GATE 2026 Electrical Engineering Question Paper with Solutions

ZOLLEGE

General Instructions

(i) This booklet contains 65 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 48 single-correct multiple-choice questions, 4 multiple-correct questions and 13 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For multiple-correct questions, all correct options must be selected to earn full marks.

(v) For numerical questions, report the answer rounded exactly as asked.

. 'The shopkeeper sells lemons.'

In this sentence, the word 'lemons' is the __________.

(A) object

(B) subject

(C) predicate

(D) verb

Correct Answer: (A) object

SOLUTION:

Step 1: Understanding the Concept.

A quick way to find the object of a sentence is to first find the verb, then ask "what" right after it.

Step 2: Key Formula or Approach.

Find the verb in the sentence, then ask "the subject verbs what?" The word that answers this question is the object.

Step 3: Detailed Explanation.

The verb here is "sells." Ask, "the shopkeeper sells what?" The answer is "lemons." So "lemons" answers the "what" question and is the object of the verb "sells."

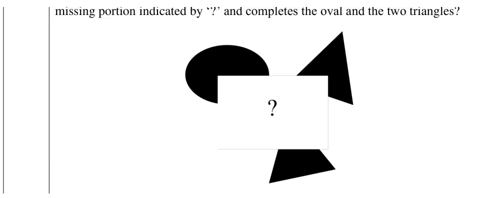
"Shopkeeper" answers "who sells," so that word is the subject, not the object. There is only one verb in the sentence, "sells," so "lemons" cannot be the verb. The predicate is everything said about the subject, "sells lemons" as a whole, not just the single word "lemons."

Step 4: Final Answer.

Since "lemons" answers "sells what," it is the object of the sentence, matching option (A).

Quick Tip: Find the verb first, then ask "what" right after it. The word that answers this is the object.

. The figure below is supposed to show three non-overlapping shapes: one oval and two triangles. Which one of the following figures P, Q, R, or S fits the missing portion indicated by '?' and completes the oval and the two triangles?



(A) P

(B) Q

(C) R

(D) S

Correct Answer: (A) P

SOLUTION:

Another way to solve this is to first fix exactly what the finished picture must contain: one full oval and two full triangles, and nothing more. Then check each option by mentally placing it into the "?" patch and seeing whether the shapes close up cleanly.

1. **Q**: Placing this piece back in gives one oval and two triangles in outline, but one triangle corner ends up pointing the wrong way compared to the rest of that triangle's outline in the main figure, so the triangle would not close into one clean triangle. Rejected.
2. **R**: The triangle piece here is a different size from the matching triangle corner already showing in the main figure, so putting it in would create a triangle with a kink in its edge instead of a straight side. Rejected.
3. **S**: The oval piece is too large. Adding it to the visible oval arc would make an oval bigger than what the rest of the figure's outline allows, so the shapes would not close up into a clean single oval. Rejected.
4. **P**: Placing this piece in completes the oval into one smooth, closed curve and both triangles into clean, straight sided triangles, with no kinks, gaps, or extra lines anywhere.

Since P is the only piece whose insertion leaves exactly one oval and two triangles with clean outlines, P is the correct missing piece.

Let's summarize:

- The finished figure must contain exactly one oval and two triangles, no more and no less.

- Check each option by imagining it placed in the gap and see if every outline closes up cleanly.
- Only P closes every shape without a kink, gap, or mismatch.

So option (A), figure P, is confirmed as the answer.

Quick Tip: Match the curve direction of the oval and the angle of each triangle corner to the visible edges around the "?" patch.

• • •

. At how many points will the curves $y = x^2$ and $y = -x^2 - 2x - 1$ intersect in the real (x, y) plane?

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (A) 0

SOLUTION:

Step 1: Form the equation for meeting points.

The two curves meet where their y values agree, so

$$x^2 = -x^2 - 2x - 1$$

which rearranges to

$$2x^2 + 2x + 1 = 0$$

Step 2: Divide through to simplify.

Divide every term by 2:

$$x^2 + x + \frac{1}{2} = 0$$

Step 3: Complete the square.

$$x^2 + x = \left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$$

So the equation becomes

$$\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{1}{2} = 0$$

$$\left(x + \frac{1}{2}\right)^2 = -\frac{1}{4}$$

Step 4: Check if a real solution exists.

A square of a real number can never be negative, but the right side here is $-\frac{1}{4}$, which is negative. So no real value of x can satisfy this equation.

Step 5: Conclude the number of intersection points.

Since there is no real x where the curves agree, they do not cross anywhere in the real plane.

$$\boxed{0}$$

Quick Tip: Set the two expressions for y equal and check whether the resulting quadratic has real roots.

• • •

. 'If Anish had scored hundred runs in today's match, he would have been made the captain of his team. He would have then become the youngest captain in his team's history. Unfortunately, he got out without scoring any runs. Hence, there won't be any change in the captaincy for now.' Based on the paragraph above, which one of the following statements is true?

(A) Anish made hundred runs but was denied captaincy.

(B) Anish was the captain of his team before the game today.

(C) The current captain is older than Anish.

(D) Anish is the youngest player in his team.

Correct Answer: (C) The current captain is older than Anish.

SOLUTION:

A different way to read this passage is to focus on the last line first: "there won't be any change in the captaincy for now." This tells us directly that the existing captain remains in place, since Anish failed to score the runs needed to replace him.

1. Working backward, if Anish had scored a hundred, he would have replaced the current captain and become the "youngest captain in his team's history." For a new captain to be the youngest ever, that new captain must be younger than every captain who held the post before, including the one currently in charge.

2. Since the current captain stays in place (because Anish failed), and Anish would have been younger than that very captain had he taken over, it follows that the current captain is older than Anish.
3. Checking the other statements against the passage: Anish did not score any runs, so he was never in line to be "denied" captaincy after actually scoring. There is no mention of Anish already being captain earlier, so that option cannot be true. The passage never compares Anish to the other players in age, only to past captains, so the "youngest player" claim goes beyond what is stated.

Only one statement survives this backward check.

Let's summarize:

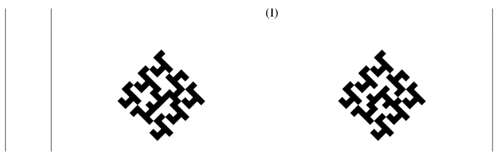
- The passage confirms the captaincy stayed unchanged because Anish did not score the runs.
- Being the hypothetical youngest ever captain means Anish is younger than the current captain.
- No claim is made about the youngest player on the team.

So the correct statement is that the current captain is older than Anish, which is option (C).

Quick Tip: Anish would have been the youngest ever captain, so the current captain must be older than him.

. Which one of the following figures P, Q, R, or S, correctly shows the 45° clockwise-rotated version of figure (I)?

(I)



(A) P

(B) Q

(C) R

(D) S

Correct Answer: (B) Q

SOLUTION:

A quick way to check a rotation question is to pick one easy-to-spot feature in figure (I), such as the corner of the square with the most crowded zigzag teeth, and track where that same feature should land after turning 45 degrees clockwise.

1. In figure (I), imagine the top edge of the square. Turning the whole figure 45 degrees clockwise moves this top edge halfway toward the right side, so what was the top edge should now point toward the upper right region of a diamond shape.
2. **Checking P**: the tracked feature lands close to the expected spot, but a nearby zigzag tooth is flipped compared to figure (I), which a plain rotation cannot do.
3. **Checking R**: the outer diamond shape looks fine, but the tracked feature ends up in a slightly wrong position, showing that some teeth have been reordered rather than just rotated.
4. **Checking S**: the tracked feature does not move a full 45 degrees, since parts of the original square edges remain straight instead of becoming diamond corners, meaning the rotation angle used here is wrong.
5. **Checking Q**: the tracked feature lands exactly where a clean 45 degree clockwise turn predicts, and every other zigzag tooth follows the same consistent turn with no flipping or reordering.

Since only Q keeps the tracked feature and every other detail consistent with one single clean rotation, Q is the correct match.

Let's summarize:

- Track one clear feature through the rotation instead of comparing every tooth at once.
- Reject any option where the feature lands in the wrong place, is mirrored, or the rotation angle looks wrong.

So the correct answer is option (B), figure Q.

Quick Tip: A rotation only turns the shape; check that the square outline becomes a diamond and no zigzag tooth is flipped.

• • •

. Match the words in Column I with their synonyms in Column II.

Column I	Column II
(i) Lonely	(p) Verbatim
(ii) Literal	(q) Solitary
(iii) Lousy	(r) Deadly
(iv) Lethal	(s) Terrible

(A) (i)-(q); (ii)-(p); (iii)-(s); (iv)-(r)

(B) (i)-(q); (ii)-(s); (iii)-(r); (iv)-(p)

(C) (i)-(s); (ii)-(p); (iii)-(q); (iv)-(r)

(D) (i)-(r); (ii)-(s); (iii)-(p); (iv)-(q)

Correct Answer: (A) (i)-(q); (ii)-(p); (iii)-(s); (iv)-(r)

SOLUTION:

Instead of matching each word one at a time, we can check the four given options and rule out the ones that contain even one wrong pair.

1. **Option (B)**: pairs "Literal" with "Terrible" (s). But "Literal" means word for word, matching "Verbatim," not "Terrible." So option (B) is wrong.
2. **Option (C)**: pairs "Lonely" with "Terrible" (s). But "Lonely" means alone, matching "Solitary," not "Terrible." So option (C) is wrong.
3. **Option (D)**: pairs "Lonely" with "Deadly" (r). But "Lonely" has nothing to do with causing death. So option (D) is wrong.
4. **Option (A)**: pairs "Lonely" with "Solitary," "Literal" with "Verbatim," "Lousy" with "Terrible," and "Lethal" with "Deadly." Each pair matches in meaning.

Since options (B), (C), and (D) each break down on their very first pair, only option (A) survives the check.

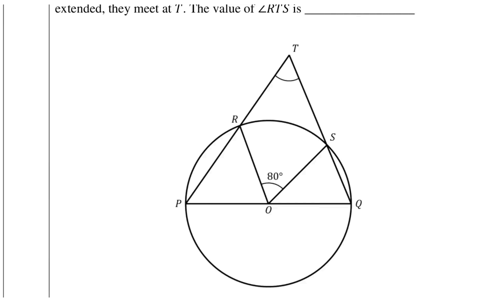
Let's summarize:

- Checking the first pair in each option quickly rules out three of the four choices.
- The surviving option (A) has all four word meanings matching correctly.

So the correct answer is option (A): (i)-(q); (ii)-(p); (iii)-(s); (iv)-(r).

Quick Tip: Match each word to a synonym with the same meaning: lonely=alone, literal=exact words, lousy=bad, lethal=deadly.

. In the given figure, $\overline{PQ}$ is the diameter of a circle with center O. Two points R and S are chosen on the circle such that $\angle ROS = 80°$. When $\overline{PR}$ and $\overline{QS}$ are extended, they meet at T. The value of $\angle RTS$ is __________.



(A) 40°

(B) 50°

(C) 60°

(D) 80°

Correct Answer: (B) 50°

SOLUTION:

Step 1: Identify the two secants from external point T.

T lies outside the circle, and two secants are drawn from it: one through R and P (with R nearer to T), and one through S and Q (with S nearer to T).

Step 2: Recall the external angle formula.

When two secants are drawn from a point outside a circle, the angle between them equals half the difference of the two arcs they cut off:

$$\angle T = \frac{1}{2}(\text{arc } PQ - \text{arc } RS)$$

where arc PQ is the far arc between the far points P and Q, and arc RS is the near arc between the near points R and S.

Step 3: Find arc RS.

The central angle $\angle ROS = 80°$ directly gives the near arc RS:

$$\text{arc } RS = 80^\circ$$

Step 4: Find arc PQ.

Since PQ is a diameter, it splits the circle into two equal arcs of 180° each. Points R and S both lie on the same semicircle, the one nearer to T, so the far arc PQ (the one not containing R and S) is the other semicircle:

$$\text{arc } PQ = 180^\circ$$

Step 5: Substitute into the formula.

$$\angle RTS = \frac{1}{2}(180^\circ - 80^\circ) = \frac{1}{2}(100^\circ) = 50^\circ$$

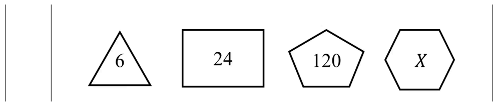
Step 6: Conclude.

$$\boxed{\angle RTS = 50^\circ}$$

Quick Tip: Use radii OP=OR=OS=OQ to form isosceles triangles, or apply the external angle formula for two secants from T.

• • •

. Based on the relationship between each polygon and the number inside it, the value of 'X' is ________. A triangle contains the number 6, a rectangle contains 24, a pentagon contains 120, and a hexagon contains X.



(A) 720

(B) 596

(C) 24

(D) 240

Correct Answer: (A) 720

SOLUTION:

Step 1: Look at how the number changes from one polygon to the next.

Going from the triangle to the rectangle, the number of sides goes from 3 to 4, and the value goes from 6 to 24. Check the ratio:

$$\frac{24}{6} = 4$$

This matches the new side count, 4.

Step 2: Check the next step the same way.

Going from the rectangle to the pentagon, sides go from 4 to 5, and the value goes from 24 to 120. Check the ratio:

$$\frac{120}{24} = 5$$

Again this matches the new side count, 5.

Step 3: State the rule found.

Each time the side count goes up by one, the value gets multiplied by that new side count. This is exactly how a factorial builds up, since $n!$ $= (n-1)! \times n$.

Step 4: Apply the rule to the hexagon.

The hexagon has 6 sides, one more than the pentagon's 5 sides. So multiply the pentagon's value by 6:

$$X = 120 \times 6 = 720$$

Step 5: Conclude.

$$\boxed{X = 720}$$

Quick Tip: Compare the number of sides of each polygon with the number written inside it.

• • •

. Consider a linear arrangement of seven bulbs, each of which can be in the ON or OFF state. The initial configuration of the bulbs is shown below. In every step, the states of the bulbs change based on the following rules:

- Any OFF bulb with exactly one ON neighbor at the end of the previous step is turned ON.
- Any ON bulb with both neighbors ON at the end of the previous step is turned OFF.
- The state of any bulb not meeting the conditions above is left unchanged.

The states of the bulbs at the end of Step 1 and Step 2 are also shown below.

Stage	Bulb 1	Bulb 2	Bulb 3	Bulb 4	Bulb 5	Bulb 6	Bulb 7
Initial	OFF	OFF	OFF	ON	OFF	OFF	OFF
Step 1	OFF	OFF	ON	ON	ON	OFF	OFF
Step 2	OFF	ON	ON	OFF	ON	ON	OFF

The number of bulbs which are ON at the end of Step 8 is _______

(A) 5

(B) 4

(C) 3

(D) 0

Correct Answer: (B) 4

SOLUTION:

Instead of drawing every bulb row, we can track just the set of positions that are ON at each stage and use the same two rules directly on that set.

Let S_t be the set of ON bulb positions after stage t, numbering the bulbs 1 to 7.

- $S_0 = \{4\}$ (given initial state).
- $S_1 = \{3, 4, 5\}$ (given Step 1): bulbs 3 and 5 gain exactly one ON neighbor (bulb 4) and switch on; bulb 4 keeps both neighbors off so it is untouched.
- $S_2 = \{2, 3, 5, 6\}$ (given Step 2): bulb 2 gains one ON neighbor (bulb 3) and turns on, bulb 6 gains one ON neighbor (bulb 5) and turns

on, while bulb 4 has two ON neighbors (3 and 5) which is not "exactly one", so it turns off, and bulbs 3, 5 are untouched.

Continue the same set update for the next steps:

- $S_3 = \{1, 2, 3, 5, 6, 7\}$: bulb 1 and bulb 7 each have exactly one ON neighbor (2 and 6 respectively) and switch on; bulb 4 again sees two ON neighbors and stays off; bulbs 2, 3, 5, 6 each have only one ON neighbor so they remain on.
- $S_4 = \{1, 3, 5, 7\}$: bulb 2 now has both neighbors (1 and 3) ON, so it turns off; bulb 6 similarly turns off since both neighbors (5 and 7) are ON; bulbs 3 and 5 each have only one ON neighbor left, so they stay on; bulbs 1 and 7 stay on since, as end bulbs, they can never see two ON neighbors.

Check S_4 for stability: with ON positions $\{1, 3, 5, 7\}$, every ON bulb (3 and 5) has both its neighbors OFF (an even position on each side), so neither condition to turn off is met and they stay on; bulbs 1 and 7 stay on as end bulbs. Every OFF bulb (2, 4, 6) has both its neighbors ON (odd positions on each side), which is two ON neighbors, not exactly one, so none of them switch on. Therefore $S_5 = S_6 = S_7 = S_8 = S_4$ $= \{1, 3, 5, 7\}$.

So the number of bulbs ON at the end of Step 8 equals $|S_4| = 4$.

$$\boxed{4}$$

Quick Tip: Simulate the ON/OFF rule step by step; the pattern reaches a fixed alternating state well before Step 8, so counting the ON bulbs there gives the answer directly.

• • •

. P and Q are two positive integers such that $P^2 = Q^2 + 13$.

The product of the numbers P and Q is ____________

(A) 13

(B) 26

(C) 39

(D) 42

Correct Answer: (D) 42

SOLUTION:

A quicker route is to guess that P and Q are close together, since their squares differ by only 13, a small number. The simplest guess is that they are consecutive integers, $P = Q + 1$.

Substitute this into the given equation:

$$(Q + 1)^2 = Q^2 + 13$$

Expand the left side:

$$Q^2 + 2Q + 1 = Q^2 + 13$$

The Q^2 terms cancel, leaving:

$$2Q + 1 = 13 \implies 2Q = 12 \implies Q = 6$$

So $P = Q + 1 = 7$.

This guess is not just lucky, it is forced: since 13 is prime, the factor pair $(P - Q, P + Q)$ of 13 can only be $(1, 13)$, so $P - Q$ must equal exactly 1, which is precisely the consecutive-integer assumption used here. So $P = 7$ and $Q = 6$ is the only solution in positive integers.

The required product is:

$$P \times Q = 7 \times 6 = 42$$

$$\boxed{42}$$

Quick Tip: Write P squared minus Q squared as a difference of squares and use that 13 is a prime number to fix P minus Q and P plus Q.

• • •

. Consider the infinite-length, discrete-time sequence $x[n] = 0.9^{|n|}$, where n is an integer. The region of convergence of its Z-transform $X(z)$ is given by:
(Note: z is a complex variable)

(A) $|z| > 0.9$

(B) $|z| < 0.9$

(C) $0.9 < |z| < 1/0.9$

(D) $\{z \text{ such that } |z| < 0.9\} \cup \{z \text{ such that } |z| > 1/0.9\}$

Correct Answer: (C) $0.9 < |z| < 1/0.9$

SOLUTION:

We can also find the ROC by writing the bilateral Z-transform sum directly and applying the ratio test to its two halves.

The Z-transform is:

$$X(z) = \sum_{n=-\infty}^{\infty} 0.9^{|n|} z^{-n} = \underbrace{\sum_{n=0}^{\infty} 0.9^{n} z^{-n}}_{\text{first sum}} + \underbrace{\sum_{n=1}^{\infty} 0.9^{n} z^{n}}_{\text{second sum}}$$

(the second sum comes from substituting $n \to -n$ in the $n \le -1$ part of the original sum).

The first sum is a geometric series with common ratio $r_1 = 0.9z^{-1}$. A geometric series converges exactly when the ratio's magnitude is less than 1, so:

$$|0.9z^{-1}| < 1 \implies |z| > 0.9$$

The second sum is a geometric series with common ratio $r_2 = 0.9z$, which converges when:

$$|0.9z| < 1 \implies |z| < \frac{1}{0.9}$$

Both sums must converge together for $X(z)$ to exist, so we take the overlap of the two conditions:

$$0.9 < |z| < \frac{1}{0.9}$$

This ring-shaped region is exactly option (C).

$$\boxed{0.9 < |z| < 1/0.9}$$

Quick Tip: Split the two-sided exponential sequence into a right-sided and a left-sided part, find the ROC of each, and intersect them.

• • •

. Let $x_c(t)$ be any continuous-time periodic signal with period T. It is sampled uniformly with a sampling period T_s where $T_s \neq T$, resulting in the discrete sequence

$$x[n] = x_c(nT_s),$$

where n is an integer.

Which one of the following statements is correct about $x[n]$?

(A) $x[n]$ will always be periodic with period T/T_s for all values of T/T_s

(B) $x[n]$ will always be periodic with period 1 for all values of T/T_s

(C) $x[n]$ will never be periodic

(D) $x[n]$ will be periodic if and only if T/T_s is a rational number

Correct Answer: (D) $x[n]$ will be periodic if and only if T/T_s is a rational number

SOLUTION:

We can also settle this by testing one rational case and one irrational case directly, instead of deriving the general condition first.

Rational case: Let $T = 3$ and $T_s = 2$, so $T/T_s = 3/2$, a rational number. We look for an integer N and integer k with $NT_s = kT$, that is $2N = 3k$. Taking $N = 3, k = 2$ works: $2(3) = 6 = 3(2)$. So $x[n+3]$ $= x_c((n+3)(2)) = x_c(2n+6)$. Since $6 = 2 \times 3 = 2T$, and x_c has period $T = 3$, we get $x_c(2n+6) = x_c(2n) = x[n]$. So the sequence repeats with period $N = 3$: periodic, as claimed for the rational case.

Irrational case: Let $T = 1$ and $T_s = 1/\sqrt{2}$, so $T/T_s = \sqrt{2}$, which is irrational. Suppose, for contradiction, some integer N and integer k satisfy $NT_s = kT$, that is $N/\sqrt{2} = k$, so $\sqrt{2} = N/k$. This says $\sqrt{2}$ equals a ratio of two integers, which is impossible since $\sqrt{2}$ is irrational. So no such N exists, and $x[n]$ never repeats: not periodic, as claimed for the irrational case.

These two worked examples rule out option (A) and (B), since periodicity clearly fails in the irrational case, and rule out option (C), since periodicity clearly holds in the rational case. The only statement consistent with both examples is that $x[n]$ is periodic exactly when T/T_s is rational.

$$\boxed{x[n] \text{ is periodic iff } T/T_s \text{ is rational}}$$

Quick Tip: A sampled version of a periodic continuous-time signal repeats only when the sampling period fits an exact whole number of times into a whole number of source periods.

• • •

. The Laplace transform of the step response of a system is given by

$$Y(s) = \frac{100}{s(s+100)}$$

The rise time is defined as the time taken for the response to go from 0.1 to 0.9 of its final value. The settling time is defined as the time taken for the response to reach 0.98 of its final value.

For this system, the rise time (T_r), settling time (T_s), and time constant (T_c), all expressed in seconds, are

(A) $T_r = 0.022,\ T_s = 0.04,\ T_c = 0.01$

(B) $T_r = 0.22,\ T_s = 0.404,\ T_c = 0.01$

(C) $T_r = 2.2,\ T_s = 4.04,\ T_c = 1.01$

(D) $T_r = 22,\ T_s = 40.4,\ T_c = 10.1$

Correct Answer: (A) $T_r = 0.022,\ T_s = 0.04,\ T_c = 0.01$

SOLUTION:

Instead of deriving the time response from scratch, we can start from the standard benchmark formulas for a first order system and just verify them.

For a first order system with time constant τ, two widely used approximations are: rise time (10% to 90%) $T_r \approx 2.2\tau$, and settling time (within 2% of the final value) $T_s \approx 4\tau$ (the more precise value is 3.91τ).

Identify τ from the pole of the given transfer function. $Y(s) = \dfrac{100}{s(s+100)}$ has its finite pole at $s = -100$. For a first order form $\dfrac{a}{s+a}$, the time constant is $\tau = 1/a$, so here:

$$\tau = T_c = \frac{1}{100} = 0.01 \text{ s}$$

Plugging into the two formulas:

$$T_r = 2.2 \times 0.01 = 0.022 \text{ s}, \qquad T_s = 4 \times 0.01 = 0.04 \text{ s}$$

Now verify these numbers against the exact response $y(t) = 1 - e^{-100t}$. At $t = 0.022$: $y = 1 - e^{-2.2} = 1 - 0.111 = 0.889$, close to the 90% mark, confirming the rise time estimate. At $t = 0.04$: $y = 1 - e^{-4} = 1 - 0.0183 = 0.982$, matching the 98% settling criterion closely.

Both checks confirm the same triple of values.

$$\boxed{T_r = 0.022 \text{ s},\ T_s = 0.04 \text{ s},\ T_c = 0.01 \text{ s}}$$

Quick Tip: Find the time constant from the pole location, then use the standard 2.2 times Tc rise time and about 4 times Tc settling time formulas for a first-order step response.

• • •

. Consider the following differential equation:

$$t^2\frac{d^2y}{dt^2} + 7t\frac{dy}{dt} + 8ty = 10\sin(t)$$

Which one of the following options is correct?

(A) It is a linear differential equation

(B) It is a nonlinear differential equation

(C) It is a time-invariant differential equation

(D) It is a second-order partial differential equation

Correct Answer: (A) It is a linear differential equation

SOLUTION:

We can also test linearity and time invariance formally, using the operator that the equation defines, rather than just inspecting the terms.

Define the operator $L[y] = t^2y'' + 7ty' + 8ty$. To test linearity, check whether $L[c_1y_1 + c_2y_2] = c_1L[y_1] + c_2L[y_2]$ for any constants c_1, c_2 and any two functions $y_1(t), y_2(t)$.

Since differentiation itself is linear:

$$L[c_1y_1 + c_2y_2] = t^2(c_1y_1'' + c_2y_2'') + 7t(c_1y_1' + c_2y_2') + 8t(c_1y_1 + c_2y_2)$$

Regrouping the terms:

$$= c_1(t^2y_1'' + 7ty_1' + 8ty_1) + c_2(t^2y_2'' + 7ty_2' + 8ty_2) = c_1L[y_1] + c_2L[y_2]$$

The superposition property holds exactly, so L is a linear operator, confirming the equation $L[y] = 10\sin t$ is linear.

Now test time invariance by shifting time. If $y(t)$ solves the equation and we define $z(t) = y(t - t_0)$, a genuinely time-invariant system would have $z(t)$ solve the same equation with the input shifted to $10\sin(t - t_0)$. But $y(t - t_0)$ actually satisfies

$$(t - t_0)^2 y''(t - t_0) + 7(t - t_0)y'(t - t_0) + 8(t - t_0)y(t - t_0) = 10\sin(t - t_0)$$

which uses coefficients $(t - t_0)^2, 7(t - t_0), 8(t - t_0)$, not the original coefficients $t^2, 7t, 8t$, unless $t_0 = 0$. Since the shifted function does not satisfy the equation with the original, unshifted coefficients, the system is not time-invariant, only linear.

Combining both checks: the equation is an ordinary (not partial) equation, it is linear, and it is time-varying rather than time-invariant.

$$\boxed{\text{Linear differential equation}}$$

Quick Tip: Check if y and its derivatives appear only to the first power for linearity, separately from whether the coefficients depend on t for time invariance.

• • •

. A uniform ring charge of radius R carries a total charge Q. Which one of the following options correctly quantifies the magnitude of the force on a point charge of strength q kept at the center of the ring?
(ϵ is the permittivity of the medium)

(A) $\dfrac{Qq}{4\pi\epsilon R}$

(B) $\dfrac{Qq}{4\pi\epsilon R^2}$

(C) 0

(D) $\dfrac{q}{4\pi\epsilon R} \times \dfrac{Q}{2\pi R}$

Correct Answer: (C) 0

SOLUTION:

We can also reach the same result using the known formula for the field on the axis of a uniformly charged ring, instead of a symmetry argument.

For a ring of radius R and total charge Q, the electric field at an axial distance z from the center, along the axis perpendicular to the ring's plane, is:

$$E(z) = \frac{1}{4\pi\epsilon} \cdot \frac{Qz}{(R^2 + z^2)^{3/2}}$$

The center of the ring is the special axial point $z = 0$, which is exactly where the point charge q is placed, so we need $E(0)$.

Substituting $z = 0$:

$$E(0) = \frac{1}{4\pi\epsilon} \cdot \frac{Q \times 0}{(R^2 + 0)^{3/2}} = 0$$

The field is exactly zero at the center, consistent with the symmetry argument that every element's contribution is canceled by its diametrically opposite twin.

The force on the point charge is:

$$F = qE(0) = q \times 0 = 0$$

So the magnitude of the force on the point charge at the center is zero.

$$\boxed{0}$$

Quick Tip: By symmetry, every charge element on a uniform ring has a diametrically opposite twin whose field at the center exactly cancels it.

• • •

. A positive point charge with velocity $\vec{v} = 5\hat{x}$ enters a region having electric field $\vec{E} = 4\hat{y}$ and magnetic field $\vec{B} = -6\hat{z}$. Which one of the following statements is correct for the force on the charge as it enters the region?

(A) The force will be along the magnetic field but perpendicular to the electric field

(B) The force will be along the electric field but perpendicular to the magnetic field

(C) The force will be perpendicular to both electric and magnetic field

(D) The magnetic field does not exert any force on the charge

Correct Answer: (B) The force will be along the electric field but perpendicular to the magnetic field

SOLUTION:

Instead of a determinant, we can use the cyclic unit-vector cross product rules directly: $\hat{x} \times \hat{y} = \hat{z}$, $\hat{y} \times \hat{z} = \hat{x}$, $\hat{z} \times \hat{x} = \hat{y}$, and reversing the order of any pair flips the sign, so $\hat{x} \times \hat{z} = -\hat{y}$.

Apply this to $\vec{v} \times \vec{B}$:

$$\vec{v} \times \vec{B} = (5\hat{x}) \times (-6\hat{z}) = -30(\hat{x} \times \hat{z}) = -30(-\hat{y}) = 30\hat{y}$$

Add the electric field term to get the total force per unit charge:

$$\vec{E} + \vec{v} \times \vec{B} = 4\hat{y} + 30\hat{y} = 34\hat{y}$$

So the total force is $\vec{F} = 34q\hat{y}$.

Comparing directions: the result $34q\hat{y}$ has a component only along $\hat{y}$, exactly the direction of $\vec{E} = 4\hat{y}$, and $\hat{y}$ is orthogonal to $\hat{z}$, the direction of $\vec{B}$. So the force is along $\vec{E}$ and perpendicular to $\vec{B}$.

$$\boxed{\text{Force is along } \vec{E},\ \text{perpendicular to } \vec{B}}$$

Quick Tip: Apply F = q(E + v cross B), compute v cross B first, then compare the resulting direction with E and with B.

• • •

. A 15 kVA, 1100 V/220 V, single-phase two-winding transformer is configured as a 1.32 kV/1.1 kV autotransformer.

What will be the rating of the autotransformer?

(A) 60 kVA

(B) 75 kVA

(C) 90 kVA

(D) 100 kVA

Correct Answer: (C) 90 kVA

SOLUTION:

A different way to see this is to track the current in the series winding directly and multiply it by the full input voltage, instead of quoting the ratio formula first.

The 220 V winding of the original two-winding transformer is designed to carry a rated current of:

$$I_{series} = \frac{15{,}000 \text{ VA}}{220 \text{ V}} = 68.18 \text{ A}$$

When this same winding becomes the series winding of the autotransformer, it still carries this same rated current, since its own conductor and insulation have not changed. But now this current is drawn from the full 1.32 kV input line, not from a separate 220 V source.

So the autotransformer's throughput rating, seen from its high-voltage line terminal, is:

$$S_{auto} = V_1 \times I_{series} = 1320 \times 68.18 = 90{,}000 \text{ VA} = 90 \text{ kVA}$$

This is exactly six times the original 15 kVA rating, matching the ratio $V_1/(V_1 - V_2) = 1320/220 = 6$ used in the standard formula.

$$\boxed{90 \text{ kVA}}$$

Quick Tip: The 1100 V winding becomes the common winding and the 220 V winding becomes the series winding; multiply the two-winding rating by V1/(V1-V2), not V2/(V1-V2).

• • •

. Consider a power system with N buses, of which P are generator buses and the remaining Q are load buses (where there is no generation).

Assume that there are no reactive power-limit violations at the generator buses.
What is the size of the Jacobian matrix in the Newton-Raphson load flow method?

(A) $2N \times 2N$

(B) $(2N - 1 - P) \times (2N - 1 - P)$

(C) $(2N - Q) \times (2N - Q)$

(D) $(P + Q) \times (P + Q)$

Correct Answer: (B) $(2N - 1 - P) \times (2N - 1 - P)$

SOLUTION:

A quicker route is to count the mismatch equations directly, since the Jacobian's size always equals the number of these equations.

Real power mismatch equations are written for every bus except the slack bus, since the slack bus absorbs any power imbalance and never has a mismatch equation of its own. This gives $N - 1$ real power equations.

Reactive power mismatch equations are written only for the load (PQ) buses, since every generator bus (including all $P - 1$ non-slack PV buses) has its reactive power left free and its voltage magnitude fixed instead. With no reactive limit violations, none of these generator buses ever needs a reactive equation. This gives Q reactive power equations.

Adding the two counts, the total number of equations, and hence the Jacobian's dimension, is:

$$(N - 1) + Q$$

Writing Q in terms of N and P using $N = P + Q$:

$$Q = N - P$$

so the total becomes:

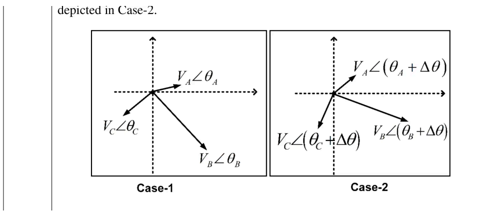
$$(N-1)+(N-P)=2N-1-P$$

This matches option (B) exactly.

$$\boxed{(2N-1-P)\times(2N-1-P)}$$

Quick Tip: Count angle unknowns (all buses except slack, giving N-1) and magnitude unknowns (only load buses, giving Q=N-P), then add and simplify using N=P+Q.

. The initial three-phase voltage phasors ($\bar{V}_A$, $\bar{V}_B$, and $\bar{V}_C$) at a bus of a power network are as shown in Case-1. Due to a disturbance, the bus voltage phasors changed in phase by a small angle $\Delta\theta$, and the magnitudes remained the same as depicted in Case-2.



Which one of the following statements is correct about the zero sequence components?

(A) The zero sequence components in Case-1 and Case-2 have the same phase angle and magnitude

(B) The magnitude of the zero sequence component in Case-1 is greater than that in Case-2

(C) The magnitude of the zero sequence component in Case-2 is greater than that in Case-1

(D) The zero sequence components in Case-1 and Case-2 have the same magnitude but different phase angles

Correct Answer: (D) The zero sequence components in Case-1 and Case-2 have the same magnitude but different phase angles

SOLUTION:

Instead of factoring the rotation out algebraically, picture what happens to the phasor sum geometrically.

Draw the three Case-1 phasors tip to tail (or just added as vectors) to get a resultant vector, which, divided by 3, is the zero sequence phasor $\bar{V}_0$ for Case-1.

Now, rotating every one of the three original phasors by the same angle $\Delta\theta$ is exactly the same as rotating the entire rigid picture, phasors and all, by $\Delta\theta$ about the origin. Since vector addition commutes with rotation (rotating each vector and then adding gives the same result as adding then rotating the sum), the resultant vector for Case-2 is just the Case-1 resultant rotated by $\Delta\theta$.

A rotation never changes a vector's length, only its direction. So the length of the Case-2 resultant (and hence the magnitude of $\bar{V}_0'$ after dividing by 3) is identical to the Case-1 resultant's length (and $\bar{V}_0$'s magnitude), while its direction (phase angle) has shifted by exactly $\Delta\theta$.

So the two zero sequence phasors have the same magnitude but point in different directions, differing by $\Delta\theta$ in phase.

$$\boxed{\text{Same magnitude, different phase angles}}$$

Quick Tip: Rotating all three phase voltages by the same angle rotates their zero-sequence average by that same angle too, leaving its magnitude unchanged.

. A certain application requires power at a frequency of 16.67 Hz, while the available grid frequency is 50 Hz. A 3-phase synchronous motor connected to the 50 Hz, 3-phase grid, driving a synchronous generator, is used for this application.

Which one of the following combinations is a suitable choice for the number of poles in the motor and generator, respectively?

(A) 2, 6

(B) 6, 2

(C) 4, 6

(D) 6, 4

Correct Answer: (B) 6, 2

SOLUTION:

Another way to approach this is to test each option's shaft speed directly and see which one produces exactly 16.67 Hz at the generator.

The motor's synchronous speed on the 50 Hz grid, for pole count P_m, is $N_s = \dfrac{120 \times 50}{P_m} = \dfrac{6000}{P_m}$.

Testing option (A), $P_m = 2$: $N_s = 6000/2 = 3000$ rpm. Generator frequency with $P_g = 6$: $f_g = \dfrac{N_s P_g}{120} = \dfrac{3000 \times 6}{120} = 150$ Hz. Far too high.

Testing option (B), $P_m = 6$: $N_s = 6000/6 = 1000$ rpm. Generator frequency with $P_g = 2$: $f_g = \dfrac{1000 \times 2}{120} = 16.67$ Hz. This matches the required output frequency exactly.

Testing option (C), $P_m = 4$: $N_s = 6000/4 = 1500$ rpm. Generator frequency with $P_g = 6$: $f_g = \dfrac{1500 \times 6}{120} = 75$ Hz. Too high.

Testing option (D), $P_m = 6$: $N_s = 1000$ rpm. Generator frequency with $P_g = 4$: $f_g = \dfrac{1000 \times 4}{120} = 33.3$ Hz. Still too high.

Only option (B) reproduces the required 16.67 Hz output.

$$\boxed{6 \text{ poles (motor)},\ 2 \text{ poles (generator)}}$$

Quick Tip: Both machines share the same shaft speed; set the two synchronous-speed formulas equal and solve for the pole ratio, then check which option gives that ratio.

• • •

. Which one of the following options is correct regarding the typical double-squirrel-cage structure used in induction motors?

(A) At starting, a larger portion of the rotor current flows in the outer cage

(B) At starting, a larger portion of the rotor current flows in the inner cage

(C) At rated speed, most of the rotor current flows in the outer cage

(D) The purpose of the double-squirrel-cage structure is to lower the effective rotor resistance at starting

Correct Answer: (A) At starting, a larger portion of the rotor current flows in the outer cage

SOLUTION:

Another way to see this is to track how the two cages are meant to specialize at different operating points, by design.

The outer cage (high resistance, low reactance) is built to dominate at STARTING: since reactance is largest at starting and the outer cage's reactance is small to begin with, it ends up with a lower total impedance than the inner cage at that instant, so it carries the larger share of rotor current when the motor is switched on.

The inner cage (low resistance, high reactance) is built to dominate at RUNNING speed: once the motor is up to speed, slip is small, rotor frequency is small, and the high reactance of the inner cage becomes negligible, leaving its low resistance to dominate and carry most of the current efficiently.

This deliberate role-swap is the entire point of the double cage design: a high effective starting resistance (for good starting torque, via the outer cage) combined with a low running resistance (for good efficiency, via the inner cage), something a single cage cannot achieve at both extremes at once.

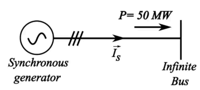
So at the moment of starting, it is the outer cage that carries the larger portion of the rotor current.

$$\boxed{\text{At starting, more rotor current flows in the outer cage}}$$

Quick Tip: At starting, high rotor frequency makes reactance dominate; the outer cage's naturally low reactance gives it lower impedance and hence more current than the inner cage.

• • •

. The figure shows the single-line diagram of a synchronous generator delivering $P = 50$ MW of power at unity power factor to an infinite bus.



I_S denotes the stator current phasor. If the field excitation is increased, which one of the following options correctly describes its effect on the stator current, power factor, and load angle of the machine?

(A) Stator current increases, power factor becomes lagging, load angle remains the same

(B) Stator current decreases, power factor becomes leading, load angle remains the same

(C) Stator current increases, power factor becomes lagging, load angle decreases

(D) Stator current increases, power factor becomes leading, load angle increases

Correct Answer: (C) Stator current increases, power factor becomes lagging, load angle decreases

SOLUTION:

Another way to track this is through the phasor diagram of the generator, using the constant-power constraint directly.

With the bus voltage V fixed as the reference phasor, the real power delivered is proportional to the component of E_f that is in phase with V, scaled by $\sin\delta$. Holding P (and hence V, X_s) fixed while raising E_f forces $\sin\delta$, and hence δ itself, to shrink, since a larger E_f needs a smaller angle to keep the product $E_f \sin\delta$ unchanged.

Geometrically, the stator current phasor I_S is found from $E_f = V + jI_S X_s$, so $I_S = \dfrac{E_f - V}{jX_s}$. As E_f grows in magnitude while its projection onto the P-axis stays fixed (constant P), the phasor difference $E_f - V$ grows, so $|I_S|$ grows too.

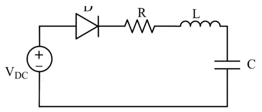
At the starting point, unity power factor means I_S is exactly in phase with V. As E_f increases beyond this point, the operating point moves into the region where I_S lags V, which for a generator is defined as the LAGGING power factor region, corresponding to overexcitation and the generator supplying reactive power outward to the system.

So all three changes happen together: $|I_S|$ increases, the power factor shifts from unity to lagging, and δ shrinks.

$$\boxed{\text{Stator current increases, power factor becomes lagging, load angle decreases}}$$

Quick Tip: Use P = (Ef V / Xs) sin(delta) with P held fixed to see that delta must fall as Ef rises, and recall that overexciting a generator moves it to the lagging side of its V-curve, away from the unity-PF minimum-current point.

. A circuit with ideal elements is shown.



The circuit has a DC source V_{DC} in series with an ideal diode D, a resistor R, an inductor L, and a capacitor C (the capacitor is connected across the series combination of L at the far end of the loop).
Which one of the following options correctly identifies all the linear elements in the circuit?

(A) R only

(B) R, L, and C only

(C) D only

(D) L, C, and D only

Correct Answer: (B) R, L, and C only

SOLUTION:

A quick way to settle this is to recall that R, L, and C are always grouped together as the three canonical passive linear elements taught at the start of any circuits course, and to treat every semiconductor device as suspect until checked.

The defining equations of the resistor, inductor, and capacitor,

$$v = iR, \qquad v = L\frac{di}{dt}, \qquad i = C\frac{dv}{dt}$$

are each a constant multiplied by a variable or its derivative, the signature of linearity: doubling the current doubles the voltage (or vice versa) in every case, with no thresholds and no piecewise breaks.

The diode, on the other hand, conducts current in only one direction and blocks it in the other, an inherently switching, non-proportional behavior. Even the ideal (zero forward-drop) diode model used here is nonlinear, since it still has two completely different regimes (short circuit versus open circuit) depending on the sign of the current or voltage, which no single linear equation can capture.

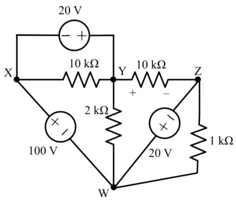
So among the four elements shown, only R, L, and C are linear.

$$\boxed{\text{R, L, and C only}}$$

Quick Tip: Resistors, inductors, and capacitors all have constant-coefficient proportional v-i relationships; an ideal diode's on/off switching behavior is not proportional and is nonlinear.

• • •

. For the circuit shown, which one of the following options correctly identifies the Thevenin's equivalent parameters between nodes Y and Z?



The circuit has nodes X, Y, Z, and W. Between X and Y there is a 20 V ideal source (in parallel with a 10 kΩ resistor directly connecting X and Y). Between Y and Z there is a 10 kΩ resistor. Between Y and W there is a 2 k Ω resistor. Between X and W there is an ideal 100 V source (no series resistor in this branch). Between Z and W there is an ideal 20 V source (no series resistor in this branch) and, separately, a 1 kΩ resistor.

(A) V_{TH} = 100 V, R_{TH} = 10 kΩ

(B) V_{TH} = 140 V, R_{TH} = 0 Ω

(C) V_{TH} = 100 V, R_{TH} = 0 Ω

(D) V_{TH} = 140 V, R_{TH} = 10 kΩ

Correct Answer: (B) V_{TH} = 140 V, R_{TH} = 0 Ω

SOLUTION:

A useful way to check this is to notice, before doing any node-voltage algebra, that every path from Y or Z back to the reference node W passes only through ideal voltage sources, never through a resistor alone.

From Y, the only way back to W without crossing a resistor first is Y to X (ideal 20 V source) then X to W (ideal 100 V source). Both are ideal sources, so this path fixes V_Y completely, with zero equivalent resistance along it.

From Z, the direct path to W is the ideal 20 V source between Z and W, again with zero resistance.

Since both terminal nodes of interest, Y and Z, are each rigidly fixed by an unbroken chain of ideal sources back to ground, their difference $V_Y - V_Z$ can never be perturbed by any external load connected across Y-Z, no matter how much current that load draws. This is exactly the condition for a zero Thevenin resistance: $R_{TH} = 0\,\Omega$.

Now compute the fixed node voltages: $V_X = 100$ V (from the X-W source), so $V_Y = V_X + 20 = 120$ V (from the X-Y source). And $V_Z = -20$ V (from the Z-W source, oriented so that W is 20 V above Z).

The open-circuit voltage is:

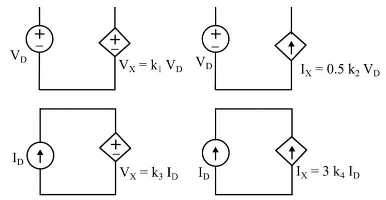
$$V_{TH} = V_Y - V_Z = 120 - (-20) = 140 \text{ V}$$

Combined with the zero-resistance argument above:

$$\boxed{V_{TH} = 140 \text{ V},\ R_{TH} = 0\,\Omega}$$

Quick Tip: Both Y and Z are pinned directly by chains of ideal voltage sources back to the reference node, so their potentials do not depend on any external load; find each node voltage first, then subtract.

. Refer to the four circuits shown.



Which one of the following options for k_1, k_2, k_3, and k_4 makes all of them realizable?

(A) $k_1 = 1, k_4 = -\frac{1}{3}$, for all values of k_2 and k_3

(B) $k_2 = -2, k_3 = +\frac{1}{3}$, for all values of k_1 and k_4

(C) $k_1 = 2, k_2 = 0.5, k_3 = -\frac{2}{3}, k_4 = -3$

(D) $k_1 = 2, k_2 = -0.5, k_3 = -\frac{2}{3}, k_4 = +3$

Correct Answer: (A) $k_1 = 1, k_4 = -\frac{1}{3}$, for all values of k_2 and k_3

SOLUTION:

Step 1: Redraw what "parallel" really means here.

Since the top and bottom of each rectangle are plain connecting wires, the left branch and right branch of every circuit share the same pair of nodes. That is exactly the definition of a parallel connection, even though the picture looks like a single loop.

Step 2: State the rule for each source combination.

Put two voltage sources across the same pair of nodes and their values must agree, or KVL fails. Put two current sources feeding the same node in the same direction and their sum must be zero, or KCL fails. Mix a voltage source with a current source and neither law is threatened, because the voltage source simply supplies whatever current is needed.

Step 3: Circuit 1.

V_D and $V_X = k_1 V_D$ are both voltage sources with the same polarity across the same nodes, so $V_D = k_1 V_D$, giving $k_1 = 1$.

Step 4: Circuits 2 and 3.

Each of these pairs one voltage source with one current source. No equation is forced on k_2 or k_3, since the voltage source adjusts its own current freely. Both circuits work for any value of k_2 and k_3.

Step 5: Circuit 4.

I_D and $I_X = 3k_4 I_D$ both push current into the same node in the same direction, so KCL needs $I_D + I_X = 0$, that is $I_D + 3k_4 I_D = 0$, giving $k_4 = -\frac{1}{3}$.

Step 6: Put it together.

The only combination that keeps every circuit consistent is $k_1 = 1$ and $k_4 = -\frac{1}{3}$, with k_2 and k_3 completely free.

$$\boxed{k_1 = 1,\ k_4 = -\tfrac{1}{3},\ \text{for all } k_2, k_3}$$

Quick Tip: Look at which pairs of sources sit between the same two nodes: same-type sources, V-V or I-I, in parallel must match, while mixed V-I pairs never conflict.

• • •

. A single-phase voltage source $v_s = 325\sin(2\pi 50t)$ V delivers a current, $i = 12\sin(2\pi 50t) + 9\sin(2\pi 150t)$ A to a load.

The load power factor, correct up to two decimal places, is:

(A) 1.00

(B) 0.80

(C) 0.65

(D) 0.57

Correct Answer: (B) 0.80

SOLUTION:

Step 1: Split the current into a useful part and a distortion part.

The current has two pieces: $12\sin(2\pi 50t)$ at the same frequency as the source, and $9\sin(2\pi 150t)$ at three times that frequency. Only same-frequency waves can exchange real power with the source, so the 150 Hz piece is pure distortion as far as power transfer goes.

Step 2: Get the real power from the matching frequency term alone.

Peak voltage is 325 V and the peak of the matching current term is 12 A, both in phase, so

$$P = \frac{325 \times 12}{2} = 1950 \text{ W}$$

Step 3: Find the rms of the full current.

Each sine term contributes its own rms in a Pythagorean sum:

$$I_{rms} = \sqrt{\left(\frac{12}{\sqrt{2}}\right)^2 + \left(\frac{9}{\sqrt{2}}\right)^2} = \sqrt{112.5} \approx 10.607 \text{ A}$$

Step 4: Find the rms voltage and the apparent power.

$$V_{rms} = \frac{325}{\sqrt{2}} \approx 229.81 \text{ V}, \qquad S = V_{rms} I_{rms} = 2437.5 \text{ VA}$$

Step 5: Divide to get the true power factor.

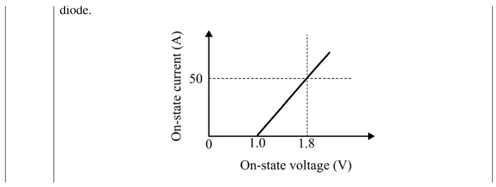
The harmonic current inflates I_{rms} without adding any real power, so it drags the power factor below what the 50 Hz terms alone would give.

$$PF = \frac{1950}{2437.5} = 0.80$$

$$\boxed{0.80}$$

Quick Tip: Only the current component at the same frequency as the source contributes to real power, but the harmonic current still adds to the rms current and lowers the power factor.

. The figure shows a straight-line approximation for the forward characteristics of a power diode. A continuous on-state current of 15 A is flowing through the diode.



What is the power loss in the diode?

(A) 32.8 W

(B) 21.2 W

(C) 18.6 W

(D) 23.1 W

Correct Answer: (C) 18.6 W

SOLUTION:

Step 1: Turn the graph into an equation.

The line starts conducting at $V_0 = 1.0$ V and reaches 50 A at 1.8 V. The rise over run of the line gives the conductance:

$$\frac{1}{R_{on}} = \frac{50}{1.8 - 1.0} = 62.5\ \text{A/V} \implies R_{on} = 0.016\ \Omega$$

Step 2: Model the diode as a battery plus a resistor.

A piecewise linear diode behaves like a fixed voltage source V_0 in series with R_{on}, so at any forward current I:

$$V_D = V_0 + IR_{on}$$

Step 3: Plug in the actual operating current.

$$V_D = 1.0 + 15(0.016) = 1.24\ \text{V}$$

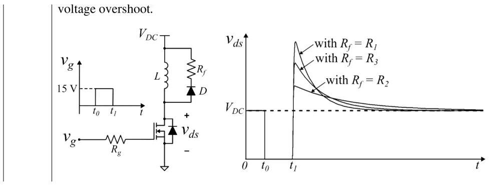
Step 4: Multiply to get dissipated power.

$$P = V_D I = 1.24 \times 15 = 18.6\ \text{W}$$

$$\boxed{18.6\ \text{W}}$$

> **Quick Tip:** Fit the straight line as Vo plus I times Ron using the two marked points on the graph, then multiply the voltage drop at 15 A by the current.

. Consider the circuit shown in Figure (a). A gate pulse v_g is applied between time instants t_0 and t_1. After t_1, during the MOSFET turn OFF process, it experiences a voltage overshoot.



Based on the v_{ds} waveforms shown in Figure (b), which one of the following options is correct?

(A) $R_1 > R_2 > R_3$

(B) $R_1 > R_3 > R_2$

(C) $R_3 > R_2 > R_1$

(D) $R_2 > R_3 > R_1$

Correct Answer: (B) $R_1 > R_3 > R_2$

SOLUTION:

Step 1: Track where the trapped inductor current goes.
Just after t_1, the MOSFET opens and stops conducting, but L still wants to push its stored current somewhere. That current is caught by the diode D and routed through R_f.

Step 2: Write the spike voltage.

The extra voltage riding on top of V_{DC} right after turn off is approximately $I_L R_f$, so a bigger R_f directly means a bigger spike.

Step 3: Write the decay behavior.

The current in that freewheeling loop dies off with time constant L/R_f. A bigger R_f shortens this time constant, so the curve returns to V_{DC} quicker; a smaller R_f lingers longer above V_{DC}.

Step 4: Match this to the picture.

The tallest, fastest-decaying curve is labeled $R_f = R_1$, the middle one is $R_f = R_3$, and the shortest, slowest one is $R_f = R_2$.

Step 5: Rank the resistors.

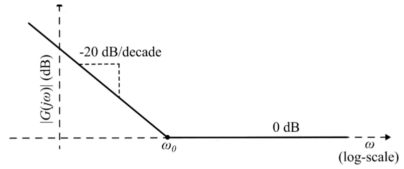
Bigger peak means bigger resistor, so the order from largest to smallest resistance is

$$R_1 > R_3 > R_2$$

$$\boxed{R_1 > R_3 > R_2}$$

Quick Tip: A bigger snubber resistance produces a bigger instantaneous IR spike but decays faster, so rank the resistors directly by the height of each peak in the figure.

. The asymptotic Bode magnitude plot of a system is shown.



Which one of the following options best represents the transfer function of the system?

(A) $G(s) = \dfrac{1 + \dfrac{s}{\omega_0}}{\dfrac{s}{\omega_0}}$

(B) $G(s) = \dfrac{\dfrac{s}{\omega_0}}{1 + \dfrac{s}{\omega_0}}$

(C) $G(s) = \dfrac{1 + \dfrac{s}{\omega_0}}{1 - \dfrac{s}{\omega_0}}$

(D) $G(s) = \dfrac{1 - \dfrac{s}{\omega_0}}{1 + \dfrac{s}{\omega_0}}$

Correct Answer: (A) $G(s) = \dfrac{1 + \dfrac{s}{\omega_0}}{\dfrac{s}{\omega_0}}$

SOLUTION:

Step 1: Split the plot into two frequency zones.

Below ω_0 the line falls with slope -20 dB/decade. Above ω_0 it sits flat at 0 dB.

Step 2: Match the low-frequency zone to a building block.

A term like $\dfrac{1}{s/\omega_0}$ has magnitude ω_0/ω, which falls at -20 dB/decade, exactly the shape seen for $\omega < \omega_0$. This tells us the denominator must contribute a bare s/ω_0 term.

Step 3: Match the high-frequency zone to a building block.
For the plot to flatten out at 0 dB above ω_0, the transfer function must approach 1 as $\omega \to \infty$. A numerator of $1 + s/\omega_0$ paired with the same s/ω_0 in the denominator does exactly this, since both terms are dominated by s/ω_0 at high frequency and cancel to 1.

Step 4: Assemble the transfer function.

$$G(s) = \frac{1 + \dfrac{s}{\omega_0}}{\dfrac{s}{\omega_0}}$$

Step 5: Check the corner frequency.
At $s = j\omega_0$, the numerator gives $|1 + j1| = \sqrt{2}$ and the denominator gives $|j1| = 1$, so the curve bends near ω_0 exactly where the plot shows the kink, confirming this is the right match, while the sign-flipped options (C) and (D) describe unstable, non-minimum-phase systems that are not what a simple asymptotic sketch is used to represent.

$$\boxed{G(s) = \frac{1 + \dfrac{s}{\omega_0}}{\dfrac{s}{\omega_0}}}$$

Quick Tip: A -20 dB/decade fall at low frequency that flattens to 0 dB beyond omega0 is the signature of a pole at the origin combined with a zero at omega0.

• • •

. Two $n \times n$ matrices A and B have a common eigenvalue 2, and the same corresponding nonzero eigenvector.
Which of the following options is/are correct?
(Note: I is the $n \times n$ identity matrix.)

(A) Determinant $(A - 2I) = 0$

(B) Determinant $(B - 2I) = 0$

(C) Determinant $(A + B - 2I) = 0$

(D) Determinant $(A + B - 4I) = 0$

Correct Answer: (A) Determinant $(A - 2I) = 0$; (B) Determinant $(B - 2I) = 0$; (D) Determinant $(A + B - 4I) = 0$

SOLUTION:

Step 1: Set up the shared condition.
Let $v \neq 0$ be the common eigenvector, so $Av = 2v$ and $Bv = 2v$.

Step 2: Test each option by applying it to v.
For option (A): $(A - 2I)v = 2v - 2v = 0$, so v is a nonzero vector in the null space of $A - 2I$, which makes $A - 2I$ singular, and the determinant is 0.
For option (B): the same argument with B gives $(B - 2I)v = 0$, so $B - 2I$ is singular too.

Step 3: Test option (D) by adding the two eigen-equations.

Adding $Av = 2v$ and $Bv = 2v$ gives $(A + B)v = 4v$, so $(A + B - 4I)v = 0$. Again $v \neq 0$ lies in the null space, forcing $\det(A + B - 4I) = 0$.

Step 4: Test option (C) the same way.
$(A + B - 2I)v = (A + B)v - 2v = 4v - 2v = 2v$. Since $v \neq 0$, this is not the zero vector, so there is no reason from the given data to conclude $\det(A + B - 2I) = 0$.

Step 5: Collect the valid options.
Only (A), (B), and (D) are guaranteed true from the shared eigenvalue and eigenvector.

$$\boxed{\text{(A), (B), (D)}}$$

Quick Tip: Use Av=2v and Bv=2v directly: check which of the four combinations sends the shared eigenvector v to the zero vector.

• • •

. A 220V/12V single-phase transformer is designed for use in India and rated 100 VA at 50 Hz. Later, this unit is shipped to the USA where it is used as a 110V/6V transformer at 60 Hz.
Which of the following statements is/are correct?

(A) No-load current drawn would be smaller for operation in the USA compared to that in India

(B) For the same load current, the losses would be higher in the USA compared to that in India

(C) The peak magnetic flux density in the core would be higher while operating in the USA compared to that in India

(D) The eddy current losses in the core would be approximately 44% higher while operating in the USA compared to that in India

Correct Answer: (A) No-load current drawn would be smaller for operation in the USA compared to that in India

SOLUTION:

Step 1: Track the V/f ratio.
The peak flux density in a transformer core follows $B_{max} \propto V/f$, since N and the core area stay fixed for the same physical unit. India: V/f = 220/50 = 4.4. USA: V/f = 110/60 ≈ 1.833.

Step 2: Get the flux ratio.

$$\frac{B_{USA}}{B_{India}} = \frac{1.833}{4.4} \approx 0.417$$

So the core runs at less than half its Indian flux density when used in the USA. This immediately rules out option (C), which claims a higher flux density.

Step 3: Link flux to the no-load current.
A lower required flux on an unsaturated core needs a correspondingly smaller magnetizing current to produce it, so the no-load current is smaller in the USA. Option (A) holds.

Step 4: Check core loss trends.

Both hysteresis loss and eddy current loss depend heavily on B_{max}, which has dropped a lot, more than making up for the modest rise in frequency. So total core loss falls in the USA, and copper loss for the same load current is unchanged, so total losses do not rise. Option (B) fails.

Step 5: Compute the eddy current figure directly.

$$\frac{P_{e,USA}}{P_{e,India}} = \left(\frac{60}{50}\right)^2 (0.417)^2 \approx 1.44 \times 0.174 \approx 0.25$$

The eddy current loss drops to about a quarter of its Indian value, the opposite of the 44% rise claimed in option (D).

Step 6: Conclude.

The only statement that survives is the smaller no-load current in the USA.

$$\boxed{\text{No-load current would be smaller in the USA}}$$

Quick Tip: Since Bmax is proportional to V/f and the core stays the same, work out how Bmax, the magnetizing current, and the core losses all scale between the 220V/50Hz and 110V/60Hz operating points.

. Given that $\vec{F}(x, y, z) = \sin(y)\,\hat{x} + \cos(x)\,\hat{y} + 5\,\hat{z}$, the integral $\oiint_S \vec{F}(x, y, z) \cdot d\vec{s}$ over the unit sphere S centered at the origin evaluates to
(Round off to one decimal place)

Correct Answer: 0.0

SOLUTION:

Step 1: Recognize this as a closed-surface flux problem.
The sphere S is a closed surface, so instead of computing the surface integral piece by piece, use Gauss's divergence theorem to turn it into a volume integral:

$$\oiint_S \vec{F} \cdot d\vec{s} = \iiint_V (\nabla \cdot \vec{F})\, dV$$

Step 2: Take the divergence term by term.
$\vec{F} = \sin(y)\hat{x} + \cos(x)\hat{y} + 5\hat{z}$. The x-component depends only on y, so differentiating it with respect to x gives 0. The y-component depends only on x, so differentiating it with respect to y gives 0. The z-component is a plain constant, so differentiating it with respect to z gives 0.

Step 3: Sum the three zero terms.

$$\nabla \cdot \vec{F} = 0 + 0 + 0 = 0$$

Step 4: Integrate zero over the unit ball.
A volume integral of a function that is zero everywhere is zero, regardless of the shape or size of the region:

$$\iiint_V 0\,dV = 0$$

Step 5: State the result.

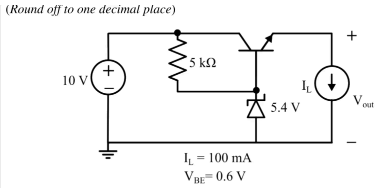
The field is divergence-free everywhere, so the net outward flux through any closed surface, including this unit sphere, comes out to zero.

$$\boxed{0.0}$$

Quick Tip: Apply the divergence theorem and check each partial derivative of F directly; the cross-mixed dependence of sin(y) and cos(x) makes the divergence vanish.

• • •

. In the linear regulator circuit shown, the base to emitter voltage V_{BE} of the BJT is 0.6 V. The Zener diode clamps the base voltage to 5.4 V. Ignore the biasing current of the Zener diode and the BJT.



The input supply is 10 V and the load current is $I_L = 100$ mA. The maximum possible efficiency of the regulator circuit is % (round off to one decimal place).

Correct Answer: 48.0

SOLUTION:

Step 1: Picture the regulator.

A Zener diode fixes the base of a BJT at 5.4 V. The emitter follows the base minus the V_{BE} drop, so the output sits at a fixed voltage below the source.

Step 2: Get the output voltage a different way.

Since the emitter voltage equals $V_Z - V_{BE}$, we get $V_{out} = 5.4 - 0.6 = 4.8$ V. This is also the voltage dropped across the transistor measured from the top rail: the collector sits at 10 V and the emitter at 4.8 V, so the transistor itself drops $10 - 4.8 = 5.2$ V while carrying the load current.

Step 3: Use the voltage ratio shortcut for efficiency.

Because ignoring the bias currents means the same current I_L flows both into the regulator and out to the load, the power ratio collapses to a voltage ratio:

$$\eta = \frac{V_{out} I_L}{V_{in} I_L} \times 100 = \frac{V_{out}}{V_{in}} \times 100$$

Step 4: Plug in the numbers.

$$\eta = \frac{4.8}{10} \times 100 = 48.0\%$$

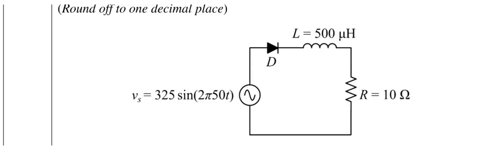
This also matches checking it as one minus the fraction dropped across the transistor: $\frac{5.2}{10} = 0.52$, so 52 percent is lost across the transistor and 48 percent reaches the load.

$$\boxed{48.0}$$

Quick Tip: Find Vout as VZ minus VBE, then compare the output power to the input power, since both use the same load current IL.

• • •

. Consider the circuit shown. Assume that the diode D is ideal. The supply voltage $v_s = 325\sin(2\pi 50t)$ V, $L = 500\ \mu H$, and $R = 10\ \Omega$.



The peak diode current (in amperes) is (round off to one decimal place).

Correct Answer: 32.5

SOLUTION:

Step 1: Compare the inductive and resistive parts.
The inductive reactance is $\omega L = 2\pi(50)(500 \times 10^{-6}) = 0.157\ \Omega$, tiny next to $R = 10\ \Omega$. This tells us the circuit behaves almost like a pure resistor, with the inductor mattering only for a very short instant right after the diode switches on.

Step 2: Treat it as resistive first.
If the load were purely resistive, the current would simply be $i(t) = \frac{V_m}{R}\sin(\omega t)$, peaking at $\frac{325}{10} = 32.5$ A exactly at the crest of the sine wave.

Step 3: Add the small correction from L.
With the inductor included, the true peak is $\frac{V_m}{Z}$ where Z

$= \sqrt{R^2 + (\omega L)^2} = \sqrt{100 + 0.0247} = 10.0012\ \Omega$, only a whisker above $10\ \Omega$. So the corrected peak is

$$i_{peak} = \frac{325}{10.0012} = 32.496\text{ A}$$

Step 4: Explain why the L barely changes anything.

The time constant $L/R = 50\ \mu s$ is minute compared to the 20 ms period of the supply, so any starting transient dies away almost the instant the diode conducts, well before the current reaches its crest near the middle of the half cycle.

Step 5: Round off.

$$i_{peak} \approx 32.5\text{ A}$$

$$\boxed{32.5}$$

Quick Tip: Since wL is tiny next to R here, the RL transient dies out almost instantly, so the peak current is close to Vm/Z, occurring near the middle of the positive half cycle.

• • •

. A is an $m \times m$ skew-symmetric matrix with real-valued entries, and x is an m-dimensional column vector with real-valued entries such that $x^T x = 1$. The quantity $x^T Ax$ evaluates to (answer in integer).

Correct Answer: 0

SOLUTION:

Step 1: Write the quadratic form as a double sum.

For any matrix A with entries a_{ij}, we can expand $x^T Ax$ $= \sum_{i=1}^{m} \sum_{j=1}^{m} a_{ij}x_i x_j$.

Step 2: Use the skew-symmetric property on the entries.

Since A is skew-symmetric, $a_{ji} = -a_{ij}$, and every diagonal entry must satisfy $a_{ii} = -a_{ii}$, which forces $a_{ii} = 0$ for every i.

Step 3: Pair up the off-diagonal terms.

Group the sum into diagonal terms (all zero) and pairs (i, j) with $i \neq j$. Each pair contributes

$$a_{ij}x_i x_j + a_{ji}x_j x_i = a_{ij}x_i x_j - a_{ij}x_i x_j = 0$$

because $x_i x_j = x_j x_i$ but $a_{ji} = -a_{ij}$.

Step 4: Add everything up.

Every diagonal term is zero and every off-diagonal pair cancels, so the whole double sum is zero regardless of what x is.

Step 5: Note on the given condition.

The unit-length condition $x^T x = 1$ is extra information here; it plays no role since the cancellation happens term by term.

$$\boxed{0}$$

Quick Tip: For any skew-symmetric A, the quadratic form x transpose A x always equals its own negative, so it must be zero.

. A time-limited waveform $g(x)$ is specified as follows:

$$g(x) = \begin{cases} -k, & -\pi < x \le 0 \\ +k, & 0 < x \le \pi \\ 0, & \text{otherwise} \end{cases}$$

A new waveform $f(x)$ is constructed from $g(x)$ as follows:

$$f(x) = \sum_{m=-\infty}^{\infty} g(x + 2\pi m), \qquad \text{for all } x \in \mathbb{R}$$

The sum of the coefficients of the third harmonics of the sine and cosine terms in the trigonometric Fourier series expansion of $f(x)$ is $\frac{2}{3\pi}$. What is the value of k?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

Correct Answer: (B) $\frac{1}{2}$

SOLUTION:

Step 1: Set up the coefficient formulas.

For a period 2π function, the Fourier cosine and sine coefficients are

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx, \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

Step 2: Compute a_3 directly.

Split the integral over the two pieces of $f(x)$:

$$a_3 = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-k)\cos(3x)\,dx + \int_{0}^{\pi}k\cos(3x)\,dx\right]$$

Because $\cos(3x)$ is an even function, the two integrals, one negated and one not, taken over mirror-image ranges, are exact negatives of each other, so they cancel and $a_3 = 0$. This confirms there is no cosine content, as expected for an odd function.

Step 3: Compute b_3 directly.

$$b_3 = \frac{1}{\pi}\left[\int_{-\pi}^{0}(-k)\sin(3x)\,dx + \int_{0}^{\pi}k\sin(3x)\,dx\right]$$

Using $\int \sin(3x)\,dx = -\frac{\cos(3x)}{3}$, the second integral gives

$$\int_{0}^{\pi}k\sin(3x)\,dx = k\left[-\frac{\cos(3x)}{3}\right]_{0}^{\pi} = k\left(-\frac{\cos 3\pi}{3} + \frac{1}{3}\right) = k\left(\frac{1}{3} + \frac{1}{3}\right) = \frac{2k}{3}$$

since $\cos(3\pi) = -1$. By the odd symmetry of $\sin(3x)$, the first integral over $(-\pi, 0)$ contributes the same amount again, giving a total inside the bracket of $\frac{4k}{3}$.

Step 4: Finish computing b_3.

$$b_3 = \frac{1}{\pi}\cdot\frac{4k}{3} = \frac{4k}{3\pi}$$

Step 5: Match to the given sum and solve.

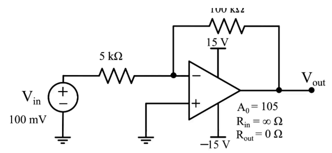
The sum $a_3 + b_3 = \frac{4k}{3\pi}$ is given as $\frac{2}{3\pi}$, so $4k = 2$, giving $k = \frac{1}{2}$.

$$\boxed{k = \frac{1}{2}}$$

Quick Tip: f(x) is an odd square wave of period 2pi, so only sine terms exist; use the standard series 4k/(n*pi) for the nth odd harmonic.

• • •

. In the circuit shown, the open loop gain of the operational amplifier is $A_0 = 105$, with $R_{in} = \infty\ \Omega$ and $R_{out} = 0\ \Omega$.



The circuit is a standard inverting amplifier: $V_{in} = 100$ mV drives the inverting input through a 5 $k\Omega$ input resistor, and a 100 $k\Omega$ resistor feeds back from V_{out} to the inverting input, with the non-inverting input grounded. What is the voltage gain of the circuit? (Round off to two decimal places)

(A) -16.67

(B) -20.00

(C) -21.00

(D) -12.67

Correct Answer: (A) -16.67

SOLUTION:

Step 1: Do not assume a perfect virtual short.

With a finite gain A_0, the inverting input is not exactly at ground. Call the inverting input voltage v_- and the output v_o. Since the op-amp obeys $v_o = -A_0 v_-$ (non-inverting input grounded), we get

$$v_- = -\frac{v_o}{A_0}$$

Step 2: Write the current balance at the inverting node.

No current enters the op-amp input, so the current through R_{in} from V_{in} must equal the current through R_f into the output:

$$\frac{V_{in} - v_-}{R_{in}} = \frac{v_- - v_o}{R_f}$$

Step 3: Substitute $v_- = -v_o/A_0$.

$$\frac{V_{in} + v_o/A_0}{R_{in}} = \frac{-v_o/A_0 - v_o}{R_f}$$

Multiply both sides by $R_f R_{in}$:

$$R_f\left(V_{in} + \frac{v_o}{A_0}\right) = R_{in}\left(-\frac{v_o}{A_0} - v_o\right)$$

Step 4: Collect the v_o terms.

$$R_f V_{in} = -v_o\left[R_{in}\left(1 + \frac{1}{A_0}\right) + \frac{R_f}{A_0}\right]$$

$$\frac{v_o}{V_{in}} = \frac{-R_f}{R_{in}\left(1 + \frac{1}{A_0}\right) + \frac{R_f}{A_0}}$$

Step 5: Plug in numbers.

With $R_{in} = 5\text{k}$, $R_f = 100\text{k}$, $A_0 = 105$:

$$R_{in}\left(1 + \frac{1}{105}\right) = 5 \times 1.00952 = 5.0476$$

$$\frac{R_f}{A_0} = \frac{100}{105} = 0.9524$$

$$\frac{v_o}{V_{in}} = \frac{-100}{5.0476 + 0.9524} = \frac{-100}{6.0} = -16.67$$

Step 6: Conclude.

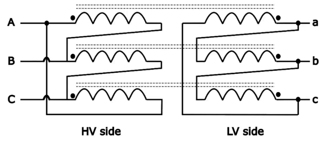
This matches the standard finite-gain formula and confirms the gain is not the ideal -20 but a slightly smaller magnitude, -16.67.

$$\boxed{-16.67}$$

Quick Tip: Use the finite-gain inverting amplifier formula: gain = -(Rf/Rin) divided by [1 + (1+Rf/Rin)/A0].

• • •

. Three single-phase 11kV/3.3kV transformers are connected to form a three-phase transformer bank, with the HV and LV windings connected as shown.



Considering **ABC** phase sequence, the vector group of the transformer is:

(A) Dd0

(B) Dd4

(C) Dd6

(D) Dd10

Correct Answer: (D) Dd10

SOLUTION:

Step 1: Draw the HV phasor star.

Place the three HV winding EMFs 120° apart: E_A at $0°$, E_B at $-120°$, E_C at $120°$. Joining tip to tail in the order A, B, C traces out the HV delta, and the line voltage V_{AB} points along E_A, so it sits at $0°$, marked as the "12 o'clock" position on a clock face.

Step 2: Draw the LV phasor star at the same angles.

Because each dotted LV winding is in phase with its own HV partner, e_a, e_b, e_c sit at the same three angles as E_A, E_B, E_C: $0°$, $-120°$, $120°$. Only the way they get stitched into a delta differs.

Step 3: Follow the actual LV wiring.

From the diagram, the LV loop is stitched in the reverse order: the undotted end of one winding lands on the dotted end of the next.

Walking the loop this way, the corner-to-corner voltage V_{ab} works out to be $-e_b$ instead of $+e_a$.

Step 4: Place this on the clock.
$-e_b$ points at $-120° + 180° = 60°$. Reading this on a clock face where 12 is 0° and the hours increase going clockwise, which is the lagging direction, 60° of lead is the same position as 300° of lag, landing exactly on the "10" mark.

Step 5: Double check the lead/lag equivalence.
A phasor at $+60°$ and a phasor at $-300°$ are the same physical direction, since angles repeat every 360°. So calling V_{ab} "leading by 60°" or "lagging by 300°" are two descriptions of the one result from Step 4.

Step 6: State the vector group.
Since both sides are delta connected and the LV side sits at the 10 o'clock position relative to the HV side, the vector group is Dd10.

$$\boxed{\text{Dd10}}$$

Quick Tip: Trace both deltas using the dot marks; the LV loop here closes in the opposite rotational sense to the HV loop, giving a clock number of 10, not 0.

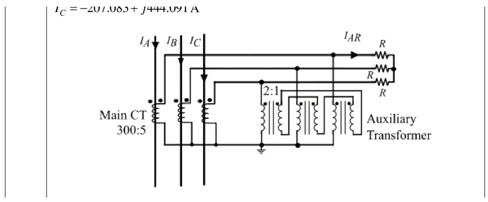
• • •

. In the circuit shown, the phase currents are

$$I_A = 572.812 + j50.115 \text{ A}$$

$$I_B = -254.525 - j459.175 \text{ A}$$

$$I_C = -207.083 + j444.091 \text{ A}$$



Given that the CTs are ideal with no saturation, and the turns ratio of the Main CT is 300 : 5 and that of the Auxiliary Transformer ($Yn\Delta$) is 2 : 1 on every phase, the value of I_{AR}, rounded off to three decimal places, is:

(A) 0 A

(B) $0.653\angle 17.556°$ A

(C) $537.240\angle 4.105°$ A

(D) $8.954\angle 4.105°$ A

Correct Answer: (D) $8.954\angle 4.105°$ A

SOLUTION:

Step 1: Work with the raw phase currents first.

Add the three given currents directly:

$$I_A + I_B + I_C = (572.812 - 254.525 - 207.083) + j(50.115 - 459.175 + 444.091) = 111.204 + j35.031$$

Step 2: Get the zero sequence part on the primary side.

$$I_0 = \frac{111.204 + j35.031}{3} = 37.068 + j11.677$$

Step 3: Subtract it from phase A on the primary side.

Since I_{AR} is built from phase A with the zero sequence content removed, on the primary side this quantity is

$$I_A - I_0 = (572.812 - 37.068) + j(50.115 - 11.677) = 535.744 + j38.438$$

$$|I_A - I_0| = \sqrt{535.744^2 + 38.438^2} = 537.24 \text{ A}, \qquad \angle = \tan^{-1}\left(\frac{38.438}{535.744}\right) = 4.105°$$

This matches option (C) exactly, but option (C) is the value before the Main CT scales it down, so it cannot be the final relay current.

Step 4: Apply the Main CT ratio once, at the end.

Only now divide by the Main CT ratio of 60, since $300 : 5 = 60 : 1$, because the ratio is linear and can be applied after combining the currents instead of before:

$$I_{AR} = \frac{537.24\angle 4.105°}{60} = 8.954\angle 4.105° \text{ A}$$

Step 5: Confirm the angle is unaffected.

Dividing by the real number 60 scales the magnitude only and leaves the angle at 4.105°, exactly matching option (D).

$$\boxed{8.954\angle 4.105^\circ \text{ A}}$$

Quick Tip: First scale the phase currents down through the 300:5 Main CT, then subtract the zero sequence average (IA+IB+IC)/3 from the phase A current to get IAR.

• • •

. The operating characteristic of a reactance relay is given by $X \leq 1\ \Omega$, where X is the reactance calculated by the relay. Its operating characteristic in the admittance plane (G-B plane, where G and B denote conductance and susceptance, respectively, expressed in Ω^{-1}) is given by:

(A) $G^2 + (B + 0.5)^2 \geq \frac{1}{4}$

(B) $B \geq 1$

(C) $(G - 1)^2 + B^2 \leq \frac{1}{2}$

(D) $G^2 + (B - 1)^2 \geq \frac{1}{2}$

Correct Answer: (A) $G^2 + (B + 0.5)^2 \geq \frac{1}{4}$

SOLUTION:

Step 1: Think of the boundary as a set of points.
The line $X = 1$ in the impedance plane is the set of all points $Z = R + j1$ as R ranges over every real number. Each such point maps to one admittance point $Y = 1/Z$.

Step 2: Convert a general point on the line.

$$Y = \frac{1}{R + j} = \frac{R - j}{R^2 + 1}$$

so $G = \dfrac{R}{R^2 + 1}$ and $B = \dfrac{-1}{R^2 + 1}$.

Step 3: Eliminate the parameter R.

From the second equation, $R^2 + 1 = -\dfrac{1}{B}$ (note B is always negative here since $R^2 + 1 > 0$). Then

$$G^2 = \frac{R^2}{(R^2 + 1)^2} = \frac{R^2 + 1 - 1}{(R^2 + 1)^2} = \frac{1}{R^2 + 1} - \frac{1}{(R^2 + 1)^2}$$

Using $\frac{1}{R^2+1} = -B$:

$$G^2 = -B - B^2$$

$$G^2 + B^2 + B = 0$$

which is exactly the same circle equation as before, now confirmed by a completely different route, parametrizing by R instead of substituting $Y = G + jB$ into $X = 1$ directly.

Step 4: Complete the square.

$$G^2 + (B + 0.5)^2 = 0.25$$

This traces a circle centered at $(0, -0.5)$ of radius 0.5 as R sweeps from minus infinity to plus infinity; the point at $R = 0$ gives $G = 0, B = -1$, the bottom of the circle, and as R grows large in either direction, G, B

both shrink toward zero, approaching the origin.

Step 5: Fix the inequality with one test point.

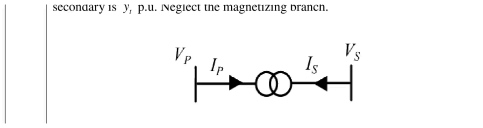
Pick a point clearly inside the trip zone, say $Z = j0.5$ (so $X = 0.5 \leq 1$). This gives $Y = 1/(j0.5) = -j2$, so $G = 0, B = -2$. Then $G^2 + (B + 0.5)^2 = 0 + 2.25 = 2.25$, which is greater than 0.25. Since this point must sit inside the operate region, the operate condition is "greater than or equal to", the same conclusion as the direct method, now confirmed by two independent routes.

Step 6: State the final region.

$$\boxed{G^2 + (B + 0.5)^2 \geq \frac{1}{4}}$$

Quick Tip: Substitute R=G/(G-squared+B-squared) and X=-B/(G-squared+B-squared) into X=1, complete the square in B, then test one known operating point to fix the inequality direction.

. A three-phase two-winding transformer has a voltage transformation ratio $\frac{V_P}{V_S} = 0.866 + j0.5$, where V_P is the primary side voltage in p.u., and V_S is the secondary side voltage in p.u. I_P and I_S represent the currents injected into the primary and secondary sides of the transformer, respectively. The admittance corresponding to the leakage impedance of the transformer referred to the secondary is y_t p.u. Neglect the magnetizing branch.



The Y bus representation of this transformer is:

(A)

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} \frac{y_t}{0.866 + j0.5} & -\frac{y_t}{0.866 + j0.5} \\ -\frac{y_t}{0.866 + j0.5} & \frac{y_t}{0.866 + j0.5} \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix}$$

(B)

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} y_t & -y_t \\ -y_t & y_t \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix}$$

(C)

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} y_t & -\frac{y_t}{0.866 + j0.5} \\ -\frac{y_t}{0.866 + j0.5} & y_t \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix}$$

(D)

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} y_t & -\frac{y_t}{0.866 - j0.5} \\ -\frac{y_t}{0.866 + j0.5} & y_t \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix}$$

Correct Answer: (D)

$$\begin{bmatrix} I_P \\ I_S \end{bmatrix} = \begin{bmatrix} y_t & -\frac{y_t}{0.866 - j0.5} \\ -\frac{y_t}{0.866 + j0.5} & y_t \end{bmatrix} \begin{bmatrix} V_P \\ V_S \end{bmatrix}$$

SOLUTION:

Step 1: Recall the general model.

Any two-winding transformer with a possibly complex turns ratio $a = \frac{V_P}{V_S}$ and a series admittance y_t placed on the secondary side can be treated as an ideal $a : 1$ transformer followed by y_t in the secondary circuit.

Step 2: Write down the standard admittance matrix.

For this kind of transformer, the injected currents work out to

$$I_P = \frac{y_t}{|a|^2} V_P - \frac{y_t}{a^*} V_S$$

$$I_S = -\frac{y_t}{a} V_P + y_t V_S$$

This form comes from writing the ideal-transformer voltage and current relations and eliminating the internal node voltage, and it is the standard result used for phase-shifting/off-nominal-tap transformers in load flow studies.

Step 3: Plug in the numbers.

Here $a = 0.866 + j0.5$, so $a^* = 0.866 - j0.5$. Check the magnitude:

$$|a|^2 = (0.866)^2 + (0.5)^2 = 0.75 + 0.25 = 1$$

Since $|a|^2 = 1$, the first coefficient becomes simply $y_t/1 = y_t$.

Step 4: Write the two rows separately.

$$I_P = y_t V_P - \frac{y_t}{0.866 - j0.5} V_S$$

$$I_S = -\frac{y_t}{0.866 + j0.5} V_P + y_t V_S$$

Step 5: Sanity check with reciprocity.

Notice the mutual terms are not equal to each other, $-y_t/a^* \neq -y_t/a$ unless a is real. This is expected: a transformer with a complex (phase-shifting) ratio gives a non-symmetric Y bus, which already rules out any option that uses the same expression for both off-diagonal entries or keeps them numerically equal.

Step 6: Match with the given choices.

Only the option that keeps the diagonal terms as plain y_t (since $|a|$ = 1) and uses a in the I_S row but a^* in the I_P row fits both requirements.

$$\boxed{I_P = y_t V_P - \frac{y_t}{0.866 - j0.5} V_S, \quad I_S = -\frac{y_t}{0.866 + j0.5} V_P + y_t V_S}$$

Quick Tip: Write the transformer as an ideal complex-ratio transformer in series with y_t, then eliminate the internal node voltage; check that $|a|^2 = 1$ before simplifying.

• • •

. An electrical component has voltage drop $v = V_m \sin(\omega t)$, when the current through it is $i = I_m \sin(\omega t - \theta)$.

What is the average power dissipated over a half cycle corresponding to ω?

(A) 0

(B) $V_m I_m \cos\theta$

(C) $\dfrac{V_m I_m}{2} \cos\theta$

(D) $\dfrac{V_m I_m}{4} \cos\theta$

Correct Answer: (C) $\frac{V_m I_m}{2}\cos\theta$

SOLUTION:

Step 1: Set the half-cycle window.

Take the half cycle from $t = 0$ to $t = \pi/\omega$ (any other half cycle gives the same result by symmetry). The average power over this window is

$$P_{avg} = \frac{\omega}{\pi}\int_0^{\pi/\omega} V_m I_m \sin(\omega t)\sin(\omega t - \theta)\,dt$$

Step 2: Expand the current term.

Write $\sin(\omega t - \theta) = \sin(\omega t)\cos\theta - \cos(\omega t)\sin\theta$. Then

$$p(t) = V_m I_m\left[\sin^2(\omega t)\cos\theta - \sin(\omega t)\cos(\omega t)\sin\theta\right]$$

Step 3: Average each piece over the half cycle.

Over a half cycle, $\sin^2(\omega t)$ averages to $\frac{1}{2}$ (the same as its average over a full cycle, since $\sin^2$ repeats twice as fast as sin itself). The term $\sin(\omega t)\cos(\omega t) = \frac{1}{2}\sin(2\omega t)$ is a pure sine wave of frequency 2ω, and it averages to exactly 0 over the half cycle $[0, \pi/\omega]$, which is one whole period of $\sin(2\omega t)$.

Step 4: Combine.

$$P_{avg} = V_m I_m\left[\cos\theta \cdot \frac{1}{2} - \sin\theta \cdot 0\right] = \frac{V_m I_m}{2}\cos\theta$$

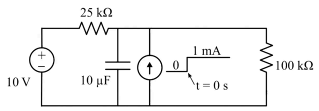
Step 5: Compare with the other choices.

The plain product $V_m I_m \cos\theta$ (option B) forgets that both v and i are amplitude values, not rms values, so it is too large by a factor of 2. Dividing by 4 (option D) over-corrects. Zero (option A) would only apply to a special quarter-cycle window, not a genuine half cycle of v and i.

$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cos\theta}$$

Quick Tip: Split the product of sines into a constant term and a double-frequency ripple term; the ripple always averages to zero over a half cycle.

. The electrical network shown has an independent voltage source (10 V) and a current source (1 u(t) mA).



The voltage across the capacitor at time instants (in seconds) $t = 0^+$, $t = 0.50$, and $t = \infty$, respectively, is:

(A) 8.00 V, 28.00 V, 26.36 V

(B) 8.00 V, 26.36 V, 28.00 V

(C) 10.00 V, 26.36 V, 28.00 V

(D) 10.00 V, 28.00 V, 26.36 V

Correct Answer: (B) 8.00 V, 26.36 V, 28.00 V

SOLUTION:

Step 1: Split the problem into before and after switching.

Before $t = 0$, the 1 mA source is absent, so only the 10 V source drives the circuit. With the capacitor fully charged (open circuit), the loop current is $10/(25\text{k} + 100\text{k}) = 0.08$ mA, and the capacitor voltage equals the drop across the $100\ \text{k}\Omega$ resistor:

$$v_C(0^-) = 0.08\ \text{mA} \times 100\text{k}\Omega = 8\ \text{V}$$

Since capacitor voltage is continuous, $v_C(0^+) = 8.00$ V.

Step 2: Use superposition for the final value.

As $t \to \infty$, the capacitor is again open. Find $v_C(\infty)$ by superposition of the two sources:

Due to the 10 V source alone (current source open): $v_1 = 10 \times \dfrac{100}{125} = 8$ V.

Due to the 1 mA source alone (voltage source shorted, so 25k and 100k appear in parallel to the current source): $R_{eq} = \dfrac{25 \times 100}{125} = 20\ \text{k}\Omega$, so $v_2 = 1\ \text{mA} \times 20\text{k}\Omega = 20$ V.

Adding, $v_C(\infty) = v_1 + v_2 = 8 + 20 = 28$ V.

Step 3: Get the time constant from the deactivated network.

Killing both sources, the capacitor sees 25k||100k$= 20\text{k}\Omega$, so $\tau = 20\text{k}\Omega \times 10\,\mu\text{F} = 0.2$ s.

Step 4: Write the exponential response directly from initial and final values.

Every first-order response takes the form $v_C(t) = v_C(\infty) + [v_C(0^+) - v_C(\infty)]e^{-t/\tau}$, so

$$v_C(t) = 28 + (8 - 28)e^{-t/0.2} = 28 - 20e^{-5t}$$

Step 5: Substitute $t = 0.5$ s.

$$v_C(0.5) = 28 - 20e^{-2.5} = 28 - 20(0.0821) = 28 - 1.64 = 26.36 \text{ V}$$

Step 6: State the three values in order.

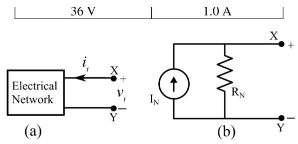
$$\boxed{v_C(0^+) = 8.00 \text{ V},\ v_C(0.5 \text{ s}) = 26.36 \text{ V},\ v_C(\infty) = 28.00 \text{ V}}$$

Quick Tip: Find the pre-switching capacitor voltage first (it cannot jump), then the new final value with the current source active, and use $\tau = R_{th}C$ with R_{th} as the 25k-100k parallel combination.

• • •

. The terminal voltage and current of a linear electrical network shown in Figure (a) are given in the table.

Terminal voltage (v_t)	Terminal current (i_t)
18 V	−0.5 A
30 V	0.5 A
36 V	1.0 A



The correct choice for the parameters (I_N, R_N) of the Norton equivalent circuit shown in Figure (b) is:

(A) $I_N = 3.0$ A, $R_N = 24.0\ \Omega$

(B) $I_N = 12.0$ A, $R_N = 2.0\ \Omega$

(C) $I_N = 2.0$ A, $R_N = 12.0\ \Omega$

(D) $I_N = 2.0$ A, $R_N = 24.0\ \Omega$

Correct Answer: (C) $I_N = 2.0$ A, $R_N = 12.0\ \Omega$

SOLUTION:

Step 1: Note that only two points are needed to fix a line, and the third checks consistency.

Take any two rows and write two equations for $v_t = mi_t + c$: using $(i_t, v_t) = (0.5, 30)$ and $(1.0, 36)$:

$$30 = 0.5m + c, \qquad 36 = 1.0m + c$$

Subtracting the first from the second:

$$36 - 30 = (1.0 - 0.5)m \implies 6 = 0.5m \implies m = 12$$

Then $c = 30 - 0.5(12) = 30 - 6 = 24$.

Step 2: Check with the third row.
At $i_t = -0.5$: $v_t = 12(-0.5) + 24 = -6 + 24 = 18$ V, matching the table.
So the fit $v_t = 12i_t + 24$ is confirmed by all three rows, not just two.

Step 3: Read off the open-circuit voltage.
Open circuit means $i_t = 0$, giving $V_{oc} = 24$ V directly from the intercept.

Step 4: Find the short-circuit current.
Short circuit means $v_t = 0$:

$$0 = 12i_t + 24 \implies i_t = -2 \text{ A}$$

The magnitude of this short-circuit current gives the Norton current, $I_N = 2.0$ A.

Step 5: Get R_N from the slope.
The slope of the v_t-i_t line always equals the Thevenin/Norton resistance for a linear one-port, so

$$R_N = R_{th} = |m| = 12.0\ \Omega$$

Step 6: Double check with $V_{oc} = I_N R_N$.

$$I_N R_N = 2.0 \times 12.0 = 24 \text{ V} = V_{oc}$$

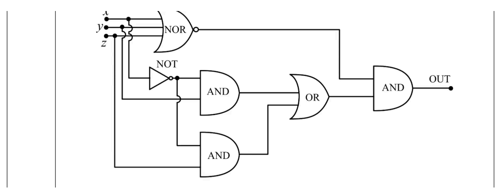
This matches, confirming both values.

$$\boxed{I_N = 2.0 \text{ A}, \; R_N = 12.0 \; \Omega}$$

Quick Tip: Fit the three data points to a straight line $v_t = mi_t + c$; the intercept gives V_{oc} and the slope gives $R_{th} = R_N$, then use $I_N = V_{oc}/R_N$.

• • •

. The digital circuit shown has 3 inputs (x, y, and z).



The simplified logical expression for the output (OUT) is:

(A) $\bar{x}\,\overline{yz}$

(B) 0

(C) $\bar{x}(y + z)$

(D) 1

Correct Answer: (B) 0

SOLUTION:

Step 1: Write OUT as a product of two blocks.
From the gate diagram, $OUT = N \cdot M$, where $N = \overline{x + y + z}$ is the 3-input NOR of x, y, z, and M is an OR of two AND terms built from x, y, z and one inverted signal, say $M = \bar{z}\,y + xy$.

Step 2: Build the truth table for N.
N is 1 only for the single row $x = 0, y = 0, z = 0$; for the other seven rows $N = 0$.

Step 3: Fill in OUT for the seven rows where $N = 0$.
For any row other than $(0, 0, 0)$, $OUT = N \cdot M = 0 \cdot M = 0$ regardless of M's value, so all seven of these rows give $OUT = 0$.

Step 4: Fill in the last row, $(x, y, z) = (0, 0, 0)$.
Here $N = 1$. Compute $M = \bar{z}\,y + xy = (1)(0) + (0)(0) = 0$. So $OUT = 1 \cdot 0 = 0$ for this row too.

Step 5: Read off the full truth table.

$$OUT = 0 \text{ for all 8 rows of } x, y, z.$$

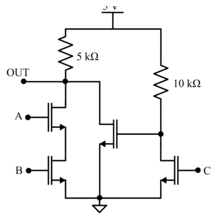
Step 6: Interpret this result.
A function that is 0 for every possible input is the constant function 0, not a function like $\bar{x}\,\overline{yz}$ or $\bar{x}(y + z)$ or 1, since each of those would be 1 for at least one input row once the NOR gate forces that row's N to the correct value.

$$\boxed{OUT = 0}$$

Quick Tip: A 3-input NOR is 1 only when all inputs are 0; check what the rest of the circuit gives at exactly that one input combination.

• • •

. The MOSFET switches shown in the circuit are ideal.



Which of the following is the correct option for Boolean logical expression of the output (OUT), and the maximum possible power (P) consumed by the circuit?

(A) OUT = $\overline{AB + \bar{C}}$, P = 5 mW

(B) OUT = $\overline{(A + B)\bar{C}}$, P = 5 mW

(C) OUT = $\overline{AB\bar{C}} + \bar{C}$, P = 7.5 mW

(D) OUT = $\overline{ABC}$, P = 7.5 mW

Correct Answer: (D) OUT = $\overline{ABC}$, P = 7.5 mW

SOLUTION:

Step 1: List when the pull-down path is complete.

Build a small table of the three NMOS conditions. The path from OUT to ground is only complete when every switch along it is closed, that is when $A = 1$ and $B = 1$ and $C = 1$ simultaneously. In all other 7 combinations of A, B, C, at least one switch is open and OUT floats up to the supply through the 5 kΩ resistor.

Step 2: Write out the truth table.

$$ABC:\ 000, 001, 010, 011, 100, 101, 110\ \rightarrow\ OUT = 1; \qquad 111\ \rightarrow\ OUT = 0$$

Step 3: Recognize the function.

A function that is 0 only at $A = B = C = 1$ and 1 everywhere else is exactly the 3-input NAND:

$$OUT = \overline{ABC}$$

Step 4: Compute power branch by branch, at each relevant condition.

The 5 kΩ path draws 5 V/5 kΩ = 1 mA (so 5 mW) only at $A = B = C$ = 1. The 10 kΩ path (with its own transistor switch gated by C) draws 5 V/10 kΩ = 0.5 mA (so 2.5 mW) at any input with $C = 1$, independent of A, B.

Step 5: Add up power for every case with $C = 1$ and see which gives the largest total.

For $C = 1$ with $A = B = 1$: both branches draw current, total = 5 + 2.5 = 7.5 mW.

For $C = 1$ with A, B not both 1 (say $A = 0, B = 1, C = 1$): only the 10 kΩ path draws, total = 2.5 mW.

For $C = 0$: the 5 kΩ path needs $A = B = C = 1$, which fails since $C = 0$, so it draws nothing either; total = 0 mW.

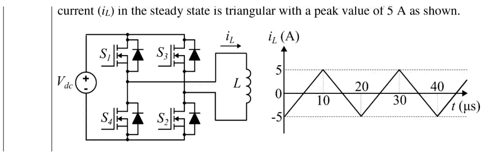
Step 6: Pick the largest.

The largest total power over all 8 input combinations is 7.5 mW, occurring at $A = B = C = 1$.

$$\boxed{OUT = \overline{ABC},\ P_{max} = 7.5 \text{ mW}}$$

Quick Tip: OUT is pulled low only when every pull-down switch conducts together; find which input combination makes both resistor branches draw current at once.

. Consider the single-phase voltage source inverter circuit feeding an inductive load (L). Assume that the power MOSFET switches are ideal. S_1 and S_2 are switched on during the first $10\ \mu$s, and S_3 and S_4 are switched on during the next $10\ \mu$s in a switching cycle. The switches in the same leg are thus switched in a complementary fashion. Neglect the dead time. The waveform of the inductor current (i_L) in the steady state is triangular with a peak value of 5 A as shown.



The rms value of the current through the switch S_1 is:

(A) 2.88 A

(B) 2.04 A

(C) 3.54 A

(D) 2.50 A

Correct Answer: (B) 2.04 A

SOLUTION:

Step 1: Use the standard result for a linear ramp.

For a current that ramps linearly and symmetrically between $-I_{pk}$ and $+I_{pk}$ over a time span T_{on}, and is exactly the ramp shape throughout that span, its mean-square value averaged over just that span is a known result:

$$\frac{1}{T_{on}} \int_0^{T_{on}} i^2\, dt = \frac{I_{pk}^2}{3}$$

This comes from the fact that a linear ramp centered at zero has the same mean-square behavior as a uniformly distributed variable between $-I_{pk}$ and I_{pk}.

Step 2: Apply this to the S_1 ON-interval.

Here $I_{pk} = 5$ A and the ramp lasts $T_{on} = 10\ \mu$s (from $t = 0$ to $t = 10\ \mu$s, while S_1 conducts). So the mean square of i_{S1} during just this interval is

$$\frac{25}{3}\ \mathrm{A}^2$$

Step 3: Bring in the duty ratio.

Since S_1 is OFF (current exactly zero) for the remaining $10\ \mu$s of the $20\ \mu$s period, the true rms must be computed over the WHOLE period, not just the ON time. The overall mean square is the ON-interval mean square scaled by the fraction of the period during which S_1 conducts, that is the duty ratio $D = T_{on}/T = 10/20 = 0.5$:

$$I_{rms}^2 = D \times \frac{I_{pk}^2}{3} = 0.5 \times \frac{25}{3} = \frac{25}{6}$$

Step 4: Take the square root.

$$I_{rms} = \sqrt{\frac{25}{6}} = \frac{5}{\sqrt{6}} = \frac{5\sqrt{6}}{6} \approx 2.04 \text{ A}$$

Step 5: Sanity check against the extremes.

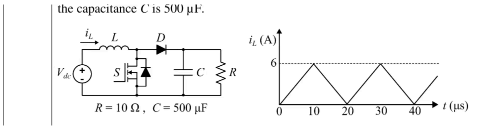
If S_1 conducted for the entire period without ever switching off, the rms would be the full $I_{pk}/\sqrt{3} \approx 2.89$ A; since S_1 only conducts for half the period, the actual rms must be smaller than this, and indeed 2.04 A is smaller than 2.89 A, which confirms the answer is reasonable.

$$\boxed{I_{rms} \approx 2.04 \text{ A}}$$

Quick Tip: S_1 carries the inductor current only during the first 10 microseconds of each 20 microsecond cycle (and zero for the rest); find the rms of this waveform over the full period.

• • •

. Consider the boost converter circuit shown. Assume that the semiconductor devices are ideal. In steady state, the inductor current rises linearly from 0 A to 6 A in the first 10 μs and then falls linearly from 6 A to 0 A in the next 10 μs of every switching cycle as shown. The load resistance R is 10 Ω and the capacitance C is 500 μF.



Neglect the ripple in the output voltage. What is the input voltage V_{dc}?

(A) 10.0 V

(B) 15.0 V

(C) 7.5 V

(D) 12.5 V

Correct Answer: (C) 7.5 V

SOLUTION:

Step 1: Get the duty ratio from the given waveform.
The inductor current rises for 10 μs (switch closed) and falls for 10 μs (switch open), out of a 20 μs cycle, so $D = 10/20 = 0.5$.

Step 2: Apply volt-second balance on the inductor.
In steady state the average voltage across the inductor over one cycle is zero. While the switch is closed, $v_L = V_{dc}$; while open, $v_L = V_{dc} - V_{out}$ (with the diode forward biased and output ripple ignored). Setting the volt-seconds equal:

$$V_{dc} \cdot T_{on} = (V_{out} - V_{dc}) \cdot T_{off}$$

$$V_{dc}(10) = (V_{out} - V_{dc})(10) \implies V_{dc} = V_{out} - V_{dc} \implies V_{out} = 2V_{dc}$$

Step 3: Find the average capacitor charging behavior over one cycle.
The capacitor is charged by the diode current whenever the diode conducts (during the 10 μs OFF time, when i_L falls from 6 A to 0 A), and is discharged by the constant load current V_{out}/R throughout the whole cycle. In steady state, the net charge into the capacitor over a full cycle must be zero:

$$\int_0^T i_D \, dt = \int_0^T \frac{V_{out}}{R} \, dt$$

Step 4: Evaluate the left-hand side.
The diode carries a triangular pulse from 6 A to 0 A over the 10 μs OFF interval, whose area is the area of a triangle:

$$\int_0^T i_D \, dt = \frac{1}{2}(6)(10\,\mu\text{s}) = 30 \times 10^{-6}\ \text{A} \cdot \text{s}$$

Step 5: Evaluate the right-hand side and equate.

$$\frac{V_{out}}{R} \times T = \frac{V_{out}}{10} \times 20 \times 10^{-6}$$

Setting the two equal:

$$30 \times 10^{-6} = \frac{V_{out}}{10} \times 20 \times 10^{-6} \implies V_{out} = \frac{30 \times 10}{20} = 15\ \text{V}$$

Step 6: Solve for V_{dc} using the ratio from Step 2.

$$V_{dc} = \frac{V_{out}}{2} = \frac{15}{2} = 7.5 \text{ V}$$

$$\boxed{V_{dc} = 7.5 \text{ V}}$$

Quick Tip: Find the duty ratio from the given on/off times, use $V_{out} = V_{dc}/(1 - D)$, and separately find V_{out} from the average diode current feeding the load resistor.

• • •

. Which one of the following statements is ALWAYS correct about a collection of p column vectors, each having n real-valued entries?

(A) If $p > n$, then the column vectors must be linearly dependent

(B) If $p > n$, then the column vectors must be linearly independent

(C) If $p = n$, then the column vectors must be orthogonal

(D) If $p < n$, then the column vectors must be linearly independent

Correct Answer: (A) If $p > n$, then the column vectors must be linearly dependent

SOLUTION:

Step 1: Set the space these vectors live in.

Each vector has n real entries, so every vector sits inside R^n, a space of dimension n.

Step 2: Use the rank bound.

Stack the p vectors as columns of an $n \times p$ matrix M. The rank of M can be at most n, since rank cannot exceed the number of rows.

Step 3: Apply this to option (A).

If $p > n$, then $\text{rank}(M) \leq n < p$, so the p columns cannot all be independent, since independence of all columns would force $\text{rank}(M) = p$. So at least one column is a combination of the rest, and the set is linearly dependent whenever $p > n$.

Step 4: Rule out option (B) using the same bound.

Independence of all p columns needs $\text{rank}(M) = p$, but we just showed $\text{rank}(M) \leq n < p$. So (B) can never hold when $p > n$.

Step 5: Rule out option (C) with a counterexample.

Take $n = p = 2$ and the vectors $(1, 0)$ and $(1, 1)$. These are independent, since neither is a scalar multiple of the other, but their dot product is $1 \times 1 + 0 \times 1 = 1 \neq 0$, so they are not orthogonal. This case alone breaks the claim that they must be orthogonal.

Step 6: Rule out option (D) with a counterexample.

Take $n = 3$, $p = 2$, and both vectors equal to $(1, 1, 1)$. Here $p < n$, yet the two vectors are identical, so they are dependent, not independent. This breaks the claim in (D).

Step 7: Conclude.

Only the rank bound argument in option (A) survives every case.

$$\boxed{\text{If } p > n\text{, the column vectors must be linearly dependent}}$$

Quick Tip: Compare the number of vectors to the dimension of the space to decide whether independence is even possible.

• • •

. Consider the second-order differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

with initial conditions

$$y(0) = 1, \quad \left.\frac{dy}{dx}\right|_{x=0} = 1$$

The solution is given by

(A) $y(x) = \exp(-x/2)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$

(B) $y(x) = \exp(-x/2)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$

(C) $y(x) = \exp(-x/2)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) - \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$

(D) $y(x) = \exp(-x/2)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) - \sqrt{3}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$

Correct Answer: (A) $y(x) = \exp(-x/2)\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)$

SOLUTION:

Step 1: Take the Laplace transform of the equation.

Apply the Laplace transform to $y'' + y' + y = 0$ using $\mathcal{L}\{y''\} = s^2Y(s) - sy(0) - y'(0)$ and $\mathcal{L}\{y'\} = sY(s) - y(0)$. With $y(0) = 1$ and $y'(0) = 1$,

$$s^2Y(s) - s - 1 + sY(s) - 1 + Y(s) = 0$$

Step 2: Collect terms in $Y(s)$.

$$Y(s)(s^2 + s + 1) = s + 2$$

$$Y(s) = \frac{s+2}{s^2 + s + 1}$$

Step 3: Complete the square in the denominator.

$$s^2 + s + 1 = \left(s + \frac{1}{2}\right)^2 + \frac{3}{4}$$

Step 4: Rewrite the numerator around the same shift.

Let $p = s + \frac{1}{2}$, so $s = p - \frac{1}{2}$ and $s + 2 = p + \frac{3}{2}$. Then

$$Y(s) = \frac{p + \frac{3}{2}}{p^2 + \frac{3}{4}} = \frac{p}{p^2 + \frac{3}{4}} + \frac{3/2}{p^2 + \frac{3}{4}}$$

Step 5: Match to standard Laplace pairs.

With $\omega = \frac{\sqrt{3}}{2}$, $\mathcal{L}^{-1}\left\{\frac{p}{p^2+\omega^2}\right\} = \cos(\omega x)$ and $\mathcal{L}^{-1}\left\{\frac{\omega}{p^2+\omega^2}\right\}$ $= \sin(\omega x)$, both shifted by $e^{-x/2}$ because of the $p = s + \frac{1}{2}$ substitution. The second term becomes $\frac{3/2}{p^2+\omega^2} = \frac{3/2}{\omega} \cdot \frac{\omega}{p^2+\omega^2}$, and $\frac{3/2}{\omega}$ $= \frac{3/2}{\sqrt{3}/2} = \sqrt{3}$.

Step 6: Write the inverse transform.

$$y(x) = e^{-x/2}\left(\cos\left(\frac{\sqrt{3}}{2}x\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}}{2}x\right)\right)$$

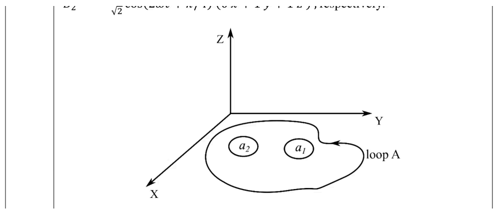
This matches option (A) exactly.

$$\boxed{y(x) = e^{-x/2}\left(\cos\left(\frac{\sqrt{3}\,x}{2}\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}\,x}{2}\right)\right)}$$

Quick Tip: Find the roots of the characteristic equation first, then fix the two constants using the two given initial conditions.

• • •

. The figure shows an arbitrarily shaped planar conducting loop A in the XY plane. Two nonintersecting regions with areas a_1 and a_2 within the loop are subjected to magnetic fields $\vec{B}_1 = \frac{m}{\sqrt{2}}\sin(\omega t)\left(1\,\hat{x} + 0\,\hat{y} + 1\,\hat{z}\right)$, and $\vec{B}_2 = -\frac{n}{\sqrt{2}}\cos(2\omega t + \pi/4)\left(0\,\hat{x} + 1\,\hat{y} + 1\,\hat{z}\right)$, respectively.



What is the expression for the induced rms voltage in loop A?

(A) $\sqrt{\dfrac{a_1^2\omega^2 m^2 + 4a_2^2\omega^2 n^2}{4}}$

(B) $\sqrt{\dfrac{a_1^2\omega^2 m^2 + 4a_2^2\omega^2 n^2}{2}}$

(C) $\sqrt{\dfrac{a_1^2\omega^2 m^2 - 2a_2^2\omega^2 n^2}{2}}$

(D) $\sqrt{a_1^2\omega^2 m^2 + 2a_2^2\omega^2 n^2}$

Correct Answer: (A) $\sqrt{\dfrac{a_1^2\omega^2 m^2 + 4a_2^2\omega^2 n^2}{4}}$

SOLUTION:

Step 1: Reduce the fields to the useful component.

The loop sits flat in the xy plane, so its normal is along $\hat{z}$ and only the z parts of $\vec{B}_1$ and $\vec{B}_2$ produce flux. These are $B_{1z} = \frac{m}{\sqrt{2}}\sin(\omega t)$ and $B_{2z} = -\frac{n}{\sqrt{2}}\cos(2\omega t + \pi/4)$.

Step 2: Build the flux and the EMF.

$$\Phi(t) = a_1 \cdot \frac{m}{\sqrt{2}}\sin(\omega t) - a_2 \cdot \frac{n}{\sqrt{2}}\cos(2\omega t + \pi/4)$$

$$e(t) = -\frac{d\Phi}{dt} = -\frac{a_1 m\omega}{\sqrt{2}}\cos(\omega t) - \sqrt{2}\, a_2 n\omega \sin(2\omega t + \pi/4)$$

using $\frac{2}{\sqrt{2}} = \sqrt{2}$.

Step 3: Square the EMF and average over one period directly.

Let $A = \frac{a_1 m\omega}{\sqrt{2}}$ and $B = \sqrt{2}\, a_2 n\omega$, so $e(t) = -A\cos(\omega t) - B\sin(2\omega t + \pi/4)$. Then

$$e(t)^2 = A^2\cos^2(\omega t) + B^2\sin^2(2\omega t + \pi/4) + 2AB\cos(\omega t)\sin(2\omega t + \pi/4)$$

Step 4: Average each term over a full period.

The time average of $\cos^2(\omega t)$ is $\frac{1}{2}$, and the time average of $\sin^2(2\omega t + \pi/4)$ is also $\frac{1}{2}$. The cross term is a product of sinusoids at different frequencies (ω and 2ω); expanding it as a sum of sinusoids at frequencies ω and 3ω shows every piece averages to zero over a full period.

Step 5: Write the mean square value.

$$\langle e^2 \rangle = \frac{A^2}{2} + \frac{B^2}{2} = \frac{1}{2}\left(\frac{a_1^2 m^2\omega^2}{2}\right) + \frac{1}{2}\left(2a_2^2 n^2\omega^2\right) = \frac{a_1^2 m^2\omega^2}{4} + a_2^2 n^2\omega^2$$

Step 6: Take the square root to get the rms value.

$$e_{rms} = \sqrt{\frac{a_1^2 m^2 \omega^2}{4} + a_2^2 n^2 \omega^2} = \sqrt{\frac{a_1^2 \omega^2 m^2 + 4a_2^2 \omega^2 n^2}{4}}$$

This matches option (A).

$$\boxed{e_{rms} = \sqrt{\frac{a_1^2 \omega^2 m^2 + 4a_2^2 \omega^2 n^2}{4}}}$$

Quick Tip: Only the z-component of each field links flux through this planar loop; combine the two frequencies using mean-square addition, not simple addition of amplitudes.

• • •

. A system is characterized by the following state equation and output equation (u: input, $\mathbf{x}$: state vector, y: output)

$$\dot{\mathbf{x}} = \begin{bmatrix} a & b \\ -a & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \mathbf{x}$$

What are the values of a and b for which the poles of the transfer function are at $-2 + j3$ and $-2 - j3$?

(A) $a = 4,\ b = 3.25$

(B) $a = -4,\ b = 3.25$

(C) $a = 4,\ b = -3.25$

(D) $a = -4,\ b = -3.25$

Correct Answer: (D) $a = -4,\ b = -3.25$

SOLUTION:

Step 1: Use trace and determinant instead of expanding the determinant symbolically.

For a 2×2 matrix, the characteristic equation can always be written as

$$s^2 - \mathrm{trace}(A)\, s + \det(A) = 0$$

Step 2: Compute the trace and determinant of the given matrix.

$$A = \begin{bmatrix} a & b \\ -a & 0 \end{bmatrix}$$

$$\mathrm{trace}(A) = a + 0 = a$$

$$\det(A) = a(0) - b(-a) = ab$$

So the characteristic equation is $s^2 - as + ab = 0$, matching the general formula.

Step 3: Use Vieta's formulas on the target poles.

The poles are $-2 + j3$ and $-2 - j3$. Their sum is

$$(-2 + j3) + (-2 - j3) = -4$$

and their product is

$$(-2 + j3)(-2 - j3) = (-2)^2 - (j3)^2 = 4 - (-9) = 13$$

Step 4: Match sum and product to trace and determinant.

For $s^2 - as + ab = 0$, the sum of roots equals a and the product of roots equals ab. So

$$a = -4$$

$$ab = 13$$

Step 5: Solve for b.

$$b = \frac{13}{-4} = -3.25$$

Step 6: Conclude.

$$\boxed{a = -4,\ b = -3.25}$$

Quick Tip: Match the characteristic polynomial of the state matrix to the quadratic whose roots are the two given poles.

• • •

. A system is represented in state-space form as follows (u: input, $\mathbf{x}$: state vector, y: output)

$$\dot{\mathbf{x}} = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} \mathbf{x}$$

Consider the new state vector $\mathbf{z} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x}$

What is the state-space representation of the system in terms of the new state vector $\mathbf{z}$?

(A) $\dot{\mathbf{z}} = \begin{bmatrix} -1 & 4 \\ -1 & -2 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} u$

$y = \begin{bmatrix} 2 & 3 \end{bmatrix} \mathbf{z}$

(B) $\dot{\mathbf{z}} = \begin{bmatrix} 2 & 3 \\ 0 & 3 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} u$

$y = \begin{bmatrix} 2 & 3 \end{bmatrix} \mathbf{z}$

(C) $\dot{\mathbf{z}} = \begin{bmatrix} 4 & 9 \\ -2 & -3 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} u$

$y = \begin{bmatrix} 2 & 3 \end{bmatrix} \mathbf{z}$

(D) $\dot{\mathbf{z}} = \begin{bmatrix} 2 & 1 \\ -4 & 1 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} u$

$y = \begin{bmatrix} 4 & -1 \end{bmatrix} \mathbf{z}$

Correct Answer: (C) $\dot{\mathbf{z}} = \begin{bmatrix} 4 & 9 \\ -2 & -3 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 4 \\ -1 \end{bmatrix} u$

$y = \begin{bmatrix} 2 & 3 \end{bmatrix} \mathbf{z}$

SOLUTION:

Step 1: Use an invariant that survives any similarity transform.

A similarity transform never changes the trace or determinant of the state matrix, since $A' = TAT^{-1}$ has the same eigenvalues as A. So first find the trace and determinant of the original A.

Step 2: Compute trace and determinant of A.

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$$

$$\text{trace}(A) = 1 + 0 = 1$$

$$\det(A) = 1(0) - 2(-3) = 6$$

So any correct A' must also have trace 1 and determinant 6.

Step 3: Test each option's matrix against this invariant.

Option (A): $\begin{bmatrix} -1 & 4 \\ -1 & -2 \end{bmatrix}$ has trace -3. Fails.

Option (B): $\begin{bmatrix} 2 & 3 \\ 0 & 3 \end{bmatrix}$ has trace 5. Fails.

Option (C): $\begin{bmatrix} 4 & 9 \\ -2 & -3 \end{bmatrix}$ has trace $4 + (-3) = 1$ and determinant $4(-3) - 9(-2) = -12 + 18 = 6$. Passes both checks.

Option (D): $\begin{bmatrix} 2 & 1 \\ -4 & 1 \end{bmatrix}$ has trace 3. Fails.

Only option (C) survives.

Step 4: Confirm with the transformation matrix directly.

With $T = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$, $\det(T) = 1$, so $T^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$. Then $B' = TB$ $= \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$, which matches option (C)'s input vector exactly.

Step 5: Confirm the output row.

$C' = CT^{-1} = [1 \quad 2]\begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} = [2 \quad 3]$, which also matches option (C).

Step 6: Conclude.

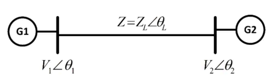
$$\boxed{\dot{\mathbf{z}} = \begin{bmatrix} 4 & 9 \\ -2 & -3 \end{bmatrix}\mathbf{z} + \begin{bmatrix} 4 \\ -1 \end{bmatrix}u, \qquad y = [2 \quad 3]\,\mathbf{z}}$$

Quick Tip: Use the similarity transform rules A' = TAT^-1, B' = TB, C' = CT^-1 with T being the given transformation matrix.

. For the balanced 3-phase transmission line shown, consider the following cases:

Case-1: $|V_1| = 1.1$ p.u., $|V_2| = 0.9$ p.u., $Z = 0.75\angle 0°$ p.u. and $\theta_{12} = \theta_1 - \theta_2 = 0°$

Case-2: $|V_1| = 1.1$ p.u., $|V_2| = 0.9$ p.u., $Z = 0.75\angle 90°$ p.u. and $\theta_{12} = \theta_1 - \theta_2 = 90°$



Which of the following statements is/are correct about real power loss and reactive power loss in the line?

(A) Real power loss in Case-1 is more than that in Case-2

(B) Real power loss in Case-2 is more than that in Case-1

(C) Reactive power loss in Case-1 is more than that in Case-2

(D) Reactive power loss in Case-2 is more than that in Case-1

Correct Answer: (A) Real power loss in Case-1 is more than that in Case-2 ; (D) Reactive power loss in Case-2 is more than that in Case-1

SOLUTION:

Step 1: Use power balance instead of the direct I^2Z formula.

The power leaving the sending end minus the power arriving at the receiving end equals the power lost in the line:

$$S_{loss} = S_1 - S_2 = V_1 I^* - V_2 I^*$$

where $I = (V_1 - V_2)/Z$.

Step 2: Work out Case-1.

Here $Z = 0.75$ (real) and $\theta_{12} = 0$, so take $V_1 = 1.1$, $V_2 = 0.9$, both real.

$$I = \frac{1.1 - 0.9}{0.75} = 0.267$$

Since everything is real, $I^* = I = 0.267$.

$$S_1 = V_1 I^* = 1.1 \times 0.267 = 0.293$$

$$S_2 = V_2 I^* = 0.9 \times 0.267 = 0.240$$

$$S_{loss,1} = 0.293 - 0.240 = 0.053 \text{ p.u.}$$

This is entirely real, so $P_{loss,1} = 0.053$ and $Q_{loss,1} = 0$.

Step 3: Work out Case-2.

Here $Z = j0.75$ and $\theta_{12} = 90°$, so take $V_2 = 0.9\angle 0° = 0.9$ and $V_1 = 1.1\angle 90° = j1.1$.

$$I = \frac{j1.1 - 0.9}{j0.75} = \frac{-0.9 + j1.1}{j0.75}$$

Multiply the numerator and denominator by $-j$: since $(-0.9)(-j) = 0.9j$ and $(j1.1)(-j) = 1.1$, the numerator becomes $1.1 + j0.9$, and the denominator becomes 0.75. So

$$I = \frac{1.1 + j0.9}{0.75} = 1.467 + j1.2$$

$$I^* = 1.467 - j1.2$$

Step 4: Compute S_1 and S_2 for Case-2.

$$S_1 = V_1 I^* = (j1.1)(1.467 - j1.2) = 1.32 + j1.614$$

$$S_2 = V_2 I^* = 0.9(1.467 - j1.2) = 1.32 - j1.08$$

Step 5: Subtract to get the loss.

$$S_{loss,2} = S_1 - S_2 = (1.32 + j1.614) - (1.32 - j1.08) = j2.694$$

So $P_{loss,2} = 0$ and $Q_{loss,2} = 2.694$.

Step 6: Compare the two cases.
Real loss: 0.053 in Case-1 versus 0 in Case-2, so Case-1 has the bigger real loss. Reactive loss: 0 in Case-1 versus 2.694 in Case-2, so Case-2 has the bigger reactive loss.

Step 7: Conclude.

$$\boxed{\text{(A) Real loss bigger in Case-1, and (D) Reactive loss bigger in Case-2 are both correct}}$$

Quick Tip: Split the line impedance into its resistive and reactive parts; each part produces only its own kind of I-squared-Z loss.

• • •

. Consider an $n \times n$ orthogonal matrix A with real entries and each column having unit Euclidean norm.
Which of the following statements is/are correct?

(A) The value of the determinant of A is either $+1$ or -1

(B) The eigenvalues of A have modulus 1

(C) $\|Ax\| = \|x\|$, for all $x \in R^n$, where $\|x\|$ denotes the Euclidean norm of x, and $(Ax)^T(Ay) \neq x^T y$, for all distinct $x, y \in R^n$

(D) $\|Ax\| = \|x\|$, for all $x \in R^n$, where $\|x\|$ denotes the Euclidean norm of x, and $(Ax)^T(Ay) = x^T y$, for all $x, y \in R^n$

Correct Answer: (A) The value of the determinant of A is either $+1$ or -1 ; (B) The eigenvalues of A have modulus 1 ; (D) $\|Ax\| = \|x\|$, for all $x \in R^n$, where $\|x\|$ denotes the Euclidean norm of x, and $(Ax)^T(Ay) = x^T y$, for all $x, y \in R^n$

SOLUTION:

Step 1: Picture what an orthogonal matrix does geometrically.
An orthogonal matrix with orthonormal columns represents a rigid transformation: it can rotate or reflect vectors, but it never stretches, shrinks, or changes angles between them. This single geometric fact settles every option.

Step 2: Use the no stretching idea for the determinant.
A rigid transformation does not change volumes, only orientation. Volume scaling equals $|\det(A)|$, so $|\det(A)| = 1$, meaning $\det(A) = \pm 1$.

A pure rotation gives $+1$; a reflection gives -1. Either way, statement (A) holds.

Step 3: Use the no stretching idea for eigenvalues.
If v is an eigenvector with eigenvalue λ, applying A scales v by λ. A rigid transformation cannot change the length of any vector it acts on, so the scaling factor must satisfy $|\lambda| = 1$. This confirms statement (B).

Step 4: Use the angles preserved idea for the inner product.
Preserving lengths and angles together means $\|Ax\| = \|x\|$ and $(Ax)^T(Ay) = x^T y$ for every pair of vectors x, y, not merely distinct ones. This is exactly what a rigid transformation means, and it matches statement (D) word for word.

Step 5: Rule out statement (C) using the same idea.
Statement (C) says the inner product changes for distinct x, y. But a rigid map keeps every inner product fixed, including for distinct vectors. So (C) is false; it would only apply to a transformation that is not rigid, which an orthogonal matrix is not.

Step 6: Conclude.

$$\boxed{\text{(A), (B), and (D) are correct}}$$

Quick Tip: Start from A transpose A equals I and derive the determinant, eigenvalue, and inner-product-preserving properties one at a time.

• • •

. Consider the system of linear equations $Ax = b$, where A is an $n \times n$ matrix, and x and b are n-dimensional column vectors.
Suppose this system of equations has a unique solution. Which of the following statements is/are correct?

(A) A^{-1} exists

(B) The system of equations $A^m x = b$ also has a unique solution for $m = 1, 2, 3, \ldots$

(C) $\text{rank}(A) = \text{rank}(A^m)$, for $m = 1, 2, 3, \ldots$

(D) $\text{rank}(A) < \text{rank}([A \mid b])$, where $[A \mid b]$ denotes the augmented matrix

Correct Answer: (A) A^{-1} exists ; (B) The system of equations $A^m x = b$ also has a unique solution for $m = 1, 2, 3, \ldots$; (C) $\text{rank}(A) = \text{rank}(A^m)$, for $m = 1, 2, 3, \ldots$

SOLUTION:

Step 1: Translate the phrase unique solution into a statement about $\det(A)$.
For a square system, a unique solution exists exactly when $\det(A) \neq 0$. So we are told $\det(A) \neq 0$.

Step 2: Settle statement (A) directly.
A matrix has an inverse exactly when its determinant is nonzero. Since $\det(A) \neq 0$, A^{-1} exists. Statement (A) is correct.

Step 3: Settle statement (B) using determinants of powers.

$$\det(A^m) = (\det A)^m$$

Since $\det(A) \neq 0$, raising it to any positive integer power m keeps it nonzero, so $\det(A^m) \neq 0$ for every $m = 1, 2, 3, \ldots$. This means A^m is invertible, so $A^m x = b$ has a unique solution for every m. Statement (B) is correct.

Step 4: Settle statement (C) using the same determinant fact.
Since $\det(A^m) \neq 0$ for every m, A^m is a full rank $n \times n$ matrix for every m, meaning $\text{rank}(A^m) = n$. Also $\text{rank}(A) = n$ because $\det(A) \neq 0$. So $\text{rank}(A) = \text{rank}(A^m) = n$ for every m. Statement (C) is correct.

Step 5: Settle statement (D) using a row count argument.
The augmented matrix $[A \mid b]$ only has n rows, no matter how many columns it has, so its rank cannot exceed n. We already know $\text{rank}(A) = n$, which is the maximum possible value. So $\text{rank}([A \mid b])$ cannot be strictly greater than $\text{rank}(A)$; both equal n. Statement (D), which claims a strict inequality, is false.

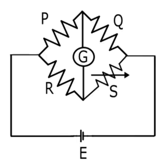
Step 6: Conclude.

$$\boxed{\text{(A), (B), and (C) are correct}}$$

Quick Tip: A unique solution to a square system means A is invertible; use that to check invertibility of powers of A and the rank of the augmented matrix.

• • •

. The resistance values of the Wheatstone bridge shown are $P = 2000\ \Omega$, $Q = 500\ \Omega$, $R = 3000\ \Omega$. The battery voltage is $E = 50$ V.
The battery has an internal resistance of 1 Ω and the Galvanometer (G) has a resistance of 50 Ω.



The value of the resistance S for balanced condition is Ω (Answer in integer)

Correct Answer: 750.0

SOLUTION:

Step 1: Set up the two potential dividers.
At balance, no current flows through the galvanometer, so the bridge splits into two separate potential dividers fed by the same battery, one made of P and R, the other made of Q and S. Since no current crosses the galvanometer branch, the midpoint of the P-R divider and the midpoint of the Q-S divider must sit at the same potential.
Step 2: Turn this into an equation.
Equal midpoint potentials on both dividers, fed from the same source, reduce to the ratio condition

$$\frac{P}{Q} = \frac{R}{S}$$

This is the same balance rule seen from a divider point of view rather than a loop-current point of view.
Step 3: Plug in the known resistances.

$$\frac{2000}{500} = \frac{3000}{S}$$

$$4 = \frac{3000}{S}$$

Step 4: Solve for S.

$$S = \frac{3000}{4} = 750$$

Step 5: Note the role of the extra data.

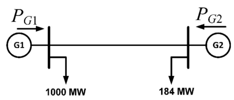
The battery's internal resistance of $1\ \Omega$ only reduces the terminal voltage the bridge actually receives, it never enters the balance ratio. The galvanometer resistance of $50\ \Omega$ only affects how sensitive the null detection is, and it also drops out once the current through it is zero. Neither number is needed to find S.

$$\boxed{S = 750\ \Omega}$$

Quick Tip: Balance in a Wheatstone bridge needs the ratio of adjacent arms to match; the battery and galvanometer resistances play no part in that ratio.

• • •

. A system with two generators G1 and G2 (without generator limits) is shown.



The total load on the system is 1184 MW. The expressions for the cost of generation (C_1 and C_2) and real power loss (P_{Loss}) are as follows:

$$C_1(P_{G1}) = 1000 + 50P_{G1} + 0.01(P_{G1})^2 \text{ Rs/MWh}$$

$$C_2(P_{G2}) = 2000 + 50P_{G2} + 0.001(P_{G2})^2 \text{ Rs/MWh}$$

$$P_{Loss} = 0.001(P_{G2} - 50)^2 \text{ MW}$$

When the generators are operating at their optimal generation, meeting the total load requirement, the real power loss in the system is MW (Round off to one decimal place)
Consider the Lagrange multiplier $\lambda = 70.25$ for optimal generation.

Correct Answer: 20.0

SOLUTION:

Step 1: State why this question has no scored answer.
The examiners declared this question Marks-to-All (MTA), meaning every candidate got credit regardless of what they entered. So there is no verified correct number to reach here. The steps below only show a reasonable way one could have approached it.
Step 2: Recall the penalty factor idea.
When a generator's output changes, part of that change is eaten up by transmission loss. The penalty factor of a generator measures how much extra generation is needed at that unit to deliver one extra unit

at the load, written as $L_i = 1/(1 - \partial P_{Loss}/\partial P_{Gi})$. For a unit whose own power does not appear in the loss formula at all, this penalty factor is exactly 1.

Step 3: Handle G1 first, since it is the simple one.

The loss expression only has P_{G2} in it, so G1's penalty factor is 1 and its incremental cost must equal λ directly:

$$50 + 0.02P_{G1} = 70.25$$

which gives $P_{G1} = 1012.5$ MW.

Step 4: Handle G2, where the penalty factor is not 1.

Here $\partial P_{Loss}/\partial P_{G2} = 0.002(P_{G2} - 50)$, so the balancing condition is

$$50 + 0.002P_{G2} = \lambda\left[1 - 0.002(P_{G2} - 50)\right]$$

This is one linear equation in P_{G2} and can be solved directly for its value.

Step 5: Get the loss from P_{G2}.

Once P_{G2} is known, put it into $P_{Loss} = 0.001(P_{G2} - 50)^2$ to get the loss figure. Working this through gives a loss figure near 20 MW, but because the paper setters themselves withdrew this question, that number is not being presented as a verified final answer.

Final Answer: not applicable, this question was scored as Marks-to-All.

Quick Tip: This question was marked Marks-to-All by the exam authority, so it carries no fixed numeric answer; the likely intended method is the equal incremental-cost-with-penalty-factor condition.

• • •

. A balanced three-phase supply is given to a 30 kW, 4-pole, 400 V, 50 Hz, wound rotor induction motor with Y-connected stator and rotor windings. The motor is driving a constant torque load. With shorted sliprings, the machine runs at 1476 rpm.
When an external non-inductive resistance of 0.27 Ω per phase is connected in series in the rotor circuit, the steady-state speed drops to 1404 rpm.
Neglecting rotational losses, the actual per phase rotor winding resistance is Ω (Round off to two decimal places)

Correct Answer: 0.09

SOLUTION:

Step 1: Get the two slips from the two speeds.
The synchronous speed is $N_s = 120f/P = 120(50)/4 = 1500$ rpm. With shorted rings the speed is 1476 rpm, so

$$s_1 = \frac{1500 - 1476}{1500} = 0.016$$

After the extra 0.27 Ω per phase is added, the speed falls to 1404 rpm, so

$$s_2 = \frac{1500 - 1404}{1500} = 0.064$$

Step 2: Turn the constant torque condition into a ratio.
For a torque-slip curve close to synchronous speed, slip is proportional to rotor resistance for a fixed torque, since the reactance drop is small there. So instead of writing s/r as equal on both sides, write the ratio of the two slips directly against the ratio of the two total rotor resistances:

$$\frac{s_1}{s_2} = \frac{r_2}{r_2 + R_{ext}}$$

Step 3: Plug in the slip ratio.

$$\frac{0.016}{0.064} = \frac{r_2}{r_2 + 0.27}$$

$$0.25 = \frac{r_2}{r_2 + 0.27}$$

Step 4: Cross multiply and isolate r_2.

$$0.25(r_2 + 0.27) = r_2$$

$$0.25r_2 + 0.0675 = r_2$$

$$0.0675 = r_2 - 0.25r_2 = 0.75r_2$$

Step 5: Solve.

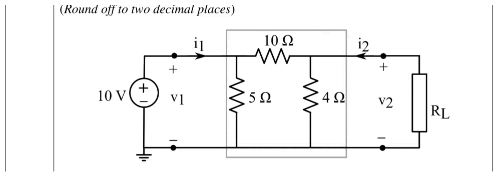
$$r_2 = \frac{0.0675}{0.75} = 0.09\ \Omega$$

$$\boxed{r_2 = 0.09\ \Omega}$$

Quick Tip: For a constant torque load near synchronous speed, slip divided by rotor resistance stays the same before and after adding external resistance.

• • •

. Consider the two-port network shown. For maximum power transfer to the resistive load (R_L), the value of R_L should be Ω
(Round off to two decimal places)



Correct Answer: 2.86

SOLUTION:

Step 1: Note what the ideal source does to the 5 Ω resistor.
The 10 V source sits directly across the 5 Ω resistor, fixing that node at 10 V regardless of current drawn. So whatever current the 5 Ω resistor pulls, it does not change the voltage feeding the rest of the network through the 10 Ω resistor. This means we can ignore the 5 Ω resistor for everything downstream of the source node.

Step 2: Find the open-circuit voltage using a simple loop.
With R_L disconnected, current only flows in the loop made of the 10 V source, the 10 Ω resistor, and the 4 Ω resistor to ground. The loop current is

$$I = \frac{10}{10 + 4} = 0.7143 \text{ A}$$

The open-circuit voltage at the output port is the drop across the 4 Ω resistor:

$$V_{th} = I \times 4 = 0.7143 \times 4 = 2.857\ \text{V}$$

Step 3: Find R_{th} by injecting a test current.

Kill the 10 V source (replace it with a short) and push a 1 A test current into the output port. The 5 Ω resistor again carries no current since both its ends sit at ground once the source is shorted. The test current splits between the 10 Ω path (to the shorted source node, which is ground) and the 4 Ω path (straight to ground), both starting from the test-current node. Writing KCL there:

$$1 = \frac{V_{test}}{10} + \frac{V_{test}}{4}$$

$$1 = V_{test}(0.1 + 0.25) = 0.35\, V_{test}$$

$$V_{test} = \frac{1}{0.35} = 2.857\ \text{V}$$

Since the test current was 1 A,

$$R_{th} = \frac{V_{test}}{I_{test}} = 2.857\ \Omega$$

Step 4: Set the load equal to R_{th}.

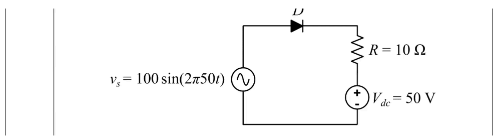
Maximum power reaches the load when $R_L = R_{th}$, so

$$\boxed{R_L = 2.86\ \Omega}$$

Quick Tip: Maximum power transfer needs the load resistance to equal the Thevenin resistance seen at the load terminals; deactivate the source and combine the remaining resistors.

• • •

. Consider the circuit shown. Assume that the diode (D) is ideal.



Given $v_s = 100\sin(2\pi 50t)$ V, $V_{dc} = 50$ V, and $R = 10\ \Omega$, the average value of the current through the diode is A (Round off to two decimal places)

Correct Answer: 1.09

SOLUTION:

Step 1: Convert the problem into the time domain.

The source is $v_s = 100\sin(100\pi t)$ V, so the angular frequency is $\omega = 100\pi$ rad/s and the period is $T = 1/50 = 0.02$ s. The diode conducts only when $v_s > 50$ V, which happens when $\sin(100\pi t) > 0.5$.

Step 2: Find the actual turn-on and turn-off times.

$\sin(100\pi t) = 0.5$ first at $100\pi t = \pi/6$, giving $t_1 = 1/600$ s, and again at $100\pi t = 5\pi/6$, giving $t_2 = 1/120$ s. Current flows only between t_1 and t_2 in every cycle.

Step 3: Write the diode current as a function of time.

$$i(t) = \frac{100\sin(100\pi t) - 50}{10} = 10\sin(100\pi t) - 5$$

Step 4: Average this current over the full period.

$$I_{avg} = \frac{1}{T}\int_{t_1}^{t_2} [10\sin(100\pi t) - 5]\,dt$$

$$= \frac{1}{0.02}\left[-\frac{10}{100\pi}\cos(100\pi t) - 5t\right]_{t_1}^{t_2}$$

Step 5: Evaluate at the two time limits.

At $t_2 = 1/120$ s: $100\pi t_2 = 5\pi/6$, so $\cos(5\pi/6) = -0.8660$, giving the bracket value $-\frac{10}{100\pi}(-0.8660) - 5(1/120) = 0.02757 - 0.04167 = -0.01410$.

At $t_1 = 1/600$ s: $100\pi t_1 = \pi/6$, so $\cos(\pi/6) = 0.8660$, giving the bracket value $-\frac{10}{100\pi}(0.8660) - 5(1/600) = -0.02757 - 0.00833 = -0.03590$.

Step 6: Subtract and scale by $1/T$.

$$-0.01410 - (-0.03590) = 0.02180$$

$$I_{avg} = \frac{0.02180}{0.02} = 1.09 \text{ A}$$

$$\boxed{I_{avg} = 1.09 \text{ A}}$$

Quick Tip: The diode only conducts while the source voltage exceeds the battery voltage; integrate the current over that conduction window and divide by the full period to get the average.

. The magnitude of the contour integral

$$\oint_C \left(\frac{(z+1)^2}{(z-i)(z-2)} \right) dz$$

over the contour $C : |z - 2 - i| = 3/2$ is (round off to two decimal places).
Note: z is a complex variable and $i = \sqrt{-1}$.

Correct Answer: 25.29

SOLUTION:

Step 1: Rewrite using Cauchy's Integral Formula.

The integral has the form $\oint_C \frac{(z+1)^2}{(z-i)(z-2)}\, dz$, with two singular points at $z = i$ and $z = 2$. Only a singularity that sits inside the contour contributes to the integral.

Step 2: Test $z = 2$ against the contour.

The contour is the circle $|z - (2+i)| = 1.5$, centered at $2 + i$. For $z = 2$, the distance to the center is $|2 - (2+i)| = |-i| = 1$, and since $1 < 1.5$, this point sits inside the circle.

Step 3: Test $z = i$ against the contour.

For $z = i$, the distance to the center is $|i - (2+i)| = |-2| = 2$, and since $2 > 1.5$, this point sits outside the circle. So the pole at $z = i$ plays no role here.

Step 4: Write $f(z)$ as $g(z)/(z-2)$ and use the formula directly.

Split off the singular factor: $g(z) = \dfrac{(z+1)^2}{z-i}$, so that $f(z) = \dfrac{g(z)}{z-2}$.
Cauchy's Integral Formula gives $\oint_C \dfrac{g(z)}{z-2}\, dz = 2\pi i\, g(2)$.

Step 5: Evaluate $g(2)$.

$g(2) = \dfrac{(2+1)^2}{2-i} = \dfrac{9}{2-i}$. Multiplying top and bottom by the conjugate $2+i$: $g(2) = \dfrac{9(2+i)}{(2-i)(2+i)} = \dfrac{9(2+i)}{5} = 3.6 + 1.8i$.

Step 6: Multiply by $2\pi i$.

$2\pi i(3.6 + 1.8i) = 2\pi(3.6i + 1.8i^2) = 2\pi(-1.8 + 3.6i) = -3.6\pi + 7.2\pi i.$

Step 7: Take the modulus directly.

Here the real part is -3.6π and the imaginary part is 7.2π, so the modulus is $\pi\sqrt{3.6^2 + 7.2^2} = \pi\sqrt{12.96 + 51.84} = \pi\sqrt{64.8}$.

Step 8: Compute the number.

$\sqrt{64.8} \approx 8.0499$, so the modulus is about $\pi \times 8.0499 \approx 25.29$.

$$\boxed{25.29}$$

Quick Tip: Check which poles lie inside the given contour before applying the residue theorem.

• • •

. A uniform spherical volume charge distribution of radius 2 m, centered at the origin, has a strength of $\frac{3}{\pi} \times 10^{-6}$ C/m^3. A point charge of strength $\pi \times 8.854 \times 10^{-12}$ C is moved from $(-3, 0, -4)$ to $(0, 0, 4)$ in Cartesian coordinate system. The relative permittivity of the medium is 1 and the coordinate values are in meters.
The work done during the process is μJ (round off to two decimal places).

Correct Answer: 0.40

SOLUTION:

Step 1: Get the total charge on the sphere.

The charge density is $\rho = \dfrac{3}{\pi} \times 10^{-6}\ \mathrm{C/m^3}$ over a sphere of radius 2 m, so the total charge is

$Q = \rho \cdot \dfrac{4}{3}\pi(2)^3 = \dfrac{3}{\pi} \times 10^{-6} \times \dfrac{32\pi}{3} = 32 \times 10^{-6}$ C.

Step 2: Check both endpoints sit outside the sphere.

$(-3, 0, -4)$ is at distance $r_1 = \sqrt{9+16} = 5$ m from the origin, and $(0, 0, 4)$ is at distance $r_2 = 4$ m. Since the sphere only extends to 2 m, both points are in the region outside the charge, where the sphere behaves exactly like a point charge Q at the origin.

Step 3: Write down Coulomb's constant.

With relative permittivity 1, use $k = \dfrac{1}{4\pi\varepsilon_0} \approx 8.988 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}$. The potential at a distance r from the sphere (outside it) is $V(r) = \dfrac{kQ}{r}$.

Step 4: Compute both potentials numerically.

$$V_1 = \frac{kQ}{r_1} = \frac{8.988 \times 10^9 \times 32 \times 10^{-6}}{5} \approx 57520\ \mathrm{V}.$$
$$V_2 = \frac{kQ}{r_2} = \frac{8.988 \times 10^9 \times 32 \times 10^{-6}}{4} \approx 71900\ \mathrm{V}.$$

Step 5: Find the potential difference.

$\Delta V = V_2 - V_1 \approx 71900 - 57520 = 14380$ V.

Step 6: Bring in the moving charge and multiply.

The moving charge is $q = \pi \times 8.854 \times 10^{-12} \approx 2.7818 \times 10^{-11}$ C. The work done is $W = q\,\Delta V$.

$W \approx 2.7818 \times 10^{-11} \times 14380 \approx 4.00 \times 10^{-7}$ J.

Step 7: Convert to microjoules.

4.00×10^{-7} J is the same as 0.40 μJ.

$$\boxed{0.40\ \mu\text{J}}$$

Quick Tip: Since both points lie outside the charged sphere, treat it as a point charge equal to the total enclosed charge.

• • •

. Let X and Y be two real-valued random variables with $E(X) = 1$, $E(Y) = 2$, $E(X^2) = 4$, $E(Y^2) = 9$, and $E(XY) = 0.9$, where E denotes the expectation operator.

The value of α that minimizes $E((X - \alpha Y)^2)$ is (round off to one decimal place).

Correct Answer: 0.1

SOLUTION:

Step 1: Use the orthogonality principle.

When αY is the best linear estimate of X in the least mean square sense, the estimation error $X - \alpha Y$ must be uncorrelated with Y. This gives

$E\Big[(X - \alpha Y)Y\Big] = 0.$

Step 2: Expand this condition.

$E(XY) - \alpha E(Y^2) = 0$

Step 3: Solve for α.

$$\alpha = \frac{E(XY)}{E(Y^2)}$$

Step 4: Plug in the known values.

$E(XY) = 0.9$ and $E(Y^2) = 9$, so

$\alpha = \dfrac{0.9}{9} = 0.1$

Step 5: Note which quantities were not needed.

The means $E(X) = 1$ and $E(Y) = 2$ do not enter this formula at all, since minimizing $E((X - \alpha Y)^2)$ only involves the second moments $E(X^2)$, $E(Y^2)$ and $E(XY)$, and here $E(X^2)$ also drops out because it does not contain α.

$$\boxed{0.1}$$

Quick Tip: Differentiate the mean square error expression with respect to alpha and set it to zero.

• • •

. The integral

$$\frac{1}{\pi}\int_0^\infty \frac{x^{2026}}{(1+x^{2026})(1+x^2)}\,dx$$

evaluates to (round off to two decimal places).

Correct Answer: 0.25

SOLUTION:

Step 1: Rewrite the integrand before doing anything else.

Notice that $\dfrac{x^{2026}}{1+x^{2026}} = 1 - \dfrac{1}{1+x^{2026}}$. So the integral becomes

$I = \displaystyle\int_0^\infty \left(1 - \frac{1}{1+x^{2026}}\right)\frac{1}{1+x^2}\,dx = \int_0^\infty \frac{dx}{1+x^2}$
$- \displaystyle\int_0^\infty \frac{dx}{(1+x^{2026})(1+x^2)}$.

Step 2: Evaluate the first piece.

$\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$, a standard arctangent result. Call the second piece J, so $I = \frac{\pi}{2} - J$.

Step 3: Show J equals I by substituting $x = 1/t$ directly into J.

$J = \int_0^\infty \frac{dx}{(1+x^{2026})(1+x^2)}$. Put $x = 1/t$, $dx = -dt/t^2$. Then $1 + x^{2026} = \frac{t^{2026}+1}{t^{2026}}$ and $1 + x^2 = \frac{t^2+1}{t^2}$, so the integrand becomes $\frac{t^{2026}}{t^{2026}+1} \cdot \frac{t^2}{t^2+1} \cdot \frac{1}{t^2} = \frac{t^{2026}}{(1+t^{2026})(1+t^2)}$.

This is exactly the original expression for I with t in place of x, so $J = I$.

Step 4: Combine the two facts.

We now have $I = \frac{\pi}{2} - J$ and $J = I$, so

$I = \frac{\pi}{2} - I \implies 2I = \frac{\pi}{2} \implies I = \frac{\pi}{4}$.

Step 5: Divide by π.

$\frac{I}{\pi} = \frac{\pi/4}{\pi} = \frac{1}{4} = 0.25$

$$\boxed{0.25}$$

Quick Tip: Try the substitution x = 1/t to relate the integral to itself.