GATE 2026 Electronics & Communication Engineering Question Paper with Solutions

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General Instructions

(i) This booklet contains 65 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 42 single-correct multiple-choice questions, 10 multiple-correct questions and 13 numerical / integer-type questions.

(iii) These questions follow the GATE Electronics and Communication Engineering examination pattern.

(iv) Questions carry 1 or 2 marks. MCQs have negative marking (1/3 or 2/3 of the marks); MSQ and numerical-answer (NAT) questions carry no negative marking.

(v) GATE is a 3-hour paper of 65 questions (100 marks).

(vi) A virtual scientific calculator is provided; physical calculators are not allowed.

(vii) Attempt each question on your own before reviewing the given solution.

(viii) For multiple-correct questions, all correct options must be selected to earn full marks.

(ix) For numerical questions, report the answer rounded exactly as asked.

1. Among the following options, the antonym of the word 'nocturnal' is _____.

(A) normal

(B) diurnal

(C) abnormal

(D) exceptional

Correct Answer: (B) diurnal

Solution:

The word "nocturnal" describes an animal or activity that is active during the night. To find its antonym, we need a word that means active during the day, not simply a word built from a prefix like "non-" or "un-".

1. **normal**: This word means usual or typical. It says nothing about day or night, so it does not describe the opposite of "nocturnal".
2. **diurnal**: This word means active during the day. It is the direct opposite of "nocturnal", which is why animals such as most birds are called diurnal while owls are called nocturnal.
3. **abnormal**: This word means not normal or unusual. It has no connection to the timing of activity, so it cannot be the antonym here.
4. **exceptional**: This word means unusually good or rare. Again, it does not relate to day or night activity.

The correct antonym is "diurnal", since it directly names daytime activity as opposed to nighttime activity described by "nocturnal".

Let's summarize:

- "Nocturnal" means active at night.
- "Diurnal" means active during the day, making it the true antonym.
- The other three options describe unrelated ideas such as usualness or rarity, not a time of activity.

So the word that correctly completes the blank as the antonym of "nocturnal" is "diurnal".

Quick Tip: Look for a word that names daytime activity, since "nocturnal" names nighttime activity.

2. The statements (S1), (S2), and (S3) pertain to the scores obtained by students in an exam. The maximum possible marks in the exam is 150.
(S1) The highest score is 100.
(S2) The fourth highest score is 76.
(S3) There are at least four students whose scores are within 25 of each other.
Which one of the following options is necessarily correct?

(A) (S1) and (S2) together imply (S3)

(B) (S1) and (S3) together imply (S2)

(C) (S2) and (S3) together imply (S1)

(D) (S1) implies (S3)

Correct Answer: (A) (S1) and (S2) together imply (S3)

Solution:

Step 1: Understand what "within 25 of each other" means.
A group of students has scores "within 25 of each other" when the difference between the highest and the lowest score in that group is at most 25. We need to check which combination of statements forces such a group of at least four students to exist.

Step 2: Test option (A), combining (S1) and (S2).
If the highest score is 100 and the fourth highest score is 76, then the top four scores (1st, 2nd, 3rd, and 4th highest) all lie between 76 and 100.

The 2nd and 3rd highest scores must lie between the 1st and 4th, so they also fall in this same range.

Step 3: Compute the spread of these four scores.

$$100 - 76 = 24$$

Since all four scores lie within a band of width **24**, which is less than **25**, every pair among these four students has a score difference of at most **24**.
So these four students automatically satisfy (S3): there are at least four students whose scores are within 25 of each other.
Hence (S1) and (S2) together always force (S3) to be true, no matter what the other students score.

Step 4: Test option (B), combining (S1) and (S3).
Knowing only that the highest score is **100** and that some four students are within 25 of each other does not fix the fourth highest score at **76**. The four clustered students could just as well have scores like **40, 45, 50, 55**, which has nothing to do with the fourth highest mark. So (S2) does not have to follow.

Step 5: Test option (C), combining (S2) and (S3).
Knowing the fourth highest score is **76** and that some four students are close together does not force the highest score to be exactly **100**. The highest score could be **76** itself (if the top four are tied) or any value above **76**, so (S1) is not guaranteed.

Step 6: Test option (D), using (S1) alone.
Knowing only that the highest score is **100** tells us nothing about how

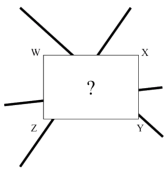
the other 99 students are spread out. They could all be spaced more than 25 marks apart from each other and from the topper, so (S3) does not have to follow from (S1) alone.

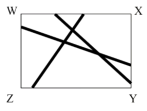
Step 7: Final conclusion.
Only the combination in option (A) is forced to be true in every possible case.

(S1) and (S2) together imply (S3)

Quick Tip: Work out how close together the top four scores must be once you fix the 1st and 4th highest values.

3. The figure below has exactly three intersecting line segments with a rectangular portion WXYZ missing. Which one of the following options P, Q, R, and S is the missing portion WXYZ?





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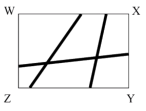

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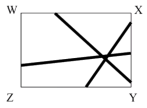

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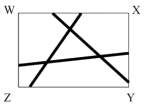


S

(A) P

(B) Q

(C) R

(D) S

Correct Answer: (D) S

Solution:

Concept:

The figure shows three straight line segments that pass through a rectangular window WXYZ. Each of these three lines is visible outside the rectangle on both sides. Since every line is straight, the hidden piece inside the rectangle must be the exact continuation of each visible arm, with no bend and no change of direction at the boundary of the rectangle.

Step 1: Identify the three lines from their visible arms.

Two arms come from the top of the figure and cross each other above the rectangle before entering it, one arm entering close to corner W and the other close to corner X. On the outside below the rectangle, one steep arm leaves near corner Z and another steep arm leaves near corner Y. A third pair of arms leaves from the middle of the left side and the middle of the right side, running almost horizontally.

Step 2: Match each pair of arms into one straight line.

The arm entering near W keeps the same downward-to-the-right slope it had outside, so inside the rectangle it must continue down to the arm leaving near Y, giving one diagonal from the W corner region to the Y corner region.

The arm entering near X keeps its downward-to-the-left slope, so it

must continue inside to meet the arm leaving near Z, giving the second diagonal from the X corner region to the Z corner region.
The two almost horizontal arms on the left and right sides belong to the third line, which must run nearly straight across the middle of the rectangle connecting them.

Step 3: Check each option for a straight, unbroken match.
In option P, the line meant to join the W side to Y and the line meant to join X to Z do not keep a constant slope matching the outside arms, and the near-horizontal line bends where it should stay straight.
In option Q, the two diagonal lines do not line up with the direction the outside arms are travelling in for one of the four corner regions.
In option R, the crossing point of the two diagonals sits too close to one corner instead of matching the position implied by the entry points of the outside arms, so at least one diagonal is off.
In option S, all three lines connect their correct pair of outside arms with a constant slope and no kink at the rectangle's boundary, exactly reproducing three uninterrupted straight lines.

Step 4: Final conclusion.
The missing portion WXYZ that keeps all three line segments straight is option S.

S

Quick Tip: Each visible arm outside the rectangle must continue inside with exactly the same slope, since all three lines are straight.

4. Real numbers y, p, and n (all greater than 1) satisfy

$$\left(\log_{p^{1/n}} y\right)\left(\log_{y^{1/n}} p\right) = 16,$$

where the logarithms are taken to the bases $p^{1/n}$ and $y^{1/n}$.
The value of n is _____

(A) 2

(B) 4

(C) 8

(D) 16

Correct Answer: (B) 4

Solution:

Step 1: Rewrite the first logarithm using change of base.

$$\log_{p^{1/n}} y = \frac{\ln y}{\ln\left(p^{1/n}\right)} = \frac{\ln y}{\frac{1}{n}\ln p} = \frac{n \ln y}{\ln p}$$

Step 2: Rewrite the second logarithm the same way.

$$\log_{y^{1/n}} p = \frac{\ln p}{\ln\left(y^{1/n}\right)} = \frac{\ln p}{\frac{1}{n}\ln y} = \frac{n \ln p}{\ln y}$$

Step 3: Multiply the two expressions together.

$$\left(\log_{p^{1/n}} y\right)\left(\log_{y^{1/n}} p\right) = \frac{n \ln y}{\ln p} \times \frac{n \ln p}{\ln y}$$

The terms $\ln y$ and $\ln p$ cancel exactly, leaving

$$= n^2$$

Step 4: Apply the given equation.

We are told this product equals 16, so

$$n^2 = 16$$

Step 5: Solve for n.

$$n = \pm 4$$

Since y, p, and n are all real numbers greater than 1, n must be positive, so we discard $n = -4$.

Step 6: Final conclusion.

$$\boxed{n = 4}$$

Hence the correct option is (B).

Quick Tip: Change both logarithms to a common base and see how the product simplifies to a clean power of n.

5. The following observation is made about the scores obtained by 100 students in an exam:
'For each student, there exists another student in the class such that their scores are at most ten marks away.'
If the above statement is **false**, which one of the following statements is necessarily true?

(A) For each student, the scores of all the other students are more than 10 marks away.

(B) There exists at least one student in the class for whom the scores of all the other students are more than 10 marks away.

(C) There is exactly one student in the class for whom the scores of some students are more than 10 marks away.

(D) For each student, the score of exactly one other student is more than 10 marks away.

Correct Answer: (B) There exists at least one student in the class for whom the scores of all the other students are more than 10 marks away.

Solution:

Step 1: Write the given statement using quantifiers.
Let the students be labelled 1 to 100, and let $s(i)$ be the score of student i. The given statement says:
"For every student i, there exists some other student j such that $|s(i) - s(j)| \leq 10$."
In symbols, this is: for all i, there exists $j \neq i$ with $|s(i) - s(j)| \leq 10$.

Step 2: Recall how to negate a "for all, there exists" statement.
The negation of "for all i, there exists j such that $P(i, j)$" is "there exists an i such that for all j, $P(i, j)$ is false." The "for all" flips to "there exists", the "there exists" flips to "for all", and the inner condition gets

negated.

Step 3: Apply this to our statement.
The inner condition here is $|s(i) - s(j)| \leq 10$, whose negation is $|s(i) - s(j)| > 10$.
So the negation of the whole statement is: "there exists a student i such that for every other student j, $|s(i) - s(j)| > 10$."
In plain words: there is at least one student whose score is more than 10 marks away from every other student's score.

Step 4: Match this to the options.
This is exactly what option (B) says: "There exists at least one student in the class for whom the scores of all the other students are more than 10 marks away."

Step 5: Check why option (A) is wrong.
Option (A) says this holds for EVERY student, not just for at least one. Negating a "there exists" statement only gives us one such student, not all 100 of them.

Step 6: Check why option (C) is wrong.
Option (C) restricts the count to "exactly one" student and also weakens the condition to "some students" instead of "all other students". Neither restriction is forced by the negation.

Step 7: Check why option (D) is wrong.
Option (D) says "for each student, exactly one other student is more than 10 marks away," which is a completely different statement about counting distances for every student, not about the existence of one isolated student. It is not what the negation produces.

Step 8: Final conclusion.

There exists at least one student for whom the scores of all other students are more than 10 marks away.

Hence the correct option is (B).

Quick Tip: Negating "for every student there is a close one" flips the quantifiers: it becomes "some student has no close one".

6. Each one of the following clues contains a keyword that is partially filled.
Clue 1: Synonym of recognize (8 letters): _D_NT_FY
Clue 2: A story long enough to fill a book (5 letters): _ _ _ EL
Clue 3: Two of something (6 letters): _ _ _ PLE
Clue 4: A fraction of something, split equally into two parts (4 letters): _ _ _ F
The first letter of each of the keywords can be rearranged to form a four-letter word.
Which one of the options below is a possible choice for the four-letter word?

(A) CHIN

(B) COIN

(C) ITCH

(D) NOSE

Correct Answer: (A) CHIN

Solution:

Step 1: Solve Clue 1.
We need an 8 letter synonym of "recognize" that fits the pattern *_D_NT_FY*. The word IDENTIFY fits perfectly: I-D-E-N-T-I-F-Y, matching the given letters D, N, T, F, Y in their exact positions. Its first letter is I.

Step 2: Solve Clue 2.
We need a 5 letter word for "a story long enough to fill a book" fitting *___EL*. This is a NOVEL: N-O-V-E-L. Its first letter is N.

Step 3: Solve Clue 3.
We need a 6 letter word for "two of something" fitting *___PLE*. This is a COUPLE: C-O-U-P-L-E. Its first letter is C.

Step 4: Solve Clue 4.
We need a 4 letter word for "a fraction split equally into two parts" fitting *___F*. This is HALF: H-A-L-F. Its first letter is H.

Step 5: Collect the first letters.
The first letters of the four keywords are I, N, C, and H.

Step 6: Rearrange these letters into a four letter word.
The letters I, N, C, H can be rearranged to spell CHIN, which is a valid English word.

Step 7: Check the other options.

(B) COIN: Uses the letter O, which is not among I, N, C, H. Not possible.

(C) ITCH: Uses the letter T, which is not among I, N, C, H. Not

possible.

(D) NOSE: Uses the letters O, S, and E, none of which are among I, N, C, H. Not possible.

Step 8: Final conclusion.
Only CHIN can be formed from the exact letters I, N, C, and H.

CHIN

Quick Tip: Solve each clue first, then look at just the first letters of the four answers.

7. Three children P, Q, R and two grown-ups X, Y play a badminton doubles tournament. X and Y are parents to two of the children playing. The child of X is not the same as the child of Y. Exactly one of the children does not have a parent playing in the tournament. The following rules are followed:
(i) A parent and his/her child cannot be on the same team.
(ii) A match can feature at most one parent and his/her child, that is, a maximum of one parent-child pair can play in a match.
The following matches were played:

	TEAM 1	TEAM 2
MATCH 1	P and X	Q and R
MATCH 2	P and R	X and Y
MATCH 3	R and X	Q and Y

Which one of the following options is correct?

(A) P does not have any parent playing

(B) Q does not have any parent playing

(C) R does not have any parent playing

(D) X does not have a child playing

Correct Answer: (C) R does not have any parent playing

Solution:

Step 1: Set up the parent-child assignment.
X and Y are each the parent of exactly one of P, Q, R, and these two children are different from each other (the child of X is not the child of Y). So two of the three children have a parent in the tournament, and exactly one child has no parent playing. Call the child of X as C_X, the child of Y as C_Y, and the child with no parent as C_0.

Step 2: Use Match 1 to rule out one possibility.
In Match 1, Team 1 is "P and X". Rule (i) says a parent cannot be on the same team as his or her own child. If C_X were P, then X and his child P would both be on Team 1, breaking rule (i). So C_X cannot be P; it must be Q or R.

Step 3: Use Match 2 to narrow down C_0.
In Match 2, all four of P, R, X, and Y are playing (Q sits out). If both C_X and C_Y were chosen from $\{P, R\}$, then both parent-child pairs would be present together in this one match, since both children and both parents are all playing at once. That directly breaks rule (ii), which allows at most one parent-child pair per match. So C_X and C_Y cannot both lie in $\{P, R\}$, which means C_0 cannot be Q; the child left without a parent must be P or R.

Step 4: Combine Steps 2 and 3.

From Step 2, $C_X \in \{Q, R\}$. From Step 3, $C_0 \in \{P, R\}$, meaning Q is definitely one of C_X or C_Y (Q always has a parent playing).

This leaves two possible cases:

Case 1: $C_0 = P$, so $\{C_X, C_Y\} = \{Q, R\}$.

Case 2: $C_0 = R$, so $\{C_X, C_Y\} = \{P, Q\}$, and since $C_X \neq P$ (Step 2), we must have $C_X = Q$ and $C_Y = P$.

Step 5: Test Case 1 against Match 3.

In Match 3, Team 1 is "R and X" and Team 2 is "Q and Y" (P sits out).

In Case 1, C_X is Q or R.

If $C_X = R$, then X and his child R are on the same Team 1, breaking rule (i). So within Case 1 we would need $C_X = Q$ and $C_Y = R$.

But then check rule (ii) in Match 3: X's child Q is present (Team 2) along with X (Team 1), giving one parent-child pair; and Y's child R is present (Team 1) along with Y (Team 2), giving a second parent-child pair. Two pairs appear together in the same match, which breaks rule (ii).

So Case 1 fails no matter how we assign Q and R between C_X and C_Y.

Step 6: Confirm Case 2 works.

In Case 2, $C_X = Q$, $C_Y = P$, and $C_0 = R$ (R has no parent playing).

Check all three matches:

Match 1 (P,X vs Q,R): X's child Q is on the opposite team from X, so rule (i) holds. Y is not playing, so there is nothing to check for Y. Only one parent-child pair (X-Q) appears, satisfying rule (ii).

Match 2 (P,R vs X,Y): the two children P, R are together on one team and the two parents X, Y are together on the other, so rule (i) is automatically satisfied. Q is sitting out, so only Y's child P is present alongside Y, giving exactly one parent-child pair, satisfying rule (ii).

Match 3 (R,X vs Q,Y): X's child Q is on the opposite team from X,

satisfying rule (i). P is sitting out, so Y's child P is not present in this match, leaving only one parent-child pair (X-Q), satisfying rule (ii). Every rule holds in every match, so Case 2 is the valid arrangement.

Step 7: Read off the answer.
In this valid arrangement, R is the child with no parent playing, X's child is Q, and Y's child is P.
Checking the options: (A) is false since P has a parent (Y). (B) is false since Q has a parent (X). (D) is false since X's child Q does play in Matches 1 and 3. Only (C) is true.

Step 8: Final conclusion.

R does not have any parent playing

Hence the correct option is (C).

Quick Tip: Use the fact that a parent's team cannot include his or her own child to test each match one at a time.

8. Let P_k represent the perimeter of a square with sides of length k. The value of the expression $P_1 + P_2 + P_3 + \cdots + P_{10}$ is _____

(A) 55

(B) 110

(C) 220

(D) 440

Correct Answer: (C) 220

Solution:

Step 1: Write the perimeter of a square in terms of its side.

A square with side length k has four equal sides, so its perimeter is

$$P_k = 4k$$

Step 2: Write out the required sum using this formula.

$$P_1 + P_2 + P_3 + \cdots + P_{10} = 4(1) + 4(2) + 4(3) + \cdots + 4(10) = 4(1 + 2 + 3 + \cdots + 10)$$

Step 3: Find the sum of the first 10 natural numbers.

Using the formula for the sum of the first n natural numbers,

$$1 + 2 + 3 + \cdots + 10 = \frac{10 \times 11}{2} = 55$$

Step 4: Multiply by 4.

$$P_1 + P_2 + \cdots + P_{10} = 4 \times 55 = 220$$

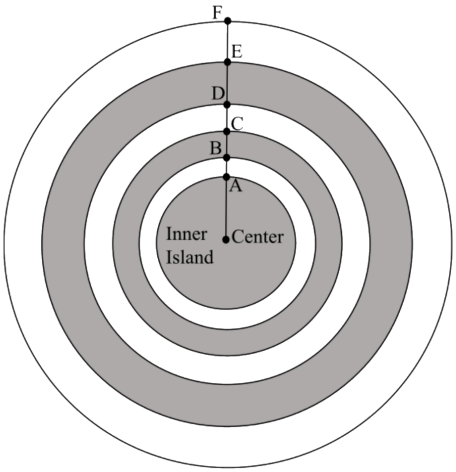
Step 5: Final conclusion.

$$\boxed{220}$$

Hence the correct option is (C).

Quick Tip: Write each perimeter as 4k, then pull the 4 out and sum the numbers 1 through 10.

9. The city of Atlantis was crafted by the God of the seas, Poseidon. It was made of alternating concentric circular rings of land (shaded) and water (not shaded) as represented in the figure (not to scale). The radius of Inner Island was **2.5** stades (a unit of length used in ancient Greece). The water surrounding Inner Island was one stade wide (length AB). This was surrounded by two pairs of alternating rings of land and water. The first pair of land and water was two stades wide each (lengths BC and CD), and the outer pair is three stades wide each (lengths DE and EF).



The ratio of the surface area of the land to that of the water in the city of Atlantis is _____ (round off to two decimal places).

(A) 0.45

(B) 0.60

(C) 0.75

(D) 0.90

Correct Answer: (C) 0.75

Solution:

Step 1: List the radii of every ring boundary.

The Inner Island is a solid circle of radius 2.5 stades. Moving outward, each named segment adds its own width to the running radius.

Radius up to the Inner Island (land): 2.5.

Radius up to B, after the water ring AB of width 1: $2.5 + 1 = 3.5$.

Radius up to C, after the land ring BC of width 2: $3.5 + 2 = 5.5$.

Radius up to D, after the water ring CD of width 2: $5.5 + 2 = 7.5$.

Radius up to E, after the land ring DE of width 3: $7.5 + 3 = 10.5$.

Radius up to F, after the water ring EF of width 3: $10.5 + 3 = 13.5$.

Step 2: Sort the rings into land and water.

Land regions are the Inner Island (radius 0 to 2.5), ring BC (radius 3.5 to 5.5) and ring DE (radius 7.5 to 10.5). Water regions are ring AB (radius 2.5 to 3.5), ring CD (radius 5.5 to 7.5) and ring EF (radius 10.5 to 13.5).

Step 3: Work out each land area using $\pi(R_{out}^2 - R_{in}^2)$.

$$\text{Inner Island} = \pi(2.5)^2 = 6.25\pi$$

$$\text{Ring BC} = \pi\left(5.5^2 - 3.5^2\right) = \pi(30.25 - 12.25) = 18\pi$$

$$\text{Ring DE} = \pi\left(10.5^2 - 7.5^2\right) = \pi(110.25 - 56.25) = 54\pi$$

Step 4: Add up the land area.

$$\text{Total land} = 6.25\pi + 18\pi + 54\pi = 78.25\pi$$

Step 5: Work out and add the water areas the same way.

$$\text{Ring AB} = \pi\left(3.5^2 - 2.5^2\right) = \pi(12.25 - 6.25) = 6\pi$$

$$\text{Ring CD} = \pi\left(7.5^2 - 5.5^2\right) = \pi(56.25 - 30.25) = 26\pi$$

$$\text{Ring EF} = \pi\left(13.5^2 - 10.5^2\right) = \pi(182.25 - 110.25) = 72\pi$$

$$\text{Total water} = 6\pi + 26\pi + 72\pi = 104\pi$$

Step 6: Form the ratio and check the other options.

$$\frac{\text{Land}}{\text{Water}} = \frac{78.25\pi}{104\pi} = \frac{78.25}{104} \approx 0.7524$$

Rounded to two decimal places this is **0.75**. Option (A) **0.45** and option (B) **0.60** undercount the land rings, and option (D) **0.90** would need more land area than the rings actually give; none of these match the ring-by-ring computation.

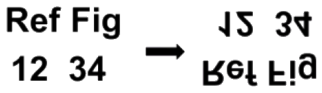
Step 7: Final conclusion.

$$\boxed{0.75}$$

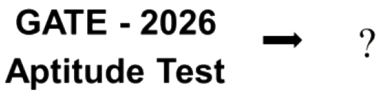
Quick Tip: Add up the ring widths to get every boundary radius, then use pi(R_out^2 - R_in^2) for each ring's area before forming the ratio.

10. Consider the visual pattern rule shown below.

If

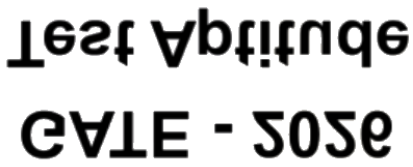


Then

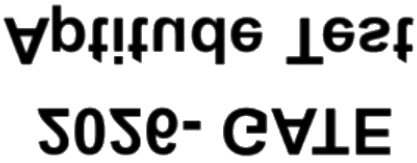


If the label "Ref Fig / 12 34" transforms into the pattern shown after the arrow under a fixed rule, then applying the same rule to the label "GATE - 2026 / Aptitude Test" produces which one of the patterns P, Q, R, S shown below?

(A) Pattern P

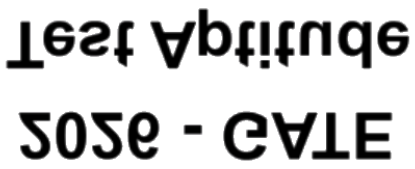


(B) Pattern Q

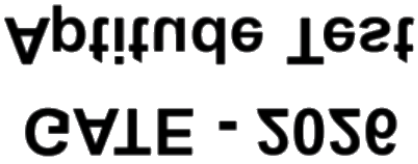


(C) Pattern R

(D) Pattern S



Correct Answer: (C) Pattern R



Solution:

Step 1: Read off the rule from the example.
The example takes the two-line label "Ref Fig" over "12 34" and turns it into the pattern shown after the arrow. In that result, the line that was on the bottom ("12 34") now sits on top, and the line that was on top ("Ref Fig") now sits at the bottom. Inside each line, every letter and digit is flipped upside down and the left-to-right order of the characters is reversed.

Step 2: Name this rule.
Swapping the order of the two lines while turning every character upside down and reversing its order along the line is exactly what happens when the whole label is rotated by 180° about its own centre, the way a printed card looks after being spun half way around flat on a table.

Step 3: Apply the same rule to the new label.
The new label is "GATE - 2026" over "Aptitude Test". Rotating this whole block by 180° puts the "Aptitude Test" line on top and the "GATE - 2026" line at the bottom, with every letter, digit and hyphen turned upside down and read in reverse order along each line.

Step 4: Compare with the four patterns.
Patterns P, Q and S each break the rule in one place: either the two lines keep their original order instead of swapping, or the characters are only mirror-reflected left to right instead of being rotated a full 180° (a plain mirror reflection does not turn a character fully upside down), or the year digits do not match a genuine 180° turn of "2026". Pattern R is the only one showing "Aptitude Test" on top, "GATE - 2026" at the bottom, and every character rotated a full 180° with its order reversed, matching exactly what the example did to "Ref Fig / 12 34".

Step 5: Final conclusion.

Pattern R

Quick Tip: Work out precisely what the example figure does to "Ref Fig / 12 34" (line order swap plus a 180-degree turn of every character), then apply that same rule to "GATE - 2026 / Aptitude Test".

11. Consider the differential equation $\dot{\vec{w}} = A\vec{w}$, with $\vec{w}(t=0) = \begin{bmatrix}1\\1\end{bmatrix}$. If $\vec{w}(t) = e^{t}\vec{u}_x + e^{-2t}\vec{u}_y$ is the solution to the equation, where $\vec{u}_x$ and $\vec{u}_y$ are unit vectors along the positive x and y axes respectively, then which of the following options is the correct matrix representing A?

(A) $\begin{bmatrix}1 & 0\\0 & -2\end{bmatrix}$

(B) $\begin{bmatrix}-1 & 0\\0 & 2\end{bmatrix}$

(C) $\begin{bmatrix}0 & -2\\1 & 0\end{bmatrix}$

(D) $\begin{bmatrix}0 & 2\\-1 & 0\end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix}1 & 0\\0 & -2\end{bmatrix}$

Solution:

Step 1: Write the solution as a column vector.

Since $\vec{u}_x$ and $\vec{u}_y$ are the unit vectors along the x and y axes, the given solution can be written as

$$\vec{w}(t) = \begin{bmatrix} e^{t} \\ e^{-2t} \end{bmatrix}$$

Step 2: Check the initial condition.

At $t = 0$, $\vec{w}(0) = \begin{bmatrix} e^{0} \\ e^{0} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, which matches the given initial condition, so this form is consistent.

Step 3: Differentiate the solution component by component.

$$\dot{\vec{w}}(t) = \begin{bmatrix} \frac{d}{dt} e^{t} \\ \frac{d}{dt} e^{-2t} \end{bmatrix} = \begin{bmatrix} e^{t} \\ -2e^{-2t} \end{bmatrix}$$

Step 4: Compare with $A\vec{w}$.

We need A such that $A \begin{bmatrix} e^{t} \\ e^{-2t} \end{bmatrix} = \begin{bmatrix} e^{t} \\ -2e^{-2t} \end{bmatrix}$. The first row must multiply e^{t} by 1 and e^{-2t} by 0, giving the row $[1 \;\; 0]$. The second row must multiply e^{t} by 0 and e^{-2t} by -2, giving the row $[0 \;\; -2]$.

Step 5: Write out A and rule out the other options.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

Option (B) has the signs of 1 and -2 swapped, which would give $w_1(t) = e^{-t}$ and $w_2(t) = e^{2t}$, the opposite growth behaviour to the one given. Options (C) and (D) are off-diagonal matrices; they mix w_1 and w_2 together and would produce coupled or oscillating solutions, not the two independent exponentials e^{t} and e^{-2t} seen here.

Step 6: Final conclusion.

$$\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

Quick Tip: Since u_x and u_y are the coordinate unit vectors, write w(t) as the column vector [e^t, e^{-2t}] and differentiate it directly.

12. A surface is given by $z^2 = 2x^2 - y^2$ and $\vec{n}$ and $-\vec{n}$ are unit normal vectors to the surface at the point $\vec{P} = \hat{\imath} + \sqrt{2}\,\hat{k}$. Which of the following vectors can be $\vec{n}$, where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are the unit vectors along x, y and z axes, respectively?

(A) $\hat{\imath} - \sqrt{2}\,\hat{k}$

(B) $\frac{2}{3}\hat{\imath} - \frac{1}{3}\hat{k}$

(C) $\sqrt{2}\hat{\imath} - \sqrt{3}\,\hat{k}$

(D) $\frac{\sqrt{2}\hat{\imath} - \hat{k}}{\sqrt{3}}$

Correct Answer: (D) $\frac{\sqrt{2}\hat{\imath} - \hat{k}}{\sqrt{3}}$

Solution:

Step 1: Write the surface as a level set.

Rewrite the surface equation as $F(x, y, z) = 2x^2 - y^2 - z^2 = 0$. For a surface $F(x, y, z) = 0$, the gradient ∇F always points along the normal direction to the surface.

Step 2: Compute the gradient.

$$\nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right) = (4x, -2y, -2z)$$

Step 3: Evaluate the gradient at the given point.

The point is $\vec{P} = \hat{\imath} + \sqrt{2}\hat{k}$, so $x = 1, y = 0, z = \sqrt{2}$.

$$\nabla F\big|_P = (4(1), -2(0), -2\sqrt{2}) = (4, 0, -2\sqrt{2})$$

Step 4: Find the length of this normal vector.

$$|\nabla F| = \sqrt{4^2 + 0^2 + (2\sqrt{2})^2} = \sqrt{16 + 8} = \sqrt{24} = 2\sqrt{6}$$

Step 5: Divide by the length to get a unit normal.

$$\vec{n} = \frac{(4, 0, -2\sqrt{2})}{2\sqrt{6}} = \left(\frac{2}{\sqrt{6}}, 0, -\frac{\sqrt{2}}{\sqrt{6}}\right)$$

Simplify each component: $\frac{2}{\sqrt{6}} = \frac{\sqrt{2}}{\sqrt{3}}$ and $\frac{\sqrt{2}}{\sqrt{6}} = \frac{1}{\sqrt{3}}$. So

$$\vec{n} = \frac{\sqrt{2}}{\sqrt{3}}\hat{\imath} - \frac{1}{\sqrt{3}}\hat{k} = \frac{\sqrt{2}\hat{\imath} - \hat{k}}{\sqrt{3}}$$

Step 6: Check the other options.

Option (A) is the raw gradient direction with z-component $-\sqrt{2}$ but is

not divided by its own length, since its magnitude is $\sqrt{1+2} = \sqrt{3} \neq 1$. Option (B) uses the wrong ratio 2/3 to 1/3 instead of $\sqrt{2}/\sqrt{3}$ to $1/\sqrt{3}$. Option (C) is not a unit vector either, since $\sqrt{(\sqrt{2})^2 + (\sqrt{3})^2} = \sqrt{5} \neq 1$.

Step 7: Final conclusion.

$$\boxed{\vec{n} = \frac{\sqrt{2}\hat{\imath} - \hat{k}}{\sqrt{3}}}$$

Quick Tip: The gradient of F(x,y,z)=2x^2-y^2-z^2 at a point gives the normal direction; divide it by its own magnitude to get a unit vector.

13. The Laplace Transform of the signal $x(t) = u(t-2) * (t\,u(t))$ is given by which of the following expressions? ["$*$" represents the convolution operator]

(A) $\dfrac{e^{-2s}}{s^2(s-2)}$

(B) $\dfrac{e^{-2(s-2)}}{s^3}$

(C) $\dfrac{se^{-2s}}{(s-2)^2}$

(D) $\dfrac{e^{-2s}}{s^3}$

Correct Answer: (D) $\dfrac{e^{-2s}}{s^3}$

Solution:

Step 1: Recall the convolution property of the Laplace Transform.

If $x(t) = x_1(t) * x_2(t)$, then convolution in time becomes plain

multiplication in the Laplace domain: $X(s) = X_1(s)X_2(s)$. Here $x_1(t) = u(t-2)$ and $x_2(t) = t\,u(t)$, so we just need to transform each one separately and multiply.

Step 2: Transform the shifted step $u(t-2)$.

The Laplace Transform of $u(t)$ is $1/s$. Using the time-shift property, $\mathcal{L}\{u(t-2)\} = e^{-2s} \cdot \frac{1}{s} = \frac{e^{-2s}}{s}$.

Step 3: Transform the ramp $t\,u(t)$.

A standard Laplace pair is $\mathcal{L}\{t\,u(t)\} = \frac{1}{s^2}$.

Step 4: Multiply the two transforms.

$$X(s) = \frac{e^{-2s}}{s} \cdot \frac{1}{s^2} = \frac{e^{-2s}}{s^3}$$

Step 5: Check why the other options are wrong.

Option (A) has an extra factor $(s-2)$ in the denominator, as if a frequency-shift property was mixed in with the time-shift by mistake. Option (B) alters the exponent to $e^{-2(s-2)}$, which would only appear if the signal carried an extra factor e^{2t}, which it does not. Option (C) has an extra s in the numerator and a squared $(s-2)$ term, neither of which arises from a plain time delay and a ramp. Since $u(t-2)$ is a pure time delay with no exponential attached to it, no $(s-2)$ term should appear anywhere.

Step 6: Final conclusion.

$$\boxed{\frac{e^{-2s}}{s^3}}$$

Quick Tip: Convolution in time becomes multiplication in the Laplace domain; transform u(t-2) and t.u(t) separately, then multiply the results.

14. Consider carrier transport in a Zener diode in the breakdown region. Which is the dominant transport mechanism for current flow in this case?

(A) Drift

(B) Diffusion

(C) Tunneling

(D) Ballistic transport

Correct Answer: (C) Tunneling

Solution:

Step 1: Recall how a Zener diode is built.
A Zener diode is a p-n junction diode with both the p-side and n-side doped heavily, which makes its depletion region very thin compared to an ordinary diode.

Step 2: Recall what happens under reverse bias near breakdown.
As the reverse voltage increases, the electric field across this thin depletion region becomes extremely high, of the order of 10^6 V/cm or more.

Step 3: Identify the transport mechanism at this field strength.
At such a high field, the depletion region is thin enough and the field strong enough that valence electrons on the p-side can quantum

mechanically tunnel straight across the narrow energy barrier into the conduction band on the n-side, without needing extra thermal energy to climb over the barrier. This direct barrier crossing is called tunneling, and it is the mechanism behind what is named Zener breakdown.

Step 4: Rule out the other options.
Drift is charge motion caused by an electric field acting on free carriers already present in a band; it describes ordinary current flow in the diode, not the barrier-crossing event itself. Diffusion is motion driven by a concentration gradient, and it dominates minority-carrier current in the neutral regions, not the breakdown of the thin junction. Ballistic transport describes carriers moving without scattering over distances shorter than their mean free path, a concept used for very short channel devices, not the physics of Zener breakdown.

Step 5: Final conclusion.

Tunneling

Quick Tip: A Zener diode is doped heavily on both sides, giving a thin depletion region where carriers can cross the barrier directly under a high field.

15. Two analog signals $x_1(t)$ and $x_2(t)$ (t in second) are sampled at a rate $F_s = 40$ Hz, where

$$x_1(t) = \cos(20\pi t),\ t \geq 0, \qquad x_2(t) = \cos(100\pi t),\ t \geq 0.$$

The first ten samples (starting from $t = 0$) are considered for the analysis. Which of the following statements is TRUE?

(A) All of the first three samples of $x_1(t)$ are greater than the corresponding samples of $x_2(t)$.

(B) All of the last three samples of $x_1(t)$ are greater than the corresponding samples of $x_2(t)$.

(C) All of the samples of $x_2(t)$ are greater than the corresponding samples of $x_1(t)$.

(D) All of the fourth to seventh samples of $x_1(t)$ are equal to the corresponding samples of $x_2(t)$.

Correct Answer: (D) All of the fourth to seventh samples of $x_1(t)$ are equal to the corresponding samples of $x_2(t)$.

Solution:

Step 1: Write the sampling instants.

Sampling at $F_s = 40$ Hz means the sampling period is $T_s = 1/40 = 0.025$ s, so samples are taken at $t_n = nT_s = 0.025n$ for $n = 0, 1, 2, \ldots, 9$.

Step 2: Write the sampled sequence for $x_1(t)$.

$$x_1[n] = \cos(20\pi \cdot 0.025n) = \cos\left(\frac{\pi n}{2}\right)$$

Step 3: Write the sampled sequence for $x_2(t)$.

$$x_2[n] = \cos(100\pi \cdot 0.025n) = \cos(2.5\pi n)$$

Since $2.5\pi n = 2\pi n + \frac{\pi n}{2}$ and $2\pi n$ is always a whole number of full cycles for integer n, the cosine repeats:

$$x_2[n] = \cos\left(2\pi n + \frac{\pi n}{2}\right) = \cos\left(\frac{\pi n}{2}\right)$$

Step 4: Compare the two sequences.

Both simplify to the exact same formula, $\cos(\pi n/2)$, so $x_1[n] = x_2[n]$ for every integer n. This happens because the 50 Hz frequency of x_2 aliases down to $50 - 40 = 10$ Hz when sampled at 40 Hz, exactly matching the 10 Hz frequency of x_1.

Step 5: List the first ten sample values.

$$n = 0, 1, 2, \ldots, 9: \quad \cos\left(\frac{\pi n}{2}\right) = 1,\ 0,\ -1,\ 0,\ 1,\ 0,\ -1,\ 0,\ 1,\ 0$$

Step 6: Check each option against these values.

(A) The first three samples of both signals are $1, 0, -1$; they are equal, not greater, so (A) is false. (B) The last three samples ($n = 7, 8, 9$) are $0, 1, 0$ for both signals, again equal, so (B) is false. (C) Since every sample of x_2 equals the corresponding sample of x_1, none is strictly greater, so (C) is false. (D) The fourth to seventh samples are the same list of values, $0, 1, 0, -1$, for both $x_1[n]$ and $x_2[n]$, so they are indeed equal. This statement is TRUE.

Step 7: Final conclusion.

All of the fourth to seventh samples of $x_1(t)$ are equal to the corresponding samples of $x_2(t)$.

Quick Tip: Sample both cosines at t_n = n/40 seconds and simplify cos(2.5*pi*n) using the fact that 2*pi*n is always a whole number of cycles.

16. The response of a discrete time system $y[n]$ obeys the following relation:

$$y[n] = \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2] + x[n].$$

The input to the system is $x[n] = \delta[n] - \frac{1}{3}\delta[n-1]$. Which of the following options is TRUE for $y[n]$?

(A) Stable and causal response

(B) Stable and non-causal response

(C) Unstable and causal response

(D) Unstable and non-causal response

Correct Answer: (A) Stable and causal response

Solution:

Step 1: Write the difference equation in standard form.

Rearranging the given relation, $y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n]$.

Step 2: Take the z-transform to get the system function.

Using $y[n-k] \leftrightarrow z^{-k}Y(z)$,

$$Y(z)\left(1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}\right) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}$$

Step 3: Factor the denominator to find the poles.

$$1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2} = \left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{3}z^{-1}\right)$$

Expanding this back gives $1 - \left(\frac{1}{2} + \frac{1}{3}\right)z^{-1} + \frac{1}{6}z^{-2} = 1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}$, which matches, so the poles of $H(z)$ are at $z = \frac{1}{2}$ and $z = \frac{1}{3}$.

Step 4: Decide causality.

The difference equation expresses $y[n]$ using only $y[n-1]$, $y[n-2]$ and the present input $x[n]$, with no dependence on any future sample. Solving this forward in n starting from rest gives the causal solution, whose region of convergence (ROC) is the exterior of the outermost pole: $|z| > \frac{1}{2}$.

Step 5: Decide stability.

A discrete LTI system is stable exactly when its ROC includes the unit circle $|z| = 1$. Since both poles have magnitude $1/2$ and $1/3$, both strictly less than 1, the causal ROC $|z| > 1/2$ does include the unit circle.

Step 6: Combine the two conclusions.

The system is both causal (ROC is the exterior of the outermost pole, matching a difference equation that only uses past outputs and the present input) and stable (the unit circle lies inside that ROC). The given input $x[n] = \delta[n] - \frac{1}{3}\delta[n-1]$ is itself a finite, causal signal, so it does not change this conclusion.

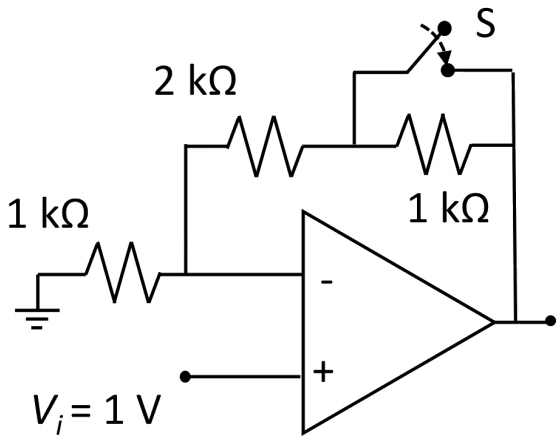
Step 7: Rule out the other options.
Non-causal responses (B, D) would need an anti-causal or two-sided ROC, which does not fit a difference equation using only past outputs and the present input. Unstable responses (C, D) would need the ROC to exclude the unit circle, which does not happen since both poles lie strictly inside the unit circle.

Step 8: Final conclusion.

Stable and causal response

Quick Tip: Factor the characteristic polynomial to find the pole locations, then check that both poles lie strictly inside the unit circle for the causal ROC.

17. The ideal OP-AMP circuit shown in the Figure produces output voltage $V_o = x$ when the Switch, S, is open.



Which of the options represents the output voltage when S is closed?

(A) x

(B) $\frac{2}{3}x$

(C) $\frac{3}{4}x$

(D) $\frac{1}{2}x$

Correct Answer: (C) $\frac{3}{4}x$

Solution:

Step 1: Identify the op-amp configuration.

The input $V_i = 1\ \text{V}$ is applied to the non-inverting (+) terminal, and the inverting (-) terminal connects to a $1\ \text{k}\Omega$ resistor to ground and to a $2\ \text{k}\Omega$ feedback resistor. Since the op-amp is ideal and the feedback is negative, the two input terminals sit at the same voltage (virtual short), so the inverting node is always at $1\ \text{V}$, whether S is open or closed.

Step 2: Write the general non-inverting gain formula.

For a non-inverting amplifier with a resistor R_1 from the inverting node to ground and a feedback resistor R_f from the inverting node to the output,

$$V_o = V_i\left(1 + \frac{R_f}{R_1}\right)$$

Here $R_1 = 1\ \text{k}\Omega$ in both cases. Only R_f, the path from the inverting node to V_o, changes with the switch.

Step 3: Find R_f when S is open.

With S open, the only path from the inverting node to V_o is through the $2\ \text{k}\Omega$ resistor and then the $1\ \text{k}\Omega$ resistor in series, since the switch

branch is broken and current must pass through both resistors one after another. So

$$R_f = 2\,\text{k}\Omega + 1\,\text{k}\Omega = 3\,\text{k}\Omega$$

$$x = V_o(\text{S open}) = V_i\left(1 + \frac{3\text{k}}{1\text{k}}\right) = 1 \times 4 = 4\,\text{V}$$

Step 4: Find R_f when S is closed.

Closing S places a short circuit directly across the $1\,\text{k}\Omega$ resistor that sits between the feedback node and V_o. A resistor in parallel with a short circuit behaves as a short circuit, so that resistor no longer contributes any resistance. The feedback path is now just the $2\,\text{k}\Omega$ resistor.

$$R_f = 2\,\text{k}\Omega$$

Step 5: Compute the output voltage when S is closed.

$$V_o(\text{S closed}) = V_i\left(1 + \frac{2\text{k}}{1\text{k}}\right) = 1 \times 3 = 3\,\text{V}$$

Step 6: Express the answer as a fraction of x.

Since $x = 4\,\text{V}$ and the closed-switch output is $3\,\text{V}$,

$$V_o(\text{S closed}) = \frac{3}{4} \times 4\,\text{V} = \frac{3}{4}x$$

The other options do not come from a valid resistor combination here: option (A) would mean the switch changes nothing, option (B) matches a wrong series value, and option (D) would follow only if the whole $3\,\text{k}\Omega$ had been halved instead of correctly reduced to $2\,\text{k}\Omega$.

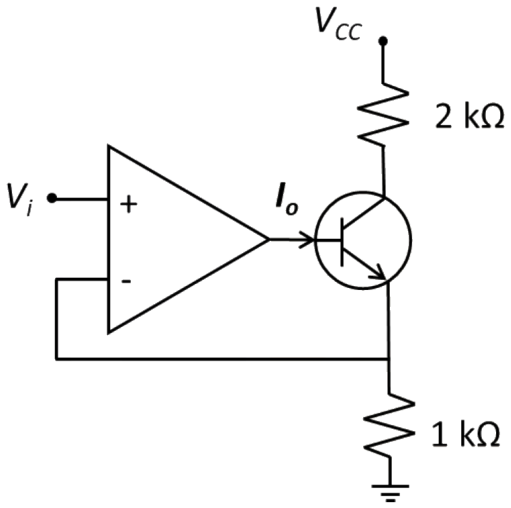
Final Answer:

The output voltage when S is closed is $\frac{3}{4}x$.

$$\boxed{\frac{3}{4}x}$$

Quick Tip: Shorting S removes the outer $1\,\text{k}\Omega$ resistor from the feedback path, so only the feedback resistance in the gain formula changes between the two cases.

18. Consider the circuit shown in the Figure with $V_i = 3\,\text{V}$ and $V_{CC} = 12\,\text{V}$.



Assume $V_{BE} = 0.7\,\text{V}$ and $\beta_{dc} = 99$ for the BJT.

Which of the following options is the correct value of the current I_o?

(A) $60\,\mu A$

(B) $6\,\mu A$

(C) $3\,\mu A$

(D) $30\,\mu A$

Correct Answer: (D) $30\,\mu A$

Solution:

Step 1: Understand what the op-amp feedback forces.

The op-amp output I_o drives the base of the NPN transistor, and the inverting input of the op-amp is wired to the transistor's emitter node. Because the op-amp has negative feedback through this connection, the emitter voltage is forced to equal the non-inverting input voltage.

$$V_E = V_i = 3\,\text{V}$$

Step 2: Find the emitter current using the emitter resistor.

The emitter connects to ground through a $1\,\text{k}\Omega$ resistor, so the emitter current is

$$I_E = \frac{V_E}{1\,\text{k}\Omega} = \frac{3\,\text{V}}{1\,\text{k}\Omega} = 3\,\text{mA}$$

Step 3: Relate the base current to the emitter current.

For a BJT, the emitter current splits into the base and collector currents, giving

$$I_E = I_B + I_C = I_B(1 + \beta_{dc})$$

since $I_C = \beta_{dc} I_B$. Here I_o is the current supplied by the op-amp output into the base, so $I_o = I_B$.

Step 4: Solve for the base current.

$$I_B = \frac{I_E}{1 + \beta_{dc}} = \frac{3\ \text{mA}}{1 + 99} = \frac{3\ \text{mA}}{100} = 30\,\mu\text{A}$$

Step 5: Check the role of V_{CC}, the $2\,\text{k}\Omega$ collector resistor, and V_{BE}. These values fix the collector voltage and confirm the transistor stays in the active region, so the simple current relations above apply, but none of them enter the calculation of I_o itself, since I_o depends only on the emitter voltage and the current-gain relation.

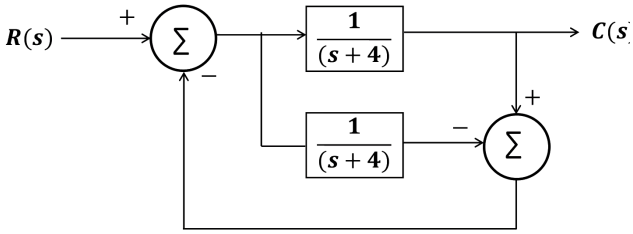
Step 6: Rule out the other options.
$60\,\mu\text{A}$ and $6\,\mu\text{A}$ do not follow from dividing $3\ \text{mA}$ by 100; they only appear from using the wrong resistor value or misplacing a decimal point. $3\,\mu\text{A}$ is off by a further factor of ten. Only dividing the correct emitter current $3\ \text{mA}$ by $(\beta_{dc} + 1) = 100$ gives a value consistent with the circuit.

Final Answer:

$$\boxed{I_o = 30\,\mu\text{A}}$$

Quick Tip: The op-amp forces the emitter voltage to equal V_i; use that to find I_E, then divide by $(\beta_{dc} + 1)$ to get the base current I_o.

19. A control system is shown in the Figure.



Which option represents the correct transfer function of the system?

(A) $\dfrac{1}{(s+4)^2}$

(B) $\dfrac{1}{(s+4)}$

(C) $\dfrac{2}{(s+4)}$

(D) $\dfrac{1}{(s^2+8s+17)}$

Correct Answer: (B) $\dfrac{1}{(s+4)}$

Solution:

Step 1: Label the signals in the diagram.

Let $E(s)$ be the output of the first summing junction, so $E(s)$ is also the signal that enters the first block $1/(s+4)$. Reading the diagram, the branch feeding the second block $1/(s+4)$ is tapped from this very same line, so the second block's input is also $E(s)$, not $C(s)$.

Step 2: Write the two block outputs.

$$C(s) = E(s) \cdot \frac{1}{s+4}$$

$$\text{Second block output} = E(s) \cdot \frac{1}{s+4}$$

Since both blocks are identical and both receive exactly the same input $E(s)$, their outputs are identical:

$$\text{Second block output} = C(s)$$

Step 3: Evaluate the second summing junction.
The second summing junction adds $C(s)$, coming off the main output line on its "+" input, and subtracts the second block's output on its "-" input. Since these two signals are equal,

$$F(s) = C(s) - C(s) = 0$$

Here $F(s)$ is the signal this second summing junction sends back to the first summing junction's negative input.

Step 4: Apply this to the first summing junction.

$$E(s) = R(s) - F(s) = R(s) - 0 = R(s)$$

The feedback contributes nothing, for every value of s, so the error signal simply equals the reference input.

Step 5: Find the overall transfer function.

$$C(s) = E(s) \cdot \frac{1}{s+4} = \frac{R(s)}{s+4}$$

$$\frac{C(s)}{R(s)} = \frac{1}{s+4}$$

Step 6: Explain why the other options are wrong.

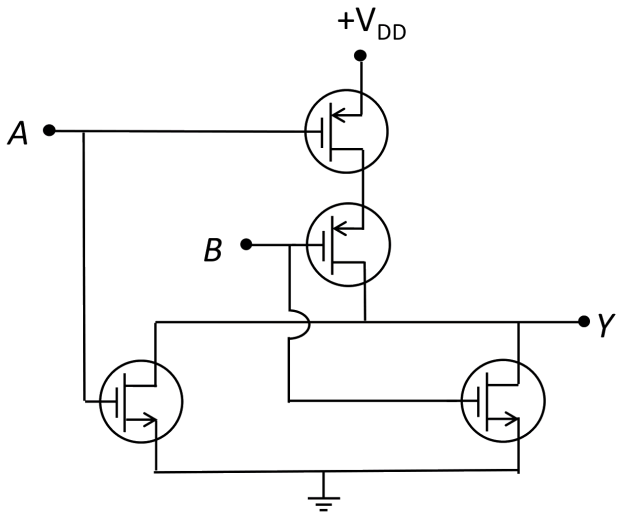
Option (A), $1/(s+4)^2$, and option (D), $1/(s^2+8s+17)$, would appear if the second block were wrongly assumed to be fed by $C(s)$ instead of $E(s)$, leading to a non-zero inner loop. Option (C), $2/(s+4)$, would follow from wrongly adding the two block outputs instead of subtracting them at the second summing junction. Careful tracing of the actual tap point shows the inner loop cancels exactly, leaving the simple cascade result.

Final Answer:

$$\boxed{\frac{1}{s+4}}$$

Quick Tip: Check exactly where the second block's input line is tapped from. It shares its input with the first block, so its output equals $C(s)$, which makes the inner feedback cancel to zero.

20. In the circuit shown in the Figure, A and B are logic inputs and Y is the logic output.



Which of the following logic operations is realized by the circuit?

(A) NOR

(B) OR

(C) XOR

(D) AND

Correct Answer: (A) NOR

Solution:

Step 1: Identify the pull-up network.

Two PMOS transistors are stacked in series between $+V_{DD}$ and the output node Y, one gated by A and one gated by B. A PMOS transistor conducts when its gate is at logic **0**. Since the two PMOS devices are in series, current can reach Y from V_{DD} only when both PMOS transistors conduct together, which needs both A and B at logic **0**.

Step 2: Identify the pull-down network.

Two NMOS transistors sit in parallel between Y and ground, one

gated by A and one gated by B. An NMOS transistor conducts when its gate is at logic 1. Because the two NMOS devices are in parallel, Y gets pulled to ground if either A or B, or both, is at logic 1.

Step 3: Build the truth table.

When $A = 0, B = 0$: both PMOS conduct, giving $Y = 1$; both NMOS are off, so there is no conflicting path to ground. When $A = 0, B = 1$: the PMOS gated by B turns off, breaking the series pull-up path, while the NMOS gated by B turns on, pulling $Y = 0$. When $A = 1, B = 0$: by the same reasoning with the roles swapped, $Y = 0$. When $A = 1, B = 1$: both PMOS are off and both NMOS conduct, so $Y = 0$.

Step 4: Match the truth table to a logic function.

Y is 1 only when both A and B are 0, and Y is 0 for every other input combination. This is exactly the truth table of the NOR function, $Y = \overline{A + B}$.

Step 5: Rule out the other options.

OR would give $Y = 1$ whenever A or B is 1, the opposite of what this circuit produces. XOR would give $Y = 1$ only when A and B differ, which does not match the row $A = 0, B = 0$ giving $Y = 1$ here. AND would need a series NMOS pull-down and a parallel PMOS pull-up, which is the reverse of this circuit's structure.

Final Answer:

The circuit realizes the NOR function. This is the standard static CMOS structure: series PMOS in the pull-up network paired with parallel NMOS in the pull-down network always gives a NOR gate.

NOR

Quick Tip: Series PMOS in the pull-up network together with parallel NMOS in the pull-down network is the standard CMOS structure for a NOR gate.

21. Consider the Friis' transmission equation $P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi D)^2}$, where P_R and P_T are the received and the transmitted powers, respectively. G_T and G_R are the gain of transmitting and receiving antennas, respectively, D is the distance between the transmitting and receiving antennas, and λ is the wavelength in free space.

Given: $G_T = G_R = 1.0$, $\lambda = 0.30$ m and $P_T = +10$ dBm.

Choose the distance (D), in km, from the following options at which the received power, $P_R = -90$ dBm?

(A) $\frac{15}{4\pi}$

(B) $\frac{15}{2\pi}$

(C) $\frac{75}{2\pi}$

(D) $\frac{3}{4\pi}$

Correct Answer: (B) $\frac{15}{2\pi}$

Solution:

Step 1: Convert the power levels to a linear ratio.

Both P_T and P_R are given in dBm, so the ratio between them in dB is

$$P_R(\text{dB}) - P_T(\text{dB}) = -90 - 10 = -100 \text{ dB}$$

Converting this decibel difference to a plain linear power ratio,

$$\frac{P_R}{P_T} = 10^{-100/10} = 10^{-10}$$

Step 2: Write the Friis equation as a power ratio.

Since $G_T = G_R = 1.0$, the Friis equation simplifies to

$$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi D}\right)^2$$

Step 3: Equate the two expressions for the ratio.

$$\left(\frac{\lambda}{4\pi D}\right)^2 = 10^{-10}$$

Taking the square root of both sides,

$$\frac{\lambda}{4\pi D} = 10^{-5}$$

Step 4: Solve for D.

$$D = \frac{\lambda}{4\pi \times 10^{-5}} = \frac{0.30}{4\pi \times 10^{-5}}\ \text{m}$$

$$D = \frac{0.30 \times 10^5}{4\pi}\ \text{m} = \frac{30000}{4\pi}\ \text{m}$$

Step 5: Convert D from metres to kilometres.

$$D = \frac{30000}{4\pi \times 1000}\ \text{km} = \frac{30}{4\pi}\ \text{km} = \frac{15}{2\pi}\ \text{km}$$

Step 6: Rule out the other options.

Option (A), $15/(4\pi)$, is exactly half of the correct value, the kind of error that appears if the square root step in Step 3 is skipped incorrectly. Option (C), $75/(2\pi)$, is five times too large. Option (D), $3/(4\pi)$, is too small by a factor of 10, consistent with a slip in the power-of-ten conversion.

Final Answer:

$$\boxed{D = \frac{15}{2\pi}\ \text{km}}$$

Quick Tip: Convert the 100 dB power difference to a linear ratio first, then use $P_R/P_T = (\lambda/4\pi D)^2$ to solve for D, remembering to take a square root.

22. Consider a discrete memoryless source with an alphabet of four source symbols. $s(t)$ is a multi-level $(-1, 0, +1, +2)$ signal representing a long sequence of random symbols from the above source which is generating 10^4 symbols per second.

Which of the following options is the correct value of equivalent Nyquist bandwidth of $s(t)$?

(A) 10 kHz

(B) 64 kHz

(C) 5 kHz

(D) 20 kHz

Correct Answer: (C) 5 kHz

Solution:

Step 1: Recall the Nyquist minimum bandwidth theorem.

For an M-ary baseband pulse signal transmitted at a symbol (baud) rate of R_s symbols per second, the minimum bandwidth needed to send it without inter-symbol interference is

$$B_{\text{Nyquist}} = \frac{R_s}{2}$$

This result depends only on the symbol rate, not on how many levels M each symbol can take. The number of amplitude levels changes how much information each symbol carries, but it does not change the minimum pulse bandwidth needed at a fixed symbol rate.

Step 2: Identify the symbol rate from the question.

The source produces 10^4 symbols per second, so

$$R_s = 10^4 \text{ symbols/s}$$

The fact that each symbol is one of four levels $(-1, 0, +1, +2)$ tells us this is a 4-ary signal, but as noted in Step 1, this does not enter the Nyquist bandwidth formula directly.

Step 3: Apply the formula.

$$B_{\text{Nyquist}} = \frac{R_s}{2} = \frac{10^4}{2} = 5 \times 10^3 \text{ Hz} = 5 \text{ kHz}$$

Step 4: Rule out the other options.

10 kHz is simply the symbol rate itself, mistaking the bandwidth for R_s rather than $R_s/2$. 20 kHz doubles the symbol rate rather than halving it. 64 kHz does not follow from either the symbol rate or the number of levels in this problem at all.

Final Answer:

$$\boxed{B_{\text{Nyquist}} = 5 \text{ kHz}}$$

Quick Tip: The Nyquist bandwidth of a baseband signal is half the symbol rate, regardless of how many amplitude levels each symbol has.

23. The relation between the input current (I) and the output voltage (V) of a circuit is governed by the equation: $C\frac{dV}{dt} = I(t) - m(t)$. The circuit is excited by $I(t) = q\,\delta(t)$, where q is a real valued constant. V at $t = 0^-$ is V_0. Which of the following is an equivalent representation of the above case?

(A) $C\frac{dV}{dt} = -m(t)$, with $V(t = 0^-) = V_0 + q/C$

(B) $C\frac{dV}{dt} = -m(t)$, with $V(t = 0^-) = V_0 + q/C + m(t = 0^-)$

(C) $C\frac{dV}{dt} = -m(t)$, with $V(t = 0^-) = V_0 - q/C + m(t = 0^-)$

(D) $C\frac{dV}{dt} = -m(t)$, with $V(t = 0^-) = q/C$

Correct Answer: (A) $C\frac{dV}{dt} = -m(t)$, with $V(t = 0^-) = V_0 + q/C$

Solution:

Step 1: Write down what the impulse does physically.

The original equation is

$$C\frac{dV}{dt} = I(t) - m(t) = q\,\delta(t) - m(t)$$

A current impulse $q\,\delta(t)$ delivers a finite, instantaneous burst of charge q to the capacitor exactly at $t = 0$. Everywhere except at that single instant, the equation just reads $C\,dV/dt = -m(t)$, since $\delta(t) = 0$ for $t \neq 0$.

Step 2: Integrate the equation across the impulse.

Integrate both sides from $t = 0^-$ to $t = 0^+$, an interval that shrinks to zero width but still contains the impulse:

$$C\int_{0^-}^{0^+} \frac{dV}{dt}\,dt = \int_{0^-}^{0^+} q\,\delta(t)\,dt - \int_{0^-}^{0^+} m(t)\,dt$$

Step 3: Evaluate each term.

The left side is $C[V(0^+) - V(0^-)]$ by the fundamental theorem of calculus. On the right, the sifting property of the delta function gives $\int_{0^-}^{0^+} q\,\delta(t)\,dt = q$. Since $m(t)$ is an ordinary, non-impulsive function, its integral over a vanishing time interval is zero:

$$\int_{0^-}^{0^+} m(t)\,dt = 0$$

Step 4: Combine these results.

$$C[V(0^+) - V(0^-)] = q - 0 = q$$

$$V(0^+) = V(0^-) + \frac{q}{C} = V_0 + \frac{q}{C}$$

Step 5: Write the reduced equation.

For every instant after the impulse has acted, the current source term is gone, since $\delta(t) = 0$ for $t > 0$, leaving

$$C\frac{dV}{dt} = -m(t)$$

This reduced equation needs its own starting value to be solved going forward, and that starting value is exactly the jumped voltage found in Step 4, that is $V_0 + q/C$.

Step 6: Rule out the other options.

Options (B) and (C) both add an extra $m(t = 0^-)$ term to the jump, but Step 3 showed the ordinary function $m(t)$ contributes nothing to the jump, since it has no impulsive content there; only the delta-function current source produces a jump. Option (D) drops V_0 entirely, which would only be correct if the capacitor started at zero voltage before the impulse arrived, which is not stated here.

Final Answer:

$$\boxed{C\frac{dV}{dt} = -m(t), \text{ starting from } V_0 + \frac{q}{C}}$$

Quick Tip: A current impulse of area q into a capacitor C causes an instantaneous voltage jump of q/C; add this jump to the pre-impulse voltage V_0 to get the new starting condition.

24. The electric field of a monochromatic plane wave travelling in a lossless isotropic and homogenous medium is given by

$$\vec{E}(z,t) = E_0\left[\hat{x}\cos(\omega t - kz) + \hat{y}\sin(\omega t - kz)\right]$$

in a right-handed orthogonal co-ordinate system.
Which of the following is the correct polarization of the electromagnetic wave?

(A) Right-handed circularly polarized

(B) Left-handed circularly polarized

(C) Linearly polarized

(D) Linearly polarized with $-45°$ angle to $\hat{x}$

Correct Answer: (A) Right-handed circularly polarized

Solution:

Step 1: Identify the direction of propagation and the field components.

The phase term $\omega t - kz$ shows the wave travels in the $+\hat{z}$ direction.
Writing $\phi = \omega t - kz$, the two transverse components are

$$E_x = E_0\cos\phi, \qquad E_y = E_0\sin\phi$$

Both components have the same amplitude E_0 and are $90°$ out of phase with each other, which already rules out linear polarization (options C and D), since a linearly polarized wave needs its two components either in phase or exactly out of phase by $180°$, tracing a straight line rather than a circle.

Step 2: Track how the tip of the field vector moves in time at a fixed plane.

Fix $z = 0$ and watch the field as t increases. At $\omega t = 0$, $(E_x, E_y) = (E_0, 0)$, pointing along $+\hat{x}$. At $\omega t = 90°$, $(E_x, E_y) = (0, E_0)$, pointing along $+\hat{y}$. So as time increases, the tip of $\vec{E}$ sweeps from $+\hat{x}$ toward $+\hat{y}$.

Step 3: Apply the handedness rule for the direction of propagation $+\hat{z}$.

Using the standard right-hand (corkscrew) convention: point the right thumb along the direction of propagation, $+\hat{z}$. The fingers of the right hand then curl from $+\hat{x}$ toward $+\hat{y}$ as seen by an observer standing further along $+\hat{z}$ and looking back toward the source, that is, looking in the $-\hat{z}$ direction. This is exactly the sense of rotation found in Step 2, sweeping from $+\hat{x}$ to $+\hat{y}$ as t grows.

Step 4: Conclude the handedness.

Since the field's rotation matches the right-hand curl direction for propagation along $+\hat{z}$, the wave is right-handed circularly polarized.

Step 5: Rule out the remaining option.

Left-handed circular polarization would require the field to sweep the opposite way, from $+\hat{x}$ toward $-\hat{y}$ as t increases, which would correspond to a minus sign in front of the $\sin\phi$ term instead of the

plus sign actually given.

Final Answer:

$$\boxed{\text{Right-handed circularly polarized}}$$

Quick Tip: Since $E_x = E_0 \cos\phi$ and $E_y = E_0 \sin\phi$ have equal amplitude and a 90 degree phase difference, the wave is circularly polarized; use the right-hand rule along the propagation direction to fix the handedness.

25. Consider a p-n junction diode when it is forward biased with 2 V.

Which of the following is/are the correct magnitude(s) of the energy difference between quasi Fermi-levels, E_{fn} in the n-side and E_{fp} in the p-side?

(A) 2 eV

(B) 1 eV

(C) 2 V

(D) 1 V

Correct Answer: (A) 2 eV

Solution:

Step 1: Recall what a quasi-Fermi level is.

In a p-n junction at equilibrium there is one common Fermi level, E_F, on both sides. Once the diode is forward biased, the carrier populations move out of equilibrium and we need two separate levels: E_{fn} for electrons and E_{fp} for holes.

Step 2: Write the carrier concentrations using quasi-Fermi levels.
The electron and hole densities anywhere in the device can be written as

$$n = n_i \exp\left(\frac{E_{fn} - E_i}{kT}\right), \qquad p = n_i \exp\left(\frac{E_i - E_{fp}}{kT}\right)$$

Multiplying these two expressions,

$$np = n_i^2 \exp\left(\frac{E_{fn} - E_{fp}}{kT}\right)$$

Step 3: Connect this product to the applied bias.
For a forward biased junction with negligible series resistance, the standard result for the junction (the law of the junction) gives, through the depletion region,

$$np = n_i^2 \exp\left(\frac{qV}{kT}\right)$$

where V is the applied forward voltage.

Step 4: Equate the two expressions for np.
Comparing the exponents,

$$\frac{E_{fn} - E_{fp}}{kT} = \frac{qV}{kT}$$

so

$$E_{fn} - E_{fp} = qV$$

The split between the two quasi-Fermi levels, measured in energy, equals the applied voltage measured in electron-volts.

Step 5: Put in the given voltage.
Here $V = 2$ V, so

$$|E_{fn} - E_{fp}| = q(2\text{ V}) = 2\text{ eV}$$

Step 6: Check the other options.

(B) 1 eV: This would need $V = 1$ V, but the diode is biased at 2 V, so this is wrong.

(C) 2 V: This mixes up units. The Fermi level split is an energy, so it must be expressed in eV, not V.

(D) 1 V: Wrong for the same unit reason as (C), and also has the wrong number.

Final Answer:
The magnitude of the energy split between the quasi-Fermi levels is

$$\boxed{2\text{ eV}}$$

Quick Tip: The split between quasi-Fermi levels equals the applied voltage converted to energy: $E_{fn} - E_{fp} = qV$.

26. Consider the matrix $M = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ -1 & a & b \end{bmatrix}$.

Which of the following options is/are TRUE if $\det(M) \neq 0$?

(A) $a = -\frac{1}{2}$ and $b = -\frac{1}{2}$

(B) $a = \frac{1}{2}$ and $b = \frac{1}{2}$

(C) $a = -3$ and $b = 0$

(D) $a = \frac{1}{2}$ and $b = -3$

Correct Answer: (B) $a = \frac{1}{2}$ and $b = \frac{1}{2}$; (D) $a = \frac{1}{2}$ and $b = -3$

Solution:

Step 1: Write the determinant using cofactor expansion along the first row.

For $M = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ -1 & a & b \end{bmatrix}$, expanding along the first row gives

$$\det(M) = 2\begin{vmatrix} 3 & 0 \\ a & b \end{vmatrix} - 1\begin{vmatrix} 1 & 0 \\ -1 & b \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ -1 & a \end{vmatrix}$$

Step 2: Evaluate each 2×2 minor.

$$\begin{vmatrix} 3 & 0 \\ a & b \end{vmatrix} = 3b, \qquad \begin{vmatrix} 1 & 0 \\ -1 & b \end{vmatrix} = b, \qquad \begin{vmatrix} 1 & 3 \\ -1 & a \end{vmatrix} = a + 3$$

Step 3: Combine the terms.

$$\det(M) = 2(3b) - 1(b) + 1(a+3) = 6b - b + a + 3$$

$$\det(M) = 5b + a + 3$$

Step 4: Apply the condition $\det(M) \neq 0$.
We need $5b + a + 3 \neq 0$ for the given pair. Check each option by plugging it in.

Step 5: Check option (A), $a = -\frac{1}{2},\ b = -\frac{1}{2}$.

$$5\left(-\tfrac{1}{2}\right) + \left(-\tfrac{1}{2}\right) + 3 = -2.5 - 0.5 + 3 = 0$$

The result is 0, so $\det(M) = 0$ here, which contradicts the given condition. Option (A) is incorrect.

Step 6: Check option (B), $a = \frac{1}{2},\ b = \frac{1}{2}$.

$$5\left(\tfrac{1}{2}\right) + \tfrac{1}{2} + 3 = 2.5 + 0.5 + 3 = 6 \neq 0$$

This keeps $\det(M) \neq 0$, so option (B) is correct.

Step 7: Check option (C), $a = -3,\ b = 0$.

$$5(0) + (-3) + 3 = 0$$

This gives $\det(M) = 0$, contradicting the condition. Option (C) is incorrect.

Step 8: Check option (D), $a = \frac{1}{2}$, $b = -3$.

$$5(-3) + \tfrac{1}{2} + 3 = -15 + 3.5 = -11.5 \neq 0$$

This keeps $\det(M) \neq 0$, so option (D) is correct.

Final Answer:

Only the pairs in options (B) and (D) keep the determinant nonzero.

$$\boxed{a = \tfrac{1}{2},\ b = \tfrac{1}{2} \text{ and } a = \tfrac{1}{2},\ b = -3}$$

Quick Tip: Expand $\det(M)$ in terms of a and b, then check which given pairs make it nonzero.

27. A binary ripple counter is designed to count $(0)_{10}$ to $(64)_{10}$.

Which of the following is/are the number of flip-flops required to design the counter?

(A) 6

(B) 7

(C) 4

(D) 5

Correct Answer: (B) 7

Solution:

Step 1: Find how many distinct states the counter must have.
The counter needs to count every value from 0 up to 64, inclusive. That is

$$64 - 0 + 1 = 65$$

distinct states.

Step 2: Recall how many states n flip-flops give.
A ripple counter built from n flip-flops can represent 2^n distinct binary states, from 0 to $2^n - 1$.

Step 3: Find the smallest n with enough states.
Try $n = 6$:

$$2^6 = 64$$

This only gives states 0 to 63, so state 64 cannot be reached. Six flip-flops are not enough.
Try $n = 7$:

$$2^7 = 128$$

This gives states 0 to 127, which comfortably covers 0 to 64.

Step 4: Check the other options.

(A) 6: Gives only 64 states (0 to 63), one short of what is needed. Incorrect.

(C) 4: Gives only 16 states, far too few. Incorrect.

(D) 5: Gives only 32 states, still too few. Incorrect.

Final Answer:
The smallest number of flip-flops that lets the counter reach all the way to 64 is

$$\boxed{7}$$

Quick Tip: You need $2^n \geq 65$ states to count from 0 to 64; find the smallest such n.

28. Which option(s) represents/represent the dielectric loss tangent of a substrate?

(A) Ratio of the real to imaginary parts of the total displacement current

(B) $(\omega\epsilon'' + \sigma)/(\omega\epsilon')$

(C) Ratio of the electric susceptibility to permittivity

(D) Ratio of the polarization vector $\vec{P}$ to the displacement vector $\vec{D}$

Correct Answer: (A) Ratio of the real to imaginary parts of the total displacement current ; (B) $(\omega\epsilon'' + \sigma)/(\omega\epsilon')$

Solution:

Step 1: Recall the standard definition of loss tangent.
For a lossy dielectric with complex permittivity $\epsilon = \epsilon' - j\epsilon''$ and conductivity σ, the loss tangent measures how much energy is dissipated compared to how much is stored. It is defined as

$$\tan\delta = \frac{\epsilon''_{eff}}{\epsilon'}$$

where the effective imaginary part folds in the ohmic loss due to conductivity,

$$\epsilon''_{eff} = \epsilon'' + \frac{\sigma}{\omega}$$

Step 2: Write the loss tangent explicitly.
Substituting,

$$\tan\delta = \frac{\epsilon'' + \frac{\sigma}{\omega}}{\epsilon'} = \frac{\omega\epsilon'' + \sigma}{\omega\epsilon'}$$

This is exactly option (B), so (B) is correct.

Step 3: Look at the total current in the dielectric.
The total current density (conduction plus displacement) in phasor form is

$$J = \sigma E + j\omega\epsilon E = \sigma E + j\omega(\epsilon' - j\epsilon'')E = (\sigma + \omega\epsilon'')E + j\omega\epsilon' E$$

The real part, $(\sigma + \omega\epsilon'')E$, is in phase with the field and represents the dissipated (lossy) part. The imaginary part, $\omega\epsilon' E$, is the reactive, lossless part that stores energy.

Step 4: Form the ratio of real to imaginary parts.

$$\frac{\text{Real part}}{\text{Imaginary part}} = \frac{\sigma + \omega\epsilon''}{\omega\epsilon'}$$

This is the same expression as the loss tangent found in Step 2. So describing the loss tangent as the ratio of the real to imaginary parts of this total current is also a valid statement, and option (A) is correct too.

Step 5: Check option (C).

The electric susceptibility χ relates polarization to field by $P = \epsilon_0 \chi E$, and permittivity is $\epsilon = \epsilon_0(1 + \chi)$. The ratio χ/ϵ is a plain real number and says nothing about any phase lag, so it cannot represent a loss. Option (C) is incorrect.

Step 6: Check option (D).

Since $D = \epsilon_0 E + P$, the ratio $|P|/|D|$ is just another way of writing $\chi/(1 + \chi)$, again a real, lossless quantity with no imaginary part involved. Option (D) is incorrect.

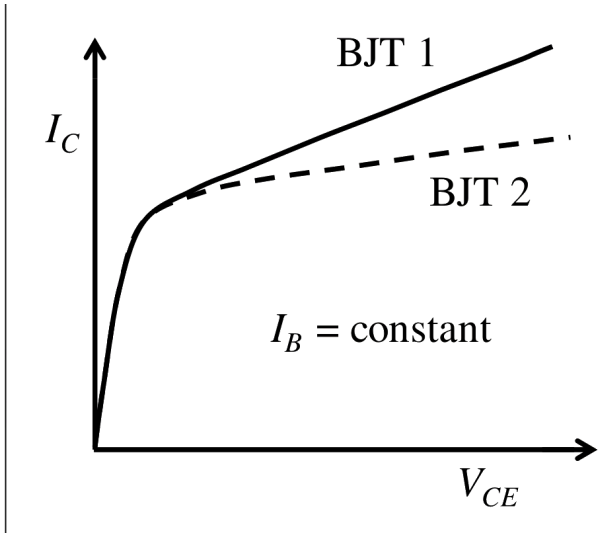
Final Answer:

The loss tangent is correctly represented by

$$\boxed{\text{options (A) and (B)}}$$

Quick Tip: Loss tangent compares the in-phase (lossy) current to the out-of-phase (reactive) current: $\tan\delta = (\omega\epsilon'' + \sigma)/(\omega\epsilon')$.

29. Figure shows the output characteristics of two different Bipolar Junction Transistors (BJT), BJT 1 with magnitude of Early voltage $|V_{A1}|$, and BJT 2 with magnitude of Early voltage $|V_{A2}|$.



Which of the following options is/are correct regarding the Early voltages?

(A) $|V_{A1}| > |V_{A2}|$

(B) $|V_{A1}|$ is infinitely large

(C) $|V_{A1}| < |V_{A2}|$

(D) $|V_{A2}|$ is finite

Correct Answer: (C) $|V_{A1}| < |V_{A2}|$; (D) $|V_{A2}|$ is finite

Solution:

Step 1: Recall what the Early voltage means on this graph.

For a BJT biased at a fixed base current, I_C rises a little with V_{CE} once the transistor is in the active region (the Early effect). Extending the sloped part of the I_C versus V_{CE} line backward, it crosses the V_{CE} axis at $V_{CE} = -V_A$, where V_A is that transistor's Early voltage.

Step 2: Connect the slope of the curve to the Early voltage.
Near a bias point, I_C can be written as

$$I_C \approx I_{C0}\left(1 + \frac{V_{CE}}{V_A}\right)$$

so the slope of the curve is

$$\frac{dI_C}{dV_{CE}} \approx \frac{I_{C0}}{V_A}$$

A larger V_A gives a smaller slope (flatter curve), and a smaller V_A gives a larger slope (steeper curve).

Step 3: Read the two curves in the figure.
BJT 1's curve rises noticeably as V_{CE} increases (steeper slope). BJT 2's curve, drawn dashed, is much closer to flat over the same range of V_{CE} (smaller slope).

Step 4: Compare the Early voltages using the slopes.
Since BJT 1 is steeper, its slope is larger, so from Step 2 its Early voltage $|V_{A1}|$ must be smaller. Since BJT 2 is flatter, its slope is smaller, so its Early voltage $|V_{A2}|$ must be larger. Hence

$$|V_{A1}| < |V_{A2}|$$

This makes option (C) correct and rules out option (A), which claims the opposite.

Step 5: Check whether either Early voltage could be infinite.
An infinite Early voltage would mean a slope of exactly zero, a

perfectly horizontal line with I_C completely independent of V_{CE}. BJT 1's line clearly slants upward, so its slope is not zero, ruling out option (B).

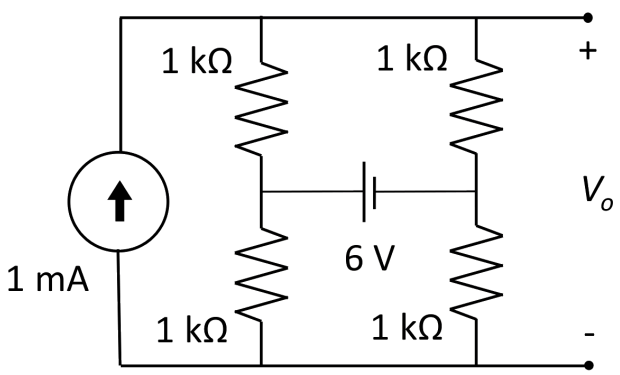
Step 6: Check option (D).
Even though BJT 2's line looks almost flat on this scale, no real transistor has a perfectly flat output characteristic; the Early effect always gives it some finite slope, however small. A truly flat line would need $|V_{A2}| \to \infty$, which does not happen in an actual device. So $|V_{A2}|$ is large but still finite, making option (D) correct.

Final Answer:

$$\boxed{|V_{A1}| < |V_{A2}| \text{ and } |V_{A2}| \text{ is finite}}$$

Quick Tip: Extend each curve's straight part back to the V_{CE} axis; a flatter curve corresponds to a larger, but still finite, Early voltage.

30. The output voltage V_o (in Volt) for the network given in the Figure is (rounded off to two decimal places).



Correct Answer: 1.00

Solution:

Step 1: Label the nodes of the network.

Call the top rail (where the current source's upper terminal, both $1\ \text{k}\Omega$ top resistors, and the $+$ terminal of V_o all meet) node T. Call the bottom rail (the current source's lower terminal, both bottom $1\ \text{k}\Omega$ resistors, and the $-$ terminal of V_o) node B. The 6 V source sits between two middle nodes, call them M_1 (left) and M_2 (right). Since the output terminals are open, no current leaves through $+$ or $-$, so $V_o = V_T - V_B$.

Step 2: Notice the resistor network is symmetric.

All four resistors are $1\ \text{k}\Omega$: T to M_1, T to M_2, M_1 to B, and M_2 to B. This is a balanced bridge with the 6 V source across the bridge's middle branch (M_1 to M_2).

Step 3: Use superposition, starting with the 6 V source acting alone.

Turn off the current source (open it) and keep only the 6 V source. With the current source open, node T only connects out through the two top resistors, and since nothing else attaches to T, the current entering T from the two top resistors must sum to zero. Writing $V_T = t$, $V_{M_1} = m_1$, $V_{M_2} = m_2$:

$$\frac{t - m_1}{1} + \frac{t - m_2}{1} = 0 \implies 2t = m_1 + m_2$$

The same reasoning at node B (nothing else attaches there either) gives

$$\frac{m_1 - 0}{1} + \frac{m_2 - 0}{1} = 0 \implies m_1 + m_2 = 0$$

Combining these, $2t = 0$, so $t = 0$, and the same steps give $V_B = 0$ too. So the 6 V source alone contributes nothing to V_o: it only pushes a circulating current inside the bridge.

Step 4: Now turn on the current source alone (short the 6 V source). With the voltage source shorted, M_1 and M_2 merge into a single node M. From T, the two $1\ \text{k}\Omega$ resistors going to M are in parallel:

$$1\ \text{k}\Omega \parallel 1\ \text{k}\Omega = 0.5\ \text{k}\Omega$$

Likewise, from M to B the two bottom resistors in parallel also give $0.5\ \text{k}\Omega$. These two pieces are in series between T and B, so the total resistance seen by the current source is

$$R_{eq} = 0.5 + 0.5 = 1\ \text{k}\Omega$$

Step 5: Compute the voltage produced by the current source. Since the +/− terminals draw no current, the whole 1 mA from the source must flow through this $1\ \text{k}\Omega$ equivalent resistance between T and B, giving

$$V_T - V_B = I \times R_{eq} = 1\ \text{mA} \times 1\ \text{k}\Omega = 1\ \text{V}$$

Step 6: Add the two contributions.
From Step 3, the 6 V source contributes 0 V to V_o. From Step 5, the current source contributes 1 V. So

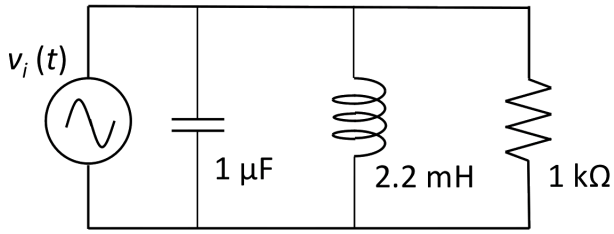
$$V_o = 0 + 1 = 1\,\text{V}$$

Final Answer:

$$\boxed{V_o = 1.00\,\text{V}}$$

Quick Tip: The bridge of four equal $1\,\text{k}\Omega$ resistors is balanced, so the 6 V source does not affect V_o; only the current source and the $1\,\text{k}\Omega$ equivalent resistance matter.

31. Consider the circuit shown in the Figure, where the input $v_i(t)$ is in Volt.



$v_i\,(t) = 12\sin(\omega t)$

The average power (in mW) dissipated in the load resistance of $1\,\text{k}\Omega$ at the resonant frequency is (rounded off to two decimal places).

Correct Answer: 72.00

Solution:

Step 1: Look at how the source is connected.

The source $v_i(t)$, the capacitor, the inductor, and the resistor are all drawn directly in parallel with each other, with no impedance in series with the ideal source. That means the voltage across every one of these branches, including the resistor, is exactly $v_i(t)$ at every instant, no matter what the frequency is.

Step 2: Write the source explicitly.

$$v_i(t) = 12\sin(\omega t)\ \text{V}$$

so its amplitude is $V_m = 12$ V.

Step 3: Note what "resonant frequency" means here.

The resonant frequency of the parallel L-C combination is where their reactances cancel,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

At this frequency the tank draws minimum current from a source feeding it, but here the resistor is wired straight to the ideal voltage source, not fed through the tank. So the tank's resonance does not change the voltage the resistor sees; that voltage stays fixed at $v_i(t)$ regardless of ω.

Step 4: Compute the rms voltage across the resistor.

For a sinusoid of amplitude V_m,

$$V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{12}{\sqrt{2}}\ \text{V}$$

Step 5: Compute the average power dissipated in the 1 kΩ resistor.

$$P_{avg} = \frac{V_{rms}^2}{R} = \frac{\left(\frac{12}{\sqrt{2}}\right)^2}{1000} = \frac{72}{1000}\ \text{W}$$

$$P_{avg} = 0.072\ \text{W} = 72\ \text{mW}$$

Step 6: Note the role of resonance.

Even though the question mentions the resonant frequency, the resistor's power does not actually depend on ω here, because the resistor sits straight across the ideal source. Resonance only matters for the current drawn from the source itself, not for the power in the resistor.

Final Answer:

$$\boxed{P_{avg} = 72.00\ \text{mW}}$$

Quick Tip: The resistor sits directly across the ideal source, so its power depends only on V_m and R: $P = \frac{(V_m/\sqrt{2})^2}{R}$.

32. A wireless digital transmission scheme is using 16-QAM over an additive white Gaussian noise channel and a maximum-likelihood receiver. Consider the information bit rate from source to be 4×10^6 bits per second.

The minimum transmission bandwidth (in MHz) of the modulated signal necessary for optimum recovery of information at the receiver is (rounded off to two decimal places).

Correct Answer: 1.00

Solution:

Step 1: Find how many bits each 16-QAM symbol carries.

16-QAM has 16 distinct constellation points, and

$$16 = 2^4$$

so each symbol carries

$$k = \log_2(16) = 4 \text{ bits}$$

Step 2: Find the symbol rate.

The information bit rate is

$$R_b = 4 \times 10^6 \text{ bits/s}$$

Each symbol packs 4 bits, so the symbol (baud) rate is

$$R_s = \frac{R_b}{k} = \frac{4 \times 10^6}{4} = 1 \times 10^6 \text{ symbols/s}$$

Step 3: Recall the minimum bandwidth needed for a QAM signal.
For QAM, the baseband in-phase and quadrature pulse streams are each shaped to just fit in a bandwidth of $R_s/2$ (satisfying the Nyquist criterion for zero inter-symbol interference at the minimum possible bandwidth). When these two streams are placed on quadrature carriers to build the passband QAM signal, the two sidebands combine so that the total transmission bandwidth required is exactly

$$B_{min} = R_s$$

Step 4: Substitute the symbol rate.

$$B_{min} = 1 \times 10^6 \text{ Hz} = 1 \text{ MHz}$$

Final Answer:

$$\boxed{B_{min} = 1.00 \text{ MHz}}$$

Quick Tip: Find bits per symbol from $\log_2(16)$, convert bit rate to symbol rate, then use $B_{min} = R_s$ for QAM.

33. The cutoff frequency (in GHz) for the dominant TE_{10} mode of an air-filled rectangular waveguide of inner dimension 0.28 inch × 0.14 inch is . (rounded off to two decimal places)

Correct Answer: 21.09

Solution:

Step 1: Recall the cutoff frequency formula for TE10 mode.

For a rectangular waveguide with broad wall dimension a and narrow wall dimension b, the cutoff frequency of the dominant TE_{10} mode depends only on the broad dimension a. It is given by

$$f_c = \frac{c}{2a}$$

where c is the speed of light in free space, taken as 3×10^8 m/s.

Step 2: Identify the broad dimension.

The waveguide has inner dimensions 0.28 inch $\times$ 0.14 inch. The broad wall dimension is the larger one, so

$$a = 0.28 \text{ inch}$$

Step 3: Convert the dimension to metres.

Since 1 inch $= 0.0254$ m,

$$a = 0.28 \times 0.0254 = 0.007112 \text{ m}$$

Step 4: Substitute into the cutoff frequency formula.

$$f_c = \frac{3 \times 10^8}{2 \times 0.007112} = \frac{3 \times 10^8}{0.014224}$$

Step 5: Evaluate the expression.

$$f_c \approx 2.1091 \times 10^{10} \text{ Hz} = 21.09 \text{ GHz}$$

Step 6: Final answer.

The cutoff frequency for the dominant TE_{10} mode is

$$\boxed{21.09 \text{ GHz}}$$

Quick Tip: For TE10 mode the cutoff frequency depends only on the broad wall dimension a, through fc = c/(2a).

34. For a lossless passive two-port network, $|S_{11}|$ and $|S_{21}|$ intersect at -3 dB.

For a lossy passive two-port network, $|S_{11}|$ and $|S_{21}|$ intersect at -4 dB.

The percentage of power dissipated in the lossy network at the intersection frequency is .

(rounded off to two decimal places)

Correct Answer: 20.38

Solution:

Step 1: Recall power conservation for a two-port network.

For a two-port network fed at port 1 with a matched port 2, the fraction of incident power reflected is $|S_{11}|^2$ and the fraction transmitted is $|S_{21}|^2$. The fraction dissipated inside the network is

$$P_{diss} = 1 - |S_{11}|^2 - |S_{21}|^2$$

Step 2: Check the lossless case as a sanity check.
For a lossless network, no power is dissipated, so

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

At the intersection point $|S_{11}| = |S_{21}|$, so each term equals 0.5, which is $10\log_{10}(0.5) = -3.01$ dB. This matches the given -3 dB intersection for the lossless case, confirming the power relation used above.

Step 3: Convert the lossy intersection value from dB to a ratio.
For the lossy network, $|S_{11}| = |S_{21}|$ at -4 dB, so

$$|S_{11}|^2 = |S_{21}|^2 = 10^{-4/10} = 10^{-0.4}$$

Step 4: Evaluate the numeric value.

$$10^{-0.4} \approx 0.3981$$

Step 5: Find the total power leaving the network.

$$|S_{11}|^2 + |S_{21}|^2 = 2 \times 0.3981 = 0.7962$$

Step 6: Find the power dissipated inside the lossy network.

$$P_{diss} = 1 - 0.7962 = 0.2038$$

Step 7: Convert to a percentage.

$$P_{diss} \times 100 = 20.38\%$$

Step 8: Final answer.

The percentage of power dissipated in the lossy network at the intersection frequency is

$$\boxed{20.38\%}$$

Quick Tip: Use power conservation: fraction dissipated = 1 minus |S11|^2 minus |S21|^2, after converting the given dB value to a ratio.

35. The negative edge triggered JK flip-flop shown has J and K inputs tied to Logic High, and a square wave of 10 cycles/second is applied to its clock (C) input.

The frequency of the output Q (in cycles/second) is .

(rounded off to two decimal places)

Correct Answer: 5.00

Solution:

Step 1: Recall the JK flip-flop behaviour when J = K = 1.

A JK flip-flop with both inputs tied to Logic High is in toggle mode. In

this mode, the output Q flips its state, from 0 to 1 or from 1 to 0, on every active clock edge.

Step 2: Identify the active edge.
The flip-flop is negative edge triggered, so it toggles once for every falling (high-to-low) transition of the clock signal, and it does not change state on the rising edge.

Step 3: Count active edges per clock cycle.
One full clock cycle contains exactly one falling edge and one rising edge. Since only the falling edge causes a toggle, the flip-flop toggles exactly once per clock cycle.

Step 4: Relate toggle rate to output frequency.
Two toggles are needed to complete one full cycle of the output waveform, one toggle to go from 0 to 1 and a second toggle to go back from 1 to 0. Since the flip-flop toggles once per input clock cycle, it needs two input clock cycles to produce one output cycle. So the output frequency is half the clock frequency:

$$f_Q = \frac{f_{clock}}{2}$$

Step 5: Substitute the given clock frequency.

$$f_Q = \frac{10}{2} = 5 \text{ cycles/second}$$

Step 6: Final answer.
The frequency of the output Q is

5.00 cycles/second

Quick Tip: A JK flip-flop with J = K = 1 toggles once per active clock edge, so its output frequency is half the clock frequency.

36. Consider the two series, S_A and S_B, where

$$S_A = \sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

$$S_B = 1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{64} + \frac{1}{128} + \frac{1}{512} + \cdots$$

Which of the following statements is correct for the two given series?

(A) Both S_A and S_B converge.

(B) Neither S_A nor S_B converges.

(C) S_A converges but S_B does not converge.

(D) S_B converges but S_A does not converge.

Correct Answer: (A) Both S_A and S_B converge.

Solution:

Step 1: Test S_A for convergence using the ratio test.

For $S_A = \sum_{n=1}^{\infty} \frac{n^2}{2^n}$, apply the ratio test with $a_n = \frac{n^2}{2^n}$:

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \frac{(n+1)^2}{2^{n+1}} \times \frac{2^n}{n^2} = \lim_{n\to\infty} \frac{1}{2} \left(\frac{n+1}{n} \right)^2 = \frac{1}{2}$$

Since the limit $\frac{1}{2}$ is less than 1, the series S_A converges.

Step 2: Find the exact value of S_A as a check.
Using the standard identity $\sum_{n=1}^{\infty} n^2 x^n = \frac{x(1+x)}{(1-x)^3}$ at $x = \frac{1}{2}$:

$$S_A = \frac{\frac{1}{2}\left(1+\frac{1}{2}\right)}{\left(1-\frac{1}{2}\right)^3} = \frac{\frac{1}{2} \times \frac{3}{2}}{\frac{1}{8}} = \frac{\frac{3}{4}}{\frac{1}{8}} = 6$$

So S_A converges to a finite value, 6.

Step 3: Spot the pattern in S_B.
Write out the terms of S_B and look at the denominators:

$$1, \frac{1}{2}, \frac{1}{8}, \frac{1}{16}, \frac{1}{64}, \frac{1}{128}, \frac{1}{512}, \cdots$$

Splitting into odd-position terms (1st, 3rd, 5th, ...) and even-position terms (2nd, 4th, 6th, ...): odd-position terms 1, $\frac{1}{8}$, $\frac{1}{64}$, ... each equal to $\frac{1}{8}$ times the previous one, and even-position terms $\frac{1}{2}$, $\frac{1}{16}$, $\frac{1}{128}$, ... also each equal to $\frac{1}{8}$ times the previous one.

Step 4: Sum the two interleaved geometric series.
The odd-position terms form a geometric series with first term 1 and common ratio $\frac{1}{8}$:

$$1 + \frac{1}{8} + \frac{1}{64} + \cdots = \frac{1}{1-\frac{1}{8}} = \frac{8}{7}$$

The even-position terms form a geometric series with first term $\frac{1}{2}$ and common ratio $\frac{1}{8}$:

$$\frac{1}{2}+\frac{1}{16}+\frac{1}{128}+\cdots=\frac{\frac{1}{2}}{1-\frac{1}{8}}=\frac{1}{2}\times\frac{8}{7}=\frac{4}{7}$$

Step 5: Add both parts.

$$S_B=\frac{8}{7}+\frac{4}{7}=\frac{12}{7}$$

Since $\frac{12}{7}$ is a finite number, S_B also converges.

Step 6: Analyze the options.

(A) Both S_A and S_B converge: True, as shown above.

(B) Neither S_A nor S_B converges: False, both converge.

(C) S_A converges but S_B does not converge: False, S_B also converges.

(D) S_B converges but S_A does not converge: False, S_A also converges.

Step 7: Final conclusion.
Therefore, the correct statement is

$$\boxed{\text{Both } S_A \text{ and } S_B \text{ converge.}}$$

Quick Tip: Check S_A with the ratio test, then split S_B into two interleaved geometric series with common ratio 1/8 each.

37. The continuous time signal $x(t)$ is real, periodic with period T, and satisfies the Dirichlet conditions.
The Fourier series representation of $x(t)$ is

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{j\left(\frac{2\pi nt}{T}\right)}$$

and $x(t)$ satisfies the following:

$$x\left(t - \frac{T}{2}\right) = -x(t).$$

For any integer m, which of the following options is correct?

(A) $a_{2m} = 0$

(B) $a_{2m} = 1$

(C) $a_{2m} = a_{2m+1}$

(D) $a_{2m} = -1$

Correct Answer: (A) $a_{2m} = 0$

Solution:

Step 1: Write the shifted signal in terms of the Fourier coefficients.
Starting from

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{j\frac{2\pi nt}{T}},$$

replace t by $t - \frac{T}{2}$:

$$x\left(t-\frac{T}{2}\right)=\sum_{n=-\infty}^{\infty}a_n e^{j\frac{2\pi n}{T}\left(t-\frac{T}{2}\right)}=\sum_{n=-\infty}^{\infty}a_n e^{j\frac{2\pi nt}{T}}e^{-j\pi n}$$

Step 2: Simplify the extra exponential factor.

Since n is an integer,

$$e^{-j\pi n}=(-1)^n$$

So,

$$x\left(t-\frac{T}{2}\right)=\sum_{n=-\infty}^{\infty}a_n(-1)^n e^{j\frac{2\pi nt}{T}}$$

Step 3: Apply the given condition.

We are told

$$x\left(t-\frac{T}{2}\right)=-x(t)=-\sum_{n=-\infty}^{\infty}a_n e^{j\frac{2\pi nt}{T}}$$

Comparing the two Fourier series term by term (Fourier coefficients are unique), for every n:

$$a_n(-1)^n=-a_n$$

Step 4: Solve this equation for even and odd n separately.

For even n, write $n=2m$, so $(-1)^n=1$, giving

$$a_{2m}=-a_{2m}\Rightarrow 2a_{2m}=0\Rightarrow a_{2m}=0$$

For odd n, $(-1)^n = -1$, giving

$$-a_n = -a_n$$

which is always true and places no restriction on odd-indexed coefficients.

Step 5: Interpret the result.
The condition $x\left(t - \frac{T}{2}\right) = -x(t)$ is the half-wave symmetry condition. It forces every even-indexed Fourier coefficient, including the DC term a_0, to be zero, so only odd harmonics can be present in the signal.

Step 6: Analyze the options.

(A) a_2m = 0: Matches the result derived above. Correct.

(B) a_2m = 1: Contradicts the derivation, which gives zero, not one. Incorrect.

(C) a_2m = a_2m+1: There is no general equality forced between an even-indexed coefficient and the next odd-indexed one; the derivation only forces the even one to vanish. Incorrect.

(D) a_2m = -1: Contradicts the derivation. Incorrect.

Step 7: Final conclusion.
Therefore, for any integer m,

$$\boxed{a_{2m} = 0}$$

Quick Tip: Substitute t - T/2 into the Fourier series and compare coefficients with -x(t); only the even-indexed coefficients get forced to zero.

38. Let X, N, Y and Z be random variables. The variables X and N are independent of each other. X is uniformly distributed between -1 and 1; N follows Normal distribution with zero mean and unity variance.
Y and Z are defined as $Y = X + N$ and $Z = X^2 + N$.
Which of the following pairs represents the values of correlation between X and Y, and that between X and Z?

(A) $\frac{1}{3}$ and 0

(B) $\frac{1}{3}$ and $\frac{1}{9}$

(C) $\frac{1}{3}$ and $\frac{1}{3}$

(D) 1 and 0

Correct Answer: (A) $\frac{1}{3}$ and 0

Solution:

Step 1: Recall the definition of correlation (covariance) between two random variables.

For two random variables U and V, the correlation used here is the covariance

$$\text{Corr}(U, V) = E[UV] - E[U]E[V]$$

Step 2: Note the basic facts about X and N.

X is uniform on $[-1, 1]$, so

$$E[X] = 0, \qquad \text{Var}(X) = \frac{(1-(-1))^2}{12} = \frac{4}{12} = \frac{1}{3}$$

Because the density of X is symmetric about 0, every odd moment of X is zero, so

$$E[X^3] = 0$$

N has zero mean, and X, N are independent, so

$$E[XN] = E[X]E[N] = 0, \qquad E[X^2N] = E[X^2]E[N] = 0$$

Step 3: Find the correlation between X and Y.

$$\text{Corr}(X, Y) = E[XY] - E[X]E[Y] = E[X(X+N)] - 0 = E[X^2] + E[XN]$$

Since $E[X] = 0$, $E[X^2] = \text{Var}(X) = \frac{1}{3}$, and $E[XN] = 0$:

$$\text{Corr}(X, Y) = \frac{1}{3} + 0 = \frac{1}{3}$$

Step 4: Find the correlation between X and Z.

$$\text{Corr}(X, Z) = E[XZ] - E[X]E[Z] = E[X(X^2+N)] - 0 = E[X^3] + E[XN]$$

Since $E[X^3] = 0$ (odd moment of a symmetric distribution) and $E[XN]$

$= 0$ (independence):

$$\text{Corr}(X, Z) = 0 + 0 = 0$$

Step 5: Analyze the options.

(A) 1/3 and 0: Matches both computed values exactly. Correct.

(B) 1/3 and 1/9: The first value is right but the second should be 0, not $\frac{1}{9}$, since the odd moment $E[X^3]$ vanishes. Incorrect.

(C) 1/3 and 1/3: Would only hold if $E[X^3]$ equalled $\frac{1}{3}$, but it is zero by symmetry. Incorrect.

(D) 1 and 0: The correlation between X and Y is $\text{Var}(X) = \frac{1}{3}$, not 1, since X does not have unit variance. Incorrect.

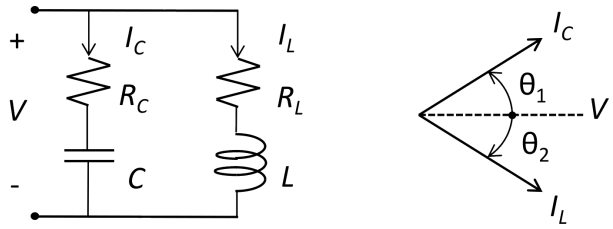
Step 6: Final conclusion.

Therefore, the required pair of correlation values is

$$\boxed{\frac{1}{3} \text{ and } 0}$$

Quick Tip: Use Corr(U,V) = E[UV] - E[U]E[V]; the odd moment E[X^3] of a symmetric uniform variable is zero.

39. In the given circuit, $L = 1\ \mu H$ and $C = 1\ \mu F$. The phasor diagram for I_C and I_L is also shown. Assume that the phase $(\theta_1 + \theta_2)$ is $90°$ at a frequency of 159.15 kHz.



Among the following options, what is the nearest integer value of $R_C \times R_L$?

(A) 0

(B) 1

(C) 2

(D) 10

Correct Answer: (B) 1

Solution:

Step 1: Identify the two parallel branches from the figure.

The source V drives two parallel branches. One branch has a resistor R_C in series with the capacitor C, carrying current I_C. The other branch has a resistor R_L in series with the inductor L, carrying current I_L.

Step 2: Write the branch impedances.

For the RC branch,

$$Z_C = R_C + \frac{1}{j\omega C} = R_C - \frac{j}{\omega C}$$

For the RL branch,

$$Z_L = R_L + j\omega L$$

Step 3: Express the phase angles of the two branch currents.

Since $I_C = \frac{V}{Z_C}$ and the phase of V is taken as the reference ($0°$), the phase of I_C is the negative of the phase of Z_C. Because Z_C has a negative imaginary part, I_C leads V by an angle θ_1 where

$$\tan\theta_1 = \frac{1}{\omega C R_C}$$

Similarly, since Z_L has a positive imaginary part, I_L lags V by an angle θ_2 where

$$\tan\theta_2 = \frac{\omega L}{R_L}$$

This matches the figure, where I_C is drawn above V at angle θ_1 and I_L is drawn below V at angle θ_2.

Step 4: Use the given condition on the angles.

We are given $\theta_1 + \theta_2 = 90°$, so $\theta_2 = 90° - \theta_1$, which means

$$\tan\theta_2 = \tan(90° - \theta_1) = \cot\theta_1 = \frac{1}{\tan\theta_1}$$

Step 5: Substitute the two tangent expressions.

$$\frac{\omega L}{R_L} = \frac{1}{\frac{1}{\omega C R_C}} = \omega C R_C$$

Step 6: Simplify to find the product R_C R_L.
Cancel ω from both sides:

$$\frac{L}{R_L} = CR_C \quad \Rightarrow \quad L = CR_CR_L \quad \Rightarrow \quad R_CR_L = \frac{L}{C}$$

Notice that this relation does not depend on the operating frequency at all, so the value of 159.15 kHz given in the question is not needed for this particular calculation; it is only there to confirm that the 90° condition genuinely occurs at some real frequency.

Step 7: Substitute the given values of L and C.

$$R_CR_L = \frac{L}{C} = \frac{1 \times 10^{-6}}{1 \times 10^{-6}} = 1$$

Step 8: Analyze the options.

(A) 0: Would mean either R_C or R_L is zero, which is not implied by the given condition. Incorrect.

(B) 1: Matches the value computed directly from $R_CR_L = L/C$. Correct.

(C) 2: Does not match the computed ratio $L/C = 1$. Incorrect.

(D) 10: Far larger than the computed value of 1. Incorrect.

Step 9: Final conclusion.

The nearest integer value of $R_C \times R_L$ is

$$\boxed{1}$$

Quick Tip: Write tan(theta1) and tan(theta2) for the two branch currents, use theta1+theta2=90 degrees, and simplify: the result is RC times RL = L/C.

40. For the control system shown in the Figure, the transfer function of a plant,

$$G(s) = \frac{1}{(s+1)(s+2)}$$

is connected in cascade with a compensator

$$C(s) = K(s+\alpha),$$

where K and α are positive real valued constants. The compensator and plant are placed in the forward path of a unity negative feedback system with input $R(s)$ and output $Y(s)$.
Which of the following pairs (K, α) represent the correct values for the closed loop system to have poles at $\left(-3 \pm j\sqrt{5}\right)$?

(A) $2, 3$

(B) $3, 4$

(C) $2, 4$

(D) $3, 3$

Correct Answer: (B) $3, 4$

Solution:

Step 1: Write the open loop transfer function.

The forward path has the compensator in cascade with the plant, so the open loop transfer function is

$$L(s) = C(s)G(s) = \frac{K(s+\alpha)}{(s+1)(s+2)}$$

Step 2: Write the closed loop characteristic equation.

For a unity negative feedback system, the characteristic equation is $1 + L(s) = 0$, that is

$$(s+1)(s+2) + K(s+\alpha) = 0$$

Step 3: Expand the equation.

$$(s+1)(s+2) = s^2 + 3s + 2$$

So the characteristic equation becomes

$$s^2 + 3s + 2 + Ks + K\alpha = 0$$

$$s^2 + (3+K)s + (2+K\alpha) = 0$$

Step 4: Use the desired pole locations to get sum and product of roots.

The desired closed loop poles are $-3 + j\sqrt{5}$ and $-3 - j\sqrt{5}$.
Sum of roots:

$$(-3 + j\sqrt{5}) + (-3 - j\sqrt{5}) = -6$$

Product of roots:

$$(-3 + j\sqrt{5})(-3 - j\sqrt{5}) = (-3)^2 - (j\sqrt{5})^2 = 9 - (-5) = 9 + 5 = 14$$

Step 5: Match with the characteristic equation coefficients.
For a quadratic $s^2 + bs + c = 0$ with these roots, sum of roots $= -b$ and product of roots $= c$. Comparing with $s^2 + (3 + K)s + (2 + K\alpha) = 0$:

$$-(3 + K) = -6 \quad \Rightarrow \quad 3 + K = 6 \quad \Rightarrow \quad K = 3$$

$$2 + K\alpha = 14 \quad \Rightarrow \quad K\alpha = 12$$

Step 6: Solve for alpha.

$$\alpha = \frac{12}{K} = \frac{12}{3} = 4$$

Step 7: Analyze the options.

(A) 2, 3: Gives $K = 2$, which does not satisfy $3 + K = 6$. Incorrect.

(B) 3, 4: Matches both $K = 3$ and $\alpha = 4$ found above. Correct.

(C) 2, 4: The value $K = 2$ fails the sum condition. Incorrect.

(D) 3, 3: $K = 3$ is right, but $\alpha = 3$ does not satisfy $K\alpha = 12$, since $3 \times 3 = 9 \neq 12$. Incorrect.

Step 8: Final conclusion.
The correct pair is

$$\boxed{(K, \alpha) = (3, 4)}$$

Quick Tip: Form the characteristic equation (s+1)(s+2)+K(s+alpha)=0 and match its sum and product of roots to the given complex pole pair.

41. The state and output equations for a control system are:

$$\dot{x} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$

$$y = [1.5 \quad 0.625]\, x$$

Which of the following expressions correctly represents the transfer function $\frac{Y(s)}{U(s)}$ of the system with zero initial conditions?

(A) $\frac{3s}{s^2 + 4s - 6}$

(B) $\frac{3s + 5}{s^2 + 4s - 6}$

(C) $\frac{3s + 5}{s^2 + 4s + 6}$

(D) $\frac{3s}{s^2 + 4s + 6}$

Correct Answer: (C) $\dfrac{3s+5}{s^2+4s+6}$

Solution:

Step 1: Identify the state space matrices.

The system is written in the standard form

$$\dot{x} = Ax + Bu, \qquad y = Cx$$

where

$$A = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad C = [1.5 \quad 0.625]$$

There is no direct feed through term here, so $D = 0$.

Step 2: Recall the transfer function formula.

For zero initial conditions, taking the Laplace transform of the state equations gives

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B$$

So the job is to build $sI - A$, invert it, and multiply by B and then by C.

Step 3: Form sI - A.

$$sI - A = \begin{bmatrix} s+4 & 1.5 \\ -4 & s \end{bmatrix}$$

Step 4: Find the determinant of sI - A.

$$\det(sI - A) = s(s+4) - (1.5)(-4) = s^2 + 4s + 6$$

This determinant becomes the denominator of the transfer function.

Step 5: Find the inverse of sI - A.

For a 2×2 matrix, the inverse is the adjoint divided by the determinant:

$$(sI - A)^{-1} = \frac{1}{s^2 + 4s + 6} \begin{bmatrix} s & -1.5 \\ 4 & s + 4 \end{bmatrix}$$

Step 6: Multiply by B.

$$(sI - A)^{-1}B = \frac{1}{s^2 + 4s + 6} \begin{bmatrix} s & -1.5 \\ 4 & s + 4 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \frac{1}{s^2 + 4s + 6} \begin{bmatrix} 2s \\ 8 \end{bmatrix}$$

Step 7: Multiply by C to get the transfer function.

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B = \frac{1.5(2s) + 0.625(8)}{s^2 + 4s + 6} = \frac{3s + 5}{s^2 + 4s + 6}$$

Step 8: Check the other options.

Options (A) and (D) show a numerator of $3s$ with the constant term 5 missing, which happens if the $0.625 \times 8 = 5$ contribution from C is dropped by mistake. Options (A) and (B) show a denominator of $s^2 + 4s - 6$, which happens if the product $(1.5)(-4) = -6$ inside the determinant is added instead of subtracted.

Step 9: Final conclusion.

The correct transfer function is

$$\boxed{\frac{3s+5}{s^2+4s+6}}$$

Quick Tip: Use $\frac{Y(s)}{U(s)} = C(sI-A)^{-1}B$ and be careful with the sign inside the determinant of $sI-A$.

42. The address of the first location of a 256 kilo byte (KB) memory is $(2500)_H$.
Choose the correct address of the last location of the memory.

(A) $(2FFF)_H$

(B) $(124FF)_H$

(C) $(424FF)_H$

(D) $(324FF)_H$

Correct Answer: (C) $(424FF)_H$

Solution:

Step 1: Convert the memory size into bytes.
A memory of 256 KB has

$$256 \times 1024 = 262144$$

bytes, since 1 KB = 1024 bytes.

Step 2: Write this size in hexadecimal.
$262144 = 4 \times 65536 = 4 \times 16^4$, so in hex the size is $(40000)_H$.

Step 3: Recall how the last address is found.

If the first address is A_1 and the memory holds S bytes, the last address is

$$A_2 = A_1 + S - 1$$

because the first byte itself already occupies one of the S locations.

Step 4: Add the starting address and the size in hex.

$$(2500)_H + (40000)_H = (42500)_H$$

Step 5: Subtract 1 to get the last address.

$$(42500)_H - 1 = (424FF)_H$$

Step 6: Check the other options.
$(2FFF)_H$ treats the memory as far too small, close to 4KB only. $(124FF)_H$ and $(324FF)_H$ come from an error in converting 256KB to hex, for example using $(20000)_H$ or $(30000)_H$ in place of the correct $(40000)_H$.

Step 7: Final conclusion.
The last address of the memory is

$$\boxed{(424FF)_H}$$

Quick Tip: Last address = first address + total bytes - 1, all worked out in hexadecimal.

43. Consider a real signal $x(t)$, $-\infty < t < \infty$, such that $x(t) = 0$ for $t < 0$, $x(t) = 2$ for $0 \le t < 1$ and $x(t) = 0$ for $t \ge 1$.

Let $E[x(t)] = \int_{-\infty}^{\infty} [x(t)]^2 \, dt$.

Which of the following options correctly represents the ratio, $E[x(t)]/E[3\, x(-3t + 5)]$?

(A) 3

(B) 1

(C) 1/3

(D) 1/9

Correct Answer: (C) 1/3

Solution:

Step 1: Compute E[x(t)].

The signal $x(t)$ is a rectangular pulse of height 2 over $0 \le t < 1$ and zero elsewhere, so

$$E[x(t)] = \int_0^1 (2)^2 \, dt = \int_0^1 4 \, dt = 4$$

Step 2: Set up the scaled and shifted signal.

Let $w(t) = 3x(-3t + 5)$, so we need $E[w(t)]$.

Step 3: Use the general scaling rule for the energy integral.

For a signal of the form $w(t) = A\, x(Bt + C)$, substitute $u = Bt + C$, so $du = B\, dt$ and $dt = du/|B|$ (the modulus takes care of the limits flipping when B is negative). Then

$$E[w] = \int [A\,x(Bt + C)]^2\,dt = A^2 \int [x(u)]^2 \frac{du}{|B|} = \frac{A^2}{|B|} E[x]$$

Step 4: Plug in the given values.

Here $A = 3$ and $B = -3$, so

$$E[w] = \frac{3^2}{|-3|} E[x] = \frac{9}{3} E[x] = 3E[x] = 3(4) = 12$$

Step 5: Form the required ratio.

$$\frac{E[x(t)]}{E[3x(-3t + 5)]} = \frac{4}{12} = \frac{1}{3}$$

Step 6: Check the other options.

Option (A), 3, is the reciprocal of the correct ratio and would come from dividing the wrong way round. Option (B), 1, ignores the scaling factor $A^2/|B|$ altogether. Option (D), 1/9, would follow only if the factor were mistakenly taken as $1/A^2$ instead of $A^2/|B|$.

Step 7: Final conclusion.

$$\boxed{\frac{1}{3}}$$

Quick Tip: For $w(t) = A\,x(Bt + C)$, the energy scales as $E[w] = \frac{A^2}{|B|} E[x]$.

44. A QPSK modulated signal from an additive white Gaussian noise (AWGN) channel is received with an $E_b/N_o = 8.4$ dB at the input of a coherent QPSK demodulator. A maximum likelihood reception method is used in the demodulator.
Assume the complementary error function

$$erfc(u) \cong \left[1/(u\sqrt{\pi})\right] \exp(-u^2).$$

Which is the nearest bit error rate (BER) at the output of the demodulator?

(A) 10^{-3}

(B) 10^{-4}

(C) 10^{-5}

(D) 10^{-6}

Correct Answer: (B) 10^{-4}

Solution:

Step 1: Convert Eb/No from dB into a plain ratio.

$$\frac{E_b}{N_o} = 10^{8.4/10} = 10^{0.84}$$

Since $10^{0.8} \approx 6.31$ and $10^{0.04} \approx 1.096$,

$$10^{0.84} \approx 6.31 \times 1.096 \approx 6.92$$

Step 2: Recall the bit error rate formula for coherent QPSK.
With Gray coded, maximum likelihood coherent detection, QPSK

behaves like two independent BPSK channels on the in-phase and quadrature axes, and the bit error probability is

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_o}}\right)$$

Step 3: Compute the argument of erfc.

$$u = \sqrt{6.92} \approx 2.63$$

Step 4: Apply the given approximation for erfc(u).

$$erfc(u) \approx \frac{\exp(-u^2)}{u\sqrt{\pi}}$$

Here $u^2 \approx 6.92$ which is just E_b/N_o itself, so

$$\exp(-6.92) \approx 9.9 \times 10^{-4}$$

and

$$u\sqrt{\pi} \approx 2.63 \times 1.772 \approx 4.66$$

So

$$erfc(u) \approx \frac{9.9 \times 10^{-4}}{4.66} \approx 2.13 \times 10^{-4}$$

Step 5: Get the bit error rate.

$$P_b = \frac{1}{2} \times 2.13 \times 10^{-4} \approx 1.06 \times 10^{-4}$$

Step 6: Round to the nearest power of ten.
1.06×10^{-4} is closest to 10^{-4}.

Step 7: Check the other options.
10^{-3} would result from forgetting the square root, that is, using $u = E_b/N_o$ directly inside erfc. 10^{-5} and 10^{-6} would need a much larger E_b/N_o or a mistaken doubling of the exponent inside the exponential term.

Step 8: Final conclusion.

$$\boxed{10^{-4}}$$

Quick Tip: QPSK behaves like two independent BPSK links, so $P_b = \frac{1}{2} erfc(\sqrt{E_b/N_o})$.

45. What is the 10's complement of $(47)_{10}$?

(A) 52

(B) 53

(C) 54

(D) 55

Correct Answer: (B) 53

Solution:

Step 1: Recall the definition of the 10's complement.

For an n-digit decimal number N, the 10's complement is defined as

$$10^n - N$$

It is used in digital systems to carry out subtraction using addition.

Step 2: Find n for the given number.

47 has 2 digits, so $n = 2$ and $10^n = 100$.

Step 3: Compute the 10's complement.

$$100 - 47 = 53$$

Step 4: Check the other options.

52 is the 9's complement of 47, found from $99 - 47 = 52$, not the 10's complement. 54 and 55 would come from an arithmetic slip such as adding an extra 1 or 2 after the subtraction.

Step 5: Final conclusion.

$$\boxed{53}$$

Quick Tip: 10's complement of an n-digit number N is $10^n - N$.

46. Consider a real baseband signal $x(t) = e^{-2t}$, for t (in seconds) ≥ 0. If 99% of the energy of $x(t)$ lies within B Hz, then which of the following options is TRUE for the value of B?

(A) $B > 1$ kHz

(B) $63/\pi$ Hz $< B < 64/\pi$ Hz

(C) $126/\pi$ Hz $< B < 128/\pi$ Hz

(D) $B < 1$ Hz

Correct Answer: (B) $63/\pi$ Hz $< B < 64/\pi$ Hz

Solution:

Step 1: Find the total energy in the time domain.

$$E_{total} = \int_0^\infty \left(e^{-2t}\right)^2 dt = \int_0^\infty e^{-4t} dt = \frac{1}{4}$$

Step 2: Find the Fourier transform of x(t).

$$X(f) = \int_0^\infty e^{-2t} e^{-j2\pi ft} dt = \frac{1}{2 + j2\pi f}$$

so the energy spectral density is

$$|X(f)|^2 = \frac{1}{4 + 4\pi^2 f^2}$$

Step 3: Write the energy contained in the band from -B to B.

$$E(B) = \int_{-B}^{B} \frac{1}{4 + 4\pi^2 f^2} df = 2\int_0^B \frac{1}{4} \cdot \frac{1}{1 + \pi^2 f^2} df$$

Step 4: Carry out the integral.

Using $\int \frac{df}{1+\pi^2 f^2} = \frac{1}{\pi}\arctan(\pi f)$,

$$E(B) = \frac{1}{2} \cdot \frac{1}{\pi}\arctan(\pi B) = \frac{1}{2\pi}\arctan(\pi B)$$

Step 5: Form the energy fraction.

$$\frac{E(B)}{E_{total}} = \frac{\frac{1}{2\pi}\arctan(\pi B)}{\frac{1}{4}} = \frac{2}{\pi}\arctan(\pi B)$$

Step 6: Set this fraction equal to 0.99 and solve for B.

$$\frac{2}{\pi}\arctan(\pi B) = 0.99 \Rightarrow \arctan(\pi B) = \frac{0.99\pi}{2} \approx 1.5551 \text{ rad}$$

Step 7: Solve for πB using the fact that 1.5551 is close to $\pi/2$.

Let $\varepsilon = \frac{\pi}{2} - 1.5551 \approx 0.0157$. Since $\tan\left(\frac{\pi}{2} - \varepsilon\right) = \cot(\varepsilon) \approx \frac{1}{\varepsilon}$ for small ε,

$$\pi B \approx \frac{1}{0.0157} \approx 63.7$$

Step 8: Read off B.

$$B \approx \frac{63.7}{\pi}$$

which lies between $\frac{63}{\pi}$ Hz and $\frac{64}{\pi}$ Hz.

Step 9: Check the other options.

Option (A) and (D) are far off in scale, since the actual bandwidth is only about 20 Hz. Option (C) is exactly double the correct range, the

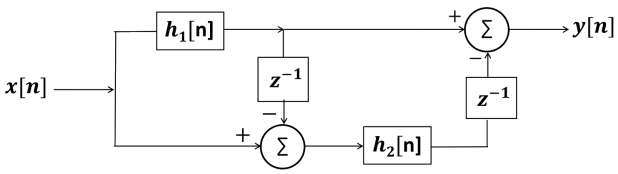
kind of error that appears if the factor of 2 from integrating over $-B$ to B is applied twice.

Step 10: Final conclusion.

$$\boxed{\frac{63}{\pi}\ \text{Hz} < B < \frac{64}{\pi}\ \text{Hz}}$$

Quick Tip: Use Parseval's theorem, integrate the energy spectral density from -B to B, and set the fraction equal to 0.99.

47. Consider the discrete time system (S) with input $x[n]$ and output $y[n]$ as shown in the figure. The two sub-systems, represented by their impulse responses $h_1[n]$ and $h_2[n]$, are linear and time invariant.



Which of the following statements is necessarily TRUE?

(A) S is causal.

(B) S is linear and time invariant.

(C) S is linear and time varying.

(D) S is non-linear.

Correct Answer: (B) S is linear and time invariant.

Solution:

Step 1: Trace the upper branch of the diagram.

The input $x[n]$ passes through $h_1[n]$ to produce $a[n] = h_1[n] * x[n]$, where $*$ denotes convolution. This $a[n]$ feeds the final summing junction and is also delayed by one sample through a z^{-1} block.

Step 2: Trace the lower branch.

At the lower summing junction, the delayed value $a[n-1]$ is subtracted from $x[n]$, giving $c[n] = x[n] - a[n-1]$. This passes through $h_2[n]$ to give $d[n] = h_2[n] * c[n]$, which is then delayed by another z^{-1} block.

Step 3: Write the final output.

At the top summing junction, the delayed $d[n-1]$ is subtracted from $a[n]$, so

$$y[n] = a[n] - d[n-1]$$

Step 4: Check linearity.

Every operation used to build $y[n]$ from $x[n]$, convolution with h_1 and h_2, unit delay, and addition or subtraction, is a linear operation. A network built only from linear building blocks connected in cascade and feedback still obeys superposition at every internal signal, so the overall system S is linear no matter what $h_1[n]$ and $h_2[n]$ actually are.

Step 5: Check time invariance.

Convolution with a fixed impulse response is a time invariant operation, and a unit delay is time invariant by definition, since delaying the input by n_0 samples simply delays every internal signal, and hence the output, by the same n_0 samples. Since every block in

the diagram is time invariant, the interconnection S is time invariant as well.

Step 6: Check causality.
Causality of S depends only on whether $h_1[n]$ and $h_2[n]$ are themselves causal. The question states only that they are LTI, with no restriction that their impulse response is zero for negative n, so S is not necessarily causal.

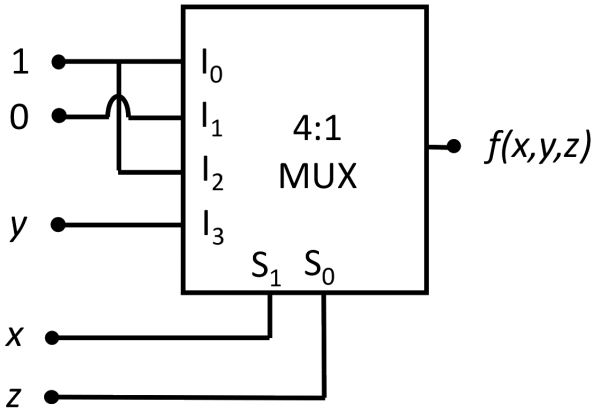
Step 7: Rule out the other options.
Option (A) fails because causality is not guaranteed by the given information. Options (C) and (D) contradict the fact, shown above, that the system is built entirely from linear, time invariant blocks.

Step 8: Final conclusion.

$$\boxed{S \text{ is linear and time invariant}}$$

Quick Tip: Convolution, addition and a fixed delay are all linear and time invariant, so any interconnection of LTI blocks stays LTI.

48. A Boolean function, $f(x, y, z)$, with x as MSB and z as LSB, is realized by a 4:1 multiplexer (MUX) with select lines S_1 and S_0 (S_1 is MSB, S_0 is LSB) and inputs I_0, I_1, I_2, I_3 as shown in the figure.



Which of the following options is the correct expression of $f(x, y, z)$?

(A) $xz + y$

(B) $x\bar{y} + z$

(C) $xy + \bar{z}$

(D) $\bar{x}y + \bar{z}$

Correct Answer: (C) $xy + \bar{z}$

Solution:

Step 1: Read the connections from the figure.

The data inputs are $I_0 = 1, I_1 = 0, I_2 = 1$ and $I_3 = y$, and the select lines are $S_1 = x$ and $S_0 = z$.

Step 2: Recall the standard 4:1 MUX output expression.

For select lines S_1 (MSB) and S_0 (LSB),

$$f = \overline{S_1}\,\overline{S_0}\,I_0 + \overline{S_1}\,S_0\,I_1 + S_1\,\overline{S_0}\,I_2 + S_1\,S_0\,I_3$$

Step 3: Substitute $S_1 = x$, $S_0 = z$ and the given inputs.

$$f = \bar{x}\bar{z}(1) + \bar{x}z(0) + x\bar{z}(1) + xz(y)$$

$$f = \bar{x}\bar{z} + x\bar{z} + xyz$$

Step 4: Simplify the first two terms.

$$\bar{x}\bar{z} + x\bar{z} = \bar{z}(\bar{x} + x) = \bar{z}$$

so

$$f = \bar{z} + xyz$$

Step 5: Absorb the last term.

Using the identity $\bar{z} + xyz = \bar{z} + xy$ (check by putting $z = 0$: both sides give 1; putting $z = 1$: both sides give xy),

$$f = xy + \bar{z}$$

Step 6: Match with the given options.

This is exactly option (C).

Step 7: Rule out the other options.

Option (A) would need $I_2 = z$ dependent behaviour rather than a fixed 1. Options (B) and (D) contain $\bar{y}$ or $\bar{x}y$ terms that do not appear from

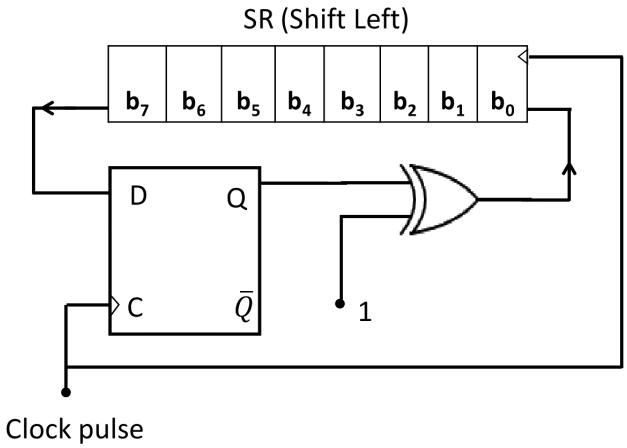
this wiring, since y only enters through I_3 when $S_1S_0 = 11$.

Step 8: Final conclusion.

$$\boxed{xy + \bar{z}}$$

Quick Tip: Write out $f = \bar{S}_1\bar{S}_0I_0 + \bar{S}_1S_0I_1 + S_1\bar{S}_0I_2 + S_1S_0I_3$ with $S_1 = x$, $S_0 = z$ and simplify.

49. A shift-left Shift Register (SR) and a D flip-flop are connected to a synchronized clock as shown in the figure. Assume that the SR and D flip-flops are initially cleared and the XOR gate has no propagation delay.



Which of the following options gives the correct binary representation $b_7b_6b_5b_4b_3b_2b_1b_0$ of the content of the shift register immediately after the 5^{th} clock transition (positive edge)?

(A) 00011111

(B) 10111111

(C) 00111111

(D) 11000011

Correct Answer: (A) 00011111

Solution:

Step 1: Read the feedback path of the circuit.

The 8-bit register shifts left on every clock edge, so on each edge every bit moves one place towards b_7 and b_7 itself is pushed out of the register. That bit which leaves b_7 is wired to the D input of the flip-flop. The flip-flop's Q output goes to one input of the XOR gate, and the other XOR input is tied permanently to logic 1. The XOR output feeds the serial input at b_0. Since the XOR gate has no propagation delay, this output is ready in time for the very same clock edge that also updates the flip-flop and the register.

Step 2: Write the update rule for each block.

Let the register content after the n^{th} clock edge be $b_7^{(n)} b_6^{(n)} \cdots b_0^{(n)}$, and let $Q^{(n)}$ be the flip-flop output after the n^{th} edge. An XOR gate with one input tied to 1 is simply an inverter on its other input, so the serial bit entering b_0 is the complement of the flip-flop's present output. This gives

$$Q^{(n+1)} = b_7^{(n)}$$

$$b_7^{(n+1)} = b_6^{(n)},\ b_6^{(n+1)} = b_5^{(n)},\ \ldots,\ b_1^{(n+1)} = b_0^{(n)}$$

$$b_0^{(n+1)} = \overline{Q^{(n)}}$$

Step 3: Set the initial condition.

Both the register and the flip-flop start cleared, so

$$b_7^{(0)} b_6^{(0)} b_5^{(0)} b_4^{(0)} b_3^{(0)} b_2^{(0)} b_1^{(0)} b_0^{(0)} = 00000000, \qquad Q^{(0)} = 0$$

Step 4: Trace edges 1 through 5.

Edge 1: $Q^{(1)} = b_7^{(0)} = 0$; the register shifts left and $b_0^{(1)} = \overline{Q^{(0)}} = \overline{0} = 1$. New state: 00000001.

Edge 2: $Q^{(2)} = b_7^{(1)} = 0$; shift left, $b_0^{(2)} = \overline{Q^{(1)}} = \overline{0} = 1$. New state: 00000011.

Edge 3: $Q^{(3)} = b_7^{(2)} = 0$; shift left, $b_0^{(3)} = \overline{Q^{(2)}} = \overline{0} = 1$. New state: 00000111.

Edge 4: $Q^{(4)} = b_7^{(3)} = 0$; shift left, $b_0^{(4)} = \overline{Q^{(3)}} = \overline{0} = 1$. New state: 00001111.

Edge 5: $Q^{(5)} = b_7^{(4)} = 0$; shift left, $b_0^{(5)} = \overline{Q^{(4)}} = \overline{0} = 1$. New state: 00011111.

Step 5: Analyze the options.

(A) 00011111: Matches the state obtained after the 5th clock edge. Correct.

(B) 10111111: Would need the MSB to turn 1 while six low bits also turn 1, which does not match a slow left shift of single 1s entering at b_0. Incorrect.

(C) 00111111: This is the state reached after 6 clock edges, one edge too many. Incorrect.

(D) 11000011: Does not match any reachable state of this feedback chain within the first five edges. Incorrect.

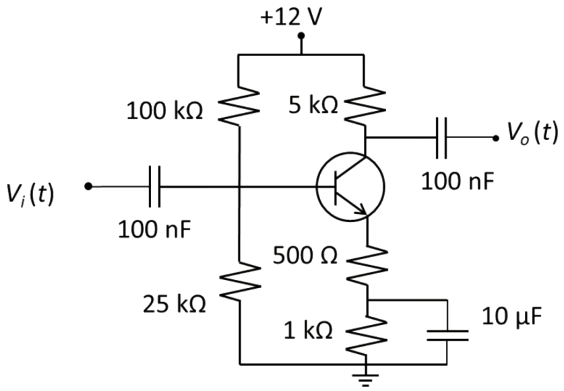
Step 6: Final conclusion.

After the 5th clock transition the register holds

00011111

Quick Tip: Trace the D flip-flop and XOR feedback state after each clock edge, five times.

50. A small signal source, $V_i(t) = A\cos(10^5 t) + B\sin(10^7 t)$ is applied to a BJT circuit as shown in the figure.



Assume zero source resistance, $V_{BE} = 0.7$ V, $\beta_{dc} = 99$, Early voltage $= 100$ V and thermal voltage $= 25$ mV. Effect of internal parasitic capacitances of the BJT may be neglected.

Which expression is the best approximation of the output voltage $V_o(t)$?

(A) $-9.1[A\cos(10^5 t) + B\sin(10^7 t)]$

(B) $9.1[A\cos(10^5 t) - B\sin(10^7 t)]$

(C) $-190.4[A\cos(10^5 t) + B\sin(10^7 t)]$

(D) $190.4[A\cos(10^5 t) - B\sin(10^7 t)]$

Correct Answer: (A) $-9.1[A\cos(10^5 t) + B\sin(10^7 t)]$

Solution:

Step 1: Find the DC bias point.

The base divider is $R_1 = 100\ k\Omega$ from V_{CC} to the base and $R_2 = 25\ k\Omega$ from the base to ground. The Thevenin voltage and resistance seen by the base are

$$V_{th} = V_{CC} \cdot \frac{R_2}{R_1 + R_2} = 12 \times \frac{25}{125} = 2.4\ \text{V}$$

$$R_{th} = R_1 \parallel R_2 = \frac{100 \times 25}{125} = 20\ k\Omega$$

The total DC emitter resistance is $500 + 1000 = 1500\ \Omega$ (the bypass capacitor blocks DC, so both emitter resistors carry the bias current). Writing the base loop with $\beta_{dc} = 99$:

$$I_B = \frac{V_{th} - V_{BE}}{R_{th} + (\beta_{dc} + 1)(1500)} = \frac{2.4 - 0.7}{20000 + 100 \times 1500} = \frac{1.7}{170000} = 10\ \mu A$$

So $I_C = \beta_{dc} I_B \approx 0.99\ \text{mA} \approx 1\ \text{mA}$ and $I_E = (\beta_{dc} + 1) I_B = 1\ \text{mA}$.

Step 2: Find the small-signal parameters.

$$r_e = \frac{V_T}{I_E} = \frac{25\ \text{mV}}{1\ \text{mA}} = 25\ \Omega, \qquad r_o = \frac{V_A}{I_C} = \frac{100}{0.001} = 100\ k\Omega$$

Step 3: Check which part of the emitter resistor is bypassed at each frequency.

The bypass capacitor is $10\ \mu F$ across the $1\ k\Omega$ resistor only, the $500\ \Omega$ resistor is never bypassed. Its reactance at the two signal frequencies is

$$X_C(10^5) = \frac{1}{\omega C} = \frac{1}{10^5 \times 10 \times 10^{-6}} = 1\ \Omega, \qquad X_C(10^7) = \frac{1}{10^7 \times 10 \times 10^{-6}} = 0.01\ \Omega$$

Both values are tiny next to $1\ k\Omega$, so at both $\omega = 10^5$ and $\omega = 10^7$ the $1\ k\Omega$ resistor is fully shorted by the capacitor. The unbypassed emitter resistance is therefore the same $500\ \Omega$ for both frequency components of the input.

Step 4: Check the coupling capacitors.
Each $100\ nF$ coupling capacitor has reactance

$$X_C(10^5) = \frac{1}{10^5 \times 10^{-7}} = 100\ \Omega, \qquad X_C(10^7) = \frac{1}{10^7 \times 10^{-7}} = 1\ \Omega$$

Both are small compared to the input resistance the amplifier presents (several $k\Omega$) and there is no load resistor on the output side, so neither coupling capacitor causes any noticeable attenuation or phase shift at either frequency.

Step 5: Compute the mid-band voltage gain.
With $500\ \Omega$ of unbypassed emitter resistance, the common-emitter gain with emitter degeneration is

$$A_v = -\frac{R_C \parallel r_o}{r_e + R_{E,ac}}$$

$$R_C \parallel r_o = \frac{5000 \times 100000}{105000} = 4761.9\ \Omega$$

$$A_v = -\frac{4761.9}{25 + 500} = -\frac{4761.9}{525} \approx -9.1$$

Step 6: Apply the same gain to both terms of $V_i(t)$.
Because the effective emitter resistance and the coupling capacitor behaviour are the same at $\omega = 10^5$ and at $\omega = 10^7$, both the $A\cos(10^5 t)$ part and the $B\sin(10^7 t)$ part of the input see the identical gain of about -9.1, with the same inverting sign.

Step 7: Analyze the options.

(A) $-9.1[A\cos(10^5 t) + B\sin(10^7 t)]$**:** Matches the gain found for both terms with the correct inverting sign. Correct.

(B) $9.1[A\cos(10^5 t) - B\sin(10^7 t)]$**:** Has the wrong overall sign and wrongly flips the sign of only one term. Incorrect.

(C) $-190.4[\ldots]$**:** This magnitude would follow from forgetting the $500\ \Omega$ emitter degeneration entirely and using only r_e in the denominator. Incorrect.

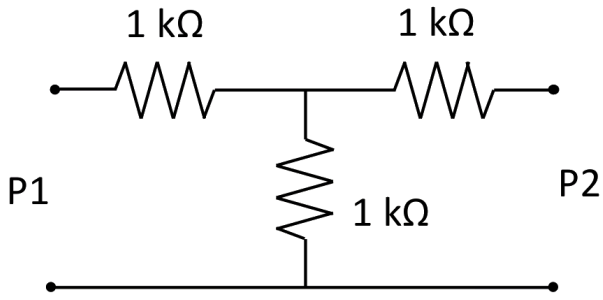
(D) $190.4[\ldots]$**:** Same wrong magnitude as (C), with the wrong sign as well. Incorrect.

Step 8: Final conclusion.

$$\boxed{V_o(t) \approx -9.1\left[A\cos(10^5 t) + B\sin(10^7 t)\right]}$$

Quick Tip: Check whether the coupling and bypass capacitors act as short circuits at both signal frequencies.

51. Consider the two-port network as shown in the figure.



Which of the following options provides the correct set of values of A, B, C and D parameters?

(A) $A = \frac{1}{2}, B = 3 \times 10^3\ \Omega, C = 10^{-3}\ \Omega^{-1}, D = \frac{1}{2}$

(B) $A = 2, B = 3\ \Omega, C = 1\ \Omega^{-1}, D = 2$

(C) $A = 2, B = 6 \times 10^3\ \Omega, C = 2 \times 10^{-3}\ \Omega^{-1}, D = 2$

(D) $A = 2, B = 3 \times 10^3\ \Omega, C = 10^{-3}\ \Omega^{-1}, D = 2$

Correct Answer: (D) $A = 2, B = 3 \times 10^3\ \Omega, C = 10^{-3}\ \Omega^{-1}, D = 2$

Solution:

Step 1: Identify the network topology.

The circuit is a symmetric T-section: a $1\ k\Omega$ resistor in series on the input side, a $1\ k\Omega$ resistor as the shunt (middle) arm, and a $1\ k\Omega$ resistor in series on the output side. Call these $Z_1 = Z_3 = 1000\ \Omega$ (the two series arms) and $Z_2 = 1000\ \Omega$ (the shunt arm).

Step 2: Recall the ABCD parameters of a T-network.

For a general T-section with series arms Z_1, Z_3 and shunt arm Z_2,

$$A = 1 + \frac{Z_1}{Z_2}, \qquad B = Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2}, \qquad C = \frac{1}{Z_2}, \qquad D = 1 + \frac{Z_3}{Z_2}$$

Step 3: Substitute the resistor values.

$$A = 1 + \frac{1000}{1000} = 2, \qquad D = 1 + \frac{1000}{1000} = 2$$

$$C = \frac{1}{1000} = 10^{-3}\ \Omega^{-1}$$

$$B = 1000 + 1000 + \frac{1000 \times 1000}{1000} = 1000 + 1000 + 1000 = 3000\ \Omega = 3 \times 10^{3}\ \Omega$$

Step 4: Analyze the options.

(A) $A = 1/2,\ B = 3 \times 10^{3}\ \Omega,\ C = 10^{-3}\ \Omega^{-1},\ D = 1/2$**:** The B and C values match but A and D are inverted. Incorrect.

(B) $A = 2,\ B = 3\ \Omega,\ C = 1\ \Omega^{-1},\ D = 2$**:** A and D are right but B and C are off by a factor of 1000, a common unit slip. Incorrect.

(C) $A = 2,\ B = 6 \times 10^{3}\ \Omega,\ C = 2 \times 10^{-3}\ \Omega^{-1},\ D = 2$**:** B and C are exactly double the correct values. Incorrect.

(D) $A = 2,\ B = 3 \times 10^{3}\ \Omega,\ C = 10^{-3}\ \Omega^{-1},\ D = 2$**:** Matches every computed value. Correct.

Step 5: Final conclusion.

$$\boxed{A = 2,\ B = 3 \times 10^{3}\ \Omega,\ C = 10^{-3}\ \Omega^{-1},\ D = 2}$$

Quick Tip: Treat the network as a T-section and use the standard T-to-ABCD conversion formulas.

52. A complex load (in Ω) is represented as $\Gamma_L = 0.5\angle 30°$ on the Smith chart. A co-axial cable with a characteristic impedance of $50\ \Omega$ is connected to the load. The new input impedance of the load now moves to a diametrically opposite point on the same Γ circle on the Smith chart. Which option is the nearest input impedance of the cable connected load (in Ω)?

(A) $20.7 - j5.1$

(B) $17.7 - j11.8$

(C) $97.5 - j65.0$

(D) $97.5 + j65.0$

Correct Answer: (B) $17.7 - j11.8$

Solution:

Step 1: Convert the given reflection coefficient to rectangular form.

$$\Gamma_L = 0.5\angle 30° = 0.5\cos(30°) + j\,0.5\sin(30°) = 0.433 + j0.25$$

Step 2: Interpret "diametrically opposite on the same Γ circle".
On a Smith chart, moving along a lossless transmission line keeps $|\Gamma|$ fixed and rotates its angle by $-2\beta l$, tracing a circle centred at the origin. A diametrically opposite point on that same circle is reached when the angle changes by exactly $180°$, which happens for a line

length of a quarter wavelength ($l = \lambda/4$, since $2\beta l = 2 \cdot \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \pi$). So the new reflection coefficient at the cable input is

$$\Gamma_{in} = \Gamma_L e^{-j\pi} = -\Gamma_L$$

Step 3: Compute Γ_{in}.

$$\Gamma_{in} = -(0.433 + j0.25) = -0.433 - j0.25 = 0.5\angle 210^\circ$$

Step 4: Convert Γ_{in} to the input impedance.

$$Z_{in} = Z_0 \cdot \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$$

$$1 + \Gamma_{in} = 0.567 - j0.25, \qquad 1 - \Gamma_{in} = 1.433 + j0.25$$

Multiply numerator and denominator by the conjugate of the denominator:

$$\frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = \frac{(0.567 - j0.25)(1.433 - j0.25)}{(1.433)^2 + (0.25)^2} = \frac{0.75 - j0.5}{2.116} = 0.354 - j0.236$$

Step 5: Scale by $Z_0 = 50\ \Omega$.

$$Z_{in} = 50(0.354 - j0.236) = 17.7 - j11.8\ \Omega$$

Step 6: Analyze the options.

(A) $20.7 - j5.1$**:** Does not match either the real or the imaginary part found above. Incorrect.

(B) $17.7 - j11.8$**:** Matches the computed real and imaginary parts. Correct.

(C) $97.5 - j65.0$**:** This is what results from mistakenly using the conjugate of Γ_L as Γ_{in} instead of applying the diametric flip. Incorrect.

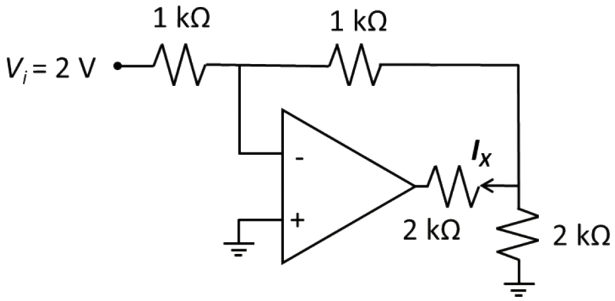
(D) $97.5 + j65.0$**:** This is what results from forgetting the diametric flip altogether and using Γ_L itself as Γ_{in}. Incorrect.

Step 7: Final conclusion.

$$\boxed{Z_{in} \approx 17.7 - j11.8\ \Omega}$$

Quick Tip: A diametrically opposite point on the same reflection coefficient circle means the sign of Gamma flips.

53. A circuit using an ideal OP-AMP is shown in the figure.



Which of the following options gives the correct value of the current I_X?

(A) 2.0 mA

(B) 1.5 mA

(C) 3.0 mA

(D) 0 mA

Correct Answer: (C) 3.0 mA

Solution:

Step 1: Identify the virtual ground.

The op-amp is ideal and its non-inverting input is grounded, so the inverting input node (call it node A, where V_i connects through the first $1\ k\Omega$ resistor) sits at $0\ V$. No current enters the op-amp's input terminals.

Step 2: Find the current leaving V_i into node A.

$$I_1 = \frac{V_i - 0}{1\ k\Omega} = \frac{2}{1000} = 2\ mA$$

Since no current can enter the op-amp, this entire $2\ mA$ must flow onward through the second $1\ k\Omega$ resistor, into the node where the two $2\ k\Omega$ resistors meet (call this node D).

Step 3: Find the voltage at node D.

Because a fixed current of $2\ mA$ flows from A (at $0\ V$) through the $1\ k\Omega$ resistor into D,

$$V_D = V_A - I_1(1000) = 0 - 2 \times 10^{-3} \times 1000 = -2\ V$$

Step 4: Write KCL at node D.

Node D connects to three branches: the $1\ k\Omega$ resistor back to A (carrying the incoming $2\ mA$), the $2\ k\Omega$ resistor straight to ground, and the $2\ k\Omega$ resistor I_X leading to the op-amp output V_o. Current in equals current out:

$$2\ mA = \frac{V_D - 0}{2000} + \frac{V_D - V_o}{2000}$$

Step 5: Solve for V_o.

$$2 \times 10^{-3} = \frac{-2}{2000} + \frac{-2 - V_o}{2000}$$

$$2 \times 10^{-3} \times 2000 = -2 + (-2 - V_o)$$

$$4 = -4 - V_o$$

$$V_o = -8\ V$$

Step 6: Compute I_X.

$$I_X = \frac{V_D - V_o}{2000} = \frac{-2-(-8)}{2000} = \frac{6}{2000} = 3 \times 10^{-3}\, A = 3.0\, mA$$

Step 7: Analyze the options.

(A) 2.0 mA: This is only the current I_1 through the input resistor, not the current I_X through the feedback branch. Incorrect.

(B) 1.5 mA: Would follow from wrongly treating node D as a virtual ground itself. Incorrect.

(C) 3.0 mA: Matches the value found from the correct nodal analysis at D. Correct.

(D) 0 mA: Would only be true if V_D and V_o were equal, which they are not here. Incorrect.

Step 8: Final conclusion.

$$\boxed{I_X = 3.0\, mA}$$

Quick Tip: The node between the two feedback resistors is not a virtual ground, so use nodal analysis there.

54. Consider an LED based on a direct bandgap semiconductor material with energy bandgap 1.3 eV.

Given: Planck's constant, $h = 6.63 \times 10^{-34}$ J s and speed of light in free space is 3×10^{8} m s^{-1}.

In which of the following wavelength ranges the LED will NOT emit?

(A) 1410 ± 20 nm

(B) 1090 ± 20 nm

(C) 950 ± 20 nm

(D) 510 ± 20 nm

Correct Answer: (A) 1410 ± 20 nm ; (B) 1090 ± 20 nm ; (D) 510 ± 20 nm

Solution:

Step 1: Recall how an LED emits light.

In a direct bandgap semiconductor, an electron in the conduction band recombines with a hole in the valence band and releases the energy difference as a photon. The photon energy released in this band-to-band recombination is very close to the bandgap energy E_g, with only a small spread of a few kT around it coming from the thermal spread of carrier energies. An LED does not efficiently emit photons whose energy is far below E_g (no such transition is available) or far above E_g (very few carriers sit that high above the band edge).

Step 2: Convert the bandgap energy into a wavelength.

The photon energy and wavelength are related by

$$E_g = \frac{hc}{\lambda_g} \Rightarrow \lambda_g = \frac{hc}{E_g}$$

Using $h = 6.63 \times 10^{-34}\, J\, s$, $c = 3 \times 10^{8}\, m\, s^{-1}$, and converting E_g

$= 1.3\ eV$ to joules with $1\ eV = 1.6 \times 10^{-19}\ J$:

$$E_g = 1.3 \times 1.6 \times 10^{-19} = 2.08 \times 10^{-19}\ J$$

$$\lambda_g = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2.08 \times 10^{-19}} = \frac{1.989 \times 10^{-25}}{2.08 \times 10^{-19}} \approx 9.56 \times 10^{-7}\ m = 956\ nm$$

Step 3: Compare each option's range to $\lambda_g \approx 956\ nm$.

(A) 1410 ± 20 nm: This is far longer than 956 nm, corresponding to a photon energy well below E_g. Direct band-to-band recombination cannot supply photons with energy less than the bandgap, so the LED will NOT emit here.

(B) 1090 ± 20 nm: Also longer than 956 nm (energy below E_g), so this too is below the smallest photon energy the recombination can give. The LED will NOT emit here either.

(C) 950 ± 20 nm: This range straddles the calculated $\lambda_g \approx 956$ nm almost exactly, which is where band-edge recombination peaks. The LED DOES emit in this range.

(D) 510 ± 20 nm: This corresponds to a photon energy of roughly 2.4 eV, nearly double $E_g = 1.3$ eV. Ordinary band-edge recombination in this material has no mechanism to release almost twice the bandgap energy as a single photon, so the LED will NOT emit here.

Step 4: Final conclusion.

The wavelength ranges where the LED will NOT emit are

$$\boxed{1410 \pm 20\ nm,\ \ 1090 \pm 20\ nm,\ \ 510 \pm 20\ nm}$$

that is, options (A), (B) and (D).

Quick Tip: An LED emits photons with energy close to its bandgap, not far above or below it.

55. Let the relevant bandwidth (B) of a digital communication system be 1 MHz and $kT = -174$ dBm/Hz, where k is Boltzmann's constant and T is the equivalent noise temperature of the receiver. The power (S) of signal received through an additive Gaussian channel is -80 dBm.
Which of the following options is/are TRUE about Shannon capacity (C) of the channel?

(A) $C = B$

(B) $C = 2B$

(C) $C > 3B$

(D) $C < B$

Correct Answer: (C) $C > 3B$

Solution:

Step 1: Recall the Shannon capacity formula.
For a channel with bandwidth B and signal-to-noise ratio SNR, the Shannon capacity is

$$C = B\log_2(1 + SNR)$$

Step 2: Find the noise power in the given bandwidth.

The noise power spectral density is $kT = -174\ dBm/Hz$. Over a bandwidth $B = 1\ MHz = 10^6\ Hz$, the total noise power in dB is

$$N = kT + 10\log_{10}(B) = -174 + 10\log_{10}(10^6) = -174 + 60 = -114\ dBm$$

Step 3: Find the SNR in dB, then convert to a plain ratio.

$$SNR(dB) = S - N = -80 - (-114) = 34\ dB$$

$$SNR = 10^{34/10} = 10^{3.4} \approx 2512$$

Step 4: Compute the Shannon capacity.

$$C = B\log_2(1 + SNR) = 10^6 \times \log_2(2513)$$

Since $2^{11} = 2048$ and $2^{12} = 4096$, $\log_2(2513)$ sits between 11 and 12, close to 11.3:

$$C \approx 10^6 \times 11.3 = 11.3 \times 10^6 \text{ bits/s} = 11.3\ B$$

Step 5: Analyze the options.

(A) $C = B$**:** Would mean an SNR of only 1 (0 dB), far below the $34\ dB$ actually present. Incorrect.

(B) $C = 2B$**:** Would need $SNR = 3$, also far too small. Incorrect.

(C) $C > 3B$**:** Since $C \approx 11.3B$, which is indeed greater than $3B$, this statement holds. Correct.

(D) $C < B$**:** This would need a negative or zero SNR, which contradicts the large positive SNR here. Incorrect.

Step 6: Final conclusion.

$$\boxed{C \approx 11.3B, \text{ so } C > 3B \text{ is the TRUE statement}}$$

Quick Tip: Convert the given dBm values into a linear SNR before applying the Shannon capacity formula.

56. Consider the four-variable Boolean function,

$$f(w, x, y, z) = \sum m(0, 2, 5, 7, 8, 10, 13, 14, 15)$$

with w as MSB and z as LSB.
Which of the following expressions is/are the valid form(s) of $f(w, x, y, z)$?

(A) $\bar{x}\bar{z} + xz + wxy$

(B) $xz + wxy + w\bar{x}\bar{z} + \bar{w}xy\bar{z}$

(C) $\bar{x}\bar{z} + wxy + w\bar{x}\bar{z} + \bar{w}xy\bar{z}$

(D) $\bar{x}\bar{z} + xz + wy\bar{z}$

Correct Answer: (A) $\bar{x}\bar{z} + xz + wxy$; (D) $\bar{x}\bar{z} + xz + wy\bar{z}$

Solution:

Step 1: Build the K-map for the given minterms.

$$f(w,x,y,z) = \sum m(0,2,5,7,8,10,13,14,15)$$

Placing 1s at these minterms and 0s elsewhere in a standard 4-variable K-map (rows wx, columns yz, Gray code order) gives 1s at exactly $m_0, m_2, m_5, m_7, m_8, m_{10}, m_{13}, m_{14}, m_{15}$, and 0s at every other minterm, including m_6, m_4, m_9, m_{11}, m_{12}, m_1, m_3.

Step 2: Find three valid groups that cover exactly the given minterms.

The group $\bar{x}\bar{z}$ ($x = 0, z = 0$) covers m_0, m_2, m_8, m_{10}, all four of which are 1s. The group xz ($x = 1, z = 1$) covers m_5, m_7, m_{13}, m_{15}, also all 1s. The group wxy ($w = 1, x = 1, y = 1$) covers m_{14}, m_{15}, both 1s. Together these three groups cover

$$\{0,2,8,10\} \cup \{5,7,13,15\} \cup \{14,15\} = \{0,2,5,7,8,10,13,14,15\}$$

which is exactly the given minterm list, with nothing extra.

Step 3: Check option (A), $\bar{x}\bar{z} + xz + wxy$.

This is precisely the set of groups found in Step 2, and it reproduces the onset exactly. Valid.

Step 4: Check option (B), $xz + wxy + w\bar{x}\bar{z} + \bar{w}xy\bar{z}$.

Here xz gives $\{5,7,13,15\}$, wxy gives $\{14,15\}$, and $w\bar{x}\bar{z}$ ($w = 1, x = 0, z = 0$) gives $\{8,10\}$. This union is $\{5,7,8,10,13,14,15\}$, missing minterms 0 and 2 entirely. On top of that, the last term $\bar{w}xy\bar{z}$ fixes $w = 0, x = 1, y = 1, z = 0$, which is exactly m_6, a cell that is 0 in the function. So this

expression both misses required minterms and wrongly turns on m_6. Invalid.

Step 5: Check option (C), $\bar{x}\bar{z} + wxy + w\bar{x}\bar{z} + \bar{w}xy\bar{z}$.
$\bar{x}\bar{z}$ gives $\{0, 2, 8, 10\}$, wxy gives $\{14, 15\}$, $w\bar{x}\bar{z}$ repeats $\{8, 10\}$, and $\bar{w}xy\bar{z}$ again wrongly adds m_6. There is no term of the form xz, so m_5, m_7, m_{13} are never covered. This expression both misses required minterms and adds the wrong m_6. Invalid.

Step 6: Check option (D), $\bar{x}\bar{z} + xz + wy\bar{z}$.
$\bar{x}\bar{z}$ gives $\{0, 2, 8, 10\}$ and xz gives $\{5, 7, 13, 15\}$, same as before. The term $wy\bar{z}$ ($w = 1, y = 1, z = 0$, x free) covers m_{10} and m_{14}, both 1s. The union is

$$\{0, 2, 8, 10\} \cup \{5, 7, 13, 15\} \cup \{10, 14\} = \{0, 2, 5, 7, 8, 10, 13, 14, 15\}$$

which again matches the onset exactly, with m_{10} simply covered twice. Valid.

Step 7: Final conclusion.
The valid forms of $f(w, x, y, z)$ are

$$\boxed{\bar{x}\bar{z} + xz + wxy \text{ (A) and } \bar{x}\bar{z} + xz + wy\bar{z} \text{ (D)}}$$

Quick Tip: Build the K-map for the given minterms and check each product term against every cell it touches.

57. Consider a real, narrowband signal $x(t) = A(t)\cos[2\pi f_c t + \theta(t)]$ where the maximum frequency components of $A(t)$ and $\theta(t)$ are f_M and f_c ($= 1000 f_M$), respectively. Which of the following statements is/are correct for $-\infty < t < \infty$?

(A) $x(t)$ represents a PSK modulated signal for suitable choices of $A(t)$ and $\theta(t)$.

(B) $x(t)$ represents an amplitude modulated signal for suitable choices of $A(t)$ and $\theta(t)$.

(C) $x(t)$ represents a band-limited Gaussian noise process.

(D) $x(t)$ never represents a narrowband FM signal.

Correct Answer: (A) $x(t)$ represents a PSK modulated signal for suitable choices of $A(t)$ and $\theta(t)$. ; (B) $x(t)$ represents an amplitude modulated signal for suitable choices of $A(t)$ and $\theta(t)$. ; (C) $x(t)$ represents a band-limited Gaussian noise process.

Solution:

Step 1: Recall the general form of a narrowband signal.
Any real narrowband (bandpass) signal can be written in the canonical envelope and phase form

$$x(t) = A(t)\cos[2\pi f_c t + \theta(t)]$$

where $A(t)$ is the slowly varying envelope and $\theta(t)$ is the slowly varying phase, both bandlimited compared with the carrier frequency f_c.

Step 2: Check option (A), PSK.
In phase shift keying the envelope is held constant, so we pick $A(t)$ $=$ constant, and the data is carried entirely by discrete jumps in $\theta(t)$ (for example 0 and π for BPSK). This is exactly a special case of the

given form.
So option (A) is correct.

Step 3: Check option (B), amplitude modulation.
In amplitude modulation we set $\theta(t) = 0$ and let the envelope carry the message,

$$A(t) = 1 + k\,m(t)$$

where $m(t)$ is the message signal. This is again a special case of the same canonical form.
So option (B) is correct.

Step 4: Check option (C), band-limited Gaussian noise.
A narrowband Gaussian noise process also has a well known canonical envelope-phase decomposition,

$$n(t) = R(t)\cos[2\pi f_c t + \psi(t)]$$

where $R(t)$ is a Rayleigh distributed envelope and $\psi(t)$ is a uniformly distributed phase, both of which are bandlimited random processes. Picking $A(t) = R(t)$ and $\theta(t) = \psi(t)$ reproduces this.
So option (C) is correct.

Step 5: Check option (D), narrowband FM.
In narrowband FM the envelope is held constant, $A(t) = \text{constant}$, and the phase carries the message through

$$\theta(t) = 2\pi k_f \int m(\tau)\,d\tau$$

This is again a special case of the same canonical form, so $x(t)$ can represent a narrowband FM signal. The claim in option (D) that it never does so is false.

So option (D) is incorrect.

Step 6: Final conclusion.

The signal $x(t) = A(t)\cos[2\pi f_c t + \theta(t)]$ is the general canonical form for any narrowband signal, so it can represent PSK, AM, and band-limited Gaussian noise, and it can also represent narrowband FM, which rules out option (D).

$$\boxed{\text{(A), (B), (C)}}$$

Quick Tip: Recall that any narrowband signal has a general envelope-phase form that covers PSK, AM, FM, and even noise as special cases.

58. Let $x_1(t) = \cos(2\pi nt)$ and $x_2(t) = 2\sin(4\pi nt)$ represent two sinusoids for a positive integer n and $-\infty < t < \infty$. Which of the following statements about $x_1(t)$ and $x_2(t)$ is/are valid?

(A) $x_1(t)$ and $x_2(t)$ are orthogonal to each other over $0 \le t < 1/n$.

(B) $x_1(t)$ and $x_2(t)$ are orthonormal to each other over $0 \le t < 1/n$.

(C) $x_2(t)$ is a harmonic of $x_1(t)$.

(D) $x_1(t)$ and $x_2(t)$ are non-orthogonal to each other over $0 \le t < 1/(2n)$.

Correct Answer: (A) $x_1(t)$ and $x_2(t)$ are orthogonal to each other over $0 \le t < 1/n$. ; (C) $x_2(t)$ is a harmonic of $x_1(t)$. ; (D) $x_1(t)$ and $x_2(t)$ are non-orthogonal to each other over $0 \le t < 1/(2n)$.

Solution:

Step 1: Write the product in a sum of sines.

We need

$$x_1(t)x_2(t) = \cos(2\pi nt) \cdot 2\sin(4\pi nt)$$

Using $2\cos A \sin B = \sin(A+B) + \sin(B-A)$ with $A = 2\pi nt$ and $B = 4\pi nt$,

$$x_1(t)x_2(t) = \sin(6\pi nt) + \sin(2\pi nt)$$

Step 2: Test orthogonality over $0 \le t < 1/n$.

The period of $x_1(t)$ is $1/n$, so we integrate over one full period,

$$\int_0^{1/n} \sin(6\pi nt)\,dt = \left[\frac{-\cos(6\pi nt)}{6\pi n}\right]_0^{1/n} = \frac{-\cos(6\pi) + \cos(0)}{6\pi n} = 0$$

$$\int_0^{1/n} \sin(2\pi nt)\,dt = \left[\frac{-\cos(2\pi nt)}{2\pi n}\right]_0^{1/n} = \frac{-\cos(2\pi) + \cos(0)}{2\pi n} = 0$$

Both terms vanish, so

$$\int_0^{1/n} x_1(t)x_2(t)\,dt = 0$$

So $x_1(t)$ and $x_2(t)$ are indeed orthogonal over $0 \le t < 1/n$, and option (A) is correct.

Step 3: Test orthonormality over the same interval.

Orthonormal needs unit energy on top of orthogonality. The energy of $x_1(t)$ over one period is

$$\int_0^{1/n} \cos^2(2\pi nt)\,dt = \frac{1}{2} \cdot \frac{1}{n} = \frac{1}{2n}$$

which equals 1 only for a special value of n, not for a general positive integer n. So option (B) is incorrect.

Step 4: Check the harmonic relation.

The frequency of $x_1(t)$ is n and the frequency of $x_2(t)$ is $2n$, an integer multiple of n. A signal whose frequency is an integer multiple of a fundamental frequency is a harmonic of it.

So option (C) is correct.

Step 5: Test the shorter interval $0 \leq t < 1/(2n)$.

Integrating the same product over this half period,

$$\int_0^{1/(2n)} \sin(6\pi nt)\,dt = \frac{-\cos(3\pi)+\cos(0)}{6\pi n} = \frac{1+1}{6\pi n} = \frac{1}{3\pi n}$$

$$\int_0^{1/(2n)} \sin(2\pi nt)\,dt = \frac{-\cos(\pi)+\cos(0)}{2\pi n} = \frac{1+1}{2\pi n} = \frac{1}{\pi n}$$

Adding these,

$$\int_0^{1/(2n)} x_1(t)x_2(t)\,dt = \frac{1}{3\pi n} + \frac{1}{\pi n} = \frac{4}{3\pi n} \neq 0$$

This is nonzero, so $x_1(t)$ and $x_2(t)$ are non-orthogonal over $0 \leq t < 1/(2n)$, and option (D) is correct.

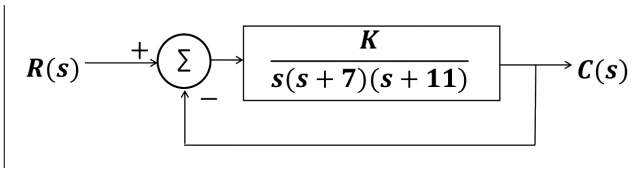
Step 6: Final conclusion.

$$\boxed{\text{(A), (C), (D)}}$$

Quick Tip: Turn the product of the two sinusoids into a sum of sines, then integrate over each interval to test orthogonality.

59.

Consider the unity negative feedback control system shown in the figure below, where the forward path transfer function is $G(s) = \dfrac{K}{s(s+7)(s+11)}$. The value of gain K (> 0) at which the given system will remain marginally stable is . (Answer in integer)



Correct Answer: 1386

Solution:

Step 1: Write the closed loop characteristic equation.

For unity negative feedback with forward path

$$G(s) = \frac{K}{s(s+7)(s+11)}$$

the characteristic equation is

$$1 + G(s) = 0 \implies s(s+7)(s+11) + K = 0$$

Step 2: Expand the polynomial.

$$(s+7)(s+11) = s^2 + 18s + 77$$

$$s(s^2 + 18s + 77) = s^3 + 18s^2 + 77s$$

So the characteristic equation is

$$s^3 + 18s^2 + 77s + K = 0$$

Step 3: Build the Routh array.

The rows for s^3, s^2, s^1, s^0 are

$$s^3 : 1,\ 77 \qquad s^2 : 18,\ K \qquad s^1 : \frac{18 \times 77 - K}{18},\ 0 \qquad s^0 : K$$

Step 4: Apply the marginal stability condition.

A system is marginally stable when a full row of the Routh array becomes zero, placing a pair of roots exactly on the imaginary axis. This happens when the s^1 row vanishes,

$$18 \times 77 - K = 0$$

$$1386 - K = 0$$

$$K = 1386$$

Step 5: Verify with the auxiliary equation.

The auxiliary equation comes from the s^2 row,

$$18s^2 + K = 0 \implies 18s^2 + 1386 = 0 \implies s^2 = -77 \implies s = \pm j\sqrt{77}$$

These are purely imaginary roots, which confirms sustained oscillation and marginal stability at this value of K.

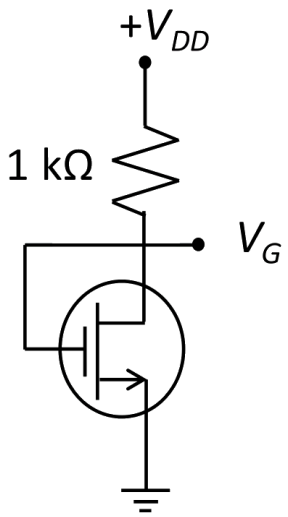
Step 6: Final answer.

$$\boxed{K = 1386}$$

Quick Tip: Build the Routh array for $s^3 + 18s^2 + 77s + K = 0$ and find the K that makes a row vanish.

60.

An n-channel MOSFET is connected as shown in the figure below. Assume $V_{TH} = 1\text{ V}$, $V_{DD} = 5\text{ V}$, and $\mu C_{ox}\left(\frac{W}{L}\right) = 2\text{ mA V}^{-2}$, and neglect channel length modulation effects. The gate voltage V_G of the n-channel MOSFET (in Volt) is . (Rounded off to two decimal places)



Correct Answer: 2.56

Solution:

Step 1: Identify the MOSFET connection.

The gate of the MOSFET is tied directly to its drain, which is the standard diode-connected configuration. The source is grounded, and a 1 kΩ resistor connects V_{DD} to the common gate-drain node, which is V_G.

Since gate and drain are the same node here,

$$V_{GS} = V_{DS} = V_G$$

Step 2: Confirm the region of operation.

For a diode-connected NMOS, $V_{DS} = V_{GS}$, so

$$V_{DS} - (V_{GS} - V_{TH}) = V_{TH} > 0$$

This means V_{DS} always exceeds the saturation boundary once the device is on, so the transistor operates in saturation.

Step 3: Write the saturation current equation.

$$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2 = \frac{1}{2}(2\ \text{mA V}^{-2})(V_G - 1)^2 = (V_G - 1)^2\ \text{mA}$$

Step 4: Apply KVL through the resistor.

The same current I_D flows through the $1\ \text{k}\Omega$ resistor from V_{DD}, so

$$V_{DD} = I_D R + V_G$$

$$5 = I_D(1) + V_G$$

$$I_D = 5 - V_G$$

Step 5: Combine the two current expressions.

$$(V_G - 1)^2 = 5 - V_G$$

$$V_G^2 - 2V_G + 1 = 5 - V_G$$

$$V_G^2 - V_G - 4 = 0$$

Step 6: Solve the quadratic.

$$V_G = \frac{1 \pm \sqrt{1+16}}{2} = \frac{1 \pm \sqrt{17}}{2}$$

Since $\sqrt{17} \approx 4.123$, the two roots are approximately 2.5616 and -1.5616. A gate voltage cannot be negative here, since the transistor must be on with $V_G > V_{TH} = 1$, so we reject the negative root.

Step 7: Final answer.

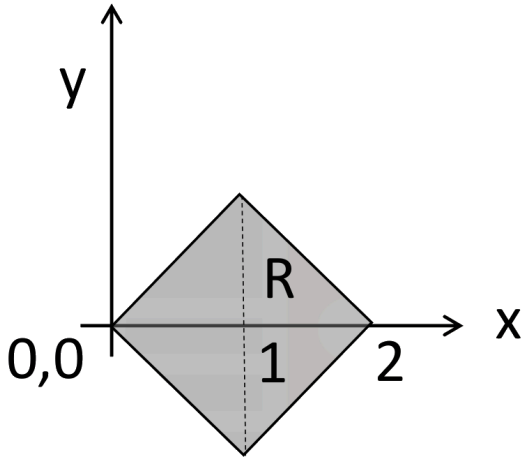
$$V_G \approx \boxed{2.56\ \text{V}}$$

Quick Tip: The gate tied to the drain forces $V_{GS} = V_{DS}$, so the device stays in saturation; combine this with the resistor's KVL equation.

61.

Consider the square region R in the X-Y plane as shown with the dark shading in the figure below. The value of $\iint_R (x^2 + y^2 - 1)\,dx\,dy$ is .

(Rounded off to two decimal places)



Correct Answer: 0.67

Solution:

Step 1: Identify the region from the figure.

The square (diamond shaped) region R has its four vertices at $(0, 0)$, $(1, 1)$, $(2, 0)$, and $(1, -1)$, so it can be written as

$$R: \ |x - 1| + |y| \leq 1$$

Step 2: Shift to a centered coordinate.

Let

$$u = x - 1, \qquad x = u + 1$$

The region becomes the standard diamond

$$|u| + |y| \leq 1$$

Step 3: Rewrite the integrand.

$$x^2 + y^2 - 1 = (u+1)^2 + y^2 - 1 = u^2 + 2u + 1 + y^2 - 1 = u^2 + y^2 + 2u$$

Step 4: Drop the odd term by symmetry.

The region $|u| + |y| \leq 1$ is symmetric about $u = 0$, while $2u$ is an odd function of u. Hence

$$\iint_R 2u \, du \, dy = 0$$

Step 5: Reduce to a single symmetric integral.

The remaining integral is

$$I = \iint_{|u|+|y|\leq 1} (u^2 + y^2) \, du \, dy$$

Since the diamond is symmetric under swapping u and y,

$$\iint u^2 \, du \, dy = \iint y^2 \, du \, dy$$

so

$$I = 2 \iint_{|u|+|y|\leq 1} u^2 \, du \, dy$$

Step 6: Evaluate the inner integral over y.

For a fixed $u \in [-1, 1]$, y ranges over $[-(1 - |u|), 1 - |u|]$, a strip of length $2(1 - |u|)$. So

$$\iint u^2 \, du \, dy = \int_{-1}^{1} u^2 \cdot 2(1 - |u|) \, du = 4 \int_{0}^{1} u^2 (1 - u) \, du$$

Step 7: Evaluate the remaining single integral.

$$\int_0^1 (u^2 - u^3) \, du = \left[\frac{u^3}{3} - \frac{u^4}{4} \right]_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

So

$$\iint u^2 \, du \, dy = 4 \times \frac{1}{12} = \frac{1}{3}$$

Step 8: Combine everything.

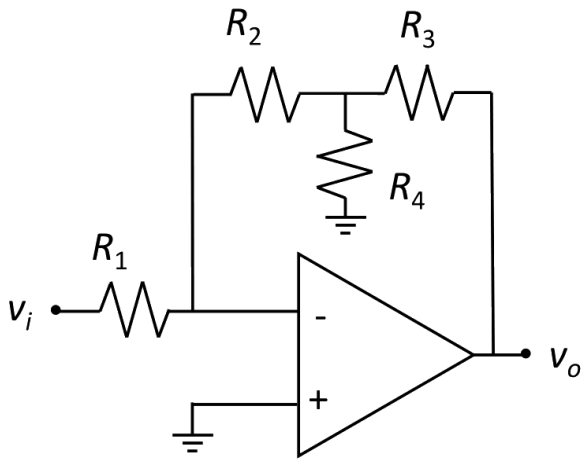
$$I = 2 \times \frac{1}{3} = \frac{2}{3} \approx 0.67$$

Step 9: Final answer.

$$\boxed{I \approx 0.67}$$

Quick Tip: Shift the region so its center sits at the origin, then use symmetry to simplify the double integral.

62. Consider an ideal OP-AMP circuit as shown in the figure.



The resistances $R_1 = R_2 = R_3 = R_4 = 50\ \text{k}\Omega$.
The magnitude of the closed loop gain is (rounded off to two decimal places).

Correct Answer: 3.00

Solution:

Step 1: Look at the circuit topology.
The signal v_i drives the inverting input of the op-amp through R_1. The non-inverting input is tied to ground, so by the virtual short property of an ideal op-amp in negative feedback, the inverting input also sits at 0 V.
The feedback path is not a single resistor. It is a T-network: R_2 runs from the inverting node to an internal node, call it X, then R_4 runs from X down to ground, and R_3 runs from X to the output v_o. This T-network lets the circuit reach a gain larger than 1 using only equal valued resistors.

Step 2: Apply KCL at the inverting node.

No current enters the ideal op-amp input, so whatever current comes in through R_1 must leave through R_2 toward node X.

$$\frac{v_i - 0}{R_1} = \frac{0 - V_X}{R_2}$$

This gives

$$V_X = -\frac{R_2}{R_1} v_i$$

Step 3: Apply KCL at node X.

At node X, the current that arrived from R_2 splits into two paths: one through R_4 to ground, and one through R_3 to the output.

$$\frac{0 - V_X}{R_2} = \frac{V_X - 0}{R_4} + \frac{V_X - v_o}{R_3}$$

Step 4: Put in the equal resistor values.

Here $R_1 = R_2 = R_3 = R_4 = R = 50\ \text{k}\Omega$. From Step 2, $V_X = -v_i$.

Substitute this into the Step 3 equation and cancel the common R:

$$-V_X = V_X + V_X - v_o$$

$$v_o = 3V_X$$

Step 5: Solve for the gain.

$$v_o = 3(-v_i) = -3v_i$$

So the closed loop gain is

$$\frac{v_o}{v_i} = -3$$

The negative sign only shows that the circuit is inverting, so the magnitude asked in the question is

$$\boxed{3.00}$$

Quick Tip: Redraw the R2-R3-R4 T-network as one equivalent feedback resistor, then use the usual inverting-amplifier gain formula.

63. The average bit error rate at the input of a $(7, 4, 1)$ Hamming decoder is 0.10.
The probability that the decoder will fail to decode a received word correctly is (rounded off to two decimal places).

Correct Answer: 0.15

Solution:

Step 1: Understand what the code can and cannot fix.
A $(7, 4, 1)$ Hamming code sends a 7-bit codeword built from 4 message bits, and the 1 means it can correct at most 1 bit error in each received word. If 2 or more bits in a 7-bit word are flipped, the decoder either corrects to the wrong codeword or cannot fix the word at all. So the

decoder fails exactly when 2 or more of the 7 bits are in error.

Step 2: Set up the bit error model.
Each of the 7 bits is wrong independently with probability $p = 0.10$ and correct with probability $1 - p = 0.90$. The number of wrong bits in a 7-bit word follows a binomial distribution with $n = 7$ trials.

Step 3: Write the failure probability.

$$P(\text{fail}) = 1 - P(0 \text{ errors}) - P(1 \text{ error})$$

because the decoder only succeeds when the word has 0 errors, or exactly 1 error that it can correct.

Step 4: Compute the probability of zero errors.

$$P(0 \text{ errors}) = (1 - p)^7 = (0.9)^7$$

$$(0.9)^7 = 0.4783$$

Step 5: Compute the probability of exactly one error.
There are 7 ways to choose which single bit is wrong, so

$$P(1 \text{ error}) = \binom{7}{1} p(1 - p)^6 = 7(0.1)(0.9)^6$$

$$(0.9)^6 = 0.5314$$

$$P(1 \text{ error}) = 7(0.1)(0.5314) = 0.3720$$

Step 6: Combine and subtract from 1.

$$P(0 \text{ or } 1 \text{ error}) = 0.4783 + 0.3720 = 0.8503$$

$$P(\text{fail}) = 1 - 0.8503 = 0.1497$$

Step 7: Round the answer.

Rounded to two decimal places, the probability of decoding failure is

$$\boxed{0.15}$$

Quick Tip: The decoder fails only when 2 or more of the 7 bits are wrong, so subtract the 0-error and 1-error binomial probabilities from 1.

64. Consider that the concentration of electrons in a semiconductor bar varies linearly from $2 \times 10^{17}\ \text{cm}^{-3}$ at $x = 1\ \mu\text{m}$ to $1 \times 10^{16}\ \text{cm}^{-3}$ at $x = 4\ \mu\text{m}$ along the x-direction. Assume that the concentration of electrons does not vary along the y- and z-directions.

Given: the mobility of electron is $1400\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$, the thermal voltage is $25\ \text{mV}$ and the electronic charge is 1.6×10^{-19} Coulomb.

The density of electron diffusion current (in A/mm^2) is (rounded off to two decimal places).

Correct Answer: -35.47

Solution:

Step 1: Write down the diffusion current formula.

When electron concentration n changes with position, electrons move by diffusion from the region of high concentration to the region of low concentration. The resulting conventional current density carried by this electron diffusion is

$$J_{n,\text{diff}} = qD_n \frac{dn}{dx}$$

where q is the electronic charge, D_n is the electron diffusion coefficient, and $\frac{dn}{dx}$ is the rate of change of electron concentration with position.

Step 2: Find the diffusion coefficient using the Einstein relation.

The Einstein relation connects mobility and the diffusion coefficient through the thermal voltage V_T:

$$D_n = \mu_n V_T$$

Here $\mu_n = 1400\ \text{cm}^2\text{V}^{-1}\text{s}^{-1}$ and $V_T = 25\ \text{mV} = 0.025\ \text{V}$.

$$D_n = 1400 \times 0.025 = 35\ \text{cm}^2/\text{s}$$

Step 3: Compute the concentration gradient.

The concentration changes linearly from $n_1 = 2 \times 10^{17}\ \text{cm}^{-3}$ at $x_1 = 1\ \mu\text{m}$ to $n_2 = 1 \times 10^{16}\ \text{cm}^{-3}$ at $x_2 = 4\ \mu\text{m}$. Converting positions to

centimeters, $1\ \mu m = 10^{-4}$ cm, so $x_1 = 1 \times 10^{-4}$ cm and $x_2 = 4 \times 10^{-4}$ cm.

$$\frac{dn}{dx} = \frac{n_2 - n_1}{x_2 - x_1} = \frac{1 \times 10^{16} - 2 \times 10^{17}}{4 \times 10^{-4} - 1 \times 10^{-4}}$$

$$= \frac{-1.9 \times 10^{17}}{3 \times 10^{-4}} = -6.333 \times 10^{20}\ \text{cm}^{-4}$$

Step 4: Substitute into the diffusion current formula.

$$J_{n,\text{diff}} = qD_n \frac{dn}{dx} = (1.6 \times 10^{-19})(35)(-6.333 \times 10^{20})$$

$$= (5.6 \times 10^{-18})(-6.333 \times 10^{20}) = -3546.67\ \text{A/cm}^2$$

Step 5: Convert to A/mm^2.

Since $1\ \text{cm} = 10\ \text{mm}$, $1\ \text{cm}^2 = 100\ \text{mm}^2$, so dividing an A/cm^2 value by 100 gives A/mm^2:

$$J_{n,\text{diff}} = \frac{-3546.67}{100} = -35.4667\ \text{A/mm}^2$$

Step 6: Round the answer.

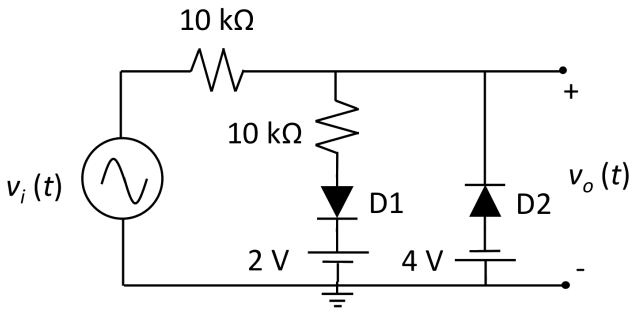
Rounded to two decimal places, the density of electron diffusion current is

$$\boxed{-35.47\ \text{A/mm}^2}$$

The negative sign only shows that this diffusion current flows in the negative x-direction, opposite to the direction along which the concentration is falling.

Quick Tip: Get Dn from the Einstein relation Dn = mu_n VT, then use Jn = q Dn (dn/dx) with the gradient in cm and cm^-3 units before converting to A/mm^2.

65. Consider the ideal diodes D_1 and D_2 as shown in the figure with cut-in voltage $V_\gamma = 0$ Volt and $v_i(t)$ is in Volt.



The maximum voltage (Volt) of the output $v_o(t)$ is (rounded off to two decimal places).

Correct Answer: 4.00

Solution:

Step 1: Understand the circuit.

The source $v_i(t) = 6\sin(\omega t)$ drives a $10\ \text{k}\Omega$ resistor R_1 to a node A, which is where the output $v_o(t)$ is measured with respect to the ground rail. From node A, a second $10\ \text{k}\Omega$ resistor R_2 leads to a node N. Node N connects to ground through two parallel ideal-diode

branches: D_1 in series with a 2 V battery, and D_2 in series with a 4 V battery. Since $V_\gamma = 0$, each diode turns on the instant the voltage across it tries to go positive.

Step 2: Find when the diodes are off.

If both diodes are off, no current flows anywhere in the circuit, so there is no drop across R_1 or R_2, and node A and node N both sit at $v_i(t)$. This holds as long as $v_i(t) < 2$ V, since that keeps D_1, the lower clamp, reverse biased. In this range,

$$v_o(t) = v_i(t)$$

Step 3: Find what happens once D1 turns on.

As $v_i(t)$ rises past 2 V, node N tries to rise above 2 V too, which turns D_1 on. Since D_1 is ideal, it pins node N at exactly 2 V, and current now flows from the source through R_1 and R_2 into the 2 V battery. Because 2 V is smaller than 4 V, node N never gets a chance to reach 4 V, so D_2 stays off the whole time. Only D_1 is active.

Step 4: Write the current when D1 conducts.

With D_1 on, the same current I flows through the series pair $R_1 + R_2$ between the source and the clamped node:

$$I = \frac{v_i(t) - 2}{R_1 + R_2} = \frac{v_i(t) - 2}{20\ \text{k}\Omega}$$

Step 5: Find the output voltage in this range.

The output is measured at node A, before R_2, so

$$v_o(t) = v_i(t) - IR_1 = v_i(t) - \frac{R_1}{R_1 + R_2}\big(v_i(t) - 2\big)$$

With $R_1 = R_2$, $\frac{R_1}{R_1 + R_2} = \frac{1}{2}$, so

$$v_o(t) = v_i(t) - \frac{1}{2}\big(v_i(t) - 2\big) = \frac{1}{2}v_i(t) + 1$$

Step 6: Find the maximum of v_o(t).

Since $v_o(t) = \frac{1}{2}v_i(t) + 1$ grows with $v_i(t)$ in this region, and $v_i(t) = 6\sin(\omega t)$ reaches its largest value of 6 V at its peak, the maximum output occurs there:

$$v_{o,\max} = \frac{1}{2}(6) + 1 = 3 + 1 = 4\,\text{V}$$

Step 7: Final conclusion.

For the lower part of the swing, $v_i(t) < 2$ V, including all negative values down to -6 V, $v_o(t) = v_i(t)$ stays below 2 V, so it never beats the clamped region. The overall maximum output voltage is

$$\boxed{4.00\ \text{V}}$$

Quick Tip: D1 (the 2 V branch) clamps node N before D2 ever gets a chance, so only D1 is active; find vo as a voltage divider once D1 conducts.