GATE Data Science and Artificial Intelligence

Sample Paper – 3

Duration: 128 Minutes **Maximum Marks: 72**

Instructions

- This paper contains **46 questions** covering the core **Data Science and Artificial Intelligence** subject of the GATE paper conducted by the IITs and IISc (General Aptitude and Engineering Mathematics are provided as separate papers).
- **Q1–Q20 carry 1 mark each** and **Q21–Q46 carry 2 marks each**, for a total of **72 marks**.
- Each question carries **negative marking of one-third of its allotted marks**: a wrong 1-mark question costs **0.33** and a wrong 2-mark question costs **0.67**. Unattempted questions carry no penalty.
- Every question here is a single-correct **MCQ**. The real GATE paper also has Multiple Select (MSQ) and Numerical Answer Type (NAT) questions, and **those carry no negative marking**.
- Attempt this paper in one timed sitting of about **128 minutes**, the share this core subject earns at GATE's overall pace of 180 minutes for 65 questions. All logarithms in information-theoretic quantities (entropy, information gain) are to base 2 and expressed in bits unless stated otherwise.

Probability & Statistics

Q1. A bag contains 3 red and 2 blue balls. Two balls are drawn one after another *without* replacement. The probability that *both* balls drawn are red is: **[1 mark]**

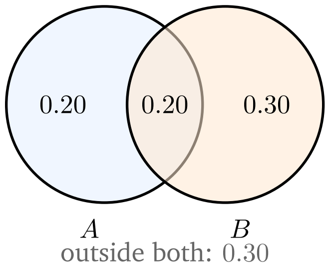
(A) 0.3

(B) 0.4

(C) 0.6

(D) 0.5

Q2. For two events A and B the probabilities are shown on the Venn diagram below (the region outside both circles has probability 0.30). The conditional probability $P(B \mid A)$ is: **[1 mark]**



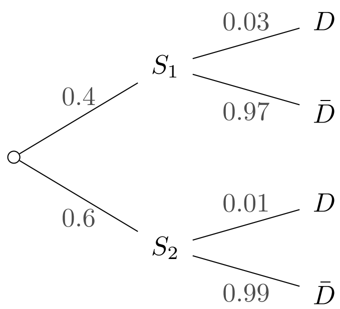
(A) 0.40

(B) 0.20

(C) 0.50

(D) 0.667

Q3. A store receives the same component from two suppliers. Supplier S_1 provides 40% of the stock with a defect rate of 3%; supplier S_2 provides the remaining 60% with a defect rate of 1%. The probability tree is shown. A component picked at random is found to be defective. The probability that it came from S_2 is: **[1 mark]**



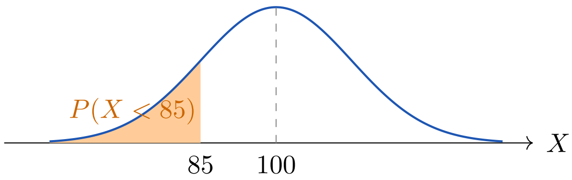
(A) 0.667

(B) 0.012

(C) 0.400

(D) 0.333

Q4. A feature X is normally distributed with mean $\mu = 100$ and standard deviation $\sigma = 15$. Using the shaded left tail of the standard normal curve shown, the probability $P(X < 85)$ is nearest to: **[1 mark]**



(A) 0.1587

(B) 0.8413

(C) 0.3413

(D) 0.5000

Q5. Defects appear on a production line following a Poisson process at an average rate of 2 defects per hour. The probability of observing *exactly* 3 defects in a given one-hour window is nearest to (take $e^{-2} = 0.1353$): **[1 mark]**

(A) 0.271

(B) 0.180

(C) 0.135

(D) 0.090

Q6. A discrete random variable X takes the values $1, 2, 3$ with probabilities $0.5, 0.3, 0.2$ respectively. The expected value $E[X]$ is: **[1 mark]**

(A) 1.5

(B) 1.7

(C) 2.0

(D) 0.61

Q7. A data set consists of the five values $2, 4, 6, 8, 10$. Treating these as the whole population, the (population) variance of the data set is: **[1 mark]**

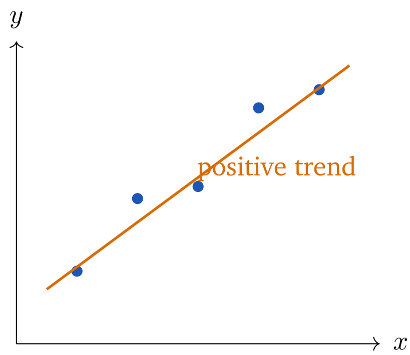
(A) 6

(B) 10

(C) 5.66

(D) 8

Q8. For a bivariate data set the sums of squares and cross-products about the means are $S_{xx} = 25$, $S_{yy} = 100$ and $S_{xy} = 40$, with the scatter shown below. The Pearson correlation coefficient r between x and y is: **[1 mark]**



(A) 1.6

(B) 0.4

(C) 0.64

(D) 0.8

Q9. A fair coin (probability 0.5 of heads) is tossed 5 times independently. The probability of obtaining *exactly* 2 heads is: **[1 mark]**

(A) 0.5000

(B) 0.15625

(C) 0.3125

(D) 0.2500

Q10. The lifetime of a component follows an exponential distribution with a mean of 4 hours. The probability that the component fails *within* the first 4 hours, i.e. $P(X < 4)$, is (take $e^{-1} = 0.3679$): **[1 mark]**

(A) 0.3679

(B) 0.1353

(C) 0.5000

(D) 0.6321

Linear Algebra, Calculus & Optimization

Q11. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 5 \end{bmatrix}$. The rank of A is: **[1 mark]**

(A) 1

(B) 2

(C) 3

(D) 0

Q12. The determinant of the matrix $B = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 3 \end{bmatrix}$ is: **[1 mark]**

(A) 18

(B) 15

(C) 12

(D) 9

Q13. The larger eigenvalue of the matrix $C = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$ is: **[1 mark]**

(A) 1

(B) 5

(C) 2

(D) 6

Q14. For a real matrix A, the matrix $A^{\top}A$ has eigenvalues 16 and 4. The *smallest* singular value of A is: **[1 mark]**

(A) 2

(B) 4

(C) 16

(D) 8

Q15. For the vector $v = (2, -3, 6)$, the Euclidean (ℓ_2) norm $\|v\|_2$ is: **[1 mark]**

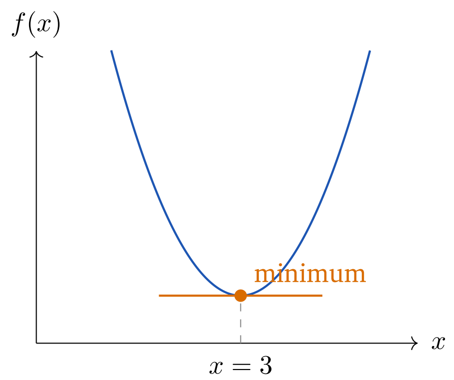
(A) 11

(B) 6

(C) 49

(D) 7

Q16. The function $f(x) = x^2 - 6x + 11$ is minimized over the real line, as suggested by its convex graph below. The minimum *value* of f is: **[1 mark]**



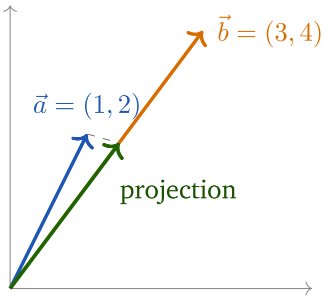
(A) 2

(B) 11

(C) 3

(D) 0

Q17. The scalar projection (component) of $\vec{a} = (1, 2)$ onto $\vec{b} = (3, 4)$, as shown, is: **[1 mark]**



(A) 11

(B) 5

(C) 2.2

(D) 1.1

Q18. Gradient descent is used to minimize $f(x) = x^2 + 2x$ with a fixed learning rate $\eta = 0.2$. Starting from $x_0 = 3$, the value of x_1 after one update $x_1 = x_0 - \eta f'(x_0)$ is: **[1 mark]**

(A) 2.4

(B) 1.4

(C) 3.6

(D) 0.6

Q19. Consider the function $f(x, y) = x^2 + y^2 + 3xy$ on $\mathbb{R}^2$. Based on its Hessian, the function is: **[1 mark]**

(A) neither convex nor concave

(B) convex everywhere

(C) concave everywhere

(D) linear (affine)

Programming, Data Structures & Algorithms

Q20. What is the output of the following Python snippet?

```
a = [1, 2, 3, 4, 5]
print(a[::2])
```

[1 mark]

(A) `[2, 4]`

(B) `[1, 2]`

(C) `[1, 2, 3, 4, 5]`

(D) `[1, 3, 5]`

Q21. Consider the following Python code:

```
s = [i*i for i in range(4)]
print(s[2])
```

The value printed is: **[2 marks]**

(A) 4

(B) 9

(C) 2

(D) 1

Q22. The following operations are performed on an initially empty *queue*, in order: `enqueue(5)`, `enqueue(10)`, `dequeue()`, `enqueue(15)`, `dequeue()`. The value returned by the *last* `dequeue()` is: **[2 marks]**

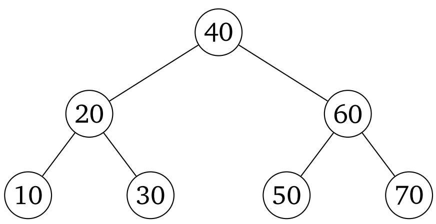
(A) 10

(B) 5

(C) 15

(D) 20

Q23. Starting from an empty binary search tree, the keys $40, 20, 60, 10, 30, 50, 70$ are inserted in the given order, producing the tree shown. The *height* of the resulting tree (the number of edges on the longest root-to-leaf path) is: **[2 marks]**



(A) 3

(B) 7

(C) 2

(D) 1

Q24. A hash table of size 7 uses the hash function $h(k) = k \bmod 7$ with *linear probing* for collision resolution. The keys 8, 15, 22 are inserted in this order into an otherwise empty table. The final index occupied by key 22 is: **[2 marks]**

(A) 1

(B) 2

(C) 22

(D) 3

Q25. Binary search is performed on a sorted array of 255 distinct elements. The maximum number of key comparisons required in the worst case is: **[2 marks]**

(A) 255

(B) 8

(C) 128

(D) 16

Q26. The worst-case time complexity of the standard Merge Sort algorithm on an array of n elements is: **[2 marks]**

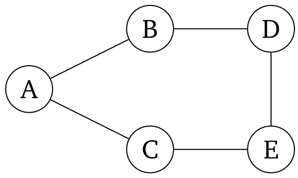
(A) $O(n^2)$

(B) $O(n \log n)$

(C) $O(n)$

(D) $O(\log n)$

Q27. A depth-first search (DFS) is started from vertex A in the undirected graph shown, always visiting adjacent vertices in increasing alphabetical order. The order in which the vertices are first visited is: **[2 marks]**



(A) A, B, C, D, E

(B) A, C, E, D, B

(C) A, B, D, E, C

(D) A, B, C, E, D

Q28. The running time of an algorithm satisfies the recurrence $T(n) = 4T(n/2) + n$, with $T(1) = 1$. By the Master theorem, $T(n)$ is: **[2 marks]**

(A) $\Theta(n^2)$

(B) $\Theta(n \log n)$

(C) $\Theta(n)$

(D) $\Theta(n^2 \log n)$

Database Management & Warehousing

Q29. Relation R has 3 attributes and 6 tuples; relation S has 4 attributes and 5 tuples. The *degree* (number of attributes) of the Cartesian product $R \times S$ is: **[2 marks]**

(A) 30

(B) 11

(C) 7

(D) 9

Q30. A table `EMP` has 8 rows. The `dept` column holds the values `'A'` in 3 rows, `'B'` in 3 rows and `'C'` in 2 rows. What does the query `SELECT COUNT(DISTINCT dept) FROM EMP;` return? **[2 marks]**

(A) 8

(B) 2

(C) 6

(D) 3

Q31. A relation $R(A, B, C)$ has the functional dependencies $AB \to C$ and $C \to B$. The candidate keys are $\{A, B\}$ and $\{A, C\}$. The highest normal form that R satisfies is: **[2 marks]**

(A) BCNF

(B) 2NF

(C) 3NF

(D) 1NF

Q32. Relation $R(A, B)$ contains the tuples $\{(1, x), (2, x), (3, y)\}$ and relation $S(B, C)$ contains $\{(x, p), (x, q), (z, r)\}$. The number of tuples in the natural join $R \bowtie S$ (on the common attribute B) is: **[2 marks]**

(A) 3

(B) 4

(C) 6

(D) 9

Q33. In OLAP, the operation that navigates from a detailed (finer) level of a data cube to a summarized (coarser) level, such as from monthly totals to yearly totals, is called: **[2 marks]**

(A) roll-up

(B) drill-down

(C) slice

(D) pivot

Q34. The figure shows an internal node of a B$^+$ tree of order $m = 6$ (each internal node holds up to m child pointers, drawn as •, separated by keys).

For a *non-root* internal node, the *minimum* number of child pointers it must hold is: **[2 marks]**

•	k_1	•	k_2	•	$\cdots$

order $m = 6$: up to 6 pointers (•), 5 keys

(A) 6

(B) 5

(C) 3

(D) 2

Q35. A relation has attributes $\{A, B, C, D, E\}$ with functional dependencies $A \to B$, $B \to C$ and $D \to E$. The attribute closure A^+ is: **[2 marks]**

(A) $\{A, B, C\}$

(B) $\{A, B, C, D, E\}$

(C) $\{A\}$

(D) $\{A, B\}$

Machine Learning & Artificial Intelligence

Q36. A logistic regression model has weight vector $w = (2, -1)$ and bias $b = 0$, and uses the sigmoid $\sigma(z) = 1/(1 + e^{-z})$. For the input $x = (1, 1)$, the predicted probability $\sigma(w^{\mathsf{T}}x + b)$ is nearest to (take $e^{-1} = 0.3679$): **[2 marks]**

(A) 0.500

(B) 0.731

(C) 0.269

(D) 0.880

Q37. A very simple model gives poor accuracy on *both* the training set and the test set, and adding more training data barely helps. In terms of the bias–variance decomposition, this behaviour is best described as: **[2 marks]**

(A) high bias, low variance

(B) low bias, high variance

(C) low bias, low variance

(D) high bias, high variance

Q38. A 1-nearest-neighbour classifier (Euclidean distance) is trained on the labelled points $A(1,1)$: Class 0, $B(4,4)$: Class 1, $C(1,2)$: Class 0, $D(5,5)$: Class 1. The query point $Q(2,2)$ is classified as: **[2 marks]**

(A) Class 1

(B) a tie (undecided)

(C) Class 0

(D) cannot be determined

Q39. A training set for a binary classification task contains 10 examples, of which 5 are positive and 5 are negative. The entropy of this set (in bits) is: **[2 marks]**

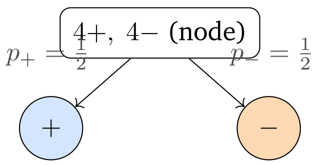
(A) 0.811

(B) 1.000

(C) 0.500

(D) 0.000

Q40. A node in a decision tree contains 4 positive and 4 negative examples, as shown. The Gini impurity of this node is: **[2 marks]**



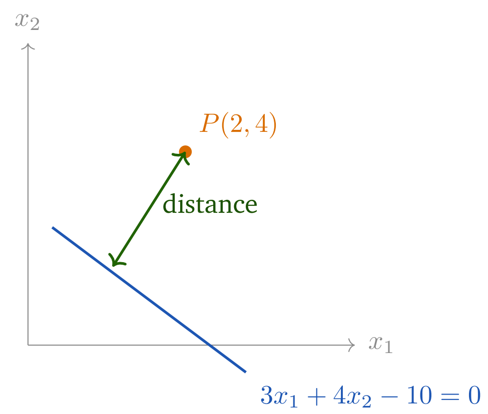
(A) 0.25

(B) 0.811

(C) 0.50

(D) 1.00

Q41. A linear support vector machine has the decision hyperplane $3x_1 + 4x_2 - 10 = 0$, drawn below. The perpendicular distance from the data point $P(2,\ 4)$ to this hyperplane is: **[2 marks]**



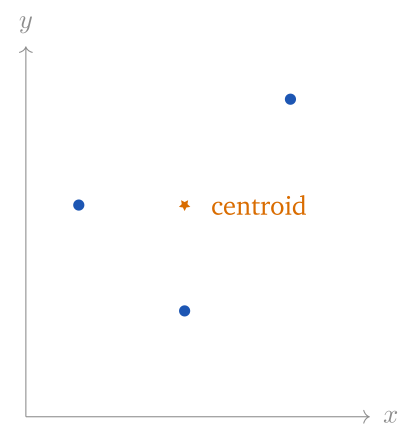
(A) 12

(B) 2.4

(C) 5

(D) 1.2

Q42. In one iteration of the k-means algorithm, a cluster is assigned the three points $(1,4)$, $(3,2)$ and $(5,6)$, shown below. The updated centroid of this cluster is: **[2 marks]**



(A) $(3,\ 3)$

(B) $(9,\ 12)$

(C) $(3,\ 4)$

(D) $(2,\ 4)$

Q43. Principal component analysis is applied to a data set whose covariance matrix has eigenvalues $5, 3, 2$. The fraction of the total variance explained by the *first* principal component alone is: **[2 marks]**

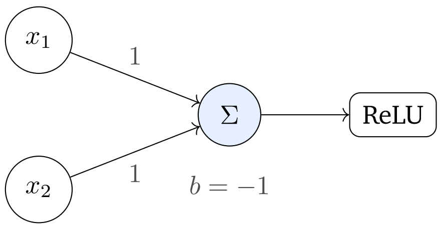
(A) 50%

(B) 80%

(C) 30%

(D) 20%

Q44. A single neuron with a ReLU activation computes ReLU$(w_1x_1 + w_2x_2 + b)$, as shown, where ReLU$(z) = \max(0, z)$. With inputs $x_1 = 2$, $x_2 = 1$, weights $w_1 = 1$, $w_2 = 1$ and bias $b = -1$, the output of the neuron is: **[2 marks]**



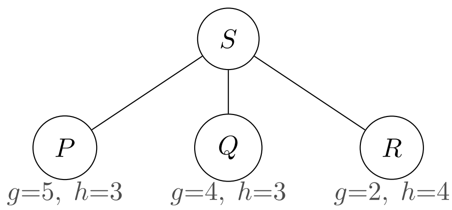
(A) 0

(B) 2

(C) 0.5

(D) 1

Q45. A* search maintains three frontier nodes P, Q and R with g (path cost so far) and h (heuristic) values shown in the tree below, where $f = g + h$. The node A* selects for expansion next is: **[2 marks]**



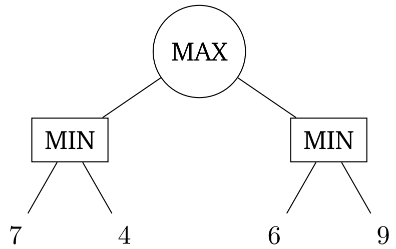
(A) P

(B) Q

(C) cannot be determined

(D) R

Q46. In the two-ply game tree shown, the root is a MAX node and the second level are MIN nodes; the leaf values are given. The minimax value backed up to the root is: **[2 marks]**



(A) 4

(B) 7

(C) 9

(D) 6

Detailed Solutions

Q1.

Solution

Concept – probability of a sequence without replacement: Multiply the probability of the first red by the probability of a second red given the first was red.

Step 1 – probability the first ball is red: There are 3 red out of 5 balls, so $P(\text{1st red}) = \frac{3}{5}$.

Step 2 – probability the second ball is red given the first was red: Now 2 red remain out of 4 balls, so $P(\text{2nd red} \mid \text{1st red}) = \frac{2}{4}$.

Step 3 – multiply the two: $P(\text{both red}) = \frac{3}{5} \times \frac{2}{4}$

$= \frac{6}{20}$

Step 4 – simplify: $= 0.3$

Final Answer: $P(\text{both red}) = 0.3 \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q1**

Q2.

Solution

Concept – conditional probability: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$.

Step 1 – read the overlap from the Venn diagram: $P(A \cap B) = 0.20$ (the shared region).

Step 2 – read $P(A)$ as the whole of circle A: $P(A) = 0.20 + 0.20 = 0.40$

Step 3 – substitute: $P(B \mid A) = \dfrac{0.20}{0.40}$

Step 4 – divide: $P(B \mid A) = 0.50$

Step 5 – sanity check the whole sample space: $0.20 + 0.20 + 0.30 + 0.30 = 1.00$, so the diagram is consistent.

Final Answer: $P(B \mid A) = 0.50 \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q2**

Q3.

Solution

Concept – Bayes' theorem (reverse the tree): $P(S_2 \mid D) = \dfrac{P(S_2)\,P(D \mid S_2)}{P(S_1)\,P(D \mid S_1) + P(S_2)\,P(D \mid S_2)}$.

Step 1 – probability from the S_1 branch: $P(S_1)\,P(D \mid S_1) = 0.40 \times 0.03 = 0.012$

Step 2 – probability from the S_2 branch: $P(S_2)\,P(D \mid S_2) = 0.60 \times 0.01 = 0.006$

Step 3 – total probability of a defective item: $P(D) = 0.012 + 0.006 = 0.018$

Step 4 – apply Bayes' theorem: $P(S_2 \mid D) = \dfrac{0.006}{0.018}$

Step 5 – divide: $P(S_2 \mid D) = 0.333$

Final Answer: $P(S_2 \mid D) = 0.333 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q3

Q4.

Solution

Concept – standardize, then use the normal table: Convert X to the standard normal $Z = \dfrac{X - \mu}{\sigma}$.

Step 1 – standardize the cut-off $X = 85$: $Z = \dfrac{85 - 100}{15}$

$Z = \dfrac{-15}{15} = -1$

Step 2 – write the required probability in Z: $P(X < 85) = P(Z < -1)$

Step 3 – use symmetry of the standard normal: $P(Z < -1) = P(Z > 1) = 1 - P(Z \le 1)$

Step 4 – substitute $P(Z \le 1) = 0.8413$: $P(Z < -1) = 1 - 0.8413 = 0.1587$

Final Answer: $P(X < 85) = 0.1587 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q4

Q5.

Solution

Concept – Poisson probability: $P(X = k) = \dfrac{e^{-\lambda}\,\lambda^k}{k!}$, with mean rate λ.

Step 1 – list the parameters: $\lambda = 2$ per hour, $k = 3$.

Step 2 – substitute into the formula: $P(X = 3) = \dfrac{e^{-2}\,2^3}{3!}$

Step 3 – evaluate the powers and factorial: $2^3 = 8$ and $3! = 6$

$P(X = 3) = \frac{e^{-2} \times 8}{6}$

Step 4 – substitute $e^{-2} = 0.1353$**:** $P(X = 3) = \frac{0.1353 \times 8}{6} = \frac{1.0824}{6}$

Step 5 – divide: $P(X = 3) = 0.180$

Final Answer: $P(X = 3) \approx 0.180 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q5

Q6.

Solution

Concept – expected value of a discrete random variable: $E[X] = \sum_i x_i P(X = x_i)$.

Step 1 – multiply each value by its probability: $1 \times 0.5 = 0.5$

$2 \times 0.3 = 0.6$

$3 \times 0.2 = 0.6$

Step 2 – add the products: $E[X] = 0.5 + 0.6 + 0.6$

Step 3 – finish the arithmetic: $E[X] = 1.7$

Final Answer: $E[X] = 1.7 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept – population variance: $\sigma^2 = \frac{1}{N} \sum_i (x_i - \bar{x})^2$, where $\bar{x}$ is the mean.

Step 1 – compute the mean: $\bar{x} = \frac{2 + 4 + 6 + 8 + 10}{5} = \frac{30}{5} = 6$

Step 2 – compute the deviations from the mean: $2 - 6 = -4$, $4 - 6 = -2$, $6 - 6 = 0$, $8 - 6 = 2$, $10 - 6 = 4$.

Step 3 – square the deviations: $(-4)^2 = 16$, $(-2)^2 = 4$, $0^2 = 0$, $2^2 = 4$, $4^2 = 16$.

Step 4 – sum the squared deviations: $16 + 4 + 0 + 4 + 16 = 40$

Step 5 – divide by $N = 5$**:** $\sigma^2 = \frac{40}{5} = 8$

Final Answer: population variance $= 8 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q7

Q8.

Solution

Concept – Pearson correlation coefficient: $r = \dfrac{S_{xy}}{\sqrt{S_{xx}\,S_{yy}}}$.

Step 1 – list the sums of squares: $S_{xx} = 25$, $S_{yy} = 100$, $S_{xy} = 40$.

Step 2 – compute the product under the root: $S_{xx}\,S_{yy} = 25 \times 100 = 2500$

Step 3 – take the square root: $\sqrt{2500} = 50$

Step 4 – form the ratio: $r = \dfrac{40}{50}$

Step 5 – simplify: $r = 0.8$

Final Answer: $r = 0.8 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept – binomial probability: $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$.

Step 1 – list the parameters: $n = 5$, $k = 2$, $p = 0.5$, $1 - p = 0.5$.

Step 2 – evaluate the binomial coefficient: $\binom{5}{2} = \dfrac{5 \times 4}{2 \times 1} = 10$

Step 3 – evaluate the powers: $p^2(1-p)^3 = 0.5^2 \times 0.5^3 = 0.5^5$

$0.5^5 = 0.03125$

Step 4 – multiply: $P(X = 2) = 10 \times 0.03125$

$= 0.3125$

Final Answer: $P(X = 2) = 0.3125 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q9

Q10.

Solution

Concept – exponential cumulative distribution: For an exponential variable with rate λ, $P(X < t) = 1 - e^{-\lambda t}$.

Step 1 – find the rate from the mean: Mean $= \dfrac{1}{\lambda} = 4$ hours, so $\lambda = \dfrac{1}{4} = 0.25\ \text{h}^{-1}$.

Step 2 – form the exponent: $\lambda t = 0.25 \times 4 = 1$

Step 3 – write the CDF: $P(X < 4) = 1 - e^{-1}$

Step 4 – substitute $e^{-1} = 0.3679$**:** $P(X < 4) = 1 - 0.3679 = 0.6321$

Final Answer: $P(X < 4) = 0.6321 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q10

Q11.

Solution

Concept – rank is the number of independent rows: Reduce the rows and count how many are linearly independent.

Step 1 – compare rows 1 **and** 2**:** Row $2 = (2, 4, 2) = 2 \times (1, 2, 1) = 2\,R_1$, so row 2 is dependent on row 1.

Step 2 – eliminate row 2**:** $R_2 \leftarrow R_2 - 2R_1 = (0, 0, 0)$

Step 3 – test row 3 **against row** 1**:** Row $3 = (3, 6, 5)$; a multiple of $(1, 2, 1)$ giving $3, 6$ would need the third entry 3, but it is 5, so row 3 is independent.

Step 4 – count the non-zero independent rows: Independent rows are R_1 and R_3, giving rank $= 2$.

Final Answer: $\text{rank}(A) = 2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q11

Q12.

Solution

Concept – cofactor expansion along the first column: Expanding down column 1 is efficient because it has a zero entry.

Step 1 – first-column entries: $b_{11} = 2$, $b_{21} = 1$, $b_{31} = 1$.

Step 2 – minor and sign for b_{11} **(sign** $+$**):** $+2\begin{vmatrix} 3 & 2 \\ 0 & 3 \end{vmatrix} = 2\big[(3)(3) - (2)(0)\big] = 2(9) = 18$

Step 3 – minor and sign for b_{21} **(sign** $-$**):** $-1\begin{vmatrix} 0 & 1 \\ 0 & 3 \end{vmatrix} = -1\big[(0)(3) - (1)(0)\big] = -1(0) = 0$

Step 4 – minor and sign for b_{31} **(sign** $+$**):** $+1\begin{vmatrix} 0 & 1 \\ 3 & 2 \end{vmatrix} = 1\big[(0)(2) - (1)(3)\big] = 1(-3) = -3$

Step 5 – add the three contributions: $\det B = 18 + 0 + (-3) = 15$

Final Answer: $\det B = 15 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q12

Q13.

Solution

Concept – eigenvalues from trace and determinant: For a 2×2 matrix, $\lambda^2 - \mathrm{tr}(C)\,\lambda + \det(C) = 0$.

Step 1 – trace and determinant of C: $\mathrm{tr}(C) = 5 + 2 = 7$ and $\det(C) = (5)(2) - (2)(2) = 10 - 4 = 6$.

Step 2 – write the characteristic equation: $\lambda^2 - 7\lambda + 6 = 0$

Step 3 – factor: $(\lambda - 6)(\lambda - 1) = 0$

Step 4 – read the roots: $\lambda = 6$ or $\lambda = 1$.

Step 5 – pick the larger: The larger eigenvalue is 6.

Final Answer: $\lambda_{\max} = 6 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q13

Q14.

Solution

Concept – singular values are square roots of the eigenvalues of $A^\top A$: $\sigma_i = \sqrt{\lambda_i(A^\top A)}$.

Step 1 – take the square roots of the given eigenvalues: $\sigma_1 = \sqrt{16} = 4$ and $\sigma_2 = \sqrt{4} = 2$.

Step 2 – identify the smallest singular value: $\sigma_{\min} = \min(4, 2) = 2$.

Step 3 – state the result: The smallest singular value of A is 2.

Final Answer: $\sigma_{\min} = 2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q14

Q15.

Solution

Concept – the ℓ_2 norm is the square root of the sum of squares: $\|v\|_2 = \sqrt{\sum_i v_i^2}$.

Step 1 – square each component: $2^2 = 4, \quad (-3)^2 = 9, \quad 6^2 = 36.$

Step 2 – add the squares: $4 + 9 + 36 = 49$

Step 3 – take the square root: $\|v\|_2 = \sqrt{49} = 7$

Step 4 – contrast with the other norms: $\|v\|_1 = 2 + 3 + 6 = 11$ and $\|v\|_\infty = \max(2, 3, 6) = 6$; the ℓ_2 value is 7.

Final Answer: $\|v\|_2 = 7 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q15

Q16.

Solution

Concept – minimize by setting the derivative to zero: For a convex parabola, the stationary point is the global minimum.

Step 1 – differentiate: $f'(x) = 2x - 6$

Step 2 – set the derivative to zero: $2x - 6 = 0$

$x = 3$

Step 3 – confirm it is a minimum: $f''(x) = 2 > 0$, so the point is a minimum.

Step 4 – evaluate f at $x = 3$: $f(3) = 3^2 - 6(3) + 11$

$= 9 - 18 + 11$

$= 2$

Final Answer: minimum value $= 2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q16

Q17.

Solution

Concept – scalar projection: The component of $\vec{a}$ along $\vec{b}$ is $\text{comp}_{\vec{b}}\,\vec{a} = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}$.

Step 1 – compute the dot product: $\vec{a} \cdot \vec{b} = (1)(3) + (2)(4) = 3 + 8 = 11$

Step 2 – compute the magnitude of $\vec{b}$: $\|\vec{b}\| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Step 3 – form the scalar projection: $\text{comp}_{\vec{b}}\,\vec{a} = \dfrac{11}{5}$

Step 4 – divide: $\dfrac{11}{5} = 2.2$

Final Answer: scalar projection $= 2.2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q17

Q18.

Solution

Concept – one gradient-descent step: $x_1 = x_0 - \eta\, f'(x_0)$.

Step 1 – differentiate the objective: $f(x) = x^2 + 2x \Rightarrow f'(x) = 2x + 2$

Step 2 – evaluate the gradient at $x_0 = 3$: $f'(3) = 2(3) + 2 = 6 + 2 = 8$

Step 3 – apply the update with $\eta = 0.2$: $x_1 = 3 - 0.2 \times 8$

Step 4 – finish the arithmetic: $x_1 = 3 - 1.6 = 1.4$

Final Answer: $x_1 = 1.4 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q18

Q19.

Solution

Concept – convexity via the Hessian: A twice-differentiable function is convex iff its Hessian is positive semi-definite everywhere; if the Hessian is indefinite, the function is neither convex nor concave.

Step 1 – compute the second partials: $f_{xx} = 2,\ f_{yy} = 2,\ f_{xy} = f_{yx} = 3$.

Step 2 – write the Hessian: $H = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$

Step 3 – test the leading principal minors: First minor: $2 > 0$. Second minor: $\det H = (2)(2) - (3)(3) = 4 - 9 = -5 < 0$.

Step 4 – interpret the sign of the determinant: A negative determinant means H has one positive and one negative eigenvalue, so H is indefinite.

Step 5 – conclude: An indefinite Hessian means f is neither convex nor concave (it has a saddle shape).

Final Answer: f is neither convex nor concave $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q19

Q20.

Solution

Concept – extended slice with a step: `a[start:stop:step]` takes every `step`-th element; `a[::2]` starts at index 0 and takes every second element.

Step 1 – index the list: $\texttt{a} = [\underbrace{1}_{0}, \underbrace{2}_{1}, \underbrace{3}_{2}, \underbrace{4}_{3}, \underbrace{5}_{4}]$

Step 2 – apply the step of 2 from index 0: Take indices $0,\ 2,\ 4$.

Step 3 – read off the elements: Index $0 \to 1$, index $2 \to 3$, index $4 \to 5$, giving `[1, 3, 5]`.

Final Answer: `[1, 3, 5]` $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept – list comprehension: `[i*i for i in range(4)]` builds a list of the squares of $0, 1, 2, 3$.

Step 1 – expand `range(4)`: `range(4)` yields $0,\ 1,\ 2,\ 3$.

Step 2 – square each value: $0^2 = 0, \quad 1^2 = 1, \quad 2^2 = 4, \quad 3^2 = 9.$

Step 3 – form the list: $\texttt{s} = [0,\ 1,\ 4,\ 9]$

Step 4 – read `s[2]`: Index 2 holds the value 4.

Final Answer: $4 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept – a queue is first-in, first-out (FIFO): `dequeue()` always removes the element that has waited longest (the front).

Step 1 – `enqueue(5)`, `enqueue(10)`: Queue (front→rear): $[5,\ 10]$.

Step 2 – `dequeue()`: Removes the front 5; queue becomes $[10]$.

Step 3 – `enqueue(15)`: Queue becomes $[10,\ 15]$.

Step 4 – `dequeue()` (the last one): Removes the front 10; this is the value returned.

Final Answer: last `dequeue()` returns $10 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q22

Q23.

Solution

Concept – height of a BST: The height is the number of edges on the longest path from the root down to a leaf.

Step 1 – place the root: 40 is inserted first and becomes the root (level 0).

Step 2 – place the second level: $20 < 40$ goes left; $60 > 40$ goes right (both at level 1).

Step 3 – place the remaining keys: $10 < 20$ (left of 20); 30 is between 20 and 40 (right of 20); 50 is between 40 and 60 (left of 60); $70 > 60$ (right of 60). All are at level 2.

Step 4 – find the longest root-to-leaf path: The deepest nodes are at level 2, e.g. $40 \to 20 \to 10$, which uses 2 edges.

Step 5 – state the height: Height $= 2$ edges.

Final Answer: height $= 2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q23

Q24.

Solution

Concept – linear probing scans the next free slot: On a collision at index i, try $i+1, i+2, \ldots$ (modulo table size).

Step 1 – insert 8: $h(8) = 8 \bmod 7 = 1$; slot 1 is free, so 8 goes to index 1.

Step 2 – insert 15: $h(15) = 15 \bmod 7 = 1$; index 1 is occupied, so probe index 2; it is free, so 15 goes to index 2.

Step 3 – insert 22: $h(22) = 22 \bmod 7 = 1$; index 1 occupied $\to$ probe index 2 (occupied) $\to$ probe index 3 (free).

Step 4 – place 22: 22 is stored at index 3.

Final Answer: index $3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q24

Q25.

Solution

Concept – binary search halves the range each step: The worst-case number of comparisons is $\lfloor \log_2 n \rfloor + 1$.

Step 1 – take the base-2 logarithm of n: $\log_2 255 \approx 7.99$

Step 2 – take the floor: $\lfloor 7.99 \rfloor = 7$

Step 3 – add one: $7 + 1 = 8$

Step 4 – cross-check by powers of two: $2^7 = 128 \le 255 < 256 = 2^8$, so 8 halvings suffice.

Final Answer: 8 comparisons $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q25

Q26.

Solution

Concept – Merge Sort always splits evenly: Merge Sort divides the array into two halves, sorts each, and merges in linear time, regardless of the input order.

Step 1 – write the recurrence: $T(n) = 2\,T(n/2) + O(n)$

Step 2 – identify the merge cost: Merging two sorted halves of total length n takes $O(n)$ comparisons in the worst case.

Step 3 – solve the recurrence: The tree has $\log_2 n$ levels, each doing $O(n)$ work, so $T(n) = O(n \log n)$.

Step 4 – note it is the same in every case: Because the split is always balanced, the best, average, and worst cases are all $O(n \log n)$.

Final Answer: worst case $= O(n \log n) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept – DFS goes as deep as possible before backtracking: Using a stack (or recursion), always follow the smallest-labelled unvisited neighbour first.

Step 1 – start at A: Visit A. Its neighbours are B and C; choose B (alphabetically first).

Step 2 – go deep from B: Visit B. Its unvisited neighbour is D. Visit D.

Step 3 – go deep from D: Visit D. Its unvisited neighbour is E. Visit E.

Step 4 – continue from E: E's neighbours are C and D; D is visited, so visit C.

Step 5 – assemble the order: $A,\ B,\ D,\ E,\ C$.

Final Answer: $A, B, D, E, C \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q27

Q28.

Solution

Concept – Master theorem: For $T(n) = a\,T(n/b) + f(n)$, compare $f(n)$ with $n^{\log_b a}$.

Step 1 – identify the parameters: $a = 4$, $b = 2$, $f(n) = n$.

Step 2 – compute the critical exponent: $n^{\log_b a} = n^{\log_2 4} = n^2$

Step 3 – compare $f(n)$ with $n^{\log_b a}$: $f(n) = n = O\big(n^{2-\varepsilon}\big)$ for some $\varepsilon > 0$, which is Master-theorem *Case 1*.

Step 4 – apply Case 1: $T(n) = \Theta\big(n^{\log_b a}\big) = \Theta(n^2)$

Final Answer: $T(n) = \Theta(n^2) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q28

Q29.

Solution

Concept – degree of a Cartesian product: The Cartesian product concatenates the attributes of both relations, so its degree is the *sum* of the two degrees (its cardinality would be the product of the tuple counts).

Step 1 – read the two degrees: R has 3 attributes; S has 4 attributes.

Step 2 – add the degrees: Degree of $R \times S = 3 + 4 = 7$

Step 3 – note the cardinality (not asked, for contrast): Number of tuples would be $6 \times 5 = 30$, but the question asks for the degree.

Final Answer: degree $= 7 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q29

Q30.

Solution

Concept – `COUNT(DISTINCT col)` counts distinct non-null values: It collapses duplicate values and ignores NULLs.

Step 1 – list the distinct values in `dept`: The column holds `'A'`, `'B'` and `'C'`.

Step 2 – ignore how many times each occurs: `'A'` appears 3 times, `'B'` 3 times, `'C'` 2 times, but each distinct value is counted once.

Step 3 – count the distinct values: There are 3 distinct department codes.

Final Answer: `COUNT(DISTINCT dept)` returns $3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q30

Q31.

Solution

Concept – 3NF versus BCNF: BCNF requires the left side of every non-trivial FD to be a superkey; 3NF relaxes this when the right side is a *prime* attribute.

Step 1 – identify prime attributes: The candidate keys are $\{A, B\}$ and $\{A, C\}$, so A, B and C are all prime attributes.

Step 2 – check the FD $AB \to C$: AB is a candidate key, hence a superkey, so this FD does not violate BCNF.

Step 3 – check the FD $C \to B$: C alone is *not* a superkey, so $C \to B$ violates BCNF.

Step 4 – test that FD against 3NF: For 3NF, a violating FD is allowed if its right side is a prime attribute. Here B *is* prime, so $C \to B$ satisfies 3NF.

Step 5 – conclude the highest normal form: R is in 3NF but not in BCNF, so the highest normal form is 3NF.

Final Answer: highest form $=$ 3NF $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q31

Q32.

Solution

Concept – natural join matches tuples on the common attribute: Combine a tuple of R with a tuple of S whenever their B values are equal.

Step 1 – match R's tuple $(1, x)$: S has (x, p) and (x, q) with $B = x$, giving $(1, x, p)$

and $(1, x, q)$ – 2 tuples.

Step 2 – match R's tuple $(2, x)$: S again has (x, p) and (x, q) with $B = x$, giving $(2, x, p)$ and $(2, x, q)$ – 2 tuples.

Step 3 – match R's tuple $(3, y)$: S has no tuple with $B = y$ (it only has x and z), giving 0 tuples.

Step 4 – total the matches: $2 + 2 + 0 = 4$

Final Answer: 4 tuples $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q32

Q33.

Solution

Concept – OLAP navigation operations: Roll-up aggregates to a coarser level; drill-down moves to finer detail.

Step 1 – describe the direction of movement: Going from monthly totals to yearly totals *removes* detail (more aggregation).

Step 2 – match the operation: Moving to a more summarized level is **roll-up**.

Why the other options are wrong: Drill-down is the reverse (more detail); slice fixes one dimension to a single value; pivot rotates the cube's axes.

Final Answer: roll-up $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q33

Q34.

Solution

Concept – minimum occupancy of a non-root internal node: In a B^+ tree of order m, a non-root internal node must be at least half full, holding at least $\lceil m/2 \rceil$ child pointers.

Step 1 – write the rule: Minimum child pointers $= \lceil m/2 \rceil$.

Step 2 – substitute $m = 6$: $\lceil 6/2 \rceil = \lceil 3 \rceil$

Step 3 – evaluate: $= 3$

Step 4 – sanity check against the maximum: The maximum is $m = 6$ pointers, and $3 \leq 6$, so the minimum 3 is consistent.

Final Answer: minimum 3 child pointers $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q34

Q35.

Solution

Concept – attribute closure: A^+ is the set of all attributes functionally determined by A, found by repeatedly applying the FDs.

Step 1 – start with the attribute itself: $A^+ = \{A\}$

Step 2 – apply $A \to B$**:** A determines B, so $A^+ = \{A, B\}$.

Step 3 – apply $B \to C$**:** B is now in the closure, so it brings in C: $A^+ = \{A, B, C\}$.

Step 4 – check $D \to E$**:** D is *not* in the closure, so $D \to E$ cannot be applied; neither D nor E is added.

Step 5 – stop: No further FD adds anything, so $A^+ = \{A, B, C\}$.

Final Answer: $A^+ = \{A, B, C\} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q35

Q36.

Solution

Concept – logistic regression prediction: Compute the linear score $z = w^\top x + b$, then apply the sigmoid.

Step 1 – compute the linear score: $z = (2)(1) + (-1)(1) + 0$

$= 2 - 1 + 0$

$= 1$

Step 2 – write the sigmoid: $\sigma(1) = \dfrac{1}{1 + e^{-1}}$

Step 3 – substitute $e^{-1} = 0.3679$**:** $\sigma(1) = \dfrac{1}{1 + 0.3679} = \dfrac{1}{1.3679}$

Step 4 – divide: $\sigma(1) = 0.731$

Final Answer: $\sigma(1) \approx 0.731 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q36

Q37.

Solution

Concept – underfitting in bias–variance terms: A model too simple to capture the pattern makes systematic errors everywhere: high bias, low variance.

Step 1 – interpret the poor training accuracy: Failing even on the training set means the model cannot represent the underlying relationship, i.e. *high bias*.

Step 2 – interpret the stability across data: Because the model is so simple, its predictions change little from one training sample to another, i.e. *low variance*.

Step 3 – combine: Poor on both train and test, insensitive to the data, is the classic signature of underfitting: high bias, low variance.

Final Answer: high bias, low variance $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q37

Q38.

Solution

Concept – 1-NN assigns the label of the single closest point: Compute the distance from Q to each training point and take the label of the nearest.

Step 1 – distance $Q(2,2)$ to $A(1,1)$: $\sqrt{(2-1)^2+(2-1)^2} = \sqrt{1+1} = \sqrt{2} = 1.41$

Step 2 – distance to $C(1,2)$: $\sqrt{(2-1)^2+(2-2)^2} = \sqrt{1+0} = 1.00$

Step 3 – distance to $B(4,4)$: $\sqrt{(2-4)^2+(2-4)^2} = \sqrt{4+4} = \sqrt{8} = 2.83$

Step 4 – distance to $D(5,5)$: $\sqrt{(2-5)^2+(2-5)^2} = \sqrt{9+9} = \sqrt{18} = 4.24$

Step 5 – pick the single nearest point: The smallest distance is 1.00 to C, which is Class 0.

Final Answer: Q is Class $0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q38

Q39.

Solution

Concept – binary entropy: $H = -p\log_2 p - (1-p)\log_2(1-p)$, with p the fraction of positives.

Step 1 – compute the class proportions: $p = \frac{5}{10} = 0.5$ positive, $1-p = \frac{5}{10} = 0.5$ negative.

Step 2 – take the base-2 logs: $\log_2 0.5 = -1$.

Step 3 – form each term: $-0.5 \times (-1) = 0.5$ (positives)

$-0.5 \times (-1) = 0.5$ (negatives)

Step 4 – add the terms: $H = 0.5 + 0.5 = 1.0$ bit

Step 5 – interpret: A perfectly balanced two-class set has maximum entropy, 1 bit.

Final Answer: $H = 1.0$ bit $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q39

Q40.

Solution

Concept – Gini impurity: Gini $= 1 - \sum_i p_i^2$, where p_i is the fraction of class i.

Step 1 – compute the class proportions: $p_+ = \frac{4}{8} = 0.5$ and $p_- = \frac{4}{8} = 0.5$.

Step 2 – square the proportions: $p_+^2 = 0.25$ and $p_-^2 = 0.25$.

Step 3 – sum the squares: $0.25 + 0.25 = 0.5$

Step 4 – subtract from 1: Gini $= 1 - 0.5 = 0.5$

Final Answer: Gini impurity $= 0.5 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q40

Q41.

Solution

Concept – distance from a point to a hyperplane: For a line $ax_1 + bx_2 + c = 0$, the distance from (x_1^0, x_2^0) is $\frac{|ax_1^0 + bx_2^0 + c|}{\sqrt{a^2 + b^2}}$.

Step 1 – identify the coefficients: $a = 3$, $b = 4$, $c = -10$; the point is $(2, 4)$.

Step 2 – evaluate the numerator: $|3(2) + 4(4) - 10| = |6 + 16 - 10| = |12| = 12$

Step 3 – evaluate the denominator: $\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Step 4 – form the ratio: distance $= \frac{12}{5}$

Step 5 – divide: $= 2.4$

Final Answer: distance $= 2.4 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q41

Q42.

Solution

Concept – k-means centroid update: The new centroid is the coordinate-wise mean of the points assigned to the cluster.

Step 1 – average the x-coordinates: $\bar{x} = \frac{1+3+5}{3} = \frac{9}{3} = 3$

Step 2 – average the y-coordinates: $\bar{y} = \frac{4+2+6}{3} = \frac{12}{3} = 4$

Step 3 – assemble the centroid: $(\bar{x}, \bar{y}) = (3,\ 4)$

Final Answer: centroid $= (3, 4) \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q42

Q43.

Solution

Concept – variance explained in PCA: Each principal component explains variance proportional to its eigenvalue; the fraction is that eigenvalue over the total.

Step 1 – total variance (sum of all eigenvalues): $5 + 3 + 2 = 10$

Step 2 – variance in the first component: The largest eigenvalue is 5.

Step 3 – form the fraction: $\frac{5}{10} = 0.50$

Step 4 – express as a percentage: $0.50 = 50\%$

Final Answer: 50% of the variance $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q43

Q44.

Solution

Concept – neuron forward pass with ReLU: Compute the weighted sum plus bias, then apply $\text{ReLU}(z) = \max(0, z)$.

Step 1 – weighted sum of inputs: $w_1x_1 + w_2x_2 = (1)(2) + (1)(1)$

$= 2 + 1$

$= 3$

Step 2 – add the bias: $z = 3 + (-1) = 2$

Step 3 – apply ReLU: $\text{ReLU}(2) = \max(0,\ 2) = 2$

Final Answer: output $= 2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q44

Q45.

Solution

Concept – A* expands the node of least $f = g + h$: Compute f for each frontier node and pick the minimum.

Step 1 – f for node P: $f(P) = g + h = 5 + 3 = 8$

Step 2 – f for node Q: $f(Q) = 4 + 3 = 7$

Step 3 – f for node R: $f(R) = 2 + 4 = 6$

Step 4 – choose the smallest f: $\min(8, 7, 6) = 6$, which belongs to node R.

Final Answer: A* expands $R \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q45

Q46.

Solution

Concept – minimax back-up: MIN nodes take the minimum of their children; the MAX root takes the maximum of the MIN results.

Step 1 – evaluate the left MIN node: $\min(7, 4) = 4$

Step 2 – evaluate the right MIN node: $\min(6, 9) = 6$

Step 3 – evaluate the MAX root: $\max(4, 6) = 6$

Final Answer: minimax value $= 6 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q46

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	C	3	D	4	A	5	B
6	B	7	D	8	D	9	C	10	D
11	B	12	B	13	D	14	A	15	D
16	A	17	C	18	B	19	A	20	D
21	A	22	A	23	C	24	D	25	B
26	B	27	C	28	A	29	C	30	D
31	C	32	B	33	A	34	C	35	A
36	B	37	A	38	C	39	B	40	C
41	B	42	C	43	A	44	B	45	D
46	D								