GATE 2026 Electronics & Communication Engineering Question Paper with Solutions

ZOLLEGE

General Instructions

(i) This booklet contains 65 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 42 single-correct multiple-choice questions, 10 multiple-correct questions and 13 numerical / integer-type questions.

(iii) These questions follow the GATE Electronics and Communication Engineering examination pattern.

(iv) Questions carry 1 or 2 marks. MCQs have negative marking (1/3 or 2/3 of the marks); MSQ and numerical-answer (NAT) questions carry no negative marking.

(v) GATE is a 3-hour paper of 65 questions (100 marks).

(vi) A virtual scientific calculator is provided; physical calculators are not allowed.

(vii) Attempt each question on your own before reviewing the given solution.

(viii) For multiple-correct questions, all correct options must be selected to earn full marks.

(ix) For numerical questions, report the answer rounded exactly as asked.

1. Among the following options, the antonym of the word 'nocturnal' is __________.

(A) normal

(B) diurnal

(C) abnormal

(D) exceptional

Correct Answer: (B) diurnal

SOLUTION:

Step 1: Break down the word "nocturnal".
The root here relates to "night" (from the Latin "nox"), so nocturnal means happening or active at night. Owls, bats and cats are common examples of nocturnal animals.

Step 2: Find the matching root for "day".
The Latin root for day is "dies", and the adjective built from it is "diurnal", meaning happening or active during the day. This directly opposes "nocturnal" in meaning.

Step 3: Check the remaining options against this root test.
"Normal" comes from a root meaning a standard or rule, not day or night. "Abnormal" just negates "normal", again unrelated to time. "Exceptional" relates to being an exception or rare case, also unrelated to time of day. None of these three carry any link to daytime activity.

Step 4: Final answer.
Since diurnal is built from the root for day and directly contrasts with the night based root of nocturnal, it is the correct antonym.

$$\boxed{\text{diurnal}}$$

Quick Tip: Look for a word that names daytime activity, since "nocturnal" names nighttime activity.

2. The statements (S1), (S2), and (S3) pertain to the scores obtained by students in an exam. The maximum possible marks in the exam is 150.
(S1) The highest score is 100.
(S2) The fourth highest score is 76.
(S3) There are at least four students whose scores are within 25 of each other.
Which one of the following options is necessarily correct?

(A) (S1) and (S2) together imply (S3)

(B) (S1) and (S3) together imply (S2)

(C) (S2) and (S3) together imply (S1)

(D) (S1) implies (S3)

Correct Answer: (A) (S1) and (S2) together imply (S3)

SOLUTION:

Step 1: Turn (S1) and (S2) into a range.
If the top score is 100 and the fourth highest is 76, then the 2nd and 3rd highest scores are squeezed between them, since the ranking already fixes their order. So the top four scores all sit inside $[76, 100]$.

Step 2: Measure that range.
The width of this range is $100 - 76 = 24$, which is under 25. So any two of the top four students differ by at most 24 marks, and (S3) is automatically satisfied by these four students alone. This makes option (A) always true.

Step 3: Try to break option (B) with an example.
Take a class where the topper scores 100, and four low scoring students happen to sit close together, say at $30, 32, 34, 36$. This satisfies (S1) and (S3), but the fourth highest score here could easily be

something other than 76, say 85. So (S1) and (S3) do not force (S2).

Step 4: Try to break option (C) with an example.
Let four students all score exactly 76, satisfying (S2) (the fourth highest is 76) and (S3) (these four are within 0 of each other). Nothing stops the highest score in the class from also being 76, so (S1), which needs the highest score to be 100, is not forced.

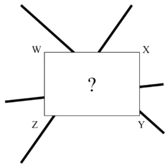
Step 5: Try to break option (D) with an example.
Let the topper score 100, but space every other student out by more than 25 marks from everyone else, for example $100, 74, 48, 22, \ldots$ This keeps (S1) true while making (S3) false, so (S1) alone cannot force (S3).

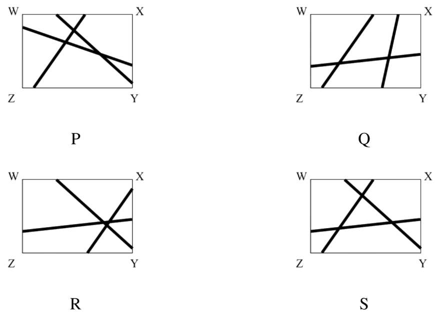
Step 6: Conclude.
Only option (A) survives every attempt to break it, because the arithmetic in Step 1 and Step 2 shows it must hold in general, not just in one example.

$$\boxed{\text{(S1) and (S2) together imply (S3)}}$$

Quick Tip: Work out how close together the top four scores must be once you fix the 1st and 4th highest values.

3. The figure below has exactly three intersecting line segments with a rectangular portion WXYZ missing. Which one of the following options P, Q, R, and S is the missing portion WXYZ?





(A) P

(B) Q

(C) R

(D) S

Correct Answer: (D) S

SOLUTION:

Step 1: Label what a straight line demands.

A straight line cannot change direction. So for each of the three lines crossing the missing rectangle, the piece hidden inside must leave each side of the rectangle heading in exactly the same direction the visible arm outside is already heading.

Step 2: Look at the corners one at a time.

Near corner W and corner X at the top, two arms enter the rectangle after crossing each other above it. Near corner Z and corner Y at the bottom, two steep arms leave the rectangle. Reading the slopes, the arm at W must connect through to the arm at Y, and the arm at X must connect through to the arm at Z, since swapping this pairing would force a line to bend.

Step 3: Add the third, flatter line.

A nearly flat arm leaves the middle of the left edge, and a matching nearly flat arm leaves the middle of the right edge. Being almost horizontal, this third line has to travel close to straight across, only tilting slightly, to join these two side arms without a kink.

Step 4: Rule out P, Q, and R.

Checking each candidate rectangle against this plan, P and Q each show at least one diagonal that does not point toward the correct corner once you follow its slope from the visible arm, and R places the meeting point of its two diagonals too far toward one corner instead of centrally, so none of these three keep all three lines straight through the missing block.

Step 5: Confirm S.

Only the rectangle labelled S carries all three lines through with the same slope and position that the six visible outside arms demand, joining W to Y, X to Z, and the two side arms, all without a bend.

$$\boxed{\text{S}}$$

Quick Tip: Each visible arm outside the rectangle must continue inside with exactly the same slope, since all three lines are straight.

4. Real numbers y, p, and n (all greater than 1) satisfy

$$\left(\log_{p^{1/n}} y\right)\left(\log_{y^{1/n}} p\right) = 16,$$

where the logarithms are taken to the bases $p^{1/n}$ and $y^{1/n}$.
The value of n is ___________

(A) 2

(B) 4

(C) 8

(D) 16

Correct Answer: (B) 4

SOLUTION:

Step 1: Substitute for the bases.
Let $u = p^{1/n}$ and $v = y^{1/n}$, so that $p = u^n$ and $y = v^n$.

Step 2: Rewrite the first log using this substitution.

$$\log_u y = \log_u (v^n) = n \log_u v$$

Step 3: Rewrite the second log the same way.

$$\log_v p = \log_v (u^n) = n \log_v u$$

Step 4: Multiply and use the reciprocal log identity.

$$(\log_u y)(\log_v p) = n^2 (\log_u v)(\log_v u)$$

Since $\log_u v$ and $\log_v u$ are reciprocals of each other, their product is always 1. So the whole expression reduces to n^2.

Step 5: Use the given value.

$$n^2 = 16$$

As n describes an exponent applied to positive reals greater than 1, it must be positive, so $n = 4$, not -4.

$$\boxed{n = 4}$$

Quick Tip: Change both logarithms to a common base and see how the product simplifies to a clean power of n.

• • •

5. The following observation is made about the scores obtained by 100 students in an exam:
'For each student, there exists another student in the class such that their scores are at most ten marks away.'
If the above statement is **false**, which one of the following statements is necessarily true?

(A) For each student, the scores of all the other students are more than 10 marks away.

(B) There exists at least one student in the class for whom the scores of all the other students are more than 10 marks away.

(C) There is exactly one student in the class for whom the scores of some students are more than 10 marks away.

(D) For each student, the score of exactly one other student is more than 10 marks away.

Correct Answer: (B) There exists at least one student in the class for whom the scores of all the other students are more than 10 marks away.

SOLUTION:

Step 1: Restate the claim in plain terms.

The original claim says every single student has at least one "buddy" somewhere in the class whose score sits within 10 marks of their own.

Step 2: Think about what makes this claim false.

The claim fails only if we can point to at least one student who has no such buddy at all, meaning every other student's score is more than 10 marks away from this particular student's score. We do not need this to happen for every student, just for one.

Step 3: Rule out the wrong quantities.

Option (A) wrongly demands that this "no buddy" situation happens for every student in the class, which is a much stronger claim than what a single failure requires.

Option (C) wrongly limits it to exactly one student and softens "all other students" down to just "some students," which is weaker than the falseness we found.

Option (D) talks about exactly one distant student per person, which describes a totally different kind of pattern, not the breakdown of the original claim.

Step 4: Match the correct reading.
The only reading that matches "the claim breaks down for at least one student, and for that student, every other student is far away" is option (B).

Step 5: Conclude.

$$\boxed{\text{There exists at least one student for whom the scores of all other students are more than 10 marks away.}}$$

Quick Tip: Negating "for every student there is a close one" flips the quantifiers: it becomes "some student has no close one".

• • •

6. Each one of the following clues contains a keyword that is partially filled.
Clue 1: Synonym of recognize (8 letters): _D_NT_FY
Clue 2: A story long enough to fill a book (5 letters): _ _ _ EL
Clue 3: Two of something (6 letters): _ _ _ PLE
Clue 4: A fraction of something, split equally into two parts (4 letters): _ _ _ F
The first letter of each of the keywords can be rearranged to form a four-letter word.
Which one of the options below is a possible choice for the four-letter word?

(A) CHIN

(B) COIN

(C) ITCH

(D) NOSE

Correct Answer: (A) CHIN

SOLUTION:

Step 1: Solve the four clues first.

Clue 1 wants an 8 letter word for "recognize" matching $\_D\_NT\_FY$: that is IDENTIFY, starting with I.

Clue 2 wants a 5 letter word for a book length story matching $\_\_\_EL$: that is NOVEL, starting with N.

Clue 3 wants a 6 letter word for "two of something" matching $\_\_\_PLE$: that is COUPLE, starting with C.

Clue 4 wants a 4 letter word for an equal half matching $\_\_\_F$: that is HALF, starting with H.

Step 2: Fix the target letter set.

The four first letters give the set $\{I, N, C, H\}$, and any valid answer must use exactly these four letters, no more and no less.

Step 3: Test each option against this set.

CHIN breaks into C, H, I, N, which is exactly the set $\{I, N, C, H\}$ with no extra letters. This passes.

COIN breaks into C, O, I, N. The letter O does not belong to the set, so this fails.

ITCH breaks into I, T, C, H. The letter T does not belong to the set, so this fails.

NOSE breaks into N, O, S, E. Three of these four letters, O, S and E, do not belong to the set, so this fails badly.

Step 4: Conclude.

Only CHIN survives the letter check, so it is the word we are looking for.

$$\boxed{\text{CHIN}}$$

Quick Tip: Solve each clue first, then look at just the first letters of the four answers.

• • •

7. Three children P, Q, R and two grown-ups X, Y play a badminton doubles tournament. X and Y are parents to two of the children playing. The child of X is not the same as the child of Y. Exactly one of the children does not have a parent playing in the tournament. The following rules are followed:

(i) A parent and his/her child cannot be on the same team.

(ii) A match can feature at most one parent and his/her child, that is, a maximum of one parent-child pair can play in a match.

The following matches were played:

	TEAM 1	**TEAM 2**
MATCH 1	P and X	Q and R
MATCH 2	P and R	X and Y
MATCH 3	R and X	Q and Y

Which one of the following options is correct?

(A) P does not have any parent playing

(B) Q does not have any parent playing

(C) R does not have any parent playing

(D) X does not have a child playing

Correct Answer: (C) R does not have any parent playing

SOLUTION:

Step 1: List the three possible cases for who has no parent playing.
Exactly one of P, Q, R has no parent in the tournament, so there are three cases to test: no-parent child is P, or Q, or R. In each case, the other two children are shared between X and Y as their respective children, in some order.

Step 2: Test "Q has no parent."
Then P and R are the children of X and Y, in some order. Look at Match 2, where P, R, X, and Y all play together (P, R on one team, X, Y on the other). Since both P and R are children of the two parents present, both parent-child pairs would appear together in this single match, which is not allowed since a match may contain at most one such pair. This case fails.

Step 3: Test "P has no parent."
Then Q and R are the children of X and Y, in some order. Match 1 has Team 1 as "P and X" and Team 2 as "Q and R". If X's child were R, look at Match 3 (Team 1: R, X; Team 2: Q, Y): X would share a team with his own child R, breaking the same-team rule. So X's child must be Q here, leaving R as Y's child. Now recheck Match 3: X's child Q sits on Team 2 while Y's child R sits on Team 1, so BOTH parent-child pairs are present in Match 3 together (X-Q and Y-R), again breaking

the one-pair-per-match rule. So this case also fails.

Step 4: Test "R has no parent."

Then P and Q are the children of X and Y, in some order. From Match 1 (Team 1: P, X), if X's child were P they would share a team, which is forbidden, so X's child must be Q, leaving P as Y's child.

Check Match 2 (Team 1: P, R; Team 2: X, Y): Q is sitting out, so X's child Q is absent from this match, and only Y's child P is present alongside Y. That is one pair, which is allowed.

Check Match 3 (Team 1: R, X; Team 2: Q, Y): P is sitting out, so Y's child P is absent from this match, and only X's child Q is present alongside X, on the opposite team. That is again one pair, allowed, and no same-team conflict.

Every match checks out cleanly in this case.

Step 5: Conclude.

Since the first two cases both broke a rule and this last case satisfies every match, R must be the child without a parent playing, with X's child being Q and Y's child being P.

$$\boxed{\text{R does not have any parent playing}}$$

Quick Tip: Use the fact that a parent's team cannot include his or her own child to test each match one at a time.

• • •

8. Let P_k represent the perimeter of a square with sides of length k. The value of the expression $P_1 + P_2 + P_3 + \cdots + P_{10}$ is ____________

(A) 55

(B) 110

(C) 220

(D) 440

Correct Answer: (C) 220

SOLUTION:

Step 1: Compute each perimeter directly.

Since a square of side k has perimeter $4k$, the ten perimeters are

$$4, 8, 12, 16, 20, 24, 28, 32, 36, 40$$

Step 2: Pair the terms from opposite ends.

Pairing the smallest with the largest, the second smallest with the second largest, and so on:

$$(4+40)=44, \quad (8+36)=44, \quad (12+32)=44, \quad (16+28)=44, \quad (20+24)=44$$

Every pair adds up to the same value, 44.

Step 3: Count the pairs and multiply.

There are 10 numbers, which makes 5 such pairs. So the total sum is

$$5 \times 44 = 220$$

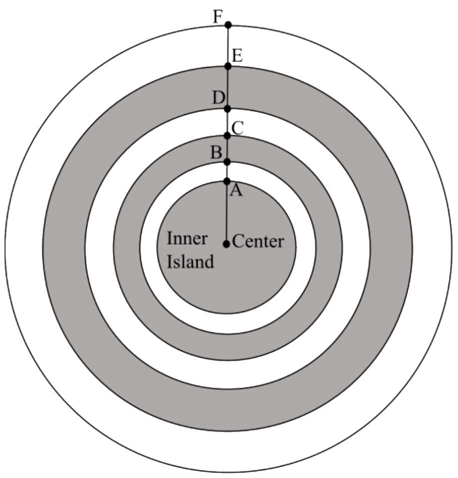
Step 4: Conclude.

$\boxed{220}$

Quick Tip: Write each perimeter as 4k, then pull the 4 out and sum the numbers 1 through 10.

• • •

9. The city of Atlantis was crafted by the God of the seas, Poseidon. It was made of alternating concentric circular rings of land (shaded) and water (not shaded) as represented in the figure (not to scale). The radius of Inner Island was 2.5 stades (a unit of length used in ancient Greece). The water surrounding Inner Island was one stade wide (length AB). This was surrounded by two pairs of alternating rings of land and water. The first pair of land and water was two stades wide each (lengths BC and CD), and the outer pair is three stades wide each (lengths DE and EF).



The ratio of the surface area of the land to that of the water in the city of Atlantis is ___________ (round off to two decimal places).

(A) 0.45

(B) 0.60

(C) 0.75

(D) 0.90

Correct Answer: (C) 0.75

SOLUTION:

Step 1: Use the sum-times-difference shortcut for ring area.

A ring between radius R_{in} and R_{out} has area $\pi(R_{out}^2 - R_{in}^2) = \pi(R_{out} + R_{in})(R_{out} - R_{in})$. The second factor is just the given width of the ring, so this avoids squaring big decimals.

Step 2: List every boundary radius.

Starting from the centre: $0,\ 2.5,\ 3.5,\ 5.5,\ 7.5,\ 10.5,\ 13.5$, built by adding the Inner Island radius and then each ring width in turn.

Step 3: Apply the shortcut to each ring.

Water ring AB: $(3.5 + 2.5)(1) = 6$. Land ring BC: $(5.5 + 3.5)(2) = 18$. Water ring CD: $(7.5 + 5.5)(2) = 26$. Land ring DE: $(10.5 + 7.5)(3) = 54$. Water ring EF: $(13.5 + 10.5)(3) = 72$. Each of these is the bracket value only; multiply by π for the actual area.

Step 4: Add the Inner Island and total each side.

Land (in units of π): $2.5^2 + 18 + 54 = 6.25 + 18 + 54 = 78.25$. Water (in units of π): $6 + 26 + 72 = 104$.

Step 5: Take the ratio.

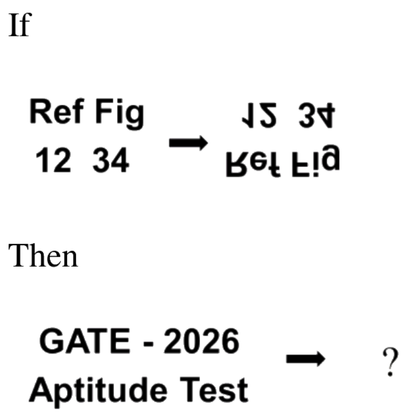
$$\frac{78.25}{104} = 0.7524\ldots \approx 0.75$$

$\boxed{0.75}$

Quick Tip: Add up the ring widths to get every boundary radius, then use pi(R_out^2 - R_in^2) for each ring's area before forming the ratio.

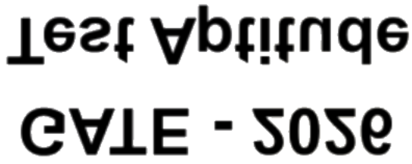
• • •

10. Consider the visual pattern rule shown below.

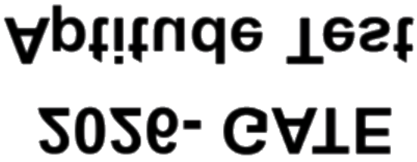


If the label "Ref Fig / 12 34" transforms into the pattern shown after the arrow under a fixed rule, then applying the same rule to the label "GATE - 2026 / Aptitude Test" produces which one of the patterns P, Q, R, S shown below?

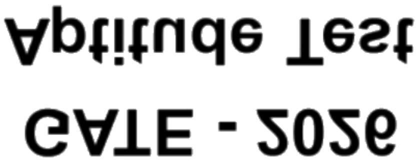
(A) Pattern P



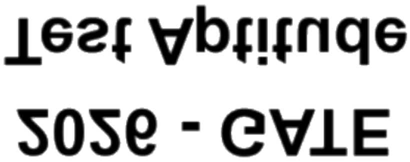
(B) Pattern Q



(C) Pattern R



(D) Pattern S



Correct Answer: (C) Pattern R

SOLUTION:

Step 1: Treat the transformation as a single geometric operation.
Instead of checking letter by letter, treat the whole two-line label as one picture. The example shows this picture after a 180° turn in its own plane, the same as picking up the label and spinning it half way around like a clock hand.

Step 2: Note the two effects a half turn always produces.
A half turn does two things at once: it puts the bottom line where the top line was, and it turns every character so it only reads correctly if you also turn your head upside down. A genuine rotation never produces just one of these effects alone.

Step 3: Use this as a filter on the four choices.
Check P, Q, R and S against both effects together. Any option that keeps the original line order, or that only mirrors the letters left to right without turning them upside down, or that changes a digit of the year in a way a genuine turn would not produce, fails this filter.

Step 4: Only one choice survives.
Only pattern R keeps both effects consistent with the example: "Aptitude Test" on top, "GATE - 2026" below it, and every character genuinely turned through 180°.

$$\boxed{\text{Pattern R}}$$

Quick Tip: Work out precisely what the example figure does to "Ref Fig / 12 34" (line order swap plus a 180-degree turn of every character), then apply that same rule to "GATE - 2026 / Aptitude Test".

- - -

11. Consider the differential equation $\dot{\vec{w}} = A\vec{w}$, with $\vec{w}(t=0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. If $\vec{w}(t) = e^t\vec{u}_x + e^{-2t}\vec{u}_y$ is the solution to the equation, where $\vec{u}_x$ and $\vec{u}_y$ are unit vectors along the positive x and y axes respectively, then which of the following options is the correct matrix representing A?

(A) $\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

(B) $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$

SOLUTION:

Step 1: Recognize the exponential form as an eigen-solution.

A linear system $\dot{\vec{w}} = A\vec{w}$ has solutions built from $e^{\lambda t}\vec{v}$, where λ is an eigenvalue of A and $\vec{v}$ is its eigenvector. The given solution is a sum of two such terms, $e^t\vec{u}_x$ and $e^{-2t}\vec{u}_y$.

Step 2: Read off the eigenvalues and eigenvectors.

The term $e^t\vec{u}_x$ gives eigenvalue $\lambda_1 = 1$ with eigenvector $\vec{u}_x = [1,0]^T$.

The term $e^{-2t}\vec{u}_y$ gives eigenvalue $\lambda_2 = -2$ with eigenvector $\vec{u}_y = [0,1]^T$.

Step 3: Build A from its eigenvalues and eigenvectors.

Since the eigenvectors are exactly the standard basis vectors, A is already diagonal in this basis, with the eigenvalues placed on the diagonal in the same order as their eigenvectors:

$$A = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

Step 4: Confirm the other matrices do not share these eigenpairs.
The matrix in (B) has eigenvalues -1 and 2, the wrong sign for both growth rates. The matrices in (C) and (D) are not diagonal in the standard basis, so $\vec{u}_x$ and $\vec{u}_y$ would not even be their eigenvectors.

$$\boxed{\begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}}$$

Quick Tip: Since u_x and u_y are the coordinate unit vectors, write w(t) as the column vector [e^t, e^{-2t}] and differentiate it directly.

• • •

12. A surface is given by $z^2 = 2x^2 - y^2$ and $\vec{n}$ and $-\vec{n}$ are unit normal vectors to the surface at the point $\vec{P} = \hat{\imath} + \sqrt{2}\,\hat{k}$. Which of the following vectors can be $\vec{n}$, where $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are the unit vectors along x, y and z axes, respectively?

(A) $\hat{\imath} - \sqrt{2}\,\hat{k}$

(B) $\frac{2}{3}\hat{\imath} - \frac{1}{3}\hat{k}$

(C) $\sqrt{2}\hat{\imath} - \sqrt{3}\,\hat{k}$

(D) $\dfrac{\sqrt{2}\hat{\imath} - \hat{k}}{\sqrt{3}}$

Correct Answer: (D) $\dfrac{\sqrt{2}\hat{\imath} - \hat{k}}{\sqrt{3}}$

SOLUTION:

Step 1: Get two tangent directions on the surface at P.

Write the surface as $z = \sqrt{2x^2 - y^2}$ near $P = (1, 0, \sqrt{2})$, treating z as a function of x and y. The partial derivatives give the slopes needed to build a normal.

$\dfrac{\partial z}{\partial x} = \dfrac{2x}{\sqrt{2x^2 - y^2}}$, which at $(1, 0)$ gives $\dfrac{2}{\sqrt{2}} = \sqrt{2}$.

$\dfrac{\partial z}{\partial y} = \dfrac{-y}{\sqrt{2x^2 - y^2}}$, which at $(1, 0)$ gives 0.

Step 2: Build the normal from these slopes.

For a surface $z = f(x, y)$, an unnormalized normal vector is $\left(-\dfrac{\partial z}{\partial x}, -\dfrac{\partial z}{\partial y}, 1\right)$. Substituting the slopes found above,

$$\vec{n}_{raw} = (-\sqrt{2},\ 0,\ 1)$$

Step 3: Normalize this vector.

Its length is $\sqrt{(\sqrt{2})^2 + 0^2 + 1^2} = \sqrt{3}$, so a unit normal is $\dfrac{(-\sqrt{2}, 0, 1)}{\sqrt{3}}$.

This points opposite to the direction found by the gradient method, but the question allows either $\vec{n}$ or $-\vec{n}$, so flipping the sign gives

$$\vec{n} = \frac{\sqrt{2}\,\hat{\imath} - \hat{k}}{\sqrt{3}}$$

Step 4: Match to the options.

This is exactly the vector in option (D), already scaled to length one, unlike the other three choices which are either unnormalized or use the wrong ratio between components.

$$\boxed{\frac{\sqrt{2}\hat{\imath} - \hat{k}}{\sqrt{3}}}$$

Quick Tip: The gradient of F(x,y,z)=2x^2-y^2-z^2 at a point gives the normal direction; divide it by its own magnitude to get a unit vector.

• • •

13. The Laplace Transform of the signal $x(t) = u(t-2) * (t\,u(t))$ is given by which of the following expressions? ["$*$" represents the convolution operator]

(A) $\dfrac{e^{-2s}}{s^2(s-2)}$

(B) $\dfrac{e^{-2(s-2)}}{s^3}$

(C) $\dfrac{se^{-2s}}{(s-2)^2}$

(D) $\dfrac{e^{-2s}}{s^3}$

Correct Answer: (D) $\dfrac{e^{-2s}}{s^3}$

SOLUTION:

Step 1: Do the convolution directly in the time domain.

By definition, $x(t) = u(t-2) * t\,u(t) = \int_{-\infty}^{\infty} u(\tau-2)\,(t-\tau)u(t-\tau)\,d\tau.$

The factor $u(\tau-2)$ restricts $\tau \geq 2$, and $u(t-\tau)$ restricts $\tau \leq t$, so the integral only survives for $t \geq 2$, running from $\tau = 2$ to $\tau = t$.

Step 2: Evaluate the integral.

$$x(t) = \int_2^t (t-\tau)\,d\tau, \qquad t \geq 2$$

Let $\sigma = t - \tau$; as τ runs from 2 to t, σ runs from $t-2$ down to 0. Then

$$x(t) = \int_0^{t-2} \sigma\,d\sigma = \frac{(t-2)^2}{2}, \qquad t \geq 2$$

So $x(t) = \dfrac{(t-2)^2}{2} u(t-2)$.

Step 3: Take the Laplace Transform of this time-domain result.

Since $\mathcal{L}\{t^2 u(t)\} = \dfrac{2}{s^3}$, we get $\mathcal{L}\left\{\dfrac{t^2}{2} u(t)\right\} = \dfrac{1}{s^3}$. Applying the time-shift property for the delay by 2,

$$X(s) = e^{-2s} \cdot \frac{1}{s^3} = \frac{e^{-2s}}{s^3}$$

Step 4: Match with the options.

This confirms option (D) directly, without ever needing the frequency-domain convolution shortcut.

$$\boxed{\frac{e^{-2s}}{s^3}}$$

Quick Tip: Convolution in time becomes multiplication in the Laplace domain; transform u(t-2) and t.u(t) separately, then multiply the results.

14. Consider carrier transport in a Zener diode in the breakdown region. Which is the dominant transport mechanism for current flow in this case?

(A) Drift

(B) Diffusion

(C) Tunneling

(D) Ballistic transport

Correct Answer: (C) Tunneling

SOLUTION:

Step 1: Separate the two breakdown mechanisms in reverse-biased diodes.
A reverse-biased p-n junction can break down in two distinct ways: Zener breakdown and avalanche breakdown. Knowing the difference settles the answer here.

Step 2: Describe avalanche breakdown first, as a contrast.
In lightly doped diodes with a wide depletion region, carriers accelerated by the field pick up enough kinetic energy through drift to knock other electrons loose in collisions, building current through impact ionization. This needs a wide region for the carriers to gain speed in.

Step 3: Describe Zener breakdown, which applies here.
A Zener diode is doped heavily on purpose, so its depletion region is very narrow. In such a narrow region a carrier cannot pick up much energy from drift before crossing it. Instead, the field is strong enough that electrons pass straight through the thin barrier by tunneling, a purely quantum effect that does not need the carrier to gain kinetic energy first.

Step 4: Match this to the question.

Since the question names it a Zener diode and asks specifically about its breakdown region, the dominant mechanism is this tunneling process, not drift-driven avalanche multiplication, and not diffusion or ballistic transport, neither of which describes junction breakdown at all.

$$\boxed{\text{Tunneling}}$$

Quick Tip: A Zener diode is doped heavily on both sides, giving a thin depletion region where carriers can cross the barrier directly under a high field.

• • •

15. Two analog signals $x_1(t)$ and $x_2(t)$ (t in second) are sampled at a rate $F_s = 40$ Hz, where

$$x_1(t) = \cos(20\pi t),\ t \geq 0, \qquad x_2(t) = \cos(100\pi t),\ t \geq 0.$$

The first ten samples (starting from $t = 0$) are considered for the analysis. Which of the following statements is TRUE?

(A) All of the first three samples of $x_1(t)$ are greater than the corresponding samples of $x_2(t)$.

(B) All of the last three samples of $x_1(t)$ are greater than the corresponding samples of $x_2(t)$.

(C) All of the samples of $x_2(t)$ are greater than the corresponding samples of $x_1(t)$.

(D) All of the fourth to seventh samples of $x_1(t)$ are equal to the corresponding samples of $x_2(t)$.

Correct Answer: (D) All of the fourth to seventh samples of $x_1(t)$ are equal to the corresponding samples of $x_2(t)$.

SOLUTION:

Step 1: Read off the two signal frequencies.
$x_1(t) = \cos(20\pi t)$ has frequency $f_1 = 10$ Hz, since $2\pi f_1 = 20\pi$.
Likewise $x_2(t) = \cos(100\pi t)$ has frequency $f_2 = 50$ Hz.

Step 2: Find the alias frequency of x_2 under 40 Hz sampling.
Sampling at $F_s = 40$ Hz can only distinguish frequencies up to the Nyquist limit of 20 Hz; anything above that folds back. The alias of 50 Hz is found by subtracting the sampling rate until the result lands inside $[0, F_s/2]$:

$$f_{2,alias} = 50 - 40 = 10 \text{ Hz}$$

This lands exactly on 10 Hz, with no extra folding needed since 10 < 20.

Step 3: Conclude about the samples.
Since the alias frequency of x_2 exactly equals the frequency of x_1, and both are plain cosines with zero phase, the two sampled sequences

$x_1[n]$ and $x_2[n]$ are identical at every sample index n, not just approximately close.

Step 4: Compute the actual values to check the options.
With $T_s = 1/40$ s, $x_1[n] = \cos(\pi n/2)$ gives the ten values $1, 0, -1, 0, 1, 0, -1, 0, 1, 0$ for $n = 0$ to 9, and $x_2[n]$ gives the identical list. So any statement claiming one sequence is strictly greater than the other anywhere is false, while a statement claiming a block of samples are equal, such as the fourth to seventh, is true.

$$\boxed{\text{All of the fourth to seventh samples of } x_1(t) \text{ are equal to the corresponding samples of } x_2(t).}$$

Quick Tip: Sample both cosines at t_n = n/40 seconds and simplify cos(2.5*pi*n) using the fact that 2*pi*n is always a whole number of cycles.

• • •

16. The response of a discrete time system $y[n]$ obeys the following relation:

$$y[n] = \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2] + x[n].$$

The input to the system is $x[n] = \delta[n] - \frac{1}{3}\delta[n-1]$. Which of the following options is TRUE for $y[n]$?

(A) Stable and causal response

(B) Stable and non-causal response

(C) Unstable and causal response

(D) Unstable and non-causal response

Correct Answer: (A) Stable and causal response

SOLUTION:

Step 1: Solve the homogeneous recursion first.

The recursion $y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 0$ has a characteristic equation found by trying $y[n] = r^n$:

$$r^2 - \frac{5}{6}r + \frac{1}{6} = 0$$

Multiplying through by 6 gives $6r^2 - 5r + 1 = 0$, which factors as $(2r - 1)(3r - 1) = 0$, giving roots $r = \frac{1}{2}$ and $r = \frac{1}{3}$.

Step 2: Write the general causal solution form.

Because the recursion only uses past values of y and the present $x[n]$, solving it forward from $n = 0$ produces a response built from these two roots as base terms:

$$y[n] = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{3}\right)^n, \quad n \geq 0$$

plus a particular part driven by $x[n]$, which is itself a combination of shifted, scaled versions of these same decaying terms since $x[n]$ is just two impulses.

Step 3: Check boundedness directly from these terms.

Both base terms $(1/2)^n$ and $(1/3)^n$ shrink to zero as n grows, and neither blows up for any finite n. A sum of decaying geometric terms stays bounded for a bounded input, which is the direct time-domain meaning of stability, matching the earlier finding from the pole locations.

Step 4: Check causality directly from the recursion structure.

Since $y[n]$ only ever needs $y[n-1]$, $y[n-2]$ and the current $x[n]$, and never a future sample, the response at any time n can be computed purely from the past, which is exactly what causal means.

Step 5: Conclude.

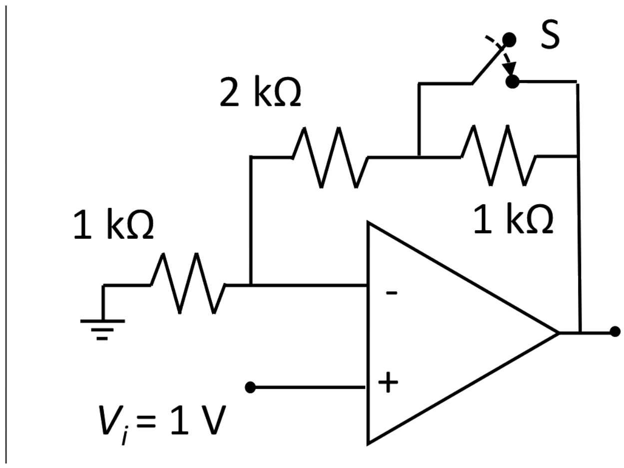
Both the decay of the geometric terms and the forward-only dependency confirm the same result reached earlier from the z-transform poles.

$$\boxed{\text{Stable and causal response}}$$

Quick Tip: Factor the characteristic polynomial to find the pole locations, then check that both poles lie strictly inside the unit circle for the causal ROC.

• • •

17. The ideal OP-AMP circuit shown in the Figure produces output voltage $V_o = x$ when the Switch, S, is open.



Which of the options represents the output voltage when S is closed?

(A) x

(B) $\frac{2}{3}x$

(C) $\frac{3}{4}x$

(D) $\frac{1}{2}x$

Correct Answer: (C) $\frac{3}{4}x$

SOLUTION:

Step 1: Note that the inverting input always sits at 1 V.

Because the op-amp has negative feedback in both switch positions, the inverting terminal is forced to the same voltage as the non-inverting terminal, so this node (call it A) is always at $V_i = 1$ V.

Step 2: Find the current leaving node A through the grounded resistor.

The 1 kΩ resistor connects node A to ground, so the current from A to ground is

$$I = \frac{1\text{ V}}{1\text{ k}\Omega} = 1\text{ mA}$$

Since no current enters the op-amp input, this same 1 mA must arrive at node A from the output side through the 2 kΩ resistor.

Step 3: Find the voltage at the midpoint node B, between the 2 kΩ resistor and the switch/1 kΩ combination.

$$V_B = V_A + I \times 2\text{ k}\Omega = 1 + 1 \times 2 = 3\text{ V}$$

Step 4: Case S open, find x.

With S open, the 1 mA continues from B to the output through the remaining 1 kΩ resistor, so

$$x = V_B + I \times 1\,\text{k}\Omega = 3 + 1 = 4\,\text{V}$$

Step 5: Case S closed, find the new output.

Closing S ties node B directly to V_o with zero resistance, so now $V_o = V_B = 3$ V, since there is no longer a resistor between B and the output to drop any voltage.

Step 6: Write the closed-switch output as a fraction of x.

We found $x = 4$ V and $V_o(\text{closed}) = 3$ V, so

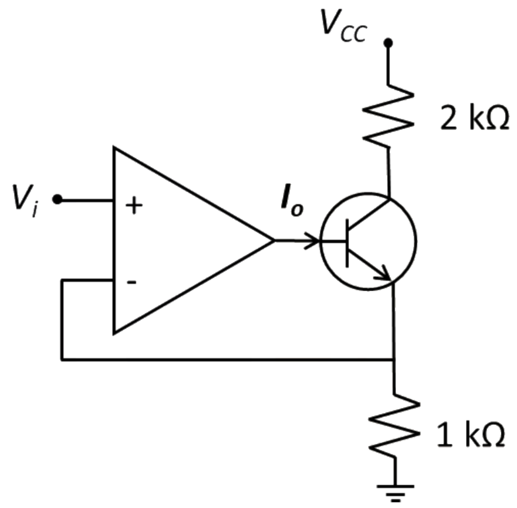
$$\frac{V_o(\text{closed})}{x} = \frac{3}{4}$$

$$\boxed{V_o(\text{closed}) = \frac{3}{4}x}$$

Quick Tip: Shorting S removes the outer $1\,\text{k}\Omega$ resistor from the feedback path, so only the feedback resistance in the gain formula changes between the two cases.

• • •

18. Consider the circuit shown in the Figure with $V_i = 3$ V and $V_{CC} = 12$ V.



Assume $V_{BE} = 0.7$ V and $\beta_{dc} = 99$ for the BJT.
Which of the following options is the correct value of the current I_o?

(A) $60\,\mu$A

(B) $6\,\mu$A

(C) $3\,\mu$A

(D) $30\,\mu$A

Correct Answer: (D) $30\,\mu$A

SOLUTION:

Step 1: Pin down the emitter node voltage.
The feedback loop ties the op-amp's minus input straight to the emitter, and an ideal op-amp with negative feedback keeps its two inputs equal. So the emitter sits at the same level as the plus input, $V_E = V_i = 3$ V.

Step 2: Get the emitter current from Ohm's law.
A 1 kΩ resistor takes the emitter to ground, so

$$I_E = \frac{3\ \text{V}}{1\ \text{k}\Omega} = 3\ \text{mA}$$

Step 3: Bring in the common base current gain α.

For a BJT, $\alpha = \frac{\beta_{dc}}{1 + \beta_{dc}}$, and the base current is the leftover fraction of the emitter current not carried by the collector, that is $I_B = (1 - \alpha)I_E$.

Step 4: Work out $1 - \alpha$.

$$\alpha = \frac{99}{100} = 0.99$$

$$1 - \alpha = 0.01 = \frac{1}{100}$$

Step 5: Compute the base current.

$$I_o = I_B = (1 - \alpha)I_E = \frac{1}{100} \times 3\ \text{mA} = 30\,\mu\text{A}$$

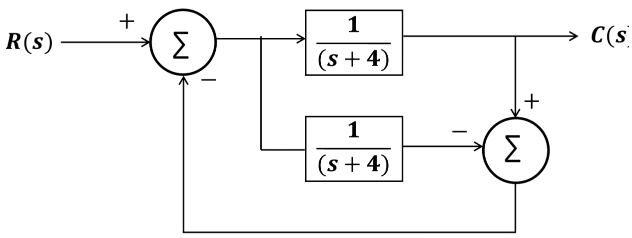
This matches the direct $I_E/(\beta_{dc} + 1)$ route exactly, since $1 - \alpha = 1/(1 + \beta_{dc})$ algebraically.

$$\boxed{I_o = 30\,\mu\text{A}}$$

Quick Tip: The op-amp forces the emitter voltage to equal V_i; use that to find I_E, then divide by $(\beta_{dc} + 1)$ to get the base current I_o.

• • •

19. A control system is shown in the Figure.



Which option represents the correct transfer function of the system?

(A) $\dfrac{1}{(s+4)^2}$

(B) $\dfrac{1}{(s+4)}$

(C) $\dfrac{2}{(s+4)}$

(D) $\dfrac{1}{(s^2+8s+17)}$

Correct Answer: (B) $\dfrac{1}{(s+4)}$

SOLUTION:

Step 1: Draw the signal flow graph nodes.

Call the reference R, the error node E, the forward output C, the second block's output D, and the inner summing node output F, which feeds back to the error node.

Step 2: Write the branch gains.

E to C: gain $\dfrac{1}{s+4}$. E to D: gain $\dfrac{1}{s+4}$, since the second block taps the same node E, not C. C and D combine at the inner summer as $F = C - D$. Finally $E = R - F$.

Step 3: Substitute to collapse the inner loop.

Since both C and D equal $E \cdot \dfrac{1}{s+4}$, we get $F = E \cdot \dfrac{1}{s+4} - E \cdot \dfrac{1}{s+4}$

$= 0$ identically, for any E.

Step 4: Reduce the graph.

With $F \equiv 0$, the feedback branch carries no signal at all, so effectively there is no loop; the loop gain of this inner loop is zero and Mason's gain formula reduces to just the forward path from R to E to C.

Step 5: Write the forward path gain.

$E = R$, since $F = 0$, and $C = E \cdot \dfrac{1}{s+4} = \dfrac{R}{s+4}$.

Step 6: State the transfer function.

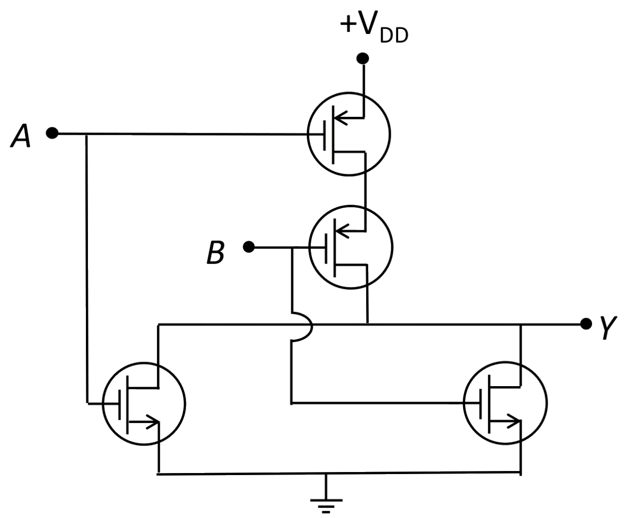
$$\frac{C(s)}{R(s)} = \frac{1}{s+4}$$

$$\boxed{\frac{1}{s+4}}$$

Quick Tip: Check exactly where the second block's input line is tapped from. It shares its input with the first block, so its output equals $C(s)$, which makes the inner feedback cancel to zero.

• • •

20. In the circuit shown in the Figure, A and B are logic inputs and Y is the logic output.



Which of the following logic operations is realized by the circuit?

(A) NOR

(B) OR

(C) XOR

(D) AND

Correct Answer: (A) NOR

SOLUTION:

Step 1: Write the pull-up condition in Boolean form.

A PMOS conducts on a low gate signal, so the PMOS gated by A conducts when $\overline{A}$ is true, and the one gated by B conducts when $\overline{B}$ is true. Since these two PMOS devices are wired in series, the pull-up path is complete only when both conduct together, so the pull-up condition is the AND of the two: $\overline{A} \cdot \overline{B}$.

Step 2: Connect the pull-up condition to the output.

In a fully complementary CMOS gate, Y is high exactly when the pull-up network conducts, so

$$Y = \overline{A} \cdot \overline{B}$$

Step 3: Simplify with De Morgan's theorem.

$$\overline{A} \cdot \overline{B} = \overline{A + B}$$

so

$$Y = \overline{A + B}$$

Step 4: Check this against the pull-down network.
The pull-down side has an NMOS gated by A and one gated by B wired in parallel, each conducting on a high gate signal. This pulls Y low whenever $A + B$ is true, which is exactly the complementary condition to the pull-up expression above, confirming consistency between the two networks.

Step 5: Name the function.
$Y = \overline{A + B}$ is, by definition, the NOR function of A and B.

$$\boxed{Y = \overline{A + B} = \text{NOR}(A, B)}$$

Quick Tip: Series PMOS in the pull-up network together with parallel NMOS in the pull-down network is the standard CMOS structure for a NOR gate.

21. Consider the Friis' transmission equation $P_R = \frac{P_T G_T G_R \lambda^2}{(4\pi D)^2}$, where P_R and P_T are the received and the transmitted powers, respectively. G_T and G_R are the gain of transmitting and receiving antennas, respectively, D is the distance between the transmitting and receiving antennas, and λ is the wavelength in free space.

Given: $G_T = G_R = 1.0$, $\lambda = 0.30$ m and $P_T = +10$ dBm.

Choose the distance (D), in km, from the following options at which the received power, $P_R = -90$ dBm?

(A) $\frac{15}{4\pi}$

(B) $\frac{15}{2\pi}$

(C) $\frac{75}{2\pi}$

(D) $\frac{3}{4\pi}$

Correct Answer: (B) $\frac{15}{2\pi}$

SOLUTION:

Step 1: Express the Friis equation in decibels.

Taking $10\log_{10}$ of both sides of $P_R = P_T G_T G_R \left(\frac{\lambda}{4\pi D}\right)^2$ turns the squared ratio into a $20\log_{10}$ term:

$$P_R(\text{dBm}) = P_T(\text{dBm}) + 10\log_{10} G_T + 10\log_{10} G_R + 20\log_{10}\left(\frac{\lambda}{4\pi D}\right)$$

Step 2: Drop the zero-gain terms.

Since $G_T = G_R = 1.0$, a linear gain of exactly 1, we get $10\log_{10}(1) = 0$ dB for each, so those terms vanish.

Step 3: Plug in the known powers.

$$-90 = 10 + 20\log_{10}\left(\frac{\lambda}{4\pi D}\right)$$

$$20\log_{10}\left(\frac{\lambda}{4\pi D}\right) = -100$$

$$\log_{10}\left(\frac{\lambda}{4\pi D}\right) = -5$$

Step 4: Remove the logarithm.

$$\frac{\lambda}{4\pi D} = 10^{-5}$$

Step 5: Solve for D in metres, then convert to km.

$$D = \frac{\lambda \times 10^5}{4\pi} = \frac{0.30 \times 10^5}{4\pi}\ \mathrm{m} = \frac{30000}{4\pi}\ \mathrm{m}$$

Dividing by 1000 to get kilometres,

$$D = \frac{30}{4\pi}\ \mathrm{km} = \frac{15}{2\pi}\ \mathrm{km}$$

$$\boxed{D = \frac{15}{2\pi}\ \mathrm{km}}$$

Quick Tip: Convert the 100 dB power difference to a linear ratio first, then use $P_R/P_T = (\lambda/4\pi D)^2$ to solve for D, remembering to take a square root.

• • •

22. Consider a discrete memoryless source with an alphabet of four source symbols. $s(t)$ is a multi-level $(-1, 0, +1, +2)$ signal representing a long sequence of random symbols from the above source which is generating 10^4 symbols per second.

Which of the following options is the correct value of equivalent Nyquist bandwidth of $s(t)$?

(A) 10 kHz

(B) 64 kHz

(C) 5 kHz

(D) 20 kHz

Correct Answer: (C) 5 kHz

SOLUTION:

Step 1: Find the time slot given to each symbol.

If 10^4 symbols go out every second, each symbol occupies a time slot of

$$T = \frac{1}{R_s} = \frac{1}{10^4}\ \text{s} = 100\ \mu\text{s}$$

Step 2: Recall how pulse duration links to bandwidth.

A pulse of duration T shaped to avoid inter symbol interference needs a minimum bandwidth of

$$B = \frac{1}{2T}$$

This comes from the idea that the smallest bandwidth channel that can still carry one independent pulse every T seconds without the pulses blurring into each other is $1/(2T)$.

Step 3: Plug in the symbol period.

$$B = \frac{1}{2 \times 100\ \mu\text{s}} = \frac{1}{200\ \mu\text{s}}$$

Step 4: Work out the number.

$$B = \frac{1}{200 \times 10^{-6}} = 5000\ \text{Hz} = 5\ \text{kHz}$$

Step 5: Note that the four signal levels do not change this.
Whether each pulse takes one of 2 levels or 4 levels only changes how many bits ride on each pulse, not the physical pulse rate or its bandwidth requirement.

$$\boxed{B = 5\ \text{kHz}}$$

Quick Tip: The Nyquist bandwidth of a baseband signal is half the symbol rate, regardless of how many amplitude levels each symbol has.

• • •

23. The relation between the input current (I) and the output voltage (V) of a circuit is governed by the equation: $C\frac{dV}{dt} = I(t) - m(t)$. The circuit is excited by $I(t) = q\,\delta(t)$, where q is a real valued constant. V at $t = 0^-$ is V_0. Which of the following is an equivalent representation of the above case?

(A) $C\dfrac{dV}{dt} = -m(t)$, with $V(t = 0^-) = V_0 + q/C$

(B) $C\dfrac{dV}{dt} = -m(t)$, with $V(t = 0^-) = V_0 + q/C + m(t = 0^-)$

(C) $C\dfrac{dV}{dt} = -m(t)$, with $V(t = 0^-) = V_0 - q/C + m(t = 0^-)$

(D) $C\dfrac{dV}{dt} = -m(t)$, with $V(t = 0^-) = q/C$

Correct Answer: (A) $C\dfrac{dV}{dt} = -m(t)$, with $V(t = 0^-) = V_0 + q/C$

SOLUTION:

Step 1: Think in terms of charge instead of the differential equation.

A capacitor's charge and voltage are linked by $Q = CV$. The current $I(t) = q\,\delta(t)$ is not an ordinary current profile, it is a burst that dumps a fixed total charge q onto the capacitor in essentially zero time, since the area under a delta function of strength q is exactly q.

Step 2: Find the charge on the capacitor just before the burst.

Just before $t = 0$, the capacitor voltage is V_0, so the stored charge is

$$Q(0^-) = CV_0$$

Step 3: Add the impulse charge.

The impulse adds charge q almost instantly, and in that same vanishing instant the $m(t)$ term, which is not impulsive, cannot move any extra charge, since it needs a nonzero time interval to do so. So right after the burst,

$$Q(0^+) = Q(0^-) + q = CV_0 + q$$

Step 4: Convert the new charge back to a voltage.

$$V(0^+) = \frac{Q(0^+)}{C} = \frac{CV_0 + q}{C} = V_0 + \frac{q}{C}$$

Step 5: State the equivalent system.

After the burst, the current source term is spent, so the governing equation returns to $C\,dV/dt = -m(t)$, now starting from the bumped-up voltage found above.

$$\boxed{C\frac{dV}{dt} = -m(t), \quad V_{\text{start}} = V_0 + \frac{q}{C}}$$

Quick Tip: A current impulse of area q into a capacitor C causes an instantaneous voltage jump of q/C; add this jump to the pre-impulse voltage V_0 to get the new starting condition.

• • •

24. The electric field of a monochromatic plane wave travelling in a lossless isotropic and homogenous medium is given by

$$\vec{E}(z,t) = E_0\left[\hat{x}\cos(\omega t - kz) + \hat{y}\sin(\omega t - kz)\right]$$

in a right-handed orthogonal co-ordinate system.

Which of the following is the correct polarization of the electromagnetic wave?

(A) Right-handed circularly polarized

(B) Left-handed circularly polarized

(C) Linearly polarized

(D) Linearly polarized with $-45°$ angle to $\hat{x}$

Correct Answer: (A) Right-handed circularly polarized

SOLUTION:

Step 1: Write the field as the real part of a rotating phasor.

Using $\cos\phi = \mathrm{Re}(e^{j\phi})$ and $\sin\phi = \mathrm{Re}(-je^{j\phi})$, the field becomes

$$\vec{E} = \mathrm{Re}\Big[E_0(\hat{x} - j\hat{y})e^{j(\omega t - kz)}\Big]$$

Step 2: Recognize the standard phasor form for circular polarization.

A transverse field phasor of the form $E_0(\hat{x} - j\hat{y})$ is the textbook signature of a circularly polarized wave, since $\hat{x}$ and $-j\hat{y}$ have equal magnitude and are in phase quadrature, a 90 degree gap contained in the $-j$ factor. Had the sign been $+j$ instead of $-j$, the handedness would flip.

Step 3: Match the sign convention to handedness for $+z$ propagation.

For a wave travelling in $+z$, the standard convention assigns the phasor form $\hat{x} - j\hat{y}$ to right-handed circular polarization and $\hat{x} + j\hat{y}$ to left-handed circular polarization.

Step 4: Compare with the given field.

Our phasor is exactly $E_0(\hat{x} - j\hat{y})$, matching the right-handed case term for term.

Step 5: Cross check using the real-time snapshot.
At $t = 0$, $z = 0$: $\vec{E} = E_0\hat{x}$. A short time later the field has swung toward $+\hat{y}$, consistent with the right-handed rotation sense identified from the phasor.

$$\boxed{\text{Right-handed circularly polarized}}$$

Quick Tip: Since $E_x = E_0 \cos\phi$ and $E_y = E_0 \sin\phi$ have equal amplitude and a 90 degree phase difference, the wave is circularly polarized; use the right-hand rule along the propagation direction to fix the handedness.

• • •

25. Consider a p-n junction diode when it is forward biased with 2 V.

Which of the following is/are the correct magnitude(s) of the energy difference between quasi Fermi-levels, E_{fn} in the n-side and E_{fp} in the p-side?

(A) 2 eV

(B) 1 eV

(C) 2 V

(D) 1 V

Correct Answer: (A) 2 eV

SOLUTION:

Step 1: Picture what happens under forward bias.
At zero bias, electrons and holes share one Fermi level all through the diode. Push the diode into forward bias and the picture splits into two levels: E_{fn} describing the electrons and E_{fp} describing the holes.

Step 2: Recall the shortcut result for a forward biased diode.

A well known consequence of forward biasing a p-n junction is that the product np inside the depletion layer rises above its equilibrium value n_i^2 by a factor $\exp(qV/kT)$:

$$np = n_i^2\, e^{qV/kT}$$

Step 3: Read this in terms of the two Fermi levels.

Writing n and p with their own quasi-Fermi levels always gives $np = n_i^2 \exp\left((E_{fn} - E_{fp})/kT\right)$. Matching the exponents of the two expressions for np tells us directly that

$$E_{fn} - E_{fp} = qV$$

Step 4: Plug in numbers.

The bias is $V = 2$ V, so the split is $q(2\text{ V}) = 2$ eV. This is an energy, so eV is the natural unit here, not V.

Step 5: Rule out the rest.

1 eV is off by a factor of two. 2 V and 1 V both quote a voltage where an energy is asked for, so both are wrong even before checking the number.

$$\boxed{2\text{ eV}}$$

Quick Tip: The split between quasi-Fermi levels equals the applied voltage converted to energy: $E_{fn} - E_{fp} = qV$.

• • •

26. Consider the matrix $M = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ -1 & a & b \end{bmatrix}$.

Which of the following options is/are TRUE if $\det(M) \neq 0$?

(A) $a = -\frac{1}{2}$ and $b = -\frac{1}{2}$

(B) $a = \frac{1}{2}$ and $b = \frac{1}{2}$

(C) $a = -3$ and $b = 0$

(D) $a = \frac{1}{2}$ and $b = -3$

Correct Answer: (B) $a = \frac{1}{2}$ and $b = \frac{1}{2}$; (D) $a = \frac{1}{2}$ and $b = -3$

SOLUTION:

Step 1: Use the diagonal (Sarrus) rule for a 3×3 determinant.

For $M = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 0 \\ -1 & a & b \end{bmatrix}$, Sarrus' rule gives

$$\det(M) = 2(3)(b) + 1(0)(-1) + 1(1)(a) - 1(3)(-1) - 2(0)(a) - 1(1)(b)$$

Step 2: Simplify term by term.

$$\det(M) = 6b + 0 + a + 3 - 0 - b$$

$$\det(M) = 5b + a + 3$$

This matches the cofactor result, as it must.

Step 3: Test each candidate pair directly.

Plug each option's (a, b) into $5b + a + 3$ and check whether it survives as nonzero.

Option A: $5(-0.5) + (-0.5) + 3 = 0$, killed.

Option B: $5(0.5) + 0.5 + 3 = 6$, survives.

Option C: $5(0) + (-3) + 3 = 0$, killed.

Option D: $5(-3) + 0.5 + 3 = -11.5$, survives.

Step 4: Read off the answer.

Only options B and D keep the matrix nonsingular.

$$\boxed{a = \frac{1}{2}, b = \frac{1}{2} \text{ and } a = \frac{1}{2}, b = -3}$$

Quick Tip: Expand $\det(M)$ in terms of a and b, then check which given pairs make it nonzero.

• • •

27. A binary ripple counter is designed to count $(0)_{10}$ to $(64)_{10}$.

Which of the following is/are the number of flip-flops required to design the counter?

(A) 6

(B) 7

(C) 4

(D) 5

Correct Answer: (B) 7

SOLUTION:

Step 1: Count how many codes are needed.

Counting from 0 to 64 needs 65 different codes, since both end points are included.

Step 2: Match this to binary capacity.

With n flip-flops a ripple counter can show 2^n codes. We need the smallest n with $2^n \geq 65$.

Step 3: Test powers of two.

$2^5 = 32$, $2^6 = 64$, $2^7 = 128$. The first power that reaches at least 65 is $2^7 = 128$, since $2^6 = 64$ falls one short of the 65 codes we need.

Step 4: Conclude.

So the counter needs 7 flip-flops, and the smaller options 4, 5, and 6 all under count the required states.

$$\boxed{7}$$

Quick Tip: You need $2^n \geq 65$ states to count from 0 to 64; find the smallest such n.

• • •

28. Which option(s) represents/represent the dielectric loss tangent of a substrate?

(A) Ratio of the real to imaginary parts of the total displacement current

(B) $(\omega\epsilon'' + \sigma) / (\omega\epsilon')$

(C) Ratio of the electric susceptibility to permittivity

(D) Ratio of the polarization vector $\vec{P}$ to the displacement vector $\vec{D}$

Correct Answer: (A) Ratio of the real to imaginary parts of the total displacement current ; (B) $(\omega\epsilon'' + \sigma) / (\omega\epsilon')$

SOLUTION:

Step 1: Start from the constitutive relation with loss.

Write the permittivity as $\epsilon = \epsilon' - j\epsilon''$ and add a real conductivity σ for ohmic loss. The full current density becomes $J = (\sigma + j\omega\epsilon)E$.

Step 2: Split into real and reactive pieces.

$$J = \sigma E + j\omega\epsilon' E + \omega\epsilon'' E = (\sigma + \omega\epsilon'')E + j\omega\epsilon' E$$

The first bracket is in phase with E, this is where power is actually used up. The second is 90° out of phase, this part just stores and returns energy.

Step 3: Define the loss tangent as this ratio.

By definition, loss tangent is in-phase current over out-of-phase current, or real over imaginary:

$$\tan\delta = \frac{\sigma + \omega\epsilon''}{\omega\epsilon'}$$

This matches both statement (A), the ratio of real to imaginary parts of the total current, and statement (B), the algebraic form $(\omega\epsilon'' + \sigma)/(\omega\epsilon')$.

Step 4: Test the remaining options.
Susceptibility to permittivity (χ/ϵ) and polarization to displacement ($|P|/|D|$) are both purely real ratios built from magnitudes at a single instant. Neither carries the 90° phase lag that defines a loss, so neither is the loss tangent.

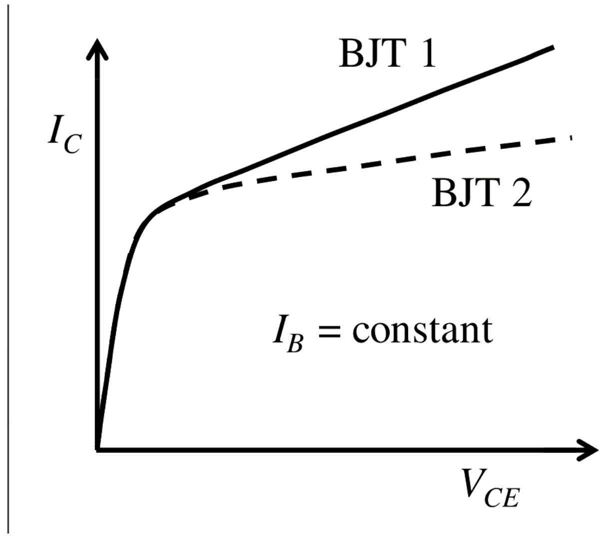
Step 5: Conclude.

$$\boxed{\text{options (A) and (B)}}$$

Quick Tip: Loss tangent compares the in-phase (lossy) current to the out-of-phase (reactive) current: $\tan\delta = (\omega\epsilon'' + \sigma)/(\omega\epsilon')$.

• • •

29. Figure shows the output characteristics of two different Bipolar Junction Transistors (BJT), BJT 1 with magnitude of Early voltage $|V_{A1}|$, and BJT 2 with magnitude of Early voltage $|V_{A2}|$.



Which of the following options is/are correct regarding the Early voltages?

(A) $|V_{A1}| > |V_{A2}|$

(B) $|V_{A1}|$ is infinitely large

(C) $|V_{A1}| < |V_{A2}|$

(D) $|V_{A2}|$ is finite

Correct Answer: (C) $|V_{A1}| < |V_{A2}|$; (D) $|V_{A2}|$ is finite

SOLUTION:

Step 1: Recall the geometric picture of the Early effect.

Draw a line through the sloped part of a BJT's I_C-V_{CE} curve at fixed I_B and extend it backward past the origin; it hits the V_{CE} axis at $-V_A$. A curve that climbs fast has a small V_A. A curve that is almost level has a large V_A.

Step 2: Compare the steepness of the two curves.

In the figure, BJT 1's solid curve climbs clearly as V_{CE} grows. BJT 2's dashed curve barely climbs at all over the same span. So BJT 1 is the steep one and BJT 2 is the flat one.

Step 3: Translate steepness into Early voltage size.

Steep means small V_A, so $|V_{A1}|$ is the smaller of the two. Flat means large V_A, so $|V_{A2}|$ is the bigger one. This gives $|V_{A1}| < |V_{A2}|$, matching option C and ruling out option A.

Step 4: Rule out an infinite Early voltage for BJT 1.

A truly infinite V_{A1} would need the curve to be dead flat, with zero slope. BJT 1's curve is visibly slanted, so V_{A1} is some finite, moderate number, not infinite. This kills option B.

Step 5: Confirm BJT 2's Early voltage is still finite.

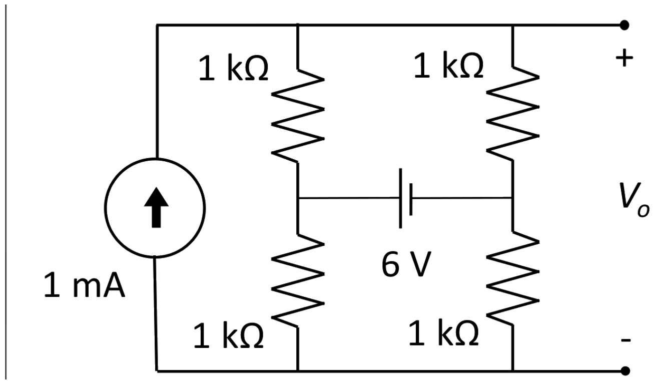
BJT 2 looks nearly flat, but a real device always has some Early effect, so its output resistance, however large, is never infinite. That keeps $|V_{A2}|$ a large but finite number, so option D holds.

Step 6: Conclude.

$$\boxed{|V_{A1}| < |V_{A2}| \text{ and } |V_{A2}| \text{ is finite}}$$

Quick Tip: Extend each curve's straight part back to the V_{CE} axis; a flatter curve corresponds to a larger, but still finite, Early voltage.

30. The output voltage V_o (in Volt) for the network given in the Figure is (rounded off to two decimal places).



Correct Answer: 1.00

SOLUTION:

Step 1: Set up node voltages directly.

Take the bottom rail B as the reference, $V_B = 0$. Let $V_T = t$ be the top rail (this is also V_o since the output is open), and let m_1, m_2 be the two middle nodes joined by the 6 V source.

Step 2: Write KCL at node T.

All 1 mA from the current source arrives at T and only leaves through the two 1 kΩ resistors (the output terminal draws nothing):

$$\frac{t - m_1}{1} + \frac{t - m_2}{1} = 1 \implies 2t - m_1 - m_2 = 1$$

Step 3: Write KCL at node B.

The same 1 mA that entered at T must leave at B through the two bottom resistors:

$$\frac{m_1 - 0}{1} + \frac{m_2 - 0}{1} = 1 \implies m_1 + m_2 = 1$$

Step 4: Combine the two equations.

Substituting $m_1 + m_2 = 1$ into the equation from Step 2:

$$2t - 1 = 1 \implies t = 1$$

Notice this did not need the value or the polarity of the 6 V source at all, since m_1 and m_2 only ever appear as the sum $m_1 + m_2$ in these two equations.

Step 5: Read off V_o.

Since $V_o = V_T - V_B = t - 0 = t$, we get

$$V_o = 1\text{ V}$$

Step 6: Sanity check with the bridge symmetry.

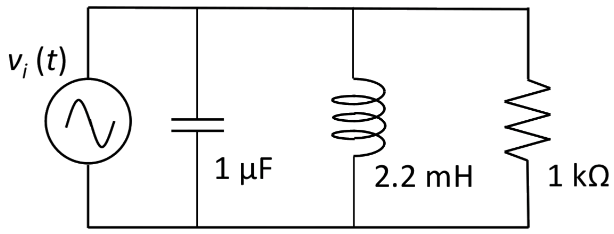
This makes sense: the network is a balanced bridge of four equal 1 kΩ arms, so a voltage source sitting across the bridge's middle branch cannot push any net current into or out of the T and B rails, it only changes how the 1 mA splits between the left and right arms.

$$\boxed{V_o = 1.00\text{ V}}$$

Quick Tip: The bridge of four equal 1 kΩ resistors is balanced, so the 6 V source does not affect V_o; only the current source and the 1 kΩ equivalent resistance matter.

• • •

31. Consider the circuit shown in the Figure, where the input $v_i(t)$ is in Volt.



$v_i\,(t) = 12\,\sin(\omega t)$

The average power (in mW) dissipated in the load resistance of 1 kΩ at the resonant frequency is (rounded off to two decimal places).

Correct Answer: 72.00

SOLUTION:

Step 1: Identify what fixes the resistor's voltage.

$v_i(t)$ is an ideal voltage source with the capacitor, inductor and resistor all hanging directly off it in parallel. An ideal voltage source forces its own voltage across everything connected straight to its terminals, so the resistor always has $v_i(t)$ across it, at any frequency, resonant or not.

Step 2: Pull out the amplitude.

From $v_i(t) = 12\sin(\omega t)$, the peak voltage is $V_m = 12$ V.

Step 3: Get the rms value.

For any sine wave, $V_{rms} = V_m/\sqrt{2}$, so here

$$V_{rms} = \frac{12}{\sqrt{2}} = 6\sqrt{2}\,\text{V}$$

Step 4: Apply the power formula for a resistor.

$$P = \frac{V_{rms}^2}{R} = \frac{(6\sqrt{2})^2}{1000} = \frac{72}{1000}\ \text{W}$$

Step 5: Convert to milliwatts.

$$P = 0.072\ \text{W} = 72\ \text{mW}$$

Step 6: Note why the $1\ \mu$F and 2.2 mH values were a distraction here. They would matter if we needed the current drawn from the source, since at resonance the tank's impedance peaks and the source current is smallest. But the question only asks for power in the resistor, and that is set purely by the source amplitude and R.

$$\boxed{P_{avg} = 72.00\ \text{mW}}$$

Quick Tip: The resistor sits directly across the ideal source, so its power depends only on V_m and R: $P = \dfrac{(V_m/\sqrt{2})^2}{R}$.

32. A wireless digital transmission scheme is using 16-QAM over an additive white Gaussian noise channel and a maximum-likelihood receiver. Consider the information bit rate from source to be 4×10^6 bits per second.

The minimum transmission bandwidth (in MHz) of the modulated signal necessary for optimum recovery of information at the receiver is (rounded off to two decimal places).

Correct Answer: 1.00

SOLUTION:

Step 1: Work out bits per symbol.
A 16-QAM constellation has 16 points, and since $2^4 = 16$, every symbol transmitted carries 4 bits of information.

Step 2: Convert the bit rate into a symbol rate.
Given the bit rate $R_b = 4 \times 10^6$ bits/s, dividing by the 4 bits carried per symbol gives the baud rate:

$$R_s = \frac{4 \times 10^6}{4} = 10^6 \text{ symbols/s}$$

Step 3: Recall the Nyquist minimum bandwidth rule for QAM.
For an ideal (Nyquist, zero-ISI) pulse shape, a passband QAM signal needs a transmission bandwidth equal to the symbol rate, $B_{min} = R_s$, since the in-phase and quadrature channels share the same passband through quadrature multiplexing instead of doubling it.

Step 4: Plug in the numbers.

$$B_{min} = 10^6 \text{ Hz} = 1 \text{ MHz}$$

Step 5: Conclude.

$$\boxed{B_{min} = 1.00 \text{ MHz}}$$

Quick Tip: Find bits per symbol from $\log_2(16)$, convert bit rate to symbol rate, then use $B_{min} = R_s$ for QAM.

• • •

33. The cutoff frequency (in GHz) for the dominant TE_{10} mode of an air-filled rectangular waveguide of inner dimension 0.28 inch × 0.14 inch is . (rounded off to two decimal places)

Correct Answer: 21.09

SOLUTION:

Step 1: Note what decides the cutoff for TE10.
In a rectangular guide, the TE10 mode is the one with the lowest cutoff, and its cutoff frequency uses only the wider side a of the cross section, not the narrower side b. The formula is $f_c = \dfrac{c}{2a}$.

Step 2: Pick out the wider side.
The cross section is 0.28 inch by 0.14 inch, so $a = 0.28$ inch is the wide side.

Step 3: Change inches to metres.
One inch is 0.0254 m, so

$$a = 0.28 \times 0.0254 = 7.112 \times 10^{-3} \text{ m}$$

Step 4: Plug numbers into the formula.

Taking $c = 3 \times 10^8$ m/s,

$$f_c = \frac{3 \times 10^8}{2 \times 7.112 \times 10^{-3}} = \frac{3 \times 10^8}{1.4224 \times 10^{-2}}$$

Step 5: Simplify the division.

$$f_c \approx 2.109 \times 10^{10} \text{ Hz} = 21.09 \text{ GHz}$$

$$\boxed{21.09 \text{ GHz}}$$

Quick Tip: For TE10 mode the cutoff frequency depends only on the broad wall dimension a, through fc = c/(2a).

• • •

34. For a lossless passive two-port network, $|S_{11}|$ and $|S_{21}|$ intersect at -3 dB.

For a lossy passive two-port network, $|S_{11}|$ and $|S_{21}|$ intersect at -4 dB.

The percentage of power dissipated in the lossy network at the intersection frequency is .

(rounded off to two decimal places)

Correct Answer: 20.38

SOLUTION:

Step 1: Set up the power balance.

At any port pair, the power that goes in either comes back as reflection, passes through as transmission, or is lost inside the network. So

$$1 = |S_{11}|^2 + |S_{21}|^2 + P_{diss}$$

where P_{diss} is the dissipated fraction.

Step 2: Turn the given dB numbers into plain ratios.

A value of x dB means the power ratio is $10^{x/10}$. For the lossy case both curves meet at -4 dB, so

$$|S_{11}|^2 = |S_{21}|^2 = 10^{-4/10} = 10^{-0.4}$$

Step 3: Work out the number.

$$10^{-0.4} \approx 0.3981$$

Step 4: Add the two equal fractions.

$$|S_{11}|^2 + |S_{21}|^2 = 2(0.3981) = 0.7962$$

Step 5: Get the dissipated share.

$$P_{diss} = 1 - 0.7962 = 0.2038$$

As a check, the lossless curve meets at -3 dB, giving $2 \times 10^{-0.3} = 2 \times 0.5012 \approx 1$, so nothing is lost there, exactly as expected for a lossless network.

Step 6: State the percentage.

$$\boxed{20.38\%}$$

Quick Tip: Use power conservation: fraction dissipated = 1 minus |S11|^2 minus |S21|^2, after converting the given dB value to a ratio.

• • •

35. The negative edge triggered JK flip-flop shown has J and K inputs tied to Logic High, and a square wave of 10 cycles/second is applied to its clock (C) input.
The frequency of the output Q (in cycles/second) is .
(rounded off to two decimal places)

Correct Answer: 5.00

SOLUTION:

Step 1: Read what J = K = 1 does.
Tying both J and K to logic high puts the flip-flop into toggle mode, so the state flips at every active clock edge instead of holding or loading a new value.

Step 2: Track the state over one clock period.
Say Q starts at 0. At the first falling edge it flips to 1. At the next falling

edge, one full clock period later, it flips back to 0. So it takes two clock periods, that is two falling edges, to bring Q back to where it started.

Step 3: Turn this into a frequency ratio.

Completing one full cycle of Q needs two periods of the clock, so the period of Q is twice the clock period, which means the frequency of Q is half the clock frequency:

$$f_Q = \frac{f_{clock}}{2}$$

Step 4: Plug in the numbers.

$$f_Q = \frac{10}{2} = 5 \text{ cycles/second}$$

$$\boxed{5.00 \text{ cycles/second}}$$

Quick Tip: A JK flip-flop with J = K = 1 toggles once per active clock edge, so its output frequency is half the clock frequency.

• • •

36. Consider the two series, S_A and S_B, where

$$S_A = \sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

$$S_B = 1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{64} + \frac{1}{128} + \frac{1}{512} + \cdots$$

Which of the following statements is correct for the two given series?

(A) Both S_A and S_B converge.

(B) Neither S_A nor S_B converges.

(C) S_A converges but S_B does not converge.

(D) S_B converges but S_A does not converge.

Correct Answer: (A) Both S_A and S_B converge.

SOLUTION:

Step 1: Apply the root test to S_A.

For $S_A = \sum \frac{n^2}{2^n}$, look at $\left(\frac{n^2}{2^n}\right)^{1/n} = \frac{n^{2/n}}{2}$. As $n \to \infty$, $n^{2/n} \to 1$, so the limit is $\frac{1}{2}$, which is below 1. By the root test, S_A converges.

Step 2: Group the terms of S_B in pairs.

Pair up the terms of S_B two at a time:

$$\left(1 + \frac{1}{2}\right) + \left(\frac{1}{8} + \frac{1}{16}\right) + \left(\frac{1}{64} + \frac{1}{128}\right) + \cdots$$

The first pair sums to $\frac{3}{2}$, the second pair sums to $\frac{3}{16}$, the third pair sums to $\frac{3}{128}$.

Step 3: Spot the ratio between pair sums.

$$\frac{3/16}{3/2} = \frac{1}{8}, \qquad \frac{3/128}{3/16} = \frac{1}{8}$$

So the pair sums form a geometric series with first term $\frac{3}{2}$ and ratio $\frac{1}{8}$.

Step 4: Sum this geometric series.

$$S_B = \frac{3/2}{1 - 1/8} = \frac{3/2}{7/8} = \frac{3}{2} \times \frac{8}{7} = \frac{12}{7}$$

This is a finite number, so S_B converges too.

Step 5: Conclude.

Since both series settle to finite values, 6 for S_A and $\frac{12}{7}$ for S_B, the correct statement is that both series converge.

$$\boxed{\text{Both } S_A \text{ and } S_B \text{ converge.}}$$

Quick Tip: Check S_A with the ratio test, then split S_B into two interleaved geometric series with common ratio 1/8 each.

• • •

37. The continuous time signal $x(t)$ is real, periodic with period T, and satisfies the Dirichlet conditions.
The Fourier series representation of $x(t)$ is

$$x(t) = \sum_{n=-\infty}^{\infty} a_n e^{j\left(\frac{2\pi nt}{T}\right)}$$

and $x(t)$ satisfies the following:

$$x\left(t - \frac{T}{2}\right) = -x(t).$$

For any integer m, which of the following options is correct?

(A) $a_{2m} = 0$

(B) $a_{2m} = 1$

(C) $a_{2m} = a_{2m+1}$

(D) $a_{2m} = -1$

Correct Answer: (A) $a_{2m} = 0$

SOLUTION:

Step 1: Name the symmetry.
The rule $x(t - T/2) = -x(t)$ says the wave repeats itself upside down after half a period. This is called half-wave symmetry, and it is well known to kill off all even harmonics in a Fourier series, including the average value a_0.

Step 2: Prove it directly from the series.
Shift the exponential series by $T/2$:

$$x\left(t-\frac{T}{2}\right)=\sum_n a_n e^{j\frac{2\pi nt}{T}}\, e^{-j\pi n}=\sum_n a_n(-1)^n e^{j\frac{2\pi nt}{T}}$$

because $e^{-j\pi n}=(-1)^n$ for integer n.

Step 3: Match this to $-x(t)$.

Since the two expansions must agree coefficient by coefficient,

$$a_n(-1)^n=-a_n \quad \text{for every } n$$

Step 4: Split by parity of n.

If n is even, say $n=2m$, then $(-1)^n=1$ and the equation becomes $a_{2m}=-a_{2m}$, forcing $a_{2m}=0$. If n is odd, $(-1)^n=-1$ and the equation becomes $-a_n=-a_n$, which holds automatically and tells us nothing new about odd coefficients.

Step 5: Read off the answer.

So every even-indexed coefficient must vanish, while odd-indexed coefficients are free.

$$\boxed{a_{2m}=0}$$

Quick Tip: Substitute t - T/2 into the Fourier series and compare coefficients with -x(t); only the even-indexed coefficients get forced to zero.

• • •

38. Let X, N, Y and Z be random variables. The variables X and N are independent of each other. X is uniformly distributed between -1 and 1; N follows Normal distribution with zero mean and unity variance.
Y and Z are defined as $Y = X + N$ and $Z = X^2 + N$.
Which of the following pairs represents the values of correlation between X and Y, and that between X and Z?

(A) $\frac{1}{3}$ and 0
(B) $\frac{1}{3}$ and $\frac{1}{9}$
(C) $\frac{1}{3}$ and $\frac{1}{3}$
(D) 1 and 0

Correct Answer: (A) $\frac{1}{3}$ and 0

SOLUTION:

Step 1: Write correlation as a covariance.
For random variables U, V, take correlation to mean $E[UV] - E[U]E[V]$. Both X and N have zero mean here, which simplifies things a lot.

Step 2: Collect the moments of X we need.
X is uniform on $[-1, 1]$. Its mean is 0. Its variance is

$$\mathrm{Var}(X) = \frac{(1-(-1))^2}{12} = \frac{1}{3}$$

Because the density is an even function, all odd powers of X average to zero, so $E[X^3] = 0$ as well.

Step 3: Handle the noise term N.

N has zero mean and is independent of X, so any product like $E[XN]$ or $E[X^2N]$ factors and gives zero.

Step 4: Correlate X with Y.

$$E[XY] = E[X(X+N)] = E[X^2] + E[XN] = \frac{1}{3} + 0 = \frac{1}{3}$$

Since $E[X]E[Y] = 0$, the correlation between X and Y is $\frac{1}{3}$.

Step 5: Correlate X with Z.

$$E[XZ] = E[X(X^2+N)] = E[X^3] + E[XN] = 0 + 0 = 0$$

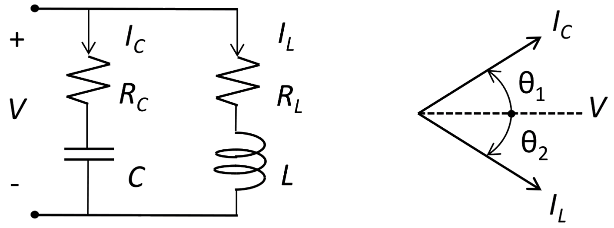
So the correlation between X and Z is 0.

Step 6: State the pair.

$$\boxed{\frac{1}{3} \text{ and } 0}$$

Quick Tip: Use Corr(U,V) = E[UV] - E[U]E[V]; the odd moment E[X^3] of a symmetric uniform variable is zero.

39. In the given circuit, $L = 1\ \mu H$ and $C = 1\ \mu F$. The phasor diagram for I_C and I_L is also shown. Assume that the phase $(\theta_1 + \theta_2)$ is 90° at a frequency of 159.15 kHz.



Among the following options, what is the nearest integer value of $R_C \times R_L$?

(A) 0

(B) 1

(C) 2

(D) 10

Correct Answer: (B) 1

SOLUTION:

Step 1: Read off the two branches.

Looking at the network, current I_C flows through R_C in series with C, and current I_L flows through R_L in series with L, with both branches sitting across the same source V.

Step 2: Write each current's phase relative to V.

For the capacitive branch, the impedance is $R_C - \dfrac{j}{\omega C}$, a negative imaginary part, so I_C sits ahead of V by angle θ_1, with

$$\tan\theta_1 = \frac{1/(\omega C)}{R_C}$$

For the inductive branch, the impedance is $R_L + j\omega L$, a positive imaginary part, so I_L sits behind V by angle θ_2, with

$$\tan\theta_2 = \frac{\omega L}{R_L}$$

Step 3: Use the 90 degree fact as a product rule.

Two positive angles that add to 90° are complementary, and for complementary angles $\tan\theta_1 \tan\theta_2 = 1$. So

$$\left(\frac{1}{\omega C R_C}\right)\left(\frac{\omega L}{R_L}\right) = 1$$

Step 4: Cancel omega and rearrange.

$$\frac{L}{C\,R_C R_L} = 1 \quad \Rightarrow \quad R_C R_L = \frac{L}{C}$$

This is frequency independent, so the exact value 159.15 kHz given in the problem is only there to assure us that such a frequency exists, not to be plugged into the formula.

Step 5: Put in L and C.

$$R_C R_L = \frac{1\,\mu\mathrm{H}}{1\,\mu\mathrm{F}} = \frac{1\times10^{-6}}{1\times10^{-6}} = 1$$

$$\boxed{1}$$

Quick Tip: Write tan(theta1) and tan(theta2) for the two branch currents, use theta1+theta2=90 degrees, and simplify: the result is RC times RL = L/C.

• • •

40. For the control system shown in the Figure, the transfer function of a plant,

$$G(s) = \frac{1}{(s+1)(s+2)}$$

is connected in cascade with a compensator

$$C(s) = K(s+\alpha),$$

where K and α are positive real valued constants. The compensator and plant are placed in the forward path of a unity negative feedback system with input $R(s)$ and output $Y(s)$.
Which of the following pairs (K, α) represent the correct values for the closed loop system to have poles at $\left(-3 \pm j\sqrt{5}\right)$?

(A) $2, 3$

(B) $3, 4$

(C) $2, 4$

(D) $3, 3$

Correct Answer: (B) $3, 4$

SOLUTION:

Step 1: Get the closed loop denominator.

With plant $G(s) = \dfrac{1}{(s+1)(s+2)}$ and compensator $C(s) = K(s+\alpha)$ in a unity feedback loop, the characteristic polynomial is

$$(s+1)(s+2) + K(s+\alpha) = 0$$

Step 2: Build the quadratic directly from the desired poles.

If the poles must be $-3 \pm j\sqrt{5}$, the quadratic with these roots is

$$(s-(-3+j\sqrt{5}))(s-(-3-j\sqrt{5})) = (s+3)^2 + 5 = s^2 + 6s + 9 + 5 = s^2 + 6s + 14$$

Step 3: Expand the actual characteristic equation.

$$(s+1)(s+2) + K(s+\alpha) = s^2 + (3+K)s + (2+K\alpha)$$

Step 4: Match coefficient by coefficient.

Comparing with $s^2 + 6s + 14$:

$$3 + K = 6 \;\Rightarrow\; K = 3$$

$$2 + K\alpha = 14 \;\Rightarrow\; K\alpha = 12 \;\Rightarrow\; \alpha = \frac{12}{3} = 4$$

Step 5: Conclude.

$$\boxed{(K,\alpha) = (3,4)}$$

Quick Tip: Form the characteristic equation (s+1)(s+2)+K(s+alpha)=0 and match its sum and product of roots to the given complex pole pair.

• • •

41. The state and output equations for a control system are:

$$\dot{x} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$

$$y = [1.5 \quad 0.625]\, x$$

Which of the following expressions correctly represents the transfer function $\frac{Y(s)}{U(s)}$ of the system with zero initial conditions?

(A) $\dfrac{3s}{s^2 + 4s - 6}$

(B) $\dfrac{3s + 5}{s^2 + 4s - 6}$

(C) $\dfrac{3s + 5}{s^2 + 4s + 6}$

(D) $\dfrac{3s}{s^2 + 4s + 6}$

Correct Answer: (C) $\dfrac{3s + 5}{s^2 + 4s + 6}$

SOLUTION:

Step 1: Write out the two state equations.

Let x_1 and x_2 be the two state variables. From the given matrices, x_1' $= -4x_1 - 1.5x_2 + 2u$ and $x_2' = 4x_1$. The output is $y = 1.5x_1 + 0.625x_2$.

Step 2: Eliminate x_1 using the second equation.

Since $x_2' = 4x_1$, we get $x_1 = \dfrac{x_2'}{4}$, and differentiating, $x_1' = \dfrac{x_2''}{4}$.

Step 3: Substitute into the first equation.

$$\frac{x_2''}{4} = -4\left(\frac{x_2'}{4}\right) - 1.5x_2 + 2u$$

Multiply through by 4:

$$x_2'' = -4x_2' - 6x_2 + 8u$$

which rearranges to a single second order equation

$$x_2'' + 4x_2' + 6x_2 = 8u$$

Step 4: Write the output in terms of x_2.

Using $x_1 = x_2'/4$, the output becomes

$$y = 1.5\left(\frac{x_2'}{4}\right) + 0.625x_2 = 0.375x_2' + 0.625x_2$$

Step 5: Take the Laplace transform of both equations with zero initial conditions.

$$X_2(s)(s^2 + 4s + 6) = 8U(s) \;\Rightarrow\; X_2(s) = \frac{8U(s)}{s^2 + 4s + 6}$$

$$Y(s) = (0.375s + 0.625)X_2(s)$$

Step 6: Combine the two results.

$$Y(s) = (0.375s + 0.625) \cdot \frac{8U(s)}{s^2 + 4s + 6} = \frac{3s + 5}{s^2 + 4s + 6} U(s)$$

Step 7: Read off the transfer function.

$$\frac{Y(s)}{U(s)} = \frac{3s + 5}{s^2 + 4s + 6}$$

This matches the matrix method and confirms option (C).

$$\boxed{\frac{3s + 5}{s^2 + 4s + 6}}$$

Quick Tip: Use $\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B$ and be careful with the sign inside the determinant of $sI - A$.

• • •

42. The address of the first location of a 256 kilo byte (KB) memory is $(2500)_H$.

Choose the correct address of the last location of the memory.

(A) $(2FFF)_H$

(B) $(124FF)_H$

(C) $(424FF)_H$

(D) $(324FF)_H$

Correct Answer: (C) $(424FF)_H$

SOLUTION:

Step 1: Convert the starting address to decimal.
$(2500)_H = 2 \times 16^3 + 5 \times 16^2 + 0 \times 16 + 0 = 8192 + 1280 = 9472$ in decimal.

Step 2: Find the memory size in decimal.
$256\text{ KB} = 256 \times 1024 = 262144$ bytes.

Step 3: Find the last address in decimal.

$$9472 + 262144 - 1 = 271615$$

Step 4: Convert 271615 back to hexadecimal.
Dividing repeatedly by powers of 16: $271615 = 4 \times 65536 + 9471$, then $9471 = 2 \times 4096 + 1279$, then $1279 = 4 \times 256 + 255$, and $255 = 15 \times 16 + 15$. Reading the digits gives $4, 2, 4, 15, 15$, that is, $(424FF)_H$ since 15 in hex is F.

Step 5: Conclude.

$$\boxed{(424FF)_H}$$

This confirms the same answer reached through pure hex addition.

Quick Tip: Last address = first address + total bytes - 1, all worked out in hexadecimal.

• • •

43. Consider a real signal $x(t)$, $-\infty < t < \infty$, such that $x(t) = 0$ for $t < 0$, $x(t) = 2$ for $0 \le t < 1$ and $x(t) = 0$ for $t \ge 1$.

Let $E[x(t)] = \int_{-\infty}^{\infty} [x(t)]^2 \, dt$.

Which of the following options correctly represents the ratio, $E[x(t)]/E[3 x(-3t + 5)]$?

(A) 3

(B) 1

(C) 1/3

(D) 1/9

Correct Answer: (C) 1/3

SOLUTION:

Step 1: Find where $x(t) = 2$.

By definition $x(t) = 2$ only for $0 \le t < 1$, and $x(t) = 0$ everywhere else.

Step 2: Find the interval where $x(-3t + 5)$ is non-zero.

We need $0 \le -3t + 5 < 1$. Subtracting 5 throughout gives $-5 \le -3t < -4$. Dividing by -3 flips both inequalities:

$$\frac{5}{3} \ge t > \frac{4}{3}$$

So $x(-3t + 5) = 2$ for t in the interval $\left(\frac{4}{3}, \frac{5}{3}\right]$, whose length is $\frac{1}{3}$.

Step 3: Find $w(t) = 3x(-3t + 5)$ on this interval.

On $\left(\frac{4}{3}, \frac{5}{3}\right]$, $w(t) = 3(2) = 6$, so $[w(t)]^2 = 36$. Outside this interval $w(t) = 0$.

Step 4: Integrate to get $E[w]$.

$$E[w] = \int_{4/3}^{5/3} 36\,dt = 36 \times \frac{1}{3} = 12$$

Step 5: Recompute $E[x]$ directly.

$$E[x] = \int_{0}^{1} (2)^2\,dt = 4$$

Step 6: Form the ratio.

$$\frac{E[x]}{E[w]} = \frac{4}{12} = \frac{1}{3}$$

$$\boxed{\frac{1}{3}}$$

This matches the result from the general scaling rule.

Quick Tip: For $w(t) = A\,x(Bt + C)$, the energy scales as $E[w] = \frac{A^2}{|B|}E[x]$.

44. A QPSK modulated signal from an additive white Gaussian noise (AWGN) channel is received with an $E_b/N_o = 8.4$ dB at the input of a coherent QPSK demodulator. A maximum likelihood reception method is used in the demodulator.
Assume the complementary error function

$$erfc(u) \cong \left[1/(u\sqrt{\pi})\right]\exp(-u^2).$$

Which is the nearest bit error rate (BER) at the output of the demodulator?

(A) 10^{-3}

(B) 10^{-4}

(C) 10^{-5}

(D) 10^{-6}

Correct Answer: (B) 10^{-4}

SOLUTION:

Step 1: Convert dB to a ratio using natural logarithms.

$$\frac{E_b}{N_o} = 10^{0.84} = e^{0.84\ln 10}$$

Since $\ln 10 \approx 2.3026$, the exponent is $0.84 \times 2.3026 \approx 1.934$, so

$$\frac{E_b}{N_o} \approx e^{1.934} \approx 6.92$$

Step 2: Write the QPSK error probability using the Q function first.
For coherent QPSK, $P_b = Q(\sqrt{2E_b/N_o})$, and using $Q(x) = \frac{1}{2}erfc(x/\sqrt{2})$ this becomes

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{2E_b/N_o}{2}}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_o}}\right)$$

which is the same expression used before, confirming the formula.

Step 3: Evaluate the argument.

$$u = \sqrt{6.92} \approx 2.63, \qquad u^2 \approx 6.92$$

Step 4: Evaluate $\exp(-u^2)$ using the split $6.92 = 7 - 0.08$.

$$\exp(-6.92) = \exp(-7)\exp(0.08) \approx (9.12 \times 10^{-4})(1.083) \approx 9.88 \times 10^{-4}$$

Step 5: Divide by $u\sqrt{\pi}$.

$$u\sqrt{\pi} \approx 4.66, \qquad erfc(u) \approx \frac{9.88 \times 10^{-4}}{4.66} \approx 2.12 \times 10^{-4}$$

Step 6: Halve to get the BER.

$$P_b \approx 1.06 \times 10^{-4}$$

which rounds to

$$\boxed{10^{-4}}$$

This matches the value obtained directly from the decibel split method.

Quick Tip: QPSK behaves like two independent BPSK links, so $P_b = \frac{1}{2} erfc(\sqrt{E_b/N_o})$.

• • •

45. What is the 10's complement of $(47)_{10}$?

(A) 52

(B) 53

(C) 54

(D) 55

Correct Answer: (B) 53

SOLUTION:

Step 1: Recall the link between the 9's and 10's complement.
For any decimal number, the 10's complement is just the 9's complement plus 1.

Step 2: Find the 9's complement of 47 digit by digit.
Subtract each digit of 47 from 9: the tens digit gives $9 - 4 = 5$ and the units digit gives $9 - 7 = 2$, so the 9's complement is 52.

Step 3: Add 1 to get the 10's complement.

$$52 + 1 = 53$$

Step 4: Conclude.

$$\boxed{53}$$

This matches the direct subtraction $100 - 47$.

Quick Tip: 10's complement of an n-digit number N is $10^n - N$.

• • •

46. Consider a real baseband signal $x(t) = e^{-2t}$, for t (in seconds) ≥ 0. If 99% of the energy of $x(t)$ lies within B Hz, then which of the following options is TRUE for the value of B?

(A) $B > 1$ kHz

(B) $63/\pi$ Hz $< B < 64/\pi$ Hz

(C) $126/\pi$ Hz $< B < 128/\pi$ Hz

(D) $B < 1$ Hz

Correct Answer: (B) $63/\pi$ Hz $< B < 64/\pi$ Hz

SOLUTION:

Step 1: Recall the standard result for a one sided exponential pulse.

For $x(t) = e^{-at}u(t)$, the fraction of energy contained within B Hz is a known result:

$$\frac{E(B)}{E_{total}} = \frac{2}{\pi}\arctan\left(\frac{2\pi B}{a}\right)$$

Step 2: Plug in the decay constant.

Here $a = 2$, so

$$\frac{E(B)}{E_{total}} = \frac{2}{\pi}\arctan(\pi B)$$

which matches the expression derived directly from the Fourier transform.

Step 3: Use the complement, since 99% inside means 1% outside.

$$\frac{2}{\pi}\arctan(\pi B) = 0.99 \Rightarrow \frac{\pi}{2} - \arctan(\pi B) = \frac{\pi}{2}(1-0.99) = 0.005\pi \approx 0.0157$$

Step 4: Use the identity $\frac{\pi}{2} - \arctan(x) = \arctan(1/x)$ for $x > 0$.

$$\arctan\left(\frac{1}{\pi B}\right) \approx 0.0157$$

For a small angle, $\arctan(y) \approx y$, so

$$\frac{1}{\pi B} \approx 0.0157 \Rightarrow \pi B \approx 63.7$$

Step 5: Conclude.

$$B \approx \frac{63.7}{\pi}$$

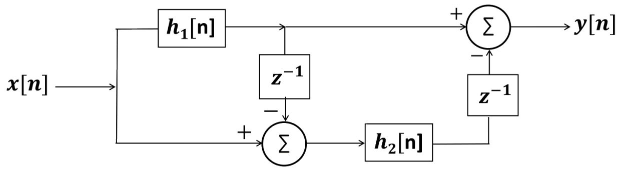
$$\boxed{\frac{63}{\pi}\text{ Hz} < B < \frac{64}{\pi}\text{ Hz}}$$

This matches the value found by direct integration.

Quick Tip: Use Parseval's theorem, integrate the energy spectral density from -B to B, and set the fraction equal to 0.99.

• • •

47. Consider the discrete time system (S) with input $x[n]$ and output $y[n]$ as shown in the figure. The two sub-systems, represented by their impulse responses $h_1[n]$ and $h_2[n]$, are linear and time invariant.



Which of the following statements is necessarily TRUE?

(A) S is causal.

(B) S is linear and time invariant.

(C) S is linear and time varying.

(D) S is non-linear.

Correct Answer: (B) S is linear and time invariant.

SOLUTION:

Step 1: Represent each branch in the z-domain.

Let $X(z)$, $Y(z)$, $H_1(z)$ and $H_2(z)$ denote the z-transforms of $x[n]$, $y[n]$, $h_1[n]$ and $h_2[n]$. Convolution in time becomes multiplication in the z-domain, and a z^{-1} block simply multiplies by z^{-1}.

Step 2: Write the upper branch.

$$A(z) = H_1(z)X(z)$$

Step 3: Write the lower branch.

$$C(z) = X(z) - z^{-1}A(z), \qquad D(z) = H_2(z)C(z)$$

Step 4: Combine at the output junction.

$$Y(z) = A(z) - z^{-1}D(z)$$

Step 5: Substitute to get a single relation between Y(z) and X(z).

$$Y(z) = H_1(z)X(z) - z^{-1}H_2(z)\left[X(z) - z^{-1}H_1(z)X(z)\right]$$

$$Y(z) = X(z)\left[H_1(z) - z^{-1}H_2(z) + z^{-2}H_1(z)H_2(z)\right]$$

Step 6: Interpret this relation.

The bracketed term is a single fixed function of z, call it $H(z)$, so $Y(z) = H(z)X(z)$ for every input, with $H(z)$ not depending on the time index n at all. A relation of the form $Y(z) = H(z)X(z)$ is exactly the z-domain signature of an LTI system, since it corresponds to $y[n] = h[n] * x[n]$ in time.

Step 7: Note what is not guaranteed.

$H(z)$ may contain positive powers of z (from the two delay terms), so the equivalent impulse response $h[n]$ need not be zero for $n < 0$, meaning causality of S is not guaranteed.

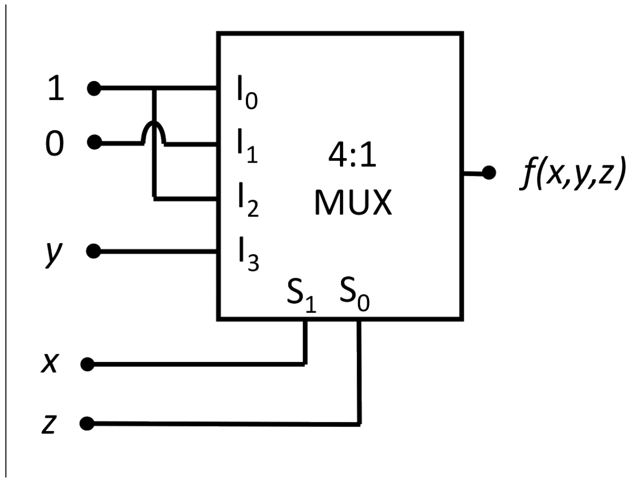
Step 8: Conclude.

$$\boxed{S \text{ is linear and time invariant}}$$

Quick Tip: Convolution, addition and a fixed delay are all linear and time invariant, so any interconnection of LTI blocks stays LTI.

• • •

48. A Boolean function, $f(x, y, z)$, with x as MSB and z as LSB, is realized by a 4:1 multiplexer (MUX) with select lines S_1 and S_0 (S_1 is MSB, S_0 is LSB) and inputs I_0, I_1, I_2, I_3 as shown in the figure.



Which of the following options is the correct expression of $f(x, y, z)$?

(A) $xz + y$

(B) $x\bar{y} + z$

(C) $xy + \bar{z}$

(D) $\bar{x}y + \bar{z}$

Correct Answer: (C) $xy + \bar{z}$

SOLUTION:

Step 1: Decide which input is chosen for each combination of x and z.

Since $S_1 = x$ and $S_0 = z$: when $x = 0, z = 0$ the MUX picks $I_0 = 1$; when $x = 0, z = 1$ it picks $I_1 = 0$; when $x = 1, z = 0$ it picks $I_2 = 1$; when $x = 1, z = 1$ it picks $I_3 = y$.

Step 2: Build the full 8-row truth table over x, y, z.

$(0,0,0) \to 1$, $(0,0,1) \to 0$, $(0,1,0) \to 1$, $(0,1,1) \to 0$, $(1,0,0) \to 1$, $(1,0,1) \to y = 0$, $(1,1,0) \to 1$, $(1,1,1) \to y = 1$.

Step 3: Compare this table against option (C), $f = xy + \bar{z}$.

For every row where $z = 0$, $\bar{z} = 1$ so $f = 1$, matching the four rows $(0,0,0)$, $(0,1,0)$, $(1,0,0)$, $(1,1,0)$. For $z = 1$, $f = xy$, which is 0 unless both $x = 1$ and $y = 1$, matching $(0,0,1) \to 0$, $(0,1,1) \to 0$, $(1,0,1) \to 0$ and $(1,1,1) \to 1$.

Step 4: Conclude.

Every row matches, so

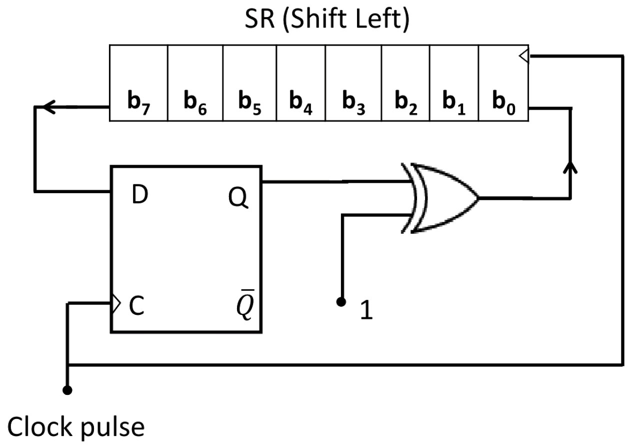
$$\boxed{f = xy + \bar{z}}$$

confirming option (C) directly from the truth table, without algebraic simplification.

Quick Tip: Write out $f = \bar{S}_1\bar{S}_0I_0 + \bar{S}_1S_0I_1 + S_1\bar{S}_0I_2 + S_1S_0I_3$ with $S_1 = x,\ S_0 = z$ and simplify.

• • •

49. A shift-left Shift Register (SR) and a D flip-flop are connected to a synchronized clock as shown in the figure. Assume that the SR and D flip-flops are initially cleared and the XOR gate has no propagation delay.



Which of the following options gives the correct binary representation $b_7b_6b_5b_4b_3b_2b_1b_0$ of the content of the shift register immediately after the 5^{th} clock transition (positive edge)?

(A) 00011111

(B) 10111111

(C) 00111111

(D) 11000011

Correct Answer: (A) 00011111

SOLUTION:

Step 1: Note what feeds what.

The XOR gate has a constant 1 on one leg, so it behaves as a plain inverter on the other leg. That means the bit going into b_0 every edge is just the flip-flop output flipped. The flip-flop itself samples whatever bit is currently sitting in b_7, the bit that is about to leave the register on that same edge.

Step 2: Build a running table.

Start everything at 0: register 00000000 and flip-flop $Q = 0$. On every edge, first copy b_7 into a "next Q" slot, then shift the whole register one place left, and finally drop $\overline{Q}$ (the OLD Q, before this edge updates it) into b_0.

Step 3: Run five edges.

Edge 1: old $Q = 0$, so b_0 gets $\overline{0} = 1$. Register becomes 00000001; new $Q = 0$ (old $b_7 = 0$).

Edge 2: old $Q = 0$, so b_0 gets 1 again. Register becomes 00000011; new $Q = 0$.

Edge 3: same pattern, register becomes 00000111; Q stays 0.

Edge 4: register becomes 00001111; Q stays 0.

Edge 5: register becomes 00011111; Q stays 0.

Step 4: Notice why Q never changes.

Since b_7 stays 0 right through the fifth edge (the block of 1s takes five edges to even reach position b_3, let alone b_7), the flip-flop keeps sampling a 0 and keeps outputting 0, so the inverter keeps injecting fresh 1s at b_0 every single edge. This is exactly why the pattern grows one 1 at a time from the right.

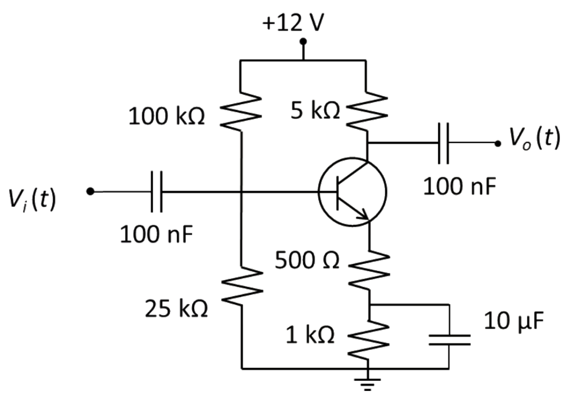
Step 5: Read off the answer.

After five edges the register reads $b_7b_6b_5b_4b_3b_2b_1b_0 = 00011111$.

$$\boxed{00011111}$$

Quick Tip: Trace the D flip-flop and XOR feedback state after each clock edge, five times.

50. A small signal source, $V_i(t) = A\cos(10^5 t) + B\sin(10^7 t)$ is applied to a BJT circuit as shown in the figure.



Assume zero source resistance, $V_{BE} = 0.7$ V, $\beta_{dc} = 99$, Early voltage = 100 V and thermal voltage = 25 mV. Effect of internal parasitic capacitances of the BJT may be neglected.
Which expression is the best approximation of the output voltage $V_o(t)$?

(A) $-9.1[A\cos(10^5 t) + B\sin(10^7 t)]$

(B) $9.1[A\cos(10^5 t) - B\sin(10^7 t)]$

(C) $-190.4[A\cos(10^5 t) + B\sin(10^7 t)]$

(D) $190.4[A\cos(10^5 t) - B\sin(10^7 t)]$

Correct Answer: (A) $-9.1[A\cos(10^5 t) + B\sin(10^7 t)]$

SOLUTION:

Step 1: Settle the operating point first.
Replace the base network by its Thevenin equivalent: $V_{th} = 12 \times \frac{25k}{125k} = 2.4$ V and $R_{th} = 100k \parallel 25k = 20k\Omega$. The full emitter leg for DC is 500 + 1000 = 1500Ω, since a capacitor carries no steady current. Solving the base-emitter loop,

$$I_B = \frac{2.4 - 0.7}{20000 + 100(1500)} = 10\ \mu A$$

so $I_C \approx 1$ mA and $I_E = 1$ mA. From these, $r_e = 25/1 = 25\Omega$ and $r_o = 100/0.001 = 100k\Omega$.

Step 2: Ask a simpler question first, is the $1k\Omega$ ever visible to the signal.

Compare its reactance to 1000Ω at the two given frequencies rather than plugging into a gain formula straight away. At $\omega = 10^5$, $X_C = 1/(10^5 \cdot 10^{-5}) = 1\Omega$; at $\omega = 10^7$, $X_C = 1/(10^7 \cdot 10^{-5}) = 0.01\Omega$. Both are two to five orders of magnitude smaller than 1000Ω, so the $10\mu F$ capacitor shorts that resistor out completely at both frequencies. Only the 500Ω resistor, which has no capacitor across it, ever appears in the AC path.

Step 3: Do the same check for the two $100nF$ coupling caps.

At $\omega = 10^5$, $X_C = 1/(10^5 \cdot 10^{-7}) = 100\Omega$; at $\omega = 10^7$, $X_C = 1/(10^7 \cdot 10^{-7}) = 1\Omega$. Since the base side always looks like several $k\Omega$ and the output has no load resistor at all, these small reactances drop no meaningful voltage at either frequency. Both signal components pass through the coupling capacitors essentially untouched.

Step 4: Now write the gain, once, for both frequencies together.

Because steps 2 and 3 show the AC circuit looks identical at 10^5 rad/s and at 10^7 rad/s, one gain expression covers both terms:

$$A_v = -\frac{R_C \parallel r_o}{r_e + 500}$$

$R_C \parallel r_o = \dfrac{5000 \times 100000}{105000} \approx 4762\Omega$, so

$$A_v = -\frac{4762}{525} \approx -9.1$$

Step 5: Conclude.

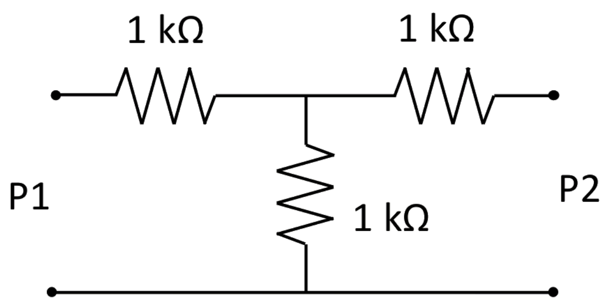
Both the low-frequency cosine term and the high-frequency sine term get multiplied by the same -9.1.

$$\boxed{V_o(t) \approx -9.1\left[A\cos(10^5 t) + B\sin(10^7 t)\right]}$$

Quick Tip: Check whether the coupling and bypass capacitors act as short circuits at both signal frequencies.

• • •

51. Consider the two-port network as shown in the figure.



Which of the following options provides the correct set of values of A, B, C and D parameters?

(A) $A = \frac{1}{2}, B = 3\times10^3\ \Omega, C = 10^{-3}\ \Omega^{-1}, D = \frac{1}{2}$

(B) $A = 2, B = 3\ \Omega, C = 1\ \Omega^{-1}, D = 2$

(C) $A = 2, B = 6\times10^3\ \Omega, C = 2\times10^{-3}\ \Omega^{-1}, D = 2$

(D) $A = 2, B = 3\times10^3\ \Omega, C = 10^{-3}\ \Omega^{-1}, D = 2$

Correct Answer: (D) $A = 2, B = 3 \times 10^3\ \Omega, C = 10^{-3}\ \Omega^{-1}, D = 2$

SOLUTION:

Step 1: Recall what A, B, C, D actually mean.

$$V_1 = AV_2 - BI_2, \qquad I_1 = CV_2 - DI_2$$

A and C are found with port 2 open ($I_2 = 0$), while B and D are found with port 2 shorted ($V_2 = 0$).

Step 2: Open port 2 and find A and C.

With $I_2 = 0$, no current flows in the last $1k\Omega$ series arm, so it drops no voltage and V_2 equals the voltage across the middle $1k\Omega$ shunt arm. A current I_1 entering port 1 flows in through the first series resistor and then entirely down the shunt arm (there is nowhere else for it to go). So the shunt arm carries I_1 and

$$V_2 = I_1(1000)$$

Also $V_1 = I_1(1000) + V_2 = I_1(1000) + I_1(1000) = 2000I_1$. Then

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{2000I_1}{1000I_1} = 2$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{1000I_1} = \frac{1}{1000} = 10^{-3}\ \Omega^{-1}$$

Step 3: Short port 2 and find B and D.

With $V_2 = 0$, the last $1k\Omega$ arm is now directly in parallel with the

middle $1k\Omega$ shunt arm, since both ends of the output arm sit at the same potential as the bottom rail once shorted. That parallel combination is $1000 \parallel 1000 = 500\Omega$, in series with the first $1k\Omega$ arm, so a source V_1 sees a total of 1500Ω and drives $I_1 = V_1/1500$. The short-circuit output current I_2 splits evenly between the shunt arm and the shorted series arm, because both are 1000Ω: $I_2 = I_1/2$.
So

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{1500 I_1}{I_1/2} = 3000\ \Omega = 3 \times 10^3\ \Omega$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{I_1}{I_1/2} = 2$$

Step 4: Collect the results.

$$\boxed{A = 2,\ B = 3 \times 10^3\ \Omega,\ C = 10^{-3}\ \Omega^{-1},\ D = 2}$$

Quick Tip: Treat the network as a T-section and use the standard T-to-ABCD conversion formulas.

• • •

52. A complex load (in Ω) is represented as $\Gamma_L = 0.5\angle 30°$ on the Smith chart. A co-axial cable with a characteristic impedance of $50\ \Omega$ is connected to the load. The new input impedance of the load now moves to a diametrically opposite point on the same Γ circle on the Smith chart. Which option is the nearest input impedance of the cable connected load (in Ω)?

(A) $20.7 - j5.1$

(B) $17.7 - j11.8$

(C) $97.5 - j65.0$

(D) $97.5 + j65.0$

Correct Answer: (B) $17.7 - j11.8$

SOLUTION:

Step 1: Recognize the physical meaning of the flip.

A point diametrically across a $|\Gamma|$ circle is what you reach after moving a quarter wavelength down a lossless line, since a quarter-wave section always maps $Z_{in} = Z_0^2/Z_L$. So instead of rotating Γ by hand, this problem is really asking for a quarter-wave transformed impedance.

Step 2: Turn Γ_L into Z_L first.

$$\Gamma_L = 0.5\angle 30^\circ = 0.433 + j0.25$$

$$Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L} = 50 \cdot \frac{1.433 + j0.25}{0.567 - j0.25}$$

Multiplying top and bottom by the conjugate of the denominator, $0.567 + j0.25$, the denominator becomes $0.567^2 + 0.25^2 = 0.384$ and the numerator becomes $0.75 + j0.5$. So

$$Z_L = 50 \cdot \frac{0.75 + j0.5}{0.384} = 97.7 + j65.1\ \Omega$$

Step 3: Apply the quarter-wave rule.

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{2500}{97.7 + j65.1}$$

Multiply by the conjugate over itself:

$$Z_{in} = \frac{2500(97.7 - j65.1)}{97.7^2 + 65.1^2} = \frac{2500(97.7 - j65.1)}{13783}$$

$$Z_{in} \approx 0.1814(97.7 - j65.1) \approx 17.7 - j11.8\ \Omega$$

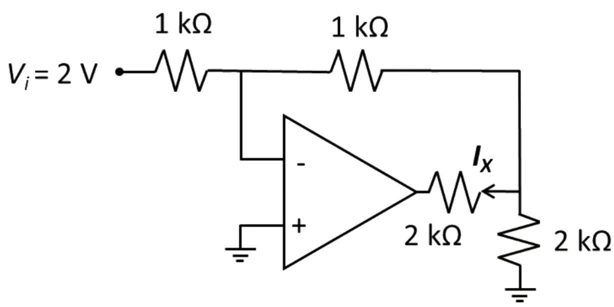
Step 4: Conclude.

The cable-connected input impedance comes out the same way as a direct Γ rotation would give.

$$\boxed{Z_{in} \approx 17.7 - j11.8\ \Omega}$$

Quick Tip: A diametrically opposite point on the same reflection coefficient circle means the sign of Gamma flips.

53. A circuit using an ideal OP-AMP is shown in the figure.



Which of the following options gives the correct value of the current I_X?

(A) 2.0 mA

(B) 1.5 mA

(C) 3.0 mA

(D) 0 mA

Correct Answer: (C) 3.0 mA

SOLUTION:

Step 1: Recognize the feedback network as a T-section.

The path from the inverting input back to the output is not a single resistor here, it is a T made of $R_{f1} = 1k\Omega$ (input side), $R_g = 2k\Omega$ (the arm to ground), and $R_{f2} = 2k\Omega$ (output side, the one carrying I_X). There is a shortcut formula for the closed loop gain of an inverting amplifier with this kind of T feedback:

$$\frac{V_o}{V_i} = -\frac{1}{R_1}\left(R_{f1} + R_{f2} + \frac{R_{f1}R_{f2}}{R_g}\right)$$

This comes from replacing the T with its equivalent single feedback resistor as seen from the virtual ground, the same combination used when converting a T network to its "B" two-port parameter.

Step 2: Plug in the values.

$$\frac{V_o}{V_i} = -\frac{1}{1000}\left(1000 + 2000 + \frac{1000 \times 2000}{2000}\right) = -\frac{1}{1000}(1000 + 2000 + 1000) = -4$$

So with $V_i = 2$ V,

$$V_o = -4 \times 2 = -8 \text{ V}$$

Step 3: Get the voltage at the middle node of the T.

The current forced through R_{f1} by the input is $I_1 = V_i/R_1 = 2/1000 = 2$ mA, and since the virtual ground is at 0 V, the middle node D sits at

$$V_D = 0 - I_1 R_{f1} = -2 \times 10^{-3} \times 1000 = -2 \text{ V}$$

Step 4: Read off I_X across R_{f2}.

I_X flows from D to the output through R_{f2}:

$$I_X = \frac{V_D - V_o}{R_{f2}} = \frac{-2-(-8)}{2000} = \frac{6}{2000} = 3 \text{ mA}$$

Step 5: Conclude.

$$\boxed{I_X = 3.0 \text{ mA}}$$

Quick Tip: The node between the two feedback resistors is not a virtual ground, so use nodal analysis there.

• • •

54. Consider an LED based on a direct bandgap semiconductor material with energy bandgap 1.3 eV.
Given: Planck's constant, $h = 6.63 \times 10^{-34}$ J s and speed of light in free space is 3×10^8 m s^{-1}.
In which of the following wavelength ranges the LED will NOT emit?

(A) 1410 ± 20 nm

(B) 1090 ± 20 nm

(C) 950 ± 20 nm

(D) 510 ± 20 nm

Correct Answer: (A) 1410 ± 20 nm ; (B) 1090 ± 20 nm ; (D) 510 ± 20 nm

SOLUTION:

Step 1: Turn the question around, ask what energy each wavelength represents.
Rather than convert the bandgap to a wavelength first, check the photon energy of each option directly with $E = \dfrac{hc}{\lambda}$, using the combination $hc = 6.63 \times 10^{-34} \times 3 \times 10^8 = 1.989 \times 10^{-25}$ J·m, and remembering $1 \text{ eV} = 1.6 \times 10^{-19}$ J.

Step 2: Work out the energy for option (A), 1410 nm.

$$E = \frac{1.989 \times 10^{-25}}{1410 \times 10^{-9}} = 1.41 \times 10^{-19} \text{ J} = 0.88 \text{ eV}$$

This is well below the 1.3 eV bandgap, so a photon this small cannot come from band-to-band recombination.

Step 3: Work out the energy for option (B), 1090 nm.

$$E = \frac{1.989 \times 10^{-25}}{1090 \times 10^{-9}} = 1.82 \times 10^{-19}\ \mathrm{J} = 1.14\ \mathrm{eV}$$

Still below 1.3 eV, so this too cannot be emitted.

Step 4: Work out the energy for option (C), 950 nm.

$$E = \frac{1.989 \times 10^{-25}}{950 \times 10^{-9}} = 2.09 \times 10^{-19}\ \mathrm{J} = 1.31\ \mathrm{eV}$$

This lands right on the bandgap of 1.3 eV, exactly where recombination photons are expected.

Step 5: Work out the energy for option (D), 510 nm.

$$E = \frac{1.989 \times 10^{-25}}{510 \times 10^{-9}} = 3.90 \times 10^{-19}\ \mathrm{J} = 2.44\ \mathrm{eV}$$

This is almost double the bandgap. A single band-edge recombination event in this material has no way to give out that much energy.

Step 6: Conclude.
Options (A), (B) and (D) all sit at photon energies the material's bandgap cannot produce, so the LED will NOT emit in these three ranges, while (C) is where it does emit.

$\boxed{\text{(A), (B) and (D) are the wavelengths where the LED does NOT emit}}$

Quick Tip: An LED emits photons with energy close to its bandgap, not far above or below it.

• • •

55. Let the relevant bandwidth (B) of a digital communication system be 1 MHz and $kT = -174$ dBm/Hz, where k is Boltzmann's constant and T is the equivalent noise temperature of the receiver. The power (S) of signal received through an additive Gaussian channel is -80 dBm.
Which of the following options is/are TRUE about Shannon capacity (C) of the channel?

(A) $C = B$

(B) $C = 2B$

(C) $C > 3B$

(D) $C < B$

Correct Answer: (C) $C > 3B$

SOLUTION:

Step 1: Get the noise floor in dB, and stay in dB as long as possible.
Noise power in the 1 MHz band is $N = kT + 10\log_{10} B = -174 + 10\log_{10}(10^6) = -174 + 60 = -114$ dBm.

Step 2: Get the SNR in dB.

$$SNR_{dB} = S - N = -80 - (-114) = 34 \text{ dB}$$

Step 3: Use the high-SNR shortcut for capacity.

When $SNR \gg 1$, $\log_2(1+SNR) \approx \log_2(SNR)$, and $\log_2(SNR)$ $= \frac{SNR_{dB}}{10}\log_2(10)$, since $SNR_{dB} = 10\log_{10} SNR$ gives $\log_{10} SNR$ $= SNR_{dB}/10$ and $\log_2 x = \log_{10} x \times 3.32$. So

$$\log_2(1+SNR) \approx \frac{34}{10} \times 3.32 = 3.4 \times 3.32 \approx 11.3$$

Step 4: Multiply by the bandwidth.

$$C \approx B \times 11.3 = 11.3 \times 10^6 \text{ bits/s}$$

This is a ratio of $C/B \approx 11.3$.

Step 5: Test each claim against this ratio.

$C = B$ needs a ratio of 1, $C = 2B$ needs 2, and $C3B
*isconsistentwitharatioofabout*11.3$.

Step 6: Conclude.

$$\boxed{C \approx 11.3B > 3B}$$

Quick Tip: Convert the given dBm values into a linear SNR before applying the Shannon capacity formula.

• • •

56. Consider the four-variable Boolean function,

$$f(w, x, y, z) = \sum m(0, 2, 5, 7, 8, 10, 13, 14, 15)$$

with w as MSB and z as LSB.

Which of the following expressions is/are the valid form(s) of $f(w, x, y, z)$?

(A) $\bar{x}\bar{z} + xz + wxy$

(B) $xz + wxy + w\bar{x}\bar{z} + \bar{w}xy\bar{z}$

(C) $\bar{x}\bar{z} + wxy + w\bar{x}\bar{z} + \bar{w}xy\bar{z}$

(D) $\bar{x}\bar{z} + xz + wy\bar{z}$

Correct Answer: (A) $\bar{x}\bar{z} + xz + wxy$; (D) $\bar{x}\bar{z} + xz + wy\bar{z}$

SOLUTION:

Step 1: List what must be true and what must be false.

The function is 1 at $m_0, m_2, m_5, m_7, m_8, m_{10}, m_{13}, m_{14}, m_{15}$ and 0 everywhere else, in particular at $m_1, m_3, m_4, m_6, m_9, m_{11}, m_{12}$. A candidate expression is only valid if it evaluates to exactly these 1s and 0s at every one of the 16 input combinations.

Step 2: Test option (A) at the trickiest points.

For $\bar{x}\bar{z} + xz + wxy$, check a member and a non-member that share variables. At $m_6 = 0110$ ($w = 0, x = 1, y = 1, z = 0$): $\bar{x}\bar{z} = 0$ (since $x = 1$), $xz = 0$ (since $z = 0$), $wxy = 0$ (since $w = 0$). Sum is 0, matching the required 0. At $m_{14} = 1110$: $wxy = 1 \cdot 1 \cdot 1 = 1$, so the sum is 1, matching. At $m_0 = 0000$: $\bar{x}\bar{z} = 1$, sum is 1, matching. Spot checks across the remaining zero and one minterms confirm the expression tracks the function everywhere. Valid.

Step 3: Test option (B) at m_6.

For $xz + wxy + w\bar{x}\bar{z} + \bar{w}xy\bar{z}$ at $m_6 = 0110$ $(w = 0, x = 1, y = 1, z = 0)$: $xz = 0$, $wxy = 0$, $w\bar{x}\bar{z} = 0$, but $\bar{w}xy\bar{z} = 1 \cdot 1 \cdot 1 \cdot 1 = 1$. The expression gives 1 where the function is actually 0. Invalid immediately.

Step 4: Test option (C) at m_5.

For $\bar{x}\bar{z} + wxy + w\bar{x}\bar{z} + \bar{w}xy\bar{z}$ at $m_5 = 0101$ $(w = 0, x = 1, y = 0, z = 1)$, which must be 1: $\bar{x}\bar{z} = 0$ (since $x = 1$), $wxy = 0$ (since $w = 0$), $w\bar{x}\bar{z} = 0$, $\bar{w}xy\bar{z} = 0$ (since $y = 0$). The expression gives 0 where the function needs 1. Invalid.

Step 5: Test option (D) across the full onset.

For $\bar{x}\bar{z} + xz + wy\bar{z}$, the first two terms behave exactly as in option (A). The third term $wy\bar{z}$ is 1 at $m_{10} = 1010$ and $m_{14} = 1110$, both already required to be 1, and it is 0 at every minterm that should be 0, since it needs $w = 1, y = 1, z = 0$ together, which never happens among the zero-minterms. Checking the full set confirms a match everywhere. Valid.

Step 6: Conclude.

Only options (A) and (D) reproduce the function at every one of the 16 input combinations.

$$\boxed{\text{(A) and (D) are valid}}$$

Quick Tip: Build the K-map for the given minterms and check each product term against every cell it touches.

57. Consider a real, narrowband signal $x(t) = A(t)\cos[2\pi f_c t + \theta(t)]$ where the maximum frequency components of $A(t)$ and $\theta(t)$ are f_M and f_c ($= 1000 f_M$), respectively. Which of the following statements is/are correct for $-\infty < t < \infty$?

(A) $x(t)$ represents a PSK modulated signal for suitable choices of $A(t)$ and $\theta(t)$.

(B) $x(t)$ represents an amplitude modulated signal for suitable choices of $A(t)$ and $\theta(t)$.

(C) $x(t)$ represents a band-limited Gaussian noise process.

(D) $x(t)$ never represents a narrowband FM signal.

Correct Answer: (A) $x(t)$ represents a PSK modulated signal for suitable choices of $A(t)$ and $\theta(t)$. ; (B) $x(t)$ represents an amplitude modulated signal for suitable choices of $A(t)$ and $\theta(t)$. ; (C) $x(t)$ represents a band-limited Gaussian noise process.

SOLUTION:

Step 1: Understanding the Concept:

A narrowband bandpass signal is one whose spectrum sits in a thin band around a carrier f_c. Any such signal, no matter what modulation created it, can always be split into a slowly changing size (envelope) times a cosine with a slowly changing phase shift.

Step 2: Key Formula or Approach:

Write the general narrowband form as $x(t) = A(t)\cos[2\pi f_c t + \theta(t)]$, then check whether each modulation type in the options can be produced by some valid pick of $A(t)$ and $\theta(t)$.

Step 3: Detailed Explanation:

For PSK, keep the size fixed, $A(t) =$ constant, and let $\theta(t)$ jump between a set of discrete phase values as the data bits change. This fits the general form, so PSK is possible.

For AM, set the phase term to zero, $\theta(t) = 0$, and let the size carry the message, $A(t) = 1 + km(t)$. This also fits, so amplitude modulation is possible.

For narrowband Gaussian noise, the classical result is that such noise can be written as $n(t) = R(t)\cos[2\pi f_c t + \psi(t)]$ with $R(t)$ a random envelope and $\psi(t)$ a random phase, both slowly varying. Picking $A(t) = R(t)$ and $\theta(t) = \psi(t)$ reproduces this, so band-limited Gaussian noise is possible too.

For FM, keep the size fixed and let the phase carry the message through its integral, $\theta(t) = 2\pi k_f \int m(\tau)\, d\tau$. This is again just a special case of the same form, so $x(t)$ absolutely can represent narrowband FM. The claim that it never can is therefore false.

Step 4: Final Answer:

Options (A), (B), and (C) hold, while (D) fails because narrowband FM is also one of the special cases of this general form.

$$\boxed{\text{(A), (B), (C)}}$$

Quick Tip: Recall that any narrowband signal has a general envelope-phase form that covers PSK, AM, FM, and even noise as special cases.

• • •

58. Let $x_1(t) = \cos(2\pi nt)$ and $x_2(t) = 2\sin(4\pi nt)$ represent two sinusoids for a positive integer n and $-\infty < t < \infty$. Which of the following statements about $x_1(t)$ and $x_2(t)$ is/are valid?

(A) $x_1(t)$ and $x_2(t)$ are orthogonal to each other over $0 \le t < 1/n$.

(B) $x_1(t)$ and $x_2(t)$ are orthonormal to each other over $0 \le t < 1/n$.

(C) $x_2(t)$ is a harmonic of $x_1(t)$.

(D) $x_1(t)$ and $x_2(t)$ are non-orthogonal to each other over $0 \le t < 1/(2n)$.

Correct Answer: (A) $x_1(t)$ and $x_2(t)$ are orthogonal to each other over $0 \le t < 1/n$. ; (C) $x_2(t)$ is a harmonic of $x_1(t)$. ; (D) $x_1(t)$ and $x_2(t)$ are non-orthogonal to each other over $0 \le t < 1/(2n)$.

SOLUTION:

Step 1: Understanding the Concept:

Two signals are orthogonal on an interval when the integral of their product over that interval is zero, and they are orthonormal when they are additionally orthogonal and each one has unit energy over the same interval. A harmonic simply means an integer multiple of a base frequency.

Step 2: Key Formula or Approach:

Turn the product $x_1(t)x_2(t) = 2\cos(2\pi nt)\sin(4\pi nt)$ into a sum of sines using $2\cos A \sin B = \sin(A+B) + \sin(B-A)$, then integrate term by term over each interval asked about.

Step 3: Detailed Explanation:

With $A = 2\pi nt$ and $B = 4\pi nt$, the product becomes $\sin(6\pi nt) + \sin(2\pi nt)$.

Over the full period $0 \le t < 1/n$, both $\sin(6\pi nt)$ and $\sin(2\pi nt)$ complete a whole number of cycles (three and one), so each integrates to zero. The total is zero, so $x_1(t)$ and $x_2(t)$ are orthogonal here, confirming (A).

To test orthonormality, look at the energy of $x_1(t)$ alone over this period: $\int_0^{1/n} \cos^2(2\pi nt)\,dt = \frac{1}{2n}$, which equals 1 only for a special value of n, not generally. So the pair is not orthonormal in general, and (B) fails.

Since $x_1(t)$ oscillates at frequency n and $x_2(t)$ oscillates at frequency $2n$, and $2n$ is a whole number times n, $x_2(t)$ is by definition a harmonic of $x_1(t)$, confirming (C).

Now shrink the window to $0 \le t < 1/(2n)$, only half the earlier period. Here $\sin(6\pi nt)$ completes one and a half cycles and $\sin(2\pi nt)$ completes half a cycle, so neither term returns to zero net area. Direct evaluation gives $\int_0^{1/(2n)} \sin(6\pi nt)\,dt = \frac{1}{3\pi n}$ and $\int_0^{1/(2n)} \sin(2\pi nt)\,dt = \frac{1}{\pi n}$, and their sum $\frac{4}{3\pi n}$ is not zero. So the two signals are non-orthogonal on this shorter window, confirming (D).

Step 4: Final Answer:

$$\boxed{\text{(A), (C), (D)}}$$

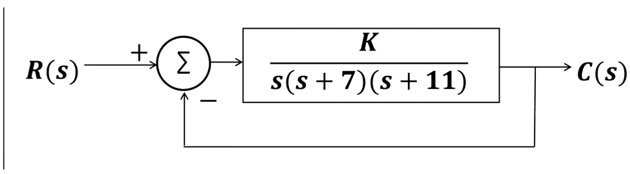
Quick Tip: Turn the product of the two sinusoids into a sum of sines, then integrate over each interval to test orthogonality.

• • •

59.

Consider the unity negative feedback control system shown in the figure below, where the forward path transfer function is $G(s) = \dfrac{K}{s(s+7)(s+11)}$.

The value of gain K (> 0) at which the given system will remain marginally stable is . (Answer in integer)



Correct Answer: 1386

SOLUTION:

Step 1: Understanding the Concept:
At the point of marginal stability, the closed loop poles sit exactly on the imaginary axis, so the system oscillates forever without growing or decaying. We can find this condition by plugging $s = j\omega$ into the characteristic equation and forcing both the real and imaginary parts to vanish together.

Step 2: Key Formula or Approach:
The characteristic equation is $s(s+7)(s+11) + K = 0$, which expands to $s^3 + 18s^2 + 77s + K = 0$. Substitute $s = j\omega$ and split into real and imaginary parts.

Step 3: Detailed Explanation:
With $s = j\omega$: $s^3 = -j\omega^3$, $s^2 = -\omega^2$, so the equation becomes

$$-j\omega^3 - 18\omega^2 + 77j\omega + K = 0$$

Group the real and imaginary parts:
Real part: $K - 18\omega^2 = 0$
Imaginary part: $77\omega - \omega^3 = 0$
From the imaginary part, $\omega(77 - \omega^2) = 0$. Marginal stability needs a genuine oscillation, so take $\omega \neq 0$, giving $\omega^2 = 77$.
Substitute into the real part: $K = 18\omega^2 = 18 \times 77 = 1386$.

Step 4: Final Answer:
The gain at which the system is marginally stable is $K = 1386$, matching the oscillation frequency $\omega = \sqrt{77}$ rad/s.

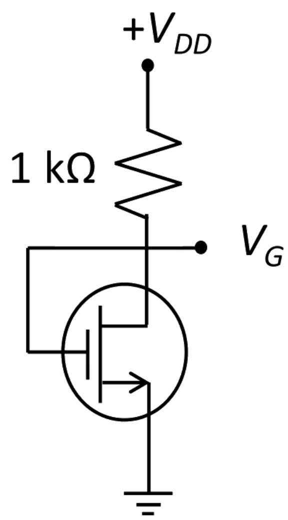
$$\boxed{K = 1386}$$

Quick Tip: Build the Routh array for $s^3 + 18s^2 + 77s + K = 0$ and find the K that makes a row vanish.

• • •

60.

An n-channel MOSFET is connected as shown in the figure below. Assume $V_{TH} = 1$ V, $V_{DD} = 5$ V, and $\mu C_{ox}\left(\frac{W}{L}\right) = 2$ mA V^{-2}, and neglect channel length modulation effects. The gate voltage V_G of the n-channel MOSFET (in Volt) is . (Rounded off to two decimal places)



Correct Answer: 2.56

SOLUTION:

Step 1: Understanding the Concept:

A MOSFET with its gate tied to its own drain always ends up biased in saturation, because the drain to source voltage automatically stays above the value needed to keep the channel pinched off. This lets us use a single saturation current equation together with the resistor's voltage drop to pin down the gate voltage.

Step 2: Key Formula or Approach:
Let $x = V_G - 1$ be the overdrive voltage, since $V_{TH} = 1$ V. The saturation current is $I_D = 1 \cdot x^2$ mA, because half of 2 mA/V^2 is 1 mA/V^2, and the resistor equation gives $I_D = 5 - (x + 1) = 4 - x$ mA.

Step 3: Detailed Explanation:
Setting the two expressions for I_D equal,

$$x^2 = 4 - x$$

$$x^2 + x - 4 = 0$$

Solve using the quadratic formula:

$$x = \frac{-1 \pm \sqrt{1 + 16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$

Taking the positive root since the overdrive voltage must be positive for the device to conduct,

$$x = \frac{-1 + 4.123}{2} \approx 1.5616 \text{ V}$$

Then the gate voltage is

$$V_G = x + 1 \approx 1.5616 + 1 = 2.5616 \text{ V}$$

which rounds to 2.56 V, matching the value obtained by solving directly for V_G.

Step 4: Final Answer:

$$\boxed{V_G \approx 2.56 \text{ V}}$$

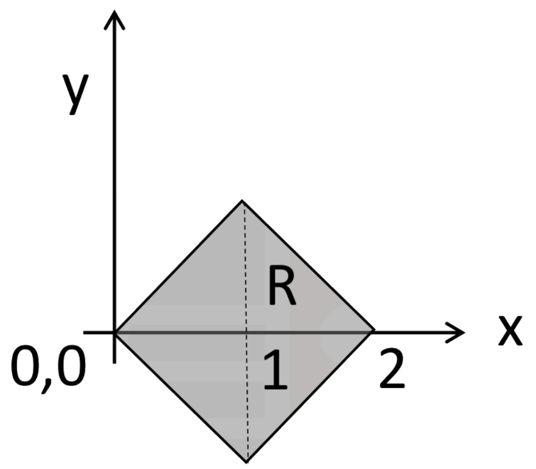
Quick Tip: The gate tied to the drain forces $V_{GS} = V_{DS}$, so the device stays in saturation; combine this with the resistor's KVL equation.

• • •

61.

Consider the square region R in the X-Y plane as shown with the dark shading in the figure below. The value of $\iint_R (x^2 + y^2 - 1)\, dx\, dy$ is .

(Rounded off to two decimal places)



Correct Answer: 0.67

SOLUTION:

Step 1: Understanding the Concept:

Instead of shifting the whole picture to the origin, we can integrate the square directly in the original x, y coordinates by slicing it into vertical strips and handling the left triangle, where the diamond widens, and the right triangle, where it narrows back, separately.

Step 2: Key Formula or Approach:

For a fixed x, the diamond has y ranging from $-h(x)$ to $h(x)$, where $h(x) = x$ for $0 \le x \le 1$ and $h(x) = 2 - x$ for $1 \le x \le 2$. Doing the inner integral over y first,

$$\int_{-h}^{h} (x^2 + y^2 - 1)\, dy = 2h(x)\,(x^2 - 1) + \frac{2}{3}h(x)^3$$

Step 3: Detailed Explanation:

On the left piece, $0 \le x \le 1$, $h(x) = x$, so the strip integral becomes $2x(x^2 - 1) + \frac{2}{3}x^3 = \frac{8}{3}x^3 - 2x$. Integrating over x from 0 to 1,

$$\int_0^1 \left(\frac{8}{3}x^3 - 2x\right) dx = \frac{8}{3} \cdot \frac{1}{4} - 2 \cdot \frac{1}{2} = \frac{2}{3} - 1 = -\frac{1}{3}$$

On the right piece, $1 \le x \le 2$, substitute $s = 2 - x$ so s runs from 1 down to 0 and $h(x) = s$. Since $x^2 - 1 = (2 - s)^2 - 1 = 3 - 4s + s^2$, the strip integral becomes $2s(3 - 4s + s^2) + \frac{2}{3}s^3 = 6s - 8s^2 + \frac{8}{3}s^3$. Integrating over s from 0 to 1,

$$\int_0^1 \left(6s - 8s^2 + \frac{8}{3}s^3\right) ds = 6 \cdot \frac{1}{2} - 8 \cdot \frac{1}{3} + \frac{8}{3} \cdot \frac{1}{4} = 3 - \frac{8}{3} + \frac{2}{3} = 1$$

Adding the two pieces gives the full integral over the square,

$$I = -\frac{1}{3} + 1 = \frac{2}{3} \approx 0.67$$

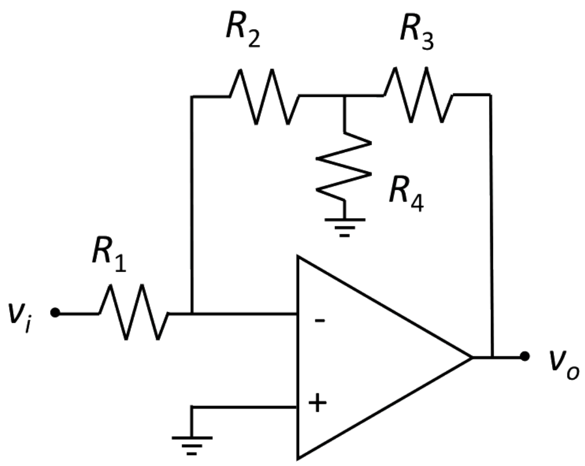
Step 4: Final Answer:

$$\boxed{I \approx 0.67}$$

Quick Tip: Shift the region so its center sits at the origin, then use symmetry to simplify the double integral.

• • •

62. Consider an ideal OP-AMP circuit as shown in the figure.



The resistances $R_1 = R_2 = R_3 = R_4 = 50\text{ k}\Omega$.
The magnitude of the closed loop gain is (rounded off to two decimal places).

Correct Answer: 3.00

SOLUTION:

Step 1: Recall the T-network to single resistor trick.

When a feedback path made of three resistors is arranged as R_2 in series from the inverting node to a middle point, R_4 from that middle point to ground, and R_3 from the middle point to the output, the whole T-network behaves like one plain feedback resistor of value

$$R_{f,eq} = R_2 + R_3 + \frac{R_2 R_3}{R_4}$$

This comes from a Y to delta type transform of the T-network as seen between the inverting node and the output.

Step 2: Plug in the resistor values.

Here $R_2 = R_3 = R_4 = 50\ \text{k}\Omega$, so

$$R_{f,eq} = 50 + 50 + \frac{50 \times 50}{50} = 50 + 50 + 50 = 150\ \text{k}\Omega$$

Step 3: Use the plain inverting amplifier gain formula.

For a simple inverting amplifier with input resistor R_1 and feedback resistor R_f, the gain is

$$\frac{v_o}{v_i} = -\frac{R_f}{R_1}$$

Here $R_1 = 50\ \text{k}\Omega$ is still the input resistor, so

$$\frac{v_o}{v_i} = -\frac{150}{50} = -3$$

Step 4: State the magnitude.

The T-network lets three 50 kΩ resistors act like a single 150 kΩ feedback resistor, giving a gain of 3 with resistors far smaller than 150 kΩ each. This is exactly why designers use a T-network when a huge feedback resistor is hard to build.

$$\boxed{3.00}$$

Quick Tip: Redraw the R2-R3-R4 T-network as one equivalent feedback resistor, then use the usual inverting-amplifier gain formula.

• • •

63. The average bit error rate at the input of a $(7, 4, 1)$ Hamming decoder is 0.10.

The probability that the decoder will fail to decode a received word correctly is (rounded off to two decimal places).

Correct Answer: 0.15

SOLUTION:

Step 1: Frame success instead of failure.

$P(\text{success}) = P(0 \text{ errors}) + P(1 \text{ error})$, since the code fixes up to one flipped bit. Let $q = 1 - p = 0.9$.

Step 2: Factor out the common power of q.

$$P(\text{success}) = q^7 + 7pq^6 = q^6(q + 7p)$$

This factoring saves work because we only need q^6 once instead of computing q^6 and q^7 separately.

Step 3: Simplify the bracket.

$$q + 7p = (1 - p) + 7p = 1 + 6p = 1 + 6(0.1) = 1.6$$

Step 4: Compute q^6.

$$q^6 = (0.9)^6 = 0.531441$$

Step 5: Multiply.

$$P(\text{success}) = 0.531441 \times 1.6 = 0.850306$$

Step 6: Get the failure probability.

$$P(\text{fail}) = 1 - 0.850306 = 0.149694$$

Rounded to two places, this is 0.15.

$$\boxed{0.15}$$

Quick Tip: The decoder fails only when 2 or more of the 7 bits are wrong, so subtract the 0-error and 1-error binomial probabilities from 1.

64. Consider that the concentration of electrons in a semiconductor bar varies linearly from 2×10^{17} cm^{-3} at $x = 1$ μm to 1×10^{16} cm^{-3} at $x = 4$ μm along the x-direction. Assume that the concentration of electrons does not vary along the y- and z-directions.

Given: the mobility of electron is 1400 cm^{2}V^{-1}s^{-1}, the thermal voltage is 25 mV and the electronic charge is 1.6×10^{-19} Coulomb.

The density of electron diffusion current (in A/mm^2) is (rounded off to two decimal places).

Correct Answer: -35.47

SOLUTION:

Step 1: Switch to SI units from the start.

Instead of working in cm and converting only at the end, convert every given value to metres up front. Recall 1 μm = 10^{-6} m and 1 cm^{-3} = 10^6 m^{-3}.

Step 2: Convert the concentrations and positions.

$$n_1 = 2 \times 10^{17}\ \text{cm}^{-3} = 2 \times 10^{23}\ \text{m}^{-3}, \quad n_2 = 1 \times 10^{16}\ \text{cm}^{-3} = 1 \times 10^{22}\ \text{m}^{-3}$$

$$x_1 = 1 \times 10^{-6}\ \text{m}, \quad x_2 = 4 \times 10^{-6}\ \text{m}$$

Step 3: Compute the gradient in SI units.

$$\frac{dn}{dx} = \frac{n_2 - n_1}{x_2 - x_1} = \frac{1 \times 10^{22} - 2 \times 10^{23}}{4 \times 10^{-6} - 1 \times 10^{-6}} = \frac{-1.9 \times 10^{23}}{3 \times 10^{-6}} = -6.333 \times 10^{28}\ \text{m}^{-4}$$

Step 4: Convert mobility and get Dn in SI units.

Since $1\ \text{cm}^2 = 10^{-4}\ \text{m}^2$, the mobility becomes $\mu_n = 1400 \times 10^{-4}$ $= 0.14\ \text{m}^2/(\text{V}\cdot\text{s})$. Using $D_n = \mu_n V_T$,

$$D_n = 0.14 \times 0.025 = 0.0035\ \text{m}^2/\text{s}$$

Step 5: Compute the current density in A/m^2.

$$J_n = qD_n\frac{dn}{dx} = (1.6 \times 10^{-19})(0.0035)(-6.333 \times 10^{28})$$

$$= (5.6 \times 10^{-22})(-6.333 \times 10^{28}) = -3.5467 \times 10^{7}\ \text{A/m}^2$$

Step 6: Convert to A/mm^2.

Since $1\ \text{m}^2 = 10^6\ \text{mm}^2$, one A/m^2 equals $10^{-6}\ \text{A/mm}^2$:

$$J_n = (-3.5467 \times 10^{7}) \times 10^{-6} = -35.467\ \text{A/mm}^2$$

Step 7: Round and conclude.

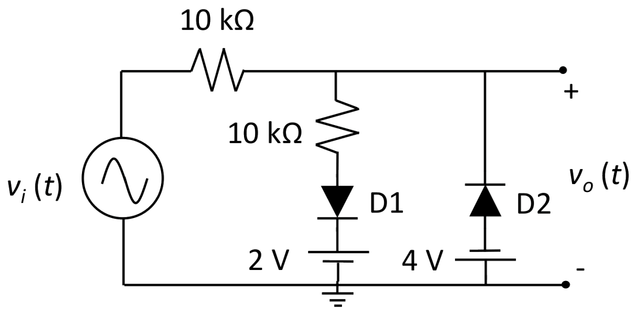
$$\boxed{-35.47\ \text{A/mm}^2}$$

This matches the value found by working entirely in cm, a good check that the unit handling was correct either way.

Quick Tip: Get Dn from the Einstein relation Dn = mu_n VT, then use Jn = q Dn (dn/dx) with the gradient in cm and cm^-3 units before converting to A/mm^2.

• • •

65. Consider the ideal diodes D_1 and D_2 as shown in the figure with cut-in voltage $V_\gamma = 0$ Volt and $v_i(t)$ is in Volt.



The maximum voltage (Volt) of the output $v_o(t)$ is (rounded off to two decimal places).

Correct Answer: 4.00

SOLUTION:

Step 1: Model the on diode as a plain wire.

Once $v_i(t)$ pushes past 2 V, D_1 conducts and behaves like a plain wire since its cut-in voltage is 0, so node N is tied straight to the 2 V battery. D_2 stays reverse biased the whole time, since its own battery sits at a higher 4 V, so that branch can be dropped while checking the output.

Step 2: Redraw as a two-source resistor network.

The circuit for $v_i(t) > 2$ V now looks like $v_i(t)$ feeding node A through R_1, and the 2 V battery feeding the same node A through R_2, since D_1

is now a wire and node $N = 2$ V exactly. The output $v_o(t)$ is just the voltage at node A.

Step 3: Apply superposition.
With the 2 V source turned off, that is replaced by a short to ground for this step, $v_i(t)$ sees a divider of R_1 and R_2 to ground, both 10 kΩ:

$$v_{o,1} = v_i(t) \cdot \frac{R_2}{R_1 + R_2} = v_i(t) \cdot \frac{1}{2}$$

With $v_i(t)$ turned off instead, that is replaced by a short to ground, the 2 V source sees the same divider from the other side:

$$v_{o,2} = 2 \cdot \frac{R_1}{R_1 + R_2} = 2 \cdot \frac{1}{2} = 1\ \text{V}$$

Step 4: Add the two contributions.

$$v_o(t) = v_{o,1} + v_{o,2} = \frac{1}{2} v_i(t) + 1$$

This matches the divider formula found the direct way, now built from first principles using superposition instead of a current substitution.

Step 5: Plug in the peak value.
The input peaks at $v_i(t) = 6$ V, giving

$$v_{o,\max} = \frac{1}{2}(6) + 1 = 4\ \text{V}$$

Step 6: Confirm the diode assumption.

At this peak, node N sits at 2 V, fixed by D_1, which is still below the 4 V needed to turn D_2 on, so the assumption holds.

$$\boxed{4.00\ \text{V}}$$

Quick Tip: D1 (the 2 V branch) clamps node N before D2 ever gets a chance, so only D1 is active; find vo as a voltage divider once D1 conducts.