GATE 2026 Agricultural Engineering Question Paper with Solutions

General Instructions

(i) This booklet contains 65 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 44 single-correct multiple-choice questions and 21 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For numerical questions, report the answer rounded exactly as asked.

1. Suresh said, "I did it yesterday." Which one of the following options is the correct form of this sentence in indirect speech?

(A) Suresh said that I did it yesterday.

(B) Suresh says I did it yesterday.

(C) Suresh says that he did it the day before.

(D) Suresh said that he had done it the day before.

Correct Answer: (D) Suresh said that he had done it the day before.

Solution:

This question checks the standard rules for converting direct speech into indirect (reported) speech, especially the backshift of tense and the change of time expressions.

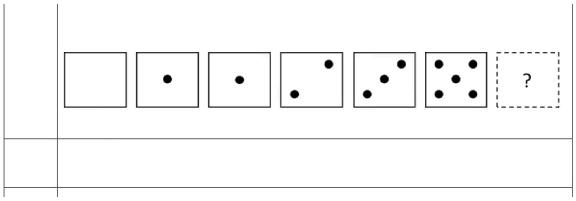
1. **Suresh said that I did it yesterday.**: This keeps the pronoun "I" and the tense unchanged, so it still reads as a direct quote and is wrong.

2. **Suresh says I did it yesterday.**: The reporting verb "says" is present tense, but the original sentence uses "said", so the tense of the reporting verb is wrong.
3. **Suresh says that he did it the day before.**: The reporting verb "says" does not match "said" in the direct speech, so this option fails on tense agreement.
4. **Suresh said that he had done it the day before.**: The pronoun changes from "I" to "he", the past tense "did" shifts back to past perfect "had done", and "yesterday" becomes "the day before". Every rule of reported speech is applied correctly.

So option (D), "Suresh said that he had done it the day before," is the correct indirect speech form.

Quick Tip: Check how the pronoun, the verb tense and the time word "yesterday" each change in reported speech.

2. To continue the sequence of tiles shown, the tile indicated by the question mark should be



(A)

(B)

(C)

(D)

Correct Answer: (C)

Solution:

Step 1: Read the dot count on each tile.

The tiles in the picture carry **0, 1, 1, 2, 3, 5** dots in order, one dot count per tile, moving left to right.

Step 2: Spot the rule linking consecutive tiles.

Add the two tiles just before a given tile: $0 + 1 = 1$, $1 + 1 = 2$, $1 + 2 = 3$, $2 + 3 = 5$. Each new tile's count is the sum of the two tiles right before it, the Fibonacci rule.

Step 3: Apply the rule to the missing tile.

The last two known tiles carry **3** and **5** dots, so the missing tile must

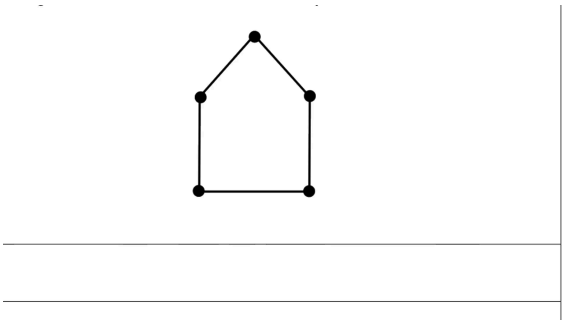
carry $3 + 5 = 8$ dots.

Final Answer:
Only the option showing a tile with eight dots continues the pattern correctly, which is option (C).

8 dots, option (C)

Quick Tip: Count the dots on each tile and look for a running-sum pattern between consecutive tiles.

3. Consider an art gallery whose walkways are shown as lines in the diagram. A black dot represents a junction of two walkways. A guard may be placed at a junction to watch over the walkways that join at that junction. The minimum number of guards needed to watch all the walkways is _____.



(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (B) 3

Solution:

Step 1: Model the picture as a graph.

Each black dot is a vertex (a junction) and each walkway is an edge joining two junctions. Watching a walkway means covering the edge that represents it.

Step 2: Recognize the problem type.

Placing a guard at a junction covers every walkway touching that junction, so we need the smallest set of vertices that touches every edge, which is the minimum vertex cover of the graph.

Step 3: Cover the edges of the house-shaped graph.

The diagram is a five-junction "house" shape: a square base with a triangular roof on top, giving six edges in total. Picking only two junctions between the base and the roof leaves some edges uncovered, so two guards are not enough. Picking three junctions, for example the two bottom corners and the roof apex, touches all six edges between them.

Final Answer:

Three guards, placed at the right junctions, are enough to watch every walkway, and no set of two junctions can do it.

$$\boxed{3}$$

Quick Tip: Treat junctions as vertices and walkways as edges, then find the smallest vertex set touching every edge.

4. The 2nd of June is a Thursday in a certain year. Which day of the week is the 3rd of July in that year?

(A) Thursday

(B) Friday

(C) Saturday

(D) Sunday

Correct Answer: (D) Sunday

Solution:

Step 1: Count the days between the two dates.

From June 2 to June 30 is $30 - 2 = 28$ days, and then from June 30 to July 3 is 3 more days, so the total gap is $28 + 3 = 31$ days.

Step 2: Convert the day gap into full weeks and a remainder.

Divide 31 by 7: $31 = 4 \times 7 + 3$, so 31 days is exactly four full weeks plus 3 extra days. Four full weeks bring the day of the week back to Thursday.

Step 3: Step forward by the remainder.

Starting from Thursday and moving 3 days forward: Thursday to Friday is 1, Friday to Saturday is 2, Saturday to Sunday is 3.

Final Answer:

Three days after Thursday lands on Sunday, so July 3 falls on a Sunday.

Sunday

Quick Tip: Count the exact number of days from June 2 to July 3 and reduce it modulo 7 from Thursday.

5. A coin with heads facing up is shown as (H) and a coin with tails facing up is shown as (T). Six coins are placed in the Starting Arrangement, as shown below. A "step" is defined as interchanging a pair of adjacent coins without flipping them. The minimum number of steps needed to go from the Starting Arrangement to the Final Arrangement, as shown below, is _____.

Starting Arrangement: (H) (H) (H) (T) (T) (T)

Final Arrangement: (T) (T) (T) (H) (H) (H)

(A) 3

(B) 6

(C) 9

(D) 12

Correct Answer: (C) 9

Solution:

Step 1: Label the coins by type and position.

The starting row is $H_1, H_2, H_3, T_1, T_2, T_3$ and the target row is $T_1, T_2, T_3, H_1, H_2, H_3$, so every head must move from the left half to the right half and every tail must move from the right half to the left half.

Step 2: Count how many tails each head must cross.

An adjacent swap only exchanges the positions of one head and one tail sitting next to each other, so each head-tail pair that ends up crossing over needs exactly one swap between that specific head and

that specific tail. There are 3 heads and 3 tails, and in the final order every one of the 3 heads must end up to the right of every one of the 3 tails.

Step 3: Multiply the crossings.
Each of the 3 heads has to cross past each of the 3 tails once, giving $3 \times 3 = 9$ required adjacent swaps in total, and no swap can do the work of two crossings at once.

Final Answer:
The minimum number of steps to fully reverse the two blocks of three coins is nine.

$\boxed{9}$

Quick Tip: Count how many head-tail pairs must cross past each other to fully reverse the two blocks.

6. Exacerbate : Mitigate :: _____
Choose the option with the correct pair of words to fill the blank.

(A) Aggravate : Alleviate

(B) Alleviate : Precipitate

(C) Aggravate : Precipitate

(D) Emancipate : Exonerate

Correct Answer: (A) Aggravate : Alleviate

Solution:

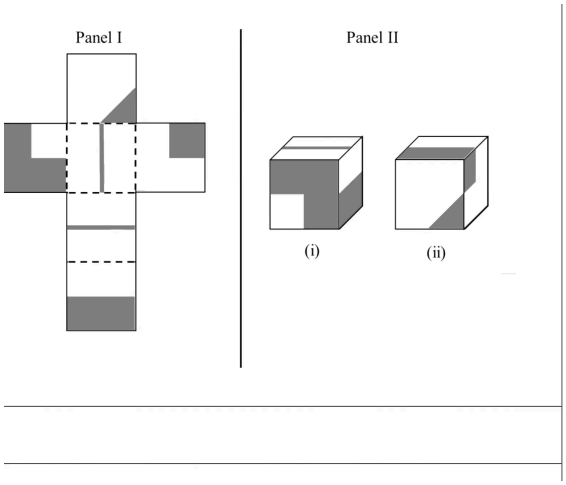
"Exacerbate" means to make a problem worse, and "mitigate" means to make it less severe, so the pair is a pair of opposites: worsen versus ease.

1. **Aggravate : Alleviate**: "aggravate" means to make worse, and "alleviate" means to make less severe, the exact same worsen-versus-ease relationship. This fits.
2. **Alleviate : Precipitate**: "alleviate" means to ease, but "precipitate" means to bring on suddenly, not to worsen an existing problem, so the pair is not opposite in the same way.
3. **Aggravate : Precipitate**: "aggravate" fits the worsen side, but "precipitate" does not mean to ease, so the second word breaks the pattern.
4. **Emancipate : Exonerate**: "emancipate" means to set free and "exonerate" means to clear of blame, both are positive actions and not opposites of each other.

Only "Aggravate : Alleviate" repeats the same worsen-and-ease opposition as "Exacerbate : Mitigate", so option (A) is correct.

Quick Tip: First fix the exact relationship between exacerbate and mitigate, then look for a pair with the same relationship.

7. A paper shown in Panel I is folded along the dashed lines (- - -) to construct a cube. The shaded regions shown in Panel I appear on the outer surface of the cube. Referring to cubes shown in Panel II, which one of the options is correct?



(A) Only (i) can correspond to the unfolded cube in Panel I.

(B) Only (ii) can correspond to the unfolded cube in Panel I.

(C) Both (i) and (ii) can correspond to the unfolded cube in Panel I.

(D) Neither (i) nor (ii) can correspond to the unfolded cube in Panel I.

Correct Answer: (B) Only (ii) can correspond to the unfolded cube in Panel I.

Solution:

Panel I shows a flat paper net with some faces shaded, and Panel II shows two folded cubes, (i) and (ii). Folding the net in your head, face by face, tells you which shaded faces end up next to each other and which end up opposite each other, and that pattern must match one of the two solid cubes.

1. **Option (A), Only (i)**: tracing the fold shows that cube (i) places two shaded faces adjacent to each other in a way the net does not allow, so (i) fails to match the net.

2. **Option (B), Only (ii)**: following the same fold, the shaded faces in cube (ii) land in exactly the positions the net predicts, both which faces touch and which faces sit opposite each other. This is consistent.
3. **Option (C), Both (i) and (ii)**: since (i) already breaks the adjacency pattern from the net, both cubes cannot be correct together.
4. **Option (D), Neither**: this is ruled out because (ii) does match the net once the fold is traced carefully.

Since only cube (ii) reproduces the shading pattern of the folded net, option (B) is correct.

Quick Tip: Fold the net one face at a time and track which shaded faces end up adjacent versus opposite.

8. In a population, patients who have high cholesterol also have high blood-pressure (BP). Some patients with high BP also have diabetes. There are no patients who have both high cholesterol and diabetes. Furthermore,
1. the total number of patients with at least one of these conditions is 75,
2. the number of patients with high cholesterol is 10,
3. the number of patients with high BP is 45, and
4. the number of patients with only high BP and no other conditions is 20.
Then the number of patients who have both diabetes and high BP is

_____

(A) 0

(B) 15

(C) 20

(D) 10

Correct Answer: (B) 15

Solution:

Step 1: Set up the regions from the given conditions.

Every patient with high cholesterol also has high BP, so the cholesterol group sits entirely inside the BP group. No patient has both cholesterol and diabetes, so cholesterol and diabetes never overlap. This leaves high BP split into three non-overlapping parts: cholesterol and BP (no diabetes), BP only (no other condition), and BP and diabetes (no cholesterol).

Step 2: Write the total-BP equation.

Total high BP patients $= 45$. Patients with cholesterol and BP $= 10$ (all 10 cholesterol patients also have BP). Patients with BP only $= 20$. So the BP-and-diabetes group is

$$45 = 10 + 20 + x \implies x = 45 - 30 = 15.$$

Step 3: Sanity check with the total count.

Let y be patients with diabetes only (no BP, no cholesterol). The total with at least one condition is $10 + 20 + 15 + y = 75$, giving $y = 30$, a valid non-negative count, so the setup is consistent.

Final Answer:

The number of patients with both diabetes and high BP works out to fifteen.

$$\boxed{15}$$

Quick Tip: Split the high BP group into cholesterol-overlap, BP-only and diabetes-overlap pieces that must sum to 45.

9. Four people P, Q, R, and S, of different ages, make the following observations.
P: I am younger than S.
Q: I am neither the youngest nor the oldest.
R: P is older than me.
Based on these observations, the youngest person is _____.

(A) P

(B) Q

(C) R

(D) S

Correct Answer: (C) R

Solution:

Step 1: Write each statement as an inequality.
P says "I am younger than S", so $P < S$.
R says "P is older than me", so $P > R$, which means $R < P$.
Q says "I am neither the youngest nor the oldest", so Q is not the smallest and not the largest age among all four.

Step 2: Combine the P and R statements.

From $R < P$ and $P < S$ we get $R < P < S$.

So among P, R, and S, R is already the smallest.

Step 3: Place Q using its own statement.

Q cannot be the youngest overall, so Q cannot sit below R, since that would make Q the youngest, which contradicts Q's own claim.

Q cannot be the oldest overall, so Q cannot sit above S.

So Q sits between R and S, which means $R < Q$ as well.

Step 4: Check who is youngest.

We now have $R < P$, $R < Q$, and $R < S$, so R is smaller than every other person.

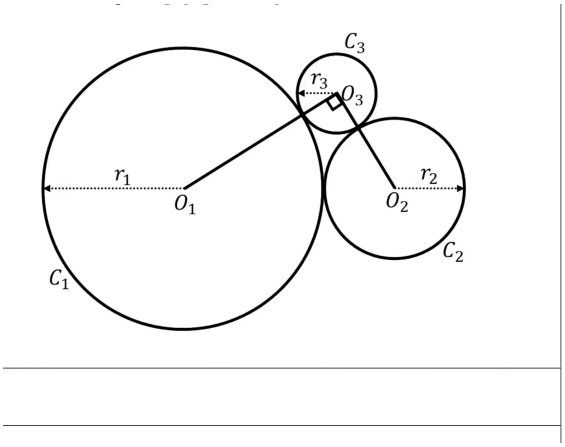
Final Answer:

R has the smallest age among P, Q, R, and S, so R is the youngest.

$$\boxed{R}$$

Quick Tip: Turn each statement into an inequality between ages and compare all four.

10. Circles C_1, C_2, and C_3, with centers O_1, O_2, and O_3, and radii r_1, r_2, and r_3, respectively, touch each other as shown in the following figure.



Given $r_1 = 2$ cm, $r_2 = 1$ cm and the angle $\angle O_1O_3O_2$ is 90°, $r_3 =$ _____ cm.

(A) $\frac{1}{2}(-3+\sqrt{17})$

(B) $\frac{1}{2}(3+\sqrt{17})$

(C) $\frac{1}{2}(-2+\sqrt{17})$

(D) $\frac{1}{2}(-3+2\sqrt{17})$

Correct Answer: (A) $\frac{1}{2}(-3+\sqrt{17})$

Solution:

Step 1: Write the triangle sides using tangency.

When two circles touch each other from outside, the distance between their centers equals the sum of their radii.

So $O_1O_2 = r_1 + r_2 = 2 + 1 = 3$ cm.

$O_1O_3 = r_1 + r_3 = 2 + r_3$ cm and $O_2O_3 = r_2 + r_3 = 1 + r_3$ cm.

Step 2: Apply the right angle at O3.

The angle $\angle O_1O_3O_2 = 90°$ means triangle $O_1O_3O_2$ is right angled at O_3, with O_1O_2 as the hypotenuse.

By the Pythagoras rule: $O_1O_2^2 = O_1O_3^2 + O_2O_3^2$.

Step 3: Substitute and expand.

$3^2 = (2 + r_3)^2 + (1 + r_3)^2$

$9 = 4 + 4r_3 + r_3^2 + 1 + 2r_3 + r_3^2$

$9 = 5 + 6r_3 + 2r_3^2$

$2r_3^2 + 6r_3 - 4 = 0$, which gives $r_3^2 + 3r_3 - 2 = 0$.

Step 4: Solve the quadratic.

$$r_3 = \frac{-3 \pm \sqrt{9 + 8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$$

Since a radius must be positive, take the + root.

Final Answer:

$r_3 = \frac{1}{2}(-3 + \sqrt{17})$ cm, so option (A) is correct.

$$\boxed{r_3 = \tfrac{1}{2}(-3 + \sqrt{17})}$$

Quick Tip: Use external tangency to write each side of triangle O1O2O3 in terms of the radii, then apply the Pythagoras rule at the right angle.

11. Identify the following partial differential equations in the given order based on their generic form where c is a constant, and u_1 and u_2 are functions of (x, t) and (x, y), respectively:

(i)

$$\frac{\partial^2 u_1}{\partial t^2} = c^2 \frac{\partial^2 u_1}{\partial x^2}$$

(ii)

$$\frac{\partial u_1}{\partial t} = c^2 \frac{\partial^2 u_1}{\partial x^2}$$

(iii)

$$\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} = 0$$

(A) (i) Wave equation, (ii) Heat equation, (iii) Laplace equation

(B) (i) Heat equation, (ii) Laplace equation, (iii) Wave equation

(C) (i) Laplace equation, (ii) Wave equation, (iii) Heat equation

(D) (i) Heat equation, (ii) Wave equation, (iii) Laplace equation

Correct Answer: (A) (i) Wave equation, (ii) Heat equation, (iii) Laplace equation

Solution:

This question checks if you can recognize the three classic second-order PDEs from their generic forms.

1. **Equation (i)**: $u_{1tt} = c^2 u_{1xx}$ has a second derivative in time equal to a constant times the second derivative in space. This is the standard wave equation, which models vibrations and traveling waves.

2. **Equation (ii)**: $u_{1t} = c^2 u_{1xx}$ has only a first derivative in time balanced against a second derivative in space. This is the heat, or diffusion, equation, which models temperature or concentration spreading over time.
3. **Equation (iii)**: $u_{2xx} + u_{2yy} = 0$ has no time variable at all, just the sum of two pure second spatial derivatives set to zero. This is the Laplace equation, which describes steady-state fields.

So the order is wave, heat, Laplace, which matches option (A).

Quick Tip: Match each equation to its standard form: second time derivative for wave, first time derivative for heat, and pure spatial Laplacian equal to zero for Laplace.

12. A box contains one red ball and two blue balls. One ball is drawn at random, its colour is noted and the ball is returned to the box. The process is repeated. In the first two draws, blue ball appears each time. The probability of drawing blue ball again in the third draw is

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) 1

Correct Answer: (A) $\frac{2}{3}$

Solution:

Step 1: Note the box composition.

The box has 1 red ball and 2 blue balls, so 3 balls in total.

Step 2: Use the "with replacement" rule.
Each ball goes back into the box after being drawn, so the box has the exact same 1 red and 2 blue balls before every single draw.
This means each draw is independent, and past outcomes do not change the odds of the next draw.

Step 3: Compute the probability for the third draw.
The information that the first two draws were blue does not change the box, so it does not change the probability for the third draw.

$$P(\text{blue on draw 3}) = \frac{\text{number of blue balls}}{\text{total balls}} = \frac{2}{3}$$

Final Answer:
The probability of drawing a blue ball on the third draw is $\frac{2}{3}$.

$$\boxed{\frac{2}{3}}$$

Quick Tip: Remember the ball is returned to the box each time, so every draw is independent of the earlier ones.

13. The general solution to the ordinary differential equation $\frac{d^2y}{dx^2} = \cos 2x$ is

(A) $y = -\frac{\cos 2x}{4} + C_1x + C_2$, where C_1 and C_2 are constants

(B) $y = \frac{\sin 2x}{2} + C_1$, where C_1 is a constant

(C) $y = \frac{\cos 2x}{4} + C_1x + C_2$, where C_1 and C_2 are constants

(D) $y = -\frac{\sin 2x}{2} + C_1$, where C_1 is a constant

Correct Answer: (A) $y = -\frac{\cos 2x}{4} + C_1x + C_2$, where C_1 and C_2 are constants

Solution:

Step 1: Integrate once.

Start from $\frac{d^2y}{dx^2} = \cos 2x$.

Integrate both sides with respect to x: $\frac{dy}{dx} = \int \cos 2x\, dx = \frac{\sin 2x}{2} + C_1$.

Step 2: Integrate a second time.

$y = \int \left(\frac{\sin 2x}{2} + C_1\right) dx$

$y = -\frac{\cos 2x}{4} + C_1x + C_2$, where C_2 is the new constant from this integration.

Step 3: Check by differentiating back.

$\frac{dy}{dx} = \frac{\sin 2x}{2} + C_1$, and $\frac{d^2y}{dx^2} = \cos 2x$, which matches the original equation.

Final Answer:

The general solution is $y = -\frac{\cos 2x}{4} + C_1x + C_2$.

$$\boxed{y = -\frac{\cos 2x}{4} + C_1 x + C_2}$$

Quick Tip: Integrate both sides of the equation twice with respect to x, adding a constant each time.

14. Two square matrices **A** and **B** of the same order are such that $\det[\mathbf{A} \cdot \mathbf{B}] = -80$ and $\det[\mathbf{A}] = 5$. Here, $\mathbf{A} \cdot \mathbf{B}$ is the product of **A** and **B**. The value of $\det[\mathbf{B}^T]$ is

(A) -16

(B) $-\frac{1}{16}$

(C) -400

(D) 28

Correct Answer: (A) -16

Solution:

Step 1: Use the product rule for determinants.

For any two square matrices of the same order, $\det[\mathbf{A} \cdot \mathbf{B}] = \det[\mathbf{A}] \cdot \det[\mathbf{B}]$.

Given $\det[\mathbf{A} \cdot \mathbf{B}] = -80$ and $\det[\mathbf{A}] = 5$.

Step 2: Solve for det[B].

$$\det[\mathbf{B}] = \frac{\det[\mathbf{A} \cdot \mathbf{B}]}{\det[\mathbf{A}]} = \frac{-80}{5} = -16$$

Step 3: Use the transpose rule.

A matrix and its transpose always have the same determinant, so

$\det[\mathbf{B}^T] = \det[\mathbf{B}]$.

Final Answer:

$\det[\mathbf{B}^T] = -16$.

$$\boxed{-16}$$

Quick Tip: Use det(AB) = det(A) det(B) first, then remember a matrix and its transpose share the same determinant.

15. Consider the simultaneous equations

$$2x + 4y = -7; \quad 3x + 5y = 1$$

While solving using Gauss elimination without pivoting, after the forward elimination, the diagonal elements of the coefficient matrix are

(A) 2 and -1

(B) 3 and 2

(C) 5 and -2

(D) 3 and 4

Correct Answer: (A) 2 and -1

Solution:

Step 1: Write the coefficient matrix.

The system is $2x + 4y = -7$ and $3x + 5y = 1$, so the coefficient matrix is

$$\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$

Step 2: Find the elimination factor.

Without pivoting, the first row stays as is, and the first entry 2 is the pivot.

The factor to clear the 3 in row 2 is $m = \frac{3}{2}$.

Step 3: Apply the row operation.

New row 2 equals row 2 minus m times row 1:

$3 - \frac{3}{2}(2) = 0$ for the first entry, as expected.

$5 - \frac{3}{2}(4) = 5 - 6 = -1$ for the second entry.

Step 4: Read off the diagonal.

After forward elimination, the coefficient matrix becomes

$$\begin{bmatrix} 2 & 4 \\ 0 & -1 \end{bmatrix}$$

so the diagonal elements are 2 and -1.

Final Answer:

The diagonal elements after forward elimination are 2 and -1.

$$\boxed{2, \ -1}$$

Quick Tip: Eliminate x from the second equation using the first row's pivot, then look at the resulting coefficient of y.

16. A plough attachment, which is a miniature plough bottom that cuts a narrow, shallow furrow ahead of the shin and moves trash and roots toward the main furrow so that they are completely covered by the mouldboard, is called as

(A) Jointer

(B) Coulter

(C) Subsoiler

(D) Weed hook

Correct Answer: (A) Jointer

Solution:

This question checks knowledge of plough attachments used in tillage implements and their specific jobs.

1. **Jointer**: a small miniature plough bottom mounted ahead of the shin. It cuts a narrow, shallow furrow and throws trash and roots into the main furrow so the mouldboard buries them completely. This matches the description exactly.
2. **Coulter**: a rotating disc or knife fitted ahead of the share that cuts the furrow slice vertically from the unploughed land, but it does not handle trash burial the way a jointer does.
3. **Subsoiler**: a heavy tool used to break up compacted soil layers well below normal ploughing depth, unrelated to surface trash covering.
4. **Weed hook**: a simple attachment used to clear weeds and trash caught on the plough beam or standard, not a miniature plough bottom.

The description of a miniature plough bottom that pre-cuts a shallow furrow and pushes trash under the mouldboard is the textbook

definition of a jointer, so option (A) is correct.

Quick Tip: Think about which small plough attachment specifically pre-cuts a shallow furrow to bury surface trash under the mouldboard.

17. Fuel property that predominantly governs 'hard / cold starting' in SI engine is P; and the property that predominantly governs 'ignition delay' in CI engine is Q. Identify the correct option for P and Q combination.

(A) P: Octane number
Q: Calorific value

(B) P: Volatility
Q: Cetane number

(C) P: Flash point
Q: Viscosity

(D) P: Calorific value
Q: Octane number

Correct Answer: (B) P: Volatility
Q: Cetane number

Solution:

This question checks which fuel property controls cold starting in a spark ignition (SI) engine and which one controls ignition delay in a compression ignition (CI) engine.

1. **Octane number and Calorific value**: Octane number measures knock resistance, not cold starting ability. Calorific value gives energy content, not ignition delay. This pairing does not fit.

2. **Volatility and Cetane number**: A fuel with low volatility does not vaporize well at low temperature, so the SI engine cranks hard when cold. Cetane number rates how fast a CI fuel self-ignites after injection, so a low cetane number means a longer ignition delay. Both parts match the physics.
3. **Flash point and Viscosity**: Flash point relates to fire safety during storage, not starting. Viscosity affects spray atomization, not the ignition delay period directly. This pairing does not fit.
4. **Calorific value and Octane number**: Neither property controls the named phenomena. Calorific value sets power output and octane number resists knock, so this pairing is wrong.

Volatility governs hard and cold starting in an SI engine because a fuel that vaporizes easily forms a combustible mixture even at low cranking speed and temperature. Cetane number governs ignition delay in a CI engine because a higher cetane number shortens the delay between injection and self-ignition. So option (B) is correct.

Quick Tip: Think about which fuel property helps vaporization at low temperature and which one rates self-ignition readiness.

18. A tractor engine delivers 382 N.m brake torque at a rated speed of 2000 rpm. The radiator cooling fan draws 5% of the engine brake power. The fan pushes $2.8\ \mathrm{m^3 s^{-1}}$ of air against 0.9 kPa static pressure rise. Assuming air as incompressible, and ignoring other losses, the fan efficiency, in %, is nearest to (Take $\pi = 3.14$)

(A) 58

(B) 60

(C) 63

(D) 65

Correct Answer: (C) 63

Solution:

Step 1: Find the engine brake power.

Brake power comes from torque and speed: $P_b = \frac{2\pi NT}{60}$, with $T = 382$ N.m and $N = 2000$ rpm.

$$P_b = \frac{2 \times 3.14 \times 2000 \times 382}{60} = \frac{4797920}{60} = 79965.3 \text{ W}$$

Step 2: Find the power drawn by the cooling fan.

The fan takes 5% of the brake power to run.

$$P_{fan} = 0.05 \times 79965.3 = 3998.3 \text{ W}$$

Step 3: Find the useful (air) power delivered by the fan.

For an incompressible flow, useful fan power equals flow rate times static pressure rise.

$$P_{air} = Q \times \Delta p = 2.8 \times 900 = 2520 \text{ W}$$

Step 4: Find the fan efficiency.

Fan efficiency is the useful air power divided by the power the fan actually draws.

$$\eta_{fan} = \frac{P_{air}}{P_{fan}} \times 100 = \frac{2520}{3998.3} \times 100 = 63.0\%$$

Final Answer:

The fan efficiency works out close to 63%.

$$\boxed{\eta_{fan} \approx 63\%}$$

Quick Tip: Find brake power from torque and speed, then compare the fan's flow-pressure power to the power it actually draws.

19. A 2WD tractor's driving axle experiences total dynamic normal load of 28 kN. The driving wheels have 0.60 m rolling radius. The total driving axle torque measured is 10.7 kN.m. At 18% wheel slip, the coefficient of net traction is 0.35. Tractive efficiency of the driving wheels, in %, is nearest to

(A) 45

(B) 35

(C) 55

(D) 58

Correct Answer: (A) 45

Solution:

Step 1: Find the rim pull produced by the axle torque.

Torque and rolling radius give the driving force at the rim: $F = \frac{T}{r} = \frac{10.7}{0.60} = 17.83$ kN.

Step 2: Find the net drawbar pull from the traction coefficient.

The coefficient of net traction relates net pull to dynamic weight: $P = C_T \times W = 0.35 \times 28 = 9.8$ kN.

Step 3: Relate axle power and drawbar power through velocities.

Tractive efficiency is drawbar power over axle power, $\eta_t = \dfrac{P \times V_a}{F \times V_t}$, and slip links the two speeds by $V_a = V_t(1 - s)$, so $\eta_t = \dfrac{P}{F}(1 - s)$.

Step 4: Substitute the numbers.

$$\eta_t = \frac{9.8}{17.83} \times (1 - 0.18) = 0.5497 \times 0.82 = 0.4507$$

Final Answer:

Tractive efficiency comes out close to 45%.

$$\boxed{\eta_t \approx 45\%}$$

Quick Tip: Combine the rim-pull to axle-force ratio with the slip factor to get tractive efficiency.

20. The RMS sound pressure is increased by 50%. This increases the sound pressure level, in dB, nearly by

(A) 3.52

(B) 8.11

(C) 100.21

(D) 43.52

Correct Answer: (A) 3.52

Solution:

Step 1: Write the sound pressure level formula.

Sound pressure level in dB is $L = 20 \log_{10}\left(\dfrac{p}{p_{ref}}\right)$, so a change in level depends only on the ratio of pressures.

Step 2: Express the new pressure as a ratio of the old one.
A 50% increase means the new RMS pressure is $p_2 = 1.5\, p_1$.

Step 3: Find the change in level.
$$\Delta L = 20 \log_{10}\left(\frac{p_2}{p_1}\right) = 20 \log_{10}(1.5)$$
$$\Delta L = 20 \times 0.1761 = 3.52 \text{ dB}$$

Final Answer:
The sound pressure level rises by about 3.52 dB.

$$\boxed{\Delta L \approx 3.52 \text{ dB}}$$

Quick Tip: Use the 20 log10 relation between pressure ratio and decibel change.

21. A trapezoidal channel carries 5 $\mathrm{m^3 s^{-1}}$ of water under uniform flow condition. The channel has a bottom width of 2 m, side slope of 2:1 (horizontal: vertical), and a bed slope of 1%. If the Manning roughness coefficient is 0.03, the conveyance of the channel, in $\mathrm{m^3 s^{-1}}$, is

(A) 2

(B) 5

(C) 15

(D) 50

Correct Answer: (D) 50

Solution:

Step 1: Recall the definition of channel conveyance.

Manning's equation for uniform flow is $Q = \frac{1}{n} A R^{2/3} S^{1/2}$, and conveyance is defined as $K = \frac{1}{n} A R^{2/3}$, so $Q = K S^{1/2}$.

Step 2: Rearrange for conveyance.

$$K = \frac{Q}{\sqrt{S}}$$

Step 3: Substitute the given discharge and bed slope.

Bed slope $S = 1\% = 0.01$, and $Q = 5\ \text{m}^3\text{s}^{-1}$.

$$K = \frac{5}{\sqrt{0.01}} = \frac{5}{0.1} = 50\ \text{m}^3\text{s}^{-1}$$

Final Answer:

The channel carries this flow with a conveyance of 50 m3/s; the bottom width, side slope and n are not even needed once Q and S are known.

$$\boxed{K = 50\ \text{m}^3\text{s}^{-1}}$$

Quick Tip: Recall that conveyance links discharge and bed slope directly through Manning's equation.

22. In wind erosion, the amount of soil transported varies

(A) directly as the cube of the threshold wind velocity and inversely as the square of the mean soil particle diameter

(B) directly as the cube of the actual wind velocity and inversely as the square of the mean soil particle diameter

(C) directly as the cube of the difference in actual and threshold wind velocity and the square of the mean soil particle diameter

(D) directly as the cube of the difference in actual and threshold wind velocity and the square root of the mean soil particle diameter

Correct Answer: (D) directly as the cube of the difference in actual and threshold wind velocity and the square root of the mean soil particle diameter

Solution:

Soil transport by wind erosion depends on how far the wind speed exceeds the threshold speed needed to start moving particles, and on the size of the particles themselves.

1. **Cube of threshold wind velocity, inverse square of particle diameter**: The threshold velocity alone does not tell us the driving force once the wind is already blowing above it, so this option misses the actual excess velocity that does the work.
2. **Cube of actual wind velocity, inverse square of particle diameter**: Using only the actual velocity ignores the fact that no transport happens until wind speed crosses the threshold, so this is not the correct dependence.
3. **Cube of the velocity difference, and the square of particle diameter**: This gets the excess velocity term right but pairs it with the square of the particle diameter, which does not match the accepted relation for particle size.

4. **Cube of the velocity difference, and the square root of particle diameter**: The soil transport rate rises with the cube of the difference between actual and threshold wind velocity, since this excess velocity supplies the energy for saltation and creep, and it varies with the square root of the mean particle diameter, reflecting how easily grains of a given size get moved.

The relation that matches the physics of wind erosion transport is option (D).

Quick Tip: Think about which velocity term actually drives particle movement once the threshold is crossed.

23. When the bed slope of the discharge carrier in a chute spillway changes from steeper to milder, we provide a

(A) concave vertical curve

(B) convex vertical curve

(C) reverse curve

(D) converging transition

Correct Answer: (A) concave vertical curve

Solution:

A chute spillway carries water down a steep slope to the stilling basin, and the shape of the transition between slope changes controls whether the water stays in contact with the bed.

1. **Concave vertical curve**: When the slope flattens from steep to mild, the flow direction must bend upward gradually. A concave

curve, sagging like the bottom of a bowl, keeps the high-speed jet pressed against the bed and avoids the flow lifting off the surface.

2. **Convex vertical curve**: A convex curve is used when the slope steepens, not when it flattens, because on a convex curve fast flow tends to separate from the bed and can cause negative pressure or cavitation.
3. **Reverse curve**: A reverse curve is used at the toe of the chute to turn the flow from steep to horizontal at the stilling basin, not to blend two straight slopes of different steepness along the chute.
4. **Converging transition**: This term describes a change in channel width, not a change in bed slope, so it does not apply here.

Since the bed slope goes from steeper to milder, the correct provision is a concave vertical curve, option (A).

Quick Tip: Picture how the flow must bend when a steep bed flattens out.

24. A natural colloidal clay, having the characteristics of absorbing water and swelling 8 to 15 times its dry size, used as an efficient and effective sealing material for farm ponds, is

(A) EPDM (Ethylene Propylene Diene Monomer)

(B) Butyl

(C) Bentonite

(D) Soil cement

Correct Answer: (C) Bentonite

Solution:

Farm pond linings need a material that seals the pond bed against seepage, and several materials are used depending on cost and availability.

1. **EPDM (Ethylene Propylene Diene Monomer)**: EPDM is a synthetic rubber sheet liner, not a natural clay, so it does not fit the description of a colloidal clay that swells with water.
2. **Butyl**: Butyl is also a synthetic rubber membrane liner used for ponds, not a naturally occurring clay mineral.
3. **Bentonite**: Bentonite is a natural colloidal clay, rich in montmorillonite, that absorbs water and swells to several times its dry volume, typically about 8 to 15 times, forming a tight, low permeability seal on the pond bed.
4. **Soil cement**: Soil cement is a mixture of soil and cement compacted as a lining, an engineered blend rather than a natural swelling clay.

The natural colloidal clay described here is bentonite, option (C).

Quick Tip: Recall which farm pond liner material is a natural clay rather than a synthetic sheet.

25. By depth-area-duration (DAD) analysis, the maximum average depth of rainfall over a 10^3 km^2 catchment due to a 6-hour storm is 80 mm. For the same storm, the maximum average depth of rainfall for a 10^4 km^2 catchment, in mm, is

(A) greater than 88

(B) lesser than 80

(C) equal to 80

(D) between 81 and 88

Correct Answer: (B) lesser than 80

Solution:

Step 1: Recall what a DAD curve shows.

A depth-area-duration (DAD) analysis plots how the maximum average rainfall depth for a fixed storm duration falls as the catchment area considered gets larger.

The storm data come from actual isohyetal maps, and the average depth is found by area-weighting the rainfall inside successively bigger boundaries drawn around the storm centre.

Step 2: Apply this to the given areas.

Here the duration is fixed at 6 hours, and the average depth for a 10^3 km^2 area around the storm centre is 80 mm.

The 10^4 km^2 catchment is ten times larger and includes zones further from the storm centre, where rainfall intensity is lower.

Averaging this low-intensity outer rain together with the high-intensity rain near the centre always pulls the average depth down as area grows.

Final Answer:

The average depth for the larger 10^4 km^2 catchment must be less than 80 mm.

$$\boxed{\text{lesser than 80 mm}}$$

Quick Tip: Think about how DAD curves behave as the area around a storm centre grows for a fixed duration.

26. Dimensionless parameter among the following options is

(A) heat exchanger effectiveness

(B) humid heat of air-water vapour mixture

(C) humid volume of air-water vapour mixture

(D) overall heat transfer coefficient

Correct Answer: (A) heat exchanger effectiveness

Solution:

Step 1: Check the units of each option.

Heat exchanger effectiveness is defined as the ratio of actual heat transfer rate to the maximum possible heat transfer rate, $\varepsilon = \frac{Q_{actual}}{Q_{max}}$.

Both quantities in this ratio carry units of watts, so they cancel and effectiveness is a pure number.

Step 2: Rule out the other options.

Humid heat has units of $\mathbf{kJ.kg^{-1}.K^{-1}}$, humid volume has units of $\mathbf{m^3.kg^{-1}}$, and overall heat transfer coefficient has units of $\mathbf{W.m^{-2}.K^{-1}}$.

None of these three cancel out to a pure number, so all three carry dimensions.

Final Answer:

Only heat exchanger effectiveness is dimensionless.

heat exchanger effectiveness

Quick Tip: Write the defining formula for each option and check whether the units cancel.

27. It is given that the water activity of a food is equal to the relative humidity of the atmosphere that is in equilibrium with the food. The partial pressure of water vapour of the food at a specific temperature is

(A) equal to the partial pressure of water vapour in air

(B) lesser than the partial pressure of water vapour in air

(C) greater than the partial pressure of water vapour in air

(D) not dependent on the partial pressure of water vapour in air

Correct Answer: (A) equal to the partial pressure of water vapour in air

Solution:

Step 1: Write the definition of water activity.

Water activity is defined as $a_w = \frac{p_w}{p_{ws}}$, the ratio of the partial pressure of water vapour exerted by the food to the saturation vapour pressure of pure water at the same temperature.

The relative humidity of the surrounding air, in equilibrium with the food, is $RH = \frac{p_{air}}{p_{ws}} \times 100$.

Step 2: Use the equilibrium condition given.

The question states $a_w = RH/100$ for this food-air system at equilibrium.

Since p_{ws} is the same reference saturation pressure in both ratios,

equating a_w and $RH/100$ forces $p_w = p_{air}$.
This is exactly what equilibrium moisture means: no net moisture transfer, so the driving vapour pressures on both sides must match.

Final Answer:
The partial pressure of water vapour of the food equals the partial pressure of water vapour in the surrounding air.

$$\boxed{p_{w,food} = p_{w,air}}$$

Quick Tip: Write water activity and relative humidity as ratios to the same saturation pressure, then compare.

28. Air carrying dust particles enters a cyclone separator of 800 mm diameter at a tangential velocity of 30 $\mathrm{m.s^{-1}}$ near the wall. The average diameter of the dust particles is 100 μm. The separation factor is nearest to (Take $g = 9.81\ \mathrm{m.s^{-2}}$)

(A) 229.36

(B) 611.62

(C) 485.72

(D) 3.75

Correct Answer: (A) 229.36

Solution:

Step 1: Recall the separation factor formula.
In a cyclone separator, the separation factor compares the centrifugal force pulling a particle outward to the gravitational force pulling it

down, $S = \dfrac{v_t^2}{rg}$.

Here v_t is the tangential (whirl) velocity of the air near the cyclone wall and r is the radius at which that velocity acts.

Step 2: Substitute the given values.

The cyclone diameter is 800 mm, so the radius is $r = 0.4$ m, and the tangential velocity is $v_t = 30\ \text{m.s}^{-1}$.

$$S = \frac{(30)^2}{0.4 \times 9.81} = \frac{900}{3.924}$$

$$S \approx 229.36$$

Step 3: Note that particle size does not enter this formula.

The 100 μm particle diameter is extra information; the separation factor as defined here depends only on wall velocity and wall radius, not particle size.

Final Answer:

The separation factor is nearest to 229.36.

$$\boxed{S \approx 229.36}$$

Quick Tip: Use the separation factor formula S equals v_t squared over r g, with the cyclone radius, not diameter.

29. A circular mild steel storage bin of 8 m height contains wheat having bulk density of 800 $\mathbf{kg.m^{-3}}$. The coefficient of static friction between wheat grain and bin wall is 0.40 and hydraulic radius of the bin is 1 m. Equivalent diameter of the bin, in m, is

(A) 4

(B) 8

(C) 1

(D) 2

Correct Answer: (A) 4

Solution:

Step 1: Recall the hydraulic radius definition for a circular bin.

Hydraulic radius is defined as cross-sectional area divided by wetted perimeter, $R_h = \frac{A}{P}$.

For a circular bin of diameter D, $A = \frac{\pi D^2}{4}$ and $P = \pi D$, so $R_h = \frac{D}{4}$.

Step 2: Solve for the diameter.

Rearranging gives $D = 4R_h$.

With $R_h = 1$ m, $D = 4 \times 1 = 4$ m.

Step 3: Note the unused data.

The bin height, bulk density, and coefficient of friction are needed for Janssen's pressure equations, not for finding the equivalent diameter from hydraulic radius.

Final Answer:

The equivalent diameter of the bin is 4 m.

$$\boxed{D = 4\ \text{m}}$$

Quick Tip: For a circular bin, the hydraulic radius equals the diameter divided by four.

30. Specific heat, coefficient of viscosity, and thermal conductivity are related in the

(A) Prandtl number

(B) Reynolds number

(C) Froude number

(D) Biot number

Correct Answer: (A) Prandtl number

Solution:

Step 1: Write the definition of the Prandtl number.

The Prandtl number is defined as $Pr = \frac{c_p \mu}{k}$, where c_p is specific heat, μ is the coefficient of viscosity, and k is thermal conductivity.

It compares how fast momentum diffuses through a fluid against how fast heat diffuses through it.

Step 2: Rule out the other dimensionless numbers.

Reynolds number, $Re = \frac{\rho v D}{\mu}$, uses density, velocity, and a length scale, not specific heat or conductivity.

Froude number, $Fr = \frac{v^2}{gL}$, only involves velocity, gravity, and length.

Biot number, $Bi = \frac{hL}{k}$, uses a convective heat transfer coefficient and

length, not viscosity or specific heat directly.

Final Answer:
Only the Prandtl number combines specific heat, viscosity, and thermal conductivity together.

$$\boxed{Pr = \frac{c_p \mu}{k}}$$

Quick Tip: Match the three named properties to the defining formula of each dimensionless number listed.

31. Mechanical model/method NOT related to rheological behaviour of food products is known as

(A) Liebman's method

(B) Newton's model

(C) Maxwell's model

(D) Saint-Venant's model

Correct Answer: (A) Liebman's method

Solution:

Rheological models describe how food materials deform and flow under stress, and each named model or method has a specific role.

1. **Liebman's method**: a numerical relaxation technique used to solve heat conduction and other field equations, it has no link to describing stress-strain behaviour of food.

2. **Newton's model**: describes a Newtonian fluid where shear stress is directly proportional to shear rate, a basic rheological model.
3. **Maxwell's model**: a spring and dashpot in series that represents viscoelastic stress relaxation, a standard rheological model.
4. **Saint-Venant's model**: represents a plastic element with a yield stress, used to model plastic flow behaviour in rheology.

Since Liebman's method is a numerical solution technique and not a stress-strain model, option A is correct.

Quick Tip: Ask which of the four names is a numerical solution technique rather than a stress-strain model.

32. At an initial pressure of 100 kPa, 1 kg air is compressed reversibly from 15 litres to 1 litre at final pressure of 2000 kPa. Neglecting other losses, the Polytropic Exponent (n) is _____. (Rounded off to three decimal places)

Correct Answer: 1.106

Solution:

Step 1: Write the polytropic process relation.

For a reversible polytropic compression, $P_1V_1^n = P_2V_2^n$, where P_1, V_1 are the initial pressure and volume and P_2, V_2 are the final values.

Taking natural logs of both sides gives $n = \dfrac{\ln(P_2/P_1)}{\ln(V_1/V_2)}$.

Step 2: Substitute the given values.

$P_1 = 100$ kPa, $P_2 = 2000$ kPa, $V_1 = 15$ L, $V_2 = 1$ L.

$$n = \frac{\ln(2000/100)}{\ln(15/1)} = \frac{\ln 20}{\ln 15}$$

$$n = \frac{2.9957}{2.7081}$$

Step 3: Compute the numeric value.

$$n \approx 1.106$$

The mass of air (1 kg) is not needed here, since the polytropic exponent depends only on the pressure and volume ratios.

Final Answer:

The polytropic exponent is about 1.106, which falls between 1.100 and 1.111 as expected.

$$\boxed{n \approx 1.106}$$

Quick Tip: Use P1 V1^n equals P2 V2^n and solve for n from the pressure and volume ratios.

33. A tubewell operating 10 hours per day discharges water at a rate of $25\ m^3.h^{-1}$. The average irrigation depth is 80 mm, and the irrigation interval is 20 days. If the intensity of irrigation is 80%, the culturable command area (in hectares) is _____. (Rounded off to two decimal places)

Correct Answer: 7.81

Solution:

Step 1: Find the volume of water pumped in one irrigation interval.

The tubewell discharges water at $25\ m^3/h$ and runs for 10 hours each day.

Volume pumped per day $= 25 \times 10 = 250\ m^3$.

Over one irrigation interval of 20 days, total volume $= 250 \times 20 = 5000\ m^3$.

Step 2: Find the area that gets wetted by this volume.

The average irrigation depth applied is 80 mm, which is $0.08\ m$.

Area actually irrigated $= \frac{\text{Volume}}{\text{depth}} = \frac{5000}{0.08} = 62500\ m^2$.

Convert to hectares: $62500\ m^2 = 6.25\ ha$.

Step 3: Scale up to the full culturable command area using the irrigation intensity.

An intensity of irrigation of 80% means only 80% of the culturable command area (CCA) gets watered in this interval.

So $0.80 \times CCA = 6.25\ ha$, which gives $CCA = \frac{6.25}{0.80} = 7.8125\ ha$.

Final Answer:

The culturable command area works out close to 7.81 hectares.

$$\boxed{CCA \approx 7.81 \text{ ha}}$$

Quick Tip: Start by finding the total volume the tubewell delivers over one irrigation interval.

34. A tile drainage system having a drainage coefficient of 25 mm drains an area of $0.2\ \text{km}^2$. The average discharge from the system (in $\text{m}^3.\text{s}^{-1}$) is _____. (Rounded off to three decimal places)

Correct Answer: 0.058

Solution:

Step 1: Convert the drainage coefficient and area to SI units.

Drainage coefficient $DC = 25\ \text{mm/day} = 0.025\ \text{m/day}$.

Area $A = 0.2\ \text{km}^2 = 0.2 \times 10^6\ \text{m}^2 = 200000\ \text{m}^2$.

Step 2: Find the volume drained per day.

Volume per day $= DC \times A = 0.025 \times 200000 = 5000\ \text{m}^3/\text{day}$.

Step 3: Convert the daily volume to an average discharge in m^3/s.

One day has 86400 seconds.

$$Q = \frac{5000}{86400} = 0.05787\ \text{m}^3/\text{s}.$$

Final Answer:

The average discharge from the tile drain rounds to 0.058 cubic metres per second.

$$\boxed{Q \approx 0.058\ \text{m}^3/\text{s}}$$

Quick Tip: Convert the drainage coefficient and area into consistent SI units before finding volume per day.

35. The impeller speed of a centrifugal pump is increased by 30%. The power requirement of the pump increases by n times. The value of n is _____. (Rounded off to two decimal places)

Correct Answer: 2.20

Solution:

Step 1: Recall how pump power depends on speed.

For a centrifugal pump running at fixed efficiency, the affinity laws give discharge proportional to speed, head proportional to speed squared, and power proportional to speed cubed.

So $P \propto N^3$.

Step 2: Write the new speed in terms of the old speed.

Speed is increased by 30%, so the new speed is $N_2 = 1.30\,N_1$.

Step 3: Find the power ratio.

$$n = \frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3 = (1.30)^3.$$

$$(1.30)^3 = 1.30 \times 1.30 \times 1.30 = 1.69 \times 1.30 = 2.197.$$

Final Answer:

The power requirement rises by about 2.20 times the original value.

$$\boxed{n \approx 2.20}$$

Quick Tip: Recall how pump power scales with impeller speed under the pump affinity laws.

36. The value of the integral $I = \int_0^1 \int_0^1 x\cos(xy)\,dx\,dy$ is

(A) $1 - \cos(1)$

(B) $1 + \cos(1)$

(C) $1 - \sin(1)$

(D) $1 + \sin(1)$

Correct Answer: (A) $1 - \cos(1)$

Solution:

Step 1: Recognise that the order of integration can be swapped.

The function $x\cos(xy)$ is continuous on the square $[0,1] \times [0,1]$, so by Fubini's theorem the iterated integral gives the same value in either order.

It is easier to integrate over y first.

Step 2: Integrate with respect to y, holding x fixed.

$$\int_0^1 x\cos(xy)\,dy = \Big[\sin(xy)\Big]_{y=0}^{1} = \sin(x) - \sin(0) = \sin(x).$$

Step 3: Integrate the result with respect to x.

$$I = \int_0^1 \sin(x)\,dx = \Big[-\cos(x)\Big]_0^1 = -\cos(1) + \cos(0) = 1 - \cos(1).$$

Step 4: Check the wrong options.

Options B and D add $\cos(1)$ or $\sin(1)$ instead of subtracting, which would come from a sign slip at the limits.

Option C mixes up $\cos(1)$ with $\sin(1)$ from the wrong antiderivative of $\cos(x)$.

Final Answer:

The integral evaluates to $1 - \cos(1)$, so option A is correct.

$$\boxed{I = 1 - \cos(1)}$$

Quick Tip: Try swapping the order of integration to get a simple antiderivative.

37. The determinant of a 2×2 square matrix is 1.6. If the sum of the eigenvalues is -2.8, the eigenvalues are

(A) -2 and -0.8

(B) -1.8 and -1

(C) -3 and 0.2

(D) -1.2 and -1.6

Correct Answer: (A) -2 and -0.8

Solution:

Step 1: Write down what is given about the eigenvalues.

For a 2×2 matrix, the sum of the eigenvalues equals the trace, and the product of the eigenvalues equals the determinant.

So $\lambda_1 + \lambda_2 = -2.8$ and $\lambda_1 \lambda_2 = 1.6$.

Step 2: Form the characteristic quadratic.

The eigenvalues satisfy $\lambda^2 - (\lambda_1 + \lambda_2)\lambda + \lambda_1 \lambda_2 = 0$, which gives λ^2

$+ 2.8\lambda + 1.6 = 0.$

Step 3: Solve the quadratic.

Discriminant $= 2.8^2 - 4(1.6) = 7.84 - 6.4 = 1.44$, and $\sqrt{1.44} = 1.2$.

$\lambda = \dfrac{-2.8 \pm 1.2}{2}$, giving $\lambda = -0.8$ or $\lambda = -2$.

Step 4: Check the wrong options.

Options B, C, D each fail either the sum or the product test; for example, $-1.8 \times -1 = 1.8 \neq 1.6$.

Final Answer:

The eigenvalues are -2 and -0.8, so option A is correct.

$$\boxed{\lambda = -2, \ -0.8}$$

Quick Tip: Set up the characteristic equation from the trace and determinant of the matrix.

38. A flat belt of negligible mass and thickness drives an output pulley of 0.6 m diameter, rotating at 500 rpm without slip. The arc of contact is 190° for this pulley, while it is 170° for the smaller input pulley. The coefficient of friction between the belt and the pulleys is 0.30. The measured slack side tension is 250 N. Neglecting other losses, the power transmitted by the belt drive, in kW, is nearest to (take $\pi = 3.14$)

(A) 4.8

(B) 5.6

(C) 6.5

(D) 7.3

Correct Answer: (B) 5.6

Solution:

Step 1: Identify which pulley governs slipping.

The smaller pulley has the smaller angle of wrap, 170°, so belt slip starts there first; this angle sets the tension ratio for the whole drive.

Convert to radians: $\theta = 170 \times \frac{3.14}{180} = 2.971$ rad.

Step 2: Find the ratio of tight side to slack side tension.

The belt friction relation gives $\frac{T_1}{T_2} = e^{\mu\theta}$.

$\mu\theta = 0.30 \times 2.971 = 0.8913$, so $\frac{T_1}{T_2} = e^{0.8913} = 2.438$.

With $T_2 = 250$ N, $T_1 = 2.438 \times 250 = 609.6$ N.

Step 3: Find the belt speed at the output pulley.

$v = \frac{\pi DN}{60} = \frac{3.14 \times 0.6 \times 500}{60} = 15.70$ m/s.

Step 4: Find the power transmitted.

$P = (T_1 - T_2)v = (609.6 - 250) \times 15.70 = 359.6 \times 15.70 = 5646$ W

≈ 5.65 kW.

Step 5: Check the wrong options.

Using the larger 190° angle by mistake would push the answer up near 6.5 kW (option C), which ignores that the smaller pulley slips first.

Final Answer:

The power transmitted is nearest to 5.6 kW.

$$\boxed{P \approx 5.6 \text{ kW}}$$

Quick Tip: Identify which pulley has the smaller angle of contact, since that one governs belt slip.

39. A tractor PTO runs at a constant speed of 540 rpm to drive a rotary tiller through a flange coupling having a shear pin parallel to the shaft. The shear pin axis is located 60 mm from the PTO shaft axis. The allowable shear stress of the pin material is 200 MPa. For overload safety, the pin must fail at 150% of the rated torque to protect the gearbox. The tiller requires 40 kW under normal load at the rated PTO speed. Neglecting bending, stress concentration effects, and other losses, the pin diameter, in mm, is nearest to (take $\pi = 3.14$)

(A) 7.5

(B) 10.6

(C) 6.1

(D) 8.6

Correct Answer: (B) 10.6

Solution:

Step 1: Find the rated torque delivered by the PTO shaft.

Angular speed $\omega = \dfrac{2\pi N}{60} = \dfrac{2 \times 3.14 \times 540}{60} = 56.52$ rad/s.

Rated power $P = 40$ kW $= 40000$ W.

Rated torque $T_{rated} = \dfrac{P}{\omega} = \dfrac{40000}{56.52} = 707.7$ N.m.

Step 2: Apply the overload factor to get the failure torque.

The pin must shear at 150% of rated torque, so $T_{fail} = 1.5 \times 707.7$ $= 1061.6$ N.m.

Step 3: Convert the failure torque into a shear force on the pin.

The pin acts at a radius $r = 60\ \text{mm} = 0.06\ \text{m}$ from the shaft axis.

$$F = \frac{T_{fail}}{r} = \frac{1061.6}{0.06} = 17693\ \text{N}.$$

Step 4: Size the pin from the allowable shear stress.

Shear area needed $A = \dfrac{F}{\tau} = \dfrac{17693}{200 \times 10^6} = 8.847 \times 10^{-5}\ \text{m}^2$.

$A = \dfrac{\pi d^2}{4}$, so $d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{4 \times 8.847 \times 10^{-5}}{3.14}} = \sqrt{1.127 \times 10^{-4}}$

$= 0.01062\ \text{m}$.

Step 5: Convert to millimetres and check the wrong options.

$d \approx 10.6\ \text{mm}$.

Option A (7.5 mm) or C (6.1 mm) would come from using the rated torque without the 150% overload factor.

Final Answer:

The shear pin diameter should be nearest to 10.6 mm.

$$\boxed{d \approx 10.6\ \text{mm}}$$

Quick Tip: Work out the overload torque first, then convert it to a shear force on the pin.

40. A small horizontal axis wind turbine (HAWT) is designed to deliver 2 kW shaft power at a steady wind speed of $7\ \text{m.s}^{-1}$. The HAWT operates with an aerodynamic power coefficient of 0.36, a drivetrain efficiency of 0.90, and an air density of $1.225\ \text{kg.m}^{-3}$. Neglecting other losses, the required rotor diameter is X m. When the wind speed drops to $6\ \text{m.s}^{-1}$, the shaft power is Y kW. Identify the nearest correct option for the X and Y combination (take $\pi = 3.14$)

(A) $X = 6.12$ m
$Y = 1.28$ kW

(B) $X = 3.06$ m
$Y = 0.63$ kW

(C) $X = 6.12$ m
$Y = 2.00$ kW

(D) $X = 6.75$ m
$Y = 1.50$ kW

Correct Answer: (A) $X = 6.12$ m
$Y = 1.28$ kW

Solution:

Step 1: Write the shaft power available from the wind.
Shaft power after drivetrain losses is $P = \eta_d\, C_p\, \frac{1}{2}\rho A V^3$, where η_d is drivetrain efficiency, C_p is the power coefficient, ρ is air density, A is the swept rotor area, and V is wind speed.

Step 2: Solve for the swept area at $V = 7$ m/s.
$2000 = 0.90 \times 0.36 \times 0.5 \times 1.225 \times A \times 7^3$.
The constant multiplier is $0.90 \times 0.36 \times 0.5 \times 1.225 \times 343 = 68.07$.
$A = \dfrac{2000}{68.07} = 29.38\ \text{m}^2$.

Step 3: Convert area to rotor diameter.

$A = \frac{\pi D^2}{4}$, so $D = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 29.38}{3.14}} = \sqrt{37.43} = 6.12$ m.

So $X = 6.12$.

Step 4: Find the shaft power when wind speed drops to 6 m/s.

Keeping the same rotor, C_p, and η_d, power now scales only with V^3.

$P_2 = 0.90 \times 0.36 \times 0.5 \times 1.225 \times 29.38 \times 6^3 = 0.19845 \times 29.38 \times 216$.

$P_2 = 5.830 \times 216 = 1259$ W ≈ 1.26 kW, so $Y \approx 1.3$.

Step 5: Match to the closest option.

Options B and C keep X or Y badly off; option D overstates Y at 1.50.

The pair $X = 6.12$, $Y \approx 1.28$ fits best.

Final Answer:

The nearest correct combination is $X = 6.12$ m and $Y = 1.28$ kW.

$$\boxed{X \approx 6.12 \text{ m},\ Y \approx 1.28 \text{ kW}}$$

Quick Tip: Use the wind power formula with the power coefficient and drivetrain efficiency to size the rotor area.

41. The ratio of moisture content on wet basis to moisture content on dry basis for wheat grain is 4:5. The ratio of weight of dry matter to weight of water in the grain is

(A) 4:1

(B) 4:5

(C) 1:4

(D) 5:4

Correct Answer: (A) 4:1

Solution:

Step 1: Write the two moisture content definitions.

Moisture content on wet basis is $M_{wb} = \frac{W}{W+D}$, where W is the weight of water and D is the weight of dry matter.

Moisture content on dry basis is $M_{db} = \frac{W}{D}$.

Step 2: Form the given ratio.

The problem gives $\frac{M_{wb}}{M_{db}} = \frac{4}{5}$.

Divide the two expressions: $\frac{M_{wb}}{M_{db}} = \frac{W/(W+D)}{W/D} = \frac{D}{W+D}$.

So $\frac{D}{W+D} = \frac{4}{5}$.

Step 3: Solve for the dry matter to water ratio.

Cross multiply: $5D = 4W + 4D$, which gives $D = 4W$.

So $\frac{D}{W} = \frac{4}{1}$.

Final Answer:

The weight of dry matter to weight of water works out to 4:1, so option (A) is correct. Options (B), (C) and (D) come from mixing up which ratio belongs to wet basis and which to dry basis.

$$\boxed{D : W = 4 : 1}$$

Quick Tip: Write wet basis and dry basis moisture content as fractions of the same water and dry matter, then divide one by the other.

42. A refrigeration system based on reverse Carnot cycle operates between $-30\,^{\circ}C$ and $20\,^{\circ}C$. The refrigeration capacity of the system is 70 kW. Heat rejected at the condenser to atmosphere (hot reservoir), in kW, is nearest to

(A) 84.40

(B) 14.41

(C) 186.67

(D) 116.67

Correct Answer: (A) 84.40

Solution:

Step 1: Convert temperatures to Kelvin.

The cold reservoir (evaporator) is at $T_C = -30 + 273 = 243\,K$.

The hot reservoir (condenser) is at $T_H = 20 + 273 = 293\,K$.

Step 2: Find the Carnot COP for refrigeration.

For a reverse Carnot cycle, $COP = \dfrac{T_C}{T_H - T_C} = \dfrac{243}{293 - 243} = \dfrac{243}{50}$

$= 4.86$.

Step 3: Find the work input from the refrigeration capacity.

$COP = \dfrac{Q_C}{W}$, so $W = \dfrac{Q_C}{COP} = \dfrac{70}{4.86} = 14.4\,kW$.

Step 4: Apply the energy balance on the cycle.

Heat rejected at the condenser equals cooling load plus work input:

$Q_H = Q_C + W = 70 + 14.4 = 84.4\,kW$.

Final Answer:

The condenser rejects about 84.4 kW to the atmosphere, matching option (A). Options (C) and (D) wrongly treat 70 kW as the work input instead of the cooling load.

$$\boxed{Q_H \approx 84.40\,kW}$$

Quick Tip: Find the Carnot COP from the reservoir temperatures, get the work input, then add it to the cooling load.

43. A 2 mm diameter electric wire is insulated with 2 mm thick plastic. The plastic has a thermal conductivity of $0.5\ \text{W.m}^{-1}\text{.K}^{-1}$. The wire is exposed to air at 320 K and outside convective heat transfer coefficient is $25\ \text{W.m}^{-2}\text{.K}^{-1}$. Wire surface temperature is constant at 420 K and remains unaffected by the covering. Assuming steady state heat transfer conditions, the ratio of heat loss per metre of wire length with insulation to that without insulation is nearest to (Take $\pi = 3.14$)

(A) 2.58

(B) 0.39

(C) 1.87

(D) 0.54

Correct Answer: (A) 2.58

Solution:

Step 1: Set up the two heat loss paths.

Wire radius $r_1 = 1\,mm = 0.001\,m$. Insulation adds 2 mm, so outer radius $r_2 = 3\,mm = 0.003\,m$.

Surface temperature stays fixed at $T_s = 420\,K$, and air is at $T_\infty = 320\,K$, so $\Delta T = 100\,K$ in both cases.

Step 2: Heat loss without insulation (pure convection).

$Q_1 = h(2\pi r_1)\Delta T = 25 \times 2 \times 3.14 \times 0.001 \times 100 = 15.7\ W/m.$

Step 3: Heat loss with insulation (conduction plus convection in series).

Conduction resistance of the plastic layer: $R_{cond} = \dfrac{\ln(r_2/r_1)}{2\pi k}$

$= \dfrac{\ln 3}{2 \times 3.14 \times 0.5} = \dfrac{1.099}{3.14} = 0.350\ m.K/W.$

Convection resistance at the outer surface: $R_{conv} = \dfrac{1}{h(2\pi r_2)}$

$= \dfrac{1}{25 \times 2 \times 3.14 \times 0.003} = 2.123\ m.K/W.$

Total resistance $R = 0.350 + 2.123 = 2.473\ m.K/W$, so $Q_2 = \Delta T/R$ $= 100/2.473 = 40.4\ W/m.$

Step 4: Take the ratio asked in the question.

$\dfrac{Q_2}{Q_1} = \dfrac{40.4}{15.7} = 2.58.$

Final Answer:

The insulated wire actually loses more heat per metre because the outer radius sits well below the critical radius of insulation, giving ratio 2.58, option (A).

$$\boxed{Q_{ins}/Q_{bare} \approx 2.58}$$

Quick Tip: Compare convection alone to conduction through the insulation plus convection in series, then check against the critical radius k/h.

44. In grinding of black pepper, 80% feed passes through 6 mm screen, and 80% product passes through 0.5 mm screen. The gross energy required is 10 kW.h per ton of material. Assuming Bond's law holds good, and for the same feed conditions, if 80% product passes through 0.2 mm screen, the energy required, in kW.h per ton of material, is nearest to

(A) 18.17

(B) 9.09

(C) 5.50

(D) 11.00

Correct Answer: (A) 18.17

Solution:

Step 1: State Bond's law.

Bond's law gives the specific grinding energy as $E = K_b\left(\frac{1}{\sqrt{D_p}} - \frac{1}{\sqrt{D_f}}\right)$, where D_f and D_p are the 80% passing sizes of feed and product.

Step 2: Find the Bond constant from the first grind.

Here $D_f = 6\,mm$ and $D_{p1} = 0.5\,mm$, with $E_1 = 10\,kW.h/ton$.

$\frac{1}{\sqrt{0.5}} - \frac{1}{\sqrt{6}} = 1.414 - 0.408 = 1.006.$

So $K_b = E_1/1.006 = 10/1.006 = 9.94.$

Step 3: Use the same constant for the finer grind.

Now $D_{p2} = 0.2\,mm$, same feed size $D_f = 6\,mm$.

$$\frac{1}{\sqrt{0.2}} - \frac{1}{\sqrt{6}} = 2.236 - 0.408 = 1.828.$$

Step 4: Compute the new energy requirement.

$E_2 = K_b \times 1.828 = 9.94 \times 1.828 = 18.17\,kW.h/ton.$

Final Answer:

Grinding finer to 0.2 mm needs about 18.17 kW.h per ton, so option (A) is correct. The other options come from using Rittinger's or Kick's law instead of Bond's law.

$$\boxed{E_2 \approx 18.17\,kW.h/ton}$$

Quick Tip: Use Bond's law to find the grinding constant from the first size reduction, then reapply it for the finer product size.

45. A single effect evaporator is used to concentrate orange fruit juice with 2.7:1 concentration ratio of product solid to feed solid. The juice is fed to the evaporator at 100 $kg.h^{-1}$ and 30 °C, where the product moisture evaporates at 60 °C. The specific heat of juice is 3.9 $kJ.kg^{-1}.°C^{-1}$. The latent heat of vaporisation at 60 °C is 2450 $kJ.kg^{-1}$. Heat to accomplish the evaporation, in $MJ.h^{-1}$, is nearest to

(A) 165.96

(B) 117.06

(C) 154.26

(D) 142.56

Correct Answer: (A) 165.96

Solution:

Step 1: Find the product flow rate from the concentration ratio.

Solids are conserved: feed solids equal product solids, $Fx_f = Px_p$. The ratio $x_p/x_f = 2.7$ means $P = F/2.7$.

With $F = 100\ kg/h$, $P = 100/2.7 = 37.04\ kg/h$.

Step 2: Find the water evaporated.

Vapour removed $V = F - P = 100 - 37.04 = 62.96\ kg/h$.

Step 3: Add the sensible heat to bring feed up to boiling.

Sensible heat $Q_s = Fc_p(T_2 - T_1) = 100 \times 3.9 \times (60 - 30) = 11700\ kJ/h$.

Step 4: Add the latent heat to vaporise the water removed.

$Q_L = V \times \lambda = 62.96 \times 2450 = 154254\ kJ/h$.

Step 5: Total the heat load and convert units.

$Q = Q_s + Q_L = 11700 + 154254 = 165954\ kJ/h \approx 165.96\ MJ/h$.

Final Answer:

The evaporator needs close to 165.96 MJ per hour, matching option (A).

$$\boxed{Q \approx 165.96\ MJ.h^{-1}}$$

Quick Tip: Use the solids balance to get the evaporated water mass, then add sensible heat to reach boiling and latent heat to vaporise it.

46. At 60 °C dry bulb temperature, saturated water vapour pressure is 20 kPa and relative humidity is 20%. The corresponding absolute humidity of air at atmospheric pressure is nearest to (Take molecular weight of water = 18.02 g.mol^{-1}, molecular weight of air = 28.97 g.mol^{-1}, and atmospheric pressure = 101.325 kPa)

(A) 0.026

(B) 0.032

(C) 0.004

(D) 0.041

Correct Answer: (A) 0.026

Solution:

Step 1: Find the actual vapour pressure from relative humidity.

Relative humidity is the ratio of actual to saturated vapour pressure:

$RH = p_v/p_{sat}$.

$p_v = RH \times p_{sat} = 0.20 \times 20 = 4\ kPa$.

Step 2: Write the absolute humidity formula.

Absolute humidity (humidity ratio) is $W = \frac{M_w}{M_a} \times \frac{p_v}{P - p_v}$, where P is total atmospheric pressure.

Step 3: Substitute the known values.

$\frac{M_w}{M_a} = \frac{18.02}{28.97} = 0.622$.

$P - p_v = 101.325 - 4 = 97.325\ kPa$.

$W = 0.622 \times \frac{4}{97.325} = 0.622 \times 0.0411 = 0.0256$.

Final Answer:

The absolute humidity comes out close to 0.026 kg water per kg dry air, matching option (A).

$$\boxed{W \approx 0.026\ kg/kg}$$

Quick Tip: Get the vapour partial pressure from relative humidity, then use the molecular weight ratio to convert to humidity ratio.

47. Pasteurisation of milk can be carried out either at 91 °C for 3 s or at 73 °C for 30 s to inactivate vegetative cells of microorganism(s). The sterilisation value is 10 in both the cases. Thermal death time constant (z value) for reference temperatures of 73 °C and 91 °C, in °C, is nearest to

(A) 18.0

(B) 1.8

(C) 9.0

(D) 0.9

Correct Answer: (A) 18.0

Solution:

Step 1: Recall the thermal death time relation.

The time to reach the same lethal effect at two different temperatures follows $\log_{10}\left(\frac{t_2}{t_1}\right) = \frac{T_1 - T_2}{z}$, where T_1 is the higher temperature with the shorter time t_1.

Step 2: Identify the two time temperature pairs.

Higher temperature: $T_1 = 91\ °C$, $t_1 = 3\ s$. Lower temperature: $T_2 = 73\ °C$, $t_2 = 30\ s$.

Since both give the same sterilisation value (10), they lie on the same thermal death time curve, so the z value relation applies directly.

Step 3: Compute the log ratio of times.

$\log_{10}(30/3) = \log_{10}(10) = 1$.

Step 4: Solve for z.

$1 = \frac{91 - 73}{z} = \frac{18}{z}$, so $z = 18\,^{\circ}\text{C}$.

Final Answer:

The z value works out to exactly 18 °C, matching option (A).

$$\boxed{z \approx 18.0\,^{\circ}\text{C}}$$

Quick Tip: Use the thermal death time formula linking the log of the time ratio to the temperature difference and the z value.

48. Mass fractions of carbohydrate, protein, fat, ash, and water in banana are 0.220, 0.010, 0.005, 0.015, and 0.750, respectively. The density of carbohydrate, protein, fat, ash, and water are 1550, 1220, 900, 610, and 1000 $kg.m^{-3}$, respectively. The density of banana, in $kg.m^{-3}$, is nearest to

(A) 1116.85

(B) 1056.05

(C) 1000.00

(D) 1107.70

Correct Answer: (A) 1116.85

Solution:

Step 1: Set up the mass fraction weighted density model.

For a mixture where the composition is given as mass fractions, the bulk density is found from $\rho = \sum x_i \rho_i$, summing each component's mass fraction times its own density.

Step 2: Multiply each mass fraction by its density.

Carbohydrate: $0.220 \times 1550 = 341.0$.

Protein: $0.010 \times 1220 = 12.2$.

Fat: $0.005 \times 900 = 4.5$.

Ash: $0.015 \times 610 = 9.15$.

Water: $0.750 \times 1000 = 750.0$.

Step 3: Add the contributions.

$\rho = 341.0 + 12.2 + 4.5 + 9.15 + 750.0 = 1116.85\ kg.m^{-3}$.

Final Answer:

The density of banana comes out to 1116.85 kg per cubic metre, matching option (A).

$$\boxed{\rho \approx 1116.85\ kg.m^{-3}}$$

Quick Tip: Multiply each component's mass fraction by its own density and add all five contributions.

49. The following two vectors are adjacent sides of a parallelogram: $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = 5\hat{i} - 4\hat{k}$.

The magnitude of area of the parallelogram is _____ (Rounded off to two decimal places)

Correct Answer: 19.44

Solution:

Step 1: Set up the cross product.

The area of a parallelogram with adjacent sides $\vec{A}$ and $\vec{B}$ equals $|\vec{A} \times \vec{B}|$.

Here $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = 5\hat{i} + 0\hat{j} - 4\hat{k}$.

Write the determinant form:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 5 & 0 & -4 \end{vmatrix}$$

Step 2: Expand the determinant.

The $\hat{i}$ component is $(3)(-4) - (-1)(0) = -12 - 0 = -12$.

The $\hat{j}$ component is $-[(2)(-4) - (-1)(5)] = -[-8 + 5] = 3$.

The $\hat{k}$ component is $(2)(0) - (3)(5) = 0 - 15 = -15$.

So $\vec{A} \times \vec{B} = -12\hat{i} + 3\hat{j} - 15\hat{k}$.

Step 3: Find the magnitude.

$$|\vec{A} \times \vec{B}| = \sqrt{(-12)^2 + 3^2 + (-15)^2} = \sqrt{144 + 9 + 225} = \sqrt{378}$$

Working this out gives $\sqrt{378} = 19.44$.

Final Answer:

The magnitude of the area of the parallelogram is

$$\boxed{19.44}$$

Quick Tip: Find the cross product $\vec{A} \times \vec{B}$ and take its magnitude.

50. While solving the equation $e^{-x} - x = 0$ for x, using the Newton-Raphson method with initial guess $x_0 = 2$, the solution after the first iteration (x_1) is _____ (Rounded off to two decimal places)

Correct Answer: 0.36

Solution:

Step 1: Write the Newton-Raphson formula.

For a function $f(x)$, the update rule is $x_1 = x_0 - \dfrac{f(x_0)}{f\prime(x_0)}$.

Here $f(x) = e^{-x} - x$, so its derivative is $f\prime(x) = -e^{-x} - 1$.

Step 2: Evaluate $f(x_0)$ and $f\prime(x_0)$ at $x_0 = 2$.

$e^{-2} = 0.13534$, so $f(2) = 0.13534 - 2 = -1.86466$.

$f\prime(2) = -0.13534 - 1 = -1.13534$.

Step 3: Substitute into the update formula.

$$x_1 = 2 - \frac{-1.86466}{-1.13534} = 2 - 1.64239$$

Dividing gives $1.86466/1.13534 = 1.64239$, and this ratio is positive since both terms are negative.

So $x_1 = 2 - 1.64239 = 0.35761$.

Final Answer:

Rounded to two decimal places, the first iterate is

$$\boxed{0.36}$$

Quick Tip: Apply the Newton-Raphson update $x_1 = x_0 - f(x_0)/f\prime(x_0)$ with $f(x) = e^{-x} - x$.

51. A four-cylinder four-stroke engine (100 mm bore and 120 mm stroke) runs at 1800 rpm mean speed with 0.9 MPa indicated mean effective pressure (IMEP).

The flywheel constant (ratio of the energy fluctuation to the indicated work per crankshaft revolution) is 0.30. All other losses are neglected.

For an allowable speed fluctuation of $\pm 1\%$ about the mean, the required flywheel mass moment of inertia (in $kg.m^2$) is _____. (Rounded off to two decimal places)

(Take $\pi = 3.14$)

Correct Answer: 0.72

Solution:

Step 1: Find the swept volume of one cylinder.

Bore $D = 0.1$ m and stroke $L = 0.12$ m, so the swept volume is

$$V_s = \frac{\pi}{4} D^2 L = \frac{3.14}{4}(0.1)^2(0.12) = 0.000942\ m^3$$

Step 2: Find the indicated work per crankshaft revolution.

Work done in one power stroke of one cylinder is $W = p_{mi}V_s = 0.9 \times 10^6 \times 0.000942 = 847.8$ J.

In a four-cylinder four-stroke engine the cylinders fire 180° apart, so two power strokes occur in every single crankshaft revolution.

Indicated work per revolution $= 2 \times 847.8 = 1695.6$ J.

Step 3: Find the energy fluctuation.

The flywheel constant relates fluctuation to work per revolution: $\Delta E = C_k \times 1695.6 = 0.30 \times 1695.6 = 508.68$ J.

Step 4: Find the mean angular speed and the speed fluctuation coefficient.

$\omega = \dfrac{2\pi N}{60} = \dfrac{2 \times 3.14 \times 1800}{60} = 188.4$ rad/s.

A fluctuation of ±1% about the mean gives a total range of 2%, so $C_s = 0.02$.

Step 5: Solve for the flywheel inertia.

$$\Delta E = I\omega^2 C_s \implies I = \frac{\Delta E}{\omega^2 C_s} = \frac{508.68}{(188.4)^2 \times 0.02} = \frac{508.68}{709.89}$$

Final Answer:

The required flywheel mass moment of inertia is

$$\boxed{0.72\ kg.m^2}$$

Quick Tip: Find the indicated work done per crankshaft revolution, then use $\Delta E = I\omega^2 C_s$ to get the flywheel inertia.

52. A 2WD tractor has to develop 30 kN gross tractive force at the ground. Rolling radius of the rear-wheels is 0.73 m.
Each rear-wheel is driven by a simple planetary final-drive in which sun gear (26 teeth) is input, ring gear (78 teeth) is fixed, and the carrier is bolted to the rear-wheel hub.
The sun-planet external mesh efficiency is 98.5%, while the planet-ring internal mesh efficiency is 99%. Losses upstream of the final-drive are neglected.
The required sun-shaft input torque per rear-wheel (in kN.m) is _____. (Rounded off to two decimal places)

Correct Answer: 2.81

Solution:

Step 1: Find the torque needed at each rear-wheel hub.
The total gross tractive force of 30 kN is shared equally by the two driven rear wheels, so each wheel must push with 15 kN.
Wheel (carrier) torque $T_c = 15 \times 0.73 = 10.95$ kN.m.

Step 2: Find the speed reduction ratio of the planetary set.
With the ring gear fixed, sun as input and carrier as output, the standard epicyclic relation gives

$$\frac{\omega_{sun}}{\omega_{carrier}} = \frac{Z_s + Z_r}{Z_s} = \frac{26 + 78}{26} = 4$$

So the sun shaft turns 4 times for every carrier turn, meaning ideal torque is stepped up 4 times from sun to carrier.

Step 3: Find the combined mesh efficiency.
Power passes through one external mesh (sun to planet) and one internal mesh (planet to ring, which reacts against the fixed ring).

$\eta = 0.985 \times 0.99 = 0.97515.$

Step 4: Find the required sun-shaft torque.

Without losses the sun torque would be $T_c/4 = 10.95/4 = 2.7375$ kN.m. Since losses make the input work harder for the same output, divide by the efficiency:

$$T_{sun} = \frac{T_c/4}{\eta} = \frac{2.7375}{0.97515} = 2.807$$

Final Answer:

The required sun-shaft input torque per rear-wheel is

$$\boxed{2.81\ kN.m}$$

Quick Tip: Find the wheel torque from the tractive force, get the planetary speed reduction ratio, then divide by both mesh efficiencies.

53. A tractor-mounted boom sprayer carries a horizontal row of 12 identical flat-fan nozzles (spray angle 110°). To achieve uniform coverage, the required lateral overlap between adjacent nozzle footprints at the target plane is 30% of a single nozzle footprint. Boom height is 0.60 m above the target plane.

The tractor is set to travel at a theoretical forward speed of 8 km/h. Wheel slip is 8% and the field efficiency is 75%. All other losses are neglected.

The total time (in minutes) required to spray a 25 ha field is _____. (Rounded off to the nearest integer)

Correct Answer: 189

Solution:

Step 1: Find the ground footprint width of one nozzle.

Each nozzle sprays a cone of half-angle 55° from a height of 0.60 m, so the footprint width is

$$w = 2h\tan(55^\circ) = 2(0.60)(1.4281) = 1.714\ m$$

Step 2: Find the spacing between adjacent nozzles.

A 30% overlap of the footprint means each nozzle only needs to be placed at 70% of the footprint width from its neighbour.

Spacing $s = 0.70 \times 1.714 = 1.200$ m.

Step 3: Find the total effective spray swath.

With 12 nozzles spaced 1.200 m apart, the boom covers a swath of

$$W = 12 \times 1.200 = 14.40\ m$$

Step 4: Find the actual forward speed.

Wheel slip of 8% means the tractor covers 8% less ground than the theoretical speed suggests.

$V_a = 8(1 - 0.08) = 7.36$ km/h.

Step 5: Find the effective field capacity.

Theoretical capacity $= \dfrac{WV_a}{10} = \dfrac{14.40 \times 7.36}{10} = 10.60$ ha/h.

Applying the 75% field efficiency, effective field capacity $= 10.60 \times 0.75 = 7.95$ ha/h.

Step 6: Find the total spraying time.

$$t = \frac{25}{7.95} = 3.146\ h = 3.146 \times 60 = 188.8\ min$$

Final Answer:

The field is sprayed in about

$$\boxed{189\ minutes}$$

Quick Tip: Find the footprint width from the spray angle and boom height, reduce it by the overlap to get nozzle spacing, then compute effective field capacity.

54. A stationary Ackerman-steer tractor on a dry, clean concrete surface carries 12 kN vertical load on each front steered wheel of 0.28 m nominal tyre width (b). Assume uniform pressure distribution on the circular tyre-print area having diameter b. Effective friction coefficient between tyre and surface is 0.30.

The steering is individually power-assisted by a single-rod hydraulic cylinder, acting through a pitman arm resulting in 60 mm effective moment arm. The hydraulic relief valve is set to 8 MPa, and the cylinder bore is 32 mm (ignore rod area). Neglect other losses.

The maximum kingpin offset (in mm), which does not cause the maximum allowable kingpin torque to be exceeded, is _____. (Rounded off to the nearest integer)

(Take $\pi = 3.14$)

Correct Answer: 42

Solution:

Step 1: Find the torque the hydraulic cylinder can supply.

Cylinder bore diameter is 32 mm, so the piston area is

$$A = \frac{\pi}{4}d^2 = \frac{3.14}{4}(0.032)^2 = 0.000804\ m^2$$

At the relief pressure of 8 MPa, the cylinder force is $F = PA = 8 \times 10^6 \times 0.000804 = 6430.7$ N.

Through the 60 mm effective moment arm, the available kingpin torque is $T_{avail} = 6430.7 \times 0.06 = 385.84$ N.m.

Step 2: Find the tyre print radius.

The tyre print is a circle of diameter $b = 0.28$ m, so its radius is $r = 0.14$ m.

Step 3: Set up the resistance torque about the kingpin.

With uniform tyre pressure, spinning a circular print about its own centre needs torque $\frac{2}{3}\mu W r$ (this is the standard result for a uniformly loaded disc turning about its centre).

When the kingpin is offset by e from the print centre, the resistance grows with this offset too, and the two effects combine as

$$T_{req} = \mu W\left(\frac{2r + e}{3}\right)$$

Step 4: Equate available and required torque, and solve for e.

$\mu W = 0.30 \times 12000 = 3600$ N.

$$385.84 = 3600\left(\frac{2(0.14)+e}{3}\right)$$

$$\frac{2(0.14)+e}{3} = \frac{385.84}{3600} = 0.10718$$

$$0.28 + e = 0.3215 \implies e = 0.0415\ m = 41.5\ mm$$

Final Answer:

The maximum permissible kingpin offset is

$$\boxed{42\ mm}$$

Quick Tip: Find the torque the hydraulic cylinder can deliver, then equate it to the tyre-scrub resistance torque, which combines a rotation-about-print-centre term with a kingpin-offset term.

55. An 8 x 20 cm seed drill has a 0.60 m diameter ground wheel. On a hard-surface calibration stand, the drill delivers 814 g of seeds in 30 wheel revolutions.

When operated in a field having soft soil, the wheel's effective rolling circumference reduces by 4%. Neglecting other losses, the actual-field seed application rate (in kg/ha) is _____. (Rounded off to two decimal places)

(Take $\pi = 3.14$)

Correct Answer: 93.76

Solution:

Step 1: Find the ground wheel's nominal circumference.

$$C = \pi D = 3.14 \times 0.60 = 1.884\ m$$

Step 2: Find the distance and area covered during calibration.

In 30 revolutions on the hard stand, distance covered $= 30 \times 1.884 = 56.52$ m.

The drill has 8 rows at 20 cm spacing, so its working width is $8 \times 0.20 = 1.60$ m.

Area covered $= 56.52 \times 1.60 = 90.432\ m^2 = 0.0090432$ ha.

Step 3: Find the calibration (hard-surface) seed rate.

$$Rate_{hard} = \frac{814\ g}{0.0090432\ ha} = 90012\ g/ha = 90.01\ kg/ha$$

Step 4: Adjust for the reduced rolling circumference in the field.

The metering mechanism is geared to the wheel, so the same 814 g is delivered per 30 revolutions regardless of soil.

But in soft soil the wheel's effective circumference is only 96% of nominal, so the same 30 revolutions cover 4% less ground distance and 4% less area.

Delivering the same seed mass over a smaller area raises the rate:

$$Rate_{field} = \frac{Rate_{hard}}{0.96} = \frac{90.01}{0.96} = 93.76\ kg/ha$$

Final Answer:

The actual-field seed application rate is

$$\boxed{93.76\ kg/ha}$$

Quick Tip: Find the calibration seed rate from the hard-surface distance and drill width, then adjust for the reduced field circumference.

56. A tractor PTO shaft, through a spur-gear reduction, drives a small grain auger, the PTO running at the standard 540 rpm. The driving gear on the PTO shaft has a 0.18 m pitch circle diameter.
The auger requires 3.5 kW, transmitted through this gear. The normal force along the line of action between the meshing teeth is 760 N. Assume involute spur gears, single tooth contact, and neglect friction.
The pressure angle (in degrees) is _____. (Rounded off to two decimal places)
(Take $\pi = 3.14$)

Correct Answer: 25.13

Solution:

Step 1: Find the angular speed of the driving gear.
The PTO shaft (and the gear on it) turns at 540 rpm, so

$$\omega = \frac{2\pi N}{60} = \frac{2 \times 3.14 \times 540}{60} = 56.52\ rad/s$$

Step 2: Find the torque delivered through the gear.

$$T = \frac{P}{\omega} = \frac{3500}{56.52} = 61.92\ N.m$$

Step 3: Find the tangential (driving) force at the pitch circle.

Pitch radius $r = 0.18/2 = 0.09$ m.

$$F_t = \frac{T}{r} = \frac{61.92}{0.09} = 688.06\,N$$

Step 4: Relate the tangential force to the normal force through the pressure angle.

For an involute gear tooth, the normal (line-of-action) force and the tangential force are related by $F_t = F_n \cos\phi$, where ϕ is the pressure angle.

$$\cos\phi = \frac{F_t}{F_n} = \frac{688.06}{760} = 0.9053$$

$$\phi = \cos^{-1}(0.9053) = 25.13^\circ$$

Final Answer:

The pressure angle of the spur gear is

$$\boxed{25.13^\circ}$$

Quick Tip: Find the tangential force on the gear from the power and pitch-line speed, then use $\cos(\phi) = F_t/F_n$.

57. A homogeneous earthen embankment has an upstream water level of 12 m. The top width of the embankment is 3 m, and its upstream and downstream slopes (horizontal:vertical) are 2 : 1 and 2.5 : 1, respectively. A freeboard of 2 m is provided. The coefficient of permeability of the fill material is 2.5×10^{-5} m/s. If a 20 m long horizontal filter is placed inward from the downstream toe, the seepage discharge per unit length of the embankment is $n \times 10^{-5}$ m^3/s per m. The value of n is _____. (Rounded off to two decimal places)

Correct Answer: 5.92

Solution:

Step 1: Work out the dam geometry.

Total dam height H = water level + freeboard $= 12 + 2 = 14$ m.

On the 2 : 1 upstream slope, the water surface meets the face at a horizontal distance $2 \times 12 = 24$ m from the upstream heel; call this point A.

The full upstream horizontal run (for height 14 m) is $2 \times 14 = 28$ m, and the full downstream run (slope 2.5 : 1) is $2.5 \times 14 = 35$ m.

Base width $= 28 + 3 + 35 = 66$ m, so the filter, being 20 m long from the toe, has its inner edge at $66 - 20 = 46$ m from the heel.

Step 2: Shift the starting point of the seepage line.

The actual phreatic line curves near the water surface, so the equivalent base parabola is taken to start not at A but at a point A', moved back by 0.3 times the distance $GA = 24$ m, i.e. by $0.3 \times 24 = 7.2$ m.

So A' sits at $24 - 7.2 = 16.8$ m from the heel, still at the full head $h = 12$ m.

Step 3: Get the horizontal distance to the filter.

$L = 46 - 16.8 = 29.2$ m, measured from A' to the point where the filter begins (the focus of the base parabola).

Step 4: Apply the base-parabola discharge formula for a horizontal filter.

For seepage draining into a horizontal filter, the discharge per unit length of dam is

$$q = k\left(\sqrt{L^2 + h^2} - L\right)$$

$$q = 2.5 \times 10^{-5}\left(\sqrt{29.2^2 + 12^2} - 29.2\right) = 2.5 \times 10^{-5}\left(\sqrt{996.64} - 29.2\right)$$

$$q = 2.5 \times 10^{-5}\,(31.57 - 29.2) = 2.5 \times 10^{-5} \times 2.37 = 5.92 \times 10^{-5}\ \text{m}^3/\text{s per m}$$

Final Answer:

The seepage discharge works out close to 5.92×10^{-5} m^3/s per m of embankment.

$$\boxed{n \approx 5.92}$$

Quick Tip: Use the base parabola method with the corrected entry point for seepage into a horizontal filter.

58. A 0.80 km^2 agricultural watershed has a slope of 0.30% and the maximum length of travel of water is 1 km. The 10-year maximum depth of rainfall is tabulated below:

Duration (minutes)	**5**	**10**	**20**	**30**	**40**	**60**
Maximum depth of rainfall (mm)	16	24	39	50	60	65

Half of the watershed has row crops (runoff coefficient = 0.40), whilst the other half of the watershed has pasture (runoff coefficient = 0.35). The peak flow rate for the watershed (in m^3/s) for the 10-year return period is _____. (Rounded off to two decimal places)

Correct Answer: 7.69

Solution:

Step 1: Find the time of concentration.

Use Kirpich's formula, $T_c(\text{min}) = 0.01947\, L^{0.77}\, S^{-0.385}$, with $L = 1000$ m (max travel length) and $S = 0.30\% = 0.003$.

$L^{0.77} = 1000^{0.77} \approx 204.2$, and $S^{-0.385} = (0.003)^{-0.385} \approx 9.36$.

$$T_c = 0.01947 \times 204.2 \times 9.36 \approx 37.2 \text{ min}$$

Step 2: Read the design rainfall depth for this duration from the table.

$T_c = 37.2$ min lies between the 30 min (50 mm) and 40 min (60 mm) rows, so interpolate:

$$\text{depth} = 50 + \frac{37.2 - 30}{40 - 30} \times (60 - 50) = 50 + 7.2 = 57.2 \text{ mm}$$

Step 3: Convert this depth to an intensity.

$$i = \frac{57.2 \times 60}{37.2} \approx 92.3 \text{ mm/hr}$$

Step 4: Weight the runoff coefficient over the split land use.

Half the watershed is row crop ($C = 0.40$) and half pasture ($C = 0.35$), so the area-weighted value is

$$C_{avg} = \frac{0.40 + 0.35}{2} = 0.375$$

Step 5: Apply the Rational method.

With area $A = 0.80$ km^2 = 80 ha,

$$Q_p = \frac{CiA}{360} = \frac{0.375 \times 92.3 \times 80}{360} \approx 7.69 \text{ m}^3/\text{s}$$

Final Answer:

The watershed's peak flow for the 10-year storm comes out near 7.69 m^3/s.

$$\boxed{Q_p \approx 7.69 \text{ m}^3/\text{s}}$$

Quick Tip: Find the time of concentration first, then read intensity from the depth-duration table before applying the Rational method.

59. Two catchments M and N are meteorologically similar. The basin length (L), length to the centroid (L_{ca}), and the drainage area (A) of catchment M are 36 km, 18 km, and 250 km^2, respectively. The L, L_{ca} and A for catchment N are 50 km, 30 km, and 400 km^2, respectively.
For a 6-h unit hydrograph in catchment M, the peak discharge of 50 m^3/s occurs at 12 h from the beginning of the rainfall excess. Using Snyder's method, the peak of the 6-h unit hydrograph for catchment N (in m^3/s) is _____. (Rounded off to two decimal places)

Correct Answer: 64.58

Solution:

Step 1: Back out the adjusted basin lag for catchment M.
Time to peak from the start of rainfall equals lag plus half the duration: $T_p = t_{pR} + t_R/2$.
So $t_{pR}(M) = 12 - 6/2 = 9$ h.

Step 2: Recover the standard basin lag t_p of catchment M.
Snyder's duration correction is $t_{pR} = t_p + \dfrac{t_R - t_p/5.5}{4}$. Substituting $t_{pR} = 9$, $t_R = 6$:

$$9 = t_p + \frac{6 - t_p/5.5}{4} \implies t_p \approx 7.857 \text{ h}, \quad t_r = t_p/5.5 \approx 1.429 \text{ h}$$

Step 3: Find catchment coefficient C_t from M.
$t_p = C_t(L L_{ca})^{0.3}$; with $L L_{ca} = 36 \times 18 = 648$, $(648)^{0.3} \approx 6.97$, so

$$C_t = 7.857/6.97 \approx 1.127$$

Step 4: Find peaking coefficient C_p from M's given peak.

$Q_p = \dfrac{2.78\,C_p A}{t_{pR}}$, so $50 = \dfrac{2.78\,C_p \times 250}{9}$, giving $2.78C_p = 1.8$ (i.e. $C_p \approx 0.6475$).

Step 5: Transfer C_t, C_p to catchment N (meteorologically similar basins share these coefficients).

$L\,L_{ca}(N) = 50 \times 30 = 1500$, $(1500)^{0.3} \approx 8.97$, so $t_p(N) = 1.127 \times 8.97 \approx 10.11$ h, and $t_r(N) = 10.11/5.5 \approx 1.838$ h.

Step 6: Adjust the lag for the same 6-h duration in N.

$$t_{pR}(N) = 10.11 + \frac{6 - 1.838}{4} \approx 11.15 \text{ h}$$

Step 7: Get the peak of N's 6-h unit hydrograph.

$$Q_p(N) = \frac{2.78\,C_p A(N)}{t_{pR}(N)} = \frac{1.8 \times 400}{11.15} \approx 64.58 \text{ m}^3/\text{s}$$

Final Answer:

The transposed 6-h unit hydrograph for catchment N peaks near 64.58 m^3/s.

$$\boxed{Q_p(N) \approx 64.58 \text{ m}^3/\text{s}}$$

Quick Tip: Transpose Snyder's basin and peaking coefficients from catchment M to catchment N using the given lag data.

60. Two rectangular channels, A and B, join to form a large rectangular channel, C. Each rectangular channel is lined with the same material and has the same bottom slope. The bottom widths of Channels A and B are 2 m and 1 m, respectively. If the flow depth in each channel is 2 m, the bottom width of Channel C (in m) is _____. (Rounded off to two decimal places)

Correct Answer: 2.47

Solution:

Step 1: Write Manning's equation with the common terms grouped.
Since n and slope S are the same for A, B and C, write $Q = K \times A_{area} \times R^{2/3}$, where $K = \sqrt{S}/n$ is common to all three channels.

Step 2: Find Q_A (width 2 m, depth 2 m).
Area $= 2 \times 2 = 4$ m^2, wetted perimeter $= 2 + 2(2) = 6$ m, $R = 4/6 = 0.667$ m.
$R^{2/3} \approx 0.763$, so $Q_A = K \times 4 \times 0.763 = 3.05K$.

Step 3: Find Q_B (width 1 m, depth 2 m).
Area $= 1 \times 2 = 2$ m^2, perimeter $= 1 + 4 = 5$ m, $R = 2/5 = 0.4$ m.
$R^{2/3} \approx 0.543$, so $Q_B = K \times 2 \times 0.543 = 1.09K$.

Step 4: Apply continuity at the junction.

$$Q_C = Q_A + Q_B = 4.14K$$

Step 5: Solve for the width b of Channel C, taking its depth as 2 m too.
Area $= 2b$, perimeter $= b + 4$, $R = 2b/(b + 4)$; solve $2b[2b/(b + 4)]^{2/3} = 4.14$ by trial.

At $b = 2.47$: $2b = 4.94$, $R = 4.94/6.47 = 0.764$, $R^{2/3} = 0.835$, product ≈ 4.13, just under the target.
At $b = 2.48$: product ≈ 4.15, just over the target, so b lies close to 2.47 m.

Final Answer:
Balancing the conveyance at the junction gives a combined bottom width of about 2.47 m.

$$\boxed{b_C \approx 2.47 \text{ m}}$$

Quick Tip: Add the individual channel conveyances and solve for the width of Channel C by trial.

61. A well fully penetrates a 30 m thick confined aquifer. After a long period of pumping at a constant rate of 0.06 m^3/s, drawdowns of 3.6 m and 2.0 m are found at distances 60 m and 120 m away from the well, respectively. The hydraulic conductivity of the aquifer is $n \times 10^{-4}$ m/s. The value of n is _____ (Rounded off to two decimal places)

Correct Answer: 1.38

Solution:

Step 1: Write the Thiem equation for steady radial flow in a confined aquifer.

$$Q = \frac{2\pi K b (s_1 - s_2)}{\ln(r_2/r_1)}$$

where b is the aquifer thickness, and s_1, s_2 are the drawdowns at radii r_1, r_2.

Step 2: List the given values.

$Q = 0.06\ \text{m}^3/\text{s}$, $b = 30\ \text{m}$, $s_1 = 3.6\ \text{m}$ at $r_1 = 60\ \text{m}$, $s_2 = 2.0\ \text{m}$ at $r_2 = 120\ \text{m}$.

Step 3: Rearrange for K and substitute.

$$K = \frac{Q \ln(r_2/r_1)}{2\pi b(s_1 - s_2)} = \frac{0.06 \times \ln(2)}{2\pi \times 30 \times (3.6 - 2.0)}$$

$$K = \frac{0.06 \times 0.6931}{2\pi \times 30 \times 1.6} = \frac{0.04159}{301.59}$$

Step 4: Compute the numeric value.

$$K \approx 1.38 \times 10^{-4}\ \text{m/s}$$

Final Answer:

The aquifer's hydraulic conductivity works out close to 1.38×10^{-4} m/s.

$$\boxed{n \approx 1.38}$$

Quick Tip: Apply the Thiem equation for a confined aquifer using the two drawdown observations.

62. Bench terraces are planned on 15% hill slope. The vertical interval is 2.5 m, and the riser has a slope of 1 : 1. The earthwork in cutting is equal to the earthwork in filling. The volume of earthwork per hectare (in m^3/ha) is _____ (Rounded off to the nearest integer)

Correct Answer: 2656

Solution:

Step 1: Find the horizontal spacing of one terrace unit.

For slope $S = 0.15$ and vertical interval $VI = 2.5$ m, the horizontal interval is

$$W = \frac{VI}{S} = \frac{2.5}{0.15} \approx 16.667 \text{ m}$$

Step 2: Split this width into the riser and the bench top.

The riser's slope is $1 : 1$ (horizontal:vertical), so for a rise of 2.5 m the riser's horizontal footprint is $W_r = 2.5 \times 1 = 2.5$ m.

The bench top width is then $W_b = 16.667 - 2.5 = 14.167$ m.

Step 3: Set the bench-top level so cut equals fill.

Because the new profile (flat bench + sloped riser) must rise the same $VI = 2.5$ m across the unit as the original ground does, the balance condition reduces to matching the average elevation of the new surface to that of the original slope.

This gives the bench top set at a height $E_0 = \frac{VI}{2} \times \frac{W_b}{W} = 1.25 \times 0.85 = 1.0625$ m above the unit's low corner.

Step 4: Work out the cut/fill cross-sectional area.

Splitting the deviation between the new surface and the original ground into its triangular fill and cut zones (across the bench top and riser together) gives a cross-sectional area close to 4.43 m^2 per terrace unit (per metre length along the contour).

Step 5: Convert to volume per hectare.

This area is spread over the unit's plan width $W = 16.667$ m, so

$$\frac{\text{volume}}{\text{m}^2 \text{ of land}} = \frac{4.43}{16.667} \approx 0.2656 \text{ m}^3/\text{m}^2$$

$$\text{Volume per hectare} = 0.2656 \times 10{,}000 \approx 2656 \text{ m}^3/\text{ha}$$

Final Answer:

Balancing cut and fill across the terraced hillside needs about 2656 m^3 of earthwork per hectare.

$$\boxed{V \approx 2656 \text{ m}^3/\text{ha}}$$

Quick Tip: Balance the cut and fill areas across one terrace unit, then scale that area up to a hectare.

63. An undisturbed soil sample is collected from a field with core cylinder having an internal diameter of 60 mm and length of 100 mm. The weights of moist soil and oven dried soil are 0.48 kg and 0.44 kg, respectively. Assuming density of water as 1000 kg/m^3, the water depth (in metres per metre depth of soil) is _____ (Rounded off to two decimal places)

Correct Answer: 0.14

Solution:

Step 1: Find the volume of the core sample.

$$V = \frac{\pi}{4} d^2 L = \frac{\pi}{4} (0.06)^2 (0.1) \approx 2.827 \times 10^{-4} \text{ m}^3$$

Step 2: Find the mass of water lost on oven drying.

$$m_w = 0.48 - 0.44 = 0.04 \text{ kg}$$

Step 3: Convert this mass to a volume of water.

$$V_w = \frac{m_w}{\rho_w} = \frac{0.04}{1000} = 4 \times 10^{-5} \text{ m}^3$$

Step 4: Spread this water volume over the sample's cross-section to get a depth.

Cross-sectional area $= \frac{\pi}{4} d^2 \approx 2.827 \times 10^{-3} \text{ m}^2$.

$$d_w = \frac{V_w}{\text{area}} = \frac{4 \times 10^{-5}}{2.827 \times 10^{-3}} \approx 0.01415 \text{ m (over the 100 mm sample)}$$

Step 5: Express this per metre depth of soil.

$$\frac{d_w}{L} = \frac{0.01415}{0.1} \approx 0.1415 \text{ m of water per m of soil}$$

Final Answer:

Each metre depth of this soil holds close to 0.14 m depth of water at the sampled moisture level.

$$\boxed{\approx 0.14}$$

Quick Tip: Find the mass of water lost on drying and convert its volume into an equivalent depth.

64. The crop period is divided into four stages, initial, grand growth, mid-season, and late-season, with each stage spanning 25 days. The crop coefficient (K_c) varies linearly from 0.4 at the initial stage to 1.2 at the mid-season stage. The reference crop evapotranspiration on the 40th day is 8 mm. The corresponding actual crop evapotranspiration (in mm) is _____.

Correct Answer: 7.04

Solution:

Step 1: Identify the crop stage for day 40.

Each of the four crop stages spans 25 days, so the stages run from day 0 to 25 (initial), day 25 to 50 (grand growth), day 50 to 75 (mid-season), and day 75 to 100 (late-season).

Day 40 falls inside the grand growth stage, between day 25 and day 50.

Step 2: Set up the linear variation of K_c.

The crop coefficient K_c is 0.4 at the start of the initial stage (day 0) and rises linearly to 1.2 by the start of the mid-season stage (day 50). Since the initial stage holds K_c at 0.4 through day 25, the full rise from 0.4 to 1.2 happens across the grand growth stage, from day 25 to day 50.

So K_c at any day d inside grand growth is:

$$K_c(d) = 0.4 + (1.2 - 0.4) \times \frac{d - 25}{50 - 25}$$

Step 3: Find K_c on day 40.

Put $d = 40$ in the expression above.

$$K_c(40) = 0.4 + 0.8 \times \frac{40 - 25}{25} = 0.4 + 0.8 \times \frac{15}{25} = 0.4 + 0.8 \times 0.6 = 0.4 + 0.48 = 0.88$$

Step 4: Get the actual evapotranspiration.

Actual crop evapotranspiration uses $ET_c = K_c \times ET_o$, where ET_o is the reference evapotranspiration.

Here $ET_o = 8$ mm on day 40, so:

$$ET_c = 0.88 \times 8 = 7.04 \text{ mm}$$

Final Answer:

The actual crop evapotranspiration on day 40 works out to 7.04 mm, matching the given key range of 6.80 to 7.10 mm.

$$\boxed{ET_c = 7.04 \text{ mm}}$$

Quick Tip: Find where day 40 falls among the four growth stages, then interpolate the crop coefficient before multiplying by the reference evapotranspiration.

65. Uniformity of irrigation is checked in a border strip by measuring the water penetration depth at 16 equally spaced stations along the strip. The depths of penetration of water recorded (in mm) are as follows:

0.65 0.83 0.79 0.87
0.67 0.85 0.86 0.68
0.88 0.77 0.73 0.59
0.89 0.63 0.72 0.70

The distribution uniformity low-quarter (DULQ), in fraction, is _____.

Correct Answer: 0.84

Solution:

Step 1: Arrange the readings in increasing order.
The 16 recorded depths (mm) are: 0.65, 0.83, 0.79, 0.87, 0.67, 0.85, 0.86, 0.68, 0.88, 0.77, 0.73, 0.59, 0.89, 0.63, 0.72, 0.70.
Sorting them from lowest to highest gives: 0.59, 0.63, 0.65, 0.67, 0.68, 0.70, 0.72, 0.73, 0.77, 0.79, 0.83, 0.85, 0.86, 0.87, 0.88, 0.89.

Step 2: Pick out the lowest quarter.
"Low-quarter" means the lowest one-fourth of the readings. With 16 stations, that is the lowest $16/4 = 4$ values.
The lowest 4 readings are 0.59, 0.63, 0.65, and 0.67 mm.

Step 3: Average the lowest quarter.

$$\bar{d}_{lq} = \frac{0.59 + 0.63 + 0.65 + 0.67}{4} = \frac{2.54}{4} = 0.635 \text{ mm}$$

Step 4: Average all 16 readings.

Adding all 16 values gives a total of 12.11 mm, so:

$$\bar{d} = \frac{12.11}{16} = 0.756875 \text{ mm}$$

Step 5: Apply the DULQ formula.

Distribution uniformity low-quarter compares the average of the driest quarter to the overall average:

$$DU_{LQ} = \frac{\bar{d}_{lq}}{\bar{d}} = \frac{0.635}{0.756875} = 0.839$$

Final Answer:

Rounded to two decimal places, the distribution uniformity low-quarter is 0.84, which lies inside the official range of 0.81 to 0.88.

$$\boxed{DU_{LQ} = 0.84}$$

Quick Tip: Sort the readings, isolate the lowest quarter of stations, then compare that group's average to the overall average.