GATE Data Science and Artificial Intelligence

Sample Paper – 8

Duration: 128 Minutes **Maximum Marks: 72**

Instructions

- This paper contains **46 questions** covering the core **Data Science and Artificial Intelligence** subject of the GATE paper conducted by the IITs and IISc (General Aptitude and Engineering Mathematics are provided as separate papers).
- **Q1–Q20 carry 1 mark each** and **Q21–Q46 carry 2 marks each**, for a total of **72 marks**.
- Each question carries **negative marking of one-third of its allotted marks**: a wrong 1-mark question costs **0.33** and a wrong 2-mark question costs **0.67**. Unattempted questions carry no penalty.
- Every question here is a single-correct **MCQ**. The real GATE paper also has Multiple Select (MSQ) and Numerical Answer Type (NAT) questions, and **those carry no negative marking**.
- Attempt this paper in one timed sitting of about **128 minutes**, the share this core subject earns at GATE's overall pace of 180 minutes for 65 questions. All logarithms in information-theoretic quantities (entropy, information gain) are to base 2 and expressed in bits unless stated otherwise.

Probability & Statistics

Q1. A bag contains 3 red and 5 blue balls. Two balls are drawn one after the other *without* replacement. The probability that *both* drawn balls are blue is: **[1 mark]**

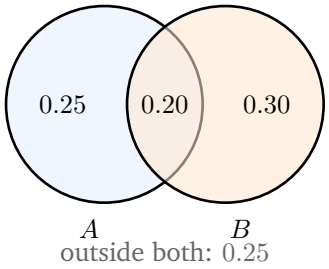
(A) $\dfrac{25}{64}$

(B) $\dfrac{3}{28}$

(C) $\dfrac{5}{14}$

(D) $\frac{5}{8}$

Q2. For two events A and B the probabilities of the disjoint regions are shown on the Venn diagram below (the region outside both circles has probability 0.25). The probability $P(A \cup B)$ is: **[1 mark]**



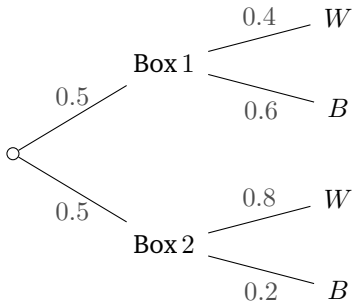
(A) 0.75

(B) 0.45

(C) 0.20

(D) 0.55

Q3. Two boxes are given: Box 1 holds 2 white and 3 black balls, and Box 2 holds 4 white and 1 black ball. A box is chosen at random (each with probability 0.5) and one ball is drawn, as shown in the probability tree. Given that the drawn ball is *white*, the probability that it came from Box 1 is: **[1 mark]**



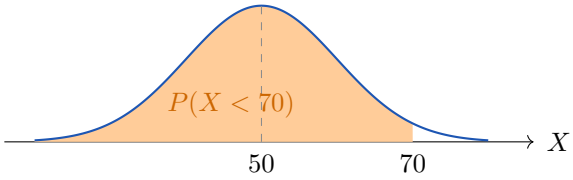
(A) 0.50

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) 0.40

Q4. A feature X is normally distributed with mean $\mu = 50$ and standard deviation $\sigma = 10$. Using the shaded left region of the standard normal curve shown, the probability $P(X < 70)$ is nearest to: **[1 mark]**



(A) 0.0228

(B) 0.5000

(C) 0.9772

(D) 0.8413

Q5. Defects occur along a production line following a Poisson process at an average rate of 2 defects per metre of cloth. The probability of finding *exactly zero* defects in a given one-metre length is nearest to (take $e^{-2} = 0.1353$): **[1 mark]**

(A) 0.2707

(B) 0.8647

(C) 0.4060

(D) 0.1353

Q6. A discrete random variable X takes the values $-1,\ 0,\ 1$ with probabilities $0.3,\ 0.4,\ 0.3$ respectively. The value of $E[X^2]$ is: **[1 mark]**

(A) 0.0

(B) 0.3

(C) 0.6

(D) 1.0

Q7. A sample of size $n = 64$ drawn from a population with known standard deviation $\sigma = 16$ has sample mean $\bar{x} = 40$. For a 95% confidence interval for the population mean (using $z_{0.025} = 1.96$), the margin of error (half-width of the interval) is: **[1 mark]**

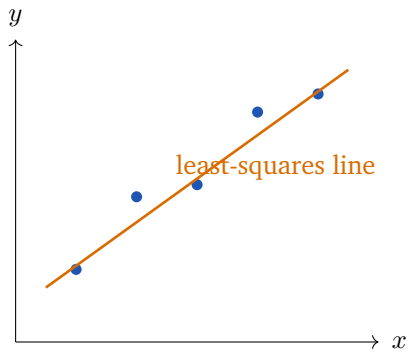
(A) 1.96

(B) 16.00

(C) 2.00

(D) 3.92

Q8. For a bivariate data set the sums of squares and cross-products about the means are $S_{xx} = 25$, $S_{yy} = 100$ and $S_{xy} = 30$, and the scatter with its least-squares line is shown. The Pearson correlation coefficient r between x and y is: **[1 mark]**



(A) 1.20

(B) 0.30

(C) 0.36

(D) 0.60

Q9. A fair coin is tossed 5 times independently. The probability of obtaining *exactly* 2 heads is: **[1 mark]**

(A) 0.5000

(B) 0.15625

(C) 0.3125

(D) 0.2500

Q10. The time until the next request arrives at a web server follows an exponential distribution with a mean of 4 seconds. The probability that the next request arrives *within* 4 seconds is (take $e^{-1} = 0.3679$): **[1 mark]**

(A) 0.368

(B) 0.632

(C) 0.135

(D) 0.865

Linear Algebra, Calculus & Optimization

Q11. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 3 & 6 & 3 \end{bmatrix}$. The rank of A is: **[1 mark]**

(A) 1

(B) 2

(C) 3

(D) 0

Q12. The determinant of the matrix $B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 3 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ is: **[1 mark]**

(A) 3

(B) 7

(C) 11

(D) 5

Q13. The *smaller* eigenvalue of the matrix $C = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ is: **[1 mark]**

(A) 4

(B) 1

(C) 5

(D) 2

Q14. For a real matrix A, the matrix $A^{\mathsf{T}}A$ has eigenvalues 16 and 9. The Frobenius norm $\|A\|_F = \sqrt{\mathrm{tr}(A^{\mathsf{T}}A)}$ is: **[1 mark]**

(A) 5

(B) 25

(C) 7

(D) 4

Q15. For the vector $v = (6,\ 8)$, the Euclidean (ℓ_2) norm $\|v\|_2$ is: **[1 mark]**

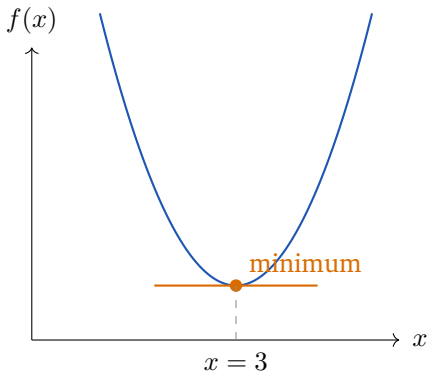
(A) 14

(B) 10

(C) 100

(D) 48

Q16. The function $f(x) = x^2 - 6x + 13$ is minimized over the real line, as suggested by its convex graph below. The minimum *value* of f is: **[1 mark]**



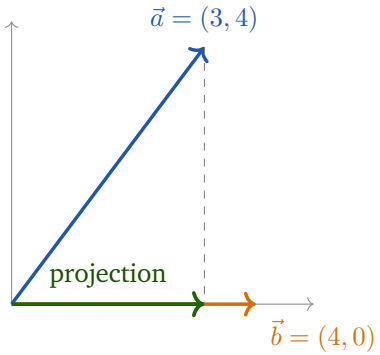
(A) 3

(B) 13

(C) 4

(D) 0

Q17. The scalar projection (component) of $\vec{a} = (3, 4)$ onto $\vec{b} = (4, 0)$, as shown, is: **[1 mark]**



(A) 4

(B) 3

(C) 5

(D) 7

Q18. Gradient descent is used to minimize $f(x) = (x-3)^2$ with a fixed learning rate $\eta = 0.2$. Starting from $x_0 = 0$, the value of x_1 after one update $x_1 = x_0 - \eta\, f'(x_0)$ is: **[1 mark]**

(A) 1.2

(B) -1.2

(C) 0.6

(D) 6.0

Q19. Consider the function $f(x, y) = 2x^2+3y^2+xy$ on $\mathbb{R}^2$. Based on its Hessian, the function is: **[1 mark]**

(A) concave everywhere

(B) neither convex nor concave

(C) convex everywhere

(D) linear (affine)

Programming, Data Structures & Algorithms

Q20. What is the output of the following Python snippet?

```
a = [1, 2, 3, 4, 5]
print(a[::2])
```

[1 mark]

(A) `[2, 4]`

(B) `[1, 3, 5]`

(C) `[1, 2, 3]`

(D) `[5, 3, 1]`

Q21. What is the output of the following Python snippet?

```
x = "hello"
print(x[::-1])
```

[2 marks]

(A) `olleh`

(B) `hello`

(C) `hell`

(D) raises an error

Q22. The following operations are performed on an initially empty *queue*, in order: `enqueue(1)`, `enqueue(2)`, `dequeue()`, `enqueue(3)`, `enqueue(4)`, `dequeue()`, `dequeue()`. The value returned by the *last* `dequeue()` is: [2 marks]

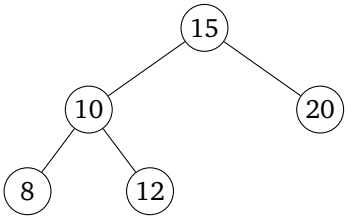
(A) 1

(B) 4

(C) 3

(D) 2

Q23. Starting from an empty binary search tree, the keys 15, 10, 20, 8, 12 are inserted in the given order, producing the tree shown. The number of *leaf* nodes in the resulting tree is: [2 marks]



(A) 2

(B) 3

(C) 4

(D) 5

Q24. A hash table of size 7 uses the hash function $h(k) = k \bmod 7$ with *linear probing* for collision resolution. The keys $15, 22, 8$ are inserted in this order into an otherwise empty table. The final index occupied by key 8 is: **[2 marks]**

(A) 1

(B) 2

(C) 3

(D) 8

Q25. Binary search is performed on a sorted array of 63 distinct elements. The maximum number of key comparisons required in the worst case is: **[2 marks]**

(A) 63

(B) 32

(C) 6

(D) 7

Q26. The worst-case time complexity of the standard Merge Sort algorithm on an array of n elements is: **[2 marks]**

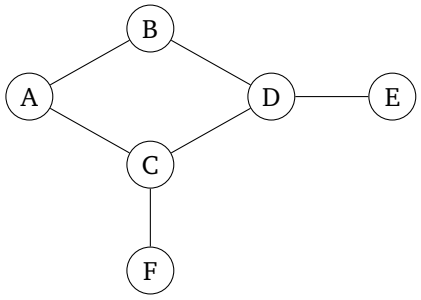
(A) $O(n^2)$

(B) $O(n)$

(C) $O(\log n)$

(D) $O(n \log n)$

Q27. A depth-first search (DFS) is started from vertex A in the undirected graph shown, always visiting adjacent vertices in increasing alphabetical order. The order in which the vertices are first visited is: **[2 marks]**



(A) A, B, C, D, E, F

(B) A, C, B, D, F, E

(C) A, B, D, E, C, F

(D) A, B, D, C, F, E

Q28. The running time of an algorithm satisfies the recurrence $T(n) = 4\,T(n/2) + n$, with $T(1) = 1$. By the Master theorem, $T(n)$ is: **[2 marks]**

(A) $\Theta(n^2)$

(B) $\Theta(n \log n)$

(C) $\Theta(n)$

(D) $\Theta(n^2 \log n)$

Database Management & Warehousing

Q29. Union-compatible relations R and S contain 8 tuples and 5 tuples respectively. The *maximum* possible number of tuples in $R \cup S$ is: **[2 marks]**

(A) 13

(B) 40

(C) 8

(D) 3

Q30. A table `T` has 6 rows, and its column `c` holds the values 10, 10, 20, `NULL`, 30, 20 (one value per row). What does the query `SELECT COUNT(DISTINCT c) FROM T;` return? **[2 marks]**

(A) 6

(B) 5

(C) 3

(D) 4

Q31. A relation $R(A, B, C)$ has the functional dependencies $AB \rightarrow C$ and $C \rightarrow B$. Its candidate keys are AB and AC. The highest normal form that R satisfies is: **[2 marks]**

(A) BCNF

(B) 3NF

(C) 2NF

(D) 1NF

Q32. Relation $R(A, B)$ contains the tuples $\{(1, p), (2, p), (3, q)\}$ and relation $S(B, C)$ contains $\{(p, x), (p, y), (q, z)\}$. The number of tuples in the natural join $R \bowtie S$ (on the common attribute B) is: **[2 marks]**

(A) 3

(B) 6

(C) 9

(D) 5

Q33. In OLAP, the operation that selects a single value for one dimension of a data cube (for example, fixing `Year` $= 2023$), thereby reducing the cube's dimensionality by one, is called: **[2 marks]**

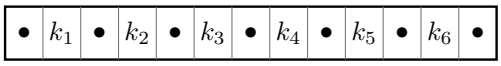
(A) roll-up

(B) drill-down

(C) dice

(D) slice

Q34. The figure shows the structure of an internal node of a B^+ tree of order $m = 7$ (each internal node holds up to m child pointers, drawn as $\bullet$, separated by keys). The maximum number of *keys* an internal node of this tree can hold is: **[2 marks]**



up to 7 pointers (•) and 6 keys

(A) 6

(B) 7

(C) 5

(D) 3

Q35. A relation has attributes $\{A, B, C, D, E\}$ with functional dependencies $A \to B$ and $B \to C$. The attribute closure A^+ is: **[2 marks]**

(A) $\{A, B, C\}$

(B) $\{A, B, C, D, E\}$

(C) $\{A\}$

(D) $\{A, B\}$

Machine Learning & Artificial Intelligence

Q36. A logistic regression model has weight vector $w = (2, -1)$ and bias $b = 0$, and uses the sigmoid $\sigma(z) = 1/(1 + e^{-z})$. For the input $x = (1, 2)$, the predicted probability $\sigma(w^{\mathsf{T}}x + b)$ is: **[2 marks]**

(A) 0.73

(B) 0.88

(C) 0.27

(D) 0.50

Q37. A very simple model attains *low* accuracy on the training set *and* similarly low accuracy on unseen test data. In terms of the bias–variance decomposition, this behaviour (underfitting) is best described as: **[2 marks]**

(A) high bias, low variance

(B) low bias, high variance

(C) low bias, low variance

(D) high bias, high variance

Q38. A 1-nearest-neighbour classifier (Euclidean distance) is trained on the labelled points $A(1,1)$: Class 0, $B(5,5)$: Class 1, $C(2,2)$: Class 0, $D(6,6)$: Class 1. The query point $Q(4,4)$ is classified as: **[2 marks]**

(A) Class 1

(B) Class 0

(C) a tie (undecided)

(D) cannot be determined

Q39. A training set for a binary classification task contains 10 examples, of which 5 are positive and 5 are negative. The entropy of this set (in bits) is: **[2 marks]**

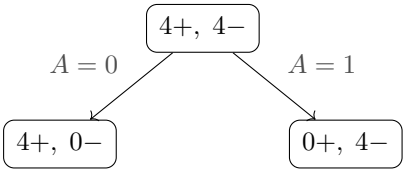
(A) 0.5

(B) 1.0

(C) 0.811

(D) 0.0

Q40. A node in a decision tree contains 4 positive and 4 negative examples. A candidate attribute A splits it into two children as shown: $A = 0$ gives $(4+,\, 0-)$ and $A = 1$ gives $(0+,\, 4-)$. The information gain of this split is: **[2 marks]**



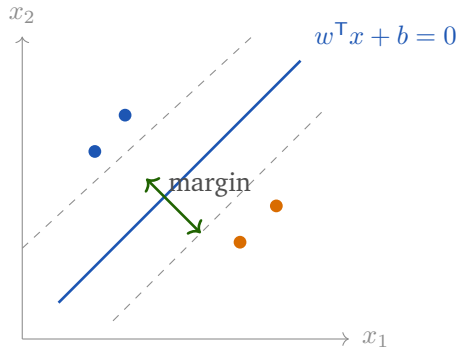
(A) 1.0

(B) 0.5

(C) 0.189

(D) 0.0

Q41. For a linear support vector machine the separating hyperplane $w^{\mathsf{T}}x + b = 0$ is shown with its two margin boundaries. If $\|w\| = 2$, the width of the margin, equal to $2/\|w\|$, is: **[2 marks]**



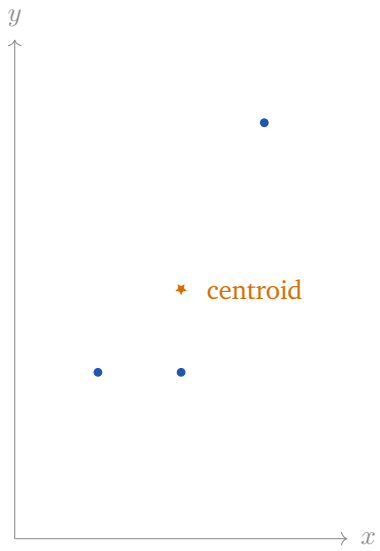
(A) 4

(B) 2

(C) 0.5

(D) 1

Q42. In one iteration of the k-means algorithm, a cluster is assigned the three points $(2, 4)$, $(4, 4)$ and $(6, 10)$, shown below. The updated centroid of this cluster is: **[2 marks]**



(A) $(4,\ 4)$

(B) $(4,\ 6)$

(C) $(6,\ 4)$

(D) $(12,\ 18)$

Q43. Principal component analysis is applied to a data set whose covariance matrix has eigenvalues $6,\ 2,\ 1,\ 1$. The fraction of the total variance explained by the *first* principal component alone is: **[2 marks]**

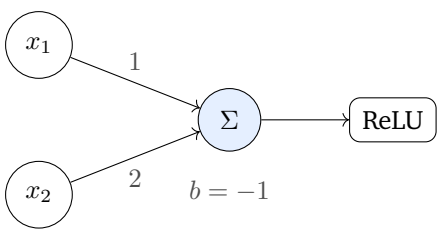
(A) 80%

(B) 20%

(C) 40%

(D) 60%

Q44. A single neuron with a ReLU activation computes $\text{ReLU}(w_1x_1 + w_2x_2 + b)$, as shown, where $\text{ReLU}(z) = \max(0, z)$. With inputs $x_1 = 2$, $x_2 = 1$, weights $w_1 = 1$, $w_2 = 2$ and bias $b = -1$, the output of the neuron is: **[2 marks]**



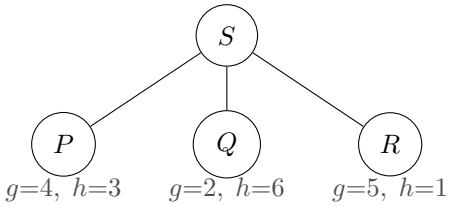
(A) 0

(B) 3

(C) 5

(D) 1

Q45. A* search maintains three frontier nodes P, Q and R with g (path cost so far) and h (heuristic) values shown in the tree below, where $f = g + h$. The node A* selects for expansion next is: **[2 marks]**



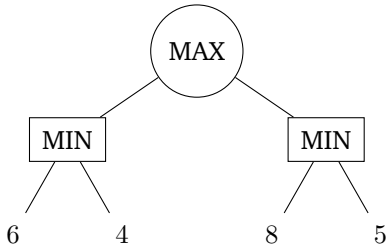
(A) P

(B) Q

(C) a tie (all equal)

(D) R

Q46. In the two-ply game tree shown, the root is a MAX node and the second level are MIN nodes; the leaf values are given. The minimax value backed up to the root is: **[2 marks]**



(A) 4

(B) 6

(C) 5

(D) 8

Detailed Solutions

Q1.

Solution

Concept – probability without replacement: Multiply the probability of the first blue by the conditional probability of the second blue.

Step 1 – count the balls: Total $= 3 + 5 = 8$ balls, of which 5 are blue.

Step 2 – probability the first ball is blue: $P(\text{1st blue}) = \frac{5}{8}$

Step 3 – probability the second is blue given the first was blue: Now 4 blue remain out of 7 balls: $P(\text{2nd blue} \mid \text{1st blue}) = \frac{4}{7}$

Step 4 – multiply: $P(\text{both blue}) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

Step 5 – simplify: $\frac{20}{56} = \frac{5}{14}$

Final Answer: $\frac{5}{14} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q1

Q2.

Solution

Concept – union from disjoint Venn regions: $P(A \cup B)$ is the sum of the "only A", "only B" and "both" regions.

Step 1 – read the three regions inside the circles: Only $A = 0.25$, both $= 0.20$, only $B = 0.30$.

Step 2 – add them: $P(A \cup B) = 0.25 + 0.20 + 0.30$

Step 3 – evaluate: $P(A \cup B) = 0.75$

Step 4 – sanity check the whole sample space: $0.75 + 0.25$ (outside) $= 1.00$, so the diagram is consistent.

Final Answer: $P(A \cup B) = 0.75 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q2

Q3.

Solution

Concept – Bayes' theorem (reverse the tree): $P(B_1 \mid W) = \dfrac{P(B_1)\,P(W \mid B_1)}{P(B_1)\,P(W \mid B_1) + P(B_2)\,P(W \mid B_2)}$.

Step 1 – white-ball probability from each box: Box 1: $\dfrac{2}{5} = 0.4$; Box 2: $\dfrac{4}{5} = 0.8$.

Step 2 – probability from the Box 1 branch: $P(B_1)\,P(W \mid B_1) = 0.5 \times 0.4 = 0.20$

Step 3 – probability from the Box 2 branch: $P(B_2)\,P(W \mid B_2) = 0.5 \times 0.8 = 0.40$

Step 4 – total probability of drawing white: $P(W) = 0.20 + 0.40 = 0.60$

Step 5 – apply Bayes' theorem: $P(B_1 \mid W) = \dfrac{0.20}{0.60} = \dfrac{1}{3}$

Final Answer: $P(B_1 \mid W) = \dfrac{1}{3} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q3

Q4.

Solution

Concept – standardize, then use the normal table: Convert X to the standard normal $Z = \dfrac{X - \mu}{\sigma}$.

Step 1 – standardize the cut-off $X = 70$: $Z = \dfrac{70 - 50}{10}$

$Z = \dfrac{20}{10} = 2$

Step 2 – write the required probability in Z: $P(X < 70) = P(Z < 2)$

Step 3 – use the standard normal value: $P(Z < 2) = 0.9772$

Step 4 – interpret the shaded region: The shaded area is the whole left region up to $Z = 2$, which is 0.9772.

Final Answer: $P(X < 70) = 0.9772 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q4

Q5.

Solution

Concept – Poisson probability: $P(X = k) = \dfrac{e^{-\lambda}\,\lambda^k}{k!}$, with mean rate λ.

Step 1 – list the parameters: $\lambda = 2$ per metre, $k = 0$.

Step 2 – substitute into the formula: $P(X=0) = \dfrac{e^{-2}\,2^0}{0!}$

Step 3 – evaluate the power and factorial: $2^0 = 1$ and $0! = 1$

$P(X=0) = e^{-2}$

Step 4 – substitute $e^{-2} = 0.1353$**:** $P(X=0) = 0.1353$

Final Answer: $P(X=0) = 0.1353 \Rightarrow$ D

Answer: (D) Go Back to Q5

Q6.

Solution

Concept – expectation of a function of X**:** $E[X^2] = \sum_i x_i^2\, P(x_i)$.

Step 1 – square each value: $(-1)^2 = 1, \quad 0^2 = 0, \quad 1^2 = 1.$

Step 2 – weight each square by its probability: $1(0.3) + 0(0.4) + 1(0.3)$

Step 3 – evaluate each term: $= 0.3 + 0 + 0.3$

Step 4 – add: $E[X^2] = 0.6$

Final Answer: $E[X^2] = 0.6 \Rightarrow$ C

Answer: (C) Go Back to Q6

Q7.

Solution

Concept – margin of error of a confidence interval: For a known-σ mean, the margin of error is $E = z\,\dfrac{\sigma}{\sqrt{n}}$.

Step 1 – compute $\sqrt{n}$**:** $\sqrt{64} = 8$

Step 2 – compute the standard error: $\dfrac{\sigma}{\sqrt{n}} = \dfrac{16}{8} = 2$

Step 3 – multiply by the critical value: $E = 1.96 \times 2$

Step 4 – evaluate: $E = 3.92$

Step 5 – interpret: The 95% interval is 40 ± 3.92, i.e. $(36.08,\ 43.92)$; its half-width is 3.92.

Final Answer: margin of error $= 3.92 \Rightarrow$ D

Answer: (D) Go Back to Q7

Q8.

Solution

Concept – Pearson correlation coefficient: $r = \dfrac{S_{xy}}{\sqrt{S_{xx}\,S_{yy}}}$.

Step 1 – list the sums of squares: $S_{xx} = 25$, $S_{yy} = 100$, $S_{xy} = 30$.

Step 2 – compute the product under the root: $S_{xx}\,S_{yy} = 25 \times 100 = 2500$

Step 3 – take the square root: $\sqrt{2500} = 50$

Step 4 – form the ratio: $r = \dfrac{30}{50}$

Step 5 – simplify: $r = 0.60$

Why the other options are wrong: 1.20 is the slope $S_{xy}/S_{xx} = 30/25$, not the correlation, and $|r|$ can never exceed 1.

Final Answer: $r = 0.60 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept – binomial probability: $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$.

Step 1 – list the parameters: $n = 5$, $k = 2$, $p = 0.5$, $1 - p = 0.5$.

Step 2 – evaluate the binomial coefficient: $\binom{5}{2} = \dfrac{5 \times 4}{2 \times 1} = 10$

Step 3 – combine the powers: $p^2(1-p)^3 = (0.5)^2(0.5)^3 = (0.5)^5$

Step 4 – evaluate the power: $(0.5)^5 = \dfrac{1}{32} = 0.03125$

Step 5 – multiply: $P(X = 2) = 10 \times 0.03125 = 0.3125$

Final Answer: $P(X = 2) = 0.3125 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q9

Q10.

Solution

Concept – exponential cumulative probability: For an exponential variable with rate λ, $P(X \le t) = 1 - e^{-\lambda t}$.

Step 1 – find the rate from the mean: Mean $= \dfrac{1}{\lambda} = 4$ seconds, so $\lambda = \dfrac{1}{4} = 0.25\ \text{s}^{-1}$.

Step 2 – form the exponent: $\lambda t = 0.25 \times 4 = 1$

Step 3 – write the cumulative probability: $P(X \leq 4) = 1 - e^{-1}$

Step 4 – substitute $e^{-1} = 0.3679$**:** $P(X \leq 4) = 1 - 0.3679 = 0.6321$

Final Answer: $P(X < 4) \approx 0.632 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept – rank is the number of independent rows: Reduce the rows and count how many are linearly independent.

Step 1 – compare rows 2 **and** 1**:** Row $2 = (2, 4, 2) = 2 \times (1, 2, 1) = 2\,R_1$, so row 2 depends on row 1.

Step 2 – compare rows 3 **and** 1**:** Row $3 = (3, 6, 3) = 3 \times (1, 2, 1) = 3\,R_1$, so row 3 also depends on row 1.

Step 3 – eliminate the dependent rows: $R_2 \leftarrow R_2 - 2R_1 = (0, 0, 0)$ and $R_3 \leftarrow R_3 - 3R_1 = (0, 0, 0)$.

Step 4 – count the non-zero independent rows: Only R_1 remains non-zero, giving rank $= 1$.

Final Answer: $\text{rank}(A) = 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q11

Q12.

Solution

Concept – cofactor expansion along the first row: $\det B = b_{11}M_{11} - b_{12}M_{12} + b_{13}M_{13}$.

Step 1 – first-row entries: $b_{11} = 1$, $b_{12} = 2$, $b_{13} = 0$.

Step 2 – minor for b_{11}**:** $\begin{vmatrix} 3 & 1 \\ 0 & 1 \end{vmatrix} = (3)(1) - (1)(0) = 3$

Step 3 – minor for b_{12}**:** $\begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix} = (0)(1) - (1)(2) = -2$

Step 4 – the third term vanishes: Since $b_{13} = 0$, its contribution is 0.

Step 5 – combine with the sign pattern: $\det B = 1(3) - 2(-2) + 0$

$= 3 + 4$

$= 7$

Final Answer: $\det B = 7 \Rightarrow$ B

Answer: (B) Go Back to Q12

Q13.

Solution

Concept – eigenvalues from trace and determinant: For a 2×2 matrix, $\lambda^2 - \mathrm{tr}(C)\,\lambda + \det(C) = 0$.

Step 1 – trace and determinant of C: $\mathrm{tr}(C) = 2 + 3 = 5$ and $\det(C) = (2)(3) - (2)(1) = 6 - 2 = 4$.

Step 2 – write the characteristic equation: $\lambda^2 - 5\lambda + 4 = 0$

Step 3 – factor: $(\lambda - 1)(\lambda - 4) = 0$

Step 4 – read the roots: $\lambda = 1$ or $\lambda = 4$.

Step 5 – pick the smaller: The smaller eigenvalue is 1.

Final Answer: $\lambda_{\min} = 1 \Rightarrow$ B

Answer: (B) Go Back to Q13

Q14.

Solution

Concept – Frobenius norm via the trace: $\|A\|_F = \sqrt{\mathrm{tr}(A^{\mathsf{T}}A)}$, and the trace equals the sum of the eigenvalues.

Step 1 – sum the eigenvalues of $A^{\mathsf{T}}A$: $\mathrm{tr}(A^{\mathsf{T}}A) = 16 + 9 = 25$

Step 2 – take the square root: $\|A\|_F = \sqrt{25}$

Step 3 – evaluate: $\|A\|_F = 5$

Why the other options are wrong: 25 is the trace itself, before taking the root; the singular values are $\sqrt{16} = 4$ and $\sqrt{9} = 3$, whose squares sum to 25.

Final Answer: $\|A\|_F = 5 \Rightarrow$ A

Answer: (A) Go Back to Q14

Q15.

Solution

Concept – the ℓ_2 norm is the square root of the sum of squares: $\|v\|_2 = \sqrt{\sum_i v_i^2}$.

Step 1 – square each component: $6^2 = 36$ and $8^2 = 64$.

Step 2 – add the squares: $36 + 64 = 100$

Step 3 – take the square root: $\|v\|_2 = \sqrt{100} = 10$

Step 4 – contrast with the other norms: $\|v\|_1 = 6 + 8 = 14$ and $\|v\|_\infty = \max(6, 8) = 8$; the ℓ_2 value is 10.

Final Answer: $\|v\|_2 = 10 \Rightarrow$ B

Answer: (B) Go Back to Q15

Q16.

Solution

Concept – minimize by setting the derivative to zero: For a convex parabola, the stationary point is the global minimum.

Step 1 – differentiate: $f'(x) = 2x - 6$

Step 2 – set the derivative to zero: $2x - 6 = 0$

$x = 3$

Step 3 – confirm it is a minimum: $f''(x) = 2 > 0$, so the point is a minimum.

Step 4 – evaluate f at $x = 3$: $f(3) = 3^2 - 6(3) + 13$

$= 9 - 18 + 13$

$= 4$

Final Answer: minimum value $= 4 \Rightarrow$ C

Answer: (C) Go Back to Q16

Q17.

Solution

Concept – scalar projection: The component of $\vec{a}$ along $\vec{b}$ is $\text{comp}_{\vec{b}}\,\vec{a} = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{b}\|}$.

Step 1 – compute the dot product: $\vec{a} \cdot \vec{b} = (3)(4) + (4)(0) = 12 + 0 = 12$

Step 2 – compute the magnitude of $\vec{b}$: $\|\vec{b}\| = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$

Step 3 – form the scalar projection: $\text{comp}_{\vec{b}}\,\vec{a} = \dfrac{12}{4}$

Step 4 – divide: $\text{comp}_{\vec{b}}\,\vec{a} = 3$

Step 5 – interpret: Since $\vec{b}$ points along the x-axis, the projection is just the x-component of $\vec{a}$, which is 3.

Final Answer: scalar projection $= 3 \Rightarrow$ B

Answer: (B) Go Back to Q17

Q18.

Solution

Concept – one gradient-descent step: $x_1 = x_0 - \eta\, f'(x_0)$.

Step 1 – differentiate the objective: $f(x) = (x-3)^2 \Rightarrow f'(x) = 2(x-3)$

Step 2 – evaluate the gradient at $x_0 = 0$: $f'(0) = 2(0-3) = -6$

Step 3 – apply the update with $\eta = 0.2$: $x_1 = 0 - 0.2 \times (-6)$

Step 4 – finish the arithmetic: $x_1 = 0 + 1.2 = 1.2$

Step 5 – sanity check the direction: The minimum is at $x = 3$; moving from 0 towards 1.2 heads in the correct direction.

Final Answer: $x_1 = 1.2 \Rightarrow$ A

Answer: (A) Go Back to Q18

Q19.

Solution

Concept – convexity via the Hessian: A twice-differentiable function is convex iff its Hessian is positive semi-definite everywhere.

Step 1 – compute the second partials: $f_{xx} = 4$, $f_{yy} = 6$, $f_{xy} = f_{yx} = 1$.

Step 2 – write the Hessian: $H = \begin{bmatrix} 4 & 1 \\ 1 & 6 \end{bmatrix}$

Step 3 – test with the leading principal minors: First minor: $4 > 0$. Second minor: $\det H = (4)(6) - (1)(1) = 24 - 1 = 23 > 0$.

Step 4 – conclude: Both leading minors are positive, so H is positive definite and f is convex everywhere.

Final Answer: f is convex $\Rightarrow$ C

Answer: (C) Go Back to Q19

Q20.

Solution

Concept – extended (step) slicing in Python: `a[start:stop:step]` takes every `step`-th element; `a[::2]` means start at 0, go to the end, step 2.

Step 1 – index the list: $\texttt{a} = [\underbrace{1}_{0}, \underbrace{2}_{1}, \underbrace{3}_{2}, \underbrace{4}_{3}, \underbrace{5}_{4}]$

Step 2 – apply the step of 2 from index 0: Take indices $0,\ 2,\ 4$.

Step 3 – read off the elements: Index $0 \to 1$, index $2 \to 3$, index $4 \to 5$, giving `[1, 3, 5]`.

Final Answer: `[1, 3, 5]` $\Rightarrow$ B

Answer: (B) Go Back to Q20

Q21.

Solution

Concept – reversing a sequence with a negative step: `x[::-1]` walks the sequence from the last element to the first, producing the reverse.

Step 1 – list the characters with their indices: `h`(0), `e`(1), `l`(2), `l`(3), `o`(4)

Step 2 – apply the step of -1: Start at the last index 4 and move backwards: $4, 3, 2, 1, 0$.

Step 3 – read off the characters in that order: `o`, `l`, `l`, `e`, `h`

Step 4 – join into a string: The result is `olleh`.

Final Answer: `olleh` $\Rightarrow$ A

Answer: (A) Go Back to Q21

Q22.

Solution

Concept – a queue is first-in, first-out (FIFO): `dequeue()` always removes the element that was enqueued earliest.

Step 1 – `enqueue(1)`, `enqueue(2)`**:** Queue (front→rear): $[1,\ 2]$.

Step 2 – `dequeue()`**:** Removes the front 1; queue becomes $[2]$.

Step 3 – `enqueue(3)`, `enqueue(4)`**:** Queue becomes $[2,\ 3,\ 4]$.

Step 4 – `dequeue()`**:** Removes the front 2; queue becomes $[3,\ 4]$.

Step 5 – `dequeue()` **(the last one):** Removes the front 3; this is the value returned.

Final Answer: last `dequeue()` returns $3 \Rightarrow$ C

Answer: (C) Go Back to Q22

Q23.

Solution

Concept – a leaf has no children: Build the BST and count nodes with no descendants.

Step 1 – place the root: 15 is inserted first and becomes the root.

Step 2 – place the next two keys: $10 < 15$ goes left; $20 > 15$ goes right.

Step 3 – place the remaining keys: $8 < 15$ then $8 < 10$ (left of 10); $12 < 15$ then $12 > 10$ (right of 10).

Step 4 – identify the leaves: The nodes with no children are $8,\ 12,\ 20$.

Step 5 – count them: There are 3 leaf nodes.

Final Answer: 3 leaves $\Rightarrow$ B

Answer: (B) Go Back to Q23

Q24.

Solution

Concept – linear probing scans the next free slot: On a collision at index i, try $i+1, i+2, \ldots$ (modulo table size).

Step 1 – insert 15**:** $h(15) = 15 \bmod 7 = 1$; slot 1 is free, so 15 goes to index 1.

Step 2 – insert 22**:** $h(22) = 22 \bmod 7 = 1$; index 1 is occupied, so probe index 2; it is free, so 22 goes to index 2.

Step 3 – insert 8**:** $h(8) = 8 \bmod 7 = 1$; index 1 occupied $\rightarrow$ probe index 2 (occupied) $\rightarrow$ probe index 3 (free).

Step 4 – place 8**:** 8 is stored at index 3.

Final Answer: index $3 \Rightarrow$ C

Answer: (C) Go Back to Q24

Q25.

Solution

Concept – binary search halves the range each step: The worst-case number of comparisons is $\lfloor \log_2 n \rfloor + 1$.

Step 1 – bracket $n = 63$ between powers of 2: $2^5 = 32$ and $2^6 = 64$, so $32 < 63 < 64$.

Step 2 – take the base-2 logarithm: $\log_2 63 \approx 5.98$

Step 3 – take the floor: $\lfloor 5.98 \rfloor = 5$

Step 4 – add one: $5 + 1 = 6$

Final Answer: 6 comparisons $\Rightarrow$ C

Answer: (C) Go Back to Q25

Q26.

Solution

Concept – Merge Sort recurrence: Merge Sort always splits the array in half and merges in linear time, giving $T(n) = 2\,T(n/2) + \Theta(n)$.

Step 1 – identify the recurrence parameters: $a = 2$, $b = 2$, $f(n) = \Theta(n)$.

Step 2 – compute the critical exponent: $n^{\log_b a} = n^{\log_2 2} = n$

Step 3 – compare $f(n)$ with $n^{\log_b a}$: $f(n) = \Theta(n) = \Theta(n^{\log_b a})$, which is Master-theorem Case 2.

Step 4 – apply Case 2: $T(n) = \Theta(n \log n)$

Why the other options are wrong: Unlike Quicksort, Merge Sort's split is always balanced, so it never degrades to $O(n^2)$; its worst case is still $O(n \log n)$.

Final Answer: $O(n \log n) \Rightarrow$ D

Answer: (D) Go Back to Q26

Q27.

Solution

Concept – DFS explores as deep as possible before backtracking: Use a stack (or recursion); always follow the smallest unvisited neighbour first.

Step 1 – adjacency lists (sorted): A: B, C; B: A, D; C: A, D, F; D: B, C, E; E: D; F: C.

Step 2 – start at A, go to B: Visit A, then its smallest neighbour B.

Step 3 – from B, go to D: B's unvisited neighbour is D. Visit D.

Step 4 – from D, go to C: D's smallest unvisited neighbour is C (since $C < E$). Visit C.

Step 5 – from C, go to F: C's unvisited neighbour is F. Visit F; it has no new neighbour, so backtrack.

Step 6 – backtrack to D, go to E: D's remaining unvisited neighbour is E. Visit E.

Step 7 – assemble the order: $A,\ B,\ D,\ C,\ F,\ E$.

Final Answer: $A,\ B,\ D,\ C,\ F,\ E \Rightarrow$ D

Answer: (D) Go Back to Q27

Q28.

Solution

Concept – Master theorem: For $T(n) = a\,T(n/b) + f(n)$, compare $f(n)$ with $n^{\log_b a}$.

Step 1 – identify the parameters: $a = 4$, $b = 2$, $f(n) = n$.

Step 2 – compute the critical exponent: $n^{\log_b a} = n^{\log_2 4} = n^2$

Step 3 – compare $f(n)$ with $n^{\log_b a}$: $f(n) = n = O(n^{2-\varepsilon})$ for $\varepsilon = 1$, so f is polynomially smaller: this is Case 1.

Step 4 – apply Case 1: $T(n) = \Theta(n^{\log_b a}) = \Theta(n^2)$

Final Answer: $T(n) = \Theta(n^2) \Rightarrow$ A

Answer: (A) Go Back to Q28

Q29.

Solution

Concept – size bounds of a set union: For union-compatible relations, $\max(|R|, |S|) \le |R \cup S| \le |R| + |S|$; the maximum occurs when the two sets are disjoint.

Step 1 – read the tuple counts: $|R| = 8$, $|S| = 5$.

Step 2 – disjoint case gives the maximum: If no tuple is shared, every tuple survives the union.

Step 3 – add: $|R \cup S|_{\max} = 8 + 5 = 13$

Why the other options are wrong: $40 = 8 \times 5$ is the Cartesian-product size, not a union; 8 is the *minimum* (when $S \subseteq R$).

Final Answer: 13 tuples $\Rightarrow$ A

Answer: (A) Go Back to Q29

Q30.

Solution

Concept – `COUNT(DISTINCT c)` counts distinct non-NULL values: It removes duplicates and ignores NULLs.

Step 1 – list the column values: 10, 10, 20, `NULL`, 30, 20.

Step 2 – drop the NULLs: The NULL is ignored, leaving 10, 10, 20, 30, 20.

Step 3 – keep only distinct values: The distinct values are $\{10, 20, 30\}$.

Step 4 – count them: There are 3 distinct non-null values.

Final Answer: `COUNT(DISTINCT c)` returns 3 $\Rightarrow$ C

Answer: (C) Go Back to Q30

Q31.

Solution

Concept – 3NF versus BCNF: 3NF allows a dependency $X \to Y$ if Y is a prime attribute; BCNF requires the left side X to be a superkey.

Step 1 – identify prime attributes: The candidate keys are AB and AC, so the prime attributes are A, B and C (all three).

Step 2 – check 2NF (partial dependency): No non-prime attribute exists, so there can be no partial dependency; R is in 2NF.

Step 3 – check 3NF for $C \to B$: B is a prime attribute, so the dependency $C \to B$ is allowed under 3NF. R is in 3NF.

Step 4 – check BCNF for $C \to B$: C alone is not a superkey (its closure $C^+ = \{B, C\}$ does not include A), so $C \to B$ violates BCNF.

Step 5 – conclude the highest normal form: R is in 3NF but not BCNF, so the highest normal form is 3NF.

Final Answer: highest form = 3NF $\Rightarrow$ B

Answer: (B) Go Back to Q31

Q32.

Solution

Concept – natural join matches tuples on the common attribute: Combine a tuple of R with a tuple of S whenever their B values are equal.

Step 1 – match R's tuple $(1, p)$: S has (p, x) and (p, y) with $B = p$, giving $(1, p, x)$ and $(1, p, y)$ – 2 tuples.

Step 2 – match R's tuple $(2, p)$: S again has (p, x) and (p, y) with $B = p$, giving $(2, p, x)$ and $(2, p, y)$ – 2 tuples.

Step 3 – match R's tuple $(3, q)$: S has (q, z) with $B = q$, giving $(3, q, z)$ – 1 tuple.

Step 4 – total the matches: $2 + 2 + 1 = 5$

Final Answer: 5 tuples $\Rightarrow$ D

Answer: (D) Go Back to Q32

Q33.

Solution

Concept – OLAP cube operations: Slice fixes one dimension to a single value; dice selects a sub-cube on several dimensions; roll-up and drill-down move between aggregation levels.

Step 1 – describe the operation: Fixing `Year` $=$ 2023 keeps only that one value of the `Year` dimension.

Step 2 – effect on dimensionality: Holding one dimension at a single value drops it from the result, cutting the cube's dimensionality by one.

Step 3 – match the name: Choosing a single value on one dimension is a **slice**.

Why the other options are wrong: Dice picks a range on *several* dimensions (dimensionality unchanged); roll-up and drill-down change the level of aggregation, not the number of dimensions.

Final Answer: slice $\Rightarrow$ D

Answer: (D) Go Back to Q33

Q34.

Solution

Concept – order and keys in a B$^+$ tree: An internal node of order m holds at most m child pointers, and the number of keys is always one fewer than the number of pointers.

Step 1 – maximum pointers: Order $m = 7 \Rightarrow$ up to 7 child pointers.

Step 2 – keys are one fewer than pointers: Maximum keys $= m - 1 = 7 - 1$

Step 3 – evaluate: $= 6$

Final Answer: 6 keys $\Rightarrow$ A

Answer: (A) Go Back to Q34

Q35.

Solution

Concept – attribute closure: A^+ is the set of all attributes functionally determined by A, found by repeatedly applying the FDs.

Step 1 – start with the attribute itself: $A^+ = \{A\}$

Step 2 – apply $A \to B$: A determines B, so $A^+ = \{A, B\}$.

Step 3 – apply $B \to C$: B is now in the closure, so it brings in C: $A^+ = \{A, B, C\}$.

Step 4 – check for more: No FD has D or E on its right-hand side reachable from these attributes, so nothing further is added.

Final Answer: $A^+ = \{A, B, C\} \Rightarrow$ A

Answer: (A) Go Back to Q35

Q36.

Solution

Concept – logistic regression prediction: Compute the linear score $z = w^{\mathsf{T}}x + b$, then apply the sigmoid.

Step 1 – compute the linear score: $z = (2)(1) + (-1)(2) + 0$

$= 2 - 2 + 0$

$= 0$

Step 2 – write the sigmoid: $\sigma(0) = \dfrac{1}{1 + e^{-0}}$

Step 3 – evaluate e^{-0}: $e^{-0} = 1$

Step 4 – finish: $\sigma(0) = \dfrac{1}{1+1} = \dfrac{1}{2} = 0.50$

Final Answer: $\sigma(0) = 0.50 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q36

Q37.

Solution

Concept – underfitting in bias–variance terms: A model too simple to capture the pattern has high bias; if it is also insensitive to the training sample, it has low variance.

Step 1 – interpret the low training accuracy: Failing to fit even the training data means the model is too rigid, i.e. *high bias*.

Step 2 – interpret the similar test accuracy: Performance barely changes across data sets, meaning it does not chase sample noise, i.e. *low variance*.

Step 3 – combine: This is the classic signature of underfitting: high bias, low variance.

Final Answer: high bias, low variance $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q37

Q38.

Solution

Concept – 1-NN takes the label of the single closest point: Compute the Euclidean distance from Q to each training point and pick the nearest.

Step 1 – distance $Q(4,4)$ to $A(1,1)$: $\sqrt{(4-1)^2+(4-1)^2} = \sqrt{9+9} = \sqrt{18} = 4.24$

Step 2 – distance to $B(5,5)$: $\sqrt{(4-5)^2+(4-5)^2} = \sqrt{1+1} = \sqrt{2} = 1.41$

Step 3 – distance to $C(2,2)$: $\sqrt{(4-2)^2+(4-2)^2} = \sqrt{4+4} = \sqrt{8} = 2.83$

Step 4 – distance to $D(6,6)$: $\sqrt{(4-6)^2+(4-6)^2} = \sqrt{4+4} = \sqrt{8} = 2.83$

Step 5 – pick the nearest point: The smallest distance is 1.41, to B, which is Class 1.

Step 6 – assign the label: With $k = 1$, Q takes B's label, Class 1.

Final Answer: Q is Class $1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q38

Q39.

Solution

Concept – binary entropy: $H = -p\log_2 p - (1-p)\log_2(1-p)$, with p the fraction of positives.

Step 1 – compute the class proportions: $p = \frac{5}{10} = 0.5$ positive, $1-p = \frac{5}{10} = 0.5$ negative.

Step 2 – take the base-2 logs: $\log_2 0.5 = -1$ for both terms.

Step 3 – form each term: $-0.5 \times (-1) = 0.5$

$-0.5 \times (-1) = 0.5$

Step 4 – add the terms: $H = 0.5 + 0.5 = 1.0$ bit

Step 5 – interpret: A perfectly balanced binary set has the maximum possible entropy of 1 bit.

Final Answer: $H = 1.0$ bit $\Rightarrow$ B

Answer: (B) Go Back to Q39

Q40.

Solution

Concept – information gain: $\text{IG} = H(\text{parent}) - \sum_{\text{children}} \frac{n_{\text{child}}}{n_{\text{parent}}} H(\text{child})$.

Step 1 – parent entropy: The parent has $4+$ and $4-$ (balanced), so $H(\text{parent}) = 1.0$ bit.

Step 2 – entropy of the $A = 0$ child: It has $4+$ and $0-$; a pure node has entropy 0.

Step 3 – entropy of the $A = 1$ child: It has $0+$ and $4-$; a pure node has entropy 0.

Step 4 – weighted child entropy: $\frac{4}{8}(0) + \frac{4}{8}(0) = 0$

Step 5 – subtract from the parent entropy: $\text{IG} = 1.0 - 0 = 1.0$

Step 6 – interpret: The attribute separates the classes perfectly, so the gain equals the full parent entropy.

Final Answer: $\text{IG} = 1.0$ bit $\Rightarrow$ A

Answer: (A) Go Back to Q40

Q41.

Solution

Concept – SVM margin width: For the hyperplane $w^{\mathsf{T}}x + b = 0$, the full margin (distance between the two boundaries) is $\dfrac{2}{\|w\|}$.

Step 1 – write the margin formula: margin $= \dfrac{2}{\|w\|}$

Step 2 – substitute $\|w\| = 2$: margin $= \dfrac{2}{2}$

Step 3 – divide: margin $= 1$

Final Answer: margin width $= 1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q41

Q42.

Solution

Concept – k-means centroid update: The new centroid is the coordinate-wise mean of the points assigned to the cluster.

Step 1 – average the x-coordinates: $\bar{x} = \dfrac{2+4+6}{3} = \dfrac{12}{3} = 4$

Step 2 – average the y-coordinates: $\bar{y} = \dfrac{4+4+10}{3} = \dfrac{18}{3} = 6$

Step 3 – assemble the centroid: $(\bar{x}, \bar{y}) = (4,\ 6)$

Why the other options are wrong: $(12, 18)$ is the coordinate sum, not the mean; the centroid divides each sum by the 3 points.

Final Answer: centroid $= (4, 6) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q42

Q43.

Solution

Concept – variance explained in PCA: Each principal component explains variance proportional to its eigenvalue; the fraction is the component's eigenvalue over the total.

Step 1 – total variance (sum of all eigenvalues): $6 + 2 + 1 + 1 = 10$

Step 2 – variance in the first component: The largest eigenvalue is 6.

Step 3 – form the fraction: $\dfrac{6}{10} = 0.60$

Step 4 – express as a percentage: $0.60 = 60\%$

Final Answer: 60% of the variance $\Rightarrow$ D

Answer: (D) Go Back to Q43

Q44.

Solution

Concept – neuron forward pass with ReLU: Compute the weighted sum plus bias, then apply $\text{ReLU}(z) = \max(0, z)$.

Step 1 – weighted sum of inputs: $w_1x_1 + w_2x_2 = (1)(2) + (2)(1)$

$= 2 + 2$

$= 4$

Step 2 – add the bias: $z = 4 + (-1) = 3$

Step 3 – apply ReLU: $\text{ReLU}(3) = \max(0,\ 3) = 3$

Final Answer: output $= 3 \Rightarrow$ B

Answer: (B) Go Back to Q44

Q45.

Solution

Concept – A* expands the node of least $f = g + h$: Compute f for each frontier node and pick the minimum.

Step 1 – f for node P: $f(P) = g + h = 4 + 3 = 7$

Step 2 – f for node Q: $f(Q) = 2 + 6 = 8$

Step 3 – f for node R: $f(R) = 5 + 1 = 6$

Step 4 – choose the smallest f: $\min(7,\ 8,\ 6) = 6$, which belongs to node R.

Final Answer: A* expands $R \Rightarrow$ D

Answer: (D) Go Back to Q45

Q46.

Solution

Concept – minimax back-up: MIN nodes take the minimum of their children; the MAX root takes the maximum of the MIN results.

Step 1 – evaluate the left MIN node: $\min(6,\ 4) = 4$

Step 2 – evaluate the right MIN node: $\min(8,\ 5) = 5$

Step 3 – evaluate the MAX root: $\max(4,\ 5) = 5$

Final Answer: minimax value $= 5 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q46

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	B	4	C	5	D
6	C	7	D	8	D	9	C	10	B
11	A	12	B	13	B	14	A	15	B
16	C	17	B	18	A	19	C	20	B
21	A	22	C	23	B	24	C	25	C
26	D	27	D	28	A	29	A	30	C
31	B	32	D	33	D	34	A	35	A
36	D	37	A	38	A	39	B	40	A
41	D	42	B	43	D	44	B	45	D
46	C								