochores

(D) Isentropic compression, constant volume heat addition, isentropic expansion, and constant volume heat rejection

Correct Answer: (C) Two reversible isotherms, and two reversible isochores

Solution:

The air-standard Stirling cycle is an idealized thermodynamic cycle originally associated with the Stirling engine, and it is defined by a specific pair of process types repeated twice around the cycle. This question checks whether that exact pair, isotherms plus isochores, can be picked out from other similar sounding cycles.

1. **Two reversible isobars, and two reversible adiabatics**: This combination, constant pressure plus isentropic legs, describes the ideal Brayton, or Joule, cycle used for gas turbines, not the Stirling cycle.
2. **Two reversible isotherms, and two reversible isobars**: Mixing constant temperature legs with constant pressure legs does not correspond to any of the standard named air-standard power cycles taught alongside Stirling, Otto, Diesel, and Brayton. This option does not describe the Stirling cycle.
3. **Two reversible isotherms, and two reversible isochores**: This is exactly the definition of the ideal Stirling cycle. Heat is added at constant temperature during isothermal expansion, then heat is absorbed internally at constant volume, then heat is rejected at constant temperature during isothermal compression, then heat is rejected at constant volume to complete the loop, aided in a real machine by a regenerator that stores and returns the heat exchanged during the two constant volume legs.
4. **Isentropic compression, constant volume heat addition, isentropic expansion, and constant volume heat rejection**: This four process description, isentropic, isochoric, isentropic, isochoric, is the ideal air-standard Otto cycle, the idealization of a spark ignition petrol engine, not the Stirling cycle.

The correct option is two reversible isotherms and two reversible isochores, since that is the precise textbook definition of the ideal air-standard Stirling cycle.

Let's sum up:

- Stirling cycle: two isotherms plus two isochores.
- Otto cycle: two isentropes plus two isochores.
- Diesel cycle: two isentropes, one isobar, one isochore.
- Brayton cycle: two isentropes plus two isobars.

So the ideal air-standard Stirling cycle is made of two reversible isotherms and two reversible isochores.

Quick Tip: The Stirling cycle is the constant temperature counterpart of the Otto cycle. It swaps the two isentropic legs of the Otto cycle for two isothermal legs, while keeping the two constant volume legs.

15. In fluid dynamics, d'Alembert's paradox refers to which one of the following?

(A) Deviation of drag from $D \propto v^2$ at very low speeds

(B) Deviation of drag from $D \propto v^2$ at high subsonic speeds

(C) Prediction of zero drag by potential flow theory

(D) Presence of shocks in transonic flows

Correct Answer: (C) Prediction of zero drag by potential flow theory

Solution:

Step 1: Recall what d'Alembert's paradox states.

In 1752, d'Alembert used potential flow theory (flow that is inviscid,

incompressible and irrotational) to find the net force on a body moving at constant speed through a fluid.
The result he got was that the drag force comes out to be exactly zero, no matter what shape the body has.

Step 2: Compare this result with real flow.
In a real fluid, a body moving through it always feels a drag force, because of viscosity and the boundary layer that forms on its surface. So potential flow theory (which ignores viscosity) predicts zero drag, while every real body actually has drag. This contradiction between theory and reality is the paradox.

Step 3: Why the other options are wrong.
Options (A) and (B) talk about the drag law $D \propto v^2$ breaking down at very low or high subsonic speeds. That is a separate topic (deviation of the drag coefficient with Reynolds number or Mach number), not d'Alembert's paradox.
Option (D) refers to shock waves in transonic flow, which is a compressibility effect and has nothing to do with the inviscid drag prediction.

Final Answer:
d'Alembert's paradox is the prediction of zero drag by potential (inviscid, irrotational) flow theory, which does not match real, viscous flow.

Option (C)

Quick Tip: Recall that potential flow theory assumes no viscosity; think about what net force it predicts on a body compared to reality.

16. The number of independent elastic constants that a fully anisotropic linear elastic material can have is .

(A) 36

(B) 21

(C) 10

(D) 2

Correct Answer: (B) 21

Solution:

Step 1: Set up the generalized Hooke's law.

For a linear elastic material, stress is related to strain through the stiffness matrix C:

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$$

Written in the compact 6 by 6 matrix (Voigt) notation, stress and strain each have 6 independent components (3 normal + 3 shear), so C is a 6 by 6 matrix, which has $6 \times 6 = 36$ entries.

Step 2: Reduce 36 using the strain energy argument.

For an elastic material, the strain energy density U is a scalar function of strain, and stress is obtained as $\sigma_i = \partial U/\partial \varepsilon_i$.

Because mixed partial derivatives of U can be taken in either order, $C_{ij} = C_{ji}$, so the stiffness matrix must be symmetric.

Step 3: Count the independent terms of a symmetric 6 by 6 matrix.

A symmetric 6×6 matrix has 6 diagonal terms plus $\frac{6\times5}{2} = 15$ off-

diagonal terms (the terms below the diagonal equal the ones above it), giving

$$6 + 15 = 21$$

independent elastic constants.

Step 4: Note why the other numbers are wrong.
36 (option A) is the raw entry count of C before using symmetry. 10 and 2 (options C, D) are smaller counts that come from extra material symmetry (such as orthotropic or isotropic materials), not from the most general, fully anisotropic (triclinic) case, which is what the question asks about.

Final Answer:
A fully anisotropic linear elastic material has 21 independent elastic constants.

$$\boxed{21}$$

Quick Tip: Use the symmetry of the stiffness matrix from strain energy to reduce the 36 raw entries of a 6 by 6 matrix.

17. A cantilever beam with an unsymmetric cross-section is subjected to a transverse shear force (P) at its free end. P acts at the shear center of the beam cross-section. Which one of the following statements is TRUE about the deformation of this beam?

(A) The beam undergoes only torsion

(B) The beam undergoes only bending

(C) The beam undergoes both torsion and bending

(D) The beam undergoes neither torsion nor bending

Correct Answer: (B) The beam undergoes only bending

Solution:

Step 1: Recall the definition of the shear center.
The shear center of a cross-section is the specific point in the plane of the cross-section through which a transverse shear force must act so that the beam bends without twisting.
If a shear force is applied anywhere else on an unsymmetric section, it produces a twisting moment (torque) in addition to bending, because the resultant of the internal shear stresses does not line up with the applied load.

Step 2: Apply this to the given beam.
Here the beam has an unsymmetric cross-section, so its shear center does not sit at the centroid, but the problem states clearly that the force P acts exactly at the shear center.
By the very definition from Step 1, when the load line passes through the shear center, the net twisting moment about that point is zero.

Step 3: Identify the resulting deformation.
With the twisting moment equal to zero, there is no torsion anywhere along the beam.
The transverse force P still produces a bending moment and a shear force distribution along the length of the cantilever, so ordinary transverse bending still occurs.

Step 4: Why the other options are wrong.

(A) is wrong because torsion needs a net twisting moment about the shear center, which is zero here.

(C) is wrong for the same reason, there is no torsion component to add to the bending.

(D) is wrong because bending is still fully present, the force still creates a bending moment along the span.

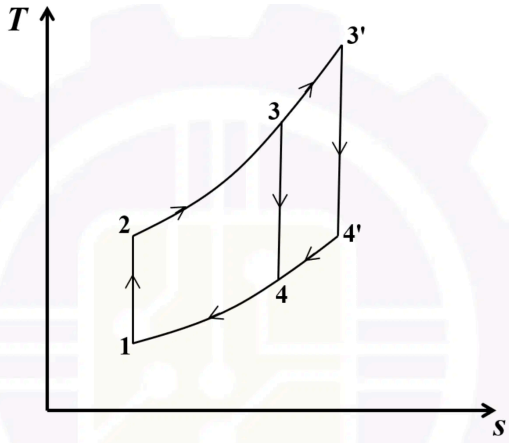
Final Answer:

Because P acts through the shear center, the beam undergoes bending only, with zero twist.

Option (B): only bending

Quick Tip: Recall the definition of the shear center: a transverse load through it produces zero twisting moment.

18. The figure below depicts two ideal gas turbine cycles, cycle 1-2-3-4-1 and cycle 1-2-3'-4'-1, on a T-s diagram. Which one of the following statements is FALSE?



(A) The thermal efficiency of the two cycles is the same

(B) The specific work of the two cycles is the same

(C) The processes 2-3 and 2-3' are isobaric

(D) The amount of heat added in the combustion process is greater for the cycle 1-2-3'-4'-1

Correct Answer: (B) The specific work of the two cycles is the same

Solution:

Step 1: Set up the ideal Brayton cycle relations.

Both cycles share the same compression process 1-2 (isentropic) and the same pressure levels, so both have the same pressure ratio r_p.

For an isentropic process in an ideal gas,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} = \frac{T_{3'}}{T_{4'}} = r_p^{(\gamma-1)/\gamma} = \tau$$

because 3-4 and 3'-4' are also isentropic expansions between the same two pressure levels as 1-2.

Step 2: Check the thermal efficiency (statement A).

The ideal Brayton cycle efficiency is

$$\eta = 1 - \frac{1}{\tau}$$

which depends only on τ (that is, only on the pressure ratio), not on the peak temperature T_3 or $T_{3'}$. Since τ is the same for both cycles, η is the same for both. So statement (A) is TRUE.

Step 3: Check the specific work (statement B).

Net specific work is $w = c_p(T_3 - T_4) - c_p(T_2 - T_1)$. Using $T_4 = T_3/\tau$,

$$w = c_p T_3 \left(1 - \frac{1}{\tau}\right) - c_p(T_2 - T_1)$$

Since $T_{3'} > T_3$ (point 3' is drawn higher on the T-s diagram), and every other term in the expression is identical for both cycles, $w' > w$. So the specific work is NOT the same for the two cycles, statement (B) is FALSE.

Step 4: Check statements C and D.
Processes 2-3 and 2-3' both happen at the same, constant combustor pressure, so they are isobaric, statement (C) is TRUE.
Heat added is $Q = c_p(T_3 - T_2)$; since $T_{3'} > T_3$, more heat is added for the 1-2-3'-4'-1 cycle, statement (D) is TRUE.

Final Answer:
Only statement (B) is false, the two cycles do not have the same specific work.

Option (B)

Quick Tip: Efficiency depends only on the pressure ratio (shared compression leg), but net work depends on the peak temperature too.

19. The velocity potential function (φ) given below represents which one of the following?

$$\varphi = 5x - 12y$$

(A) Doublet

(B) Irrotational vortex

(C) Source

(D) Uniform flow

Correct Answer: (D) Uniform flow

Solution:

Step 1: Find the velocity components from the potential function.

For 2D potential flow, velocity components come from the velocity potential as

$$u = \frac{\partial \varphi}{\partial x}, \qquad v = \frac{\partial \varphi}{\partial y}$$

Given $\varphi = 5x - 12y$,

$$u = \frac{\partial}{\partial x}(5x - 12y) = 5, \qquad v = \frac{\partial}{\partial y}(5x - 12y) = -12$$

Step 2: Interpret the result.

Both u and v come out as plain constants, they do not depend on x or y at all.

This means the velocity vector $(u, v) = (5, -12)$ is exactly the same at every point in the flow field, in both magnitude and direction.

A flow field with a constant velocity vector everywhere is, by definition, a uniform flow, with speed

$$V = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

directed at an angle $\theta = \tan^{-1}(-12/5)$ below the x-axis.

Step 3: Why the other options are wrong.
A source has $\varphi = \frac{\Lambda}{2\pi}\ln r$, which depends on the distance r from the origin, not a linear function of x and y.
An irrotational (line) vortex has $\varphi = \frac{\Gamma}{2\pi}\theta$, depending on the angle θ, again not linear in x, y.
A doublet has $\varphi = \frac{\kappa\cos\theta}{2\pi r}$, which decays with distance and is singular at the origin, unlike the given φ, which is smooth and defined everywhere.

Final Answer:
Since φ is a simple linear function of x and y, giving constant velocity components everywhere, it represents a uniform flow.

Option (D)

Quick Tip: Differentiate φ with respect to x and y to get the velocity components, then see if they are constant.

20. The fundamental purpose of the Kutta condition in the thin airfoil theory is .

(A) to determine the total strength of the source distribution

(B) to determine the speed of the uniform flow

(C) to incorporate the essential effect of viscosity in the potential flow theory

(D) to incorporate the concept of induced drag in the inviscid theory

Correct Answer: (C) to incorporate the essential effect of viscosity in the potential flow theory

Solution:

Step 1: Understanding the Concept.

In thin airfoil theory, the flow around an airfoil is modeled by placing a sheet of vortices (not sources) along the chord line, with an unknown strength distribution $\gamma(x)$.

Solving the governing integral equation for $\gamma(x)$ from potential flow alone gives infinitely many mathematically valid solutions, because inviscid theory by itself does not fix the total circulation.

Step 2: Key Formula or Approach.

The Kutta condition is the extra physical requirement that closes this gap: it states that the flow must leave the sharp trailing edge smoothly, so the velocity there stays finite (in practice, $\gamma(\text{trailing edge}) = 0$ for a cusped or sharp trailing edge).

Step 3: Detailed Explanation.

In a truly inviscid flow with no Kutta condition applied, potential theory would allow the flow to whip around the sharp trailing edge at infinite speed, which is unphysical.

In the real, viscous flow, the boundary layer and the adverse pressure gradient near the sharp trailing edge force the rear stagnation point to sit right at the trailing edge, so the flow leaves smoothly with no infinite velocity.

The Kutta condition mimics exactly this real, viscous behavior by picking out the one circulation value, out of the infinitely many mathematically possible ones, for which the inviscid model also gives a smooth, finite-velocity exit at the trailing edge.

In this sense, the Kutta condition is the one piece of borrowed viscous physics that an otherwise inviscid potential flow calculation needs, to correctly predict lift.

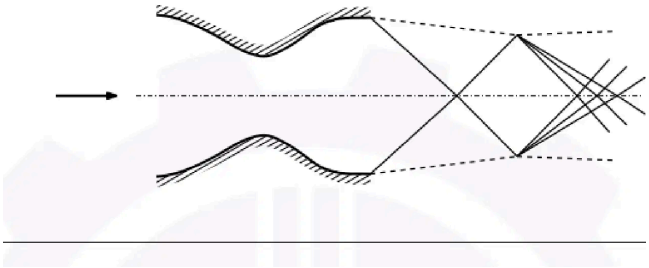
Step 4: Why the other options are wrong.
(A) is wrong because thin airfoil theory (the version producing lift and circulation) uses a vortex sheet, not a source distribution, source strength is a separate topic used for representing thickness, not this condition.
(B) is wrong because the free stream speed is a given input to the problem, not something the Kutta condition determines.
(D) is wrong because induced drag comes from the downwash of a finite, three-dimensional wing, which is outside the scope of 2D thin airfoil theory and has nothing to do with the trailing edge condition.

Final Answer:
The Kutta condition exists to bring the essential, real effect of viscosity, smooth flow off a sharp trailing edge, into the otherwise inviscid potential flow model.

Option (C)

Quick Tip: Think about why inviscid theory alone cannot fix circulation at a sharp trailing edge without an extra, physically motivated condition.

21. In the figure shown below, the flow at the nozzle exit is .



(A) overexpanded

(B) underexpanded

(C) ideally expanded

(D) subsonic

Correct Answer: (A) overexpanded

Solution:

Step 1: Recall the three exit conditions for a converging-diverging nozzle.

Let p_e be the nozzle exit pressure and p_a the surrounding (ambient/back) pressure.

If $p_e = p_a$, the jet is ideally expanded and leaves as a straight, parallel stream.

If $p_e < p_a$ (exit pressure lower than ambient), the jet is overexpanded.

If $p_e > p_a$ (exit pressure higher than ambient), the jet is underexpanded.

Step 2: Read the wave pattern shown in the figure.

The figure shows the jet boundary curving sharply inward right at the nozzle lip, with oblique waves running from the lip inward to meet on the centerline, then crossing and reflecting outward again, forming the classic repeating shock diamond pattern of a supersonic jet.

Step 3: Match the wave pattern to the physics.

When $p_e < p_a$ (overexpanded case), the higher outside pressure pushes the jet boundary inward right at the exit. This turning of the flow is achieved through oblique shock waves that start at the nozzle lip and run in towards the centerline, this is exactly the inward-curving, crossing pattern shown in the figure.

If the jet were underexpanded instead ($p_e > p_a$), the jet boundary would first bulge outward at the lip (through expansion, or Prandtl-Meyer, fan waves), the opposite of what is drawn.

If the jet were ideally expanded, there would be no waves at all at the exit, and the jet would leave as parallel, wave free flow, but the figure clearly shows a wave pattern.

Step 4: Rule out subsonic flow.

Shock waves and Mach-type wave patterns are a purely supersonic feature, a subsonic nozzle exit flow cannot produce shocks or the crossing-diamond wave pattern shown at all.

Final Answer:

The inward-turning, crossing shock pattern at the exit is the signature of overexpanded flow.

Option (A): overexpanded

Quick Tip: Compare the exit pressure to the ambient pressure and see which way the jet boundary first bends at the nozzle lip.

22. An $n \times n$ square matrix A satisfies $A^T = A^{-1}$. The determinant of this matrix may take which of the following value(s)?

(A) $+1$

(B) -1

(C) n

(D) 0

Correct Answer: (A) $+1$; (B) -1

Solution:

Step 1: Understand the Condition.

The matrix A satisfies $A^T = A^{-1}$. A matrix with this property is called an orthogonal matrix. It means the rows (and columns) of A form an orthonormal set, so $A^TA = AA^T = I$, where I is the identity matrix.

Step 2: Take the Determinant of Both Sides.

Starting from $A^T = A^{-1}$, take the determinant of each side.

$$\det(A^T) = \det(A^{-1})$$

Use two standard determinant facts here: $\det(A^T) = \det(A)$ for any square matrix, and $\det(A^{-1}) = \frac{1}{\det(A)}$ for any invertible matrix.

Step 3: Solve for the Determinant.

Substituting these facts gives

$$\det(A) = \frac{1}{\det(A)}$$

Multiply both sides by $\det(A)$:

$$[\det(A)]^2 = 1$$

So $\det(A) = +1$ or $\det(A) = -1$. Both values are possible: $\det(A) = +1$ for a proper rotation (a matrix in $SO(n)$) and $\det(A) = -1$ for an improper rotation that includes a reflection.

Step 4: Check the Other Options.

Option (C), n, cannot be a general answer. The value n has no connection to the orthogonality condition, and it does not satisfy $[\det(A)]^2 = 1$ for a general n. Option (D), 0, is also wrong: if $\det(A) = 0$ then A would be singular and A^{-1} would not even exist, contradicting the given condition $A^T = A^{-1}$.

Final Answer:

The determinant can only be $+1$ or -1.

$$\boxed{\det(A) = \pm 1 \implies \text{options (A) and (B)}}$$

Quick Tip: Take the determinant of both sides of $A^T = A^{-1}$ and use $\det(A^T) = \det(A)$, $\det(A^{-1}) = 1/\det(A)$.

23. Which of the following statements is/are TRUE about the stability of an aircraft?

(A) Static stability of an aircraft is sufficient to guarantee its dynamic stability

(B) Static stability of an aircraft is related only to its initial tendency to return towards the equilibrium position from which it is disturbed

(C) An aircraft may be dynamically unstable even if it is statically stable

(D) Dynamic stability is related to the time history of aircraft motion after being disturbed from its equilibrium position

Correct Answer: (B) Static stability of an aircraft is related only to its initial tendency to return towards the equilibrium position from which it is disturbed ; (C) An aircraft may be dynamically unstable even if it is statically stable ; (D) Dynamic stability is related to the time history of aircraft motion after being disturbed from its equilibrium position

Solution:

This question tests the difference between static stability and dynamic stability, two separate ideas that GATE often tests together. Static stability is about the very first moment after a disturbance: does the aircraft's own aerodynamics push it back toward its trim, or push it further away. Dynamic stability is about what happens over time after that first moment: does the resulting motion, an oscillation in most modes, die out, stay the same, or grow.

1. **(A) Static stability guarantees dynamic stability:** This is false. Static stability only tells us that the initial moment or force after a disturbance opposes the disturbance. It says nothing about the damping of the motion that follows. A classic example is the phugoid mode: an aircraft can be statically stable, it initially tends back to trim, and still show a phugoid oscillation whose amplitude grows with time, which is dynamic instability. So static stability is needed but not enough for dynamic stability.

2. **(B) Static stability is only about the initial tendency to return to equilibrium:** This is true, by definition. Static stability is checked using the sign of the restoring moment, for longitudinal static stability the sign of $\partial C_m/\partial \alpha$ with $C_{m,0} > 0$, right at the instant of disturbance. It makes no statement about the time history that follows.
3. **(C) An aircraft may be dynamically unstable even if it is statically stable:** This is true, and it follows directly from (A) being false. Since static stability does not control the damping of the oscillatory modes, a statically stable aircraft can still have negative damping in a mode such as the phugoid or spiral mode, making it dynamically unstable.
4. **(D) Dynamic stability is related to the time history of motion after disturbance:** This is true, by definition. Dynamic stability studies how the perturbation quantities, angle of attack, pitch rate, sideslip and so on, evolve with time, whether the oscillation amplitude decays (stable), stays constant (neutral), or grows (unstable).

So the correct statements are (B), (C) and (D); (A) is the false one, because it wrongly claims static stability by itself is enough for dynamic stability.

Let's summarize:

- Static stability is the sign of the initial restoring tendency only.
- Dynamic stability is the behaviour, growth or decay, of the resulting motion over time.
- Static stability is needed but not enough for dynamic stability, so a statically stable aircraft can still be dynamically unstable.

The TRUE statements are (B), (C) and (D).

Quick Tip: Static stability checks only the initial restoring tendency; dynamic stability checks whether the resulting motion grows or decays over time. One does not guarantee the other.

24. For a given air-standard power, the propulsive efficiency of a turbofan engine is more than that of a turbojet engine. Which of the following is/are the reason(s) for this?

(A) The mass flow rate is more for a turbofan engine

(B) The exit velocity is lower for a turbofan engine

(C) A turbofan engine operates at a lower altitude

(D) The fan of a turbofan engine consumes lesser power

Correct Answer: (A) The mass flow rate is more for a turbofan engine ; (B) The exit velocity is lower for a turbofan engine

Solution:

Step 1: Recall the Propulsive Efficiency Formula.

For an air breathing jet engine, ignoring the small fuel mass added, propulsive efficiency compares the useful thrust power to the total kinetic energy rate given to the jet:

$$\eta_p = \frac{2}{1 + \frac{V_e}{V_\infty}}$$

where V_e is the jet (exit) velocity and V_∞ is the flight speed. This shows that η_p is highest when V_e is close to V_∞, that is, when the jet velocity is only a little more than the flight speed.

Step 2: Relate Thrust, Mass Flow and Exit Velocity.

Thrust for a given air-standard power comes from the momentum change of the working fluid:

$$T \approx \dot{m}(V_e - V_\infty)$$

For a fixed thrust or a fixed power requirement, a larger mass flow rate $\dot{m}$ needs only a small velocity increment $(V_e - V_\infty)$ to produce that same thrust, while a smaller mass flow rate needs a much larger velocity increment.

Step 3: Apply This to a Turbofan.

A turbofan takes in a large bypass mass flow through the fan in addition to the core flow, so its total mass flow rate $\dot{m}$ is much larger than that of a turbojet producing the same thrust. Because of Step 2, this larger $\dot{m}$ means the turbofan only needs to accelerate the air by a small amount, so its exit velocity V_e stays much closer to V_∞ than a turbojet's does. From Step 1, a V_e closer to V_∞ directly gives a higher η_p. So both a higher mass flow rate (option A) and a lower exit velocity (option B) are the reasons for the turbofan's higher propulsive efficiency, and they are really two sides of the same cause.

Step 4: Why C and D Are Wrong.

Option (C), operating at a lower altitude, is not a general reason for higher propulsive efficiency at all. Propulsive efficiency depends on the ratio V_e/V_∞, not on altitude; turbofans are in fact widely used at high cruise altitudes. Option (D), the fan consuming lesser power, is also not the cause. The fan actually absorbs a large share of the engine's shaft power, that is exactly why the bypass flow gets its extra mass and momentum, so the fan does not consume lesser power; and

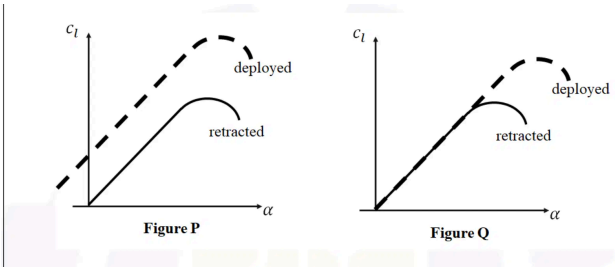
even if it did, that would be a statement about internal power distribution, not about the propulsive efficiency definition, which depends only on V_e versus V_∞.

Final Answer:

Higher mass flow rate and lower exit velocity, i.e. options (A) and (B)

Quick Tip: Propulsive efficiency rises as the jet exit velocity gets closer to the flight speed; a bigger mass flow rate needs a smaller velocity increment for the same thrust.

25. Shown below are qualitative illustrations of the lift curve for an airfoil when two different control surfaces are in their respective retracted and deployed configurations. Which of the following is/are TRUE?



(A) Figure P is for a flap

(B) Figure P is for a slat

(C) Figure Q is for a slat

(D) Figure Q is for a flap

Correct Answer: (A) Figure P is for a flap ; (C) Figure Q is for a slat

Solution:

Step 1: Recall What a Flap Does to the Lift Curve.

A trailing edge flap works mainly by adding camber to the airfoil. Adding camber shifts the whole lift curve up and to the left: the zero-lift angle of attack becomes more negative, the curve now crosses $C_l = 0$ at a negative α, and $C_{l,max}$ increases, but the slope of the linear part and the stalling angle stay roughly the same, sometimes even a little lower, because a flap does not fix the flow separation problem at the leading edge.

Step 2: Match This to Figure P.

In Figure P, the deployed (dashed) curve is shifted up and to the left of the retracted (solid) curve. The dashed curve starts from a negative angle of attack, it is already above zero lift at $\alpha = 0$, reaches a higher $C_{l,max}$, and both curves have a broadly similar shape and stall behaviour, just parallel shifted. This upward left shift with an unchanged basic shape is exactly what a camber-adding device, a flap, produces. So Figure P is for a flap, which makes option (A) TRUE and option (B), Figure P is a slat, FALSE.

Step 3: Recall What a Slat Does to the Lift Curve.

A leading edge slat works differently: it re-energises the boundary layer near the leading edge, by letting a jet of high energy air from the lower surface flow through the slot onto the upper surface, and delays flow separation. It does not add camber the same way, so the low angle of attack, linear part of the lift curve is nearly unchanged. What it does is extend that linear region to a much higher angle of attack before stall happens, giving a large increase in both $C_{l,max}$ and the stalling angle α_{stall}.

Step 4: Match This to Figure Q.

In Figure Q, the deployed (dashed) curve follows the same line as the

retracted (solid) curve for the initial linear part, then continues climbing well past the point where the retracted curve stalls, reaching a higher $C_{l,max}$ at a higher angle of attack. This is exactly the slat signature: same low angle-of-attack behaviour, extended range before stall. So Figure Q is for a slat, which makes option (C) TRUE and option (D), Figure Q is a flap, FALSE.

Final Answer:

Figure P is a flap and Figure Q is a slat, i.e. options (A) and (C)

Quick Tip: A flap shifts the whole lift curve up-left (added camber); a slat leaves the low angle-of-attack curve unchanged but extends it to a higher stall angle (delayed separation).

26. The state of stress at a point in a 2-D body, in the x-y Cartesian coordinate system, is represented in matrix form as $[\sigma]$. The transformation matrix $[Q]$ rotates the coordinate system to a new x'-y' Cartesian coordinate system. Select the CORRECT option(s) that represent(s) the state of stress in the new coordinate system.

(A) $[Q][\sigma][Q]^T$

(B) $[Q][\sigma][Q]^{-1}$

(C) $([Q]^{-1})^T[\sigma][Q]^T$

(D) $[Q]^{-1}[\sigma][Q]$

Correct Answer: (A) $[Q][\sigma][Q]^T$; (B) $[Q][\sigma][Q]^{-1}$; (C) $([Q]^{-1})^T[\sigma][Q]^T$

Solution:

Step 1: Recall the Stress Transformation Rule.

For a second order tensor like stress, the standard rule for transforming its matrix representation from an old (x, y) axis system to a new, rotated (x', y') axis system is

$$[\sigma'] = [Q][\sigma][Q]^T$$

where $[Q]$ is the rotation (direction cosine) matrix taking the old axes to the new axes. This makes option (A) correct by definition.

Step 2: Use the Key Property of a Rotation Matrix.

A rotation or transformation matrix between two Cartesian coordinate systems is always orthogonal, meaning

$$[Q]^T = [Q]^{-1}$$

This is because a rotation preserves lengths and angles, so $[Q]^T[Q] = [I]$.

Step 3: Check Option (B).

Option (B) is $[Q][\sigma][Q]^{-1}$. Substituting $[Q]^{-1} = [Q]^T$ from Step 2 turns this expression directly into $[Q][\sigma][Q]^T$, which is identical to option (A). So option (B) is just another way of writing the same correct transformation, and it is also correct.

Step 4: Check Option (C).

Option (C) is $([Q]^{-1})^T[\sigma][Q]^T$. Using $[Q]^{-1} = [Q]^T$ again, $([Q]^{-1})^T = ([Q]^T)^T = [Q]$, the transpose of a transpose returns the original matrix. Substituting this back gives $[Q][\sigma][Q]^T$, again exactly option (A). So option (C) is also correct, just written in a disguised form.

Step 5: Check Option (D).

Option (D) is $[Q]^{-1}[\sigma][Q]$. Substituting $[Q]^{-1} = [Q]^T$ gives $[Q]^T[\sigma][Q]$. This is generally NOT the same as $[Q][\sigma][Q]^T$ from Step 1, the two rotation matrices are on the wrong sides, unless $[Q]$ happens to be symmetric, which a general rotation matrix is not. So option (D) is the wrong transformation and does not represent the stress in the new coordinate system in general.

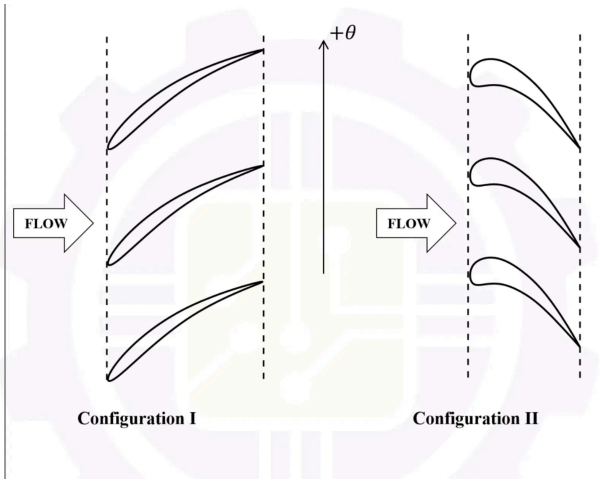
Final Answer:

Options (A), (B) and (C) all reduce to the same correct transformation once $[Q]^{-1} = [Q]^T$ is used; only (D) is wrong.

$$\boxed{\text{(A), (B), (C)}}$$

Quick Tip: Stress transforms as $[\sigma'] = [Q][\sigma][Q]^T$; since $[Q]$ is orthogonal, $[Q]^{-1} = [Q]^T$, so any option that algebraically reduces to this form is also correct.

27. The figure below shows the blading of the rotors of two different axial turbomachines under their typical operating conditions, labelled as Configuration I and Configuration II. Which of the following statements is/are TRUE?



(A) Configuration I corresponds to the rotor of a compressor and Configuration II corresponds to the rotor of a turbine

(B) Configuration I corresponds to the rotor of a turbine and Configuration II corresponds to the rotor of a compressor

(C) The rotor blades of the turbomachine in Configuration I move along the $+\theta$ direction

(D) The rotor blades of the turbomachine in Configuration II move along the $+\theta$ direction

Correct Answer: (A) Configuration I corresponds to the rotor of a compressor and Configuration II corresponds to the rotor of a turbine ; (D) The rotor blades of the turbomachine in Configuration II move along the $+\theta$ direction

Solution:

Step 1: Compare Blade Shape and Camber.

The single most reliable way to tell a compressor rotor blade from a turbine rotor blade in a cascade picture is to look at how thick and

how strongly cambered (curved) the blade is, because the two machines see opposite pressure gradients along the flow. In Configuration I, the blades are thin, aerofoil shaped sections with a modest amount of turning from inlet to exit. In Configuration II, the blades are noticeably more curved, wrapping through a much larger turning angle, and have a thicker, more cambered profile.

Step 2: Connect Blade Shape to the Pressure Gradient.
In a compressor, the flow moves against a rising static pressure (an adverse pressure gradient) as it passes through the rotor passage. An adverse pressure gradient makes the boundary layer prone to separation, so compressor blades must be kept thin and lightly cambered, turning the flow only a small amount per stage, to avoid stalling the blade passage. In a turbine, the flow accelerates through a falling static pressure (a favourable pressure gradient), which keeps the boundary layer attached even with strong curvature, so turbine blades can be thick and highly cambered, turning the flow through a much larger angle in a single stage.

Step 3: Assign the Configurations.
Configuration I's thin, lightly cambered blades match the compressor description from Step 2, so Configuration I is a compressor rotor. Configuration II's thick, strongly cambered blades match the turbine description, so Configuration II is a turbine rotor. This makes option (A) TRUE and option (B) FALSE.

Step 4: Determine the Direction of Rotor Motion.
A compressor rotor does positive work ON the fluid, so it must move with its concave, pressure side face leading the motion, the same way a fan blade pushes air ahead of its cupped face. A turbine rotor, on the other hand, is driven BY the fluid: the flow pressing on the blade's

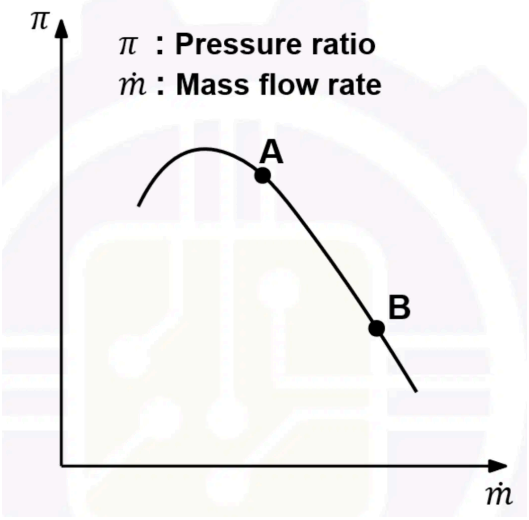
concave (pressure) face is what drives the rotor, so the blade is pushed along with its convex, suction side face leading, the pressure face trails behind. Reading the blade orientation against the marked $+\theta$ direction in the figure, the Configuration I (compressor) blades have their pressure face oriented toward the $-\theta$ side, so that rotor moves in the $-\theta$ direction, not $+\theta$; this makes option (C) FALSE. The Configuration II (turbine) blades have their suction face leading toward the $+\theta$ side, so that rotor moves in the $+\theta$ direction, making option (D) TRUE.

Final Answer:

Configuration I is a compressor, Configuration II is a turbine and moves along $+\theta$, i.e. options (A) and (D)

Quick Tip: Compare blade camber: thin and lightly cambered means compressor (adverse pressure gradient), thick and heavily cambered means turbine (favourable pressure gradient); then check which face, pressure or suction, leads the +theta direction shown.

28. A multi-stage axial compressor can be operated at two points, A and B, both of which lie on the same speed line, as shown in the figure below. If η is the isentropic efficiency of the compressor, select the statements that is/are TRUE.



(A) $\eta_A > \eta_B$

(B) $\eta_B > \eta_A$

(C) In comparison to point B, point A is closer to the surge point

(D) In comparison to point A, point B is closer to the choke point

Correct Answer: (A) $\eta_A > \eta_B$; (C) In comparison to point B, point A is closer to the surge point ; (D) In comparison to point A, point B is closer to the choke point

Solution:

Step 1: Read the Positions of A and B on the Speed Line.

The figure plots pressure ratio π against mass flow rate $\dot{m}$ for one constant speed line of a multi-stage axial compressor. As we move along this line from left to right, mass flow $\dot{m}$ increases while pressure ratio π rises to a peak and then falls. Point A sits near the top of the curve, close to the peak pressure ratio and low mass flow end. Point B sits further along the curve, at a lower pressure ratio and a higher

mass flow rate.

Step 2: Recall What Surge and Choke Mean on This Map.
Surge is an instability that occurs at the LOW mass flow, HIGH pressure ratio end of a speed line, where the flow can no longer support the adverse pressure gradient across the blade rows and breaks down. Choke happens at the opposite, HIGH mass flow end, where the flow in the narrowest passage of the compressor, usually the first rotor throat, reaches the local sonic (Mach 1) condition, so no more mass flow can pass through no matter how much the back pressure is lowered; the speed line becomes nearly vertical there.

Step 3: Locate A and B Relative to Surge and Choke.
Since A is near the peak-pressure-ratio, low mass flow end of the curve, it sits closer to the surge boundary than B does. Since B is further along toward the high mass flow, falling pressure ratio end of the curve, it sits closer to the choke condition than A does. This directly makes option (C), A is closer to surge than B, TRUE, and option (D), B is closer to choke than A, TRUE.

Step 4: Compare the Isentropic Efficiencies.
Isentropic efficiency contours on a compressor map form closed, roughly oval regions with the best efficiency located near the middle to upper part of each speed line, not right at either extreme. As operation moves away from that good-efficiency region toward choke, the relative velocities inside the narrow blade passages rise toward sonic values, so shock losses and flow separation increase quickly and efficiency drops sharply. Point B, being pushed further toward the choke end of the curve, sees this efficiency penalty, while point A, staying closer to the well-behaved part of the speed line near peak pressure ratio, retains a higher efficiency. So $\eta_A > \eta_B$, making option

(A) TRUE and option (B) FALSE.

Final Answer:

$\boxed{\eta_A > \eta_B,\ A \text{ is closer to surge},\ B \text{ is closer to choke, i.e. options (A), (C) and (D)}}$

Quick Tip: On a speed line, low mass flow / high pressure ratio is near surge and high mass flow is near choke; efficiency falls as the operating point moves toward the choke end.

29. Which of the following statements is/are TRUE regarding critical and drag divergence Mach numbers of a wing?

(A) Critical Mach number is the minimum freestream Mach number for which sonic condition is attained somewhere over the wing

(B) Drag divergence Mach number is always higher than the critical Mach number

(C) Drag divergence Mach number is the local Mach number over the wing at which the drag increases drastically

(D) Critical Mach number is independent of the angle of attack

Correct Answer: (A) Critical Mach number is the minimum freestream Mach number for which sonic condition is attained somewhere over the wing ; (B) Drag divergence Mach number is always higher than the critical Mach number

Solution:

Step 1: Recall the definitions.

The critical Mach number M_{cr} of a wing is the freestream Mach

number at which the flow accelerating over the curved upper surface first reaches sonic speed ($M = 1$) at some point on the wing, even though the freestream itself is still subsonic. Below M_{cr} the entire flow field around the wing stays subsonic.
The drag divergence Mach number M_{dd} is a freestream Mach number, higher than M_{cr}, at which the local supersonic pocket has grown large enough that a shock wave forms and causes flow separation, and the drag coefficient starts rising very sharply with further increase in Mach number.

Step 2: Check option (A).
Option (A) restates the definition of M_{cr} exactly: it is the minimum freestream Mach number at which sonic flow first appears somewhere on the wing (usually at the point of maximum thickness or maximum camber, where the local velocity peaks). This statement is TRUE.

Step 3: Check option (B).
Once M_{cr} is reached, a small sonic pocket exists but the drag rise is still gradual, because that pocket is tiny and often terminates in a weak shock. Only when the freestream Mach number increases further does the pocket grow, the shock strengthens, and separation sets in, producing the sharp drag rise. So M_{dd} always occurs after (numerically greater than) M_{cr}; the wing always passes through M_{cr} before it can reach M_{dd}. This statement is TRUE.

Step 4: Check option (C).
This option swaps freestream and local Mach number. M_{dd} is defined using the freestream Mach number, the number a pilot or a wind tunnel operator actually sets, not the local Mach number over the wing (which is already supersonic at that point, well above 1, and is not the quantity being reported as "the drag divergence Mach

number"). Because it names the wrong (local, not freestream) Mach number, this statement is FALSE.

Step 5: Check option (D).
The critical Mach number depends on how much the local flow accelerates above the freestream speed, and that acceleration is controlled by the wing section shape and by the angle of attack. Raising the angle of attack increases the suction peak on the upper surface, so the local flow reaches sonic speed at a lower freestream Mach number, lowering M_{cr}. Since M_{cr} changes with angle of attack, this statement is FALSE.

Final Answer:
Statements (A) and (B) are true, while (C) and (D) are false.

$$\boxed{\text{(A) and (B)}}$$

Quick Tip: Compare where sonic flow first appears (critical Mach) with where the shock-induced drag rise happens (drag divergence Mach); both are defined using the freestream Mach number, and both depend on angle of attack.

30. A flow is steady, inviscid and one-dimensional, with no shaft work or body forces. Which of the following is/are possible under the given conditions?

(A) Oblique shocks

(B) Sound propagation

(C) Rayleigh flow

(D) Fanno flow

Correct Answer: (B) Sound propagation ; (C) Rayleigh flow

Solution:

Step 1: List the given conditions.

The flow is steady (no time dependence), inviscid (no friction/viscosity), one-dimensional (velocity has only one component, along the flow direction) and has no shaft work (no turbine or compressor exchanging work with the fluid) and no body forces (gravity, etc. ignored). We need to check each listed phenomenon against these four conditions.

Step 2: Check option (A), oblique shocks.

An oblique shock deflects the flow through a finite turning angle, so the velocity vector changes direction across the shock. This means a transverse velocity component appears, which breaks the one-dimensional assumption (a strictly 1-D flow can only have velocity along a single line, as in a normal shock, not turned flow). So oblique shocks are not possible in a flow restricted to be one-dimensional; this option is not correct.

Step 3: Check option (B), sound propagation.

A sound wave is a small, inviscid, one-dimensional disturbance. If we look at it in a reference frame moving with the wave (the same trick used to derive the speed of sound), the wave becomes a stationary, steady control volume problem: the flow enters and leaves at slightly

different properties, with no friction, no shaft work and no body force acting on this thin control volume. This matches every given condition, so sound propagation is possible.

Step 4: Check option (C), Rayleigh flow.
Rayleigh flow is the standard model for constant-area duct flow with heat addition and negligible friction, i.e. it is explicitly inviscid, one-dimensional and steady. Heat addition is not shaft work (no mechanical device is doing work on the fluid) and body forces are still neglected, so all four conditions hold. Rayleigh flow is a valid possibility here.

Step 5: Check option (D), Fanno flow.
Fanno flow is the standard model for constant-area, adiabatic duct flow WITH friction; friction is precisely what drives the property changes along the duct in this model. Since the problem states the flow is inviscid (no friction), Fanno flow directly contradicts that condition and is not possible here.

Final Answer:
Only sound propagation and Rayleigh flow are consistent with a steady, inviscid, one-dimensional flow having no shaft work or body forces.

$$\boxed{\text{(B) and (C)}}$$

Quick Tip: Match each phenomenon's defining mechanism, friction, heat addition, flow turning, or wave propagation, against the four stated restrictions: steady, inviscid, one-dimensional, no shaft work/body force.

31. An aircraft starts gliding in power-off condition at an altitude of 4 km. Given that the maximum lift to drag ratio of the aircraft is 15, the maximum glide range that the aircraft can cover, measured along the ground, is _____ km (rounded off to the nearest integer).

Correct Answer: 60

Solution:

Step 1: Set up the glide relation.

In power-off (engine idle/off) gliding flight, the aircraft is in equilibrium under lift L, drag D and weight W, descending along a straight glide path at angle γ below the horizontal. Resolving forces along and perpendicular to the flight path gives $L = W\cos\gamma$ and $D = W\sin\gamma$, so $\tan\gamma = D/L$.

Step 2: Relate glide angle to lift to drag ratio.

Since $\tan\gamma = D/L = 1/(L/D)$, the glide angle is smallest, and hence the horizontal distance covered for a given loss of altitude is largest, when L/D is at its maximum value. Using $(L/D)_{max} = 15$:

$$\tan\gamma_{min} = \frac{1}{(L/D)_{max}} = \frac{1}{15}$$

Step 3: Relate glide range to altitude.

If the aircraft glides from height h down to the ground, the horizontal distance R (glide range) and h form the two legs of the glide-path triangle, so $\tan\gamma = h/R$, which gives $R = h/\tan\gamma$. For the maximum range case, this becomes $R_{max} = h \times (L/D)_{max}$. This is the well known result that maximum glide range equals altitude lost multiplied by the best lift to drag ratio (independent of aircraft weight or air density).

Step 4: Substitute the numbers.

The aircraft starts at $h = 4$ km and $(L/D)_{max} = 15$:

$$R_{max} = 4 \times 15 = 60 \text{ km}$$

Final Answer:

The maximum glide range measured along the ground is 60 km.

$$\boxed{60 \text{ km}}$$

Quick Tip: Maximum glide range = altitude lost multiplied by the maximum lift to drag ratio, since $\tan \gamma_{min} = 1/(L/D)_{max}$.

32. If a matrix can be written as $A = uv^T$, where both u and v are n-dimensional real-valued non-zero column vectors, then the rank of the matrix A is _____ (answer in integer).

Correct Answer: 1

Solution:

Step 1: Recall what an outer product looks like.

Here u and v are both $n \times 1$ non-zero column vectors, so v^T is a $1 \times n$ row vector. The product $A = uv^T$ is then an $n \times n$ matrix, formed as the outer product of u and v. Writing it out, the (i, j) entry of A is $A_{ij} = u_i v_j$, so every row of A is just the row vector v^T scaled by the corresponding entry of u.

Step 2: Look at the column space of A.

Every column of A can be written as $A_{:,j} = v_j\, u$, i.e. each column is a scalar multiple of the single vector u. So all columns of A lie along the one direction u; the column space of A is spanned by just the vector u.

Step 3: Determine the rank.

The rank of a matrix equals the dimension of its column space. Since every column is a multiple of the single non-zero vector u, the column space is one-dimensional (it cannot be zero-dimensional because u is non-zero, and v being non-zero guarantees at least one column has a non-zero multiple $v_j \neq 0$, so A is not the zero matrix). Therefore the column space has dimension exactly 1.

Final Answer:

The rank of $A = uv^T$, for non-zero vectors u and v, is always 1.

$$\boxed{\text{rank}(A) = 1}$$

Quick Tip: Every column of $A = uv^T$ is a scalar multiple of u, so the column space is one-dimensional whenever u, v are both non-zero.

33. The response $x(t)$ of a freely vibrating single degree of freedom underdamped system is given below. In the equation, A and ϕ are constants. The damping ratio of the system is _____ (rounded off to 3 decimal places).

$$x(t) = Ae^{-5t}\sin(10t + \phi)$$

Correct Answer: 0.447

Solution:

Step 1: Match the response with the standard underdamped form.

The free response of an underdamped single degree of freedom system is written in the standard form

$$x(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$$

where ζ is the damping ratio, ω_n is the undamped natural frequency, and $\omega_d = \omega_n\sqrt{1-\zeta^2}$ is the damped natural frequency. Comparing this with the given response

$$x(t) = Ae^{-5t}\sin(10t + \phi)$$

term by term gives two equations: the exponential decay rate $\zeta\omega_n = 5$, and the oscillation frequency $\omega_d = \omega_n\sqrt{1-\zeta^2} = 10$.

Step 2: Eliminate ω_n.

From the first equation, $\omega_n = 5/\zeta$. Substitute this into the second equation:

$$\frac{5}{\zeta}\sqrt{1-\zeta^2} = 10$$

$$\sqrt{1-\zeta^2} = 2\zeta$$

Step 3: Solve for the damping ratio.

Square both sides (valid since $0 < \zeta < 1$ for an underdamped system, so both sides are positive):

$$1 - \zeta^2 = 4\zeta^2$$

$$1 = 5\zeta^2$$

$$\zeta^2 = \frac{1}{5} = 0.2$$

$$\zeta = \sqrt{0.2} = 0.4472$$

Step 4: Round to the required precision.

Rounded off to 3 decimal places, $\zeta = 0.447$. This is comfortably inside $(0, 1)$, confirming the system is indeed underdamped, consistent with the oscillatory $\sin(\cdot)$ form given.

Final Answer:

The damping ratio of the system is 0.447.

$$\boxed{\zeta \approx 0.447}$$

Quick Tip: Compare $x(t) = Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$ with the given response: $\zeta\omega_n = 5$, $\omega_n\sqrt{1-\zeta^2} = 10$, then solve the two equations together.

34. The vortex shedding frequency behind a landing gear model is found to be 50 Hz when tested in a wind tunnel operating at 5 m/s. If the actual landing gear size is 10 times that of the model, and it is designed to operate at 50 m/s, then the expected vortex shedding frequency behind it is _____ Hz (rounded off to the nearest integer).

Correct Answer: 50

Solution:

Step 1: Identify the similarity parameter.

Vortex shedding behind a bluff body (like a landing gear strut) is characterised by the Strouhal number, a dimensionless group defined as

$$St = \frac{fD}{V}$$

where f is the shedding frequency, D is a characteristic size of the body, and V is the freestream velocity. For dynamically similar flows (same flow regime, which is the standard assumption made when scaling wind tunnel model results to full scale for this kind of problem), the Strouhal number is the same for the model and the full scale (prototype) body.

Step 2: Write the similarity condition.

Equating the Strouhal numbers of the model (subscript 1) and the actual landing gear (subscript 2):

$$\frac{f_1 D_1}{V_1} = \frac{f_2 D_2}{V_2}$$

Step 3: Substitute the given values.

The model has $f_1 = 50$ Hz, $V_1 = 5$ m/s, and some size D_1. The actual landing gear has size $D_2 = 10D_1$ and operates at $V_2 = 50$ m/s.

Substituting:

$$\frac{50 \times D_1}{5} = \frac{f_2 \times 10D_1}{50}$$

$$10D_1 = \frac{10f_2D_1}{50}$$

Step 4: Solve for f_2.

Cancel D_1 (non-zero) from both sides:

$$10 = \frac{10f_2}{50}$$

$$10 = \frac{f_2}{5}$$

$$f_2 = 50 \text{ Hz}$$

Final Answer:

The expected vortex shedding frequency behind the actual landing gear is 50 Hz.

$$\boxed{f_2 = 50 \text{ Hz}}$$

Quick Tip: Match Strouhal numbers, $St = fD/V$ constant, between the model and the actual landing gear, then solve for f_2.

36. An earth satellite has the instantaneous position vector $\vec{r}$ and velocity vector $\vec{v}$ as given below. Here $\hat{p}$ and $\hat{q}$ denote the unit vectors along the x and y axes of the perifocal frame, respectively. Assume that the value of the gravitational parameter is 398600 km^3/s^2. Which one of the following trajectories does the satellite follow?

$$\vec{r} = (8000\hat{p} + 9000\hat{q})\ \text{km and}\ \vec{v} = (-6\hat{p} + 6\hat{q})\ \text{km/s}$$

(A) Circle

(B) Hyperbola

(C) Parabola

(D) Straight line

Correct Answer: (B) Hyperbola

Solution:

Step 1: Find the magnitude of the position and velocity vectors.

The position vector is $\vec{r} = 8000\hat{p} + 9000\hat{q}$ km, so its magnitude is

$$r = \sqrt{8000^2 + 9000^2} = \sqrt{145000000} \approx 12041.6\ \text{km}$$

The velocity vector is $\vec{v} = -6\hat{p} + 6\hat{q}$ km/s, so its magnitude is

$$v = \sqrt{(-6)^2 + 6^2} = \sqrt{72} \approx 8.485\ \text{km/s}$$

Step 2: Use the vis-viva energy equation to find the specific mechanical energy.

For any two-body orbit the specific mechanical energy is

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r}$$

This single number tells us the shape of the orbit: negative energy gives an ellipse (or circle), zero energy gives a parabola, and positive energy gives a hyperbola.

Step 3: Substitute the numbers.

$$\varepsilon = \frac{72}{2} - \frac{398600}{12041.6} = 36 - 33.10 = 2.90 \text{ km}^2/\text{s}^2$$

The energy comes out positive.

Step 4: Check that the orbit is not degenerate (rule out the straight line option).

The specific angular momentum is

$$h = r_p v_q - r_q v_p = (8000)(6) - (9000)(-6) = 48000 + 54000 = 102000 \text{ km}^2/\text{s}$$

Since $h \neq 0$, the position and velocity vectors are not parallel, so the satellite is not falling on a straight radial line. This rules out option (D).

Step 5: Interpret the sign of the energy.

Positive specific energy means the satellite has more kinetic energy than it needs to stay bound to the earth. Its semi-major axis works out negative, $a = -\mu/(2\varepsilon) \approx -68765$ km, which is the signature of a

hyperbolic orbit. A circle needs $\varepsilon < 0$ with zero eccentricity, and a parabola needs $\varepsilon = 0$ exactly, so neither fits here.

Final Answer:

The satellite follows a hyperbolic trajectory.

$$\boxed{\text{Hyperbola}}$$

Quick Tip: Work out the specific mechanical energy $\varepsilon = v^2/2 - \mu/r$; its sign alone tells you the conic section (positive means hyperbola).

37. For the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, if the relation $a + b = c + d$ holds and $a, b, c, d \neq 0$, then which one of the following statements about A is FALSE?

(A) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector

(B) $\lambda = a + b$ is an eigenvalue

(C) $\lambda = d - b$ is an eigenvalue

(D) $\lambda = d + b$ is an eigenvalue

Correct Answer: (D) $\lambda = d + b$ is an eigenvalue

Solution:

Step 1: Test the vector [1, 1] against the given condition.

Multiply A by the column vector $[1, 1]^T$:

$$A \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a + b \\ c + d \end{bmatrix}$$

We are told $a + b = c + d$, so both entries of the result are the same number, $a + b$. This means

$$A\begin{bmatrix}1\\1\end{bmatrix} = (a+b)\begin{bmatrix}1\\1\end{bmatrix}$$

By definition this makes $[1, 1]^T$ an eigenvector of A, with eigenvalue $\lambda_1 = a + b$. So statements (A) and (B) are both TRUE.

Step 2: Find the second eigenvalue using the trace.
For a 2x2 matrix, the sum of the two eigenvalues always equals the trace (the sum of the diagonal entries):

$$\lambda_1 + \lambda_2 = a + d$$

We already know $\lambda_1 = a + b$, so

$$\lambda_2 = (a + d) - (a + b) = d - b$$

Step 3: Compare this with the remaining statements.
Statement (C) says $\lambda = d - b$ is an eigenvalue. That is exactly the λ_2 we just found, so (C) is TRUE.
Statement (D) says $\lambda = d + b$ is an eigenvalue. But the real second eigenvalue is $d - b$, not $d + b$. Since $b \neq 0$, $d + b$ and $d - b$ are two different numbers, so $d + b$ cannot equal λ_2 in general. It also cannot equal $\lambda_1 = a + b$ unless $a = d$, which is not guaranteed by the given condition.

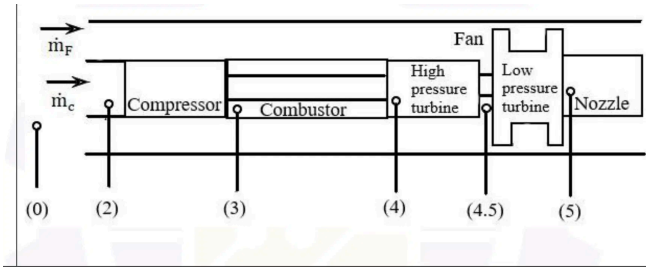
Final Answer:

Statement (D), $\lambda = d + b$ is an eigenvalue, is the FALSE one.

$$\boxed{\lambda = d + b \text{ is NOT an eigenvalue}}$$

Quick Tip: Show $[1, 1]^T$ is an eigenvector using the given relation, then use trace = sum of eigenvalues to find the second one.

38. In an ideal turbofan engine shown in the figure below, the compressor is driven by the high pressure turbine, and the fan is driven by the low pressure turbine. The stations 0, 2, 3, 4, 4.5, and 5 refer to free-stream, compressor inlet, compressor outlet, combustor exit, high pressure turbine exit, and low pressure turbine exit, respectively, and the subscript 't' refers to the total condition. Also, $\tau_r = T_{t0}/T_0$, $\tau_c = T_{t3}/T_{t2}$ and $\tau_\lambda = T_{t4}/T_0$. The total temperature ratio of the high pressure turbine $(T_{t4.5}/T_{t4})$ is given by _____.



(A) $1 - \frac{\tau_r}{\tau_\lambda}(\tau_c - 1)$

(B) $1 + \frac{\tau_r}{\tau_\lambda}(\tau_c + 1)$

(C) $1 - \frac{\tau_r}{\tau_\lambda}(\tau_c + 1)$

(D) $1 + \frac{\tau_r}{\tau_\lambda}(\tau_c - 1)$

Correct Answer: (A) $1 - \frac{\tau_r}{\tau_\lambda}(\tau_c - 1)$

Solution:

Step 1: State the power balance on the high-pressure spool.

The high pressure turbine (between stations 4 and 4.5) only exists to drive the compressor (between stations 2 and 3); no other device sits on that shaft. In an ideal engine with no mechanical loss, the power the turbine delivers must exactly equal the power the compressor absorbs:

$$\dot{m}c_p(T_{t4} - T_{t4.5}) = \dot{m}c_p(T_{t3} - T_{t2})$$

The mass flow rate and c_p cancel from both sides (ideal engine, same working fluid), leaving

$$T_{t4} - T_{t4.5} = T_{t3} - T_{t2}$$

Step 2: Write the compressor and combustor-exit temperatures using the given ratios.

By definition $\tau_r = T_{t0}/T_0$. Across an ideal inlet the total temperature does not change, so $T_{t2} = T_{t0} = \tau_r T_0$.

By definition $\tau_c = T_{t3}/T_{t2}$, so $T_{t3} = \tau_c T_{t2} = \tau_c \tau_r T_0$.

By definition $\tau_\lambda = T_{t4}/T_0$, so $T_{t4} = \tau_\lambda T_0$.

Step 3: Substitute into the power balance from Step 1.

$$T_{t4} - T_{t4.5} = T_{t3} - T_{t2} = \tau_c \tau_r T_0 - \tau_r T_0 = \tau_r T_0(\tau_c - 1)$$

So

$$T_{t4.5} = T_{t4} - \tau_r T_0(\tau_c - 1) = \tau_\lambda T_0 - \tau_r T_0(\tau_c - 1)$$

Step 4: Divide by $T_{t4} = \tau_\lambda T_0$ to get the required ratio.

$$\frac{T_{t4.5}}{T_{t4}} = 1 - \frac{\tau_r T_0(\tau_c - 1)}{\tau_\lambda T_0} = 1 - \frac{\tau_r}{\tau_\lambda}(\tau_c - 1)$$

Step 5: Check the other options.

Option (D) has the right bracket $(\tau_c - 1)$ but the wrong sign in front, which would mean the turbine adds temperature instead of extracting it, so it is wrong. Option (C) uses $(\tau_c + 1)$ instead of $(\tau_c - 1)$, which drops the correction for the compressor's own inlet temperature ratio, so it is wrong. Option (B) combines both mistakes together.

Final Answer:

$$\boxed{\frac{T_{t4.5}}{T_{t4}} = 1 - \frac{\tau_r}{\tau_\lambda}(\tau_c - 1)}$$

Quick Tip: The HP turbine only drives the compressor, so equate the compressor's temperature rise to the HP turbine's temperature drop.

39. An ideal rocket has characteristic exhaust velocity of 1200 m/s, mass flow rate of 75 kg/s, thrust coefficient of 1.5, and nozzle throat area of 0.025 m^2. The chamber pressure in kPa and the specific impulse due to gravity in seconds are _____, respectively. Assume that the acceleration due to gravity is 9.8 m/s^2.

(A) 3600 and 183.67

(B) 4600 and 190.51

(C) 3600 and 175.23

(D) 3500 and 183.67

Correct Answer: (A) 3600 and 183.67

Solution:

Step 1: Recall the definition of characteristic exhaust velocity.

The characteristic velocity c^* links the chamber pressure p_c, the throat area A_t, and the mass flow rate:

$$c^* = \frac{p_c A_t}{\dot{m}}$$

This number depends only on the propellant and chamber conditions, not on the nozzle expansion, so it is a good first quantity to use.

Step 2: Solve for the chamber pressure.

Rearranging,

$$p_c = \frac{c^* \dot{m}}{A_t} = \frac{1200 \times 75}{0.025} = \frac{90000}{0.025} = 3600000 \text{ Pa} = 3600 \text{ kPa}$$

Step 3: Find the thrust using the thrust coefficient.

The thrust coefficient relates thrust to the chamber pressure and throat area:

$$C_F = \frac{F}{p_c A_t} \implies F = C_F \, p_c A_t$$

$$F = 1.5 \times 3600000 \times 0.025 = 1.5 \times 90000 = 135000 \text{ N}$$

Step 4: Get the effective exhaust velocity, then the specific impulse.

The effective exhaust velocity is the thrust divided by the mass flow rate:

$$c = \frac{F}{\dot{m}} = \frac{135000}{75} = 1800 \text{ m/s}$$

The specific impulse due to gravity is this velocity divided by g:

$$I_{sp} = \frac{c}{g} = \frac{1800}{9.8} \approx 183.67 \text{ s}$$

Step 5: Check the other options.

Options (B) and (D) give a chamber pressure other than 3600 kPa, which does not match the direct c^* relation above. Option (C) keeps the correct chamber pressure but lands on a lower specific impulse, which does not come out of the correct thrust and mass-flow numbers.

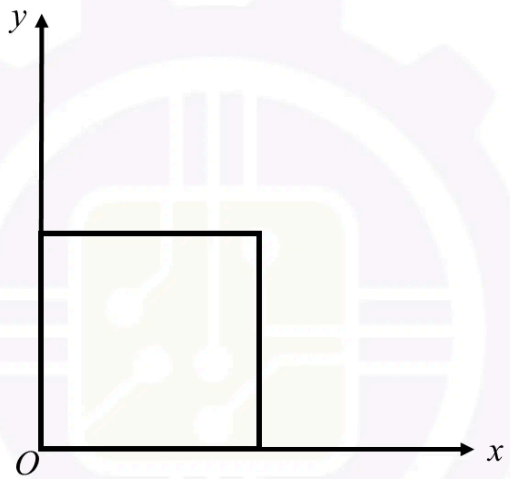
Final Answer:

The chamber pressure is 3600 kPa and the specific impulse is 183.67 s.

$$\boxed{p_c = 3600 \text{ kPa},\ I_{sp} \approx 183.67 \text{ s}}$$

Quick Tip: Use $c^* = p_c A_t/\dot{m}$ for the chamber pressure, then $F = C_F p_c A_t$ and $I_{sp} = F/(\dot{m}g)$ for the specific impulse.

40. Consider a unit square body as shown in the figure below. The body is subjected to the deformation field $u = -ay$ and $v = ax$, where 'a' is a constant. Due to the application of this deformation field, the body undergoes _____ in the x-y plane.



(A) biaxial deformation

(B) pure shear

(C) pure bending

(D) rigid body rotation

Correct Answer: (D) rigid body rotation

Solution:

Step 1: Write down the displacement gradient tensor.

The displacement field is $u = -ay$, $v = ax$. Differentiate each component with respect to x and y:

$$\frac{\partial u}{\partial x} = 0, \quad \frac{\partial u}{\partial y} = -a, \quad \frac{\partial v}{\partial x} = a, \quad \frac{\partial v}{\partial y} = 0$$

Step 2: Split the displacement gradient into its symmetric (strain) and antisymmetric (rotation) parts.

The normal strains are

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = 0, \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} = 0$$

The engineering shear strain is

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -a + a = 0$$

So the shear strain $\varepsilon_{xy} = \frac{1}{2}\gamma_{xy} = 0$ as well.

Step 3: Find the rotation part.

The (small) rotation about the z-axis is

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(a - (-a)) = a$$

This is nonzero, so the field has a rotation but no strain of any kind.

Step 4: Match this to the given options.

Since every strain component (ε_{xx}, ε_{yy}, ε_{xy}) is exactly zero, the square does not stretch along either axis (rules out biaxial deformation), does not distort its right angles (rules out pure shear), and does not bend or curve (rules out pure bending, which would need a strain varying through the body). All that remains is a plain rotation of the whole

square by angle a about the origin, which changes its orientation but not its size or shape.

Final Answer:

The body undergoes rigid body rotation in the x-y plane.

Rigid body rotation

Quick Tip: Compute the strain tensor from the displacement field first; if every strain component is zero but the antisymmetric rotation term is not, the motion is a pure rotation.

41. Consider a launch vehicle of mass 10 tons being launched vertically. The vehicle has 8 tons of propellant, which burns completely at a constant rate over 50 s. If the engine specific impulse is 250 s, and the acceleration due to gravity at sea level is g_0, the acceleration experienced by the vehicle at lift-off is _____.

(A) g_0

(B) $2g_0$

(C) $3g_0$

(D) $4g_0$

Correct Answer: (C) $3g_0$

Solution:

Step 1: Find the propellant mass flow rate.

The vehicle burns 8 tons of propellant at a constant rate over 50 s. Using 1 ton = 1000 kg, that is 8000 kg burned in 50 s, so

$$\dot{m} = \frac{8000}{50} = 160 \text{ kg/s}$$

Step 2: Find the thrust from the specific impulse.

Specific impulse links thrust, mass flow rate, and g_0 as

$$I_{sp} = \frac{F}{\dot{m} g_0} \implies F = \dot{m} I_{sp} g_0$$

$$F = 160 \times 250 \times g_0 = 40000\, g_0 \text{ N}$$

Step 3: Apply Newton's second law at the instant of lift-off.

At lift-off, none of the propellant has burned yet, so the vehicle still has its full initial mass, $m_0 = 10$ tons $= 10000$ kg. Two forces act on it: the upward thrust F and the downward weight $m_0 g_0$. The net upward force gives the net acceleration:

$$a = \frac{F - m_0 g_0}{m_0} = \frac{F}{m_0} - g_0$$

Step 4: Substitute the numbers.

$$\frac{F}{m_0} = \frac{40000\, g_0}{10000} = 4 g_0$$

$$a = 4g_0 - g_0 = 3g_0$$

Step 5: Check the other options.

Option (D), $4g_0$, is the thrust-to-mass ratio F/m_0 on its own, before subtracting the vehicle's own weight; it is not the net acceleration the vehicle actually feels, since gravity is still pulling it down throughout. Options (A) and (B) do not come from a correct force balance at all: skipping the weight term or using the wrong mass gives numbers that are too small.

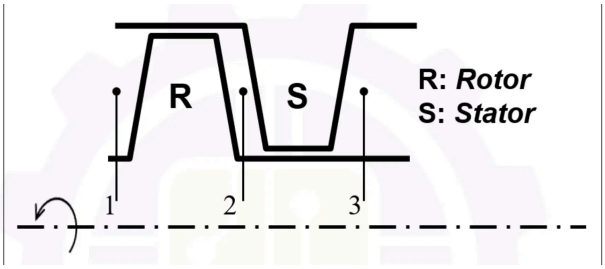
Final Answer:

The vehicle accelerates upward at $3g_0$ at the moment of lift-off.

$$\boxed{a = 3g_0}$$

Quick Tip: First find the thrust from $F = \dot{m}I_{sp}g_0$, then subtract the vehicle's own weight from the thrust before dividing by mass.

42. The figure below shows a compressor stage with station numbers 1, 2, and 3 as indicated. If p_{0i}, T_{0i}, and C_i refer to the average values of total pressure, total temperature, and absolute flow speed, respectively, at the i^{th} station, select the CORRECT option considering losses.



(A) $p_{01} < p_{02},\ p_{02} > p_{03}; T_{01} < T_{02},\ T_{02} \approx T_{03}; C_1 < C_2,\ C_2 > C_3$

(B) $p_{01} < p_{02},\ p_{02} = p_{03}; T_{01} < T_{02},\ T_{02} \approx T_{03}; C_1 < C_2,\ C_2 = C_3$

(C) $p_{01} < p_{02},\ p_{02} > p_{03}; T_{01} < T_{02},\ T_{02} < T_{03}; C_1 > C_2,\ C_2 > C_3$

(D) $p_{01} < p_{02},\ p_{02} < p_{03}; T_{01} < T_{02}, T_{02} < T_{03}; C_1 = C_2 = C_3$

Correct Answer: (A) $p_{01} < p_{02},\ p_{02} > p_{03}; T_{01} < T_{02},\ T_{02} \approx T_{03}; C_1 < C_2,\ C_2 > C_3$

Solution:

Step 1: Identify what each blade row does.

Station 1 is ahead of the rotor, station 2 is between the rotor and the stator, and station 3 is after the stator. The rotor (R) is the moving blade row that does work on the flow, and the stator (S) is a stationary blade row that only redirects and diffuses the flow.

Step 2: Track total pressure and total temperature across the rotor (station 1 to 2).

The rotor absorbs shaft work and feeds it into the air as it spins the flow up (imparts swirl). Real work is added to the fluid, so its total temperature must rise: $T_{02} > T_{01}$, always true, with or without losses, since it follows directly from the energy actually put into the flow. Total pressure also rises, $p_{02} > p_{01}$, because useful work raises stagnation pressure even though blade friction eats into part of the ideal (isentropic) rise.

Step 3: Track total pressure and total temperature across the stator (station 2 to 3).

The stator does no work at all; it just turns and slows the flow down to raise static pressure. Since no work and no heat is added, the stagnation enthalpy, and hence the total temperature, stays

essentially unchanged: $T_{02} \approx T_{03}$. Total pressure, however, always falls slightly across any real blade row because of skin friction and secondary flow losses, even though no work is involved: $p_{02} > p_{03}$.

Step 4: Track the absolute flow speed C across the stage.
The rotor, by adding tangential swirl velocity to the flow, increases the absolute speed as seen by a stationary observer: $C_2 > C_1$. The stator's whole job is to diffuse this fast, swirling flow, converting some of that kinetic energy into static pressure rise, which slows the flow back down: $C_3 < C_2$.

Step 5: Rule out the other options.
Option (B) assumes the stator is lossless ($p_{02} = p_{03}$), but the question explicitly says to consider losses, so this cannot be right. Option (C) has the temperature rising across the stator ($T_{02} < T_{03}$), which would require the stator to do work on the flow, and it also has the speed falling across the rotor ($C_1 > C_2$), the opposite of what work addition does. Option (D) has total pressure rising across the stator ($p_{02} < p_{03}$) with no work input, which violates the second law for a real, lossy blade row, and it wrongly keeps the speed constant through the whole stage.

Final Answer:

$$\boxed{p_{01} < p_{02},\ p_{02} > p_{03};\ T_{01} < T_{02},\ T_{02} \approx T_{03};\ C_1 < C_2,\ C_2 > C_3}$$

Quick Tip: The rotor adds real work (so p and T both rise), the stator adds no work (so T stays fixed) but still loses total pressure to friction while it diffuses and slows the flow.

43. An experimental study is planned to map out the low-Reynolds number incompressible steady two-dimensional aerodynamic characteristics of a promising novel airfoil. The operational parameters of the problem are the speed, density and viscosity of the freestream, the chord of the airfoil and its angle of attack. If the objective is to achieve this with the minimum number of test runs N_{min} while taking 10 equally-spaced test values of each independent parameter of the problem in a suitable range, then N_{min} is _____.

(A) 10

(B) 100

(C) 10,000

(D) 1,00,000

Correct Answer: (B) 100

Solution:

Step 1: Identify the physical variables involved.
The experiment studies the low speed aerodynamic behaviour of an airfoil section. The variables that control the result are the freestream speed V, the freestream density ρ, the freestream viscosity μ, the airfoil chord c, and the angle of attack α. That gives 5 operational parameters in total.

Step 2: Separate dimensional and non-dimensional variables.
Four of these variables, V, ρ, μ, and c, carry physical dimensions built from mass, length and time (M, L, T). The angle of attack α is already a pure number (it is a ratio, measured in radians or degrees), so it does not need any further reduction.

Step 3: Apply the Buckingham Pi theorem to the dimensional

variables.

The Buckingham Pi theorem states that the number of independent dimensionless groups equals the number of dimensional variables minus the number of fundamental dimensions they use. Here there are 4 dimensional variables (V, ρ, μ, c) and 3 fundamental dimensions (M, L, T), so the count of independent dimensionless groups is

$$4 - 3 = 1$$

This single group is the Reynolds number, $Re = \frac{\rho V c}{\mu}$. So instead of separately varying V, ρ, μ and c, the entire aerodynamic behaviour at a given angle of attack depends only on Re.

Step 4: Count the true number of independent governing parameters.

Adding back the already-dimensionless angle of attack α, the flow field is fully controlled by just 2 independent non-dimensional parameters: Re and α.

Step 5: Build the minimum test matrix.

The plan is to take 10 equally spaced test values of each independent parameter. Since there are only 2 truly independent parameters (Re and α), and every combination of the two must be tested to map out the characteristics, the minimum number of runs is

$$N_{min} = 10 \times 10 = 10^2 = 100$$

Step 6: Why the other options are wrong.

Option (A), 10, would be correct only if there were a single

independent parameter, but the angle of attack cannot be folded into the Reynolds number, so one test sweep is not enough.
Options (C) 10,000 ($= 10^4$) and (D) 1,00,000 ($= 10^5$) come from testing the 4 or 5 raw dimensional variables independently without first combining V, ρ, μ and c into the Reynolds number. That approach wastes runs, because many different combinations of V, ρ, μ, c give the same Re and therefore the same flow behaviour. Dimensional analysis is exactly what removes this redundancy.

Final Answer:
The minimum number of test runs is 100.

$$\boxed{N_{min} = 100}$$

Quick Tip: Use the Buckingham Pi theorem to reduce the 4 dimensional variables (speed, density, viscosity, chord) to the Reynolds number; the angle of attack is already dimensionless. Two independent parameters mean 10^2 runs.

44. Which of the following process(es) is/are involved in the compression of air in an ideal ramjet engine?

(A) oblique shock

(B) mechanical compression

(C) normal shock

(D) subsonic diffusion

Correct Answer: (A) oblique shock ; (C) normal shock ; (D) subsonic diffusion

Solution:

Step 1: Understand how a ramjet compresses air.

A ramjet has no compressor blades or turbine. All of the pressure rise needed before combustion comes purely from slowing down the incoming high speed air, which is called ram compression. The whole job is done by the shape of the inlet/diffuser duct.

Step 2: Compression at supersonic flight speed.

When the ramjet flies supersonically, the air entering the intake is still supersonic. An efficient supersonic diffuser first slows this air down using one or more oblique shocks. Oblique shocks give a smaller loss in stagnation pressure than a single strong shock, so real ramjet inlets are shaped to create a system of oblique shocks ahead of the throat.

Step 3: The final shock down to subsonic flow.

After the oblique shocks reduce the Mach number, the flow is still supersonic (just closer to Mach 1). A normal shock, placed near the inlet throat, then completes the job of dropping the flow from supersonic to subsonic.

Step 4: Subsonic diffusion after the shock system.

Once the flow is subsonic, it still carries more kinetic energy than the combustor can use efficiently. The diverging (widening) duct downstream of the normal shock acts as a subsonic diffuser, slowing the flow further and converting that leftover kinetic energy into an additional rise in static pressure, before the air reaches the combustion chamber.

Step 5: Why mechanical compression is excluded.

Mechanical compression means using rotating compressor blades (as

in a turbojet or turbofan) to add work to the air. A ramjet, by definition, has no compressor rotor and no turbine. It relies only on the ram effect of the incoming air, so mechanical compression plays no part in an ideal ramjet.

Final Answer:
The compression in an ideal ramjet is carried out by oblique shocks, a normal shock, and subsonic diffusion; mechanical compression is not present.

A, C, D

Quick Tip: Remember: an ideal ramjet has no compressor rotor. All compression comes from decelerating the air through oblique shocks, a normal shock, and subsonic diffusion.

45. The deformation of an open-section bar subjected to pure torsion can be solved by choosing an appropriate Prandtl stress function. Which of the following statements is/are true about the Prandtl stress function?

(A) It satisfies the equilibrium equation

(B) It is zero on the lateral surfaces of the bar

(C) It satisfies the compatibility equation

(D) It does not satisfy the equilibrium equation

Correct Answer: (A) It satisfies the equilibrium equation ; (B) It is zero on the lateral surfaces of the bar ; (C) It satisfies the compatibility equation

Solution:

Step 1: Recall why the Prandtl stress function is introduced.

In Saint-Venant torsion theory, instead of solving directly for the two shear stress components, Prandtl defined a single scalar function $\phi(x, y)$ and wrote the shear stresses as $\tau_{xz} = \frac{\partial \phi}{\partial y}$ and $\tau_{yz} = -\frac{\partial \phi}{\partial x}$. The whole point of building the stresses this way is to make one of the governing conditions automatic.

Step 2: Check the equilibrium equation.

For a bar under pure torsion with no body forces, the equilibrium equation in the cross-sectional plane reduces to $\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0$. Substituting Prandtl's definitions gives

$$\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial y \partial x} = 0$$

which is true for any smooth function ϕ, since mixed partial derivatives are equal. So equilibrium is satisfied identically, by construction, no matter what ϕ turns out to be. Statement (A) is TRUE, and statement (D), which says the opposite, is FALSE.

Step 3: Check the compatibility equation.

Equilibrium alone does not fix ϕ; a second condition, compatibility of strains (equivalently, that the warping displacement is single-valued), must also hold. Working this condition through leads to the governing Poisson equation

$$\nabla^2 \phi = -2G\theta$$

where G is the shear modulus and θ is the twist per unit length. This equation is exactly the compatibility requirement written in terms of

ϕ, so any valid Prandtl stress function must satisfy it. Statement (C) is TRUE.

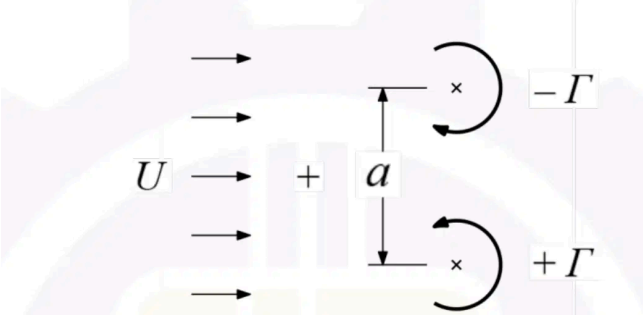
Step 4: Check the boundary condition.
The lateral surface of the bar (its outer boundary, running along the length of the bar) is stress free, since no external traction acts there, only the two end faces carry the applied torque. Working out what this stress-free condition means for ϕ shows that ϕ must stay constant along the entire boundary curve. For a solid, simply connected cross-section (a single unbroken boundary curve, as an open-section bar has), that constant can always be chosen as zero without loss of generality. So $\phi = 0$ on the lateral surface. Statement (B) is TRUE.

Final Answer:
The Prandtl stress function satisfies the equilibrium equation by construction, satisfies the compatibility equation through $\nabla^2\phi = -2G\theta$, and equals zero on the lateral surface.

$$\boxed{\text{A, B, C}}$$

Quick Tip: Recall that shear stresses are defined as derivatives of ϕ; this makes equilibrium automatic, while compatibility gives $\nabla^2\phi = -2G\theta$, and the stress-free lateral surface gives $\phi = 0$ on the boundary.

47. Consider the flow over an oval modeled using the elementary potential flows as shown below. U represents uniform flow velocity and Γ represents circulation around an irrotational line vortex.



Which of the following statements is/are TRUE for this model?

(A) Increasing U enlarges the oval

(B) Increasing Γ enlarges the oval

(C) Interchanging the sense of the two vortices does not alter the oval

(D) Moving the vortices too far apart causes the oval to break up

Correct Answer: (B) Increasing Γ enlarges the oval ; (D) Moving the vortices too far apart causes the oval to break up

Solution:

Step 1: Identify the flow model.

The picture combines a uniform stream of speed U moving in the positive x-direction with two irrotational line vortices of equal and opposite strength, placed a fixed distance a apart: the upper vortex has circulation $-\Gamma$ (clockwise) and the lower vortex has circulation $+\Gamma$ (counterclockwise). Superposing these three simple flows produces one closed streamline that encloses both vortices, an oval shaped body, in the same spirit as a source and a sink placed in a uniform stream producing a Rankine oval.

Step 2: Effect of increasing the free stream speed U.

The oval boundary is the streamline on which the velocity induced by the two vortices exactly balances the free stream at the stagnation points. If U is made larger while Γ and a are kept fixed, the vortices have to reach that balance closer to themselves, since their own induced velocity field has not changed strength. So the stagnation points, and the oval boundary they define, move inward. Increasing U shrinks the oval, it does not enlarge it. Statement (A) is FALSE.

Step 3: Effect of increasing the circulation Γ.

Now hold U and a fixed and increase Γ. A stronger vortex pair induces a larger velocity at any given distance from the vortices, so the point where this induced velocity matches the fixed free stream U moves further out. The closed streamline that separates the rotational region from the free stream therefore grows. Increasing Γ enlarges the oval. Statement (B) is TRUE.

Step 4: Effect of interchanging the sense of the two vortices.

Swapping which vortex is $+\Gamma$ and which is $-\Gamma$ reverses the direction of the velocity that each vortex induces on the flow near the other, and on the free stream in the gap between them. Since the shape and location of the closed oval streamline depends on how the induced velocities from the two vortices add to the uniform stream, reversing this sense changes that balance and does not reproduce the same oval. Statement (C), which claims the oval is unchanged, is FALSE.

Step 5: Effect of moving the vortices far apart.

The single closed oval exists only because the two vortices are close enough that their combined induced flow, together with the uniform stream, creates one continuous ring of closed streamlines enclosing both of them. If the spacing a is increased too much, each vortex

starts to behave more like an isolated vortex in a uniform stream sitting far from its partner, and the induced velocities are no longer strong enough, at the right location, to keep a single closed boundary around both vortices together. Past a critical spacing, the single oval breaks up. Statement (D) is TRUE.

Final Answer:
Increasing Γ enlarges the oval, and separating the vortices too far apart breaks the single oval up.

$$\boxed{\text{B, D}}$$

Quick Tip: The oval is the closed streamline balancing the vortex-induced velocity against U; it grows with Γ, shrinks with U, and needs the vortices close enough together to stay closed.

48. What is/are the use(s) of the single horseshoe vortex model of finite wing aerodynamic theory?

(A) It can approximate the wing pitching moment coefficient

(B) It can approximate the wing induced drag coefficient

(C) It can approximate the effect of the wing on the induced drag coefficient of a typical horizontal tail

(D) It can approximate the aerodynamic benefit/penalty of formation flight compared to isolated flight

Correct Answer: (C) It can approximate the effect of the wing on the induced drag coefficient of a typical horizontal tail ; (D) It can approximate the aerodynamic benefit/penalty of formation flight compared to isolated flight

Solution:

Step 1: Recall what a single horseshoe vortex represents.
The single horseshoe vortex model replaces a finite wing with one bound vortex line of constant strength lying along the span (usually placed at the quarter-chord), plus two trailing vortices that leave the wingtips and stretch downstream to infinity. It was the earliest and simplest version of finite-wing vortex theory, before Prandtl's full lifting-line model added many horseshoe vortices to represent a realistic, elliptic-like spanwise loading.

Step 2: Check the wing pitching moment coefficient.
Pitching moment depends on how the lift is distributed along the chord of the wing, not only on the total lift or its spanwise position. A single horseshoe vortex is placed at one chordwise location and carries the wing's total circulation, so it has no information about how the load is spread across the chord. It cannot approximate the pitching moment coefficient. Statement (A) is not correct.

Step 3: Check the wing's own induced drag coefficient.
A single horseshoe vortex assumes constant (uniform) bound circulation across the span, which corresponds to a rectangular-like loading, not the elliptical loading that real wings approach and that minimizes induced drag. Because the induced drag of a wing is very sensitive to how the loading is distributed across the span, one horseshoe vortex is too crude to give a good estimate of the wing's own induced drag; that needs many horseshoe vortices (a vortex lattice) to capture the loading shape properly. Statement (B) is not correct.

Step 4: Check the effect of the wing on a downstream tail.

For estimating the downwash that the wing induces at a horizontal tail located some distance behind and possibly below it, only the wing's overall circulation and the rough geometry of its trailing vortex sheet matter, fine details of the spanwise loading wash out quickly with distance. A single horseshoe vortex, carrying the wing's total circulation, gives a good approximation of this induced downwash, and hence of how the wing changes the tail's effective angle of attack and its own local induced drag. Statement (C) is correct.

Step 5: Check the formation flight application.
The same reasoning applies to a trailing aircraft flying in the wingtip vortex system of a leading aircraft in formation. Each aircraft can be modeled as a single horseshoe vortex, and the upwash induced by the leading aircraft's trailing vortices on the trailing aircraft's wing can be estimated well enough this way to predict the induced drag saving (or penalty) of flying in formation compared to flying alone. Statement (D) is correct.

Final Answer:
The single horseshoe vortex model is good for estimating the wing's effect on a downstream tail and the benefit of formation flight, but not for the wing's own pitching moment or its own induced drag.

C, D

Quick Tip: A single horseshoe vortex only carries total circulation, no loading detail, so it fails for the wing's own pitching moment or induced drag but works for far-field effects like tail downwash or formation flight.

49. Consider the differential equation with the initial conditions given below. If $y(x)$ is the solution of the equation, the value of the slope, $\frac{dy}{dx}$, at $x = \ln(2)$ is _____ (rounded off to three decimal places).

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0 \quad \text{with} \quad y|_{x=0} = 0 \text{ and } \left.\frac{dy}{dx}\right|_{x=0} = 1$$

Correct Answer: 0.153

Solution:

Step 1: Write the characteristic equation.

The differential equation $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ is linear, second order, with constant coefficients. Its characteristic equation is

$$r^2 + 2r + 1 = 0$$

Step 2: Solve the characteristic equation.

This factors as

$$(r + 1)^2 = 0$$

giving a repeated root $r = -1$. A repeated root means the general solution takes the form

$$y(x) = (C_1 + C_2x)e^{-x}$$

where C_1 and C_2 are constants fixed by the initial conditions.

Step 3: Apply the first initial condition.

At $x = 0$, $y(0) = 0$. Substituting $x = 0$ into the general solution:

$$y(0) = (C_1 + 0)e^0 = C_1 = 0$$

So $C_1 = 0$, and the solution simplifies to $y(x) = C_2 x e^{-x}$.

Step 4: Apply the second initial condition.
Differentiate $y(x) = C_2 x e^{-x}$ using the product rule:

$$\frac{dy}{dx} = C_2 e^{-x} - C_2 x e^{-x} = C_2(1 - x)e^{-x}$$

At $x = 0$:

$$y'(0) = C_2(1 - 0)e^0 = C_2 = 1$$

So $C_2 = 1$, and the particular solution is

$$y(x) = x e^{-x}$$

Step 5: Write the general slope and evaluate it at $x = \ln(2)$.
From Step 4, with $C_2 = 1$,

$$\frac{dy}{dx} = (1 - x)e^{-x}$$

At $x = \ln(2)$, note that $e^{-\ln(2)} = \frac{1}{e^{\ln 2}} = \frac{1}{2}$. So

$$\left.\frac{dy}{dx}\right|_{x=\ln 2} = (1 - \ln 2) \cdot \frac{1}{2}$$

Using $\ln 2 \approx 0.693147$:

$$(1 - 0.693147) \times \frac{1}{2} = 0.306853 \times \frac{1}{2} = 0.153427$$

Final Answer:

Rounded to three decimal places, the slope at $x = \ln(2)$ is 0.153.

$$\boxed{\frac{dy}{dx}\Big|_{x=\ln 2} \approx 0.153}$$

Quick Tip: Solve the repeated-root ODE to get $y = xe^{-x}$, then differentiate and evaluate at $x = \ln 2$, using $e^{-\ln 2} = 1/2$.

50. An object of mass 1 kg is launched with an initial speed of v_o into a large tank filled with a viscous liquid. The liquid exerts a resistive force (drag) of the form $D = \alpha v$ on any object that is moving inside it, where v is the instantaneous speed of the object and $\alpha = 1$ kg/s. If the effect of gravity is ignored, the time taken by the object to slow down to the speed $v_o/2$ is _____ s (rounded off to 2 decimal places). Assume that the tank is sufficiently large for the above deceleration to happen inside the tank.

Correct Answer: 0.69

Solution:

Step 1: Set up the equation of motion.

The only force on the 1 kg object once gravity is ignored is the viscous drag $D = \alpha v$, acting opposite to the motion. By Newton's second law,

$$m\frac{dv}{dt} = -\alpha v$$

with $m = 1$ kg and $\alpha = 1$ kg/s.

Step 2: Separate variables and integrate.
Rearranging,

$$\frac{dv}{v} = -\frac{\alpha}{m}\,dt$$

Integrating from the initial speed v_o at $t = 0$ to a general speed v at time t,

$$\ln\left(\frac{v}{v_o}\right) = -\frac{\alpha}{m}\,t$$

Step 3: Write the speed as a function of time.

$$v(t) = v_o\, e^{-(\alpha/m)\,t}$$

This is a simple exponential decay of speed, with time constant $\tau = m/\alpha = 1$ s.

Step 4: Apply the condition $v = v_o/2$ and solve for t.

$$\frac{v_o}{2} = v_o\, e^{-t}$$

$$\frac{1}{2} = e^{-t}$$

Taking the natural log of both sides,

$$t = \ln 2 = 0.6931 \text{ s}$$

Final Answer:

Rounded to 2 decimal places, the object takes about 0.69 s to slow from v_o to $v_o/2$.

$$\boxed{t \approx 0.69 \text{ s}}$$

Quick Tip: Set up m dv/dt = -alpha v and separate variables to get exponential decay; find t when v = v0/2.

51. The minimum value of the function $f(x) = |x| + |2x + 3|$ for real x is _____ (rounded off to 1 decimal place).

Correct Answer: 1.5

Solution:

Step 1: Find the break points of the function.

The function $f(x) = |x| + |2x + 3|$ changes its algebraic form wherever an expression inside a modulus sign changes sign. That happens at $x = 0$ and at $2x + 3 = 0$, i.e. $x = -3/2$. These two points split the real line into three regions.

Step 2: Write $f(x)$ in each region.

For $x \geq 0$: both x and $2x + 3$ are non-negative, so

$$f(x) = x + (2x + 3) = 3x + 3$$

This is increasing in x (slope $+3$), so on this region the smallest value is at $x = 0$: $f(0) = 3$.
For $-3/2 \leq x < 0$: here $x < 0$ but $2x + 3 \geq 0$, so

$$f(x) = -x + (2x + 3) = x + 3$$

This is also increasing (slope $+1$), so the smallest value on this region is at the left end $x = -3/2$: $f(-3/2) = -3/2 + 3 = 1.5$.
For $x < -3/2$: here both x and $2x + 3$ are negative, so

$$f(x) = -x - (2x + 3) = -3x - 3$$

This has slope -3 (decreasing as x increases), so as x moves from $-\infty$ up to $-3/2$, $f(x)$ keeps falling and reaches its smallest value on this region right at $x = -3/2$, which again gives $f(-3/2) = 1.5$.

Step 3: Compare the three regions.
The candidate minimum values are $f(0) = 3$ and $f(-3/2) = 1.5$ (the function is monotonic on every region, so the overall minimum can only occur at a break point). The smaller of these is 1.5.

Final Answer:

$$\boxed{f(x)_{min} = 1.5}$$

Quick Tip: Split the real line at the points where each modulus term is zero and check the sign of the slope on each piece; the minimum of a convex

piecewise-linear function is where the slope changes sign.

52. Isobutane (C_4H_{10}) is burnt completely in pure oxygen as per the reaction given below. Given that the standard heats of formation (in kcal/mole) of isobutane, carbon dioxide, and water vapour are -31.489, -94.052, and -60.150, respectively, the heat of reaction is _____ kcal (rounded off to 2 decimal places).

$$C_4H_{10} + 6.5\,O_2 \rightarrow 4\,CO_2 + 5\,H_2O$$

Correct Answer: -645.47

Solution:

Step 1: Recall Hess's law.

For any reaction, the heat of reaction is the sum of the standard heats of formation of the products minus the sum of the standard heats of formation of the reactants, each multiplied by its stoichiometric coefficient.

$$\Delta H_{rxn} = \sum \Delta H_{f,products} - \sum \Delta H_{f,reactants}$$

Oxygen (O_2) is an element in its standard state, so its heat of formation is zero.

Step 2: List the given values and coefficients.

Reactant: 1 mole isobutane, $\Delta H_f = -31.489$ kcal/mole.

Products: 4 moles CO_2, $\Delta H_f = -94.052$ kcal/mole each; 5 moles H_2O (vapour), $\Delta H_f = -60.150$ kcal/mole each.

Step 3: Sum the heats of formation of the products.

$$\Delta H_{f,products} = 4(-94.052) + 5(-60.150) = -376.208 - 300.750 = -676.958 \text{ kcal}$$

Step 4: Sum the heats of formation of the reactants.

$$\Delta H_{f,reactants} = 1(-31.489) + 6.5(0) = -31.489 \text{ kcal}$$

Step 5: Apply Hess's law.

$$\Delta H_{rxn} = -676.958 - (-31.489) = -645.469 \text{ kcal}$$

Final Answer:

Rounded to 2 decimal places, the heat of reaction is about -645.47 kcal (the negative sign shows the reaction is exothermic, it releases heat).

$$\boxed{\Delta H_{rxn} \approx -645.47 \text{ kcal}}$$

Quick Tip: Use Hess's law: heat of reaction = sum of heats of formation of products minus reactants, weighted by stoichiometric coefficients. O2 has zero heat of formation.

53. A furnace of 250 MW rating is used to melt and raise the temperature of aluminium from 25°C to 900°C. Aluminium has a solid-state specific heat, latent heat, and liquid-state specific heat of 0.9 kJ/kg-K, 390 kJ/kg, and 1.108 kJ/kg-K, respectively, and the furnace has 70% efficiency. The melting point of aluminium is 660°C. The amount of aluminium that can be processed per hour is _____ kg (rounded off to 1 decimal place).

Correct Answer: 513271.7

Solution:

Step 1: Break the heating process into three stages.

To take 1 kg of aluminium from 25 C to 900 C, it must (a) be heated as a solid from 25 C to its melting point 660 C, (b) melt at 660 C (latent heat), and (c) be heated as a liquid from 660 C to 900 C.

Step 2: Heat needed for solid-state heating.

$$q_1 = c_{p,solid} \times (660 - 25) = 0.9 \times 635 = 571.5 \text{ kJ/kg}$$

Step 3: Heat needed for melting.

$$q_2 = L = 390 \text{ kJ/kg}$$

Step 4: Heat needed for liquid-state heating.

$$q_3 = c_{p,liquid} \times (900 - 660) = 1.108 \times 240 = 265.92 \text{ kJ/kg}$$

Step 5: Total heat needed per kg of aluminium.

$$q = q_1 + q_2 + q_3 = 571.5 + 390 + 265.92 = 1227.42 \text{ kJ/kg}$$

Step 6: Find the useful heat delivered by the furnace per hour.

The furnace rating is the input power, so the useful (output) power is 70% of 250 MW:

$$P_{useful} = 0.70 \times 250{,}000 \text{ kW} = 175{,}000 \text{ kJ/s}$$

Over 1 hour (3600 s):

$$Q_{hour} = 175{,}000 \times 3600 = 630{,}000{,}000 \text{ kJ}$$

Step 7: Divide total heat available by heat needed per kg.

$$m = \frac{Q_{hour}}{q} = \frac{630{,}000{,}000}{1227.42} \approx 513{,}271.7 \text{ kg}$$

Final Answer:

About 513,271.7 kg of aluminium can be processed per hour.

$$\boxed{m \approx 513271.7 \text{ kg/hr}}$$

Quick Tip: Add up the heat needed per kg for solid heating, melting, and liquid heating, then divide the furnace's hourly useful output (rating times efficiency times 3600 s) by that value.

54. For an airfoil section the pitching moment coefficient is determined about a reference point that is 0.3 times the chord behind the leading edge. It varies with the lift coefficient as shown in the table below. The distance of the aerodynamic center from the leading edge of the airfoil as a fraction of the chord is _____ (rounded off to one decimal place).

c_l	0.2	0.4	0.6	0.8
c_m	-0.02	0	0.02	0.04

Correct Answer: 0.2

Solution:

Step 1: Recall how the pitching moment coefficient changes with reference point.

For an airfoil, the moment about the aerodynamic center (ac) does not change with lift coefficient by definition, $dc_{m,ac}/dc_l = 0$. About any other reference point located at a distance h chords behind the leading edge, the standard result is

$$c_{m,ref} = c_{m,ac} + c_l\,(h - h_{ac})$$

where h_{ac} is the location of the aerodynamic center as a fraction of chord. So the slope of the c_m versus c_l line directly gives $(h - h_{ac})$.

Step 2: Compute the slope from the table.

Using the first and last data points,

$$\frac{dc_m}{dc_l} = \frac{0.04 - (-0.02)}{0.8 - 0.2} = \frac{0.06}{0.6} = 0.1$$

This is the same for any pair of points in the table, confirming the relationship is linear, as expected.

Step 3: Solve for the aerodynamic center location.

Here $h = 0.3$ (the given reference point) and $h - h_{ac} = 0.1$, so

$$h_{ac} = h - 0.1 = 0.3 - 0.1 = 0.2$$

Final Answer:

The aerodynamic center lies 0.2 chords behind the leading edge.

$$\boxed{h_{ac} = 0.2}$$

Quick Tip: The slope of the cm versus cl line equals (href - hac); use two points from the table to find the slope, then solve for hac.

55. An earth satellite moves in an elliptical orbit with a perigee altitude of 300 km and an apogee altitude of 3000 km. Assume that the radius of the earth is 6378 km. The eccentricity of the orbit is _____ (rounded off to three decimal places).

Correct Answer: 0.168

Solution:

Step 1: Find the perigee and apogee radii from the earth's center.

Altitude is measured from the earth's surface, so add the earth's radius:

$$r_p = R_e + h_p = 6378 + 300 = 6678 \text{ km}$$

$$r_a = R_e + h_a = 6378 + 3000 = 9378 \text{ km}$$

Step 2: Recall the perigee/apogee relations for an ellipse.

For an orbit of semi-major axis a and eccentricity e,

$$r_p = a(1 - e), \qquad r_a = a(1 + e)$$

Step 3: Eliminate a to get e directly.

Dividing and rearranging,

$$e = \frac{r_a - r_p}{r_a + r_p}$$

Step 4: Substitute the numbers.

$$e = \frac{9378 - 6678}{9378 + 6678} = \frac{2700}{16056} = 0.1682$$

Final Answer:

Rounded to three decimal places, $e \approx 0.168$.

$$\boxed{e \approx 0.168}$$

Quick Tip: Convert perigee and apogee altitudes to radii from earth's center, then use e = (ra - rp)/(ra + rp).

56. Consider a finite wing of aspect ratio 10 with span effectiveness factor 0.95. Its airfoil section has a lift slope of 0.106 per degree and a zero-lift angle of attack of -1.5°. The lift coefficient of the wing at an angle of attack of 3.5° is _____ (rounded off to 2 decimal places).

Correct Answer: 0.44

Solution:

Step 1: Recall the finite-wing lift-slope correction formula.

Finite-span effects reduce the lift-curve slope compared with the 2D airfoil (section) value a_0. From lifting-line theory, the finite-wing lift-curve slope a (per radian) is

$$a = \frac{a_0}{1 + \frac{a_0}{\pi e AR}}$$

where e is the span effectiveness factor and AR is the aspect ratio, with a_0 and a both expressed per radian.

Step 2: Convert the given section lift slope to per radian.

$$a_0 = 0.106 \text{ per degree} \times \frac{180}{\pi} = 0.106 \times 57.296 = 6.073 \text{ per radian}$$

Step 3: Compute the denominator term.

$$\pi e AR = \pi \times 0.95 \times 10 = 29.845$$

$$\frac{a_0}{\pi e AR} = \frac{6.073}{29.845} = 0.2035$$

Step 4: Compute the finite-wing lift slope.

$$a = \frac{6.073}{1 + 0.2035} = \frac{6.073}{1.2035} = 5.046 \text{ per radian}$$

Converting back to per degree, $a = 5.046/57.296 = 0.0881$ per degree.

Step 5: Apply the lift equation with the zero-lift angle.

$$c_l = a(\alpha - \alpha_{L0})$$

$$\Delta\alpha = 3.5 - (-1.5) = 5^\circ$$

$$c_l = 0.0881 \times 5 = 0.440$$

Final Answer:

Rounded to 2 decimal places, the wing lift coefficient is about 0.44.

$$\boxed{c_l \approx 0.44}$$

Quick Tip: Use the finite-wing lift-slope correction a = a0/(1 + a0/(pi e AR)), convert a0 to per radian, then multiply by (alpha - alpha_L0).

57. W_A and W_B are the respective maximum take-off weights of an aircraft for two ambient air conditions given below.

Condition A: $p = 1$ bar, $T = 50°C$; Condition B: $p = 0.66$ bar, $T = -30°C$

If all other parameters relevant for take-off are kept the same in these two conditions, the ratio W_B/W_A is _____ (rounded off to 3 decimal places).

Correct Answer: 0.877

Solution:

Step 1: Identify the physical link between takeoff weight and air density.

At the moment of lift-off, the wing must generate lift equal to the aircraft weight: $L = W = \frac{1}{2}\rho V^2 S\, c_{l,max}$. The problem states that all other parameters relevant to take-off (wing area S, maximum lift coefficient $c_{l,max}$, and the take-off/rotation speed V) are kept the same between the two conditions. That leaves air density ρ as the only variable, so

$$W \propto \rho$$

$$\frac{W_B}{W_A} = \frac{\rho_B}{\rho_A}$$

Step 2: Find the air density in each condition using the ideal gas law.

$$\rho = \frac{p}{RT}, \qquad R = 287 \text{ J/(kg K)}$$

Condition A: $p_A = 1 \times 10^5$ Pa, $T_A = 50 + 273.15 = 323.15$ K:

$$\rho_A = \frac{1 \times 10^5}{287 \times 323.15} = \frac{100000}{92744.05} = 1.0782 \text{ kg/m}^3$$

Condition B: $p_B = 0.66 \times 10^5$ Pa, $T_B = -30 + 273.15 = 243.15$ K:

$$\rho_B = \frac{0.66 \times 10^5}{287 \times 243.15} = \frac{66000}{69784.05} = 0.9458 \text{ kg/m}^3$$

Step 3: Take the ratio.

$$\frac{W_B}{W_A} = \frac{\rho_B}{\rho_A} = \frac{0.9458}{1.0782} = 0.8772$$

Final Answer:

Rounded to 3 decimal places, $W_B/W_A \approx 0.877$. The lower pressure at condition B reduces air density more than the lower temperature raises it, so a lighter maximum take-off weight is possible there for the same lift-off requirements.

$$\boxed{W_B/W_A \approx 0.877}$$

Quick Tip: Since lift equals weight at rotation and S, cl_max, and V are unchanged, weight is proportional to air density; find density in each condition from p = rho R T and take the ratio.

58. A thin-walled circular tube is made of a material whose magnitude of the ultimate strength, both in tension and compression, is 200 MPa. The mean radius of the tube is 0.2 m and the wall thickness is 0.004 m. Based on the maximum stress criteria, the maximum torque that the tube can sustain is _____ kN-m (round off to the nearest integer).

Correct Answer: 201

Solution:

Step 1: Understand what "maximum stress criteria" means for this tube.

When a thin-walled tube carries only torque, every point on its wall is in a state of pure shear, stress τ. For pure shear, the two principal stresses are $\sigma_1 = \tau$ and $\sigma_2 = -\tau$, acting at 45 degrees to the tube axis. The maximum stress (maximum normal stress) theory says the material fails when the larger principal stress magnitude reaches the ultimate strength. So the allowable shear stress equals the given ultimate strength directly: $\tau_{max} = 200$ MPa.

Step 2: Write the thin-wall torsion formula.

For a thin-walled closed tube, the shear stress from a torque T is given by Bredt's formula,

$$\tau = \frac{T}{2A_m t}$$

where A_m is the area enclosed by the mean wall line and t is the wall thickness. For a circular tube of mean radius r_m, $A_m = \pi r_m^2$.

Step 3: Substitute the numbers.

$r_m = 0.2$ m, so $A_m = \pi(0.2)^2 = 0.12566$ m^2.

$$T_{max} = \tau_{max} \times 2A_m t = (200 \times 10^6)(2)(0.12566)(0.004)$$

$$T_{max} = (200 \times 10^6)(0.0010053) = 2.0106 \times 10^5 \text{ N-m}$$

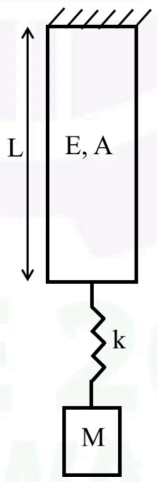
Final Answer:

$T_{max} \approx 201.06$ kN-m, which rounds to

$$\boxed{T_{max} \approx 201 \text{ kN-m}}$$

Quick Tip: Pure torsion on a thin tube gives pure shear, whose principal stresses equal the shear stress. Set that shear stress equal to the ultimate strength and use the thin-wall torsion formula $\tau = T/(2A_m t)$.

59. A system comprising a bar, spring and mass is shown in the figure below.



The bar, having negligible mass, is made of a material having Young's modulus $E = 200$ GPa, cross-sectional area $A = 100$ mm^2, and length $L = 100$ mm. The spring stiffness $k = 200$ kN/mm and the mass $M = 100$ kg. The natural frequency of free vibration of the system is _____ rad/s (rounded off to the nearest integer).

Correct Answer: 1000

Solution:

Step 1: Model the bar as a spring.

A massless elastic bar of length L, area A, and modulus E, loaded along its axis, stretches like a spring of stiffness

$$k_{bar} = \frac{EA}{L}$$

Here $E = 200 \times 10^9$ Pa, $A = 100 \times 10^{-6}$ m^2, $L = 0.1$ m.

$$k_{bar} = \frac{(200 \times 10^9)(100 \times 10^{-6})}{0.1} = 2 \times 10^8 \text{ N/m}$$

Step 2: Convert the given spring stiffness to consistent units.

$k = 200$ kN/mm. Since 1 kN/mm $= 10^6$ N/m,

$$k = 200 \times 10^6 = 2 \times 10^8 \text{ N/m}$$

Step 3: Combine the bar and the spring.

Looking at the figure, the fixed support holds the bar, the bar connects to the spring, and the spring connects to the mass. So the bar and spring carry the same force one after another, which is a series connection. For springs in series the equivalent stiffness is

$$\frac{1}{k_{eq}} = \frac{1}{k_{bar}} + \frac{1}{k} = \frac{1}{2 \times 10^8} + \frac{1}{2 \times 10^8} = \frac{1}{10^8}$$

$$k_{eq} = 1 \times 10^8 \text{ N/m}$$

Step 4: Natural frequency.

For a single mass on an equivalent spring,

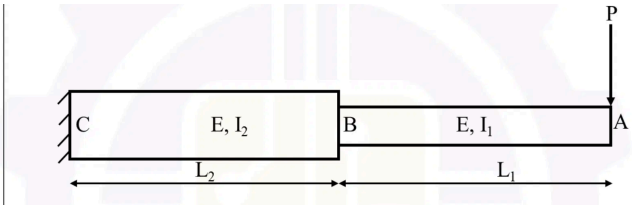
$$\omega_n = \sqrt{\frac{k_{eq}}{M}} = \sqrt{\frac{1 \times 10^8}{100}} = \sqrt{1 \times 10^6}$$

Final Answer:

$$\boxed{\omega_n = 1000 \text{ rad/s}}$$

Quick Tip: The massless bar behaves like an axial spring of stiffness EA/L. It sits in series with the given spring, so add their compliances (not their stiffnesses) to get the equivalent stiffness.

60. A stepped cantilever beam, made of a material having Young's modulus $E = 200$ GPa, is shown in the figure below.



The length and the moment of inertia of the beam from point A to B are $L_1 = 100$ mm and $I_1 = 100$ mm^4, respectively. The length and the moment of inertia of the beam from point B to C are $L_2 = 100$ mm and $I_2 = 700$ mm^4, respectively. A shear force $P = 30$ N is applied at point A of the beam. The magnitude of the deflection of the beam at point A is _____ mm (rounded off to 1 decimal place).

Correct Answer: 1.0

Solution:

Step 1: Set up the moment expression.

The beam is fixed at C and carries a single transverse force P at the free end A. For this determinate cantilever, the bending moment at any section is simply P times the distance from that section to A, regardless of how I varies along the beam. Measure x from the free end A, so at a section a distance x from A, $M(x) = Px$.

Step 2: Apply the unit-load (virtual work) formula for tip deflection.

For a cantilever with a real load P at the tip, the deflection at that same tip is

$$\delta_A = \int_0^{L_{total}} \frac{M(x)\, m(x)}{E\, I(x)}\, dx$$

where $m(x)$ is the moment from a unit virtual load at A, which has the same shape as $M(x)/P$, i.e. $m(x) = x$. So

$$\delta_A = \frac{P}{E} \int_0^{L_{total}} \frac{x^2}{I(x)}\, dx$$

Near A (from $x = 0$ to $x = L_1 = 100$ mm) the section has $I_1 = 100$ mm^4. From $x = L_1$ to $x = L_1 + L_2 = 200$ mm, the section is the BC part, with $I_2 = 700$ mm^4.

Step 3: Evaluate the two pieces of the integral.

$$\int_0^{100} \frac{x^2}{I_1}\, dx = \frac{1}{100}\left[\frac{x^3}{3}\right]_0^{100} = \frac{1{,}000{,}000/3}{100} = 3333.33 \text{ mm}^{-1}$$

$$\int_{100}^{200} \frac{x^2}{I_2}\, dx = \frac{1}{700}\left[\frac{x^3}{3}\right]_{100}^{200} = \frac{(8{,}000{,}000 - 1{,}000{,}000)/3}{700} = \frac{2{,}333{,}333.3}{700} = 3333.33 \text{ mm}^{-1}$$

Adding, the total integral is $3333.33 + 3333.33 = 6666.67$ mm^{-1}.

Step 4: Substitute E and P.

$E = 200$ GPa $= 200{,}000$ N/mm^2 (consistent with mm and N units used above).

$$\delta_A = \frac{30}{200{,}000}(6666.67) = (1.5 \times 10^{-4})(6666.67)$$

Final Answer:

$$\boxed{\delta_A = 1.0 \text{ mm}}$$

Quick Tip: Split the beam at the step. Use the unit load (virtual work) method, or superpose the tip deflection of segment AB with the deflection and slope carried over from segment BC.

61. A centrifugal compressor has a constant-width radial diffuser. The diameters at the diffuser inlet and outlet are 0.2 m and 0.3 m, respectively. The flow at the diffuser inlet and outlet is assumed to be steady and uniform. The average velocity at the diffuser inlet and outlet are $(60\,\hat{e}_r + 75\,\hat{e}_\theta)$ m/s and $(u\,\hat{e}_r + 50\,\hat{e}_\theta)$ m/s, respectively. If the flow through the diffuser is treated as steady and incompressible, the value of u is _____ (rounded off to the nearest integer).

Correct Answer: 40

Solution:

Step 1: Identify what continuity needs here.

In a radial diffuser the flow crosses circular (cylindrical) surfaces centered on the shaft axis. The area of such a surface at radius r is $A = 2\pi rb$, where b is the (constant) width of the diffuser. Only the radial velocity component v_r is normal to this surface, so only v_r carries mass flow through it; the tangential component v_θ does not.

Step 2: Write continuity between inlet (station 1) and outlet (station

2).

For steady, incompressible flow (ρ constant) with constant width b,

$$\rho(2\pi r_1 b)v_{r1} = \rho(2\pi r_2 b)v_{r2}$$

The $2\pi\rho b$ factor is the same on both sides and cancels, leaving

$$r_1 v_{r1} = r_2 v_{r2}$$

Step 3: Substitute the given values.

$r_1 = D_1/2 = 0.2/2 = 0.1$ m, $v_{r1} = 60$ m/s (the $\hat{e}_r$ component at inlet).

$r_2 = D_2/2 = 0.3/2 = 0.15$ m, $v_{r2} = u$ (the $\hat{e}_r$ component at outlet, unknown).

$$(0.1)(60) = (0.15)(u)$$

$$6 = 0.15\,u$$

Final Answer:

$$\boxed{u = 40 \text{ m/s}}$$

(The tangential components 75 m/s and 50 m/s are not needed for this mass-continuity check; they would matter if the question asked about angular momentum or diffuser efficiency instead.)

Quick Tip: Only the radial velocity component carries mass through a cylindrical control surface at radius r. Apply continuity, $\rho(2\pi rb)v_r = \text{constant}$, between inlet and outlet; the width b and density are constant so they cancel.

62. A gas mixture at a pressure of 800 kPa and a density of 5 kg/m^3 enters a turbine stage. The temperature of the gas at the nozzle exit and the stage exit are 790 K and 750 K, respectively. Assume the specific heats are constant for the gas mixture in the range of temperatures considered. The specific heat at constant pressure is 0.72 kJ/kg-K and the ratio of specific heats is 1.33. The value of the degree of reaction of the turbine stage is _____ (rounded off to 2 decimal places).

Correct Answer: 0.27

Solution:

Step 1: Recall the definition of degree of reaction.

A turbine stage has a fixed row of nozzles followed by a moving row (the rotor). The degree of reaction Λ is the fraction of the stage's total static enthalpy drop that occurs in the rotor:

$$\Lambda = \frac{h_2 - h_3}{h_1 - h_3}$$

where 1 = stage inlet, 2 = nozzle exit (rotor inlet), 3 = stage exit (rotor exit). Since c_p is constant here, enthalpy differences are just c_p times temperature differences, so the c_p cancels and

$$\Lambda = \frac{T_2 - T_3}{T_1 - T_3}$$

We already know $T_2 = 790$ K and $T_3 = 750$ K, but T_1 (stage inlet temperature) is not given directly, only p_1 and ρ_1.

Step 2: Find the gas constant R from cp and gamma.

$$R = c_p\left(\frac{\gamma - 1}{\gamma}\right) = 0.72\left(\frac{0.33}{1.33}\right) = 0.72(0.24812) = 0.17865 \text{ kJ/kg-K}$$

Step 3: Find T1 from the ideal gas law.

$$p_1 = \rho_1 R T_1 \implies T_1 = \frac{p_1}{\rho_1 R}$$

$$T_1 = \frac{800}{(5)(0.17865)} = \frac{800}{0.89324} = 895.7 \text{ K}$$

Step 4: Compute the degree of reaction.

$$\Lambda = \frac{T_2 - T_3}{T_1 - T_3} = \frac{790 - 750}{895.7 - 750} = \frac{40}{145.7}$$

Final Answer:

$$\boxed{\Lambda \approx 0.27}$$

Quick Tip: Degree of reaction is the fraction of the stage's static enthalpy drop that happens across the rotor: $\Lambda = (T_2 - T_3)/(T_1 - T_3)$. You need the

stage inlet temperature T_1 from the ideal gas law, using $R = c_p(\gamma - 1)/\gamma$.

63. Thin airfoil theory predicts the zero-lift angle of attack $\alpha_{L=0}$ of NACA 2412 airfoil as $-2.1°$. The corresponding prediction of $\alpha_{L=0}$ for NACA 5410 airfoil is _____ degrees (rounded off to 1 decimal place).

Correct Answer: -5.3

Solution:

Step 1: Decode both airfoil designations.
In the NACA 4-digit system XYZZ, X is the maximum camber as a percentage of chord (m), Y is the location of maximum camber in tenths of chord (p), and ZZ is the thickness as a percentage of chord.
NACA 2412: $m = 2\%$, $p = 0.4c$, thickness $= 12\%$.
NACA 5410: $m = 5\%$, $p = 0.4c$, thickness $= 10\%$.

Step 2: Recall what thin airfoil theory actually depends on.
Thin airfoil theory predicts lift and $\alpha_{L=0}$ purely from the mean camber line's slope dz/dx; thickness has no effect on this prediction. So the 12% versus 10% thickness difference between the two airfoils is irrelevant here.

Step 3: Use the linear scaling of the camber-line slope with m.
The NACA 4-digit camber line (both its forward and aft parabolic-arc segments) has a slope dz/dx that is directly proportional to m for a fixed camber location p. Since both airfoils share the same $p = 0.4c$, the entire camber-line slope of NACA 5410 is exactly $(5/2)$ times that of NACA 2412 at every point along the chord.

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(\cos\theta_0 - 1)\, d\theta_0$$

Because this integral is linear in dz/dx, scaling dz/dx by a constant factor scales $\alpha_{L=0}$ by the same factor:

$$\alpha_{L=0}(5410) = \frac{5}{2}\,\alpha_{L=0}(2412)$$

Step 4: Substitute.

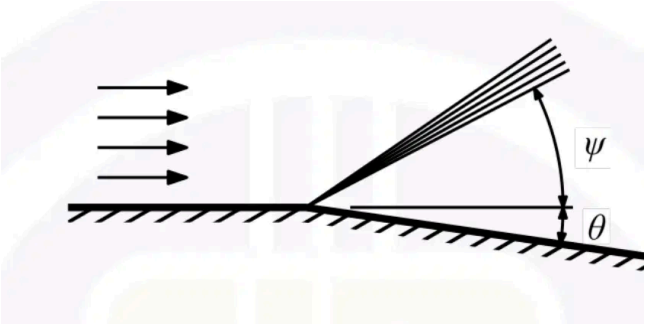
$$\alpha_{L=0}(5410) = 2.5 \times (-2.1^\circ) = -5.25^\circ$$

Final Answer:

$$\boxed{\alpha_{L=0} \approx -5.3^\circ}$$

Quick Tip: In the NACA 4-digit naming, the first digit gives the max camber (percent chord) and the second gives its location (tenths of chord). Thin airfoil theory's $\alpha_{L=0}$ is a linear function of the camber-line slope, so with the same camber location it scales directly with camber.

64. Consider a centered Prandtl-Meyer expansion fan at a $\theta = 4°$ corner in a Mach 1.78 air flow, as shown in the figure below.



The angle ψ (see figure) made by the ending wave of the fan with respect to the incoming stream is _____ degrees (rounded off to 1 decimal place).

An excerpt from the table of Prandtl-Meyer function for air is provided below.

M	ν [deg]
1.72	18.40
1.74	18.98
1.76	19.56
1.78	20.15
1.80	20.73
1.82	21.30
1.84	21.88
1.86	22.45
1.88	23.02
1.90	23.59
1.92	24.15
1.94	24.71
1.96	25.27

1.98	25.83
2.00	26.38

Correct Answer: 27.4

Solution:

Step 1: Recall the Prandtl-Meyer relation for an expansion corner.
The Prandtl-Meyer function $\nu(M)$ is the angle through which a sonic flow must turn isentropically to reach Mach number M. Across a corner that turns the flow by θ, the function simply adds:

$$\nu(M_2) = \nu(M_1) + \theta$$

Step 2: Read $\nu(M_1)$ from the table and find $\nu(M_2)$.
At $M_1 = 1.78$, the table gives $\nu_1 = 20.15°$.

$$\nu_2 = 20.15 + 4 = 24.15°$$

The table shows $\nu = 24.15°$ exactly at $M_2 = 1.92$, so $M_2 = 1.92$.

Step 3: Find the Mach angles at the start and end of the fan.
The Mach angle is $\mu = \arcsin(1/M)$.

$$\mu_1 = \arcsin\left(\frac{1}{1.78}\right) = \arcsin(0.5618) = 34.2°$$

$$\mu_2 = \arcsin\left(\frac{1}{1.92}\right) = \arcsin(0.5208) = 31.4°$$

Step 4: Relate the ending wave angle to the incoming stream.
The fan is bounded by a leading Mach wave, inclined at μ_1 to the incoming (upstream) flow direction, and an ending Mach wave, inclined at μ_2 to the outgoing (downstream) flow direction. The downstream flow direction itself has already been turned by θ away from the incoming stream direction (the flow follows the wall through the corner). So, measured from the original incoming stream direction, the ending wave sits at

$$\psi = \mu_2 - \theta = 31.4 - 4$$

Final Answer:

$$\boxed{\psi \approx 27.4^\circ}$$

Quick Tip: Use $\nu(M_2) = \nu(M_1) + \theta$ to find the downstream Mach number, then note the ending wave sits at the Mach angle μ_2 measured from the downstream (turned) flow direction, so $\psi = \mu_2 - \theta$ measured from the incoming stream.

65. A Mach 1.5 air flow enters a round duct of length 20 cm and diameter 3 cm. If the flow exits with Mach number 1.1, the average Fanning friction factor f of the duct is _____ $\times 10^{-3}$ (rounded off to 1 decimal place).

An excerpt from the Fanno flow table for air is given below.

M	1.1	1.2	1.3	1.4	1.5	1.6

$4fL^*/D \times 10^4$	99.35	336.4	648.3	997.4	1361	1724

Correct Answer: 4.7

Solution:

Step 1: Recall what the Fanno flow table gives.

In adiabatic flow with friction in a constant-area duct (Fanno flow), L^* is the extra length of duct that would be needed, from a given Mach number, to bring the flow all the way to $M = 1$. The table lists the dimensionless group $4fL^*/D$ as a function of M.

Step 2: Read the two table values needed.

At the duct inlet, $M_1 = 1.5$, giving $(4fL^*/D)_1 = 1361 \times 10^{-4}$.

At the duct exit, $M_2 = 1.1$, giving $(4fL^*/D)_2 = 99.35 \times 10^{-4}$.

Step 3: Subtract to get the actual duct's friction group.

Since the flow is supersonic and friction is pushing it toward $M = 1$ as it travels down the duct, the actual physical duct length equals the difference of the two "distance to sonic" lengths: $L = L_1^* - L_2^*$. In terms of the tabulated group,

$$\frac{4fL}{D} = \left(\frac{4fL^*}{D}\right)_1 - \left(\frac{4fL^*}{D}\right)_2 = (1361 - 99.35) \times 10^{-4} = 1261.65 \times 10^{-4} = 0.126165$$

Step 4: Substitute the given L and D.

$L = 20$ cm $= 0.2$ m, $D = 3$ cm $= 0.03$ m, so $L/D = 0.2/0.03 = 6.667$.

$$4f(6.667) = 0.126165$$

$$f = \frac{0.126165}{4 \times 6.667} = \frac{0.126165}{26.667} = 0.0047312$$

Final Answer:

$f \approx 4.73 \times 10^{-3}$, which rounds to

$$\boxed{f \approx 4.7 \times 10^{-3}}$$

Quick Tip: Use the Fanno flow subtraction rule: the actual duct's $4fL/D$ equals the tabulated $4fL^*/D$ at the inlet Mach number minus the value at the exit Mach number.