GATE 2026 Electrical Engineering Question Paper with Solutions

General Instructions

(i) This booklet contains 65 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 48 single-correct multiple-choice questions, 4 multiple-correct questions and 13 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For multiple-correct questions, all correct options must be selected to earn full marks.

(v) For numerical questions, report the answer rounded exactly as asked.

. 'The shopkeeper sells lemons.'

In this sentence, the word 'lemons' is the _____.

(A) object

(B) subject

(C) predicate

(D) verb

Correct Answer: (A) object

Solution:

Step 1: Understanding the Question.

The sentence is "The shopkeeper sells lemons." We need to find what part of the sentence the word "lemons" plays.

Step 2: Key Concept.

A simple sentence has a subject, a verb, and often an object. The

subject is who or what does the action. The verb names the action. The object is the person or thing that receives the action of the verb.

Step 3: Detailed Explanation.

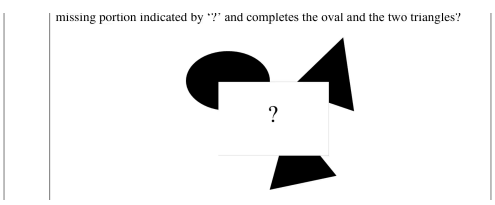
In "The shopkeeper sells lemons," the word "shopkeeper" is the subject, since the shopkeeper is the one doing the selling. The word "sells" is the verb, since it names the action. The word "lemons" comes right after the verb and names what is being sold, so it receives the action of "sells." This makes "lemons" the object of the sentence. It is not the subject, since the subject here is "shopkeeper," not "lemons." It is not the verb, since "sells" is the only action word in the sentence. It is not the predicate either, because the predicate is the whole part of the sentence that tells us what the subject does, which is "sells lemons," not the single word "lemons."

Step 4: Final Answer.

The word "lemons" receives the action of selling, so it is the object of the sentence. Hence, the correct option is (A).

Quick Tip: Find the verb first, then ask "what" right after it. The word that answers this is the object.

. The figure below is supposed to show three non-overlapping shapes: one oval and two triangles. Which one of the following figures P, Q, R, or S fits the missing portion indicated by '?' and completes the oval and the two triangles?



(A) P

(B) Q

(C) R

(D) S

Correct Answer: (A) P

Solution:

This is a figure completion question. Three shapes, one oval and two triangles, overlap at the top of the picture, and a rectangular patch marked "?" hides part of the oval and part of one triangle. We need to work out exactly what shape and orientation is hidden behind the patch, then match it to one of the four answer figures.

1. **P**: Shows a small quarter section of the oval curving in the same direction as the visible arc just outside the patch, together with triangle corners set at the same angle and direction as the visible triangle edges around the patch. Every line here continues smoothly from the main figure with no gap and no extra line.
2. **Q**: Has the same quarter oval piece, but the smaller triangle fragment is flipped so it points the opposite way from the triangle corner visible just below the patch in the main figure. This mismatch means Q does not restore the original figure correctly.
3. **R**: Uses a differently angled triangle piece that does not line up with the edge of the triangle poking out below the patch, so joining R back into the main figure would leave a broken or doubled edge.
4. **S**: Enlarges the oval piece into almost half the oval instead of the small quarter that is actually missing, which would make the finished oval far bigger than the arc already visible in the main picture.

Only option P reproduces the missing quarter oval and both triangle corners at the correct size, angle, and direction, so it is the piece that exactly completes the picture into one full oval and two full triangles with no overlaps and no extra lines.

Let's summarize:

- Match the curve direction of the oval piece to the visible arc outside the patch.
- Match the angle and direction of every triangle corner to the visible triangle edges.
- Reject any option that flips a shape, resizes it, or leaves an edge that does not continue smoothly.

The figure that satisfies all of this is P, so the correct answer is option (A).

Quick Tip: Match the curve direction of the oval and the angle of each triangle corner to the visible edges around the "?" patch.

. At how many points will the curves $y = x^2$ and $y = -x^2 - 2x - 1$ intersect in the real (x, y) plane?

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (A) 0

Solution:

Step 1: Set up the equation for intersection.

Points where the two curves meet must satisfy both equations at the same time, so set the two expressions for y equal to each other.

$$x^2 = -x^2 - 2x - 1$$

Step 2: Simplify into a single quadratic equation.

Bring every term to one side.

$$x^2 + x^2 + 2x + 1 = 0$$

$$2x^2 + 2x + 1 = 0$$

Step 3: Check the discriminant.

For a quadratic $ax^2 + bx + c = 0$, the number of real roots is decided by the discriminant $D = b^2 - 4ac$. Here $a = 2$, $b = 2$, $c = 1$.

$$D = (2)^2 - 4(2)(1) = 4 - 8 = -4$$

Step 4: Interpret the discriminant.

Since $D < 0$, the quadratic equation $2x^2 + 2x + 1 = 0$ has no real solution for x. This means there is no real value of x at which the two curves give the same y.

Step 5: Final conclusion.

Since no real x satisfies both equations, the two curves do not meet at any real point.

$$\boxed{0}$$

Hence, the correct option is (A).

Quick Tip: Set the two expressions for y equal and check whether the resulting quadratic has real roots.

. 'If Anish had scored hundred runs in today's match, he would have been made the captain of his team. He would have then become the youngest captain in his team's history. Unfortunately, he got out without scoring any runs. Hence, there won't be any change in the captaincy for now.' Based on the paragraph above, which one of the following statements is true?

(A) Anish made hundred runs but was denied captaincy.

(B) Anish was the captain of his team before the game today.

(C) The current captain is older than Anish.

(D) Anish is the youngest player in his team.

Correct Answer: (C) The current captain is older than Anish.

Solution:

This paragraph describes a conditional situation. It tells us what would have happened if Anish had scored a hundred runs, and then tells us what actually happened. We need to pick the statement that must be true based only on what is written.

1. **Anish made hundred runs but was denied captaincy**: The paragraph clearly says Anish "got out without scoring any runs," so he did not make a hundred runs. This statement contradicts the passage directly.
2. **Anish was the captain of his team before the game today**: The passage only says he would have "been made the

captain" if he had scored a hundred, which means he was not already captain before today. This statement is false.

3. **The current captain is older than Anish**: The passage says that if Anish had become captain, he "would have then become the youngest captain in his team's history." This tells us Anish is younger than every captain the team has had so far, including the one currently holding the post. So the current captain must be older than Anish. This statement follows correctly from the passage.
4. **Anish is the youngest player in his team**: The passage only talks about the youngest captain, not the youngest player overall. A team can have players younger than Anish who have never been captain. So this cannot be concluded.

The only statement fully supported by the paragraph is that the current captain is older than Anish.

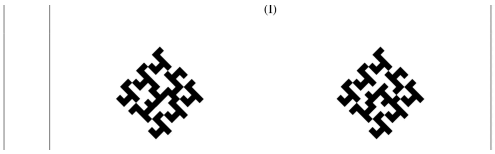
Let's summarize:

- Anish did not score a hundred runs, so the option about denied captaincy is false.
- Becoming captain was only hypothetical, so he was not already captain.
- Being the "youngest captain ever" implies he is younger than all past and current captains.
- Nothing is said about the youngest player, only the youngest captain.

So the correct answer is option (C): the current captain is older than Anish.

Quick Tip: Anish would have been the youngest ever captain, so the current captain must be older than him.

. Which one of the following figures P, Q, R, or S, correctly shows the 45° clockwise-rotated version of figure (I)?



(A) P

(B) Q

(C) R

(D) S

Correct Answer: (B) Q

Solution:

Figure (I) is an intricate zigzag pattern arranged inside a square. We need to rotate this whole pattern by 45 degrees in the clockwise direction and pick the option that matches the rotated result exactly.

1. **P**: The overall outline is turned close to the correct diamond orientation, but some of the fine zigzag teeth inside point in a direction that does not match a pure 45 degree clockwise turn of figure (I); a few teeth appear mirrored instead of rotated.
2. **Q**: The square outline of figure (I) is turned so its corners now point up, down, left, and right, forming a diamond, exactly what a 45 degree turn should do. Every zigzag tooth inside keeps the same shape as in figure (I), just turned by the same 45 degrees, with nothing flipped or mirrored.

3. **R**: The diamond outline looks right at first glance, but tracing the inner zigzag path shows a few teeth have swapped positions compared to a clean rotation of figure (I), which would not happen under a simple rotation.
4. **S**: The pattern here is turned by a different angle, not 45 degrees, since some of the straight edges of the original square are still visible instead of being turned fully into a diamond corner.

A rotation only turns a shape; it never changes its size, mirrors it, or reorders its parts. Only option Q keeps every part of the zigzag pattern in the same relative place while turning the whole figure by exactly 45 degrees clockwise.

Let's summarize:

- A correct rotation keeps every internal detail in the same relative order, just turned.
- Check the outer outline first: a 45 degree turn changes a square outline into a diamond outline.
- Then check the inner pattern is not mirrored or reordered.

Only figure Q passes both checks, so the correct answer is option (B).

Quick Tip: A rotation only turns the shape; check that the square outline becomes a diamond and no zigzag tooth is flipped.

. Match the words in Column I with their synonyms in Column II.

Column I	Column II
(i) Lonely	(p) Verbatim
(ii) Literal	(q) Solitary
(iii) Lousy	(r) Deadly
(iv) Lethal	(s) Terrible

(A) (i)-(q); (ii)-(p); (iii)-(s); (iv)-(r)

(B) (i)-(q); (ii)-(s); (iii)-(r); (iv)-(p)

(C) (i)-(s); (ii)-(p); (iii)-(q); (iv)-(r)

(D) (i)-(r); (ii)-(s); (iii)-(p); (iv)-(q)

Correct Answer: (A) (i)-(q); (ii)-(p); (iii)-(s); (iv)-(r)

Solution:

This question tests the meaning of four words in Column I by matching each with its correct synonym in Column II.

1. **Lonely**: This word describes a person or place that is isolated or without company. Its closest synonym in Column II is "Solitary" (q), which also means being alone or apart from others.
2. **Literal**: This word means following the exact words or exact meaning, without exaggeration or metaphor. Its matching synonym is "Verbatim" (p), which means word for word, in the exact original words.
3. **Lousy**: This word is an informal way of saying something is very poor in quality or unpleasant. Its matching synonym is "Terrible" (s), which also means very bad or unpleasant.
4. **Lethal**: This word describes something capable of causing death. Its matching synonym is "Deadly" (r), which means the same,

causing death or great harm.

Putting these together gives the pairing (i)-(q); (ii)-(p); (iii)-(s); (iv)-(r), which is option (A).

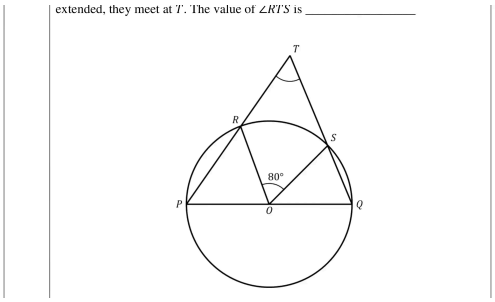
Let's summarize:

- Lonely means solitary, both describe being alone.
- Literal means verbatim, both describe an exact word-for-word sense.
- Lousy means terrible, both describe poor quality.
- Lethal means deadly, both describe something that can kill.

So the correct matching is (i)-(q); (ii)-(p); (iii)-(s); (iv)-(r), which corresponds to option (A).

Quick Tip: Match each word to a synonym with the same meaning: lonely=alone, literal=exact words, lousy=bad, lethal=deadly.

. In the given figure, $\overline{PQ}$ is the diameter of a circle with center O. Two points R and S are chosen on the circle such that $\angle ROS = 80°$. When $\overline{PR}$ and $\overline{QS}$ are extended, they meet at T. The value of $\angle RTS$ is _____.



(A) 40°

(B) 50°

(C) 60°

(D) 80°

Correct Answer: (B) 50°

Solution:

Step 1: Mark the equal radii.

Since O is the center of the circle, OP, OQ, OR, and OS are all radii of the same circle, so they are equal in length. This means triangles OPR and OQS are both isosceles.

Step 2: Set up base angles for triangle OPR.

In triangle OPR, since $OP = OR$, the base angles are equal. Let

$$\angle OPR = \angle ORP = a$$

Step 3: Set up base angles for triangle OQS.

In triangle OQS, since $OQ = OS$, the base angles are equal. Let

$$\angle OQS = \angle OSQ = b$$

Step 4: Use the straight line PQ through O.

Since P, O, Q lie on a straight line (as PQ is a diameter), the three angles $\angle POR$, $\angle ROS$, and $\angle SOQ$ together make a straight angle:

$$\angle POR + \angle ROS + \angle SOQ = 180^\circ$$

In triangle OPR, $\angle POR = 180^\circ - 2a$, and in triangle OQS, $\angle SOQ = 180^\circ$

$-2b$.

Step 5: Substitute and simplify.

$$(180^\circ - 2a) + 80^\circ + (180^\circ - 2b) = 180^\circ$$

$$440^\circ - 2a - 2b = 180^\circ$$

$$a + b = 130^\circ$$

Step 6: Use triangle PTQ.

Since R lies on line PT and S lies on line QT, the angle $\angle RTS$ is the same as $\angle PTQ$. Also, $\angle TPQ = \angle OPR = a$ and $\angle TQP = \angle OQS = b$, since O lies on segment PQ. In triangle PTQ, the angles sum to 180°:

$$a + b + \angle PTQ = 180^\circ$$

$$130^\circ + \angle PTQ = 180^\circ$$

$$\angle PTQ = 50^\circ$$

Step 7: Final conclusion.

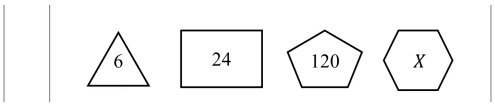
Since $\angle RTS = \angle PTQ$,

$$\boxed{\angle RTS = 50^\circ}$$

Hence, the correct option is (B).

Quick Tip: Use radii OP=OR=OS=OQ to form isosceles triangles, or apply the external angle formula for two secants from T.

. Based on the relationship between each polygon and the number inside it, the value of 'X' is _____. A triangle contains the number 6, a rectangle contains 24, a pentagon contains 120, and a hexagon contains X.



(A) 720

(B) 596

(C) 24

(D) 240

Correct Answer: (A) 720

Solution:

Step 1: List the number of sides of each polygon.

The triangle has 3 sides, the rectangle has 4 sides, the pentagon has 5 sides, and the hexagon has 6 sides.

Step 2: Compare each side count with its number.

Triangle: 3 sides, number shown is 6. Note that

$$3! = 3 \times 2 \times 1 = 6$$

Rectangle: 4 sides, number shown is 24. Note that

$$4! = 4 \times 3 \times 2 \times 1 = 24$$

Pentagon: 5 sides, number shown is 120. Note that

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Step 3: Identify the pattern.
In every case, the number inside the polygon equals the factorial of the number of sides of that polygon.
Step 4: Apply the pattern to the hexagon.
The hexagon has 6 sides, so its value should be

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$6! = 720$$

Step 5: Final conclusion.
Therefore,

$$\boxed{X = 720}$$

Hence, the correct option is (A).

Quick Tip: Compare the number of sides of each polygon with the number written inside it.

. Consider a linear arrangement of seven bulbs, each of which can be in the ON or OFF state. The initial configuration of the bulbs is shown below. In every step, the states of the bulbs change based on the following rules:

- Any OFF bulb with exactly one ON neighbor at the end of the previous step is turned ON.
- Any ON bulb with both neighbors ON at the end of the previous step is turned OFF.

- The state of any bulb not meeting the conditions above is left unchanged.

The states of the bulbs at the end of Step 1 and Step 2 are also shown below.

Stage	Bulb 1	Bulb 2	Bulb 3	Bulb 4	Bulb 5	Bulb 6	Bulb 7
Initial	OFF	OFF	OFF	ON	OFF	OFF	OFF
Step 1	OFF	OFF	ON	ON	ON	OFF	OFF
Step 2	OFF	ON	ON	OFF	ON	ON	OFF

The number of bulbs which are ON at the end of Step 8 is _____

(A) 5

(B) 4

(C) 3

(D) 0

Correct Answer: (B) 4

Solution:

Step 1: Understand the update rule.
Each bulb looks only at its own previous state and the previous state of its immediate neighbors. An OFF bulb turns ON only when exactly one of its neighbors was ON. An ON bulb turns OFF only when both of its neighbors were ON. In every other case the bulb keeps its old state. The two end bulbs, bulb 1 and bulb 7, have only one neighbor each, so an end bulb can never see "both neighbors ON" and, once it turns ON, it stays ON forever.
Step 2: Confirm the rule against the given data.
The initial row is OFF, OFF, OFF, ON, OFF, OFF, OFF, with only bulb 4 ON. Bulb 3 has one ON neighbor, bulb 4, so it turns ON. Bulb 5 has

one ON neighbor, bulb 4, so it turns ON. Bulb 4 itself has both neighbors OFF, so it is left unchanged and stays ON. Every other bulb has zero ON neighbors and stays OFF. This gives OFF, OFF, ON, ON, ON, OFF, OFF, matching the given Step 1 row, so the rule is confirmed.

Step 3: Advance from Step 1 to Step 2.

Step 1 is OFF, OFF, ON, ON, ON, OFF, OFF. Bulb 2 has one ON neighbor, bulb 3, so it turns ON. Bulb 4 has both neighbors ON, bulb 3 and bulb 5, so it turns OFF. Bulb 6 has one ON neighbor, bulb 5, so it turns ON. Bulbs 3 and 5 each have one ON neighbor and one OFF neighbor, so they stay ON. This gives OFF, ON, ON, OFF, ON, ON, OFF, which matches the given Step 2 row.

Step 4: Advance from Step 2 to Step 3.

Step 2 is OFF, ON, ON, OFF, ON, ON, OFF. Bulb 1 has one ON neighbor, bulb 2, so it turns ON, and being an end bulb it now stays ON forever. Bulb 7 similarly turns ON and stays ON forever. Bulb 4 has both neighbors ON, bulb 3 and bulb 5, so this is not "exactly one" and bulb 4 stays OFF. Bulbs 2, 3, 5, 6 each already ON have only one ON neighbor, so they stay ON. Step 3 becomes ON, ON, ON, OFF, ON, ON, ON, with 6 bulbs ON.

Step 5: Advance from Step 3 to Step 4.

Step 3 is ON, ON, ON, OFF, ON, ON, ON. Bulb 2 has both neighbors ON, bulb 1 and bulb 3, so it turns OFF. Bulb 6 has both neighbors ON, bulb 5 and bulb 7, so it turns OFF. Bulb 3 has one ON neighbor, bulb 2, and one OFF neighbor, bulb 4, so it stays ON. Bulb 5 similarly stays ON. Bulb 4 has both neighbors ON, bulb 3 and bulb 5, so it is not "exactly one" and stays OFF. Bulbs 1 and 7 stay ON as end bulbs. Step 4 becomes ON, OFF, ON, OFF, ON, OFF, ON, with 4 bulbs ON.

Step 6: Check whether Step 4 is a stable configuration.

Apply the rule to ON, OFF, ON, OFF, ON, OFF, ON again. Bulb 2 (OFF) has both neighbors ON, bulb 1 and bulb 3, so it is not "exactly

one" and stays OFF. Bulb 3 (ON) has both neighbors OFF, bulb 2 and bulb 4, so it stays ON. Bulb 4 (OFF) has both neighbors ON, so it stays OFF. Bulb 5 (ON) has both neighbors OFF, so it stays ON. Bulb 6 (OFF) has both neighbors ON, so it stays OFF. Bulbs 1 and 7 stay ON. The result is exactly ON, OFF, ON, OFF, ON, OFF, ON again, identical to Step 4. So this alternating pattern is a fixed point of the rule: once reached, it never changes again.

Step 7: Final Answer.

Since the pattern became fixed at Step 4 and stays unchanged for every later step, Step 5, Step 6, Step 7, and Step 8 are all identical to Step 4, namely ON, OFF, ON, OFF, ON, OFF, ON. The number of bulbs ON is 4 (bulbs 1, 3, 5, and 7).

$\boxed{4}$

Quick Tip: Simulate the ON/OFF rule step by step; the pattern reaches a fixed alternating state well before Step 8, so counting the ON bulbs there gives the answer directly.

. P and Q are two positive integers such that $P^2 = Q^2 + 13$.

The product of the numbers P and Q is _____

(A) 13

(B) 26

(C) 39

(D) 42

Correct Answer: (D) 42

Solution:

Step 1: Rearrange the equation as a difference of squares.

We are given

$$P^2 = Q^2 + 13$$

which rearranges to

$$P^2 - Q^2 = 13$$

Using the identity $a^2 - b^2 = (a - b)(a + b)$, this factors as

$$(P - Q)(P + Q) = 13$$

Step 2: Use the fact that 13 is prime.

Since P and Q are positive integers and $P^2 > Q^2$, we know $P > Q$, so $P - Q$ is a positive integer, and $P + Q$ is also a positive integer, larger than $P - Q$. Their product is 13, a prime number, so the only way to write 13 as a product of two positive integers is 1×13. Because $P + Q > P - Q$, we must assign

$$P - Q = 1, \qquad P + Q = 13$$

Step 3: Solve the two equations together.

Adding the two equations,

$$2P = 14 \implies P = 7$$

Subtracting the first from the second,

$$2Q = 12 \implies Q = 6$$

Step 4: Verify the solution.

Check $P^2 - Q^2 = 49 - 36 = 13$, which matches the given equation, so $P = 7$ and $Q = 6$ are correct.

Step 5: Compute the required product.

$$P \times Q = 7 \times 6 = 42$$

Step 6: Final Answer.

The value 13 in option (A) is only the given difference of squares, not the product asked for. Options (B) and (C), 26 and 39, are just 13 multiplied by 2 and by 3, with no basis in the actual solved values of P and Q. The correct product of the two integers is

$$\boxed{42}$$

Quick Tip: Write P squared minus Q squared as a difference of squares and use that 13 is a prime number to fix P minus Q and P plus Q.

. Consider the infinite-length, discrete-time sequence $x[n] = 0.9^{|n|}$, where n is an integer. The region of convergence of its Z-transform $X(z)$ is given by:

(Note: z is a complex variable)

(A) $|z| > 0.9$

(B) $|z| < 0.9$

(C) $0.9 < |z| < 1/0.9$

(D) $\{z \text{ such that } |z| < 0.9\} \cup \{z \text{ such that } |z| > 1/0.9\}$

Correct Answer: (C) $0.9 < |z| < 1/0.9$

Solution:

Step 1: Split the two-sided sequence into a right-sided and a left-sided part.

Since $x[n] = 0.9^{|n|}$, write $x[n] = x_1[n] + x_2[n]$, where

$$x_1[n] = 0.9^n u[n]$$

is the causal (right-sided) part, and

$$x_2[n] = 0.9^{-n} u[-n-1]$$

is the anticausal (left-sided) part for $n \leq -1$. At $n = 0$, $x_2[0] = 0$ and $x_1[0] = 1 = x[0]$, so the decomposition is exact.

Step 2: Find the Z-transform and ROC of the causal part.

$$X_1(z) = \sum_{n=0}^{\infty} 0.9^n z^{-n} = \frac{1}{1 - 0.9z^{-1}}$$

This geometric series converges when $|0.9z^{-1}| < 1$, that is,

$$|z| > 0.9$$

Step 3: Find the Z-transform and ROC of the anticausal part.

Writing $n = -m$ for $m \geq 1$,

$$X_2(z) = \sum_{n=-\infty}^{-1} 0.9^{-n} z^{-n} = \sum_{m=1}^{\infty} (0.9z)^m$$

This geometric series converges when $|0.9z| < 1$, that is,

$$|z| < \frac{1}{0.9}$$

Step 4: Combine the two regions.

Since $x[n] = x_1[n] + x_2[n]$, the transform $X(z)$ exists only where BOTH series converge at the same time, so the overall ROC is the intersection of $|z| > 0.9$ and $|z| < 1/0.9$, which gives the ring (annulus)

$$0.9 < |z| < \frac{1}{0.9}$$

Because $0.9 < 1/0.9$, this annulus is non-empty, exactly as expected for a genuinely two-sided decaying sequence.

Step 5: Rule out the other options.

Option (A), $|z| > 0.9$, is only the ROC of the causal half of the signal by itself, not the full two-sided sequence. Option (B), $|z| < 0.9$, is not even the ROC of either half. Option (D) takes a UNION of the two half-plane conditions rather than an intersection; a union would allow points where only one of the two series converges, but the actual $X(z)$ requires both parts of the real, physically existing two-sided sequence to converge simultaneously, so a union is not correct here.

Step 6: Final Answer.

$$\boxed{0.9 < |z| < 1/0.9}$$

Quick Tip: Split the two-sided exponential sequence into a right-sided and a left-sided part, find the ROC of each, and intersect them.

. Let $x_c(t)$ be any continuous-time periodic signal with period T. It is sampled uniformly with a sampling period T_s where $T_s \neq T$, resulting in the discrete sequence

$$x[n] = x_c(nT_s),$$

where n is an integer.

Which one of the following statements is correct about $x[n]$?

(A) $x[n]$ will always be periodic with period T/T_s for all values of T/T_s

(B) $x[n]$ will always be periodic with period 1 for all values of T/T_s

(C) $x[n]$ will never be periodic

(D) $x[n]$ will be periodic if and only if T/T_s is a rational number

Correct Answer: (D) $x[n]$ will be periodic if and only if T/T_s is a rational number

Solution:

Step 1: Recall what it means for a discrete sequence to be periodic.
$x[n]$ is periodic with period N (a positive integer) if

$$x[n+N] = x[n] \quad \text{for all } n$$

Step 2: Substitute the sampling relation.

$$x[n+N] = x_c\big((n+N)T_s\big) = x_c(nT_s + NT_s)$$

Since $x_c(t)$ is continuous-time periodic with period T, we have $x_c(nT_s + NT_s) = x_c(nT_s)$ exactly when NT_s is an integer multiple of T, that is,

$$NT_s = kT \quad \text{for some integer } k$$

Step 3: Rearrange the periodicity condition.

$$N = k\left(\frac{T}{T_s}\right)$$

For an integer N to exist satisfying this for some integer k, T/T_s must equal a ratio of two integers, that is, T/T_s must be a RATIONAL number. If $T/T_s = p/q$ in lowest terms, choosing $k = q$ gives $N = p$, an integer period.

Step 4: Show irrational T/T_s gives no periodicity.

If T/T_s is irrational, no pair of integers N, k (other than the trivial $N = 0$) can satisfy $N = k(T/T_s)$, because an irrational number can never equal a ratio of two integers. So $x[n]$ never exactly repeats and is not periodic in this case.

Step 5: Check the given options.

Option (A) claims $x[n]$ is always periodic with period T/T_s for every value of T/T_s, but T/T_s need not even be an integer, and periodicity fails entirely when T/T_s is irrational, so (A) is false. Option (B) claims the period is always 1, which would mean $x[n]$ is constant, not true in general. Option (C) is false because Step 3 shows periodicity does occur whenever T/T_s is rational. Only option (D) matches both Step 3 and Step 4.

Step 6: Final Answer.

$\boxed{x[n] \text{ is periodic if and only if } T/T_s \text{ is rational}}$

Quick Tip: A sampled version of a periodic continuous-time signal repeats only when the sampling period fits an exact whole number of times into a whole number of source periods.

. The Laplace transform of the step response of a system is given by

$$Y(s) = \frac{100}{s(s+100)}$$

The rise time is defined as the time taken for the response to go from 0.1 to 0.9 of its final value. The settling time is defined as the time taken for the response to reach 0.98 of its final value.

For this system, the rise time (T_r), settling time (T_s), and time constant (T_c), all expressed in seconds, are

(A) $T_r = 0.022,\ T_s = 0.04,\ T_c = 0.01$

(B) $T_r = 0.22,\ T_s = 0.404,\ T_c = 0.01$

(C) $T_r = 2.2,\ T_s = 4.04,\ T_c = 1.01$

(D) $T_r = 22,\ T_s = 40.4,\ T_c = 10.1$

Correct Answer: (A) $T_r = 0.022,\ T_s = 0.04,\ T_c = 0.01$

Solution:

Step 1: Identify the time constant of the system.

$$Y(s) = \frac{100}{s(s+100)} = \frac{1}{s} \cdot \frac{100}{s+100}$$

so the plant transfer function is $G(s) = 100/(s + 100)$. Writing this in the standard first order form $1/(T_c s + 1)$:

$$\frac{100}{s + 100} = \frac{1}{\frac{1}{100}s + 1}$$

so the time constant is

$$T_c = \frac{1}{100} = 0.01 \text{ s}$$

Step 2: Find the time-domain step response.
Using partial fractions,

$$Y(s) = \frac{1}{s} - \frac{1}{s + 100}$$

so

$$y(t) = 1 - e^{-100t} = 1 - e^{-t/T_c}$$

This rises from 0 toward a final value of 1, with time constant T_c $= 0.01$ s.
Step 3: Compute the rise time.
Solve $1 - e^{-t_1/T_c} = 0.1$, giving $e^{-t_1/T_c} = 0.9$, so $t_1 = T_c \ln(1/0.9) = 0.1054$ T_c. Solve $1 - e^{-t_2/T_c} = 0.9$, giving $e^{-t_2/T_c} = 0.1$, so $t_2 = T_c \ln(10)$ $= 2.3026\, T_c$. Then

$$T_r = t_2 - t_1 = (2.3026 - 0.1054)T_c \approx 2.2\, T_c$$

Substituting $T_c = 0.01$:

$$T_r \approx 2.2 \times 0.01 = 0.022 \text{ s}$$

Step 4: Compute the settling time.

Solve $1 - e^{-T_s/T_c} = 0.98$, giving $e^{-T_s/T_c} = 0.02$, so $T_s = T_c \ln(50) \approx 3.91\ T_c$, commonly rounded to $4\ T_c$. Substituting $T_c = 0.01$:

$$T_s \approx 0.0391 \approx 0.04 \text{ s}$$

Step 5: Final Answer.

So $T_c = 0.01$ s, $T_r \approx 0.022$ s, and $T_s \approx 0.04$ s, matching option (A). Options (B), (C), and (D) scale these values by factors of 10, 100, and 1000, which is what you would get by mistakenly reading the pole location as $s = 10$, $s = 1$, or $s = 0.1$ instead of the correct $s = 100$.

$$\boxed{T_r = 0.022 \text{ s},\ T_s = 0.04 \text{ s},\ T_c = 0.01 \text{ s}}$$

Quick Tip: Find the time constant from the pole location, then use the standard 2.2 times Tc rise time and about 4 times Tc settling time formulas for a first-order step response.

. Consider the following differential equation:

$$t^2 \frac{d^2y}{dt^2} + 7t \frac{dy}{dt} + 8ty = 10 \sin(t)$$

Which one of the following options is correct?

(A) It is a linear differential equation

(B) It is a nonlinear differential equation

(C) It is a time-invariant differential equation

(D) It is a second-order partial differential equation

Correct Answer: (A) It is a linear differential equation

Solution:

Step 1: Check whether the equation is ordinary or partial.

The unknown function y appears only as $y(t)$, a function of the single independent variable t, and only ordinary derivatives d^2y/dt^2 and dy/dt appear, never partial derivatives with respect to two or more variables. So this is an ordinary differential equation, not a partial differential equation, which rules out option (D).

Step 2: Check linearity.

A differential equation in y is linear when y and all of its derivatives appear only to the first power, are never multiplied together, and never pass through a nonlinear function such as y^2, $\sin(y)$, or $y \cdot y'$; the coefficients multiplying y, y', y'' are allowed to depend on the independent variable t. Here, y'', y', and y each appear to the first power, multiplied only by coefficients t^2, $7t$, $8t$ that depend on t, and the right-hand side $10\sin(t)$ is a known function of t only, not of y. So the equation is linear. This rules out option (B) and confirms option (A).

Step 3: Check time invariance.

A linear differential equation is time-invariant only when every coefficient multiplying y and its derivatives is a CONSTANT, not depending on t. Here the coefficients t^2, $7t$, and $8t$ explicitly depend on t, so this equation is linear but time-VARYING, not time-invariant. This rules out option (C). This particular equation is in fact a form of

the Euler-Cauchy (equidimensional) equation, a classic example of a linear, time-varying ODE.

Step 4: Final Answer.

Since the equation is linear in y and its derivatives, option (A) is the correct statement.

Linear differential equation

Quick Tip: Check if y and its derivatives appear only to the first power for linearity, separately from whether the coefficients depend on t for time invariance.

. A uniform ring charge of radius R carries a total charge Q. Which one of the following options correctly quantifies the magnitude of the force on a point charge of strength q kept at the center of the ring?
(ϵ is the permittivity of the medium)

(A) $\frac{Qq}{4\pi\epsilon R}$

(B) $\frac{Qq}{4\pi\epsilon R^2}$

(C) 0

(D) $\frac{q}{4\pi\epsilon R} \times \frac{Q}{2\pi R}$

Correct Answer: (C) 0

Solution:

Step 1: Set up the ring using symmetry.

Consider the uniform ring of radius R with total charge Q spread

evenly around its circumference, and the point charge q placed exactly at the geometric center O of the ring.

Step 2: Consider one small charge element on the ring.

Pick a small element of charge dq on the ring at some angle θ. By Coulomb's law, this element produces a field at the center of magnitude $dE = dq/(4\pi\epsilon R^2)$, directed along the line joining that element to the center.

Step 3: Find the diametrically opposite element.

Directly across the ring at angle $\theta + 180°$, there is another element carrying the same charge dq, since the ring is uniform. It produces a field at the center of exactly the same magnitude $dq/(4\pi\epsilon R^2)$, but pointing exactly opposite to the field from the first element, since the two elements sit on opposite ends of a diameter through the center.

Step 4: Cancel every such pair.

Since the ring is uniformly charged, every element has a diametrically opposite twin producing an equal and opposite field at the center. Adding up all such pairs around the full ring, every contribution cancels exactly, so the net electric field at the center of a uniformly charged ring is zero.

Step 5: Apply this to the force on q.

The force on the point charge is $F = qE_{center}$. Since $E_{center} = 0$, the force is $F = 0$, regardless of the values of Q, q, R, or ϵ.

Step 6: Final Answer.

Options (A) and (B) look like they come from the point-charge formula $Qq/(4\pi\epsilon R)$ or $Qq/(4\pi\epsilon R^2)$, as if the ring's charge Q were concentrated at a single point at distance R, which is not valid here since the ring's charge is spread out and the contributions cancel by symmetry rather than add up. Option (D) combines mismatched terms that do not correspond to any correct formula for this configuration.

$\boxed{0}$

Quick Tip: By symmetry, every charge element on a uniform ring has a diametrically opposite twin whose field at the center exactly cancels it.

. A positive point charge with velocity $\vec{v} = 5\hat{x}$ enters a region having electric field $\vec{E} = 4\hat{y}$ and magnetic field $\vec{B} = -6\hat{z}$. Which one of the following statements is correct for the force on the charge as it enters the region?

(A) The force will be along the magnetic field but perpendicular to the electric field

(B) The force will be along the electric field but perpendicular to the magnetic field

(C) The force will be perpendicular to both electric and magnetic field

(D) The magnetic field does not exert any force on the charge

Correct Answer: (B) The force will be along the electric field but perpendicular to the magnetic field

Solution:

Step 1: Recall the Lorentz force law.

The total electromagnetic force on a charge q moving with velocity $\vec{v}$ in fields $\vec{E}$ and $\vec{B}$ is

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Step 2: Compute $\vec{v} \times \vec{B}$ using the determinant method.

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 5 & 0 & 0 \\ 0 & 0 & -6 \end{vmatrix}$$

Expanding: the $\hat{x}$ component is $(0)(-6) - (0)(0) = 0$; the $\hat{y}$ component is $-[(5)(-6) - (0)(0)] = 30$; the $\hat{z}$ component is $(5)(0) - (0)(0) = 0$. So

$$\vec{v} \times \vec{B} = 30\,\hat{y}$$

Step 3: Add the electric force term.

$$\vec{F} = q\left(4\hat{y} + 30\hat{y}\right) = q(34\hat{y}) = 34q\,\hat{y}$$

Step 4: Interpret the direction of the total force.

The net force $34q\hat{y}$ points purely along $\hat{y}$, which is exactly the direction of the electric field $\vec{E} = 4\hat{y}$, so the force is along the electric field. Since $\hat{y}$ is perpendicular to $\hat{z}$, the direction of $\vec{B}$, the force is also perpendicular to the magnetic field.

Step 5: Rule out the other options.

Option (A) says the force is along $\vec{B}$ (the $\hat{z}$ direction), but the computed force has zero $\hat{z}$ component, so (A) is wrong. Option (C) says the force is perpendicular to both fields, but the force is actually parallel to $\vec{E}$, not perpendicular to it, so (C) is wrong. Option (D) says the magnetic field exerts no force, but the term $\vec{v} \times \vec{B} = 30\hat{y}$ is clearly nonzero; it only happens that this magnetic force adds constructively along the same direction as the electric force, which does not mean the magnetic force is absent, so (D) is wrong.

Step 6: Final Answer.

$$\boxed{\text{Force is along } \vec{E},\ \text{perpendicular to } \vec{B}}$$

Quick Tip: Apply F = q(E + v cross B), compute v cross B first, then compare the resulting direction with E and with B.

. A 15 kVA, 1100 V/220 V, single-phase two-winding transformer is configured as a 1.32 kV/1.1 kV autotransformer.

What will be the rating of the autotransformer?

(A) 60 kVA

(B) 75 kVA

(C) 90 kVA

(D) 100 kVA

Correct Answer: (C) 90 kVA

Solution:

Step 1: Identify how the two-winding transformer is reconnected.
The original two-winding transformer is rated 1100 V/220 V. As an autotransformer, its two windings are connected in series so that the combined winding spans $1100 + 220 = 1320\ \text{V} = 1.32\ \text{kV}$, and the tap between the two windings gives the 1.1 kV output. So the 1100 V winding acts as the common winding, and the 220 V winding acts as the series winding.
Step 2: Recall the standard autotransformer rating formula.
For an autotransformer built from a two-winding transformer of rating S_{2w}, with high-side voltage V_1 and low-side voltage V_2, the autotransformer's throughput rating is

$$S_{auto} = S_{2w} \times \frac{V_1}{V_1 - V_2}$$

This formula reflects that the series winding, unchanged in current rating from its two-winding operation, now carries the full autotransformer's high side line current, and this current flows through a much larger voltage V_1 at the input terminal, not just its own 220 V rated voltage.

Step 3: Substitute the given values.

$$V_1 = 1.32\ \text{kV}, \qquad V_2 = 1.1\ \text{kV}, \qquad V_1 - V_2 = 0.22\ \text{kV}$$

$$S_{auto} = 15 \times \frac{1.32}{0.22} = 15 \times 6 = 90\ \text{kVA}$$

Step 4: Sanity check using the winding current.

The two-winding transformer's 220 V winding is rated for a current of $15000/220 = 68.18$ A. As the series winding of the autotransformer, this same current of 68.18 A now flows in from the 1.32 kV line terminal, giving an autotransformer throughput of $1320 \times 68.18 = 90{,}000$ VA $= 90$ kVA, confirming Step 3.

Step 5: Rule out the other options.

Option (A), 60 kVA, would come from using the ratio $V_2/(V_1 - V_2) = 1.1/0.22 = 5$ instead of $V_1/(V_1 - V_2)$, understating the true throughput. Option (B), 75 kVA, and option (D), 100 kVA, do not follow from any correct combination of the given voltages and the 15 kVA rating.

Step 6: Final Answer.

$$\boxed{90\ \text{kVA}}$$

Quick Tip: The 1100 V winding becomes the common winding and the 220 V winding becomes the series winding; multiply the two-winding rating by V1/(V1-V2), not V2/(V1-V2).

. Consider a power system with N buses, of which P are generator buses and the remaining Q are load buses (where there is no generation).

Assume that there are no reactive power-limit violations at the generator buses.
What is the size of the Jacobian matrix in the Newton-Raphson load flow method?

(A) $2N \times 2N$

(B) $(2N-1-P) \times (2N-1-P)$

(C) $(2N-Q) \times (2N-Q)$

(D) $(P+Q) \times (P+Q)$

Correct Answer: (B) $(2N-1-P) \times (2N-1-P)$

Solution:

Step 1: Set up the bus classification.
One of the P generator buses is the slack (reference) bus; the remaining $P-1$ generator buses are PV buses. The Q load buses are PQ buses. All buses together number $N = P + Q$.
Step 2: Count the voltage angle unknowns.
Every bus except the slack bus has an unknown voltage angle in Newton-Raphson load flow, since only its scheduled real power is fixed. This gives

$$N-1$$

angle unknowns.

Step 3: Count the voltage magnitude unknowns.

Only the PQ (load) buses have an unknown voltage magnitude, since every generator bus (the slack and every PV bus) has its voltage magnitude specified and held fixed (no reactive power-limit violation means no PV bus is ever converted to a PQ bus during the iteration). This gives exactly

$$Q$$

magnitude unknowns.

Step 4: Add the two counts.

Total number of unknowns, which equals the size of the (square) Jacobian matrix:

$$(N-1)+Q$$

Step 5: Substitute $Q = N - P$.

Since $N = P + Q$, we have $Q = N - P$. Substituting,

$$(N-1)+(N-P)=2N-1-P$$

Step 6: Rule out the other options.

Option (A), $2N \times 2N$, overcounts by treating every bus as having both an unknown angle and an unknown magnitude, ignoring that generator buses have a fixed voltage magnitude. Option (C), $(2N - Q) \times (2N - Q)$, does not simplify to the correct count; substituting $Q = N$

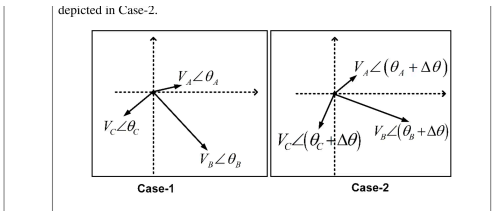
$-P$ gives $2N - Q = N + P$, which is too large. Option (D), $(P + Q) \times (P + Q) = N \times N$, also does not match, since it does not separate angle unknowns from magnitude unknowns correctly.

Step 7: Final Answer.

$$\boxed{(2N - 1 - P) \times (2N - 1 - P)}$$

Quick Tip: Count angle unknowns (all buses except slack, giving N-1) and magnitude unknowns (only load buses, giving Q=N-P), then add and simplify using N=P+Q.

. The initial three-phase voltage phasors ($\bar{V}_A$, $\bar{V}_B$, and $\bar{V}_C$) at a bus of a power network are as shown in Case-1. Due to a disturbance, the bus voltage phasors changed in phase by a small angle $\Delta\theta$, and the magnitudes remained the same as depicted in Case-2.



Which one of the following statements is correct about the zero sequence components?

(A) The zero sequence components in Case-1 and Case-2 have the same phase angle and magnitude

(B) The magnitude of the zero sequence component in Case-1 is greater than that in Case-2

(C) The magnitude of the zero sequence component in Case-2 is greater than that in Case-1

(D) The zero sequence components in Case-1 and Case-2 have the same magnitude but different phase angles

Correct Answer: (D) The zero sequence components in Case-1 and Case-2 have the same magnitude but different phase angles

Solution:

Step 1: Write the zero sequence component as a phasor sum.

For any three-phase set, the zero sequence component is

$$\bar{V}_0 = \frac{1}{3}\left(\bar{V}_A + \bar{V}_B + \bar{V}_C\right)$$

This is defined even when the three phasors are not balanced (not equal in magnitude or not exactly 120° apart), which is important here since Case-1's phasors, as drawn, are not necessarily a perfectly balanced set.

Step 2: Express Case-2's phasors in terms of Case-1's.

Each Case-2 phasor keeps the same magnitude as its Case-1 counterpart but is rotated by the same angle $\Delta\theta$:

$$\bar{V}'_A = \bar{V}_A e^{j\Delta\theta}, \qquad \bar{V}'_B = \bar{V}_B e^{j\Delta\theta}, \qquad \bar{V}'_C = \bar{V}_C e^{j\Delta\theta}$$

since multiplying a phasor by $e^{j\Delta\theta}$ rotates it by $\Delta\theta$ while leaving its

magnitude unchanged.

Step 3: Compute the zero sequence component in Case-2.

$$\bar{V}_0' = \frac{1}{3}\left(\bar{V}_A' + \bar{V}_B' + \bar{V}_C'\right) = \frac{1}{3}\left(\bar{V}_A e^{j\Delta\theta} + \bar{V}_B e^{j\Delta\theta} + \bar{V}_C e^{j\Delta\theta}\right)$$

Factor out the common term $e^{j\Delta\theta}$:

$$\bar{V}_0' = e^{j\Delta\theta} \cdot \frac{1}{3}\left(\bar{V}_A + \bar{V}_B + \bar{V}_C\right) = e^{j\Delta\theta}\,\bar{V}_0$$

Step 4: Interpret this result.

The Case-2 zero sequence phasor is simply the Case-1 zero sequence phasor multiplied by $e^{j\Delta\theta}$. Multiplying by $e^{j\Delta\theta}$ rotates a phasor by angle $\Delta\theta$ without changing its magnitude at all, since $|e^{j\Delta\theta}| = 1$. So

$$|\bar{V}_0'| = |\bar{V}_0|$$

while the phase angle of $\bar{V}_0'$ is that of $\bar{V}_0$ shifted by $\Delta\theta$.

Step 5: Rule out the other options.

Option (A) claims the phase angle also stays the same, but Step 4 shows it shifts by $\Delta\theta$ (which is nonzero, since the disturbance genuinely changes the phase). Options (B) and (C) both claim the magnitudes differ between the two cases, but Step 4 shows they are exactly equal.

Step 6: Final Answer.

Same magnitude, different phase angles

Quick Tip: Rotating all three phase voltages by the same angle rotates their zero-sequence average by that same angle too, leaving its magnitude unchanged.

. A certain application requires power at a frequency of 16.67 Hz, while the available grid frequency is 50 Hz. A 3-phase synchronous motor connected to the 50 Hz, 3-phase grid, driving a synchronous generator, is used for this application.

Which one of the following combinations is a suitable choice for the number of poles in the motor and generator, respectively?

(A) 2, 6

(B) 6, 2

(C) 4, 6

(D) 6, 4

Correct Answer: (B) 6, 2

Solution:

Step 1: Recall that the motor and generator share the same mechanical shaft speed.

Since the synchronous motor drives the synchronous generator directly, both machines rotate at the same synchronous speed N_s, in rpm.

Step 2: Write the synchronous speed formula for each machine.

For the motor, running on the 50 Hz grid with P_m poles:

$$N_s = \frac{120 \times 50}{P_m}$$

For the generator, which must produce 16.67 Hz with P_g poles, running at this same N_s:

$$N_s = \frac{120 \times 16.67}{P_g}$$

Step 3: Equate the two expressions for N_s.

$$\frac{120 \times 50}{P_m} = \frac{120 \times 16.67}{P_g}$$

Cancelling 120 from both sides:

$$\frac{50}{P_m} = \frac{16.67}{P_g} \implies \frac{P_m}{P_g} = \frac{50}{16.67} \approx 3$$

Step 4: Test option (A), 2 poles motor, 6 poles generator.

$$\frac{P_m}{P_g} = \frac{2}{6} = \frac{1}{3}$$

This is the reciprocal of what is needed; it would make the generator frequency three times the grid frequency, not one-third of it. Incorrect.

Step 5: Test option (B), 6 poles motor, 2 poles generator.

$$\frac{P_m}{P_g} = \frac{6}{2} = 3$$

This matches the required ratio from Step 3. Checking directly: $N_s = 120 \times 50/6 = 1000$ rpm; generator frequency $= N_s \times P_g/120 = 1000 \times 2/120 = 16.67$ Hz. This matches exactly. Correct.

Step 6: Rule out options (C) and (D).

Option (C), 4 and 6 poles, gives $N_s = 120 \times 50/4 = 1500$ rpm, and generator frequency $= 1500 \times 6/120 = 75$ Hz, far from 16.67 Hz.

Option (D), 6 and 4 poles, gives $N_s = 1000$ rpm and generator frequency $= 1000 \times 4/120 = 33.3$ Hz, also not matching.

Step 7: Final Answer.

$$\boxed{\text{6 poles (motor), 2 poles (generator)}}$$

Quick Tip: Both machines share the same shaft speed; set the two synchronous-speed formulas equal and solve for the pole ratio, then check which option gives that ratio.

. Which one of the following options is correct regarding the typical double-squirrel-cage structure used in induction motors?

(A) At starting, a larger portion of the rotor current flows in the outer cage

(B) At starting, a larger portion of the rotor current flows in the inner cage

(C) At rated speed, most of the rotor current flows in the outer cage

(D) The purpose of the double-squirrel-cage structure is to lower the effective rotor resistance at starting

Correct Answer: (A) At starting, a larger portion of the rotor current flows in the outer cage

Solution:

Step 1: Recall the construction of the two cages.

The outer cage sits near the rotor surface, made of higher-resistance material with LOW leakage reactance (little surrounding iron). The inner cage sits deep in the rotor slots, made of lower-resistance material with HIGH leakage reactance (surrounded by much more iron).

Step 2: Compare impedances at starting.

At starting, the rotor frequency equals the full supply frequency (slip $s = 1$), which is the highest rotor frequency the machine ever sees. Reactance ($X = 2\pi fL$) is therefore at its largest possible value for both cages at this instant, and it dominates each cage's impedance far more than resistance does.

Step 3: Compute which cage has lower impedance at starting.

The outer cage has low reactance to begin with, so even at maximum rotor frequency its total impedance stays relatively small. The inner cage's reactance, already large by design, becomes very large at starting frequency, making its total impedance much bigger than the outer cage's.

Step 4: Apply current division.

Since the two cages are effectively in parallel (both linking the same air-gap flux), the cage with the smaller impedance carries the larger share of the total rotor current. At starting, this is the outer cage.

Step 5: Rule out the other options.

Option (B) claims the opposite, contradicted by Step 4. Option (C) is about running speed, not starting, where the roles reverse (at low slip and low rotor frequency, reactance becomes negligible for both cages, and the low-resistance inner cage then dominates and carries most of the current) so (C) is false as stated for starting behavior, and even at rated speed it is the inner, not outer, cage that carries most current. Option (D) misstates the purpose: the double cage does not lower the rotor resistance at starting, it effectively RAISES the starting

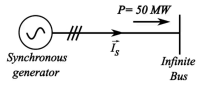
resistance seen by the machine (by forcing more current through the high-resistance outer cage), which is precisely what improves starting torque.

Step 6: Final Answer.

At starting, a larger portion of the rotor current flows in the outer cage

Quick Tip: At starting, high rotor frequency makes reactance dominate; the outer cage's naturally low reactance gives it lower impedance and hence more current than the inner cage.

. The figure shows the single-line diagram of a synchronous generator delivering $P = 50$ MW of power at unity power factor to an infinite bus.



I_S denotes the stator current phasor. If the field excitation is increased, which one of the following options correctly describes its effect on the stator current, power factor, and load angle of the machine?

(A) Stator current increases, power factor becomes lagging, load angle remains the same

(B) Stator current decreases, power factor becomes leading, load angle remains the same

(C) Stator current increases, power factor becomes lagging, load angle decreases

(D) Stator current increases, power factor becomes leading, load angle increases

Correct Answer: (C) Stator current increases, power factor becomes lagging, load angle decreases

Solution:

Step 1: Write the real power equation of the generator.

For a synchronous generator connected to an infinite bus of voltage V through synchronous reactance X_s, with excitation voltage E_f and load angle δ:

$$P = \frac{E_f V}{X_s} \sin \delta$$

Since P, V, and X_s are all held constant here (the load and the bus are unchanged), increasing E_f means

$$E_f \sin \delta = \text{constant}$$

so as E_f increases, $\sin \delta$ must decrease, which means δ decreases (for the normal range $0 < \delta < 90°$).

Step 2: Recall the generator V-curve behavior.

At the starting operating point, the machine runs at unity power factor, which is the minimum-current point on its V-curve for this value of P. Moving the excitation away from this minimum-current point in either direction (increasing OR decreasing E_f) increases the magnitude of the stator current $|I_S|$, since the V-curve is roughly U-shaped with unity power factor at its minimum. Increasing E_f moves the machine to the overexcited side of this minimum.

Step 3: Determine the power factor after overexciting.

An overexcited synchronous generator supplies reactive power to the system (behaves like a source of lagging vars from the bus's point of view), which by standard generator convention means it operates at a

LAGGING power factor. This is the mirror image of a generator's V-curve: overexcited (higher E_f) gives lagging power factor, underexcited (lower E_f) gives leading power factor.

Step 4: Combine the three effects.

Increasing the field excitation: increases $|I_S|$ (Step 2), makes the power factor lagging (Step 3), and decreases the load angle δ (Step 1).

Step 5: Rule out the other options.

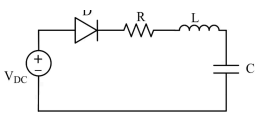
Option (A) and (D) both claim the load angle stays the same or increases, contradicting Step 1, which shows δ must decrease to keep P fixed as E_f rises. Option (B) claims the current decreases and the power factor becomes leading, but increasing excitation moves the machine toward the overexcited, lagging side, not the underexcited, leading side, and away from the unity-PF minimum-current point, so current cannot decrease.

Step 6: Final Answer.

Stator current increases, power factor becomes lagging, load angle decreases

Quick Tip: Use P = (Ef V / Xs) sin(delta) with P held fixed to see that delta must fall as Ef rises, and recall that overexciting a generator moves it to the lagging side of its V-curve, away from the unity-PF minimum-current point.

. A circuit with ideal elements is shown.



The circuit has a DC source V_{DC} in series with an ideal diode D, a resistor R, an inductor L, and a capacitor C (the capacitor is connected across the

series combination of L at the far end of the loop).
Which one of the following options correctly identifies all the linear elements in the circuit?

(A) R only

(B) R, L, and C only

(C) D only

(D) L, C, and D only

Correct Answer: (B) R, L, and C only

Solution:

Step 1: Recall what makes a circuit element linear.
An element is linear when its voltage-current relationship is proportional (or a constant-coefficient derivative/integral relation), so that scaling the current scales the voltage by the same factor, and the response to a sum of currents is the sum of the individual responses.
Step 2: Check the resistor.

$$v_R = iR$$

a direct proportionality with constant R, a textbook linear relation.
Step 3: Check the inductor.

$$v_L = L\frac{di}{dt}$$

a linear differential relation with constant coefficient L.
Step 4: Check the capacitor.

$$i_C = C\frac{dv_C}{dt}$$

similarly linear, with constant coefficient C.

Step 5: Check the diode.

An ideal diode is a switching element: it behaves like a short circuit (zero voltage drop) when forward biased and carrying current, and like an open circuit (zero current) when reverse biased, with an abrupt change of behavior at zero current/zero voltage. This piecewise, non-proportional switching behavior is fundamentally NONLINEAR; a diode never obeys a single constant-coefficient equation valid for all currents and voltages.

Step 6: Evaluate the options.

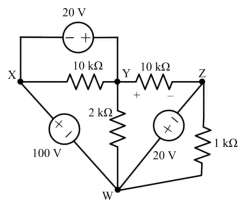
Option (A), R only, misses that L and C are also linear. Option (B), R, L, and C only, correctly includes all three passive linear elements and excludes the diode. Option (C), D only, is the opposite of the correct answer, since D is the one nonlinear element. Option (D), L, C, and D, wrongly includes the diode among the linear elements.

Step 7: Final Answer.

R, L, and C only

Quick Tip: Resistors, inductors, and capacitors all have constant-coefficient proportional v-i relationships; an ideal diode's on/off switching behavior is not proportional and is nonlinear.

. For the circuit shown, which one of the following options correctly identifies the Thevenin's equivalent parameters between nodes Y and Z?



The circuit has nodes X, Y, Z, and W. Between X and Y there is a 20 V ideal source (in parallel with a 10 kΩ resistor directly connecting X and Y). Between Y and Z there is a 10 kΩ resistor. Between Y and W there is a 2 k Ω resistor. Between X and W there is an ideal 100 V source (no series resistor in this branch). Between Z and W there is an ideal 20 V source (no series resistor in this branch) and, separately, a 1 kΩ resistor.

(A) V_{TH} = 100 V, R_{TH} = 10 kΩ

(B) V_{TH} = 140 V, R_{TH} = 0 Ω

(C) V_{TH} = 100 V, R_{TH} = 0 Ω

(D) V_{TH} = 140 V, R_{TH} = 10 kΩ

Correct Answer: (B) V_{TH} = 140 V, R_{TH} = 0 Ω

Solution:

Step 1: Take W as the reference (ground) node.

Let $V_W = 0$.

Step 2: Find V_X.

The branch between X and W is a pure ideal 100 V source with no series resistance, so it directly fixes

$$V_X = 100\ \text{V}$$

regardless of any current drawn elsewhere in the circuit, since an ideal source's terminal voltage never depends on its current.

Step 3: Find V_Y.

The branch between X and Y is a pure ideal 20 V source (in parallel with the 10 kΩ resistor, which does not affect this voltage relation since the source directly fixes the voltage across both branches). With the source's polarity giving $V_Y - V_X = 20$ V:

$$V_Y = V_X + 20 = 100 + 20 = 120 \text{ V}$$

Step 4: Find V_Z.

The branch between Z and W is a pure ideal 20 V source (separate from the 1 kΩ resistor, which is a distinct parallel branch and again does not affect this voltage). With this source's polarity giving $V_W - V_Z = 20$ V:

$$V_Z = V_W - 20 = 0 - 20 = -20 \text{ V}$$

Step 5: Compute V_{TH}.

The Thevenin voltage between Y and Z, taken with the + reference at Y and − at Z as marked in the figure, is

$$V_{TH} = V_Y - V_Z = 120 - (-20) = 140 \text{ V}$$

Step 6: Find R_{TH}.

Both V_Y and V_Z are pinned to fixed values entirely by chains of IDEAL voltage sources (with zero series resistance) tracing back to the reference node W. Since neither node's voltage depends even slightly on any current drawn from Y or Z (an ideal source's output voltage is completely independent of its load current), connecting any external load between Y and Z cannot change V_Y or V_Z at all. This means the Thevenin resistance seen from Y-Z is

$$R_{TH} = 0\ \Omega$$

Step 7: Rule out the other options.

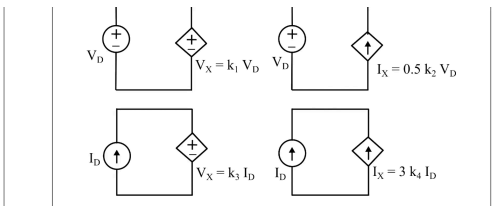
Options (A) and (D) both include a nonzero R_{TH} (10 kΩ), but this ignores that Y and Z are each rigidly pinned by an all-ideal-source path to ground, making every resistor in the circuit (10 kΩ at X-Y, 10 kΩ at Y-Z, 2 kΩ at Y-W, 1 kΩ at Z-W) irrelevant to both V_{TH} and R_{TH}. Option (C), 100 V, corresponds to only V_X (or V_Y computed incorrectly without including V_Z's contribution), missing the full $120 - (-20) = 140$ V difference.

Step 8: Final Answer.

$$\boxed{V_{TH} = 140\ \text{V},\ R_{TH} = 0\ \Omega}$$

Quick Tip: Both Y and Z are pinned directly by chains of ideal voltage sources back to the reference node, so their potentials do not depend on any external load; find each node voltage first, then subtract.

. Refer to the four circuits shown.



Which one of the following options for k_1, k_2, k_3, and k_4 makes all of them realizable?

(A) $k_1 = 1, k_4 = -\frac{1}{3}$, for all values of k_2 and k_3

(B) $k_2 = -2, k_3 = +\frac{1}{3}$, for all values of k_1 and k_4

(C) $k_1 = 2, k_2 = 0.5, k_3 = -\frac{2}{3}, k_4 = -3$

(D) $k_1 = 2, k_2 = -0.5, k_3 = -\frac{2}{3}, k_4 = +3$

Correct Answer: (A) $k_1 = 1, k_4 = -\frac{1}{3}$, for all values of k_2 and k_3

Solution:

Step 1: Look at how each pair of elements is joined.

In every one of the four circuits, the top wire and the bottom wire are plain wires with no components on them. So the left element and the right element both connect the same top node to the same bottom node. This means the two elements in each circuit are in parallel, not in series.

Step 2: Recall what parallel connection forces.

Two voltage sources connected in parallel between the same two nodes must have the same voltage, otherwise Kirchhoff's voltage law is broken. Two current sources connected in parallel between the same two nodes, feeding the node from the same side, must add up to zero at that node, otherwise Kirchhoff's current law is broken. A voltage source and a current source in parallel never conflict, because the voltage source can push or pull whatever current the current source needs, and the current source does not care what voltage appears across it.

Step 3: Analyze circuit 1, V_D parallel with $V_X = k_1 V_D$.

Both sources have the + mark on top, so they are in parallel with matching polarity. For this to hold for any nonzero V_D,

$$V_D = V_X = k_1 V_D$$

which gives

$$k_1 = 1.$$

Step 4: Analyze circuit 2, V_D parallel with $I_X = 0.5k_2 V_D$.
This is a voltage source in parallel with a current source. Whatever current the current source demands, the voltage source simply supplies or absorbs it. There is no equation linking V_D and I_X, so this circuit is realizable for every value of k_2.

Step 5: Analyze circuit 3, I_D parallel with $V_X = k_3 I_D$.
This is again a current source in parallel with a voltage source, with the roles swapped from circuit 2. The voltage source freely supplies the current I_D that the current source demands, so this circuit is realizable for every value of k_3.

Step 6: Analyze circuit 4, I_D parallel with $I_X = 3k_4 I_D$.
Both arrows point the same way into the top node, so both currents try to enter the same node through the same two branches. With nothing else connected there, Kirchhoff's current law forces

$$I_D + I_X = 0$$

so

$$I_D + 3k_4 I_D = 0 \implies k_4 = -\frac{1}{3}.$$

Step 7: Combine the results.

Circuit 1 fixes $k_1 = 1$, circuit 4 fixes $k_4 = -\frac{1}{3}$, while circuits 2 and 3 place no restriction at all on k_2 and k_3.

Step 8: Final conclusion.

Therefore all four circuits are realizable only when

$$\boxed{k_1 = 1,\ k_4 = -\frac{1}{3},\ \text{for all values of } k_2 \text{ and } k_3}$$

Hence the correct option is (A).

Quick Tip: Look at which pairs of sources sit between the same two nodes: same-type sources, V-V or I-I, in parallel must match, while mixed V-I pairs never conflict.

. A single-phase voltage source $v_s = 325\sin(2\pi 50t)$ V delivers a current, $i = 12\sin(2\pi 50t) + 9\sin(2\pi 150t)$ A to a load.

The load power factor, correct up to two decimal places, is:

(A) 1.00

(B) 0.80

(C) 0.65

(D) 0.57

Correct Answer: (B) 0.80

Solution:

Step 1: Identify why only the fundamental current matters for power.

The source voltage is a pure sinusoid at 50 Hz, $v_s = 325\sin(2\pi 50t)$. Average power is the average of the product $v_s(t)\,i(t)$ over one cycle. When a sinusoid at one frequency is multiplied by a sinusoid at a different frequency and averaged over a full period, the result is zero, because the two waves are orthogonal. So the 150 Hz component of the current, $9\sin(2\pi 150t)$, contributes nothing to the real power. Only the 50 Hz part of the current, $12\sin(2\pi 50t)$, interacts with the 50 Hz voltage.

Step 2: Compute the real power.

Both the voltage and the matching current component are pure sine terms with no phase shift between them, so they are in phase.

$$P = \frac{V_{peak}\, I_{peak,50Hz}}{2}\cos(0^\circ) = \frac{325 \times 12}{2} = 1950\ \text{W}$$

Step 3: Compute the rms voltage.

$$V_{rms} = \frac{325}{\sqrt{2}} \approx 229.81\ \text{V}$$

Step 4: Compute the rms current, including both frequencies.

For a waveform made of several sinusoids at different frequencies, the total rms value combines each term's rms value in quadrature.

$$I_{rms} = \sqrt{\left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{9}{\sqrt{2}}\right)^2} = \sqrt{\frac{144}{2} + \frac{81}{2}} = \sqrt{\frac{225}{2}} \approx 10.6066\ \text{A}$$

Step 5: Compute the apparent power.

$$S = V_{rms} \times I_{rms} = \frac{325}{\sqrt{2}} \times \sqrt{\frac{225}{2}} = 325 \times \frac{15}{2} = 2437.5 \text{ VA}$$

Step 6: Compute the load power factor.

The true power factor accounts for the full rms current, including the harmonic content, not just the fundamental.

$$\text{PF} = \frac{P}{S} = \frac{1950}{2437.5} = 0.80$$

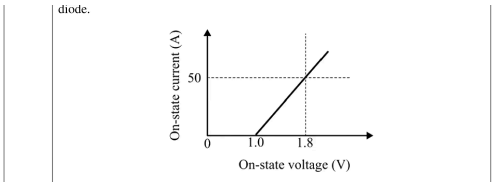
Step 7: Final conclusion.

$$\boxed{0.80}$$

Hence the correct option is (B).

Quick Tip: Only the current component at the same frequency as the source contributes to real power, but the harmonic current still adds to the rms current and lowers the power factor.

. The figure shows a straight-line approximation for the forward characteristics of a power diode. A continuous on-state current of 15 A is flowing through the diode.



What is the power loss in the diode?

(A) 32.8 W

(B) 21.2 W

(C) 18.6 W

(D) 23.1 W

Correct Answer: (C) 18.6 W

Solution:

Step 1: Read the threshold voltage and slope from the graph.
The straight-line approximation crosses the voltage axis at $V_0 = 1.0$ V, meaning the diode starts conducting only above this voltage, and it passes through the point where the on-state current is 50 A at an on-state voltage of 1.8 V.

Step 2: Find the slope of the line, which gives the inverse of the on-state resistance.

$$\text{slope} = \frac{50 - 0}{1.8 - 1.0} = \frac{50}{0.8} = 62.5 \text{ A/V}$$

So the on-state resistance is

$$R_{on} = \frac{1}{62.5} = 0.016\ \Omega$$

Step 3: Write the diode's forward voltage as a function of its current.

$$V_D = V_0 + I\,R_{on}$$

Step 4: Substitute the given on-state current of 15 A.

$$V_D = 1.0 + 15 \times 0.016 = 1.0 + 0.24 = 1.24\ \text{V}$$

Step 5: Compute the power loss.

Power loss is the product of the forward voltage drop and the current through the diode.

$$P_{loss} = V_D \times I = 1.24 \times 15 = 18.6\ \text{W}$$

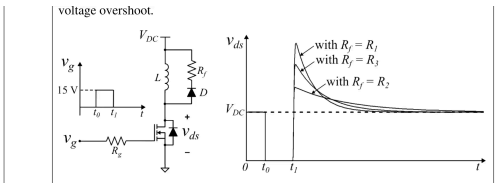
Step 6: Final conclusion.

$$\boxed{18.6\ \text{W}}$$

Hence the correct option is (C).

Quick Tip: Fit the straight line as V0 plus I times Ron using the two marked points on the graph, then multiply the voltage drop at 15 A by the current.

. Consider the circuit shown in Figure (a). A gate pulse v_g is applied between time instants t_0 and t_1. After t_1, during the MOSFET turn OFF process, it experiences a voltage overshoot.



Based on the v_{ds} waveforms shown in Figure (b), which one of the following options is correct?

(A) $R_1 > R_2 > R_3$

(B) $R_1 > R_3 > R_2$

(C) $R_3 > R_2 > R_1$

(D) $R_2 > R_3 > R_1$

Correct Answer: (B) $R_1 > R_3 > R_2$

Solution:

Step 1: Understand what happens at MOSFET turn OFF.
While the MOSFET is ON, the inductor L carries a steady current. The instant the MOSFET turns OFF, that inductor current cannot vanish immediately, so it is forced to keep flowing through the freewheeling diode D in series with the snubber resistor R_f.

Step 2: Relate the resistor value to the size of the voltage spike.
The moment the current diverts into the R_f-D branch, the voltage across R_f jumps to roughly $I_L \times R_f$, and this adds on top of V_{DC} to produce the overshoot seen on v_{ds}. A larger R_f forces a larger instantaneous voltage drop for the same trapped current, so it

produces a taller peak.

Step 3: Relate the resistor value to the decay rate.
After the initial spike, the trapped current decays with a time constant L/R_f. A larger R_f gives a smaller time constant, so the curve settles back down to V_{DC} faster; a smaller R_f decays more slowly.

Step 4: Read the ranking of peaks from the figure.
The curve marked "with $R_f = R_1$" has the tallest peak, the curve for $R_f = R_3$ has the next tallest peak, and the curve for $R_f = R_2$ has the smallest peak, sitting closest to V_{DC}.

Step 5: Convert the peak-height ranking into a resistance ranking.
Since a bigger R_f always means a bigger peak, the order of peaks directly gives the order of resistances:

$$R_1 > R_3 > R_2$$

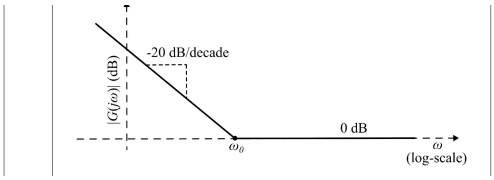
Step 6: Final conclusion.

$$\boxed{R_1 > R_3 > R_2}$$

Hence the correct option is (B).

Quick Tip: A bigger snubber resistance produces a bigger instantaneous IR spike but decays faster, so rank the resistors directly by the height of each peak in the figure.

. The asymptotic Bode magnitude plot of a system is shown.



Which one of the following options best represents the transfer function of the system?

(A) $G(s) = \dfrac{1 + \dfrac{s}{\omega_0}}{\dfrac{s}{\omega_0}}$

(B) $G(s) = \dfrac{\dfrac{s}{\omega_0}}{1 + \dfrac{s}{\omega_0}}$

(C) $G(s) = \dfrac{1 + \dfrac{s}{\omega_0}}{1 - \dfrac{s}{\omega_0}}$

(D) $G(s) = \dfrac{1 - \dfrac{s}{\omega_0}}{1 + \dfrac{s}{\omega_0}}$

Correct Answer: (A) $G(s) = \dfrac{1 + \dfrac{s}{\omega_0}}{\dfrac{s}{\omega_0}}$

Solution:

Step 1: Read the asymptotic plot.

For frequencies below ω_0, the magnitude falls at -20 dB/decade as frequency increases. For frequencies above ω_0, the magnitude is flat at 0 dB.

Step 2: Recall what a falling low-frequency asymptote means.
A slope of -20 dB/decade at low frequency is the signature of a pole at the origin, an integrator-type term, because the magnitude of $1/s$ falls exactly at that rate as frequency increases.

Step 3: Recall what a flat high-frequency asymptote at 0 dB means.
Once the magnitude becomes flat at 0 dB for large ω, the transfer function must behave like a constant of magnitude 1 at high frequency, which happens when a numerator term and a denominator term of the same order cancel out.

Step 4: Test option (A).

$$G(s) = \frac{1 + \frac{s}{\omega_0}}{\frac{s}{\omega_0}}$$

For $\omega \ll \omega_0$, the numerator is close to 1 and the denominator is close to $\frac{j\omega}{\omega_0}$, so

$$|G(j\omega)| \approx \frac{\omega_0}{\omega}$$

which falls at -20 dB/decade as ω rises, exactly as the plot shows. For $\omega \gg \omega_0$, both the numerator and denominator are dominated by the $\frac{s}{\omega_0}$ term, so $|G(j\omega)| \to 1$, giving the flat 0 dB region. This matches the plot on both sides of ω_0.

Step 5: Rule out the other options.
Option (B) is the reciprocal of (A); its magnitude would rise at $+20$ dB/decade for $\omega \ll \omega_0$, which is wrong. Options (C) and (D) both

contain a factor $1 - \frac{s}{\omega_0}$ in place of $1 + \frac{s}{\omega_0}$, which places a pole or zero in the right half of the s-plane. Such a term describes a non-minimum-phase or unstable element, which is not the simplest and standard system that a plain asymptotic magnitude sketch like this one is meant to represent. Option (A) is the well-behaved, minimum-phase choice whose magnitude asymptotes match the plot exactly.

Step 6: Final conclusion.

$$G(s) = \frac{1 + \frac{s}{\omega_0}}{\frac{s}{\omega_0}}$$

Hence the correct option is (A).

Quick Tip: A -20 dB/decade fall at low frequency that flattens to 0 dB beyond omega0 is the signature of a pole at the origin combined with a zero at omega0.

. Two $n \times n$ matrices A and B have a common eigenvalue 2, and the same corresponding nonzero eigenvector.
Which of the following options is/are correct?
(Note: I is the $n \times n$ identity matrix.)

(A) Determinant $(A - 2I) = 0$

(B) Determinant $(B - 2I) = 0$

(C) Determinant $(A + B - 2I) = 0$

(D) Determinant $(A + B - 4I) = 0$

Correct Answer: (A) Determinant $(A - 2I) = 0$; (B) Determinant $(B - 2I) = 0$; (D) Determinant $(A + B - 4I) = 0$

Solution:

Step 1: Write down what is given.

There is a nonzero vector v such that

$$Av = 2v \quad \text{and} \quad Bv = 2v$$

because both matrices share the eigenvalue 2 with the same eigenvector.

Step 2: Analyze option (A).

$$(A - 2I)v = Av - 2v = 2v - 2v = 0$$

Since v is nonzero and $(A - 2I)v = 0$, the matrix $A - 2I$ is singular. So

$$\det(A - 2I) = 0$$

Option (A) is correct.

Step 3: Analyze option (B).

By the exact same reasoning,

$$(B - 2I)v = Bv - 2v = 2v - 2v = 0$$

so $B - 2I$ is also singular and

$$\det(B - 2I) = 0$$

Option (B) is correct.

Step 4: Analyze option (C).

$$(A + B - 2I)v = Av + Bv - 2v = 2v + 2v - 2v = 2v$$

Since v is nonzero, $(A + B - 2I)v$ equals $2v$, which is not the zero vector. This shows v is not in the null space of $A + B - 2I$, so nothing in the given data forces this matrix to be singular. Option (C) is incorrect.

Step 5: Analyze option (D).

$$(A + B - 4I)v = Av + Bv - 4v = 2v + 2v - 4v = 0$$

Since v is nonzero and $(A + B - 4I)v = 0$, the matrix $A + B - 4I$ is singular, so

$$\det(A + B - 4I) = 0$$

Option (D) is correct.

Step 6: Final conclusion.

The common eigenvector v shows directly that $A - 2I$, $B - 2I$, and $A + B - 4I$ are all singular, while $A + B - 2I$ has no such guarantee.

$$\boxed{\text{(A), (B), and (D) are correct}}$$

Quick Tip: Use Av=2v and Bv=2v directly: check which of the four combinations sends the shared eigenvector v to the zero vector.

. A 220V/12V single-phase transformer is designed for use in India and rated 100 VA at 50 Hz. Later, this unit is shipped to the USA where it is used as a 110V/6V transformer at 60 Hz.
Which of the following statements is/are correct?

(A) No-load current drawn would be smaller for operation in the USA compared to that in India

(B) For the same load current, the losses would be higher in the USA compared to that in India

(C) The peak magnetic flux density in the core would be higher while operating in the USA compared to that in India

(D) The eddy current losses in the core would be approximately 44% higher while operating in the USA compared to that in India

Correct Answer: (A) No-load current drawn would be smaller for operation in the USA compared to that in India

Solution:

Step 1: Recall the EMF equation and what stays fixed on the same core.

For a transformer,

$$V = 4.44\, f\, N\, B_{max}\, A$$

where f is the frequency, N is the number of turns, B_{max} is the peak flux density, and A is the core area. Since it is physically the same

transformer moved from India to the USA, N and A do not change. So B_{max} is proportional to V/f.

Step 2: Compute the ratio of peak flux density in the two countries.

$$\frac{B_{max,USA}}{B_{max,India}} = \frac{V_{USA}/f_{USA}}{V_{India}/f_{India}} = \frac{110/60}{220/50} = \frac{1.8333}{4.4} \approx 0.4167$$

This shows the peak flux density in the USA is only about **41.7%** of what it was in India, meaning it is lower, not higher. So statement (C) is false.

Step 3: Relate no-load current to flux density.
The no-load, or magnetizing, current sets up the working flux in the core. On the same unsaturated core, a smaller required flux needs a smaller magnetizing current. Since B_{max} is lower in the USA, the no-load current needed to establish it is also smaller in the USA compared to India. So statement (A) is true.

Step 4: Check the loss statement for the same load current.
Copper loss depends only on the load current and winding resistance, so it stays unchanged between the two countries for the same load current. Core loss depends on frequency and flux density, and since B_{max} drops sharply while f rises only mildly, the core loss actually falls in the USA rather than rising. So the claim that losses would be higher in the USA, statement (B), is false.

Step 5: Check the eddy current loss statement.
Eddy current loss varies as

$$P_e \propto f^2 B_{max}^2$$

$$\frac{P_{e,USA}}{P_{e,India}} = \left(\frac{60}{50}\right)^2 \times (0.4167)^2 = 1.44 \times 0.1736 \approx 0.25$$

This means the eddy current loss in the USA is only about 25% of its value in India, a large decrease, not a 44% increase. So statement (D) is false.

Step 6: Final conclusion.
Only the statement about the smaller no-load current in the USA holds up.

No-load current drawn would be smaller for operation in the USA

Hence the correct option is (A).

Quick Tip: Since Bmax is proportional to V/f and the core stays the same, work out how Bmax, the magnetizing current, and the core losses all scale between the 220V/50Hz and 110V/60Hz operating points.

. Given that $\vec{F}(x, y, z) = \sin(y)\,\hat{x} + \cos(x)\,\hat{y} + 5\,\hat{z}$, the integral $\oiint_S \vec{F}(x, y, z) \cdot d\vec{s}$ over the unit sphere S centered at the origin evaluates to
(Round off to one decimal place)

Correct Answer: 0.0

Solution:

Step 1: Recognize the tool needed.
The integral asks for the flux of $\vec{F}$ through a closed surface, the unit

sphere. The divergence theorem converts a closed surface integral into a volume integral of the divergence:

$$\oiint_S \vec{F} \cdot d\vec{s} = \iiint_V (\nabla \cdot \vec{F})\, dV$$

Step 2: Compute the divergence of $\vec{F}$.

$$\vec{F}(x, y, z) = \sin(y)\,\hat{x} + \cos(x)\,\hat{y} + 5\,\hat{z}$$

$$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}\big(\sin(y)\big) + \frac{\partial}{\partial y}\big(\cos(x)\big) + \frac{\partial}{\partial z}(5)$$

Step 3: Evaluate each partial derivative.

$\sin(y)$ does not depend on x, so $\frac{\partial}{\partial x}\big(\sin(y)\big) = 0$. $\cos(x)$ does not depend on y, so $\frac{\partial}{\partial y}\big(\cos(x)\big) = 0$. The constant 5 does not depend on z, so $\frac{\partial}{\partial z}(5) = 0$.

Step 4: Add up the divergence.

$$\nabla \cdot \vec{F} = 0 + 0 + 0 = 0$$

The divergence is zero everywhere in space, not just on the sphere.

Step 5: Apply the divergence theorem.

Since $\nabla \cdot \vec{F} = 0$ at every point inside the unit ball,

$$\iiint_V (\nabla \cdot \vec{F})\, dV = \iiint_V 0\, dV = 0$$

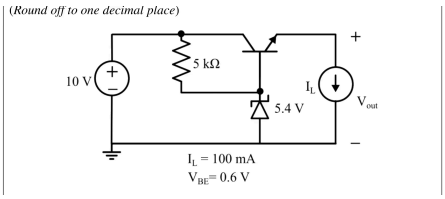
Step 6: Final conclusion.

$$\boxed{0.0}$$

Quick Tip: Apply the divergence theorem and check each partial derivative of F directly; the cross-mixed dependence of sin(y) and cos(x) makes the divergence vanish.

. In the linear regulator circuit shown, the base to emitter voltage V_{BE} of the BJT is 0.6 V. The Zener diode clamps the base voltage to 5.4 V. Ignore the biasing current of the Zener diode and the BJT.

(Round off to one decimal place)



The input supply is 10 V and the load current is $I_L = 100$ mA. The maximum possible efficiency of the regulator circuit is % (round off to one decimal place).

Correct Answer: 48.0

Solution:

Step 1: Identify the regulator topology.

This is a series pass voltage regulator built with a Zener diode and a

BJT. The 10 V source feeds a 5 kΩ resistor, which biases the Zener diode and holds the base of the transistor at a fixed voltage of 5.4 V. The transistor emitter is the regulated output, and the load draws current $I_L = 100$ mA from there.

Step 2: Find the output voltage.

The BJT works as an emitter follower here. The base sits at the Zener voltage, and the emitter is one V_{BE} drop below the base.

$$V_{out} = V_Z - V_{BE}$$

Putting in the given values,

$$V_{out} = 5.4 - 0.6 = 4.8 \text{ V}$$

Step 3: Work out the input power.

We are told to ignore the small current that biases the Zener diode and the base current of the transistor, so all the input current is really just the current the transistor pulls to supply the load. This means the current drawn from the 10 V source is simply I_L.

$$P_{in} = V_{in} \times I_L = 10 \times 0.1 = 1 \text{ W}$$

Step 4: Work out the output power.

$$P_{out} = V_{out} \times I_L = 4.8 \times 0.1 = 0.48 \text{ W}$$

Step 5: Compute the efficiency.

$$\eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{0.48}{1} \times 100 = 48.0\%$$

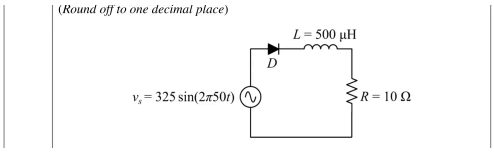
Final Answer:

The maximum possible efficiency of the regulator circuit is 48.0%.

$$\boxed{48.0}$$

Quick Tip: Find Vout as VZ minus VBE, then compare the output power to the input power, since both use the same load current IL.

. Consider the circuit shown. Assume that the diode D is ideal. The supply voltage $v_s = 325\sin(2\pi 50t)$ V, $L = 500\ \mu H$, and $R = 10\ \Omega$.



The peak diode current (in amperes) is (round off to one decimal place).

Correct Answer: 32.5

Solution:

Step 1: Set up the circuit equation.

The source, diode, inductor and resistor form one loop. Once the diode turns on, the current $i(t)$ obeys

$$L\frac{di}{dt} + Ri = V_m \sin(\omega t)$$

with $V_m = 325$ V and $\omega = 2\pi \times 50 = 314.16$ rad/s.

Step 2: Find the impedance and the phase angle.

$$\omega L = 314.16 \times 500 \times 10^{-6} = 0.1571\ \Omega$$

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + 0.1571^2} = 10.0012\ \Omega$$

$$\varphi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}(0.01571) = 0.9^\circ$$

Step 3: Write the general current expression.

For a half wave rectifier feeding an R-L load, the current while the diode conducts is

$$i(\theta) = \frac{V_m}{Z}\left[\sin(\theta - \varphi) + \sin\varphi\, e^{-\theta\cot\varphi}\right], \qquad \theta = \omega t$$

Step 4: Check how fast the extra term dies out.

Here $\cot\varphi = R/(\omega L) = 63.66$, a very large number. So the term $e^{-\theta\cot\varphi}$ collapses to almost zero within a fraction of a degree of the diode turning on. Practically, for the rest of the conduction period the current just tracks

$$i(\theta) \approx \frac{V_m}{Z}\sin(\theta - \varphi)$$

Step 5: Locate the peak.

This sine term peaks when $\theta - \varphi = 90°$, which falls well inside the conduction interval (the diode conducts for almost the whole positive half cycle here, since the tiny inductance barely delays the current zero crossing). At that point the decaying term has already vanished, so the peak value is simply

$$i_{peak} = \frac{V_m}{Z} = \frac{325}{10.0012} = 32.496 \text{ A}$$

Step 6: Round the answer.

$$\boxed{32.5}$$

The peak diode current works out to about 32.5 A.

Quick Tip: Since wL is tiny next to R here, the RL transient dies out almost instantly, so the peak current is close to Vm/Z, occurring near the middle of the positive half cycle.

. A is an $m \times m$ skew-symmetric matrix with real-valued entries, and x is an m-dimensional column vector with real-valued entries such that $x^T x = 1$. The quantity $x^T A x$ evaluates to (answer in integer).

Correct Answer: 0

Solution:

Step 1: Recall what skew-symmetric means.

A matrix A is skew-symmetric when its transpose equals its own negative:

$$A^T = -A$$

Step 2: Notice that x^TAx is just a number.

Since x is an $m \times 1$ column vector and A is $m \times m$, the product x^TAx is a 1×1 matrix, in other words a plain scalar. A scalar always equals its own transpose.

Step 3: Take the transpose of the scalar.

$$(x^TAx)^T = x^TA^Tx$$

Since the left side is a scalar, it equals the right side, so

$$x^TAx = x^TA^Tx$$

Step 4: Substitute the skew-symmetric property.

Using $A^T = -A$,

$$x^TAx = x^T(-A)x = -x^TAx$$

Step 5: Solve for the value.

$$x^TAx = -x^TAx \Rightarrow 2x^TAx = 0 \Rightarrow x^TAx = 0$$

Step 6: Note about the condition $x^T x = 1$.
This condition just says x has unit length. It does not change the result above; the quantity $x^T A x$ is zero for any real vector x, not only for unit vectors.

Final Answer:

$$\boxed{0}$$

Quick Tip: For any skew-symmetric A, the quadratic form x transpose A x always equals its own negative, so it must be zero.

. A time-limited waveform $g(x)$ is specified as follows:

$$g(x) = \begin{cases} -k, & -\pi < x \leq 0 \\ +k, & 0 < x \leq \pi \\ 0, & \text{otherwise} \end{cases}$$

A new waveform $f(x)$ is constructed from $g(x)$ as follows:

$$f(x) = \sum_{m=-\infty}^{\infty} g(x + 2\pi m), \qquad \text{for all } x \in \mathbb{R}$$

The sum of the coefficients of the third harmonics of the sine and cosine terms in the trigonometric Fourier series expansion of $f(x)$ is $\frac{2}{3\pi}$.
What is the value of k?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

Correct Answer: (B) $\frac{1}{2}$

Solution:

Step 1: Recognize the periodic waveform.

The construction $f(x) = \sum_m g(x + 2\pi m)$ simply repeats $g(x)$ every 2π, turning it into a periodic square wave of period 2π, equal to $-k$ on $(-\pi, 0]$ and $+k$ on $(0, \pi]$ in every period.

Step 2: Check the symmetry of $f(x)$.

Since $f(-x) = -f(x)$ (a negative x maps into the $-k$ region exactly when the corresponding positive x maps into the $+k$ region), $f(x)$ is an odd function. An odd periodic function has only sine terms in its Fourier series; every cosine coefficient a_n is zero.

Step 3: Recall the Fourier series of this standard square wave.

This is the classic odd square wave of amplitude k and period 2π. Its trigonometric Fourier series is

$$f(x) = \frac{4k}{\pi} \sum_{n \text{ odd}} \frac{\sin(nx)}{n}$$

so the sine coefficient of the n^{th} harmonic, for odd n, is

$$b_n = \frac{4k}{n\pi}$$

Step 4: Pick out the third harmonic.

For $n = 3$,

$$b_3 = \frac{4k}{3\pi}$$

and since $a_3 = 0$ (no cosine terms at all, as shown in Step 2), the sum of the third harmonic's sine and cosine coefficients is just

$$a_3 + b_3 = 0 + \frac{4k}{3\pi} = \frac{4k}{3\pi}$$

Step 5: Use the given value to solve for k.

We are told this sum equals $\frac{2}{3\pi}$:

$$\frac{4k}{3\pi} = \frac{2}{3\pi}$$

$$4k = 2 \Rightarrow k = \frac{1}{2}$$

Step 6: Analyze the options.

(A) 1: Would give a third harmonic sum of $4/(3\pi)$, too large. Incorrect.

(B) 1/2: Gives exactly $2/(3\pi)$, matching the given value. Correct.

(C) 1/3: Would give $4/(9\pi)$, too small. Incorrect.

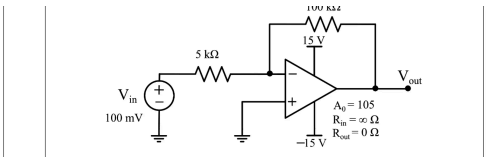
(D) 1/4: Would give $1/(3\pi)$, too small. Incorrect.

Final Answer:

$$\boxed{k = \frac{1}{2}}$$

Quick Tip: f(x) is an odd square wave of period 2pi, so only sine terms exist; use the standard series 4k/(n*pi) for the nth odd harmonic.

. In the circuit shown, the open loop gain of the operational amplifier is $A_0 = 105$, with $R_{in} = \infty\ \Omega$ and $R_{out} = 0\ \Omega$.



The circuit is a standard inverting amplifier: $V_{in} = 100$ mV drives the inverting input through a 5 $k\Omega$ input resistor, and a 100 $k\Omega$ resistor feeds back from V_{out} to the inverting input, with the non-inverting input grounded. What is the voltage gain of the circuit? (Round off to two decimal places)

(A) -16.67

(B) -20.00

(C) -21.00

(D) -12.67

Correct Answer: (A) -16.67

Solution:

Step 1: Set up the finite-gain inverting amplifier formula.

For an ideal op-amp with infinite gain, the inverting amplifier gain is simply $-R_f/R_{in}$. With a finite open loop gain A_0, a small error term must be included. The exact closed loop gain is

$$\frac{V_{out}}{V_{in}} = \frac{-R_f/R_{in}}{1 + \dfrac{1 + R_f/R_{in}}{A_0}}$$

Step 2: Plug in the resistor values.

$$\frac{R_f}{R_{in}} = \frac{100\ \text{k}\Omega}{5\ \text{k}\Omega} = 20$$

Step 3: Compute the correction term.

$$\frac{1 + R_f/R_{in}}{A_0} = \frac{1 + 20}{105} = \frac{21}{105} = 0.2$$

Step 4: Compute the denominator.

$$1 + 0.2 = 1.2$$

Step 5: Compute the gain.

$$\frac{V_{out}}{V_{in}} = \frac{-20}{1.2} = -16.67$$

Step 6: Analyze the options.

(A) -16.67: Matches the exact finite-gain result. Correct.

(B) -20.00: This is just $-R_f/R_{in}$, the ideal infinite-gain answer, which ignores the finite A_0 correction. Incorrect.

(C) -21.00: Comes from wrongly adding $1 + R_f/R_{in}$ directly to the ratio instead of dividing it by A_0 first. Incorrect.

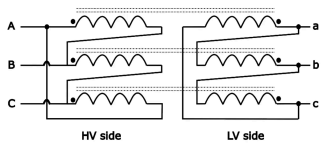
(D) -12.67: Comes from a setup error in the correction term. Incorrect.

Final Answer:

$$\boxed{-16.67}$$

Quick Tip: Use the finite-gain inverting amplifier formula: gain = -(Rf/Rin) divided by [1 + (1+Rf/Rin)/A0].

. Three single-phase 11kV/3.3kV transformers are connected to form a three-phase transformer bank, with the HV and LV windings connected as shown.



Considering **ABC** phase sequence, the vector group of the transformer is:

(A) Dd0

(B) Dd4

(C) Dd6

(D) Dd10

Correct Answer: (D) Dd10

Solution:

Step 1: Read the dot convention on each single-phase unit.
Each of the three single-phase transformers has a dot on both its HV and LV winding. The dot marks the terminal that goes positive together on both windings at the same instant, so the HV winding EMF and the LV winding EMF of the same unit are always in phase with each other.

Step 2: See how the HV delta is wired.
On the HV side, each line terminal (A, B, C) is brought out at the dot end of its own winding, and the winding's far, undotted end is looped diagonally into the dot end of the next winding, closing A to B to C and back to A. So going around the HV delta, we always travel from a winding's dot end toward its undotted end.

Step 3: See how the LV delta is wired.
On the LV side, each line terminal (a, b, c) is also brought out at the dot end of its own winding, but here the undotted end of one winding feeds the dot end of the next, so going around the LV delta we travel the opposite way, from an undotted end toward the dot end of the following winding. The LV delta is closed in the reverse rotational sense compared to the HV delta.

Step 4: Write the HV line voltages in terms of the winding EMFs.
Taking $E_A = E\angle 0°$, $E_B = E\angle -120°$, $E_C = E\angle 120°$ (balanced ABC sequence, dot-positive), tracing the HV delta gives

$$V_{AB} = E_A, \qquad V_{BC} = E_B, \qquad V_{CA} = E_C$$

Step 5: Write the LV line voltages, tracing the reversed loop.
Because the LV loop is traced in the opposite sense relative to the dots, each LV line voltage comes out as the negative of the corresponding winding EMF one step ahead:

$$V_{ab} = -e_b, \qquad V_{bc} = -e_c, \qquad V_{ca} = -e_a$$

where e_a, e_b, e_c are in phase with E_A, E_B, E_C respectively, at the same angles with a scaled-down magnitude.

Step 6: Work out the phase shift.

$$V_{ab} = -e_b = -E\angle -120° = E\angle 60°$$

Comparing to $V_{AB} = E\angle 0°$, the LV line voltage V_{ab} leads the HV line voltage V_{AB} by $60°$, which is the same as saying it lags by $360° - 60° = 300°$.

Step 7: Convert the lag angle into a clock number.
Each clock hour represents $30°$ of lag.

$$\text{Clock number} = \frac{300°}{30°} = 10$$

Step 8: Analyze the options.

(A) Dd0: Would need the LV delta wound the same rotational way as the HV delta, with dots aligned start-to-start on both sides, giving zero shift. Not the case here.

(B) Dd4: Would need a 120° lag, which does not match this connection. Incorrect.

(C) Dd6: Would need a full 180° reversal, meaning the LV dots reversed relative to HV on every leg, not just a reversed loop direction. Incorrect.

(D) Dd10: Matches the 300° lag found above. Correct.

Final Answer:

$$\boxed{\text{Dd10}}$$

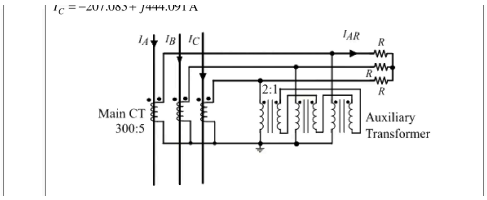
Quick Tip: Trace both deltas using the dot marks; the LV loop here closes in the opposite rotational sense to the HV loop, giving a clock number of 10, not 0.

. In the circuit shown, the phase currents are

$$I_A = 572.812 + j50.115 \text{ A}$$

$$I_B = -254.525 - j459.175 \text{ A}$$

$$I_C = -207.083 + j444.091 \text{ A}$$



Given that the CTs are ideal with no saturation, and the turns ratio of the Main CT is $300 : 5$ and that of the Auxiliary Transformer ($Yn\Delta$) is $2 : 1$ on every phase, the value of I_{AR}, rounded off to three decimal places, is:

(A) 0 A

(B) $0.653\angle 17.556°$ A

(C) $537.240\angle 4.105°$ A

(D) $8.954\angle 4.105°$ A

Correct Answer: (D) $8.954\angle 4.105°$ A

Solution:

Step 1: Reduce the phase currents through the Main CT.

The Main CT has a ratio of $300 : 5$, the same as $60 : 1$. Dividing each primary phase current by 60 gives the current available on the CT secondary side:

$$I_{A,sec} = \frac{572.812 + j50.115}{60} = 9.547 + j0.835 \text{ A}$$

$$I_{B,sec} = \frac{-254.525 - j459.175}{60} = -4.242 - j7.653 \text{ A}$$

$$I_{C,sec} = \frac{-207.083 + j444.091}{60} = -3.451 + j7.402 \text{ A}$$

Step 2: Recognize what the Auxiliary Transformer does.

The Auxiliary Transformer is connected $Yn\Delta$, meaning its three primary windings are star connected with the star point grounded, fed from the three CT secondary currents. A grounded star primary sums the three phase currents, and this sum drives a circulating current in the delta secondary. This is the standard way to extract the zero sequence part of a set of currents:

$$I_{0,sec} = \frac{I_{A,sec} + I_{B,sec} + I_{C,sec}}{3}$$

Step 3: Compute the zero sequence current.

$$I_{A,sec} + I_{B,sec} + I_{C,sec} = (9.547 - 4.242 - 3.451) + j(0.835 - 7.653 + 7.402) = 1.854 + j0.584 \text{ A}$$

$$I_{0,sec} = \frac{1.854 + j0.584}{3} = 0.618 + j0.195 \text{ A} = 0.653\angle 17.556^\circ \text{ A}$$

Step 4: Identify how I_{AR} is formed.

From the circuit, the resistor carrying I_{AR} is fed directly from the phase A CT secondary line, while the Auxiliary Transformer's delta output subtracts the zero sequence current $I_{0,sec}$ out of that line (the 2 : 1 ratio of the Auxiliary Transformer is exactly the turns ratio needed for its delta output to remove this average, $I_{0,sec}$, from the direct phase A current). So

$$I_{AR} = I_{A,sec} - I_{0,sec}$$

Step 5: Substitute the numbers.

$$I_{AR} = (9.547 + j0.835) - (0.618 + j0.195) = 8.929 + j0.640 \text{ A}$$

Step 6: Convert to polar form.

$$|I_{AR}| = \sqrt{8.929^2 + 0.640^2} = 8.954 \text{ A}$$

$$\angle I_{AR} = \tan^{-1}\left(\frac{0.640}{8.929}\right) = 4.105^\circ$$

Step 7: Analyze the options.

(A) 0 A: Would only hold if the three currents were perfectly balanced with no zero sequence at all, which is not the case here, and it is also not what I_{AR} physically represents. Incorrect.

(B) $0.653\angle 17.556^\circ$ **A:** This is actually $I_{0,sec}$, the zero sequence current itself, not I_{AR}. Incorrect.

(C) $537.240\angle 4.105^\circ$ **A:** This is 60 times the correct answer, the value obtained if the 300 : 5 Main CT ratio is never applied. Incorrect.

(D) $8.954\angle 4.105^\circ$ **A:** Matches the value found above. Correct.

Final Answer:

$$\boxed{8.954\angle 4.105^\circ \text{ A}}$$

Quick Tip: First scale the phase currents down through the 300:5 Main CT, then subtract the zero sequence average (IA+IB+IC)/3 from the phase A current to get IAR.

. The operating characteristic of a reactance relay is given by $X \leq 1\ \Omega$, where X is the reactance calculated by the relay. Its operating characteristic in the admittance plane (G-B plane, where G and B denote conductance and susceptance, respectively, expressed in Ω^{-1}) is given by:

(A) $G^2 + (B + 0.5)^2 \geq \frac{1}{4}$

(B) $B \geq 1$

(C) $(G - 1)^2 + B^2 \leq \frac{1}{2}$

(D) $G^2 + (B - 1)^2 \geq \frac{1}{2}$

Correct Answer: (A) $G^2 + (B + 0.5)^2 \geq \frac{1}{4}$

Solution:

Step 1: Write the impedance in terms of admittance.

Let $Z = R + jX$ be the impedance measured by the relay, and let $Y = G + jB = 1/Z$ be the admittance. Rationalizing,

$$Z = \frac{1}{Y} = \frac{1}{G + jB} = \frac{G - jB}{G^2 + B^2}$$

Step 2: Read off R and X.

Comparing real and imaginary parts,

$$R = \frac{G}{G^2 + B^2}, \qquad X = \frac{-B}{G^2 + B^2}$$

Step 3: Apply the boundary of the reactance characteristic.

The relay's boundary line is $X = 1$. Substituting,

$$\frac{-B}{G^2 + B^2} = 1$$

$$-B = G^2 + B^2$$

$$G^2 + B^2 + B = 0$$

Step 4: Complete the square in B.

$$G^2 + \left(B + \frac{1}{2}\right)^2 = \frac{1}{4}$$

This is a circle in the G-B plane centered at $(0, -0.5)$ with radius 0.5.

Step 5: Decide which side of the circle is the operate region.
Take a point deep inside the trip region in the impedance plane, say $R = 0,\ X = 0.5$, which satisfies $X \leq 1$. Its admittance is

$$Y = \frac{1}{j0.5} = -j2 \quad \Rightarrow \quad G = 0,\ B = -2$$

Check:

$$G^2 + (B + 0.5)^2 = 0 + (-1.5)^2 = 2.25 \geq \frac{1}{4}$$

So points that operate the relay satisfy the "greater than or equal to" inequality, meaning they lie on or outside this circle.

Step 6: Double-check with a non-operating point.
Take $R = 0,\ X = 2$, outside the trip zone since $X > 1$. Then $Y = 1/(j2) = -j0.5$, so $G = 0,\ B = -0.5$.

$$G^2 + (B + 0.5)^2 = 0 + 0^2 = 0 < \frac{1}{4}$$

This point fails the inequality, confirming that the operate region is indeed $G^2 + (B + 0.5)^2 \geq \frac{1}{4}$.

Step 7: Analyze the options.

(A) $G^2 + (B + 0.5)^2 \geq 1/4$**:** Matches the derived boundary and direction exactly. Correct.

(B) $B \geq 1$**:** A straight line condition, not a circle; it does not come from transforming $X = 1$. Incorrect.

(C) $(G-1)^2 + B^2 \leq 1/2$**:** Wrong center, wrong radius, and wrong side. Incorrect.

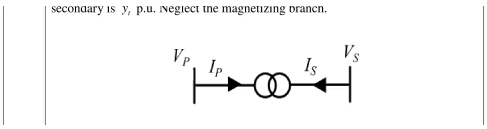
(D) $G^2 + (B-1)^2 \geq 1/2$**:** Wrong center, the sign of the B shift is flipped, and wrong radius. Incorrect.

Final Answer:

$$\boxed{G^2 + (B + 0.5)^2 \geq \frac{1}{4}}$$

Quick Tip: Substitute R=G/(G-squared+B-squared) and X=-B/(G-squared+B-squared) into X=1, complete the square in B, then test one known operating point to fix the inequality direction.

. A three-phase two-winding transformer has a voltage transformation ratio $\frac{V_P}{V_S} = 0.866 + j0.5$, where V_P is the primary side voltage in p.u., and V_S is the secondary side voltage in p.u. I_P and I_S represent the currents injected into the primary and secondary sides of the transformer, respectively. The admittance corresponding to the leakage impedance of the transformer referred to the secondary is y_t p.u. Neglect the magnetizing branch.



The Y bus representation of this transformer is:

(A)

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} \dfrac{y_t}{0.866 + j0.5} & -\dfrac{y_t}{0.866 + j0.5} \\ -\dfrac{y_t}{0.866 + j0.5} & \dfrac{y_t}{0.866 + j0.5} \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix}$$

(B)

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} y_t & -y_t \\ -y_t & y_t \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix}$$

(C)

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} y_t & -\dfrac{y_t}{0.866 + j0.5} \\ -\dfrac{y_t}{0.866 + j0.5} & y_t \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix}$$

(D)

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} y_t & -\dfrac{y_t}{0.866 - j0.5} \\ -\dfrac{y_t}{0.866 + j0.5} & y_t \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix}$$

Correct Answer: (D)

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} y_t & -\dfrac{y_t}{0.866 - j0.5} \\ -\dfrac{y_t}{0.866 + j0.5} & y_t \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix}$$

Solution:

Step 1: Set up the transformer model.

Model the transformer as an ideal transformer with complex turns ratio $a = \dfrac{V_P}{V_S} = 0.866 + j0.5$, followed by (referred to the secondary) a leakage admittance y_t that connects the internal secondary-side node to the actual secondary bus. Let V_1 be the voltage at this internal

node, right at the ideal transformer's secondary terminal, before the admittance y_t.

Step 2: Find the magnitude of the ratio.

$$|a|^2 = (0.866)^2 + (0.5)^2 = 0.75 + 0.25 = 1$$

So a has unit magnitude, meaning this transformer only shifts phase (by 30°) and does not change voltage magnitude. This fact will simplify the diagonal terms of the Y bus.

Step 3: Write the ideal transformer voltage relation.
For the ideal transformer,

$$V_P = aV_1 \implies V_1 = \frac{V_P}{a}$$

Step 4: Write the ideal transformer current relation.
An ideal transformer carries no real or reactive power itself, so the complex power on both sides is equal:

$$V_P I_P^* = V_1 I_1^*$$

where I_1 is the current leaving the ideal transformer's secondary winding into the admittance branch. Substituting $V_1 = V_P/a$ and taking the conjugate of the resulting relation gives

$$I_P = \frac{I_1}{a^*}$$

Step 5: Write the current through the admittance branch.

The admittance y_t connects node 1 to bus S, so

$$I_1 = y_t(V_1 - V_S) = y_t\left(\frac{V_P}{a} - V_S\right)$$

Step 6: Combine to get I_P.

$$I_P = \frac{I_1}{a^*} = \frac{y_t}{a^*}\left(\frac{V_P}{a} - V_S\right) = \frac{y_t}{aa^*}V_P - \frac{y_t}{a^*}V_S = \frac{y_t}{|a|^2}V_P - \frac{y_t}{a^*}V_S$$

Since $|a|^2 = 1$,

$$I_P = y_t V_P - \frac{y_t}{a^*}V_S = y_t V_P - \frac{y_t}{0.866 - j0.5}V_S$$

because the conjugate of $0.866 + j0.5$ is $0.866 - j0.5$.

Step 7: Find I_S using current continuity.

The current I_1 flows out of node 1 toward bus S; with I_S defined as the current entering the transformer branch from bus S, $I_S = -I_1$:

$$I_S = -I_1 = -y_t\left(\frac{V_P}{a} - V_S\right) = -\frac{y_t}{a}V_P + y_t V_S = -\frac{y_t}{0.866 + j0.5}V_P + y_t V_S$$

Step 8: Write the Y bus.

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} y_t & -\frac{y_t}{0.866 - j0.5} \\ -\frac{y_t}{0.866 + j0.5} & y_t \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix}$$

Step 9: Rule out the other options.

Option (B) is correct only if $a = 1$ (an ordinary transformer with no phase shift); it ignores the ratio entirely. Option (A) divides every entry of the matrix by a, which wrongly changes the diagonal terms too, when in fact $|a|^2 = 1$ keeps the diagonal terms equal to plain y_t. Option (C) uses the same denominator $0.866 + j0.5$ for both off-diagonal terms, but the two off-diagonal terms must use a and its conjugate a^* separately, since a complex (phase-shifting) ratio makes the bus admittance matrix non-symmetric.

Final Answer:

$$\boxed{I_P = y_t V_P - \frac{y_t}{0.866 - j0.5} V_S, \qquad I_S = -\frac{y_t}{0.866 + j0.5} V_P + y_t V_S}$$

This matches option (D).

Quick Tip: Write the transformer as an ideal complex-ratio transformer in series with y_t, then eliminate the internal node voltage; check that $|a|^2 = 1$ before simplifying.

. An electrical component has voltage drop $v = V_m \sin(\omega t)$, when the current through it is $i = I_m \sin(\omega t - \theta)$.

What is the average power dissipated over a half cycle corresponding to ω?

(A) 0

(B) $V_m I_m \cos\theta$

(C) $\frac{V_m I_m}{2}\cos\theta$

(D) $\frac{V_m I_m}{4}\cos\theta$

Correct Answer: (C) $\frac{V_m I_m}{2}\cos\theta$

Solution:

Step 1: Write the instantaneous power.

The instantaneous power absorbed by the component is

$$p(t) = v(t)\, i(t) = V_m I_m \sin(\omega t)\sin(\omega t - \theta)$$

Step 2: Convert the product of sines to a sum.

Using $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$ with $A = \omega t$ and $B = \omega t - \theta$,

$$p(t) = \frac{V_m I_m}{2}\Big[\cos\theta - \cos(2\omega t - \theta)\Big]$$

Step 3: Notice the two parts of this expression.

The first part, $\frac{V_m I_m}{2}\cos\theta$, does not depend on time at all, it is a constant.

The second part, $-\frac{V_m I_m}{2}\cos(2\omega t - \theta)$, oscillates at frequency 2ω, which is twice the frequency of v and i. Its own period is

$$\frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

which is exactly half the period of v and i (their period is $2\pi/\omega$).

Step 4: Average $p(t)$ over a half cycle.

A half cycle of v and i lasts π/ω seconds, which is precisely one full period of the oscillating term $\cos(2\omega t - \theta)$. The average of a cosine over exactly one of its own full periods is always zero, no matter where that interval starts. So over any half cycle,

$$\text{average of } \cos(2\omega t - \theta) = 0$$

Step 5: Find the average power.

Only the constant part survives the averaging:

$$P_{avg} = \frac{V_m I_m}{2}\cos\theta - 0 = \frac{V_m I_m}{2}\cos\theta$$

Step 6: Compare with the full-cycle average.

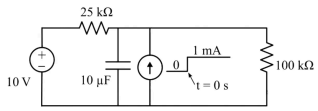
This is worth noting: the average power over a half cycle comes out to be exactly the same as the average power over one full cycle, since the ripple term always completes a whole number of its own periods within any half cycle of v and i. This rules out choice (A) (which would only hold if we averaged over a special quarter-cycle window, not a genuine half cycle), and choices (B) and (D), which correspond to using peak values directly or dividing by an extra factor of 2 by mistake.

Final Answer:

$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cos\theta}$$

Quick Tip: Split the product of sines into a constant term and a double-frequency ripple term; the ripple always averages to zero over a half cycle.

. The electrical network shown has an independent voltage source (10 V) and a current source (1 u(t) mA).



The voltage across the capacitor at time instants (in seconds) $t = 0^+$, $t = 0.50$, and $t = \infty$, respectively, is:

(A) 8.00 V, 28.00 V, 26.36 V

(B) 8.00 V, 26.36 V, 28.00 V

(C) 10.00 V, 26.36 V, 28.00 V

(D) 10.00 V, 28.00 V, 26.36 V

Correct Answer: (B) 8.00 V, 26.36 V, 28.00 V

Solution:

Step 1: Identify the circuit and label the node.

The 10 V source connects through a 25 kΩ resistor to a node that has the 10 μF capacitor, the 1 $u(t)$ mA current source (switched in at $t = 0$), and the 100 kΩ resistor, all in parallel to ground. Let v_C be the voltage at this node, across the capacitor.

Step 2: Find $v_C(0^-)$.

Before $t = 0$, the current source is not connected ($u(t) = 0$ for $t < 0$), and the circuit has been in steady state, so the capacitor carries no current and behaves as an open circuit. The only path is $10\ \text{V}$ through $25\ \text{k}\Omega$ in series with $100\ \text{k}\Omega$ to ground.

$$v_C(0^-) = 10 \times \frac{100\text{k}}{25\text{k} + 100\text{k}} = 10 \times \frac{100}{125} = 8\ \text{V}$$

Step 3: Apply continuity of capacitor voltage.

The voltage across a capacitor cannot jump instantly, so

$$v_C(0^+) = v_C(0^-) = 8.00\ \text{V}$$

This already rules out options (C) and (D), which start from $10.00\ \text{V}$.

Step 4: Find the final value $v_C(\infty)$.

For $t > 0$, the $1\ \text{mA}$ source is active and injects current into the node. Writing KCL at the node with all quantities in mA and $\text{k}\Omega$ (so that products of mA and $\text{k}\Omega$ directly give volts), and calling the node voltage V:

$$\frac{10 - V}{25} + 1 = \frac{V}{100}$$

Multiply through by 100:

$$4(10 - V) + 100 = V$$

$$40 - 4V + 100 = V \implies 140 = 5V \implies V = 28\ \text{V}$$

So $v_C(\infty) = 28.00\ \text{V}$.

Step 5: Find the Thevenin resistance seen by the capacitor.
To get the time constant, turn off the independent sources: the 10 V source becomes a short, and the 1 mA current source becomes an open circuit (it does not affect the resistance calculation since it is an ideal current source). Looking into the node from the capacitor's terminals, the resistance is 25 kΩ in parallel with 100 kΩ:

$$R_{th} = \frac{25 \times 100}{25 + 100}\text{k}\Omega = \frac{2500}{125}\text{k}\Omega = 20\ \text{k}\Omega$$

Step 6: Find the time constant and write $v_C(t)$.

$$\tau = R_{th}C = 20\text{k}\Omega \times 10\ \mu\text{F} = 0.2\ \text{s}$$

$$v_C(t) = v_C(\infty) + \left[v_C(0^+) - v_C(\infty)\right]e^{-t/\tau} = 28 - 20e^{-t/0.2}$$

Step 7: Evaluate at $t = 0.50$ s.

$$\frac{t}{\tau} = \frac{0.5}{0.2} = 2.5, \qquad e^{-2.5} \approx 0.0821$$

$$v_C(0.5) = 28 - 20(0.0821) = 28 - 1.64 \approx 26.36\ \text{V}$$

Step 8: Collect the three values in the order asked.

$$v_C(0^+) = 8.00\ \text{V}, \quad v_C(0.5) = 26.36\ \text{V}, \quad v_C(\infty) = 28.00\ \text{V}$$

This matches option (B), and rules out option (A), which places the values 28 and 26.36 in the wrong order.

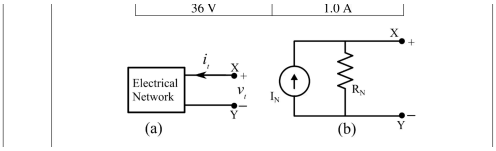
Final Answer:

$$\boxed{8.00\ \text{V},\ 26.36\ \text{V},\ 28.00\ \text{V}}$$

Quick Tip: Find the pre-switching capacitor voltage first (it cannot jump), then the new final value with the current source active, and use $\tau = R_{th}C$ with R_{th} as the 25k-100k parallel combination.

. The terminal voltage and current of a linear electrical network shown in Figure (a) are given in the table.

Terminal voltage (v_t)	Terminal current (i_t)
18 V	−0.5 A
30 V	0.5 A
36 V	1.0 A



The correct choice for the parameters (I_N, R_N) of the Norton equivalent circuit shown in Figure (b) is:

(A) $I_N = 3.0$ A, $R_N = 24.0\ \Omega$

(B) $I_N = 12.0$ A, $R_N = 2.0\ \Omega$

(C) $I_N = 2.0$ A, $R_N = 12.0\ \Omega$

(D) $I_N = 2.0$ A, $R_N = 24.0\ \Omega$

Correct Answer: (C) $I_N = 2.0$ A, $R_N = 12.0\ \Omega$

Solution:

Step 1: Check that the terminal relation is linear.

A linear network has a straight-line relationship between terminal voltage v_t and terminal current i_t, of the form $v_t = mi_t + c$. Using the first two rows of the table to find the slope:

$$m = \frac{30 - 18}{0.5 - (-0.5)} = \frac{12}{1.0} = 12$$

Step 2: Find the intercept.

$$18 = 12(-0.5) + c \implies 18 = -6 + c \implies c = 24$$

Step 3: Check the relation against the third row.

$$v_t = 12i_t + 24 \implies v_t(1.0) = 12(1.0) + 24 = 36\ \text{V}$$

This matches the third row exactly (36 V at $i_t = 1.0$ A), confirming the linear relation $v_t = 12i_t + 24$ fits all three data points.

Step 4: Interpret the two constants.
Setting $i_t = 0$ gives the open-circuit voltage:

$$V_{oc} = 24\ \text{V}$$

The coefficient of i_t is the Thevenin resistance seen from the terminals, since i_t here is defined flowing into the network at the positive terminal, and the network's driving voltage plus $i_t R_{th}$ gives the terminal voltage:

$$R_{th} = 12\ \Omega$$

Step 5: Convert Thevenin to Norton.

$$I_N = \frac{V_{oc}}{R_{th}} = \frac{24}{12} = 2.0\ \text{A}, \qquad R_N = R_{th} = 12.0\ \Omega$$

Step 6: Verify against the Norton circuit in Figure (b).
In Figure (b), applying KCL at the top node with the same current convention as i_t:

$$I_N + i_t = \frac{v_t}{R_N} \implies v_t = R_N I_N + R_N i_t$$

Substituting $I_N = 2.0$ A and $R_N = 12.0\ \Omega$:

$$v_t = 12(2.0) + 12i_t = 24 + 12i_t$$

which is exactly the fitted line $v_t = 12i_t + 24$. This confirms the values.

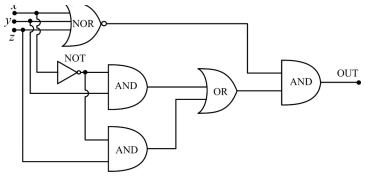
Step 7: Rule out the other options.
Options (A) and (D) use $R_N = 24\ \Omega$, which is the open-circuit voltage value mistaken for a resistance. Option (B) inverts the roles of the fitted numbers. Only $I_N = 2.0$ A with $R_N = 12.0\ \Omega$ is consistent with all three data points.

Final Answer:

$$\boxed{I_N = 2.0\text{ A},\ R_N = 12.0\ \Omega}$$

Quick Tip: Fit the three data points to a straight line $v_t = mi_t + c$; the intercept gives V_{oc} and the slope gives $R_{th} = R_N$, then use $I_N = V_{oc}/R_N$.

. The digital circuit shown has 3 inputs (x, y, and z).



The simplified logical expression for the output (OUT) is:

(A) $\bar{x}\,\overline{yz}$

(B) 0

(C) $\bar{x}(y+z)$

(D) 1

Correct Answer: (B) 0

Solution:

Step 1: Identify the gate structure.

Trace the circuit: a 3-input NOR gate takes x, y, z directly, giving

$$N = \overline{x+y+z}$$

A NOT gate takes one of the inputs (say z) and gives $\bar{z}$. Two AND gates combine this inverted signal and the remaining direct (uninverted) inputs, for example

$$P = \bar{z}\,y, \qquad Q = x\,y$$

and an OR gate combines them:

$$M = P + Q$$

Finally, the last AND gate combines N and M:

$$OUT = N \cdot M$$

Step 2: Notice when N can be 1.

A NOR gate output is 1 only when every one of its inputs is 0. So

$$N = 1 \iff x = 0,\ y = 0,\ z = 0$$

For every other input combination, $N = 0$.

Step 3: Check what happens when $N = 0$.

Whenever $N = 0$ (that is, whenever at least one of x, y, z is 1), the final AND gate immediately gives

$$OUT = N \cdot M = 0 \cdot M = 0$$

no matter what M works out to be.

Step 4: Check the one remaining case, $x = y = z = 0$.

Here $N = 1$, so we must look at M. Both AND gates that feed into M always include at least one direct (uninverted) tap of x, y, or z. Since $x = y = z = 0$ in this case, every such direct input is 0, so both AND gates output 0:

$$P = \bar{z} \cdot y = (1)(0) = 0, \qquad Q = x \cdot y = (0)(0) = 0$$

Hence

$$M = P + Q = 0 + 0 = 0$$

Step 5: Combine both cases.

So even in the one case where the NOR output is 1, the OR-of-ANDs branch M works out to 0, which again gives $OUT = N \cdot M = 1 \cdot 0 = 0$.

Step 6: State the general reason.

This circuit is built so that the NOR gate is 1 only at $x = y = z = 0$, exactly the one input combination where any AND gate built from a direct (uninverted) literal must output 0. Since the last gate is an AND of the NOR output with this OR-of-ANDs signal, the two branches can never both be 1 at the same time, for any combination of x, y, z.

Step 7: Conclude.

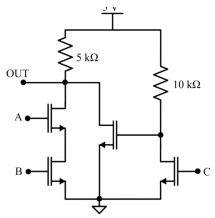
The output is 0 for every possible input combination, so OUT is identically zero, ruling out options (A), (C), and (D), which all describe an expression that is 1 for at least one input combination.

Final Answer:

$$\boxed{OUT = 0}$$

Quick Tip: A 3-input NOR is 1 only when all inputs are 0; check what the rest of the circuit gives at exactly that one input combination.

. The MOSFET switches shown in the circuit are ideal.



Which of the following is the correct option for Boolean logical expression of the output (OUT), and the maximum possible power (P) consumed by the circuit?

(A) OUT = $\overline{AB + \bar{C}}$, P = 5 mW

(B) OUT = $\overline{(A + B)\bar{C}}$, P = 5 mW

(C) OUT = $\overline{AB\bar{C}} + \bar{C}$, P = 7.5 mW

(D) OUT = $\overline{ABC}$, P = 7.5 mW

Correct Answer: (D) OUT = $\overline{ABC}$, P = 7.5 mW

Solution:

Step 1: Identify the pull-up and pull-down structure.
The OUT node is pulled up to 5 V through the 5 kΩ resistor. It is pulled down to ground through the NMOS switches gated by A and B, together with the switching action set up by the 10 kΩ resistor and the transistor gated by C, so OUT is pulled low only when a complete conducting path exists that needs all three of A, B, and C satisfied together.

Step 2: Recall how an ideal NMOS switch behaves.
An NMOS switch conducts (behaves like a short) when its gate is HIGH (logic 1) and is open (like a break) when its gate is LOW (logic 0). With the switches ideal, when the pull-down path conducts, OUT

is pulled all the way to 0 V; when no pull-down path conducts, OUT is pulled all the way up to 5 V through the resistor (no current flows, so there is no drop across the 5 kΩ).

Step 3: Combine the conditions for a conducting pull-down path.
The pull-down path conducts, forcing $OUT = 0$, only when $A = 1$, $B = 1$, and $C = 1$ all hold together; if even one of A, B, C is 0, the path is broken and no current can flow to ground, so OUT is pulled high.

Step 4: Write the Boolean expression.

$$OUT = \overline{A \cdot B \cdot C}$$

This is a 3-input NAND function; it is 1 for every input combination except $A = B = C = 1$, where it is 0.

Step 5: Find the current through the 5 kΩ branch.
This branch carries current only when $A = B = C = 1$ (the only case where OUT is pulled to 0 V):

$$I_{5k} = \frac{5\ \text{V}}{5\ \text{k}\Omega} = 1\ \text{mA} \implies P_{5k} = 5\ \text{V} \times 1\ \text{mA} = 5\ \text{mW}$$

For every other input combination this branch draws no current.

Step 6: Find the current through the 10 kΩ branch.
This branch, together with the transistor gated by C, forms a simple inverter for the internal node. It draws current whenever $C = 1$ (the transistor gated by C is then ON, pulling its node to 0 V and putting the full 5 V across the 10 kΩ):

$$I_{10k} = \frac{5\text{ V}}{10\text{ k}\Omega} = 0.5\text{ mA} \implies P_{10k} = 5\text{ V} \times 0.5\text{ mA} = 2.5\text{ mW}$$

Step 7: Find when both branches draw current at the same time.
Both branches conduct together only when $A = B = C = 1$ (this satisfies $A = B = C = 1$ for the 5 kΩ branch and $C = 1$ for the 10 kΩ branch at once). This is exactly the condition that gives the maximum total current drawn from the 5 V supply.

Step 8: Add the two powers for the maximum case.

$$P_{max} = P_{5k} + P_{10k} = 5\text{ mW} + 2.5\text{ mW} = 7.5\text{ mW}$$

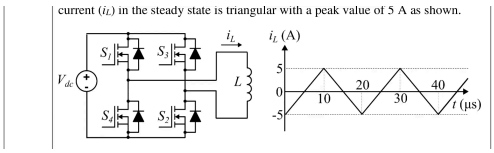
Step 9: Rule out the other options.
Options (A) and (B) both state $P = 5$ mW, which only accounts for the 5 kΩ branch and misses the extra 2.5 mW drawn through the 10 kΩ branch when $C = 1$. Option (C), on simplification, reduces to $\bar{A} + \bar{B} + C + \bar{C} = 1$ (a constant 1), since $C + \bar{C}$ is always true, so it cannot describe a circuit whose output changes with the inputs.

Final Answer:

$$\boxed{OUT = \overline{ABC}, \quad P_{max} = 7.5\text{ mW}}$$

Quick Tip: OUT is pulled low only when every pull-down switch conducts together; find which input combination makes both resistor branches draw current at once.

. Consider the single-phase voltage source inverter circuit feeding an inductive load (L). Assume that the power MOSFET switches are ideal. S_1 and S_2 are switched on during the first $10\ \mu$s, and S_3 and S_4 are switched on during the next $10\ \mu$s in a switching cycle. The switches in the same leg are thus switched in a complementary fashion. Neglect the dead time. The waveform of the inductor current (i_L) in the steady state is triangular with a peak value of 5 A as shown.



The rms value of the current through the switch S_1 is:

(A) 2.88 A

(B) 2.04 A

(C) 3.54 A

(D) 2.50 A

Correct Answer: (B) 2.04 A

Solution:

Step 1: Read the inductor current waveform.

From the graph, i_L is a symmetric triangular wave of period $20\ \mu$s: it rises linearly from -5 A at $t = 0$ to $+5$ A at $t = 10\ \mu$s (while S_1, S_2 are ON), then falls linearly from $+5$ A back to -5 A over the next $10\ \mu$s, from $t = 10\ \mu$s to $t = 20\ \mu$s (while S_3, S_4 are ON), and this pattern repeats.

Step 2: Identify when S_1 actually carries current.

S_1 is turned ON only during the first $10\ \mu s$ of every $20\ \mu s$ cycle. When S_1 and S_2 are ON (and S_3, S_4 are OFF), the entire load current path runs through S_1 and S_2, so the current through switch S_1 equals the inductor current itself during this interval, whatever its sign. During the next $10\ \mu s$, S_1 is OFF and carries no current at all.

Step 3: Write $i_{S1}(t)$ over one period.

$$i_{S1}(t) = \begin{cases} i_L(t) = -5 + t, & 0 \le t < 10\ \mu s\ (t \text{ in } \mu s) \\ 0, & 10\ \mu s \le t < 20\ \mu s \end{cases}$$

(Here $i_L(t) = -5 + t$ is just the equation of the straight line joining $(0, -5)$ to $(10, 5)$.)

Step 4: Set up the rms integral over the full period.

$$I_{rms}^2 = \frac{1}{T}\int_0^T i_{S1}^2(t)\,dt = \frac{1}{20}\left[\int_0^{10}(t-5)^2\,dt + \int_{10}^{20} 0\,dt\right]$$

Step 5: Evaluate the integral.

Substitute $u = t - 5$, so as t runs from 0 to 10, u runs from -5 to 5:

$$\int_0^{10}(t-5)^2\,dt = \int_{-5}^{5} u^2\,du = \left[\frac{u^3}{3}\right]_{-5}^{5} = \frac{125}{3} - \left(\frac{-125}{3}\right) = \frac{250}{3}$$

Step 6: Substitute back.

$$I_{rms}^2 = \frac{1}{20} \times \frac{250}{3} = \frac{250}{60} = \frac{25}{6}$$

$$I_{rms} = \sqrt{\frac{25}{6}} = \frac{5}{\sqrt{6}} \approx 2.041 \text{ A}$$

Step 7: Rule out the other options.

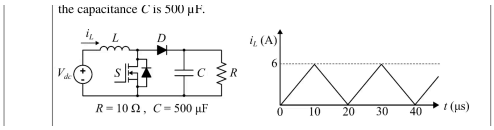
3.54 A does not correspond to any consistent calculation for this waveform. 2.88 A is close to the rms of the ramp computed only over its own 10 μs ON-time (which would be $5/\sqrt{3} \approx 2.89$), without accounting for the fact that S_1 is off (contributing zero) for the other half of the period. 2.50 A also does not follow from this waveform. Only accounting for both the shape of the ramp and the 50% duty cycle over the full period gives the correct value.

Final Answer:

$$\boxed{I_{rms} = \frac{5}{\sqrt{6}} \approx 2.04 \text{ A}}$$

Quick Tip: S_1 carries the inductor current only during the first 10 microseconds of each 20 microsecond cycle (and zero for the rest); find the rms of this waveform over the full period.

. Consider the boost converter circuit shown. Assume that the semiconductor devices are ideal. In steady state, the inductor current rises linearly from 0 A to 6 A in the first 10 μs and then falls linearly from 6 A to 0 A in the next 10 μs of every switching cycle as shown. The load resistance R is 10 Ω and the capacitance C is 500 μF.



Neglect the ripple in the output voltage. What is the input voltage V_{dc}?

(A) 10.0 V

(B) 15.0 V

(C) 7.5 V

(D) 12.5 V

Correct Answer: (C) 7.5 V

Solution:

Step 1: Identify the switching intervals.

In a boost converter, the inductor current rises while the switch S is ON (the inductor is directly across the source, diode reverse biased) and falls while S is OFF (the inductor current is forced through the diode to the output). Here the rise takes 10 μs and the fall takes 10 μs, so

$$T_{on} = 10\ \mu s, \quad T_{off} = 10\ \mu s, \quad T = 20\ \mu s, \quad D = \frac{T_{on}}{T} = 0.5$$

Step 2: Recall the ideal boost voltage relation.

For a lossless boost converter,

$$V_{out} = \frac{V_{dc}}{1 - D} = \frac{V_{dc}}{1 - 0.5} = 2V_{dc}$$

This gives the ratio between input and output but not their absolute values yet, since we still need one more equation from the load.

Step 3: Find the average diode current.
The diode conducts only while S is OFF, which is exactly when i_L falls from 6 A to 0 A over 10 μs; during the ON time the diode is reverse biased and carries no current. So the diode current waveform is a triangular pulse (peak 6 A, base 10 μs) followed by 10 μs of zero, every 20 μs.

Step 4: Average this waveform over the full period.

$$I_{D,avg} = \frac{1}{T}\left[\frac{1}{2}(6)(10\,\mu\text{s}) + 0\right] = \frac{30}{20} = 1.5\text{ A}$$

Step 5: Use the fact that the capacitor carries no net (average) current in steady state.
With the output ripple neglected, all of the average diode current flows into the load resistor:

$$I_{D,avg} = \frac{V_{out}}{R} \implies 1.5 = \frac{V_{out}}{10} \implies V_{out} = 15\text{ V}$$

Step 6: Solve for V_{dc}.

$$V_{out} = 2V_{dc} \implies V_{dc} = \frac{15}{2} = 7.5\text{ V}$$

Step 7: Check with power balance.

The average inductor current (which equals the average input current, since the inductor sits directly in the input path of a boost converter) for a triangular wave between 0 and 6 A is the average of the two extremes, $(0 + 6)/2 = 3$ A. Input power:

$$P_{in} = V_{dc} \times I_{L,avg} = 7.5 \times 3 = 22.5 \text{ W}$$

Output power:

$$P_{out} = \frac{V_{out}^2}{R} = \frac{15^2}{10} = \frac{225}{10} = 22.5 \text{ W}$$

Since $P_{in} = P_{out}$, the ideal (lossless) converter assumption is satisfied, confirming $V_{dc} = 7.5$ V.

Step 8: Rule out the other options.

15.0 V would be mistaking the output voltage itself for the input. 10.0 V and 12.5 V do not satisfy both the boost ratio $V_{out} = 2V_{dc}$ and the load power balance simultaneously.

Final Answer:

$$\boxed{V_{dc} = 7.5 \text{ V}}$$

Quick Tip: Find the duty ratio from the given on/off times, use $V_{out} = V_{dc}/(1 - D)$, and separately find V_{out} from the average diode current feeding

the load resistor.

. Which one of the following statements is ALWAYS correct about a collection of p column vectors, each having n real-valued entries?

(A) If $p > n$, then the column vectors must be linearly dependent

(B) If $p > n$, then the column vectors must be linearly independent

(C) If $p = n$, then the column vectors must be orthogonal

(D) If $p < n$, then the column vectors must be linearly independent

Correct Answer: (A) If $p > n$, then the column vectors must be linearly dependent

Solution:

Step 1: Recall what determines independence of a set of vectors.
A set of p vectors, each with n real entries, lives in R^n. The largest number of vectors that can be linearly independent in R^n is n, because the rank of any $n \times p$ matrix built from these vectors can never exceed n.

Step 2: Test option (A).
If $p > n$, we have more vectors than the dimension of the space they live in. Since at most n of them can be independent, the remaining $p - n$ vectors must be a combination of the others. So the full set cannot be linearly independent, which means it is linearly dependent. This holds for every choice of $p > n$ vectors, so option (A) is always true.

Step 3: Test option (B).
Option (B) claims the opposite of what Step 2 shows. Since $p > n$ always forces dependence, the vectors can never be independent in

this case. So option (B) is false.

Step 4: Test option (C).

When $p = n$, the vectors can form a basis of R^n if they are independent, but nothing forces them to be orthogonal. For example, the two vectors $(1, 0)$ and $(1, 1)$ in R^2 are independent but not orthogonal, and they form a valid collection with $p = n = 2$. So option (C) is false.

Step 5: Test option (D).

When $p < n$, the vectors could be independent, but they do not have to be. Take two copies of the same nonzero vector in R^3, so $p = 2$ and $n = 3$. Here $p < n$, but the two vectors are identical and hence dependent. So option (D) is false.

Step 6: Final conclusion.

Only the statement about $p > n$ forcing dependence holds in every case.

$$\boxed{\text{If } p > n, \text{ the column vectors must be linearly dependent}}$$

Quick Tip: Compare the number of vectors to the dimension of the space to decide whether independence is even possible.

. Consider the second-order differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

with initial conditions

$$y(0) = 1, \quad \left.\frac{dy}{dx}\right|_{x=0} = 1$$

The solution is given by

(A) $y(x) = \exp(-x/2)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$

(B) $y(x) = \exp(-x/2)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$

(C) $y(x) = \exp(-x/2)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) - \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$

(D) $y(x) = \exp(-x/2)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) - \sqrt{3}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$

Correct Answer: (A) $y(x) = \exp(-x/2)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$

Solution:

Step 1: Write the characteristic equation.

The equation $y'' + y' + y = 0$ has constant coefficients, so we look for solutions of the form $y = e^{rx}$. Substituting gives the characteristic equation

$$r^2 + r + 1 = 0$$

Step 2: Solve for the roots.

Using the quadratic formula,

$$r = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

The roots are complex, with real part $-\frac{1}{2}$ and imaginary part $\pm\frac{\sqrt{3}}{2}$.

Step 3: Write the general solution.

For complex roots $\alpha \pm i\beta$, the general solution has the form

$$y(x) = e^{\alpha x}\left(C_1 \cos(\beta x) + C_2 \sin(\beta x)\right)$$

Here $\alpha = -\frac{1}{2}$ and $\beta = \frac{\sqrt{3}}{2}$, so

$$y(x) = e^{-x/2}\left(C_1 \cos\left(\frac{\sqrt{3}\,x}{2}\right) + C_2 \sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$$

Step 4: Apply the first initial condition.

At $x = 0$, $\cos(0) = 1$ and $\sin(0) = 0$, so

$$y(0) = C_1 = 1$$

Thus $C_1 = 1$.

Step 5: Differentiate to apply the second initial condition.

$$y'(x) = -\frac{1}{2}e^{-x/2}\left(C_1 \cos\left(\frac{\sqrt{3}\,x}{2}\right) + C_2 \sin\left(\frac{\sqrt{3}\,x}{2}\right)\right) + e^{-x/2}\left(-C_1\frac{\sqrt{3}}{2}\sin\left(\frac{\sqrt{3}\,x}{2}\right) + C_2\frac{\sqrt{3}}{2}\cos\left(\frac{\sqrt{3}\,x}{2}\right)\right)$$

At $x = 0$, this simplifies to

$$y'(0) = -\frac{1}{2}C_1 + \frac{\sqrt{3}}{2}C_2$$

Step 6: Solve for C_2.

We are given $y'(0) = 1$ and we know $C_1 = 1$, so

$$1 = -\frac{1}{2}(1) + \frac{\sqrt{3}}{2}C_2$$

$$\frac{3}{2} = \frac{\sqrt{3}}{2}C_2$$

$$C_2 = \frac{3}{\sqrt{3}} = \sqrt{3}$$

Step 7: Write the final solution.

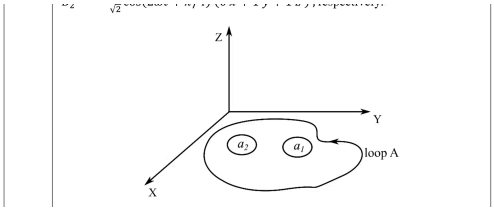
With $C_1 = 1$ and $C_2 = \sqrt{3}$,

$$\boxed{y(x) = e^{-x/2}\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)}$$

Hence, the correct option is (A).

Quick Tip: Find the roots of the characteristic equation first, then fix the two constants using the two given initial conditions.

. The figure shows an arbitrarily shaped planar conducting loop A in the XY plane. Two nonintersecting regions with areas a_1 and a_2 within the loop are subjected to magnetic fields $\vec{B}_1 = \frac{m}{\sqrt{2}}\sin(\omega t)\left(1\,\hat{x} + 0\,\hat{y} + 1\,\hat{z}\right)$, and $\vec{B}_2 = -\frac{n}{\sqrt{2}}\cos(2\omega t + \pi/4)\left(0\,\hat{x} + 1\,\hat{y} + 1\,\hat{z}\right)$, respectively.



What is the expression for the induced rms voltage in loop A?

(A) $\sqrt{\frac{a_1^2\omega^2m^2 + 4a_2^2\omega^2n^2}{4}}$

(B) $\sqrt{\frac{a_1^2\omega^2m^2 + 4a_2^2\omega^2n^2}{2}}$

(C) $\sqrt{\frac{a_1^2\omega^2m^2 - 2a_2^2\omega^2n^2}{2}}$

(D) $\sqrt{a_1^2\omega^2m^2 + 2a_2^2\omega^2n^2}$

Correct Answer: (A) $\sqrt{\frac{a_1^2\omega^2m^2 + 4a_2^2\omega^2n^2}{4}}$

Solution:

Step 1: Identify which field component matters.

Loop A lies entirely in the XY plane, so its area vector points along $\hat{z}$.

Only the z-component of each magnetic field contributes to the flux linking the loop; the $\hat{x}$ and $\hat{y}$ components lie in the plane of the loop and give zero flux.

Step 2: Extract the z-components of the two fields.

From $\vec{B}_1 = \frac{m}{\sqrt{2}}\sin(\omega t)(1\hat{x} + 0\hat{y} + 1\hat{z})$, the z-component is

$$B_{1z} = \frac{m}{\sqrt{2}}\sin(\omega t)$$

From $\vec{B}_2 = -\frac{n}{\sqrt{2}}\cos(2\omega t + \pi/4)(0\hat{x} + 1\hat{y} + 1\hat{z})$, the z-component is

$$B_{2z} = -\frac{n}{\sqrt{2}}\cos(2\omega t + \pi/4)$$

Step 3: Write the total flux through loop A.

Since a_1 and a_2 are nonintersecting regions inside the same loop, the total flux is the sum of the flux through each region:

$$\Phi = a_1 B_{1z} + a_2 B_{2z} = \frac{a_1 m}{\sqrt{2}}\sin(\omega t) - \frac{a_2 n}{\sqrt{2}}\cos(2\omega t + \pi/4)$$

Step 4: Apply Faraday's law to get the induced EMF.

$$e = -\frac{d\Phi}{dt} = -\frac{a_1 m\omega}{\sqrt{2}}\cos(\omega t) - \frac{2a_2 n\omega}{\sqrt{2}}\sin(2\omega t + \pi/4)$$

This is a sum of two sinusoids, one at frequency ω and one at frequency 2ω.

Step 5: Find the rms value of each sinusoid separately.

For a sinusoid of amplitude P, the rms value is $P/\sqrt{2}$. The first term has amplitude $\frac{a_1 m\omega}{\sqrt{2}}$, so its rms value is

$$\frac{1}{\sqrt{2}} \cdot \frac{a_1 m\omega}{\sqrt{2}} = \frac{a_1 m\omega}{2}$$

The second term has amplitude $\frac{2a_2 n\omega}{\sqrt{2}}$, so its rms value is

$$\frac{1}{\sqrt{2}} \cdot \frac{2a_2 n\omega}{\sqrt{2}} = a_2 n\omega$$

Step 6: Combine the two rms values.

Because the two terms oscillate at different frequencies (ω and 2ω), their product averages to zero over a full cycle, so the mean square values simply add:

$$e_{rms}^2 = \left(\frac{a_1 m\omega}{2}\right)^2 + (a_2 n\omega)^2 = \frac{a_1^2 m^2 \omega^2}{4} + a_2^2 n^2 \omega^2$$

Step 7: Combine into a single fraction and take the square root.

$$e_{rms}^2 = \frac{a_1^2 m^2 \omega^2 + 4a_2^2 n^2 \omega^2}{4}$$

$$\boxed{e_{rms} = \sqrt{\frac{a_1^2 \omega^2 m^2 + 4a_2^2 \omega^2 n^2}{4}}}$$

Hence, the correct option is (A).

Quick Tip: Only the z-component of each field links flux through this planar loop; combine the two frequencies using mean-square addition, not simple addition of amplitudes.

. A system is characterized by the following state equation and output equation (u: input, $\mathbf{x}$: state vector, y: output)

$$\dot{\mathbf{x}} = \begin{bmatrix} a & b \\ -a & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = [1 \quad 2]\,\mathbf{x}$$

What are the values of a and b for which the poles of the transfer function are at $-2 + j3$ and $-2 - j3$?

(A) $a = 4,\ b = 3.25$

(B) $a = -4,\ b = 3.25$

(C) $a = 4,\ b = -3.25$

(D) $a = -4,\ b = -3.25$

Correct Answer: (D) $a = -4,\ b = -3.25$

Solution:

Step 1: Write the characteristic equation of the system.

For the state matrix

$$A = \begin{bmatrix} a & b \\ -a & 0 \end{bmatrix}$$

the poles of the transfer function are the eigenvalues of A, found from $\det(sI - A) = 0$.

Step 2: Compute $sI - A$.

$$sI - A = \begin{bmatrix} s - a & -b \\ a & s \end{bmatrix}$$

Step 3: Take the determinant.

$$\det(sI - A) = (s - a)(s) - (-b)(a) = s^2 - as + ab$$

So the characteristic equation is

$$s^2 - as + ab = 0$$

Step 4: Write the desired characteristic equation from the given poles.

The poles are $-2 + j3$ and $-2 - j3$. A quadratic with these roots is

$$(s - (-2 + j3))(s - (-2 - j3)) = (s + 2)^2 + 9 = s^2 + 4s + 4 + 9 = s^2 + 4s + 13$$

Step 5: Match coefficients.

Comparing

$$s^2 - as + ab = s^2 + 4s + 13$$

gives

$$-a = 4 \quad \Rightarrow \quad a = -4$$

and

$$ab = 13$$

Step 6: Solve for b.

$$b = \frac{13}{a} = \frac{13}{-4} = -3.25$$

Step 7: Final conclusion.

$$\boxed{a = -4,\ b = -3.25}$$

Hence, the correct option is (D).

Quick Tip: Match the characteristic polynomial of the state matrix to the quadratic whose roots are the two given poles.

. A system is represented in state-space form as follows (u: input, $\mathbf{x}$: state vector, y: output)

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \mathbf{x}$$

Consider the new state vector $\mathbf{z} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x}$

What is the state-space representation of the system in terms of the new state vector $\mathbf{z}$?

(A) $\dot{\mathbf{z}} = \begin{bmatrix} -1 & 4 \\ -1 & -2 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} u$

$y = \begin{bmatrix} 2 & 3 \end{bmatrix} \mathbf{z}$

(B) $\dot{\mathbf{z}} = \begin{bmatrix} 2 & 3 \\ 0 & 3 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} u$

$y = \begin{bmatrix} 2 & 3 \end{bmatrix} \mathbf{z}$

(C) $\dot{\mathbf{z}} = \begin{bmatrix} 4 & 9 \\ -2 & -3 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} u$

$y = \begin{bmatrix} 2 & 3 \end{bmatrix} \mathbf{z}$

(D) $\dot{\mathbf{z}} = \begin{bmatrix} 2 & 1 \\ -4 & 1 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} u$

$y = \begin{bmatrix} 4 & -1 \end{bmatrix} \mathbf{z}$

Correct Answer: (C) $\dot{\mathbf{z}} = \begin{bmatrix} 4 & 9 \\ -2 & -3 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} u$

$y = \begin{bmatrix} 2 & 3 \end{bmatrix} \mathbf{z}$

Solution:

Step 1: Set up the transformation.

Let $T = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$, so the new state is $\mathbf{z} = T\mathbf{x}$, which means $\mathbf{x} = T^{-1}\mathbf{z}$.

Substituting into the original state equation gives the standard similarity transform rules

$$A' = TAT^{-1}, \qquad B' = TB, \qquad C' = CT^{-1}$$

Step 2: Find the determinant of T.

$$\det(T) = 2(0) - 1(-1) = 1$$

Step 3: Find T^{-1}.

For a 2×2 matrix $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$, the inverse is $\frac{1}{\det} \begin{bmatrix} s & -q \\ -r & p \end{bmatrix}$. Here $p = 2, q = 1, r = -1, s = 0$, so

$$T^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$$

Step 4: Compute AT^{-1}.

With $A = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$,

$$AT^{-1} = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 3 \end{bmatrix}$$

Step 5: Compute $A' = T(AT^{-1})$.

$$A' = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ -2 & -3 \end{bmatrix}$$

Step 6: Compute $B' = TB$.

With $B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$,

$$B' = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

Step 7: Compute $C' = CT^{-1}$.

With $C = [1 \quad 2]$,

$$C' = [1 \quad 2] \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} = [2 \quad 3]$$

Step 8: Assemble the final representation.

$$\boxed{\dot{\mathbf{z}} = \begin{bmatrix} 4 & 9 \\ -2 & -3 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} u, \qquad y = [2 \quad 3]\,\mathbf{z}}$$

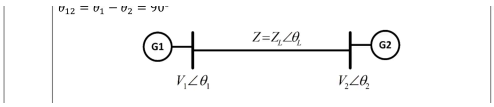
Hence, the correct option is (C).

Quick Tip: Use the similarity transform rules A' = TAT^-1, B' = TB, C' = CT^-1 with T being the given transformation matrix.

. For the balanced 3-phase transmission line shown, consider the following cases:

Case-1: $|V_1| = 1.1$ p.u., $|V_2| = 0.9$ p.u., $Z = 0.75\angle 0°$ p.u. and $\theta_{12} = \theta_1 - \theta_2 = 0°$

Case-2: $|V_1| = 1.1$ p.u., $|V_2| = 0.9$ p.u., $Z = 0.75\angle 90°$ p.u. and $\theta_{12} = \theta_1 - \theta_2 = 90°$



Which of the following statements is/are correct about real power loss and reactive power loss in the line?

(A) Real power loss in Case-1 is more than that in Case-2

(B) Real power loss in Case-2 is more than that in Case-1

(C) Reactive power loss in Case-1 is more than that in Case-2

(D) Reactive power loss in Case-2 is more than that in Case-1

Correct Answer: (A) Real power loss in Case-1 is more than that in Case-2 ; (D) Reactive power loss in Case-2 is more than that in Case-1

Solution:

Step 1: Write the general formulas needed.

The current flowing in the line is

$$I = \frac{V_1 - V_2}{Z}$$

and the complex power lost in the line impedance is

$$S_{loss} = |I|^2 Z = P_{loss} + jQ_{loss}$$

So the real power loss depends on the resistive part of Z, and the reactive power loss depends on the reactive part of Z, both scaled by $|I|^2$.

Step 2: Analyze Case-1.

Here $Z = 0.75\angle 0°$, a purely resistive line: $R_1 = 0.75, X_1 = 0$. Since $\theta_{12} = 0°$, take $\theta_1 = \theta_2 = 0°$, since only the difference matters. Then

$$V_1 - V_2 = 1.1 - 0.9 = 0.2$$

$$I_1 = \frac{0.2}{0.75} = 0.267 \text{ p.u.}$$

$$|I_1|^2 = 0.0711$$

Since Z is purely resistive,

$$P_{loss,1} = |I_1|^2 R_1 = 0.0711 \times 0.75 = 0.0533 \text{ p.u.}$$

$$Q_{loss,1} = |I_1|^2 X_1 = 0$$

Step 3: Analyze Case-2.

Here $Z = 0.75\angle 90° = j0.75$, a purely reactive line: $R_2 = 0, X_2 = 0.75$. Since $\theta_{12} = 90°$, take $\theta_2 = 0°$ and $\theta_1 = 90°$. Then

$$V_1 = 1.1\angle 90^\circ = j1.1, \qquad V_2 = 0.9\angle 0^\circ = 0.9$$

$$V_1 - V_2 = -0.9 + j1.1$$

$$|V_1 - V_2| = \sqrt{0.9^2 + 1.1^2} = \sqrt{2.02} = 1.421$$

$$|I_2| = \frac{1.421}{0.75} = 1.895 \text{ p.u.}$$

$$|I_2|^2 = 3.591$$

Since Z is purely reactive here,

$$P_{loss,2} = |I_2|^2 R_2 = 0$$

$$Q_{loss,2} = |I_2|^2 X_2 = 3.591 \times 0.75 = 2.693 \text{ p.u.}$$

Step 4: Compare real power losses.

$$P_{loss,1} = 0.0533 \text{ p.u.}, \qquad P_{loss,2} = 0$$

Since $0.0533 > 0$, the real power loss in Case-1 is more than in Case-2. So statement (A) is correct and statement (B) is incorrect.

Step 5: Compare reactive power losses.

$$Q_{loss,1} = 0, \qquad Q_{loss,2} = 2.693 \text{ p.u.}$$

Since $2.693 > 0$, the reactive power loss in Case-2 is more than in Case-1. So statement (D) is correct and statement (C) is incorrect.

Step 6: Explain the underlying reason.
When the line is purely resistive (Case-1), all of the I^2Z loss appears as real power. When the line is purely reactive (Case-2), all of that loss appears as reactive power. This is why Case-1 has real loss but no reactive loss, while Case-2 has reactive loss but no real loss.

Step 7: Final conclusion.

Correct statements: (A) Real power loss in Case-1 is more, and (D) Reactive power loss in Case-2 is more

Quick Tip: Split the line impedance into its resistive and reactive parts; each part produces only its own kind of I-squared-Z loss.

. Consider an $n \times n$ orthogonal matrix A with real entries and each column having unit Euclidean norm.
Which of the following statements is/are correct?

(A) The value of the determinant of A is either $+1$ or -1

(B) The eigenvalues of A have modulus 1

(C) $\|Ax\| = \|x\|$, for all $x \in R^n$, where $\|x\|$ denotes the Euclidean norm of x, and $(Ax)^T(Ay) \neq x^T y$, for all distinct $x, y \in R^n$

(D) $\|Ax\| = \|x\|$, for all $x \in R^n$, where $\|x\|$ denotes the Euclidean norm of x, and $(Ax)^T(Ay) = x^T y$, for all $x, y \in R^n$

Correct Answer: (A) The value of the determinant of A is either $+1$ or -1 ; (B) The eigenvalues of A have modulus 1 ; (D) $\|Ax\| = \|x\|$, for all $x \in R^n$, where $\|x\|$ denotes the Euclidean norm of x, and $(Ax)^T(Ay) = x^T y$, for all $x, y \in R^n$

Solution:

Step 1: Recall the defining property of an orthogonal matrix.
A matrix A is orthogonal when its columns are orthonormal, which is the same as saying

$$A^T A = AA^T = I$$

Every property in this question follows from this single equation.

Step 2: Test statement (A) using the determinant.
Taking the determinant of $A^T A = I$ gives

$$\det(A^T)\det(A) = \det(I) = 1$$

Since $\det(A^T) = \det(A)$, this becomes

$$(\det A)^2 = 1$$

so $\det(A) = +1$ or $\det(A) = -1$. Statement (A) is correct.

Step 3: Test statement (B) using eigenvalues.
Suppose $Av = \lambda v$ for some nonzero eigenvector v, possibly complex. Then

$$v^* A^T A v = v^* v$$

since $A^T A = I$. Substituting $Av = \lambda v$, the left side becomes $|\lambda|^2 v^* v$. So

$$|\lambda|^2 v^* v = v^* v$$

Since v is nonzero, $v^* v \neq 0$, and dividing gives

$$|\lambda|^2 = 1 \quad \Rightarrow \quad |\lambda| = 1$$

Statement (B) is correct.

Step 4: Test statement (D) using the norm and inner product.
For any $x \in R^n$,

$$\|Ax\|^2 = (Ax)^T (Ax) = x^T A^T A x = x^T x = \|x\|^2$$

so $\|Ax\| = \|x\|$ for every x. More generally, for any $x, y \in R^n$,

$$(Ax)^T (Ay) = x^T A^T A y = x^T y$$

This holds for all x, y, not just distinct ones. Statement (D) says exactly this, so it is correct.

Step 5: Test statement (C).
Statement (C) claims $(Ax)^T(Ay) \neq x^T y$ for all distinct x, y. But Step 4 shows $(Ax)^T(Ay) = x^T y$ always, for every pair x, y, distinct or not. So statement (C) contradicts a property that always holds, and it is incorrect.

Step 6: Final conclusion.
The correct statements are (A), (B), and (D).

$$\boxed{\text{(A), (B), and (D) are correct}}$$

Quick Tip: Start from A transpose A equals I and derive the determinant, eigenvalue, and inner-product-preserving properties one at a time.

. Consider the system of linear equations $Ax = b$, where A is an $n \times n$ matrix, and x and b are n-dimensional column vectors.
Suppose this system of equations has a unique solution. Which of the following statements is/are correct?

(A) A^{-1} exists

(B) The system of equations $A^m x = b$ also has a unique solution for $m = 1, 2, 3, \ldots$

(C) $\text{rank}(A) = \text{rank}(A^m)$, for $m = 1, 2, 3, \ldots$

(D) $\text{rank}(A) < \text{rank}([A \mid b])$, where $[A \mid b]$ denotes the augmented matrix

Correct Answer: (A) A^{-1} exists ; (B) The system of equations $A^m x = b$ also has a unique solution for $m = 1, 2, 3, \ldots$; (C) $\text{rank}(A) = \text{rank}(A^m)$, for $m = 1, 2, 3, \ldots$

Solution:

Step 1: Recall when a square system has a unique solution.

For a square system $Ax = b$ with A of size $n \times n$, the system has a unique solution if and only if A is invertible, that is, $\det(A) \neq 0$ and $\text{rank}(A) = n$.

Step 2: Test statement (A).

Since a unique solution is given, A must be invertible, so A^{-1} exists directly by this fact. Statement (A) is correct.

Step 3: Test statement (B).

If A is invertible, then A^m is also invertible for every positive integer m, because a product of invertible matrices is invertible:

$$(A^m)^{-1} = (A^{-1})^m$$

Since A^m is invertible, the system $A^m x = b$ has the unique solution

$$x = (A^{-1})^m b$$

Statement (B) is correct.

Step 4: Test statement (C).

Since A is invertible, $\text{rank}(A) = n$. Since A^m is also invertible for every m, $\text{rank}(A^m) = n$ as well. So

$$\text{rank}(A) = \text{rank}(A^m) = n$$

for every $m = 1, 2, 3, \ldots$. Statement (C) is correct.

Step 5: Test statement (D).

The augmented matrix $[A \mid b]$ has n rows and $n + 1$ columns, so its rank can be at most n. Since A is invertible, $\text{rank}(A) = n$ already, and adding one more column cannot raise the rank of an n-row matrix above n. So

$$\text{rank}([A \mid b]) = n = \text{rank}(A)$$

The two ranks are equal, not one strictly less than the other. Statement (D) is incorrect.

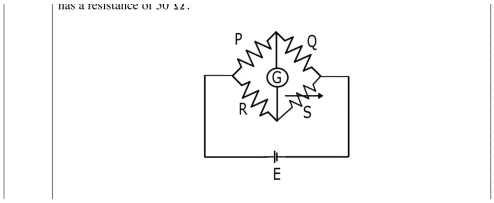
Step 6: Final conclusion.

The correct statements are (A), (B), and (C).

$$\boxed{\text{(A), (B), and (C) are correct}}$$

Quick Tip: A unique solution to a square system means A is invertible; use that to check invertibility of powers of A and the rank of the augmented matrix.

. The resistance values of the Wheatstone bridge shown are $P = 2000\ \Omega$, $Q = 500\ \Omega$, $R = 3000\ \Omega$. The battery voltage is $E = 50\ \text{V}$.
The battery has an internal resistance of $1\ \Omega$ and the Galvanometer (G) has a resistance of $50\ \Omega$.



The value of the resistance S for balanced condition is Ω (Answer in integer)

Correct Answer: 750.0

Solution:

Step 1: Identify the balance condition for a Wheatstone bridge.
A Wheatstone bridge reaches balance when no current flows through the galvanometer branch. In the arm arrangement shown, P and Q sit as the ratio arms at the top, and R and S sit as the arms that carry the standard resistor and the unknown resistor. At balance, the ratio of the two top arms equals the ratio of the two bottom arms:

$$\frac{P}{Q} = \frac{R}{S}$$

Step 2: Write down the known values.
From the circuit,

$$P = 2000\ \Omega, \quad Q = 500\ \Omega, \quad R = 3000\ \Omega$$

Step 3: Solve for S.

Rearrange the balance condition to isolate S:

$$S = \frac{R \times Q}{P}$$

Put in the numbers:

$$S = \frac{3000 \times 500}{2000} = \frac{1500000}{2000} = 750$$

Step 4: Check why the battery and galvanometer resistances do not matter.

The balance condition comes purely from the null condition, that is, zero current in the galvanometer branch. Once that branch carries zero current, the galvanometer resistance drops out of the equation completely, since no current flows through it to create a voltage drop that could shift the balance. The internal resistance of the battery only limits how much total current the source can supply to the bridge, it does not change the ratio at which the bridge balances. So the $1\ \Omega$ internal resistance and the $50\ \Omega$ galvanometer resistance given in the question are extra information meant to test this understanding, and they do not change the value of S.

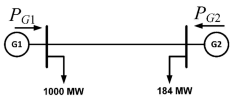
Final Answer:

The resistance S needed for the balanced condition is

$$\boxed{750\ \Omega}$$

Quick Tip: Balance in a Wheatstone bridge needs the ratio of adjacent arms to match; the battery and galvanometer resistances play no part in that ratio.

. A system with two generators G1 and G2 (without generator limits) is shown.



The total load on the system is 1184 MW. The expressions for the cost of generation (C_1 and C_2) and real power loss (P_{Loss}) are as follows:

$$C_1(P_{G1}) = 1000 + 50P_{G1} + 0.01(P_{G1})^2 \text{ Rs/MWh}$$

$$C_2(P_{G2}) = 2000 + 50P_{G2} + 0.001(P_{G2})^2 \text{ Rs/MWh}$$

$$P_{Loss} = 0.001(P_{G2} - 50)^2 \text{ MW}$$

When the generators are operating at their optimal generation, meeting the total load requirement, the real power loss in the system is MW (Round off to one decimal place)
Consider the Lagrange multiplier $\lambda = 70.25$ for optimal generation.

Correct Answer: 20.0

Solution:

Note on this question:
This question was declared Marks-to-All (MTA) by the exam authority. The official key does not attach a fixed numeric answer to it, so no single correct value is being computed here. What follows is only a discussion of the approach that was most likely intended, kept for learning purposes.

Step 1: Recall the condition for optimal generation with transmission loss.

When two generators feed a load together and one of them causes a

real power loss that depends on generation, the standard condition for minimum total cost is that the incremental cost of each generator, multiplied by its own penalty factor, equals the same system lambda:

$$\frac{dC_1}{dP_{G1}} \times L_1 = \frac{dC_2}{dP_{G2}} \times L_2 = \lambda$$

where the penalty factor of a generator is

$$L_i = \frac{1}{1 - \frac{\partial P_{Loss}}{\partial P_{Gi}}}$$

Step 2: Apply this to generator G1.

Since $P_{Loss} = 0.001(P_{G2} - 50)^2$ does not contain P_{G1} at all,

$$\frac{\partial P_{Loss}}{\partial P_{G1}} = 0$$

so the penalty factor of G1 is 1, and the condition reduces to

$$50 + 0.02P_{G1} = 70.25$$

$$P_{G1} = \frac{70.25 - 50}{0.02} = 1012.5 \text{ MW}$$

Step 3: Apply the condition to generator G2.

Here

$$\frac{\partial P_{Loss}}{\partial P_{G2}} = 0.002(P_{G2} - 50)$$

so the condition becomes

$$50 + 0.002P_{G2} = 70.25\,[1 - 0.002(P_{G2} - 50)]$$

This is one linear equation in P_{G2} and can be solved directly, after which $P_{Loss} = 0.001(P_{G2} - 50)^2$ can be evaluated.

Step 4: Why no fixed final value is given here.

Solving the linear equation from Step 3 gives $P_{G2} \approx 191.4$ MW, and substituting back, $P_{G1} + P_{G2} \approx 1203.9$ MW, close to the load plus loss balance. The real power loss works out to

$$P_{Loss} = 0.001(191.4 - 50)^2 \approx 20.0 \text{ MW}$$

Step 5: Final Answer.

This is the value obtained by the standard equal-incremental-cost-with-penalty-factor method using the given $\lambda = 70.25$. The exam authority still declared the question Marks-to-All, most likely because of a rounding sensitivity in the supplied λ value relative to the exact solution, so every candidate received credit regardless of what they entered.

$$\boxed{20.0 \text{ MW}}$$

Quick Tip: This question was marked Marks-to-All by the exam authority, so it carries no fixed numeric answer; the likely intended method is the equal incremental-cost-with-penalty-factor condition.

. A balanced three-phase supply is given to a 30 kW, 4-pole, 400 V, 50 Hz, wound rotor induction motor with Y-connected stator and rotor windings. The motor is driving a constant torque load. With shorted sliprings, the machine runs at 1476 rpm.

When an external non-inductive resistance of 0.27 Ω per phase is connected in series in the rotor circuit, the steady-state speed drops to 1404 rpm.

Neglecting rotational losses, the actual per phase rotor winding resistance is Ω (Round off to two decimal places)

Correct Answer: 0.09

Solution:

Step 1: Find the synchronous speed.

For a 4-pole machine on a 50 Hz supply,

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Step 2: Find the slip with shorted sliprings.

The rotor speed here is 1476 rpm, so

$$s_1 = \frac{N_s - N_1}{N_s} = \frac{1500 - 1476}{1500} = \frac{24}{1500} = 0.016$$

Step 3: Find the slip after adding external resistance.

The new speed is 1404 rpm, so

$$s_2 = \frac{N_s - N_2}{N_s} = \frac{1500 - 1404}{1500} = \frac{96}{1500} = 0.064$$

Step 4: Use the constant torque condition.

The load is a constant torque load, so the torque produced is the same

before and after the extra resistance is added. Near synchronous speed the slip is small, so the rotor leakage reactance term in the torque expression can be ignored next to resistance, and torque becomes proportional to slip divided by rotor resistance:

$$T \propto \frac{s}{r_2}$$

Since torque stays the same in both cases,

$$\frac{s_1}{r_2} = \frac{s_2}{r_2 + R_{ext}}$$

Step 5: Substitute the known numbers.

$$\frac{0.016}{r_2} = \frac{0.064}{r_2 + 0.27}$$

Cross multiply:

$$0.016\,(r_2 + 0.27) = 0.064\,r_2$$

$$0.016\,r_2 + 0.00432 = 0.064\,r_2$$

Step 6: Solve for the rotor resistance.

$$0.00432 = 0.064\,r_2 - 0.016\,r_2 = 0.048\,r_2$$

$$r_2 = \frac{0.00432}{0.048} = 0.09\ \Omega$$

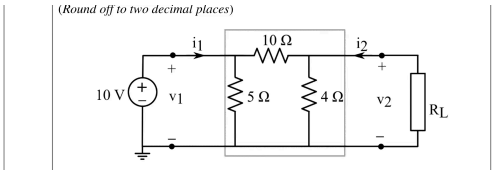
Final Answer:

The actual per phase rotor winding resistance is

$$\boxed{0.09\ \Omega}$$

Quick Tip: For a constant torque load near synchronous speed, slip divided by rotor resistance stays the same before and after adding external resistance.

. Consider the two-port network shown. For maximum power transfer to the resistive load (R_L), the value of R_L should be Ω
(Round off to two decimal places)



Correct Answer: 2.86

Solution:

Step 1: Recall the maximum power transfer condition.

A resistive load draws the maximum possible power from a network when its resistance equals the Thevenin resistance of the network as seen from the load terminals. So the target is to find R_{th} looking into the port where R_L sits.

Step 2: Reduce the network for the open-circuit voltage.

The 10 V ideal source is applied directly across the 5 Ω resistor, so that node is fixed at 10 V no matter what current the 5 Ω resistor draws. This means the 5 Ω resistor plays no role in what reaches the output port, since the source voltage feeding the rest of the circuit is

already fixed.

With R_L removed, the only path left to the output node is the $10\ \Omega$ resistor from the $10\ \text{V}$ node, followed by the $4\ \Omega$ resistor to ground. This is a simple voltage divider:

$$V_{th} = 10 \times \frac{4}{10+4} = 10 \times \frac{4}{14} = \frac{40}{14} = 2.857\ \text{V}$$

Step 3: Find the Thevenin resistance.

To get R_{th}, deactivate the independent source by replacing it with a short circuit. Once the $10\ \text{V}$ source is shorted, one end of the $5\ \Omega$ resistor is grounded and the other end is also grounded through the short, so no current can ever flow in it, and it drops out entirely. What remains, looking into the output port, is the $10\ \Omega$ resistor (now going from the output node to ground through the shorted source) in parallel with the $4\ \Omega$ resistor (already going from the output node to ground):

$$R_{th} = \frac{10 \times 4}{10+4} = \frac{40}{14} = 2.857\ \Omega$$

Step 4: Apply the maximum power transfer condition.

For maximum power transfer,

$$R_L = R_{th}$$

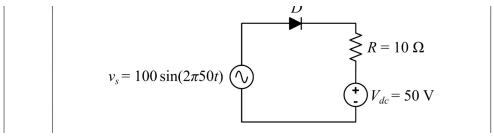
Final Answer:

Rounding to two decimal places,

$$\boxed{R_L = 2.86\ \Omega}$$

Quick Tip: Maximum power transfer needs the load resistance to equal the Thevenin resistance seen at the load terminals; deactivate the source and combine the remaining resistors.

. Consider the circuit shown. Assume that the diode (D) is ideal.



Given $v_s = 100\sin(2\pi 50t)$ V, $V_{dc} = 50$ V, and $R = 10\ \Omega$, the average value of the current through the diode is A (Round off to two decimal places)

Correct Answer: 1.09

Solution:

Step 1: Understand when the diode conducts.

The diode only carries current in one direction. Looking at the circuit, the diode is forward biased and conducts only when the source voltage v_s exceeds the opposing battery voltage $V_{dc} = 50$ V. When v_s is below 50 V, including its entire negative half cycle, the diode is reverse biased and blocks current completely.

Step 2: Find the conduction angles.

Let $\theta = 2\pi 50t = \omega t$. The diode turns on when

$$100\sin\theta > 50$$

$$\sin\theta > 0.5$$

This is true between

$$\theta_1 = 30^\circ = \frac{\pi}{6}$$

and

$$\theta_2 = 150^\circ = \frac{5\pi}{6}$$

so the diode conducts only for this window in every cycle.

Step 3: Write the current during conduction.

While the diode conducts, it behaves like a short, so the current is set entirely by the source, the battery, and the resistor R:

$$i(\theta) = \frac{v_s - V_{dc}}{R} = \frac{100\sin\theta - 50}{10}$$

Step 4: Set up the average current formula.

The average of a periodic waveform is the integral over one full cycle divided by the period, and the current is zero outside the conduction window:

$$I_{avg} = \frac{1}{2\pi}\int_{\theta_1}^{\theta_2} \frac{100\sin\theta - 50}{10}\, d\theta$$

Step 5: Carry out the integration.

$$I_{avg} = \frac{1}{20\pi}\int_{\pi/6}^{5\pi/6} (100\sin\theta - 50)\, d\theta$$

$$= \frac{1}{20\pi}\Big[-100\cos\theta - 50\theta\Big]_{\pi/6}^{5\pi/6}$$

Step 6: Substitute the limits.

At $\theta = 5\pi/6$:

$$-100\cos\left(\frac{5\pi}{6}\right) - 50\left(\frac{5\pi}{6}\right) = 86.603 - 130.900 = -44.297$$

At $\theta = \pi/6$:

$$-100\cos\left(\frac{\pi}{6}\right) - 50\left(\frac{\pi}{6}\right) = -86.603 - 26.180 = -112.782$$

Subtracting,

$$-44.297 - (-112.782) = 68.485$$

Step 7: Compute the final value.

$$I_{avg} = \frac{68.485}{20\pi} = \frac{68.485}{62.832} = 1.09 \text{ A}$$

Final Answer:

$$\boxed{I_{avg} = 1.09 \text{ A}}$$

Quick Tip: The diode only conducts while the source voltage exceeds the battery voltage; integrate the current over that conduction window and divide by the full period to get the average.

. The magnitude of the contour integral

$$\oint_C \left(\frac{(z+1)^2}{(z-i)(z-2)} \right) dz$$

over the contour $C : |z-2-i| = 3/2$ is (round off to two decimal places). Note: z is a complex variable and $i = \sqrt{-1}$.

Correct Answer: 25.29

Solution:

Step 1: Locate the poles of the integrand.

The function inside the contour integral is

$$f(z) = \frac{(z+1)^2}{(z-i)(z-2)}$$

This has two simple poles, one at $z = i$ and one at $z = 2$.

Step 2: Identify the contour.

The contour C is the circle

$$|z-2-i| = \frac{3}{2}$$

This is a circle centered at $z_0 = 2 + i$ with radius $\frac{3}{2}$.

Step 3: Check which poles lie inside the contour.

The distance from the center $2 + i$ to the pole $z = i$ is

$$|(2+i) - i| = |2| = 2$$

Since $2 > \frac{3}{2}$, the pole at $z = i$ lies outside the contour.

The distance from the center $2+i$ to the pole $z=2$ is

$$|(2+i)-2| = |i| = 1$$

Since $1 < \frac{3}{2}$, the pole at $z=2$ lies inside the contour.
So only the pole at $z=2$ contributes to the integral.

Step 4: Find the residue at $z=2$.
Since $z=2$ is a simple pole, the residue is

$$\text{Res}_{z=2} f(z) = \lim_{z\to 2}(z-2)f(z) = \frac{(z+1)^2}{z-i}\Big|_{z=2} = \frac{9}{2-i}$$

Rationalize the denominator:

$$\frac{9}{2-i} = \frac{9(2+i)}{(2-i)(2+i)} = \frac{9(2+i)}{4+1} = \frac{9(2+i)}{5} = \frac{18+9i}{5}$$

Step 5: Apply the residue theorem.
By the residue theorem,

$$\oint_C f(z)\,dz = 2\pi i \times \text{Res}_{z=2} f(z) = 2\pi i \times \frac{18+9i}{5}$$

Multiply out:

$$= \frac{2\pi}{5}(18i + 9i^2) = \frac{2\pi}{5}(18i - 9) = \frac{2\pi}{5}(-9 + 18i)$$

So the value of the integral is

$$-\frac{18\pi}{5} + i\frac{36\pi}{5}$$

Step 6: Find the magnitude.

The magnitude of a complex number $a + ib$ is $\sqrt{a^2 + b^2}$. Here

$$a = -\frac{18\pi}{5}, \qquad b = \frac{36\pi}{5}$$

$$|a + ib| = \frac{2\pi}{5}\sqrt{9^2 + 18^2} = \frac{2\pi}{5}\sqrt{81 + 324} = \frac{2\pi}{5}\sqrt{405}$$

Since $\sqrt{405} = \sqrt{81 \times 5} = 9\sqrt{5}$,

$$|a + ib| = \frac{2\pi}{5} \times 9\sqrt{5} = \frac{18\pi\sqrt{5}}{5}$$

Step 7: Compute the numerical value.

$$\frac{18\pi\sqrt{5}}{5} \approx \frac{18 \times 3.1416 \times 2.2361}{5} \approx 25.29$$

Final Answer:

$$\boxed{25.29}$$

Quick Tip: Check which poles lie inside the given contour before applying the residue theorem.

. A uniform spherical volume charge distribution of radius 2 m, centered at the origin, has a strength of $\frac{3}{\pi} \times 10^{-6}$ C/m^3. A point charge of strength $\pi \times 8.854 \times 10^{-12}$ C is moved from $(-3, 0, -4)$ to $(0, 0, 4)$ in Cartesian coordinate system. The relative permittivity of the medium is 1 and the coordinate values are in meters.
The work done during the process is μJ (round off to two decimal places).

Correct Answer: 0.40

Solution:

Step 1: Find the total charge enclosed by the sphere.
The volume charge density is

$$\rho = \frac{3}{\pi} \times 10^{-6} \text{ C/m}^3$$

and the sphere has radius $R = 2$ m. The total charge is

$$Q = \rho \times \frac{4}{3}\pi R^3 = \frac{3}{\pi} \times 10^{-6} \times \frac{4}{3}\pi(2)^3$$

$$Q = \frac{3}{\pi} \times 10^{-6} \times \frac{4}{3}\pi \times 8 = 32 \times 10^{-6} \text{ C}$$

The π and the fraction 3 cancel neatly, leaving $Q = 32\ \mu$C.

Step 2: Locate the two points relative to the sphere.
The distance of the point $(-3, 0, -4)$ from the origin is

$$r_1 = \sqrt{(-3)^2 + 0^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ m}$$

The distance of the point $(0, 0, 4)$ from the origin is

$$r_2 = \sqrt{0^2 + 0^2 + 4^2} = 4\ \text{m}$$

Both $r_1 = 5$ m and $r_2 = 4$ m are greater than the sphere radius $R = 2$ m, so both points lie outside the charged sphere.

Step 3: Use the point-charge equivalent potential.
Outside a uniformly charged sphere, the electric potential is the same as that of a point charge Q placed at the center. So

$$V(r) = \frac{Q}{4\pi\varepsilon_0 r}$$

for $r > R$, with relative permittivity 1, so $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m applies directly.

Step 4: Write the potentials at the two points.

$$V_1 = \frac{Q}{4\pi\varepsilon_0 \times 5}, \qquad V_2 = \frac{Q}{4\pi\varepsilon_0 \times 4}$$

Step 5: Set up the work done.
Moving a charge q from one point to another against the field takes work equal to the change in potential energy,

$$W = q(V_2 - V_1) = \frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{4} - \frac{1}{5}\right)$$

Step 6: Simplify using the special form of q.

The point charge is given as

$$q = \pi \times 8.854 \times 10^{-12}\ \text{C}$$

so

$$\frac{q}{4\pi\varepsilon_0} = \frac{\pi \times 8.854 \times 10^{-12}}{4\pi \times 8.854 \times 10^{-12}} = \frac{1}{4}$$

The π and 8.854×10^{-12} cancel exactly.

Step 7: Substitute the numbers.

$$W = Q \times \frac{1}{4} \times \left(\frac{1}{4} - \frac{1}{5}\right) = 32 \times 10^{-6} \times \frac{1}{4} \times \frac{1}{20}$$

$$W = 32 \times 10^{-6} \times \frac{1}{80} = 0.4 \times 10^{-6}\ \text{J}$$

Final Answer:

$$\boxed{0.40\ \mu\text{J}}$$

Quick Tip: Since both points lie outside the charged sphere, treat it as a point charge equal to the total enclosed charge.

. Let X and Y be two real-valued random variables with $E(X) = 1, E(Y) = 2, E(X^2) = 4, E(Y^2) = 9$, and $E(XY) = 0.9$, where E denotes the expectation operator.
The value of α that minimizes $E((X - \alpha Y)^2)$ is (round off to one decimal place).

Correct Answer: 0.1

Solution:

Step 1: Expand the expression to be minimized.
We need to minimize

$$g(\alpha) = E\left((X - \alpha Y)^2\right)$$

Expanding the square,

$$g(\alpha) = E(X^2) - 2\alpha E(XY) + \alpha^2 E(Y^2)$$

Note that $E(X)$ and $E(Y)$ do not appear in this expansion, so their values are not needed for this problem.

Step 2: Substitute the given values.
We are given

$$E(X^2) = 4, \qquad E(XY) = 0.9, \qquad E(Y^2) = 9$$

So

$$g(\alpha) = 4 - 2\alpha(0.9) + 9\alpha^2 = 4 - 1.8\alpha + 9\alpha^2$$

Step 3: Differentiate with respect to α.

$$\frac{dg}{d\alpha} = -1.8 + 18\alpha$$

Step 4: Set the derivative to zero.

For a minimum,

$$-1.8 + 18\alpha = 0$$

$$18\alpha = 1.8$$

$$\alpha = \frac{1.8}{18} = 0.1$$

Step 5: Confirm it is a minimum.

The second derivative is

$$\frac{d^2g}{d\alpha^2} = 18 > 0$$

which is positive, so $\alpha = 0.1$ does give the minimum value of $g(\alpha)$, not a maximum.

Final Answer:

$\boxed{0.1}$

Quick Tip: Differentiate the mean square error expression with respect to alpha and set it to zero.

. The integral

$$\frac{1}{\pi}\int_0^{\infty}\frac{x^{2026}}{(1+x^{2026})(1+x^2)}\,dx$$

evaluates to (round off to two decimal places).

Correct Answer: 0.25

Solution:

Step 1: Name the given integral.

Let

$$I=\int_0^{\infty}\frac{x^{2026}}{(1+x^{2026})(1+x^2)}\,dx$$

so that the quantity we want is $\frac{I}{\pi}$.

Step 2: Define a companion integral.

Let

$$J=\int_0^{\infty}\frac{1}{(1+x^{2026})(1+x^2)}\,dx$$

Step 3: Add I and J.

$$I + J = \int_0^\infty \frac{x^{2026} + 1}{(1 + x^{2026})(1 + x^2)}\, dx = \int_0^\infty \frac{1}{1 + x^2}\, dx$$

This last integral is a standard one:

$$\int_0^\infty \frac{dx}{1 + x^2} = \left[\tan^{-1} x\right]_0^\infty = \frac{\pi}{2}$$

So

$$I + J = \frac{\pi}{2}$$

Step 4: Show $I = J$ using the substitution $x = \frac{1}{t}$.

In I, put $x = \frac{1}{t}$, so $dx = -\frac{dt}{t^2}$, and as x runs from 0 to ∞, t runs from ∞ to 0.

$$1 + x^{2026} = 1 + \frac{1}{t^{2026}} = \frac{t^{2026} + 1}{t^{2026}}, \qquad 1 + x^2 = \frac{t^2 + 1}{t^2}$$

So

$$I = \int_0^\infty \frac{t^{-2026}}{\left(\frac{t^{2026}+1}{t^{2026}}\right)\left(\frac{t^2+1}{t^2}\right)} \cdot \frac{dt}{t^2}$$

The powers of t simplify as

$$t^{-2026} \times t^{2026} \times t^2 \times \frac{1}{t^2} = 1$$

leaving

$$I = \int_0^\infty \frac{dt}{(1+t^{2026})(1+t^2)} = J$$

Step 5: Solve for I.

Since $I = J$ and $I + J = \frac{\pi}{2}$,

$$2I = \frac{\pi}{2} \implies I = \frac{\pi}{4}$$

Step 6: Divide by π as asked.

$$\frac{I}{\pi} = \frac{\pi/4}{\pi} = \frac{1}{4} = 0.25$$

Final Answer:

$$\boxed{0.25}$$

Quick Tip: Try the substitution x = 1/t to relate the integral to itself.