GATE Data Science and Artificial Intelligence

Sample Paper – 7

Duration: 128 Minutes **Maximum Marks: 72**

Instructions

- This paper contains **46 questions** covering the core **Data Science and Artificial Intelligence** subject of the GATE paper conducted by the IITs and IISc (General Aptitude and Engineering Mathematics are provided as separate papers).
- **Q1–Q20 carry 1 mark each** and **Q21–Q46 carry 2 marks each**, for a total of **72 marks**.
- Each question carries **negative marking of one-third of its allotted marks**: a wrong 1-mark question costs **0.33** and a wrong 2-mark question costs **0.67**. Unattempted questions carry no penalty.
- Every question here is a single-correct **MCQ**. The real GATE paper also has Multiple Select (MSQ) and Numerical Answer Type (NAT) questions, and **those carry no negative marking**.
- Attempt this paper in one timed sitting of about **128 minutes**, the share this core subject earns at GATE's overall pace of 180 minutes for 65 questions. All logarithms in information-theoretic quantities (entropy, information gain) are to base 2 and expressed in bits unless stated otherwise.

Probability & Statistics

Q1. The number of distinct arrangements (words, meaningful or not) of *all* the letters of the word `GAMMA` is: **[1 mark]**

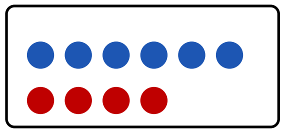
(A) 120

(B) 60

(C) 30

(D) 20

Q2. An urn contains 4 red and 6 blue balls, as shown. Two balls are drawn one after another *without replacement*. The probability that *both* drawn balls are red is: **[1 mark]**



4 red $+$ 6 blue $=$ 10 balls

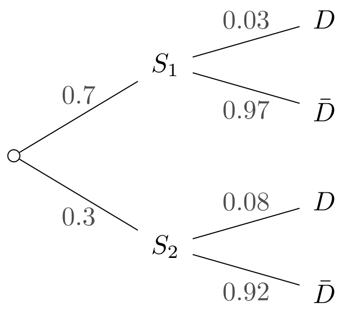
(A) $\frac{4}{25}$

(B) $\frac{2}{15}$

(C) $\frac{1}{15}$

(D) $\frac{2}{5}$

Q3. A component is bought from two suppliers. Supplier S_1 provides 70% of the stock with a 3% defect rate; supplier S_2 provides the remaining 30% with an 8% defect rate. The probability tree is shown. The probability that a randomly chosen component is defective is: **[1 mark]**



(A) 0.021

(B) 0.024

(C) 0.030

(D) 0.045

Q4. A fair six-sided die is rolled 5 times independently. The probability of obtaining *exactly* two sixes is nearest to: **[1 mark]**

(A) 0.402

(B) 0.161

(C) 0.033

(D) 0.080

Q5. A promotional game pays a prize of Rs. 50 with probability 0.2, Rs. 10 with probability 0.5, and Rs. 0 with probability 0.3. The expected prize (in rupees) per play is: **[1 mark]**

(A) 15

(B) 20

(C) 12

(D) 10

Q6. The values 2, 4, 6, 8, 10 constitute an entire (population) data set. The population standard deviation of these values is nearest to: **[1 mark]**

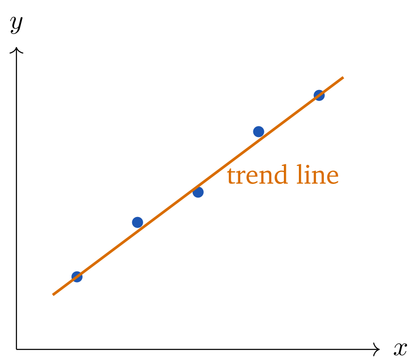
(A) 8.00

(B) 2.83

(C) 2.00

(D) 4.00

Q7. For a bivariate data set the sums of squares and cross-products about the means are $S_{xx} = 25$, $S_{yy} = 100$ and $S_{xy} = 40$; the scatter is shown. Pearson's correlation coefficient r is: **[1 mark]**



(A) 0.40

(B) 1.60

(C) 0.80

(D) 0.64

Q8. The observations $7, 3, 9, 5, 11, 1$ are recorded. The median of this data set is: **[1 mark]**

(A) 5

(B) 6

(C) 7

(D) 5.5

Q9. A continuous random variable X is uniformly distributed on the interval $[0, 10]$. The probability $P(X > 7)$ is: **[1 mark]**

(A) 0.3

(B) 0.7

(C) 0.1

(D) 0.5

Q10. A feature X is normally distributed with mean $\mu = 50$ and standard deviation $\sigma = 4$. Using the standard normal value $P(Z \leq 2) = 0.9772$, the probability $P(X < 58)$ is: **[1 mark]**

(A) 0.9772

(B) 0.0228

(C) 0.8413

(D) 0.5000

Linear Algebra, Calculus & Optimization

Q11. For the lower-triangular matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & 0 \\ 4 & 5 & 1 \end{bmatrix}$, the determinant $\det A$ is:

[1 mark]

(A) 11

(B) 9

(C) 6

(D) 0

Q12. Consider the linear system $2x + y = 5$ and $x + y = 3$. The value of x in the unique solution is: **[1 mark]**

(A) 1

(B) 3

(C) 4

(D) 2

Q13. The *smaller* eigenvalue of the symmetric matrix $C = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ is: **[1 mark]**

(A) 1

(B) 5

(C) 3

(D) 2

Q14. The rank of the matrix $D = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ is: **[1 mark]**

(A) 2

(B) 1

(C) 0

(D) 3

Q15. For the vector $v = (6, 8)$, the Euclidean (ℓ_2) norm $\|v\|_2$ is: **[1 mark]**

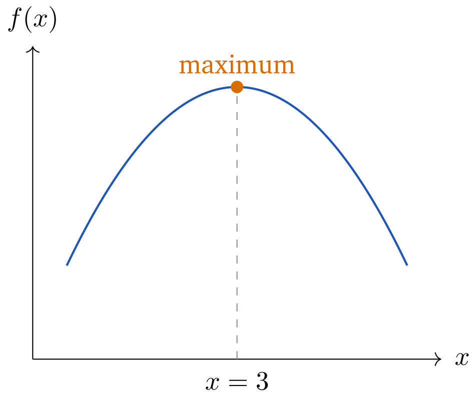
(A) 14

(B) 10

(C) 8

(D) 48

Q16. The function $f(x) = -x^2 + 6x + 1$ is maximized over the real line, as suggested by its concave graph below. The maximum *value* of f is: **[1 mark]**



(A) 3

(B) 1

(C) 6

(D) 10

Q17. For the function $f(x, y) = x^2y + 3y$, the partial derivative $\dfrac{\partial f}{\partial x}$ evaluated at the point $(2, 1)$ is: **[1 mark]**

(A) 2

(B) 7

(C) 4

(D) 5

Q18. Gradient descent is used to minimize $f(x) = x^2$ with a fixed learning rate $\eta = 0.2$. Starting from $x_0 = 5$, the value of x_1 after one update $x_1 = x_0 - \eta f'(x_0)$ is: **[1 mark]**

(A) 3

(B) 1

(C) 4

(D) 5

Q19. The Hessian of a twice-differentiable function at a point is $H = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.

This Hessian is: **[1 mark]**

(A) negative definite

(B) indefinite

(C) positive definite

(D) singular

Programming, Data Structures & Algorithms

Q20. What is the output of the following Python snippet?

```
a = [10, 20, 30, 40, 50]
print(a[-2])
```

[1 mark]

(A) 10

(B) 50

(C) 30

(D) 40

Q21. Consider the following Python code:

```
s = 0
for i in range(1, 5):
s += i
print(s)
```

The value printed is: **[2 marks]**

(A) 10

(B) 15

(C) 6

(D) 5

Q22. The following operations are performed on an initially empty *queue* (first-in, first-out), in order: `enqueue(5)`, `enqueue(15)`, `dequeue()`, `dequeue()`, `enqueue(25)`, `dequeue()`. The value returned by the *last* `dequeue()` is: **[2 marks]**

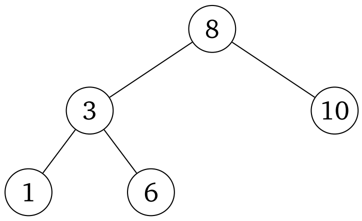
(A) 5

(B) 15

(C) 10

(D) 25

Q23. Starting from an empty binary search tree, the keys $8, 3, 10, 1, 6$ are inserted in the given order, producing the tree shown. The *inorder* traversal of the resulting tree is: **[2 marks]**



(A) $1, 3, 6, 8, 10$

(B) $8, 3, 1, 6, 10$

(C) $1, 6, 3, 10, 8$

(D) $8, 10, 6, 3, 1$

Q24. A hash table of size 7 uses the hash function $h(k) = k \bmod 7$ with *separate chaining*. The keys $10, 22, 31, 4, 15, 28$ are inserted. The number of keys that hash to index 1 is: **[2 marks]**

(A) 2

(B) 1

(C) 3

(D) 0

Q25. Merge sort is applied to an array of $n = 8$ elements. The number of *levels* of merging (the height of the recursion, i.e. $\log_2 n$) is: **[2 marks]**

(A) 8

(B) 7

(C) 4

(D) 3

Q26. The average-case time complexity of the standard Merge sort algorithm on an array of n elements is: **[2 marks]**

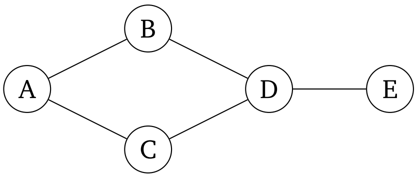
(A) $O(n \log n)$

(B) $O(n^2)$

(C) $O(n)$

(D) $O(\log n)$

Q27. A depth-first search (DFS) is started from vertex A in the undirected graph shown, always visiting adjacent vertices in increasing alphabetical order. The order in which the vertices are first visited is: **[2 marks]**



(A) A, B, C, D, E

(B) A, C, B, D, E

(C) A, B, C, E, D

(D) A, B, D, C, E

Q28. The running time of an algorithm satisfies the recurrence $T(n) = 2T(n/2) + n^2$, with $T(1) = 1$. By the Master theorem, $T(n)$ is: **[2 marks]**

(A) $\Theta(n \log n)$

(B) $\Theta(n^2)$

(C) $\Theta(n)$

(D) $\Theta(n^2 \log n)$

Database Management & Warehousing

Q29. Two union-compatible relations R and S have 8 and 5 tuples respectively, and exactly 3 tuples are common to both. The number of tuples in $R \cup S$ is: **[2 marks]**

(A) 13

(B) 5

(C) 10

(D) 3

Q30. A table `T` has 10 rows, of which 3 rows contain `NULL` in the `salary` column. What does the query `SELECT COUNT(*) FROM T;` return? **[2 marks]**

(A) 10

(B) 7

(C) 3

(D) 0

Q31. A relation $R(A, B, C, D)$ has A as its only candidate key, and the only functional dependency is $A \rightarrow BCD$. The highest normal form that R satisfies is: **[2 marks]**

(A) 2NF

(B) BCNF

(C) 1NF

(D) 3NF (but not BCNF)

Q32. Relation $R(A, B)$ contains the tuples $\{(1, x), (2, x), (3, y)\}$ and relation $S(B, C)$ contains $\{(x, p), (x, q), (y, r), (z, s)\}$. The number of tuples in the natural join $R \bowtie S$ (on the common attribute B) is: **[2 marks]**

(A) 3

(B) 9

(C) 4

(D) 5

Q33. In OLAP, the operation that reduces a data cube by fixing *one* dimension to a single value (for example, selecting only `Region = 'East'`) to obtain a sub-cube is called: **[2 marks]**

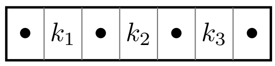
(A) roll-up

(B) slice

(C) drill-down

(D) pivot

Q34. The figure shows an internal node of a B-tree of order $m = 4$; each internal node holds up to m child pointers (drawn as •), separated by keys. The maximum number of *child pointers* an internal node of this tree can have is: **[2 marks]**



up to 4 pointers (•) and 3 keys

(A) 5

(B) 2

(C) 4

(D) 3

Q35. A relation has attributes $\{A, B, C, D, E\}$ with functional dependencies $A \to B$, $B \to C$ and $D \to E$. The attribute closure $\{A, D\}^+$ is: **[2 marks]**

(A) $\{A, B, C\}$

(B) $\{A, D\}$

(C) $\{A, B, C, D, E\}$

(D) $\{A, B, C, D\}$

Machine Learning & Artificial Intelligence

Q36. A binary classifier is evaluated on a test set, giving the confusion matrix shown (TP $= 40$, FP $= 10$, FN $= 20$, TN $= 30$). The *precision* of the classifier, $\frac{\text{TP}}{\text{TP}+\text{FP}}$, is: **[2 marks]**

	Pred +	Pred −
Act +	40	10
Act −	20	30

(A) 0.80

(B) 0.67

(C) 0.50

(D) 0.75

Q37. A binary classifier detects 30 of the actual positive cases correctly and misses 10 of them (there are no other positive cases). The *recall* (sensitivity), $\frac{\text{TP}}{\text{TP}+\text{FN}}$, is: **[2 marks]**

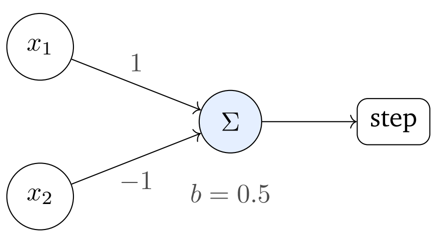
(A) 0.30

(B) 0.50

(C) 0.75

(D) 0.25

Q38. A perceptron with a step activation (output 1 if the weighted input is > 0, else 0) has weight vector $w = (1, -1)$ and bias $b = 0.5$, as shown. For the input $x = (1, 1)$, the output of the perceptron is: **[2 marks]**



(A) 0

(B) 1

(C) -1

(D) 0.5

Q39. A decision-tree node contains 3 positive and 1 negative example. The *Gini impurity* of this node, $1 - \sum_i p_i^2$, is: **[2 marks]**

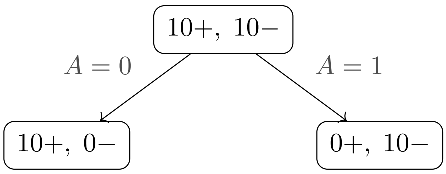
(A) 0.811

(B) 0.500

(C) 0.250

(D) 0.375

Q40. A decision-tree node with 10 positive and 10 negative examples is split by attribute A into two *pure* children as shown: $A = 0$ gives $(10+,\ 0-)$ and $A = 1$ gives $(0+,\ 10-)$. The information gain of this split is: **[2 marks]**



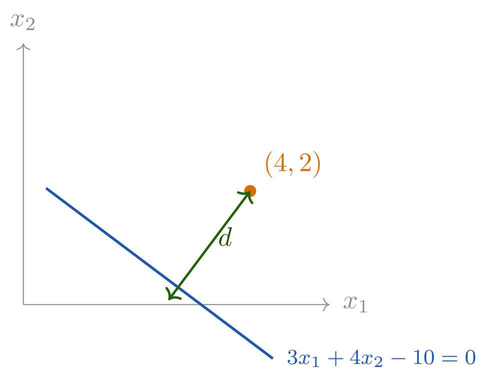
(A) 0.500

(B) 0.000

(C) 1.000

(D) 0.811

Q41. The separating hyperplane of a linear classifier is $3x_1 + 4x_2 - 10 = 0$. The perpendicular distance from the point $(4, 2)$ to this hyperplane, $\dfrac{|3x_1 + 4x_2 - 10|}{\sqrt{3^2 + 4^2}}$, is: **[2 marks]**



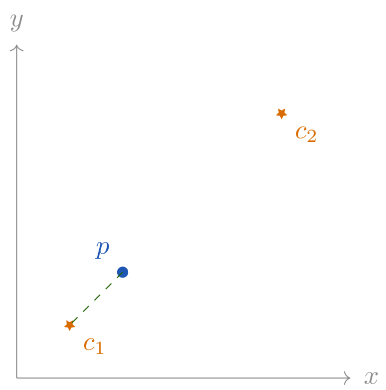
(A) 0.5

(B) 1

(C) 4

(D) 2

Q42. In the assignment step of k-means, two cluster centroids are $c_1 = (1,\ 1)$ and $c_2 = (5,\ 5)$, shown below. The point $p = (2,\ 2)$ is assigned to the nearer centroid (Euclidean distance). That centroid is: **[2 marks]**



(A) $c_1 = (1, 1)$

(B) $c_2 = (5, 5)$

(C) equidistant (a tie)

(D) cannot be determined

Q43. Principal component analysis is applied to a data set whose covariance matrix has eigenvalues $5,\ 3,\ 1,\ 1$. The fraction of the total variance explained by the *first* principal component alone is: **[2 marks]**

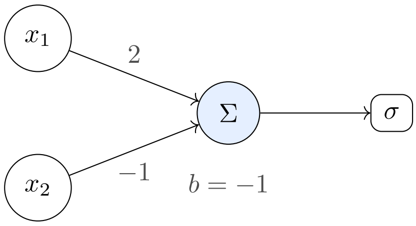
(A) 40%

(B) 50%

(C) 80%

(D) 90%

Q44. A single neuron with a sigmoid activation $\sigma(z) = 1/(1 + e^{-z})$ computes $\sigma(w_1x_1+w_2x_2+b)$, as shown. With inputs $x_1 = 1$, $x_2 = 1$, weights $w_1 = 2$, $w_2 = -1$ and bias $b = -1$, the output of the neuron is: **[2 marks]**



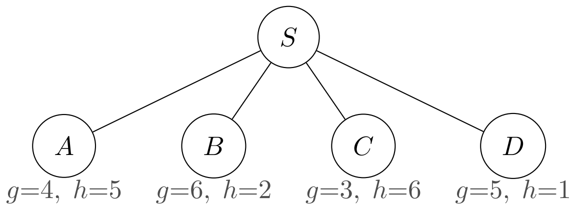
(A) 0

(B) 1

(C) 0.5

(D) 0.73

Q45. A* search maintains four frontier nodes A, B, C, D with the g (path cost so far) and h (heuristic) values shown, where $f = g + h$. The node A* selects for expansion next is: **[2 marks]**



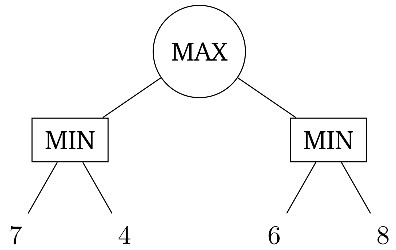
(A) A

(B) B

(C) C

(D) D

Q46. In the two-ply game tree shown, the root is a MAX node and the second level are MIN nodes; the leaf values are given. The minimax value backed up to the root is: **[2 marks]**



(A) 4

(B) 6

(C) 7

(D) 8

Detailed Solutions

Q1.

Solution

Concept – permutations of letters with repetition: For a word of n letters where a letter repeats r times, the number of distinct arrangements is $\frac{n!}{r_1!\,r_2!\cdots}$.

Step 1 – count the letters of `GAMMA`: The letters are $G,\ A,\ M,\ M,\ A$, so $n = 5$.

Step 2 – note the repeats: A appears 2 times and M appears 2 times; G appears once.

Step 3 – write the formula: Number of arrangements $= \frac{5!}{2!\ \times\ 2!}$

Step 4 – evaluate the factorials: $5! = 120$, $2! = 2$, $2! = 2$

Step 5 – divide: $\frac{120}{2 \times 2} = \frac{120}{4} = 30$

Final Answer: 30 arrangements $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q1

Q2.

Solution

Concept – probability without replacement (multiply stage by stage): $P(\text{both red}) = P(\text{1st red}) \times P(\text{2nd red} \mid \text{1st red})$.

Step 1 – probability the first ball is red: $P(\text{1st red}) = \frac{4}{10}$

Step 2 – update the counts after one red is removed: 3 red remain out of 9 balls, so $P(\text{2nd red} \mid \text{1st red}) = \frac{3}{9}$.

Step 3 – multiply the two stages: $P(\text{both red}) = \frac{4}{10} \times \frac{3}{9}$

Step 4 – multiply the numerators and denominators: $= \frac{12}{90}$

Step 5 – simplify the fraction: $\frac{12}{90} = \frac{2}{15}$

Final Answer: $P(\text{both red}) = \frac{2}{15} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept – law of total probability (sum over the branches): $P(D) = P(S_1)\,P(D \mid S_1) + P(S_2)\,P(D \mid S_2)$.

Step 1 – probability along the S_1 branch: $P(S_1)\,P(D \mid S_1) = 0.70 \times 0.03 = 0.021$

Step 2 – probability along the S_2 branch: $P(S_2)\,P(D \mid S_2) = 0.30 \times 0.08 = 0.024$

Step 3 – add the two branch probabilities: $P(D) = 0.021 + 0.024$

Step 4 – finish the addition: $P(D) = 0.045$

Final Answer: $P(D) = 0.045 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q3

Q4.

Solution

Concept – binomial probability: $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$.

Step 1 – list the parameters: $n = 5$, $k = 2$, $p = \dfrac{1}{6}$, $1 - p = \dfrac{5}{6}$.

Step 2 – evaluate the binomial coefficient: $\binom{5}{2} = \dfrac{5 \times 4}{2 \times 1} = 10$

Step 3 – evaluate the powers: $p^2 = \left(\dfrac{1}{6}\right)^2 = \dfrac{1}{36} = 0.02778$

$(1-p)^3 = \left(\dfrac{5}{6}\right)^3 = \dfrac{125}{216} = 0.57870$

Step 4 – multiply the three pieces: $P(X = 2) = 10 \times 0.02778 \times 0.57870$

Step 5 – finish the arithmetic: $= 10 \times 0.016076 = 0.16076 \approx 0.161$

Final Answer: $P(X = 2) \approx 0.161 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q4

Q5.

Solution

Concept – expected value of a discrete variable: $E[X] = \sum_i x_i\,p_i$.

Step 1 – term for the Rs. 50 **prize:** $50 \times 0.2 = 10$

Step 2 – term for the Rs. 10 **prize:** $10 \times 0.5 = 5$

Step 3 – term for the Rs. 0 **prize:** $0 \times 0.3 = 0$

Step 4 – add the three terms: $E[X] = 10 + 5 + 0 = 15$

Final Answer: expected prize $=$ Rs. $15 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q5

Q6.

Solution

Concept – population standard deviation: $\sigma = \sqrt{\frac{1}{N}\sum_i (x_i - \bar{x})^2}$.

Step 1 – compute the mean: $\bar{x} = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$

Step 2 – compute the deviations from the mean: $-4,\ -2,\ 0,\ 2,\ 4$

Step 3 – square the deviations: $16,\ 4,\ 0,\ 4,\ 16$

Step 4 – sum the squares: $16 + 4 + 0 + 4 + 16 = 40$

Step 5 – divide by $N = 5$ to get the variance: $\sigma^2 = \frac{40}{5} = 8$

Step 6 – take the square root: $\sigma = \sqrt{8} = 2.83$

Final Answer: $\sigma \approx 2.83 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept – Pearson's correlation coefficient: $r = \frac{S_{xy}}{\sqrt{S_{xx}\,S_{yy}}}$.

Step 1 – list the given sums: $S_{xx} = 25$, $S_{yy} = 100$, $S_{xy} = 40$.

Step 2 – form the product under the root: $S_{xx}\,S_{yy} = 25 \times 100 = 2500$

Step 3 – take the square root: $\sqrt{2500} = 50$

Step 4 – divide: $r = \frac{40}{50}$

Step 5 – simplify: $r = 0.80$

Final Answer: $r = 0.80 \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q7

Q8.

Solution

Concept – median of an even-sized data set: Sort the values; the median is the average of the two middle observations.

Step 1 – sort the six values in ascending order: $1,\ 3,\ 5,\ 7,\ 9,\ 11$

Step 2 – locate the two middle observations: With 6 values, the 3rd and 4th are the middle: 5 and 7.

Step 3 – average them: $\text{median} = \dfrac{5+7}{2} = \dfrac{12}{2}$

Step 4 – simplify: $\text{median} = 6$

Final Answer: $\text{median} = 6 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q8

Q9.

Solution

Concept – probability for a continuous uniform variable: For $X \sim \text{Uniform}[a, b]$, $P(X > c) = \dfrac{b-c}{b-a}$.

Step 1 – identify the interval: $a = 0$, $b = 10$, and the threshold $c = 7$.

Step 2 – length of the favourable part: $b - c = 10 - 7 = 3$

Step 3 – length of the whole interval: $b - a = 10 - 0 = 10$

Step 4 – form the ratio: $P(X > 7) = \dfrac{3}{10} = 0.3$

Final Answer: $P(X > 7) = 0.3 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept – standardize, then use the normal value: Convert X to $Z = \dfrac{X-\mu}{\sigma}$ and read the cumulative probability.

Step 1 – standardize the cut-off $X = 58$: $Z = \dfrac{58-50}{4}$

Step 2 – simplify: $Z = \dfrac{8}{4} = 2$

Step 3 – rewrite the required probability: $P(X < 58) = P(Z < 2)$

Step 4 – read the standard normal value: $P(Z < 2) = 0.9772$

Final Answer: $P(X < 58) = 0.9772 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q10

Q11.

Solution

Concept – determinant of a triangular matrix: For a lower- or upper-triangular matrix, the determinant is the product of the diagonal entries.

Step 1 – confirm A is triangular: All entries above the main diagonal are 0, so A is lower-triangular.

Step 2 – read the diagonal entries: $a_{11} = 2$, $a_{22} = 3$, $a_{33} = 1$.

Step 3 – multiply them: $\det A = 2 \times 3 \times 1$

Step 4 – evaluate: $\det A = 6$

Final Answer: $\det A = 6 \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q11

Q12.

Solution

Concept – solve a 2×2 linear system by elimination: Subtract one equation from the other to remove a variable.

Step 1 – write the two equations: $2x + y = 5 \quad \text{and} \quad x + y = 3.$

Step 2 – subtract the second from the first: $(2x + y) - (x + y) = 5 - 3$

Step 3 – simplify the left and right sides: $x = 2$

Step 4 – back-substitute to check: $x + y = 3 \Rightarrow 2 + y = 3 \Rightarrow y = 1$; then $2(2) + 1 = 5$ holds.

Final Answer: $x = 2 \Rightarrow \boxed{D}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept – eigenvalues from trace and determinant: For a 2×2 matrix, $\lambda^2 - (\text{trace})\lambda + (\det) = 0$.

Step 1 – trace and determinant of C: $\text{tr}(C) = 3 + 3 = 6$ and $\det(C) = (3)(3) - (2)(2) = 9 - 4 = 5$.

Step 2 – write the characteristic equation: $\lambda^2 - 6\lambda + 5 = 0$

Step 3 – factor: $(\lambda - 5)(\lambda - 1) = 0$

Step 4 – read the roots: $\lambda = 5$ or $\lambda = 1$.

Step 5 – pick the smaller: The smaller eigenvalue is 1.

Final Answer: $\lambda_{\text{min}} = 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept – rank counts independent rows: If one row is a scalar multiple of another, the rows are dependent.

Step 1 – compare the two rows: Row $2 = (2, 4) = 2 \times (1, 2) = 2\,R_1$.

Step 2 – eliminate row 2: $R_2 \leftarrow R_2 - 2R_1 = (0, 0)$

Step 3 – count the non-zero independent rows: Only R_1 remains non-zero, so the rank is 1.

Step 4 – cross-check with the determinant: $\det D = (1)(4) - (2)(2) = 4 - 4 = 0$, confirming rank < 2.

Final Answer: $\text{rank}(D) = 1 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q14

Q15.

Solution

Concept – the ℓ_2 norm is the square root of the sum of squares: $\|v\|_2 = \sqrt{\sum_i v_i^2}$.

Step 1 – square the components: $6^2 = 36$ and $8^2 = 64$.

Step 2 – add the squares: $36 + 64 = 100$

Step 3 – take the square root: $\|v\|_2 = \sqrt{100} = 10$

Final Answer: $\|v\|_2 = 10 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q15

Q16.

Solution

Concept – maximize by setting the derivative to zero: For a concave parabola, the stationary point is the global maximum.

Step 1 – differentiate: $f'(x) = -2x + 6$

Step 2 – set the derivative to zero: $-2x + 6 = 0$

$x = 3$

Step 3 – confirm it is a maximum: $f''(x) = -2 < 0$, so the point is a maximum.

Step 4 – evaluate f at $x = 3$: $f(3) = -(3)^2 + 6(3) + 1$

$= -9 + 18 + 1$

$= 10$

Final Answer: maximum value $= 10 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept – partial derivative treats the other variable as constant: Differentiate f with respect to x, holding y fixed, then substitute.

Step 1 – differentiate $f(x, y) = x^2y + 3y$ with respect to x: $\dfrac{\partial f}{\partial x} = 2xy + 0$

Step 2 – simplify: $\dfrac{\partial f}{\partial x} = 2xy$

Step 3 – substitute $(x, y) = (2, 1)$: $= 2 \times 2 \times 1$

Step 4 – evaluate: $= 4$

Final Answer: $\left.\dfrac{\partial f}{\partial x}\right|_{(2,1)} = 4 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q17

Q18.

Solution

Concept – one gradient-descent step: $x_1 = x_0 - \eta\, f'(x_0)$.

Step 1 – differentiate the objective: $f(x) = x^2 \Rightarrow f'(x) = 2x$

Step 2 – evaluate the gradient at $x_0 = 5$: $f'(5) = 2 \times 5 = 10$

Step 3 – apply the update with $\eta = 0.2$: $x_1 = 5 - 0.2 \times 10$

Step 4 – finish the arithmetic: $x_1 = 5 - 2 = 3$

Final Answer: $x_1 = 3 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q18

Q19.

Solution

Concept – definiteness from the leading principal minors: H is positive definite if every leading principal minor is positive.

Step 1 – first leading minor: The top-left entry is $2 > 0$.

Step 2 – second leading minor (the full determinant): $\det H = (2)(2) - (1)(1) = 4 - 1 = 3 > 0$

Step 3 – apply the test: Both leading minors are positive, so H is positive definite.

Step 4 – interpret: A positive-definite Hessian means the function is locally (and, being constant here, globally) convex.

Final Answer: H is positive definite $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q19

Q20.

Solution

Concept – negative indexing in Python: Index -1 is the last element, -2 is the second-to-last, and so on.

Step 1 – write the list with negative indices: $\texttt{a} = [\underbrace{10}_{-5}, \underbrace{20}_{-4}, \underbrace{30}_{-3}, \underbrace{40}_{-2}, \underbrace{50}_{-1}]$

Step 2 – locate index -2: Index -2 points to the second-to-last element.

Step 3 – read off the value: The second-to-last element is 40.

Final Answer: $\texttt{40} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept – `range(1, 5)` and accumulation: `range(1, 5)` yields $1, 2, 3, 4$ (the stop value 5 is excluded); the loop adds each to `s`.

Step 1 – list the loop values: $i = 1,\ 2,\ 3,\ 4.$

Step 2 – accumulate step by step: After $i = 1$: $s = 0 + 1 = 1.$

After $i = 2$: $s = 1 + 2 = 3.$

After $i = 3$: $s = 3 + 3 = 6.$

After $i = 4$: $s = 6 + 4 = 10.$

Step 3 – read the final value: $s = 10.$

Final Answer: $10 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept – a queue is first-in, first-out (FIFO): `dequeue()` always removes the element that has been in the queue longest (the front).

Step 1 – `enqueue(5)`, `enqueue(15)`: Queue (front→rear): $[5,\ 15]$.

Step 2 – `dequeue()`: Removes the front 5; queue becomes $[15]$.

Step 3 – `dequeue()`: Removes the front 15; queue becomes $[\,]$ (empty).

Step 4 – `enqueue(25)`: Queue becomes $[25]$.

Step 5 – `dequeue()` (the last one): Removes the front 25; this is the value returned.

Final Answer: last `dequeue()` returns $25 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q22

Q23.

Solution

Concept – inorder traversal of a BST is sorted: An inorder walk (left, node, right) of any binary search tree visits the keys in ascending order.

Step 1 – build the tree: 8 is the root; $3 < 8$ goes left; $10 > 8$ goes right; $1 < 3$ goes left of 3; 6 is between 3 and 8, so it goes right of 3.

Step 2 – perform the inorder walk (left, root, right): Start at the leftmost: visit 1, then its parent 3, then 3's right child 6.

Step 3 – continue to the root and right subtree: Visit the root 8, then the right child 10.

Step 4 – assemble the sequence: $1,\ 3,\ 6,\ 8,\ 10$ (which is simply the sorted order).

Final Answer: $1, 3, 6, 8, 10 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q23

Q24.

Solution

Concept – separate chaining groups keys by hash index: Compute $h(k) = k \bmod 7$ for each key and count how many land at index 1.

Step 1 – hash each key: $h(10) = 10 \bmod 7 = 3$

$h(22) = 22 \bmod 7 = 1$

$h(31) = 31 \bmod 7 = 3$

$h(4) = 4 \bmod 7 = 4$

$h(15) = 15 \bmod 7 = 1$

$h(28) = 28 \bmod 7 = 0$

Step 2 – collect the keys at index 1: Index 1 receives 22 and 15.

Step 3 – count them: There are 2 keys chained at index 1.

Final Answer: 2 keys $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q24

Q25.

Solution

Concept – merge sort halves the array each level: The number of levels of merging equals $\log_2 n$ for n a power of 2.

Step 1 – identify n: $n = 8$.

Step 2 – take the base-2 logarithm: $\log_2 8 = 3$ (since $2^3 = 8$).

Step 3 – interpret: The array splits $8 \to 4 \to 2 \to 1$, and merges back over 3 levels.

Final Answer: 3 levels $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q25

Q26.

Solution

Concept – merge sort's recurrence: $T(n) = 2\,T(n/2) + O(n)$, which the Master theorem resolves to $O(n \log n)$.

Step 1 – split cost: Each level splits the array into halves, and merging a level costs $O(n)$.

Step 2 – number of levels: There are $\log_2 n$ levels.

Step 3 – multiply per-level cost by levels: Total $= O(n) \times \log_2 n = O(n \log n)$.

Step 4 – note the case: Merge sort has the *same* $O(n \log n)$ time in the best, average and worst cases.

Final Answer: average case $= O(n \log n) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q26

Q27.

Solution

Concept – DFS explores as deep as possible before backtracking: Use a stack (or recursion); at each vertex go to the smallest unvisited neighbour first.

Step 1 – start at A: Visit A. Its neighbours are B and C; take the smaller, B.

Step 2 – go deep from B: Visit B. Its unvisited neighbour is D. Go to D.

Step 3 – go deep from D: Visit D. Unvisited neighbours are C and E; take the smaller, C.

Step 4 – process C: Visit C. Its neighbours A and D are already visited, so backtrack to D.

Step 5 – finish from D: D's remaining unvisited neighbour is E. Visit E.

Step 6 – assemble the order: $A,\ B,\ D,\ C,\ E$.

Final Answer: $A,\ B,\ D,\ C,\ E \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q27

Q28.

Solution

Concept – Master theorem: For $T(n) = a\,T(n/b) + f(n)$, compare $f(n)$ with $n^{\log_b a}$.

Step 1 – identify the parameters: $a = 2$, $b = 2$, $f(n) = n^2$.

Step 2 – compute the critical exponent: $n^{\log_b a} = n^{\log_2 2} = n^1 = n$

Step 3 – compare $f(n)$ with $n^{\log_b a}$: $f(n) = n^2$ grows polynomially *faster* than n (by a factor n^1), which is Master-theorem *Case 3*.

Step 4 – check the regularity condition: $2\,(n/2)^2 = \dfrac{n^2}{2} \le c\,n^2$ with $c = \frac{1}{2} < 1$, so Case 3 applies.

Step 5 – conclude: $T(n) = \Theta\big(f(n)\big) = \Theta(n^2)$

Final Answer: $T(n) = \Theta(n^2) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q28

Q29.

Solution

Concept – size of a set union (inclusion–exclusion): $|R \cup S| = |R| + |S| - |R \cap S|$, since relations are sets and duplicates are counted once.

Step 1 – list the sizes: $|R| = 8$, $|S| = 5$, $|R \cap S| = 3$.

Step 2 – add the two relation sizes: $8 + 5 = 13$

Step 3 – subtract the common tuples once: $13 - 3 = 10$

Final Answer: $|R \cup S| = 10 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q29

Q30.

Solution

Concept – `COUNT(*)` counts rows, NULLs included: Unlike `COUNT(column)`, the form `COUNT(*)` counts every row regardless of NULLs.

Step 1 – total rows in the table: There are 10 rows.

Step 2 – effect of the NULLs: The 3 NULLs are in a *column*; `COUNT(*)` does not look at column values, so it ignores them.

Step 3 – read the count: `COUNT(*)` returns the full row count, 10.

Final Answer: `COUNT(*)` returns $10 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q30

Q31.

Solution

Concept – BCNF test: R is in BCNF if, for every non-trivial functional dependency $X \to Y$, X is a superkey.

Step 1 – list the dependencies: The only non-trivial FD is $A \to BCD$.

Step 2 – check whether the determinant is a superkey: A is the candidate key, hence a superkey; the left side of the only FD is a superkey.

Step 3 – apply the BCNF condition: Every FD has a superkey on its left, so R is in BCNF.

Step 4 – note the hierarchy: BCNF is stronger than 3NF and 2NF, so BCNF is the *highest* form here.

Final Answer: highest form $=$ BCNF $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q31

Q32.

Solution

Concept – natural join matches tuples on the common attribute: Combine a tuple of R with a tuple of S whenever their B values are equal.

Step 1 – match R's tuple $(1, x)$: S has (x, p) and (x, q) with $B = x$, giving $(1, x, p)$ and $(1, x, q)$ – 2 tuples.

Step 2 – match R's tuple $(2, x)$: S again has (x, p) and (x, q), giving $(2, x, p)$ and $(2, x, q)$ – 2 tuples.

Step 3 – match R's tuple $(3, y)$**:** S has (y, r) with $B = y$, giving $(3, y, r)$ – 1 tuple.

Step 4 – note the unmatched S **tuple:** (z, s) has $B = z$, which no R tuple carries, so it contributes 0.

Step 5 – total the matches: $2 + 2 + 1 = 5$

Final Answer: 5 tuples $\Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q32**

Q33.

Solution

Concept – OLAP cube operations: *Slice* fixes one dimension to a single value, producing a lower-dimensional sub-cube.

Step 1 – describe the action: Selecting only `Region = 'East'` pins the Region dimension to one value.

Step 2 – match the operation: Fixing a single dimension value is a **slice**.

Why the other options are wrong: Roll-up and drill-down change the level of aggregation; pivot rotates the cube's axes; none of them fixes a single dimension value the way slice does.

Final Answer: slice $\Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q33**

Q34.

Solution

Concept – order of a B-tree bounds the number of children: An internal node of a B-tree of order m can have at most m child pointers.

Step 1 – read the order: $m = 4$.

Step 2 – apply the definition: Maximum number of child pointers $= m = 4$.

Step 3 – cross-check with the keys: The number of keys is one fewer, $m - 1 = 3$, matching the figure.

Final Answer: 4 child pointers $\Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q34**

Q35.

Solution

Concept – attribute closure: $\{A, D\}^+$ is the set of all attributes determined by $\{A, D\}$, found by repeatedly applying the FDs.

Step 1 – start with the given attributes: $\{A, D\}^+ = \{A, D\}$

Step 2 – apply $A \to B$**:** A is present, so add B: $\{A, B, D\}$.

Step 3 – apply $B \to C$**:** B is now present, so add C: $\{A, B, C, D\}$.

Step 4 – apply $D \to E$**:** D is present, so add E: $\{A, B, C, D, E\}$.

Step 5 – check for closure: All five attributes are present; no further FD adds anything.

Final Answer: $\{A, D\}^+ = \{A, B, C, D, E\} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q35

Q36.

Solution

Concept – precision: Precision $= \frac{\text{TP}}{\text{TP} + \text{FP}}$, the fraction of predicted positives that are truly positive.

Step 1 – read the required counts: $\text{TP} = 40$, $\text{FP} = 10$.

Step 2 – form the denominator: $\text{TP} + \text{FP} = 40 + 10 = 50$

Step 3 – substitute: Precision $= \frac{40}{50}$

Step 4 – simplify: $= 0.80$

Final Answer: precision $= 0.80 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q36

Q37.

Solution

Concept – recall (sensitivity): Recall $= \frac{\text{TP}}{\text{TP} + \text{FN}}$, the fraction of actual positives detected.

Step 1 – read the counts: $\text{TP} = 30$ (detected), $\text{FN} = 10$ (missed).

Step 2 – form the denominator: $\text{TP} + \text{FN} = 30 + 10 = 40$

Step 3 – substitute: Recall $= \frac{30}{40}$

Step 4 – simplify: $= 0.75$

Final Answer: recall $= 0.75 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q37

Q38.

Solution

Concept – perceptron with a step activation: Compute the weighted sum plus bias, then output 1 if it exceeds 0, else 0.

Step 1 – weighted sum of inputs: $w_1x_1 + w_2x_2 = (1)(1) + (-1)(1)$

$= 1 - 1$

$= 0$

Step 2 – add the bias: $z = 0 + 0.5 = 0.5$

Step 3 – apply the step rule: $z = 0.5 > 0$, so the output is 1.

Final Answer: output $= 1 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q38

Q39.

Solution

Concept – Gini impurity: Gini $= 1 - \sum_i p_i^2$, where p_i is the fraction of class i.

Step 1 – compute the class fractions: $p_+ = \frac{3}{4} = 0.75$ and $p_- = \frac{1}{4} = 0.25$.

Step 2 – square each fraction: $0.75^2 = 0.5625$ and $0.25^2 = 0.0625$.

Step 3 – sum the squares: $0.5625 + 0.0625 = 0.625$

Step 4 – subtract from 1: Gini $= 1 - 0.625 = 0.375$

Final Answer: Gini $= 0.375 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q39

Q40.

Solution

Concept – information gain with pure children: IG $= H(\text{parent}) - \sum_{\text{children}} \frac{n_{\text{child}}}{n_{\text{parent}}} H(\text{child})$; a pure child has entropy 0.

Step 1 – parent entropy: The parent has $10+$ and $10-$ (balanced), so

$H(\text{parent}) = 1.0$ bit.

Step 2 – entropy of each child: $A = 0$ is all positive and $A = 1$ is all negative; each child is pure, so each has entropy 0.

Step 3 – weighted child entropy: $\frac{10}{20}(0) + \frac{10}{20}(0) = 0$

Step 4 – subtract from the parent entropy: $\text{IG} = 1.0 - 0 = 1.0$

Final Answer: $\text{IG} = 1.0$ bit $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q40

Q41.

Solution

Concept – distance from a point to a line: $d = \dfrac{|ax_1 + bx_2 + c|}{\sqrt{a^2 + b^2}}$ for the line $ax_1 + bx_2 + c = 0$.

Step 1 – read the coefficients: $a = 3$, $b = 4$, $c = -10$; the point is $(x_1, x_2) = (4, 2)$.

Step 2 – evaluate the numerator: $3(4) + 4(2) - 10 = 12 + 8 - 10 = 10$, and $|10| = 10$.

Step 3 – evaluate the denominator: $\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Step 4 – divide: $d = \frac{10}{5} = 2$

Final Answer: $d = 2 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q41

Q42.

Solution

Concept – k-means assignment (nearest centroid): Assign each point to the centroid with the smallest Euclidean distance.

Step 1 – distance from $p = (2, 2)$ to $c_1 = (1, 1)$: $\sqrt{(2-1)^2 + (2-1)^2} = \sqrt{1+1} = \sqrt{2} = 1.41$

Step 2 – distance from $p = (2, 2)$ to $c_2 = (5, 5)$: $\sqrt{(2-5)^2 + (2-5)^2} = \sqrt{9+9} = \sqrt{18} = 4.24$

Step 3 – compare the distances: $1.41 < 4.24$, so c_1 is nearer.

Step 4 – assign the point: p is assigned to centroid $c_1 = (1, 1)$.

Final Answer: nearest centroid $= c_1 = (1, 1) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q42

Q43.

Solution

Concept – variance explained in PCA: Each principal component explains a fraction of variance equal to its eigenvalue divided by the total.

Step 1 – total variance (sum of all eigenvalues): $5 + 3 + 1 + 1 = 10$

Step 2 – variance in the first component: The largest eigenvalue is 5.

Step 3 – form the fraction: $\frac{5}{10} = 0.50$

Step 4 – express as a percentage: $0.50 = 50\%$

Final Answer: 50% of the variance $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q43

Q44.

Solution

Concept – neuron forward pass with sigmoid: Compute the weighted sum plus bias, then apply $\sigma(z) = \frac{1}{1+e^{-z}}$.

Step 1 – weighted sum of inputs: $w_1x_1 + w_2x_2 = (2)(1) + (-1)(1)$

$= 2 - 1$

$= 1$

Step 2 – add the bias: $z = 1 + (-1) = 0$

Step 3 – apply the sigmoid at $z = 0$: $\sigma(0) = \frac{1}{1+e^0} = \frac{1}{1+1}$

Step 4 – simplify: $\sigma(0) = \frac{1}{2} = 0.5$

Final Answer: output $= 0.5 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q44

Q45.

Solution

Concept – A* expands the node of least $f = g + h$: Compute f for each frontier node and pick the minimum.

Step 1 – f for node A: $f(A) = 4 + 5 = 9$

Step 2 – f for node B: $f(B) = 6 + 2 = 8$

Step 3 – f for node C: $f(C) = 3 + 6 = 9$

Step 4 – f for node D: $f(D) = 5 + 1 = 6$

Step 5 – choose the smallest f: $\min(9, 8, 9, 6) = 6$, which belongs to node D.

Final Answer: A* expands $D \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q45

Q46.

Solution

Concept – minimax back-up: MIN nodes take the minimum of their children; the MAX root takes the maximum of the MIN results.

Step 1 – evaluate the left MIN node: $\min(7, 4) = 4$

Step 2 – evaluate the right MIN node: $\min(6, 8) = 6$

Step 3 – evaluate the MAX root: $\max(4, 6) = 6$

Final Answer: minimax value $= 6 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q46

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	D	4	B	5	A
6	B	7	C	8	B	9	A	10	A
11	C	12	D	13	A	14	B	15	B
16	D	17	C	18	A	19	C	20	D
21	A	22	D	23	A	24	A	25	D
26	A	27	D	28	B	29	C	30	A
31	B	32	D	33	B	34	C	35	C
36	A	37	C	38	B	39	D	40	C
41	D	42	A	43	B	44	C	45	D
46	B								