GATE Data Science and Artificial Intelligence

Sample Paper – 10

Duration: 128 Minutes **Maximum Marks: 72**

Instructions

- This paper contains **46 questions** covering the core **Data Science and Artificial Intelligence** subject of the GATE paper conducted by the IITs and IISc (General Aptitude and Engineering Mathematics are provided as separate papers).
- **Q1–Q20 carry 1 mark each** and **Q21–Q46 carry 2 marks each**, for a total of **72 marks**.
- Each question carries **negative marking of one-third of its allotted marks**: a wrong 1-mark question costs **0.33** and a wrong 2-mark question costs **0.67**. Unattempted questions carry no penalty.
- Every question here is a single-correct **MCQ**. The real GATE paper also has Multiple Select (MSQ) and Numerical Answer Type (NAT) questions, and **those carry no negative marking**.
- Attempt this paper in one timed sitting of about **128 minutes**, the share this core subject earns at GATE's overall pace of 180 minutes for 65 questions. All logarithms in information-theoretic quantities (entropy, information gain) are to base 2 and expressed in bits unless stated otherwise.

Probability & Statistics

Q1. Two fair six-sided dice are rolled together. The probability that the sum of the two faces equals 7 is: **[1 mark]**

(A) $\frac{1}{12}$

(B) $\frac{1}{9}$

(C) $\frac{1}{6}$

(D) $\frac{5}{36}$

Q2. A survey of 100 people records their gender and their preferred hot drink, as summarised in the contingency table below. A person is chosen at random. Given that the chosen person is male, the probability that they prefer coffee, $P(\text{coffee} \mid \text{male})$, is: **[1 mark]**

	Coffee	**Tea**
Male	30	20
Female	25	25

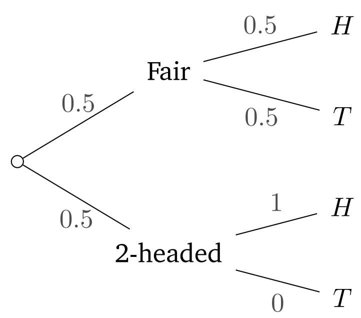
(A) 0.60

(B) 0.55

(C) 0.30

(D) 0.50

Q3. A drawer holds two coins: a fair coin (heads probability 0.5) and a two-headed coin (heads probability 1). One coin is picked at random with equal chance and tossed once, as the probability tree shows. Given that the toss shows heads, the probability that the two-headed coin was picked is: **[1 mark]**



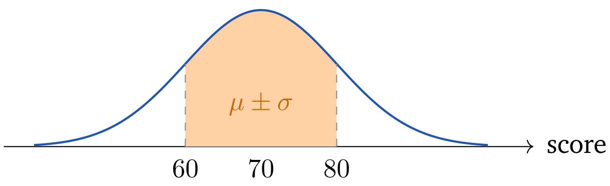
(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{3}{4}$

Q4. Test scores in a large class follow a normal distribution with mean $\mu = 70$ and standard deviation $\sigma = 10$. Using the empirical (68–95–99.7) rule, illustrated by the shaded one-standard-deviation band below, the percentage of scores lying between 60 and 80 is nearest to: **[1 mark]**



(A) 95%

(B) 99.7%

(C) 34%

(D) 68%

Q5. Data packets arrive at a router following a Poisson process at an average rate of 2 packets per millisecond. The probability of receiving *exactly* 1 packet in a given one-millisecond window is nearest to (take $e^{-2} = 0.135$): **[1 mark]**

(A) 0.135

(B) 0.271

(C) 0.406

(D) 0.500

Q6. In a promotional game a player wins Rs. 100 with probability 0.2 and loses Rs. 20 with probability 0.8. The expected gain of the player per game is: **[1 mark]**

(A) Rs. 20

(B) Rs. 80

(C) Rs. 4

(D) Rs. 16

Q7. Consider the data set $\{2, 4, 6, 8, 10\}$. Treating these five numbers as the whole population, the (population) standard deviation is nearest to: **[1 mark]**

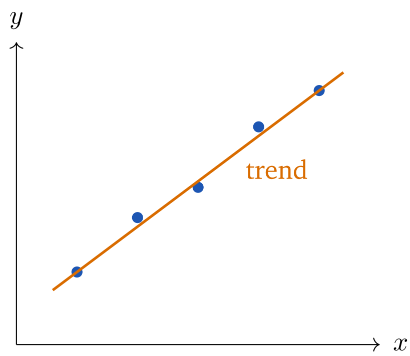
(A) 2.83

(B) 8.00

(C) 6.00

(D) 40.0

Q8. For a bivariate data set the sums of squares and cross-products about the means are $S_{xx} = 25$, $S_{yy} = 100$ and $S_{xy} = 40$; the scatter is shown below. The Pearson correlation coefficient r is: **[1 mark]**



(A) 1.00

(B) 0.80

(C) 0.64

(D) 0.40

Q9. A fair coin is tossed 5 times independently. The probability of obtaining *exactly* 2 heads is: **[1 mark]**

(A) 0.5000

(B) 0.15625

(C) 0.3125

(D) 0.2500

Q10. A random variable X is uniformly distributed on the interval $[0, 10]$. The probability $P(X > 7)$ is: **[1 mark]**

(A) 0.3

(B) 0.5

(C) 0.2

(D) 0.7

Linear Algebra, Calculus & Optimization

Q11. The trace of the matrix $A = \begin{bmatrix} 3 & 1 & 2 \\ 0 & 4 & 5 \\ 1 & 0 & 2 \end{bmatrix}$ is: **[1 mark]**

(A) 5

(B) 9

(C) 6

(D) 10

Q12. The determinant of the matrix $B = \begin{bmatrix} 3 & 4 \\ 2 & 5 \end{bmatrix}$ is: **[1 mark]**

(A) 8

(B) 23

(C) 15

(D) 7

Q13. The *smaller* eigenvalue of the symmetric matrix $C = \begin{bmatrix} 5 & 2 \\ 2 & 5 \end{bmatrix}$ is: **[1 mark]**

(A) 7

(B) 3

(C) 5

(D) 2

Q14. The Frobenius norm of the matrix $M = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, defined as $\|M\|_F = \sqrt{\sum_{i,j} m_{ij}^2}$, is nearest to: **[1 mark]**

(A) 6.00

(B) 3.16

(C) 10.0

(D) 4.24

Q15. For the vector $v = (6, 8)$, the Euclidean (ℓ_2) norm $\|v\|_2$ is: **[1 mark]**

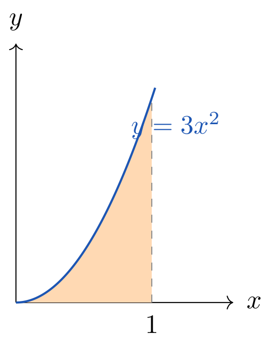
(A) 14

(B) 48

(C) 100

(D) 10

Q16. The value of the definite integral $\int_0^1 3x^2\,dx$, equal to the shaded area under the curve $y = 3x^2$ shown below, is: **[1 mark]**



(A) 3

(B) 2

(C) 1

(D) 0

Q17. For the natural-logarithm function $f(x) = \ln x$, the value of the derivative $f'(x)$ at $x = 2$ is: **[1 mark]**

(A) 2

(B) 1

(C) $\ln 2$

(D) 0.5

Q18. The function $f(x) = x^3 - 3x$ has two stationary points on the real line. The x-coordinate of its *local minimum* is: **[1 mark]**

(A) -1

(B) 0

(C) 1

(D) 3

Q19. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function used as a training objective. Which of the following statements is *always* true for such a function? **[1 mark]**

(A) it may have several local minima with different objective values

(B) its Hessian is negative definite everywhere

(C) every local minimum is also a global minimum

(D) it can have no minimum on a bounded closed set

Programming, Data Structures & Algorithms

Q20. What is the output of the following Python snippet?

```
x = "hello"
print(x[::-1])
```

[1 mark]

(A) `olleh`

(B) `hello`

(C) `hell`

(D) `ello`

Q21. Consider the following Python code:

```
d = {'a': 1, 'b': 2}
d['a'] = 5
print(sum(d.values()))
```

The value printed is: **[2 marks]**

(A) 3

(B) 7

(C) 8

(D) 6

Q22. The following operations are performed on an initially empty *queue* (first-in first-out), in order: `enqueue(5)`, `enqueue(9)`, `dequeue()`, `enqueue(7)`, `dequeue()`. The value returned by the *last* `dequeue()` is: **[2 marks]**

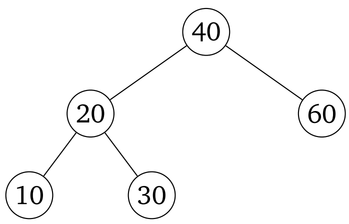
(A) 9

(B) 5

(C) 7

(D) 12

Q23. Starting from an empty binary search tree, the keys $40, 20, 60, 10, 30$ are inserted in the given order, producing the tree shown. Which sequence is the *pre-order* traversal (root, left subtree, right subtree) of this tree? **[2 marks]**



(A) $10, 20, 30, 40, 60$

(B) $40, 20, 10, 30, 60$

(C) $10, 30, 20, 60, 40$

(D) $40, 60, 20, 30, 10$

Q24. A hash table of size 7 uses the hash function $h(k) = k \bmod 7$ with *separate chaining* for collision resolution. The keys $15, 22, 8$ are inserted into an otherwise empty table. The number of keys stored in the chain at index 1 is: **[2 marks]**

(A) 1

(B) 3

(C) 2

(D) 0

Q25. Merge sort is run on an array of 8 elements. The number of levels of splitting, i.e. the depth of the recursion tree from the full array down to single-element sub-arrays, is: [2 marks]

(A) 8

(B) 4

(C) 3

(D) 2

Q26. Which of the following comparison-based sorting algorithms is *stable* and also has a *worst-case* time complexity of $O(n \log n)$? [2 marks]

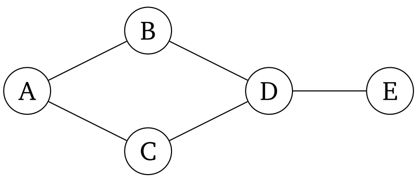
(A) Quicksort

(B) Heapsort

(C) Selection sort

(D) Merge sort

Q27. A depth-first search (DFS) is started from vertex A in the undirected graph shown, always visiting adjacent vertices in increasing alphabetical order. The order in which the vertices are first visited is: [2 marks]



(A) A, B, C, D, E

(B) A, C, B, D, E

(C) A, B, C, E, D

(D) A, B, D, C, E

Q28. The running time of an algorithm satisfies the recurrence $T(n) = 2\,T(n/2)+1$, with $T(1) = 1$. By the Master theorem, $T(n)$ is: **[2 marks]**

(A) $\Theta(n)$

(B) $\Theta(\log n)$

(C) $\Theta(n \log n)$

(D) $\Theta(n^2)$

Database Management & Warehousing

Q29. Relations R and S are union-compatible. R has 6 tuples and S has 4 tuples. The *maximum* possible number of tuples in the set union $R \cup S$ is: **[2 marks]**

(A) 24

(B) 10

(C) 6

(D) 2

Q30. A table `EMP` has a column `dept` with the six values `HR`, `IT`, `IT`, `HR`, `Finance`, `NULL` (in six rows). What does the query `SELECT COUNT(DISTINCT dept) FROM EMP;` return? **[2 marks]**

(A) 6

(B) 5

(C) 3

(D) 4

Q31. A relation $R(A, B, C, D)$ has the functional dependencies $A \to B$, $B \to C$, $C \to D$ and $D \to A$. The number of *candidate keys* of R is: **[2 marks]**

(A) 4

(B) 1

(C) 2

(D) 3

Q32. Relation $R(A, B)$ has 5 tuples and relation $S(B, C)$ has 4 tuples, where B is the only common attribute. Nothing is known about which B-values match. The *minimum* possible number of tuples in the natural join $R \bowtie S$ is: **[2 marks]**

(A) 20

(B) 5

(C) 0

(D) 4

Q33. In a data-warehouse *star schema*, the central table that stores the numeric measures together with foreign keys referencing the surrounding descriptive tables is called the: **[2 marks]**

(A) dimension table

(B) bridge table

(C) lookup table

(D) fact table

Q34. The figure shows the structure of an internal node of a B$^+$ tree of order $m = 4$ (each internal node holds up to m child pointers, drawn as •, and at least $\lceil m/2 \rceil$ of them). The *minimum* number of *keys* such an internal (non-root) node must hold is: **[2 marks]**

•	k_1	•	k_2	•

at least $\lceil 4/2 \rceil = 2$ pointers

(A) 4

(B) 3

(C) 1

(D) 2

Q35. A relation $R(A, B, C, D)$ has the functional dependencies $A \to B$ and $C \to D$. Which of the following attribute sets is a *candidate key* of R? **[2 marks]**

(A) $\{A\}$

(B) $\{C\}$

(C) $\{A, B\}$

(D) $\{A, C\}$

Machine Learning & Artificial Intelligence

Q36. A binary classifier is evaluated on 100 test examples, giving the confusion matrix shown below (TP, FN, FP, TN). The classification *accuracy* is: **[2 marks]**

	Pred +	**Pred −**
Actual +	40	20
Actual −	10	30

(A) 0.80

(B) 0.67

(C) 0.50

(D) 0.70

Q37. L_1 (Lasso) regularization is added to a linear model's training loss. Compared with L_2 (ridge) regularization, a characteristic effect of L_1 on the learned weight vector is that it tends to make the weights: **[2 marks]**

(A) sparse, driving many weights to exactly zero

(B) all large in magnitude

(C) all equal to one another

(D) always negative

Q38. The Manhattan (ℓ_1) distance between the feature vectors $u = (1,\ 2,\ 3)$ and $v = (4,\ 0,\ 3)$ is: **[2 marks]**

(A) 3

(B) 5

(C) 13

(D) 6

Q39. A decision-tree node contains 4 positive and 4 negative examples. Its Gini impurity, defined as $1 - \sum_i p_i^2$ over the class proportions p_i, is: **[2 marks]**

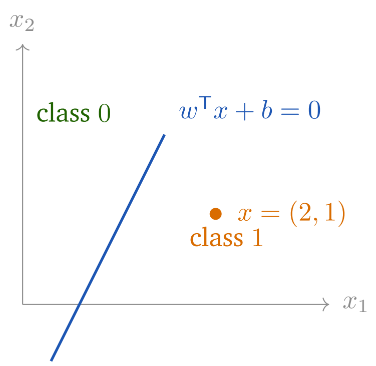
(A) 1.0

(B) 0.25

(C) 0.5

(D) 0.0

Q40. A perceptron with weight vector $w = (2, -1)$ and bias $b = -1$ predicts *class* 1 when $w^\top x + b > 0$ and *class* 0 otherwise, as sketched by the decision boundary below. For the input $x = (2, 1)$, the perceptron outputs: **[2 marks]**



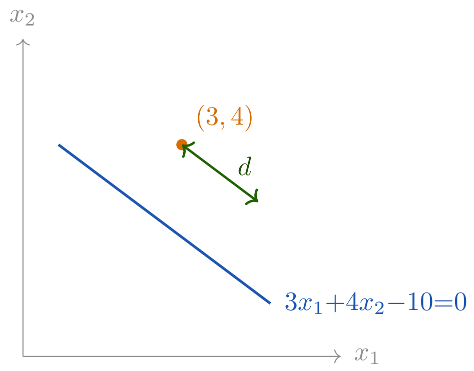
(A) class 1

(B) class 0

(C) undefined

(D) -2

Q41. For a linear classifier the decision boundary is the line $3x_1 + 4x_2 - 10 = 0$, shown below. The perpendicular distance from the point $(3, 4)$ to this line is: **[2 marks]**



(A) 5

(B) 1

(C) 15

(D) 3

Q42. In agglomerative hierarchical clustering, which linkage criterion defines the distance between two clusters as the *minimum* distance between any pair of points, one taken from each cluster? **[2 marks]**

(A) single linkage

(B) complete linkage

(C) average linkage

(D) Ward's linkage

Q43. Principal component analysis is applied to a data set whose covariance matrix has eigenvalues $5, 3, 1, 1$. The *minimum* number of principal components that must be retained to explain at least 80% of the total variance is: **[2 marks]**

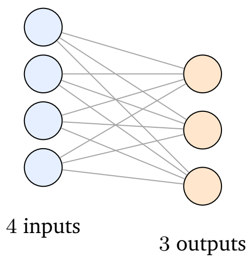
(A) 4

(B) 1

(C) 3

(D) 2

Q44. A single fully connected layer of a neural network maps 4 input units to 3 output units, and each output unit has its own bias, as shown. The total number of trainable parameters (weights plus biases) in this layer is: **[2 marks]**



(A) 12

(B) 15

(C) 7

(D) 3

Q45. To find the path with the fewest edges (the shortest path) between two vertices in an *unweighted* graph, which uninformed search strategy is guaranteed to return an optimal solution? **[2 marks]**

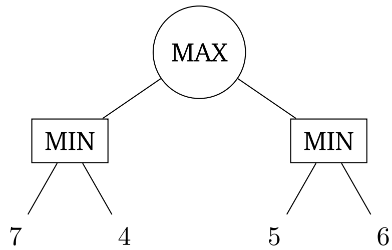
(A) depth-first search

(B) breadth-first search

(C) greedy best-first search

(D) hill climbing

Q46. In the two-ply game tree shown, the root is a MAX node and the second level are MIN nodes; the leaf values are given. The minimax value backed up to the root is: **[2 marks]**



(A) 5

(B) 4

(C) 7

(D) 6

Detailed Solutions

Q1.

Solution

Concept – equally likely outcomes: Probability $= \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$ for two fair dice.

Step 1 – count the total outcomes: Each die has 6 faces, so the total is $6 \times 6 = 36$.

Step 2 – list the outcomes that sum to 7: $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$.

Step 3 – count them: There are 6 favourable outcomes.

Step 4 – form the probability: $P(\text{sum} = 7) = \dfrac{6}{36}$

Step 5 – simplify: $\dfrac{6}{36} = \dfrac{1}{6}$

Final Answer: $P = \dfrac{1}{6} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q1

Q2.

Solution

Concept – conditional probability from a table: $P(\text{coffee} \mid \text{male}) = \dfrac{\text{male \& coffee}}{\text{all males}}$.

Step 1 – read the male row: Male $\cap$ coffee $= 30$, male $\cap$ tea $= 20$.

Step 2 – total number of males: $30 + 20 = 50$

Step 3 – form the conditional probability: $P(\text{coffee} \mid \text{male}) = \dfrac{30}{50}$

Step 4 – simplify: $\dfrac{30}{50} = 0.60$

Final Answer: $P(\text{coffee} \mid \text{male}) = 0.60 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q2

Q3.

Solution

Concept – Bayes' theorem (reverse the tree): $P(2\text{H} \mid H) = \dfrac{P(2\text{H})\,P(H \mid 2\text{H})}{P(\text{fair})\,P(H \mid \text{fair}) + P(2\text{H})\,P(H \mid 2\text{H})}$.

Step 1 – probability along the two-headed branch: $P(2\text{H})\,P(H \mid 2\text{H}) = 0.5 \times 1 =$

0.5

Step 2 – probability along the fair branch: $P(\text{fair})\,P(H \mid \text{fair}) = 0.5 \times 0.5 = 0.25$

Step 3 – total probability of heads: $P(H) = 0.5 + 0.25 = 0.75$

Step 4 – apply Bayes' theorem: $P(2\text{H} \mid H) = \dfrac{0.5}{0.75}$

Step 5 – simplify: $\dfrac{0.5}{0.75} = \dfrac{2}{3}$

Final Answer: $P(2\text{H} \mid H) = \dfrac{2}{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q3

Q4.

Solution

Concept – empirical rule for a normal distribution: About 68% of the data lie within one standard deviation of the mean.

Step 1 – locate the interval relative to the mean: $60 = 70 - 10 = \mu - \sigma$ and $80 = 70 + 10 = \mu + \sigma$.

Step 2 – recognise the band: $(60,\ 80)$ is exactly the interval $(\mu - \sigma,\ \mu + \sigma)$.

Step 3 – apply the empirical rule: The fraction inside one standard deviation is 68%.

Final Answer: about $68\% \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept – Poisson probability: $P(X = k) = \dfrac{e^{-\lambda}\,\lambda^k}{k!}$, with mean rate λ.

Step 1 – list the parameters: $\lambda = 2$ per ms, $k = 1$.

Step 2 – substitute into the formula: $P(X = 1) = \dfrac{e^{-2}\,2^1}{1!}$

Step 3 – evaluate the power and factorial: $2^1 = 2$ and $1! = 1$

$P(X = 1) = e^{-2} \times 2$

Step 4 – substitute $e^{-2} = 0.135$**:** $P(X = 1) = 0.135 \times 2$

Step 5 – multiply: $P(X = 1) = 0.271$

Final Answer: $P(X = 1) \approx 0.271 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q5

Q6.

Solution

Concept – expected value: $E[\text{gain}] = \sum(\text{value}) \times (\text{probability})$.

Step 1 – contribution of a win: $100 \times 0.2 = 20$

Step 2 – contribution of a loss: $(-20) \times 0.8 = -16$

Step 3 – add the contributions: $E[\text{gain}] = 20 + (-16)$

Step 4 – finish: $E[\text{gain}] = 4$

Final Answer: expected gain = Rs. $4 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q6

Q7.

Solution

Concept – population standard deviation: $\sigma = \sqrt{\frac{1}{N}\sum_i (x_i - \bar{x})^2}$.

Step 1 – compute the mean: $\bar{x} = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$

Step 2 – compute the squared deviations: $(2-6)^2 = 16$, $(4-6)^2 = 4$, $(6-6)^2 = 0$, $(8-6)^2 = 4$, $(10-6)^2 = 16$.

Step 3 – sum the squared deviations: $16 + 4 + 0 + 4 + 16 = 40$

Step 4 – divide by $N = 5$ to get the variance: $\sigma^2 = \frac{40}{5} = 8$

Step 5 – take the square root: $\sigma = \sqrt{8} = 2.83$

Final Answer: $\sigma \approx 2.83 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q7

Q8.

Solution

Concept – Pearson correlation coefficient: $r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}}$.

Step 1 – list the sums: $S_{xx} = 25$, $S_{yy} = 100$, $S_{xy} = 40$.

Step 2 – compute the product under the root: $S_{xx} S_{yy} = 25 \times 100 = 2500$

Step 3 – take the square root: $\sqrt{2500} = 50$

Step 4 – form the ratio: $r = \dfrac{40}{50}$

Step 5 – simplify: $r = 0.80$

Final Answer: $r = 0.80 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q8

Q9.

Solution

Concept – binomial probability: $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$.

Step 1 – list the parameters: $n = 5$, $k = 2$, $p = 0.5$, $1 - p = 0.5$.

Step 2 – evaluate the binomial coefficient: $\binom{5}{2} = \dfrac{5 \times 4}{2 \times 1} = 10$

Step 3 – evaluate the probability factor: $p^2(1-p)^3 = 0.5^2 \times 0.5^3 = 0.5^5 = \dfrac{1}{32}$

Step 4 – multiply: $P(X = 2) = 10 \times \dfrac{1}{32} = \dfrac{10}{32}$

Step 5 – convert to a decimal: $\dfrac{10}{32} = 0.3125$

Final Answer: $P(X = 2) = 0.3125 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q9

Q10.

Solution

Concept – uniform distribution probability: For $X \sim \text{Uniform}[a, b]$, $P(X > c) = \dfrac{b-c}{b-a}$.

Step 1 – identify the parameters: $a = 0$, $b = 10$, $c = 7$.

Step 2 – length of the interval above 7: $b - c = 10 - 7 = 3$

Step 3 – total length of the support: $b - a = 10 - 0 = 10$

Step 4 – form the ratio: $P(X > 7) = \dfrac{3}{10}$

Step 5 – convert to a decimal: $\dfrac{3}{10} = 0.3$

Final Answer: $P(X > 7) = 0.3 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q10

Q11.

Solution

Concept – trace of a matrix: The trace is the sum of the entries on the main diagonal.

Step 1 – read the diagonal entries: $a_{11} = 3$, $a_{22} = 4$, $a_{33} = 2$.

Step 2 – add them: $\mathrm{tr}(A) = 3 + 4 + 2$

Step 3 – finish: $\mathrm{tr}(A) = 9$

Final Answer: $\mathrm{tr}(A) = 9 \Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q11**

Q12.

Solution

Concept – determinant of a 2×2 matrix: $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$.

Step 1 – identify the entries: $a = 3$, $b = 4$, $c = 2$, $d = 5$.

Step 2 – compute the main-diagonal product: $ad = 3 \times 5 = 15$

Step 3 – compute the off-diagonal product: $bc = 4 \times 2 = 8$

Step 4 – subtract: $\det B = 15 - 8 = 7$

Final Answer: $\det B = 7 \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q12**

Q13.

Solution

Concept – eigenvalues from trace and determinant: For a 2×2 matrix, $\lambda^2 - \mathrm{tr}(C)\lambda + \det(C) = 0$.

Step 1 – trace and determinant: $\mathrm{tr}(C) = 5+5 = 10$ and $\det(C) = (5)(5)-(2)(2) = 25 - 4 = 21$.

Step 2 – write the characteristic equation: $\lambda^2 - 10\lambda + 21 = 0$

Step 3 – factor: $(\lambda - 7)(\lambda - 3) = 0$

Step 4 – read the roots: $\lambda = 7$ or $\lambda = 3$.

Step 5 – pick the smaller: The smaller eigenvalue is 3.

Final Answer: $\lambda_{\min} = 3 \Rightarrow \boxed{\text{B}}$

$\boxed{\text{Answer: (B)}}$ Go Back to Q13

Q14.

Solution

Concept – Frobenius norm: $\|M\|_F = \sqrt{\sum_{i,j} m_{ij}^2}$, the square root of the sum of squares of all entries.

Step 1 – square each entry: $1^2 = 1$, $2^2 = 4$, $2^2 = 4$, $1^2 = 1$.

Step 2 – add the squares: $1 + 4 + 4 + 1 = 10$

Step 3 – take the square root: $\|M\|_F = \sqrt{10}$

Step 4 – evaluate: $\sqrt{10} = 3.16$

Final Answer: $\|M\|_F \approx 3.16 \Rightarrow \boxed{\text{B}}$

$\boxed{\text{Answer: (B)}}$ Go Back to Q14

Q15.

Solution

Concept – Euclidean norm: $\|v\|_2 = \sqrt{\sum_i v_i^2}$.

Step 1 – square the components: $6^2 = 36$ and $8^2 = 64$.

Step 2 – add them: $36 + 64 = 100$

Step 3 – take the square root: $\|v\|_2 = \sqrt{100} = 10$

Final Answer: $\|v\|_2 = 10 \Rightarrow \boxed{\text{D}}$

$\boxed{\text{Answer: (D)}}$ Go Back to Q15

Q16.

Solution

Concept – definite integral by the power rule: $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1}$, evaluated at the limits.

Step 1 – find the antiderivative: $\displaystyle\int 3x^2\,dx = 3 \cdot \frac{x^3}{3} = x^3$

Step 2 – evaluate at the upper limit $x = 1$**:** $1^3 = 1$

Step 3 – evaluate at the lower limit $x = 0$**:** $0^3 = 0$

Step 4 – subtract: $\displaystyle\int_0^1 3x^2\,dx = 1 - 0 = 1$

Final Answer: area $= 1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q16

Q17.

Solution

Concept – derivative of the natural logarithm: $\dfrac{d}{dx}\ln x = \dfrac{1}{x}$.

Step 1 – write the derivative: $f'(x) = \dfrac{1}{x}$

Step 2 – substitute $x = 2$**:** $f'(2) = \dfrac{1}{2}$

Step 3 – convert to a decimal: $\dfrac{1}{2} = 0.5$

Final Answer: $f'(2) = 0.5 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q17

Q18.

Solution

Concept – local minimum from first and second derivatives: Set $f'(x) = 0$ and keep the root where $f''(x) > 0$.

Step 1 – differentiate: $f'(x) = 3x^2 - 3$

Step 2 – set the derivative to zero: $3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$

Step 3 – second derivative: $f''(x) = 6x$

Step 4 – test each stationary point: $f''(1) = 6 > 0$ (minimum); $f''(-1) = -6 < 0$ (maximum).

Step 5 – select the minimum: The local minimum is at $x = 1$.

Final Answer: $x = 1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q18

Q19.

Solution

Concept – key property of convex functions: A convex function has no "bad" local minima that trap optimization.

Step 1 – recall the definition of convexity: The graph lies below any chord joining two of its points, so it curves upward.

Step 2 – consequence for minima: There is no separate valley of higher value; any point that is locally lowest is lowest overall.

Step 3 – state the guaranteed property: Every local minimum of a convex function is also a global minimum.

Why the other options are wrong: A convex function cannot have local minima at different values; its Hessian is positive (not negative) semi-definite; and a continuous convex function does attain a minimum on a closed bounded set.

Final Answer: every local minimum is global $\Rightarrow \boxed{\text{C}}$

$\boxed{\textbf{Answer: (C)}}$ **Go Back to Q19**

Q20.

Solution

Concept – Python extended slicing: `x[::-1]` steps through the string with step -1, producing the reverse.

Step 1 – read the string: `x` = `"hello"`.

Step 2 – interpret the slice `[::-1]`: Start at the last character and move left one step at a time.

Step 3 – reverse the characters: `o`, `l`, `l`, `e`, `h`.

Step 4 – join them: The result is `"olleh"`.

Final Answer: `olleh` $\Rightarrow \boxed{\text{A}}$

$\boxed{\textbf{Answer: (A)}}$ **Go Back to Q20**

Q21.

Solution

Concept – dictionary update then sum of values: Assigning to an existing key overwrites its value; `sum(d.values())` adds the current values.

Step 1 – initial dictionary: `d` $= \{$`'a'`$:1,$ `'b'`$:2\}$.

Step 2 – apply `d['a'] = 5`**:** Key `'a'` is updated from 1 to 5, giving $\{$`'a'`$:5,$ `'b'`$:2\}$.

Step 3 – collect the values: The values are 5 and 2.

Step 4 – sum them: $5 + 2 = 7$

Final Answer: $7 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q21

Q22.

Solution

Concept – a queue is first-in, first-out (FIFO): Elements leave in the same order they entered; track the front after each step.

Step 1 – `enqueue(5)`, `enqueue(9)`**:** Queue (front→rear): $[5,\ 9]$.

Step 2 – `dequeue()`**:** Removes the front 5; queue becomes $[9]$.

Step 3 – `enqueue(7)`**:** Queue becomes $[9,\ 7]$.

Step 4 – `dequeue()` **(the last one):** Removes the front 9; this is the value returned.

Final Answer: last `dequeue()` returns $9 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q22

Q23.

Solution

Concept – pre-order traversal: Visit the root first, then the left subtree, then the right subtree, recursively.

Step 1 – identify the tree structure: Root 40; left child 20 (with children 10 and 30); right child 60.

Step 2 – visit the root: Output 40.

Step 3 – traverse the left subtree (rooted at 20): Visit 20, then its left child 10, then its right child 30: output $20,\ 10,\ 30$.

Step 4 – traverse the right subtree (rooted at 60): Output 60.

Step 5 – assemble the sequence: $40, 20, 10, 30, 60$.

Final Answer: $40, 20, 10, 30, 60 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q23

Q24.

Solution

Concept – separate chaining stores collisions in a linked list at the same slot: All keys with the same hash value share one bucket.

Step 1 – hash key 15: $15 \bmod 7 = 1$, so 15 goes to index 1.

Step 2 – hash key 22: $22 \bmod 7 = 1$ (since $22 = 3 \times 7 + 1$), so 22 is added to the chain at index 1.

Step 3 – hash key 8: $8 \bmod 7 = 1$, so 8 is also added to the chain at index 1.

Step 4 – count the chain at index 1: Keys $15, 22, 8$ all map to index 1, giving a chain length of 3.

Final Answer: 3 keys in the chain $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q24

Q25.

Solution

Concept – merge sort splits the array in half at each level: The number of split levels for n elements is $\log_2 n$.

Step 1 – start with the full array: Level 0: one block of 8 elements.

Step 2 – first split: Level 1: two blocks of 4.

Step 3 – second split: Level 2: four blocks of 2.

Step 4 – third split: Level 3: eight blocks of 1 (single elements, splitting stops).

Step 5 – count the splitting levels: $8 \to 4 \to 2 \to 1$ needs $\log_2 8 = 3$ splits.

Final Answer: 3 levels $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q25

Q26.

Solution

Concept – stability and worst-case bound of common sorts: A stable sort preserves the relative order of equal keys.

Step 1 – check the worst-case times: Quicksort is $O(n^2)$ worst case; heapsort and merge sort are $O(n \log n)$; selection sort is $O(n^2)$.

Step 2 – check stability: Heapsort is not stable; selection sort (standard form) is not stable; merge sort is stable.

Step 3 – combine both requirements: Only merge sort is both stable and $O(n \log n)$ in the worst case.

Final Answer: Merge sort $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q26

Q27.

Solution

Concept – DFS goes as deep as possible before backtracking: From the current vertex, follow the smallest-labelled unvisited neighbour.

Step 1 – start at A: Visit A; its neighbours are B and C; go to B first.

Step 2 – from B: Visit B; its unvisited neighbour is D; go to D.

Step 3 – from D: Visit D; its unvisited neighbours are C and E; take the smaller, C.

Step 4 – from C: Visit C; its neighbours A and D are already visited, so backtrack to D.

Step 5 – finish from D: The remaining unvisited neighbour of D is E; visit E.

Step 6 – assemble the order: $A,\ B,\ D,\ C,\ E$.

Final Answer: $A,\ B,\ D,\ C,\ E \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q27

Q28.

Solution

Concept – Master theorem: For $T(n) = a\,T(n/b) + f(n)$, compare $f(n)$ with $n^{\log_b a}$.

Step 1 – identify the parameters: $a = 2$, $b = 2$, $f(n) = 1$.

Step 2 – compute the critical exponent: $n^{\log_b a} = n^{\log_2 2} = n^1 = n$

Step 3 – compare $f(n)$ with $n^{\log_b a}$: $f(n) = 1 = O(n^{1-\varepsilon})$ grows slower than n, which is Master-theorem *Case 1*.

Step 4 – apply Case 1: $T(n) = \Theta\big(n^{\log_b a}\big) = \Theta(n)$

Why the other options are wrong: $\Theta(n \log n)$ would arise if $f(n) = \Theta(n)$; here the extra work per level is only a constant.

Final Answer: $T(n) = \Theta(n) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q28

Q29.

Solution

Concept – size of a set union: $|R \cup S| = |R| + |S| - |R \cap S|$; the union is largest when the overlap is empty.

Step 1 – write the union formula: $|R \cup S| = |R| + |S| - |R \cap S|$

Step 2 – maximise by taking no common tuples: The union is largest when $|R \cap S| = 0$ (the relations are disjoint).

Step 3 – substitute the counts: $|R \cup S|_{\text{max}} = 6 + 4 - 0$

Step 4 – add: $|R \cup S|_{\text{max}} = 10$

Final Answer: at most 10 tuples $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q29

Q30.

Solution

Concept – `COUNT(DISTINCT column)` counts distinct non-null values: NULLs are ignored, and repeated values are counted once.

Step 1 – list the values: `HR`, `IT`, `IT`, `HR`, `Finance`, `NULL`.

Step 2 – drop the NULL: The NULL is not counted, leaving `HR`, `IT`, `IT`, `HR`, `Finance`.

Step 3 – keep only distinct values: The distinct departments are `HR`, `IT`, `Finance`.

Step 4 – count them: There are 3 distinct non-null departments.

Final Answer: `COUNT(DISTINCT dept)` returns 3 $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q30

Q31.

Solution

Concept – candidate keys from a cycle of FDs: If each attribute determines the next in a closed cycle, each single attribute is a candidate key.

Step 1 – follow the closure of A: $A \to B \to C \to D \to A$, so $A^+ = \{A, B, C, D\}$; A is a key.

Step 2 – follow the closure of B: $B \to C \to D \to A \to B$, so $B^+ = \{A, B, C, D\}$; B is a key.

Step 3 – follow the closures of C and D: By the same cycle, $C^+ = \{A, B, C, D\}$ and $D^+ = \{A, B, C, D\}$; both are keys.

Step 4 – count the single-attribute candidate keys: A, B, C, D are each minimal and each a key, giving 4 candidate keys.

Final Answer: 4 candidate keys $\Rightarrow$ $\boxed{\text{A}}$

Answer: (A) Go Back to Q31

Q32.

Solution

Concept – natural join keeps only matching tuples: A tuple of R contributes only if some tuple of S shares its B-value.

Step 1 – recall when the join is empty: If no B-value of R appears in S, then no pair matches.

Step 2 – construct such a case: Choose the B-values in R and in S to be completely disjoint.

Step 3 – count the resulting tuples: With no matching B-values, the natural join has 0 tuples.

Step 4 – confirm it is the minimum: A count cannot be negative, so 0 is the least possible.

Final Answer: minimum $= 0$ tuples $\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) Go Back to Q32

Q33.

Solution

Concept – components of a star schema: A star schema has one central fact table linked to several dimension tables.

Step 1 – identify what stores the measures: Numeric measures (facts) live in the central table.

Step 2 – identify what stores the foreign keys: The central table also holds foreign keys to each dimension table.

Step 3 – name this table: A central table of measures plus dimension foreign keys is the **fact table**.

Why the other options are wrong: Dimension tables store descriptive attributes, not measures; bridge and lookup tables are auxiliary and are not the central measure store.

Final Answer: fact table $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q33

Q34.

Solution

Concept – minimum occupancy of a B$^+$ tree internal node: A non-root internal node of order m has at least $\lceil m/2 \rceil$ child pointers, and keys are one fewer than pointers.

Step 1 – minimum number of pointers: $\lceil m/2 \rceil = \lceil 4/2 \rceil = 2$.

Step 2 – relate keys to pointers: Number of keys $=$ (number of pointers) -1.

Step 3 – compute the minimum keys: $2 - 1 = 1$

Final Answer: minimum 1 key $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q34

Q35.

Solution

Concept – a candidate key's closure must be all attributes, and it must be minimal: Compute the attribute closure of each candidate set.

Step 1 – closure of $\{A\}$: $A \to B$ gives $A^+ = \{A, B\}$; this misses C and D, so A is not a key.

Step 2 – closure of $\{C\}$: $C \to D$ gives $C^+ = \{C, D\}$; this misses A and B, so C is not a key.

Step 3 – closure of $\{A, C\}$: Start with $\{A, C\}$; apply $A \to B$ to add B and $C \to D$ to add D, giving $\{A, B, C, D\}$ (all attributes).

Step 4 – check minimality of $\{A, C\}$: Neither A alone nor C alone determines all attributes (Steps 1–2), so $\{A, C\}$ is minimal.

Step 5 – conclude: $\{A, C\}$ determines every attribute and is minimal, so it is a candidate key.

Final Answer: $\{A, C\} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q35

Q36.

Solution

Concept – accuracy from a confusion matrix: accuracy $= \dfrac{\text{TP} + \text{TN}}{\text{TP} + \text{TN} + \text{FP} + \text{FN}}$.

Step 1 – read the four counts: $\text{TP} = 40$, $\text{FN} = 20$, $\text{FP} = 10$, $\text{TN} = 30$.

Step 2 – add the correct predictions: $\text{TP} + \text{TN} = 40 + 30 = 70$

Step 3 – total number of examples: $40 + 20 + 10 + 30 = 100$

Step 4 – form the ratio: accuracy $= \dfrac{70}{100}$

Step 5 – convert to a decimal: $\dfrac{70}{100} = 0.70$

Final Answer: accuracy $= 0.70 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q36

Q37.

Solution

Concept – effect of L_1 regularization: The L_1 penalty $\lambda \sum_i |w_i|$ has corners on the axes, which pushes weights exactly to zero.

Step 1 – shape of the penalty: $|w|$ is non-differentiable at zero, so the optimum often sits exactly at $w_i = 0$.

Step 2 – contrast with L_2: The L_2 penalty $\lambda \sum_i w_i^2$ only shrinks weights smoothly toward zero without setting them to zero.

Step 3 – state the characteristic effect: L_1 yields a *sparse* weight vector with many exactly-zero entries (implicit feature selection).

Final Answer: sparse weights, many exactly zero $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q37

Q38.

Solution

Concept – Manhattan (ℓ_1) distance: $d_1(u, v) = \sum_i |u_i - v_i|$.

Step 1 – component-wise differences: $|1 - 4| = 3$, $|2 - 0| = 2$, $|3 - 3| = 0$.

Step 2 – add the absolute differences: $3 + 2 + 0$

Step 3 – finish: $= 5$

Final Answer: $d_1(u, v) = 5 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q38

Q39.

Solution

Concept – Gini impurity: $G = 1 - \sum_i p_i^2$, where p_i is the proportion of class i.

Step 1 – compute the class proportions: $p_+ = \dfrac{4}{8} = 0.5$ and $p_- = \dfrac{4}{8} = 0.5$.

Step 2 – square the proportions: $0.5^2 = 0.25$ and $0.5^2 = 0.25$.

Step 3 – sum the squares: $0.25 + 0.25 = 0.5$

Step 4 – subtract from 1: $G = 1 - 0.5 = 0.5$

Final Answer: $G = 0.5 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q39

Q40.

Solution

Concept – perceptron decision rule: Compute $z = w^\top x + b$; output class 1 if $z > 0$, else class 0.

Step 1 – compute the weighted sum: $w^\top x = (2)(2) + (-1)(1) = 4 - 1 = 3$

Step 2 – add the bias: $z = 3 + (-1) = 2$

Step 3 – apply the threshold: $z = 2 > 0$, so the output is class 1.

Final Answer: class $1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q40

Q41.

Solution

Concept – distance from a point to a line: $d = \dfrac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$ for the line $ax + by + c = 0$.

Step 1 – identify the coefficients and point: $a = 3$, $b = 4$, $c = -10$; point $(x_0, y_0) = (3, 4)$.

Step 2 – evaluate the numerator: $|3(3) + 4(4) - 10| = |9 + 16 - 10| = |15| = 15$

Step 3 – evaluate the denominator: $\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Step 4 – form the ratio: $d = \dfrac{15}{5}$

Step 5 – divide: $d = 3$

Final Answer: $d = 3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q41

Q42.

Solution

Concept – linkage criteria in hierarchical clustering: Each linkage defines the between-cluster distance differently.

Step 1 – recall single linkage: Single linkage uses the *minimum* distance between any two points, one from each cluster (nearest-neighbour).

Step 2 – contrast the others: Complete linkage uses the maximum pairwise distance; average linkage uses the mean; Ward's minimises the increase in within-cluster variance.

Step 3 – match the description: "Minimum distance between any pair" is single linkage.

Final Answer: single linkage $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q42

Q43.

Solution

Concept – variance explained in PCA: Each component explains a fraction of variance equal to its eigenvalue over the total; add components until the target is met.

Step 1 – total variance: $5 + 3 + 1 + 1 = 10$

Step 2 – fraction from the first component: $\frac{5}{10} = 0.50 = 50\%$, which is below 80%.

Step 3 – fraction from the first two components: $\frac{5+3}{10} = \frac{8}{10} = 0.80 = 80\%$, which meets the target.

Step 4 – read off the minimum count: Two components are needed to reach at least 80%.

Final Answer: 2 components $\Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q43**

Q44.

Solution

Concept – parameters of a fully connected layer: A layer from n inputs to m outputs has $n \times m$ weights plus m biases.

Step 1 – count the weights: $n \times m = 4 \times 3 = 12$

Step 2 – count the biases: One per output unit: $m = 3$.

Step 3 – add weights and biases: $12 + 3 = 15$

Final Answer: 15 parameters $\Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q44**

Q45.

Solution

Concept – optimality of uninformed search: On an unweighted graph, every edge has equal cost, so the fewest-edge path is the cheapest path.

Step 1 – recall how BFS expands: Breadth-first search explores all vertices at distance 1, then distance 2, and so on.

Step 2 – link to optimality: The first time BFS reaches a vertex, it does so along a path with the fewest edges.

Step 3 – rule out the others: DFS may find a longer path first; greedy best-first and hill climbing follow heuristics and are not guaranteed optimal.

Final Answer: breadth-first search $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q45

Q46.

Solution

Concept – minimax back-up: MIN nodes take the minimum of their children; the MAX root takes the maximum of the MIN results.

Step 1 – evaluate the left MIN node: $\min(7,\ 4) = 4$

Step 2 – evaluate the right MIN node: $\min(5,\ 6) = 5$

Step 3 – evaluate the MAX root: $\max(4,\ 5) = 5$

Final Answer: minimax value $= 5 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q46

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	A	4	D	5	B
6	C	7	A	8	B	9	C	10	A
11	B	12	D	13	B	14	B	15	D
16	C	17	D	18	C	19	C	20	A
21	B	22	A	23	B	24	B	25	C
26	D	27	D	28	A	29	B	30	C
31	A	32	C	33	D	34	C	35	D
36	D	37	A	38	B	39	C	40	A
41	D	42	A	43	D	44	B	45	B
46	A								