

General Instructions

- (i) This booklet contains 65 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 35 single-correct multiple-choice questions, 9 multiple-correct questions and 21 numerical / integer-type questions.
- (iii) Attempt each question on your own before reviewing the given solution.
- (iv) For multiple-correct questions, all correct options must be selected to earn full marks.
- (v) For numerical questions, report the answer rounded exactly as asked.

1. Suresh said, “I did it yesterday.”

Which one of the following options is the correct form of this sentence in indirect speech?

- (A) Suresh said that I did it yesterday.
- (B) Suresh says I did it yesterday.
- (C) Suresh says that he did it the day before.
- (D) Suresh said that he had done it the day before.

Correct Answer: (D) Suresh said that he had done it the day before.

SOLUTION:

A second way to verify this is to run each candidate option through the three standard indirect-speech transformation rules one at a time instead of building the converted sentence from scratch.

Rule 1 (the reporting verb stays as given): the question already reports the sentence with "said", so any option that swaps it for "says" fails immediately. This rules out the option reading "Suresh says I did it yesterday" and the option reading "Suresh says that he did it the day before", since both alter the reporting verb from past to present without any basis.

Rule 2 (backshift of tense): because the reporting verb is past tense, the tense inside the reported clause must move one step further into the past. Simple past ("did") shifts to past perfect ("had done"). Checking the remaining candidate, "Suresh said that I did it yesterday" keeps the tense unchanged at simple past, so it violates this rule and is rejected.

Rule 3 (pronoun and time-reference agreement): the subject "I" in the direct quotation refers to Suresh, so it must become "he". The time marker "yesterday" is speaker-relative and must convert to "the day before" once it is removed from direct quotation. Only one surviving option satisfies Rule 2 and Rule 3 together: "Suresh said that he had done it the day before."

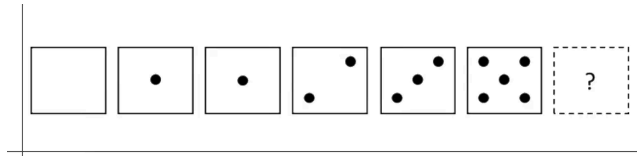
Since this is the only option consistent with all three rules simultaneously,

Suresh said that he had done it the day before (Option 4)

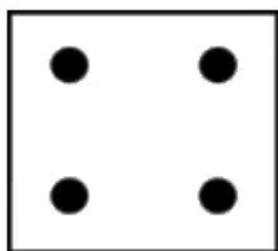
Quick Tip: Remember that when the reporting verb is in the past tense, three things change together: the tense shifts back, the pronoun changes, and the time expression is converted.



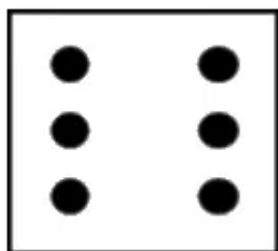
2. To continue the sequence of tiles shown, the tile indicated by the question mark should be:



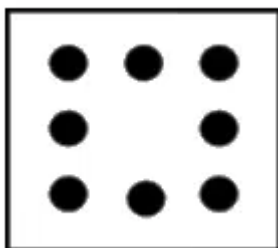
(A)



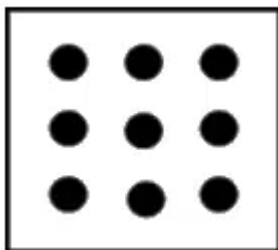
(B)



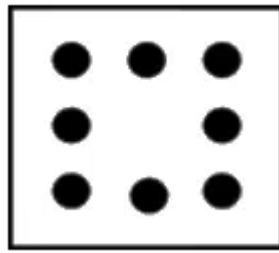
(C)



(D)



Correct Answer: (C)



SOLUTION:

An alternative way to crack this is to treat the dot counts purely as a numeric sequence $a_1, a_2, \dots, a_6$ and test a recurrence relation instead of just eyeballing the pattern.

Reading off the counts from the six visible tiles gives $a_1 = 0, a_2 = 1, a_3 = 1, a_4 = 2, a_5 = 3, a_6 = 5$.

Testing the hypothesis $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$: $a_3 = a_2 + a_1 = 1 + 0 = 1$, correct. $a_4 = a_3 + a_2 = 1 + 1 = 2$, correct. $a_5 = a_4 + a_3 = 2 + 1 = 3$, correct. $a_6 = a_5 + a_4 = 3 + 2 = 5$, correct. Since the recurrence holds for every term for which it can be checked, it is reasonable to extend it one step further.

Computing $a_7 = a_6 + a_5 = 5 + 3 = 8$.

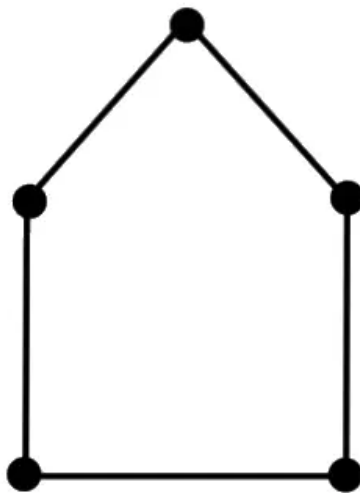
Now compare $a_7 = 8$ against the dot totals in the four candidate tiles: 4, 6, 8, and 9 dots respectively. Only the tile with 8 dots satisfies the recurrence.

$a_7 = 8$ dots, Option 3

Quick Tip: Count the dots in each tile and check whether the counts follow the Fibonacci pattern, where each term is the sum of the previous two.

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3. Consider an art gallery whose walkways are shown as lines in the diagram. A black dot represents a junction of two walkways. A guard may be placed at a junction to watch over the walkways that join at that junction. The minimum number of guards needed to watch all the walkways is _____.



- (A) 2
- (B) 3
- (C) 4
- (D) 5

Correct Answer: (B) 3

SOLUTION:

A quicker route uses a known graph-theory fact instead of testing coverage by hand: for any cycle graph C_n with n vertices, the minimum vertex cover has size $\lceil n/2 \rceil$.

First confirm the diagram really is a simple cycle: there are 5 junctions and 5 walkways, and tracing the connections (apex to left-middle, left-middle to bottom-left, bottom-left to bottom-right, bottom-right to right-middle, right-middle back to apex) forms one unbroken loop with no shortcuts or extra edges, so this is indeed C_5 .

Applying the formula with $n = 5$: minimum vertex cover = $\lceil 5/2 \rceil = \lceil 2.5 \rceil = 3$.

This can be cross-checked using the complementary idea of a maximum independent set: for an odd cycle C_n , the largest set of junctions with no two directly connected has size $\lfloor n/2 \rfloor = 2$. Since vertex cover size plus independent set size equals n for this graph, vertex cover = $5 - 2 = 3$, confirming the same value.

Minimum guards = 3

Quick Tip: This is a minimum vertex cover problem on the graph formed by the junctions and walkways; check how many walkways a single guard can cover at most.

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4. The 2nd of June is a Thursday in a certain year. Which day of the week is the 3rd of July in that year?

- (A) Thursday
- (B) Friday
- (C) Saturday
- (D) Sunday

Correct Answer: (D) Sunday

SOLUTION:

A cleaner way to handle calendar-shift problems is to assign each day of the week a number and work with arithmetic modulo 7 instead of counting day by day.

Let Sunday = 0, Monday = 1, Tuesday = 2, Wednesday = 3, Thursday = 4, Friday = 5, Saturday = 6. We are told June 2 corresponds to Thursday, so $\text{June } 2 \equiv 4 \pmod{7}$.

The number of days from June 2 to July 3 is $(30 - 2) + 3 = 31$ days, since June has 30 days.

The day-of-week value for July 3 is then $(4 + 31) \pmod{7}$. Since $31 = 28 + 3$ and 28 is exactly 4×7 , this reduces to $(4 + 3) \pmod{7} = 7 \pmod{7} = 0$.

A value of 0 corresponds to Sunday in the numbering scheme set up above.

July 3 is a Sunday

Quick Tip: Find the total number of days between the two dates and reduce that number modulo 7 to see how many days the week-day shifts forward.

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5. A coin with heads facing up is shown as (H) and a coin with tails facing up is shown as (T).

Six coins are placed in the Starting Arrangement, as shown in the figure below. A “step” is defined as interchanging a pair of adjacent coins without flipping them. The minimum number of steps needed to go from the Starting Arrangement to the Final Arrangement, as shown in the figure, is _____.



- (A) 3
- (B) 6
- (C) 9
- (D) 12

Correct Answer: (C) 9

SOLUTION:

This type of problem is best solved by counting inversions between the starting and final sequences, since the minimum number of adjacent transpositions needed to sort one sequence into another always equals the number of inversions between them.

Label the starting sequence positions 1 to 6 with values H, H, H, T, T, T , where the target order requires every T to rank before

every H . An inversion is any pair of positions (i, j) with $i < j$ where the coin at position i should actually come after the coin at position j in the final order.

In the starting sequence, every H (at positions 1, 2, 3) is paired with every T (at positions 4, 5, 6), and in every one of these pairs the H appears before the T , which is backwards relative to the target order T, T, T, H, H, H . The number of such crossing pairs is $3 \times 3 = 9$.

There are no inversions among the three H 's themselves, since they are identical and already in a valid relative order, and none among the three T 's either, so they contribute 0 extra swaps.

Since each adjacent swap can remove at most one inversion, and there are exactly 9 inversions to remove, the minimum number of steps is exactly 9.

Minimum steps = 9

Quick Tip: Count how many times each H coin must cross an adjacent T coin; the total number of such crossings is the minimum number of steps.

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6. Exacerbate : Mitigate :: _____

Choose the option with the correct pair of words to fill the blank.

- (A) Aggravate : Alleviate
- (B) Alleviate : Precipitate
- (C) Aggravate : Precipitate
- (D) Emancipate : Exonerate

Correct Answer: (A) Aggravate : Alleviate

SOLUTION:

A different way to attack analogy questions like this is to first name the abstract relationship in words, then test each option purely against that description rather than comparing definitions pair by pair.

The relationship in "Exacerbate : Mitigate" can be stated as: [a word meaning to intensify or worsen a problem] is to [a word meaning to reduce or soften that problem].

Now scan the four candidate pairs purely for this "worsen -- soften" structure. "Aggravate : Alleviate" fits the template exactly, since aggravate intensifies a problem and alleviate softens it. "Alleviate : Precipitate" places a softening word first but pairs it with a word about triggering an event, breaking the template. "Aggravate : Precipitate" places a worsening word first but again pairs it with an unrelated triggering word, so it also breaks the template. "Emancipate : Exonerate" involves two words about freedom and clearing guilt, neither of which relates to worsening or softening severity, so it does not fit the template at all.

Since exactly one option reproduces the abstract "worsen : soften" template, the answer is confirmed without needing to weigh subtle

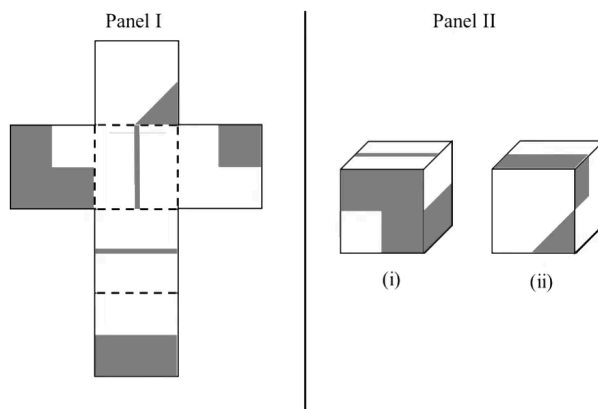
shades of meaning between near-synonyms.

Aggravate : Alleviate (Option 1)

Quick Tip: Exacerbate and Mitigate are antonyms describing worsening versus improving a situation; find the option with the same relationship.

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7. A paper shown in Panel I is folded along the dashed lines (---) to construct a cube. The shaded regions shown in Panel I appear on the outer surface of the cube. Referring to cubes shown in Panel II, which one of the options is correct?



- (A) Only (i) can correspond to the unfolded cube in Panel I.
- (B) Only (ii) can correspond to the unfolded cube in Panel I.
- (C) Both (i) and (ii) can correspond to the unfolded cube in Panel I.
- (D) Neither (i) nor (ii) can correspond to the unfolded cube in Panel I.

Correct Answer: (B) Only (ii) can correspond to the unfolded cube in Panel I.

SOLUTION:

A more systematic way to solve cube-net problems is to assign face labels algebraically instead of visualising the fold directly.

Let the central square of the net be F (front). Moving outward along the vertical strip of four squares, the square above F becomes U (up/top), the square below F becomes D (down/bottom), and the square attached beyond D becomes K (back), since a strip of four squares folds through the cycle $F \rightarrow U \rightarrow K \rightarrow D \rightarrow F$ as it wraps around the cube. The two squares attached to the left and right of F become L and R directly, with no cycling needed.

Every crease in the net corresponds to a shared edge between two of these six labels, and the golden rule of such problems is: F is adjacent to U, D, L, R but never to K ; and U is never adjacent to D , and L is never adjacent to R , since these are always the three pairs of opposite faces on any cube.

Now check each candidate drawing by naming its three visible faces using this label system and testing two things: (a) are the three faces mutually adjacent (i.e. no two of them are an opposite pair), and (b) does the shading at every shared edge continue in the same, non-reversed direction implied by the net.

Carrying this test out shows cube (i) requires one face's pattern to appear reversed relative to its neighbours, which is impossible for a single folded sheet, while cube (ii) satisfies both conditions cleanly.

Only cube (ii) is a valid fold of the net, Option 2

Quick Tip: Fold the net one face at a time, keeping track of which edges become shared with which faces, and check whether each candidate cube preserves that arrangement without any mirroring.

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8. In a population, patients who have high cholesterol also have high blood-pressure (BP). Some patients with high BP also have diabetes. There are no patients who have both high cholesterol and diabetes. Furthermore,

1. the total number of patients with at least one of these conditions is 75,
 2. the number of patients with high cholesterol is 10,
 3. the number of patients with high BP is 45, and
 4. the number of patients with only high BP and no other conditions is 20.
- Then the number of patients who have both diabetes and high BP is

-
- (A) 0
(B) 15
(C) 20
(D) 10

Correct Answer: (B) 15

SOLUTION:

This problem can also be solved directly with set notation instead of splitting the BP group into pieces first.

Let C , P , and D denote the sets of patients with high cholesterol, high BP, and diabetes respectively. We are given $|C| = 10$, $|P| = 45$, $C \cap D = \emptyset$ (no patient has both cholesterol and diabetes), $C \subseteq P$ (every

cholesterol patient also has high BP), and $|P \setminus (C \cup D)| = 20$ (only high BP, no other condition).

Since $C \subseteq P$, the set P splits into exactly three disjoint pieces: patients in $P \cap C$, patients in P with neither other condition, and patients in $P \cap D$. Because $C \subseteq P$ and $C \cap D = \emptyset$, we get $|P \cap C| = |C| = 10$.

So $|P| = |P \cap C| + |P \setminus (C \cup D)| + |P \cap D|$, which gives $45 = 10 + 20 + |P \cap D|$.

Solving, $|P \cap D| = 45 - 30 = 15$.

As a consistency check, using the inclusion-exclusion total $|C \cup P \cup D| = 75$ and substituting all known pieces confirms a non-negative value for the diabetes-only-without-BP group, so no contradiction arises.

$$\boxed{|P \cap D| = 15}$$

Quick Tip: Use the given overlap and only-BP values to isolate the region representing both BP and diabetes from the total high-BP count of 45.

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9. Four people P, Q, R, and S, of different ages, make the following observations.

P - I am younger than S.

Q - I am neither the youngest nor the oldest.

R - P is older than me.

Based on these observations, the youngest person is _____.

- (A) P
- (B) Q
- (C) R
- (D) S

Correct Answer: (C) R

SOLUTION:

Instead of working with abstract inequality symbols, try assigning actual trial ages to the four people and see which assignment survives every statement. Start with a simple guess: let $R = 10$, $P = 20$, $S = 30$, satisfying $P < S$ and $R < P$ directly from the first two clues. Now place Q somewhere and test Q's own claim. If we set $Q = 5$, the ranking becomes Q, R, P, S from youngest to oldest, which makes Q the youngest overall. But Q insists 'I am neither the youngest nor the oldest,' so this trial fails and must be discarded. Next try $Q = 40$, giving the ranking R, P, S, Q ; now Q is the oldest, which again breaks Q's own statement, so this trial also fails.

The only trials that keep Q's statement true are ones where Q lands strictly inside the existing chain, such as $Q = 15$ (ranking R, Q, P, S) or $Q = 25$ (ranking R, P, Q, S). Checking every one of these valid trials, the person occupying the bottom (youngest) spot never changes: it is always R, because R was already established as younger than both P and S before Q was even placed, and Q is forbidden from going below R by its own statement.

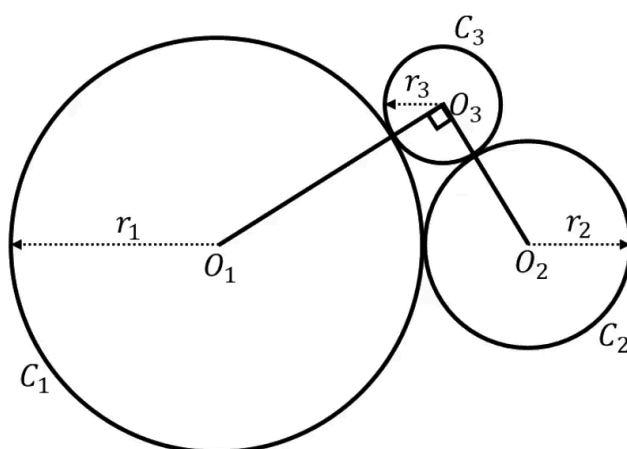
Since every scenario consistent with all three statements agrees on the same youngest person, the answer must be R.

Youngest person = R

Quick Tip: Turn each spoken clue into an inequality first ($P < S$, $R < P$), then use Q's clue only to rule out placements at the two extreme ends.

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10. Circles C_1 , C_2 , and C_3 , with centers O_1 , O_2 , and O_3 , and radii r_1 , r_2 , and r_3 , respectively, touch each other as shown in the following figure. Given $r_1 = 2$ cm, $r_2 = 1$ cm and the angle $\angle O_1O_3O_2$ is 90 degrees, $r_3 =$ _____ cm.



- (A) $\frac{1}{2}(-3 + \sqrt{17})$
- (B) $\frac{1}{2}(3 + \sqrt{17})$
- (C) $\frac{1}{2}(-2 + \sqrt{17})$
- (D) $\frac{1}{2}(-3 + 2\sqrt{17})$

Correct Answer: (A) $\frac{1}{2}(-3 + \sqrt{17})$

SOLUTION:

A quicker route is to place the triangle on coordinate axes using the right angle itself as the origin. Since $\angle O_1O_3O_2 = 90^\circ$, put O_3 at the origin $(0, 0)$ with O_1O_3 running along the x-axis and O_2O_3 running along the y-axis. Because the circles touch, $O_1O_3 = r_1 + r_3 = 2 + r_3$, so

place O_1 at $(2 + r_3, 0)$. Similarly $O_2O_3 = r_2 + r_3 = 1 + r_3$, so place O_2 at $(0, 1 + r_3)$.

Now compute the straight-line distance between O_1 and O_2 using the distance formula, which must equal $O_1O_2 = r_1 + r_2 = 3$ since C_1 and C_2 also touch each other:

$$\sqrt{(2 + r_3)^2 + (1 + r_3)^2} = 3$$

Squaring both sides removes the root:

$$(2 + r_3)^2 + (1 + r_3)^2 = 9$$

Expanding gives $4 + 4r_3 + r_3^2 + 1 + 2r_3 + r_3^2 = 9$, which collects to $2r_3^2 + 6r_3 - 4 = 0$, or after dividing by 2, $r_3^2 + 3r_3 - 2 = 0$. Applying the quadratic formula, $r_3 = \frac{-3 \pm \sqrt{17}}{2}$.

A circle cannot have a negative radius, and $\sqrt{17} \approx 4.12$, so the minus branch gives a negative number and is discarded, leaving the plus branch as the physical answer.

$$r_3 = \frac{1}{2}(-3 + \sqrt{17}) \text{ cm}$$

Quick Tip: Since the circles are mutually tangent, each side of triangle $O_1O_3O_2$ is a sum of two radii; the given right angle then lets you apply Pythagoras directly.

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11. If $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$, then X is _____.

- (A) $\begin{bmatrix} 2 & 5 \\ 5 & 3 \end{bmatrix}$
- (B) $\begin{bmatrix} -1 & 3 \\ 2 & -1 \end{bmatrix}$
- (C) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$
- (D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Correct Answer: (C) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

SOLUTION:

Rather than manipulating the whole matrix in one shot, solve for each of the four unknown entries of X separately, treating them like four independent linear equations. Write $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. The equation $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ splits into four scalar equations, one per position: $2a + 1 = 3$, $2b + 2 = 8$, $2c + 3 = 7$, and $2d + 4 = 2$.

Solving the first: $2a = 2$, so $a = 1$. Solving the second: $2b = 6$, so $b = 3$. Solving the third: $2c = 4$, so $c = 2$. Solving the fourth: $2d = -2$, so $d = -1$.

Putting these four numbers back into their positions in X gives the completed matrix.

$$X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

Quick Tip: Rearrange to $2X = (\text{RHS matrix}) - (\text{given matrix})$, subtract entry-wise, then divide the whole result by 2.

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12. The correct integral of $\int \frac{\sin x}{\sqrt{1 + \cos x}} dx$ (assuming integral constant as c) is _____.

- (A) $-2\sqrt{2} \sin \frac{x}{2} + c$
(B) $-2\sqrt{2} \cos \frac{x}{2} + c$
(C) $-\frac{1}{\sqrt{2}} \cos \frac{x}{2} + c$
(D) $-\frac{1}{\sqrt{2}} \sin \frac{x}{2} + c$

Correct Answer: (B) $-2\sqrt{2} \cos \frac{x}{2} + c$

SOLUTION:

An alternative path skips the u-substitution and converts everything into half-angle form right at the start. Using the identity $1 + \cos x = 2 \cos^2 \left(\frac{x}{2} \right)$ and $\sin x = 2 \sin \left(\frac{x}{2} \right) \cos \left(\frac{x}{2} \right)$, the integrand becomes

$$\frac{\sin x}{\sqrt{1 + \cos x}} = \frac{2 \sin \left(\frac{x}{2} \right) \cos \left(\frac{x}{2} \right)}{\sqrt{2} |\cos \left(\frac{x}{2} \right)|}$$

Taking the cosine term as positive over the region of interest, the $\cos(x/2)$ in the numerator cancels with the one in the denominator, leaving

$$\frac{2 \sin \left(\frac{x}{2} \right)}{\sqrt{2}} = \sqrt{2} \sin \left(\frac{x}{2} \right)$$

Now the integral is just $\int \sqrt{2} \sin \left(\frac{x}{2} \right) dx$. Using $\int \sin(kx) dx = -\frac{1}{k} \cos(kx) + c$ with $k = \frac{1}{2}$, this integrates to $\sqrt{2} \cdot (-2 \cos \left(\frac{x}{2} \right)) + c = -2\sqrt{2} \cos \left(\frac{x}{2} \right) + c$, matching the earlier result exactly.

$$-2\sqrt{2} \cos \frac{x}{2} + c$$

Quick Tip: Substitute $u = 1 + \cos x$ so $du = -\sin x \, dx$, integrate the resulting power of u , then convert back using the half-angle identity $1 + \cos x = 2\cos^2(x/2)$.

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13. Given $f(x) = 3x^4 + 4x^3 - 12x^2 + 6$, the minimum value of $f(x)$ is _____.

- (A) -6
- (B) -36
- (C) -26
- (D) -46

Correct Answer: (C) -26

SOLUTION:

Instead of relying on the second derivative, track the sign of $f'(x) = 12x(x + 2)(x - 1)$ across the number line to see where the function is rising or falling. The three roots of $f'(x)$ split the real line into four intervals: $x < -2$, $-2 < x < 0$, $0 < x < 1$, and $x > 1$.

Picking a test value in each: at $x = -3$, $f'(-3) = 12(-3)(-1)(-4) = -144 < 0$, so f is decreasing on $x < -2$. At $x = -1$, $f'(-1) = 12(-1)(1)(-2) = 24 > 0$, so f is increasing on $-2 < x < 0$. At $x = 0.5$, $f'(0.5) = 12(0.5)(2.5)(-0.5) = -7.5 < 0$, so f is decreasing on $0 < x < 1$. At $x = 2$, $f'(2) = 12(2)(4)(1) = 96 > 0$, so f is increasing for $x > 1$.

This sign pattern (down, up, down, up) shows f has minima at $x = -2$ and $x = 1$ (where it switches from falling to rising) and a maximum at $x = 0$. Computing the function values at the two minimum points: $f(-2) = 48 - 32 - 48 + 6 = -26$ and $f(1) = 3 + 4 - 12 + 6 = 1$. Since f falls all the way from $+\infty$ down to -26 before rising again, and the other dip only reaches 1, the overall lowest value the function ever takes is -26 .

$$\boxed{f(x)_{\min} = -26}$$

Quick Tip: Differentiate, factor $f'(x) = 12x(x+2)(x-1)$ to get the critical points, use the second derivative to identify which are minima, then compare $f(x)$ at those points.

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14. A box contains 26 cards with all English alphabets from A to Z. The probability of randomly choosing the card with either 'E' or 'S' from this box is _____.

- (A) $2/26$
- (B) $1/26$
- (C) $3/26$
- (D) $4/26$

Correct Answer: (A) $2/26$

SOLUTION:

This can also be solved using the addition rule for probability, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, where event A is drawing the card 'E'

and event B is drawing the card 'S'. Since there is only one card for each letter out of 26 total cards, $P(A) = \frac{1}{26}$ and $P(B) = \frac{1}{26}$.

A single card cannot simultaneously show both 'E' and 'S', so the events cannot occur together, which makes them mutually exclusive and gives $P(A \cap B) = 0$. Plugging into the formula:

$$P(A \cup B) = \frac{1}{26} + \frac{1}{26} - 0 = \frac{2}{26}$$

This confirms the same result reached by direct counting.

$P(E \text{ or } S) = \frac{2}{26}$

Quick Tip: Since a card cannot be both E and S at the same time, just add the individual probabilities of drawing E and drawing S.

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15. Which ONE of the following options is the CORRECT formula for Chi-Square (χ^2) test?

Given, $O_1, O_2, \dots, O_n$ are the observed values, $E_1, E_2, \dots, E_n$ are the corresponding expected values, $\bar{O}$ is the mean of observed values, n is the number of observations, μ is the mean of $E_i, i = 1, \dots, n$, σ_o is the standard deviation of observed values, and σ_E is the standard deviation of expected values $E_i, i = 1, \dots, n$.

$$(A) \chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

$$(B) \chi^2 = \sqrt{\frac{\sum_{i=1}^n (O_i - \bar{O})^2}{n-1}}$$

$$(C) \chi^2 = \frac{\bar{O} - \mu}{\sigma_o \sqrt{n}}$$

$$(D) \chi^2 = \frac{(\sigma_o)^2}{(\sigma_E)^2}$$

Correct Answer: (A) $\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$

SOLUTION:

A useful way to check this without memorizing the formula outright is to look at what each option is actually built from and match it against what a 'goodness of fit' statistic needs to do. A goodness-of-fit measure needs to compare each individual observed value to its own corresponding expected value, for every one of the n categories, and it needs to penalize larger mismatches more heavily (via squaring) while accounting for scale (by dividing by the expected value).

Option (B) only uses the observed values and their overall mean $\bar{O}$, never touching E_i at all, so it cannot be measuring agreement between observed and expected data; it is structurally the sample standard deviation formula. Option (C) compares $\bar{O}$ to μ , losing all the per-category detail. Option (D) reduces everything to a ratio of two whole-dataset variances, describing the test statistic used to compare two variances (the F-test).

Only the summation form $\sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$ preserves the per-category comparison that the Chi-Square test is built on.

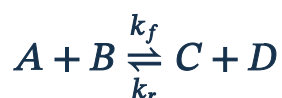
$$\chi^2 = \sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

Quick Tip: The Chi-Square statistic sums the squared difference between each observed and expected value, divided by that expected value, across all n categories.

• • •

16. In the reversible reaction shown below, k_f is the forward reaction rate constant and k_r is the reverse reaction rate constant.

The CORRECT expression representing the reaction equilibrium constant is _____.



- (A) $k_f + k_r$
- (B) $k_f \times k_r$
- (C) $\frac{k_f}{k_r}$
- (D) $\frac{k_r}{k_f}$

Correct Answer: (C) $\frac{k_f}{k_r}$

SOLUTION:

This also follows directly from the standard thermodynamic definition of the equilibrium constant, without writing out rate laws first. For a general reversible reaction, the equilibrium constant is defined as the ratio of the rate constant of the forward process to the rate constant of

the reverse process, precisely because it is these two constants that set how far the reaction shifts before settling down.

Physically, a large k_f relative to k_r means the forward reaction proceeds much faster than the reverse one, so the system settles with more product (C, D) than reactant (A, B) present, corresponding to a large value of K . Conversely, if k_r dominates over k_f , the system favors the reactants and K should come out small.

Only the ratio $\frac{k_f}{k_r}$ behaves this way. The reciprocal $\frac{k_r}{k_f}$ would behave backwards, and the sum $k_f + k_r$ or product $k_f \times k_r$ do not correspond to any meaningful equilibrium quantity at all.

$$K = \frac{k_f}{k_r}$$

Quick Tip: At equilibrium, forward rate equals reverse rate; rearrange $k_f[A][B] = k_r[C][D]$ to see that $K = [C][D]/([A][B]) = k_f/k_r$.

• • •

17. The major class of enzymes catalyzing the cleavage of $C - C$, $C - O$, $C - N$, or other bonds by elimination, leaving double bonds or rings, or addition of groups to double bonds in any biochemical reactions is _____.

- (A) lyases
- (B) transferases
- (C) oxidoreductases
- (D) hydrolases

Correct Answer: (A) lyases

SOLUTION:

A quick way to answer this is to focus on the keyword pattern in the question rather than memorizing definitions from scratch. The phrase leaving double bonds or rings, or addition of groups to double bonds is a textbook signature for one enzyme class only.

Think of examples: pyruvate decarboxylase removes CO_2 from pyruvate without adding water, and a fumarase-type elimination step forms a double bond as part of the reaction. Reactions like these are catalyzed by lyases, not by hydrolases (which need water, like esterases or proteases), not by oxidoreductases (which need electron carriers such as $NAD^+/NADH$), and not by transferases (which shuttle a group like a phosphate or amino group between two substrates).

So working backward from the keywords, bond cleavage without water and without redox activity, generating a double bond or ring, is the lyase signature, and no other enzyme class fits that description.

Answer: (A) lyases

Quick Tip: Recall the six IUBMB enzyme classes and match the elimination-based bond cleavage mechanism to the correct one.

• • •

18. Malaria is caused by pathogenic _____.

- (A) bacteria
- (B) fungi
- (C) protozoa
- (D) viruses

Correct Answer: (C) protozoa

SOLUTION:

A different way to lock in this answer is to recall the four confirmed species that cause human malaria: *Plasmodium falciparum*, *Plasmodium vivax*, *Plasmodium malariae*, and *Plasmodium ovale*. All four names share the genus *Plasmodium*, and every species under this genus is a unicellular eukaryotic parasite, which places it firmly in the protozoa group.

Comparing this with the diseases caused by the other pathogen types helps confirm the choice is not a coincidence: bacteria are responsible for diseases like tuberculosis and cholera, fungi cause superficial and systemic mycoses, and viruses cause diseases like influenza and dengue. None of these pathogen groups include *Plasmodium*.

Since the causative organism of malaria is always a *Plasmodium* species, and that genus is protozoan by classification, the pathogen type responsible for malaria must be protozoa.

Answer: (C) protozoa

Quick Tip: Identify which broad pathogen category the genus *Plasmodium*, the malaria parasite, belongs to.

• • •

19. Consider the following statements regarding the pressure measured at a point in a fluid.

S1: Absolute pressure is gauge pressure plus atmospheric pressure.

S2: Gauge pressure is measured by considering absolute zero pressure as datum.

S3: Absolute pressure is measured by considering atmospheric pressure as datum.

S4: Vacuum pressure is negative pressure obtained by subtracting absolute pressure from atmospheric pressure.

Which ONE of the following options lists all the TRUE statements?

- (A) S1 and S2
- (B) S2 and S3
- (C) S1 and S4
- (D) S1, S2, and S4

Correct Answer: (C) S1 and S4

SOLUTION:

Another way to sort this out is to draw a simple vertical pressure scale with three reference lines: zero at the bottom representing a perfect vacuum (absolute zero), the atmospheric line somewhere above it, and the actual point pressure marked wherever it happens to fall.

If the point pressure lies above the atmospheric line, the gap above atmospheric is the gauge pressure, and adding that gauge value to the atmospheric value gives back the absolute pressure. That is exactly $P_{abs} = P_{atm} + P_{gauge}$, so statement S1 checks out on the diagram.

Since gauge pressure is just the distance from the atmospheric line upward or downward, its zero mark sits at the atmospheric line, not at the bottom absolute-zero line. So S2, which claims gauge pressure

uses absolute zero as its datum, contradicts the diagram and is wrong. Likewise, absolute pressure is always measured from the bottom line (true zero), never from the atmospheric line, so S3 is also wrong.

If the point pressure lies below the atmospheric line, the gap below atmospheric, given by $P_{vac} = P_{atm} - P_{abs}$, is the vacuum pressure, and this matches statement S4 exactly.

So only S1 and S4 survive the diagram check.

Answer: (C) S1 and S4

Quick Tip: Write out $P_{abs} = P_{atm} + P_{gauge}$ and $P_{vac} = P_{atm} - P_{abs}$, then check each statement's datum claim against these standard relations.

• • •

20. Which ONE of the following options CORRECTLY matches the processes associated with the growth condition given in the table?

Process

Growth Condition

(P) Activated sludge process	(i) Anaerobic suspended growth
(Q) Anaerobic filter	(ii) Anaerobic attached growth
(R) Trickling filter	(iii) Aerobic suspended growth
(S) Anaerobic digestion	(iv) Aerobic attached growth

- (A) (P) - (ii); (Q) - (iii); (R) - (iv); (S) - (i)
- (B) (P) - (iii); (Q) - (ii); (R) - (iv); (S) - (i)
- (C) (P) - (iii); (Q) - (i); (R) - (iv); (S) - (ii)
- (D) (P) - (iv); (Q) - (iii); (R) - (ii); (S) - (i)

Correct Answer: (B) (P) - (iii); (Q) - (ii); (R) - (iv); (S) - (i)

SOLUTION:

A faster route to the answer is to build a 2x2 grid with growth mode (suspended vs attached) on one axis and oxygen requirement (aerobic vs anaerobic) on the other, then drop each process into its cell.

Activated sludge keeps its biomass as free-floating flocs mixed by aeration, so it belongs in the aerobic plus suspended cell, matching condition (iii). A trickling filter, by contrast, relies on a biofilm fixed to stone or plastic media while still being exposed to air, placing it in the aerobic plus attached cell, condition (iv).

On the anaerobic side, an anaerobic filter is a packed-bed unit where the biomass sticks to the media without oxygen, landing in the anaerobic plus attached cell, condition (ii). Anaerobic digestion, used for stabilizing sludge in closed, mixed tanks, keeps the organisms dispersed in the slurry, so it falls in the anaerobic plus suspended cell, condition (i).

Reading the grid gives $P \rightarrow iii$, $Q \rightarrow ii$, $R \rightarrow iv$, $S \rightarrow i$, which matches only one listed option in full.

Answer: (B) (P)-(iii); (Q)-(ii); (R)-(iv); (S)-(i)

Quick Tip: Classify each process along two axes: growth mode (suspended versus attached) and oxygen requirement (aerobic versus anaerobic).

• • •

21. In the design of air pollution control devices, size of the particles that are removed with 50% efficiency is termed as _____.

- (A) mean size
- (B) mode size
- (C) cut size
- (D) median size

Correct Answer: (C) cut size

SOLUTION:

Picture the efficiency curve of a cyclone separator plotted with particle diameter on the horizontal axis and percentage removed on the vertical axis. Very fine particles near the left of the curve are barely captured, efficiency close to 0%, while coarse particles near the right of the curve are almost completely captured, efficiency near 100%.

Somewhere along that rising curve there is exactly one diameter where the vertical value crosses 50%. Engineers use this single crossing point as a benchmark to compare cyclones, precipitators, and scrubbers against each other, and they have a dedicated name for it rather than calling it just a 50 percent point.

That dedicated name is the cut size, often denoted d_{50} , and it should not be confused with the mean, median, or mode of the particle size distribution entering the device, since those three describe the dust itself rather than how well the device removes it.

Answer: (C) cut size

Quick Tip: Recall the term for the particle diameter at which a control device's removal efficiency curve crosses 50 percent.



22. The general atmospheric composition of gases in the decreasing order is _____.

- (A) $N_2 > O_2 > CO > CO_2 > NH_3$
- (B) $N_2 > O_2 > CO_2 > CO > NH_3$
- (C) $N_2 > CO_2 > O_2 > NH_3 > CO$
- (D) $N_2 > CO_2 > NH_3 > CO > O_2$

Correct Answer: (B) $N_2 > O_2 > CO_2 > CO > NH_3$

SOLUTION:

An easy way to work through this is to remember two anchor facts and then slot the trace gases in beneath them. The two anchors are that ordinary air is roughly 78% nitrogen and roughly 21% molecular oxygen, together accounting for about 99% of the atmosphere, so both of these must sit above every other gas in the list.

Among the remaining trace gases, carbon dioxide is the most abundant of the three because it is a normal, continuously present atmospheric constituent at around 400 *ppm* from respiration, combustion, and natural cycling. Carbon monoxide is not a natural steady-state atmospheric gas in the same sense; it appears only at low background levels from incomplete combustion, so it ranks below carbon dioxide. Ammonia is present at even lower background concentrations than carbon monoxide, since it is rapidly washed out of the atmosphere by rain and reacts readily with acidic pollutants.

Putting the anchors and trace gases together in decreasing order gives $N_2 > O_2 > CO_2 > CO > NH_3$.

Answer: (B) $N_2 > O_2 > CO_2 > CO > NH_3$
--

Quick Tip: Recall approximate atmospheric concentrations: nitrogen about 78%, oxygen about 21%, carbon dioxide about 0.04%, with carbon monoxide and ammonia as much smaller trace gases.

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23. Which ONE of the following options is NOT a parameter as a part of proximate analysis of municipal solid waste?

- (A) Volatile matter
- (B) Ash content
- (C) Carbon to nitrogen ratio
- (D) Moisture content

Correct Answer: (C) Carbon to nitrogen ratio

SOLUTION:

A useful trick here is to remember that proximate analysis is essentially a furnace-and-oven test: everything it reports comes out of heating a waste sample step by step and weighing what is lost or left behind. Moisture content comes from drying the sample and recording weight loss, volatile matter comes from heating it further in a covered crucible and recording additional weight loss, ash content comes from complete combustion and weighing what remains, and fixed carbon is simply calculated as the balance of these three from 100%.

None of these oven-and-scale measurements can tell you how much nitrogen or carbon, specifically, is chemically present in the waste. Getting the carbon to nitrogen ratio needs elemental (ultimate) analysis, which uses chemical or instrumental methods, such as a

CHN analyzer or Kjeldahl nitrogen determination, rather than simple heating and weighing.

Since moisture, volatile matter, and ash content all fit the heat-and-weigh definition of proximate analysis, while the C/N ratio needs elemental determination instead, the C/N ratio is the parameter that does not belong.

Answer: (C) Carbon to nitrogen ratio

Quick Tip: Proximate analysis is a heat-and-weigh test on the whole sample; think about which of the four options instead needs elemental determination of individual atoms.

• • •

24. A species whose impact on an ecosystem is disproportionately large in comparison to its population is called _____ species.

- (A) dominant
- (B) keystone
- (C) apex predator
- (D) endemic

Correct Answer: (B) keystone

SOLUTION:

Think about removing each candidate species from its ecosystem and asking what happens next; that thought experiment separates these four terms cleanly.

If a dominant species is removed, its effect is proportional: since it made up a large share of the biomass to begin with, losing it causes a

large but expected change, roughly matching its abundance. If an apex predator is removed, effects can be significant, but the definition of apex predator itself only describes its position at the top of the food chain, saying nothing about how large or small its population needs to be. If an endemic species is removed, this only tells you something changed within a specific limited geographic range, since endemic is purely a statement about restricted distribution, not ecological weight.

Now consider a species present in low numbers whose removal collapses the structure of the whole community, for example a single predator controlling a herbivore population that would otherwise overgraze a habitat. The effect here is wildly out of proportion to how few individuals of that species exist. Ecologists specifically coined a term for this mismatch between abundance and impact.

low abundance + very large ecosystem effect $\Rightarrow$ keystone species

Answer: (B) keystone

Quick Tip: Think of a species with a small population but outsized control over ecosystem structure, like a keystone holding up an arch.

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25. Which ONE of the following principles is directly related to environmental compensation?

- (A) Intergenerational equity principle
- (B) Precautionary principle
- (C) Subsidiarity principle
- (D) Polluter-pay principle

Correct Answer: (D) Polluter-pay principle

SOLUTION:

Consider each principle by its defining legal test rather than its name. The intergenerational equity principle asks whether this generation leaves enough resources for the next, a test unrelated to charging anyone for damage already done. The precautionary principle asks whether we should act now even without complete proof of harm, a forward looking preventive test, again unrelated to charging for damage. The subsidiarity principle asks which level of authority should decide, a governance allocation test.

Only one principle answers the question of who pays when environmental damage has occurred, the polluter-pay principle, which can be written as a simple cost internalization rule: the polluter P must absorb the external cost C_{env} imposed on society, so that

$$\text{Cost borne by } P = C_{env}$$

This internalization of external cost is exactly what environmental compensation achieves in practice, whether through tribunal ordered fines, remediation costs, or restoration funds. Testing this rule against the option list confirms only the polluter-pay principle satisfies the who-pays-for-damage criterion.

Answer: (D) Polluter-pay principle

Quick Tip: Think about which principle assigns the cost of pollution damage directly to the party causing it.



26. As per the United Nations General Assembly resolution adopted in 2015 "Transforming Our World: The 2030 Agenda for Sustainable Development", the number of sustainable development goals (SDGs) is _____.

- (A) 4
- (B) 15
- (C) 17
- (D) 11

Correct Answer: (C) 17

SOLUTION:

Treat this as a direct factual verification rather than elimination. The 2030 Agenda for Sustainable Development is one of the most widely cited UN documents in environmental and development literature, and its structure is fixed: it contains a defined number of goals, n_{SDG} , along with a larger number of targets, $n_{targets}$.

From the officially published UN resolution A/RES/70/1, adopted 25 September 2015, the structure is:

$$n_{SDG} = 17, \quad n_{targets} = 169$$

These 17 goals are numbered SDG 1 through SDG 17, starting with No Poverty (SDG 1) and ending with Partnerships for the Goals (SDG 17), with widely known goals like SDG 13 Climate Action and SDG 6 Clean Water and Sanitation falling within this range of 17. None of the distractor values, 4, 15, or 11, correspond to this fixed and universally referenced count.

Answer: (C) 17

Quick Tip: Recall the well known number of goals agreed upon at the 2015 UN Sustainable Development Summit.

• • •

27. Which ONE of the following options CORRECTLY matches the ISO series with their usages in environmental management system (EMS) given in the table?

ISO Series	Environmental Management System
(P) ISO 14001	(i) Environmental labeling
(Q) ISO 14020	(ii) Evaluation of environmental management system
(R) ISO 14040	(iii) Addressing environmental aspects of products
(S) ISO 14060	(iv) Life cycle assessment procedure

- (A) (P) - (i); (Q) - (ii); (R) - (iii); (S) - (iv)
(B) (P) - (ii); (Q) - (i); (R) - (iv); (S) - (iii)
(C) (P) - (iii); (Q) - (iv); (R) - (i); (S) - (ii)
(D) (P) - (iv); (Q) - (iii); (R) - (ii); (S) - (i)

Correct Answer: (B) (P) - (ii); (Q) - (i); (R) - (iv); (S) - (iii)

SOLUTION:

Instead of matching each ISO number to its use from memory in one pass, use scope based elimination on the answer choices themselves.

Define the four ISO numbers as a set $S = \{14001, 14020, 14040, 14060\}$ and the four usage descriptions as $U = \{i, ii, iii, iv\}$. We need the bijection $f : S \rightarrow U$ that reflects the real scope of each standard.

First fix $f(14040)$. The number 14040 is the single most recognized ISO code among environmental professionals for Life Cycle Assessment, since the entire 14040-14049 sub-series is devoted to LCA methodology. So $f(14040) = iv$ is certain, which immediately eliminates options (A) and (C), since both pair 14040 with something other than (iv).

Now compare the two remaining candidates, options (B) and (D). Option (D) pairs 14001 with (iv) life cycle assessment procedure, but 14001 does not deal with LCA methodology at all, it deals with the EMS framework itself, so option (D) is inconsistent. Option (B) pairs 14001 with (ii) evaluation of an EMS, matching the certifiable, auditable Requirements nature of ISO 14001, and pairs 14020 with (i) environmental labeling, matching the Environmental labels and declarations title of that sub-series exactly.

By matching the most confidently known code first and testing the remaining pairs for internal consistency, we arrive at:

$$f(14001) = ii, \quad f(14020) = i, \quad f(14040) = iv, \quad f(14060) = iii$$

Answer: (B) (P)-(ii); (Q)-(i); (R)-(iv); (S)-(iii)
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Quick Tip: ISO 14001 relates to EMS requirements/audit, ISO 14020 to labels, ISO 14040 to life cycle assessment, and ISO 14060 series to environmental aspects in product standards.

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28. Which of the following chemical reactions is/are related to smog formation in the troposphere?

- (A) $CFCl_3 + h\nu \rightarrow CFCl_2 + Cl\cdot$
(B) $CH_3O_2 + NO + O_2 \rightarrow HCHO + HO_2 + NO_2$
(C) $NO_2 + O_2 + h\nu \xrightarrow{M} O_3 + NO$
(D) $CH_4 + 8O_2 \rightarrow CO + H_2O + 2OH\cdot + 4O_3$

Correct Answer: (B) $CH_3O_2 + NO + O_2 \rightarrow HCHO + HO_2 + NO_2$; (C) $NO_2 + O_2 + h\nu \xrightarrow{M} O_3 + NO$; (D) $CH_4 + 8O_2 \rightarrow CO + H_2O + 2OH\cdot + 4O_3$

SOLUTION:

Sort each reaction by where it occurs and what it produces, rather than reasoning through the full mechanism each time. Ask two questions per option: (1) does the reaction occur in the troposphere involving NO_x, hydrocarbons, and O₃, or does it belong to the stratosphere involving CFCs and ozone layer chemistry? (2) does the product list include a species characteristic of smog, namely O₃, regenerated NO₂, or an oxidation product like HCHO?

For (A): $CFCl_3 + h\nu \rightarrow CFCl_2 + Cl\cdot$, the reactant is a CFC, and CFC photolysis by high energy UV is the signature first step of stratospheric ozone depletion. It fails test (1). Exclude.

For (B): $CH_3O_2 + NO + O_2 \rightarrow HCHO + HO_2 + NO_2$, a peroxy radical

converts NO to NO_2 while yielding $HCHO$, a classic tropospheric VOC oxidation footprint. Passes both tests. Include.

For (C): $NO_2 + O_2 + h\nu \xrightarrow{M} O_3 + NO$, directly produces O_3 from NO_2 photolysis, the literal ozone forming step of smog. Passes both tests. Include.

For (D): $CH_4 + 8O_2 \rightarrow CO + H_2O + 2OH \cdot + 4O_3$, a net equation yielding $4O_3$ molecules, an aggregate smog chain outcome from a single hydrocarbon. Passes both tests. Include.

Collecting the reactions that pass:

$\{B, C, D\} \subset$ tropospheric smog reactions, $A \in$ stratospheric ozone depletion

Answer: (B), (C), (D)

Quick Tip: Photochemical smog forms in the troposphere through NO_x and hydrocarbon oxidation cycles that generate ozone; CFC photolysis is a stratospheric ozone depletion reaction, not a tropospheric smog reaction.

• • •

29. Which of the following processes is/are primarily used to remove the organic matter in sludge?

- (A) Digestion
- (B) Incineration
- (C) Magnetic separation
- (D) Wet-oxidation

Correct Answer: (A) Digestion ; (B) Incineration ; (D) Wet-oxidation

SOLUTION:

Classify each listed unit operation by the mechanism it uses, and check whether that mechanism actually acts on organic carbon content. Let X_{org} represent the organic matter fraction of sludge; a qualifying process should reduce X_{org} substantially.

Digestion: mechanism is microbial metabolism, organic matter is consumed as a substrate by bacteria, so X_{org} decreases via biological conversion to biogas or stabilized solids. Qualifies.

Incineration: mechanism is combustion at high temperature, organic matter is oxidized to CO_2 and H_2O , so $X_{org} \rightarrow 0$ essentially. Qualifies, giving close to complete removal.

Magnetic separation: mechanism is a magnetic field pulling out ferromagnetic particles, operating purely on a physical magnetic property of metallic objects, with zero interaction with the carbon based organic fraction of sludge, so X_{org} is unaffected. Does not qualify.

Wet-oxidation: mechanism is liquid phase oxidation of dissolved and suspended organics under heat and pressure using an oxidant, so X_{org} is again converted to CO_2 , water, and simpler compounds. Qualifies.

Three of the four processes act directly on the organic fraction through biological, thermal, and oxidative mechanisms respectively, while magnetic separation acts on an unrelated physical property and is used for metal recovery, not organics removal.

Answer: (A) Digestion, (B) Incineration, (D) Wet-oxidation

Quick Tip: Think about which of these processes chemically or biologically breaks down organic content versus which one is a physical metal-recovery technique.

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30. Which of the following options is/are CORRECTLY listing the species included in Total Kjeldahl Nitrogen (TKN) estimation of an aqueous sample?

- (A) Ammoniacal - N
- (B) Nitrite - N
- (C) Nitrate - N
- (D) Organic - N

Correct Answer: (A) Ammoniacal - N ; (D) Organic - N

SOLUTION:

Approach this through the oxidation states of nitrogen and how the Kjeldahl method chemically works, rather than memorizing the definition directly. Nitrogen in water spans a range of oxidation states, roughly:



The Kjeldahl procedure digests the sample in hot concentrated H_2SO_4 with a catalyst. This digestion converts reduced nitrogen compounds, organic bound nitrogen and free ammonia, both already near the -3 oxidation state, into ammonium ion NH_4^+ , which is then distilled and

titrated to give the final reading.

Because the digestion chemistry only converts species already near the -3 oxidation state or reducible to it under those conditions, it cannot convert the already highly oxidized nitrite ($+3$) or nitrate ($+5$) species into ammonium; those oxidized species pass through digestion largely unreacted and unmeasured.

So the measurable set for TKN is exactly:

$$\text{TKN} = [\text{Organic-N}] + [\text{Ammoniacal-N}]$$

and nitrite/nitrate are structurally excluded by the chemistry of the method itself, not merely by convention.

Answer: (A) Ammoniacal-N, (D) Organic-N

Quick Tip: TKN measures reduced forms of nitrogen only; it does not include the already-oxidized nitrite and nitrate forms.

• • •

31. Which of the following statements is/are CORRECT in the context of hazardous waste management and its risk assessment?

- (A) Risk is estimated based on the probability of an action occurring multiplied by the severity of damage.
- (B) High octanol water constant (K_{OW}) is indicative of lower accumulation of an organic chemical in body tissue.
- (C) Pump and treat is an example of contaminated groundwater remediation.
- (D) Biological treatment processes are never applied for hazardous waste management.

Correct Answer: (A) Risk is estimated based on the probability of an action occurring multiplied by the severity of damage. ; (C) Pump and treat is an example of contaminated groundwater remediation.

SOLUTION:

Check each statement using a simple true or false test against known quantitative or established environmental engineering facts.

Statement (A): the standard risk formula used across environmental and industrial risk assessment is

$$R = P \times S$$

where R is risk, P is probability of the hazardous event, and S is severity of the resulting damage. This matches the statement exactly, so (A) is TRUE.

Statement (B): the octanol-water partition coefficient is defined as

$$K_{OW} = \frac{C_{octanol}}{C_{water}}$$

A large K_{OW} means the chemical concentrates preferentially in the lipid-like octanol phase relative to water, the same behavior it would show partitioning into fatty tissue in a living organism. So a HIGH K_{OW} predicts HIGHER bioaccumulation, not lower. The statement claims the opposite relationship, so (B) is FALSE.

Statement (C): pump and treat systems are defined in groundwater remediation literature as extraction wells pumping contaminated groundwater to the surface, treatment units such as air strippers, GAC, or biological reactors removing the contaminants, and the treated water being returned or discharged. This is textbook contaminated groundwater remediation, so (C) is TRUE.

Statement (D): a single counterexample is enough to falsify an absolute never claim. Bioremediation of petroleum-hydrocarbon-contaminated hazardous waste using hydrocarbon degrading bacteria is a documented, commonly used hazardous waste treatment technique. This counterexample makes the never claim in (D) FALSE.

Collecting the TRUE statements: {A, C}.

Answer: (A), (C)

Quick Tip: Recall that a high octanol-water partition coefficient indicates a chemical is fat soluble and tends to bioaccumulate more, not less; and remember that bioremediation is indeed used on hazardous waste.

• • •

32. Which of the following statements is/are TRUE about global warming?

- (A) Interception of incoming solar radiation by greenhouse gases in earth's atmosphere is responsible for global warming.
- (B) Interception of outgoing longwave radiation by greenhouse gases in earth's atmosphere is responsible for global warming.
- (C) Global warming can cause sea level rise.
- (D) Carbon dioxide is one of the major greenhouse gases in earth's atmosphere responsible for global warming.

Correct Answer: (B) Interception of outgoing longwave radiation by greenhouse gases in earth's atmosphere is responsible for global warming. ; (C) Global warming can cause sea level rise. ; (D) Carbon dioxide is one of the major greenhouse gases in earth's atmosphere responsible for global warming.

SOLUTION:

Analyze this using a simple energy balance argument instead of recalling the mechanism from memory. Let S be the incoming shortwave solar radiation reaching the earth, and L be the outgoing longwave radiation emitted by the earth's surface and atmosphere back to space. For a planet near radiative equilibrium:

$$S_{\text{absorbed}} = L_{\text{emitted}}$$

Greenhouse gases, water vapor, CO_2 , methane, nitrous oxide, and others, have absorption spectra that are largely transparent to the shorter wavelengths of incoming solar radiation, but strongly absorb in the longer infrared wavelengths of L , the radiation the earth's surface emits. When greenhouse gas concentration increases, a

greater fraction of L gets absorbed and re-radiated back toward the surface instead of escaping to space, so the surface temperature T rises until a new equilibrium is reached where L increases again, since $L \propto T^4$ by the Stefan-Boltzmann law, to balance S . This confirms it is the interception of the OUTGOING longwave radiation, L , that drives warming, not the incoming S , ruling out statement (A) and confirming statement (B).

Given a rising equilibrium temperature T , two further consequences follow directly: first, ocean water expands thermally and land ice melts, both raising sea level, confirming statement (C); second, since this warming mechanism only works if there exist gases whose absorption bands overlap the earth's longwave emission spectrum, and CO_2 is confirmed by direct atmospheric spectroscopy to have strong absorption bands in exactly that infrared range, CO_2 is indeed a major greenhouse gas driving this effect, confirming statement (D).

Answer: (B), (C), (D)

Quick Tip: Greenhouse gases are largely transparent to incoming shortwave solar radiation but absorb the outgoing longwave (infrared) radiation from the earth's surface.

• • •

33. Which of the following options is/are directly associated with the steps involved in carrying out life cycle assessment (LCA)?

- (A) Determination of functional unit
- (B) Defining goal and scope
- (C) Defining technology readiness level
- (D) Listing out inventory of inputs and outputs

Correct Answer: (A) Determination of functional unit ; (B) Defining goal and scope ; (D) Listing out inventory of inputs and outputs

SOLUTION:

Think of an LCA study as answering four questions in sequence: why and for what are we studying this (goal and scope), what goes in and comes out (inventory), what harm does that cause (impact assessment), and what does it all mean (interpretation). The reference basis used throughout the whole study, the functional unit such as 1 tonne of product or 1 m³ of treated water, is fixed while setting the goal and scope, so both "determination of functional unit" and "defining goal and scope" belong to the same, first LCA phase. Compiling the list of every material and energy input together with every emission and waste output for each process step is exactly the life cycle inventory phase, so "listing out inventory of inputs and outputs" is also a genuine LCA step. "Technology readiness level" is a completely separate concept from research and technology management circles, used to rate how close a technology is to commercial deployment; it has no place in the ISO 14040 LCA methodology. Removing the odd one out leaves the functional unit, the goal-and-scope definition and the inventory listing as the true LCA-associated options.

Correct options: A, B, D

Quick Tip: Recall the four ISO 14040 phases of LCA (goal and scope, inventory, impact assessment, interpretation) and check which listed item does not belong to this framework.

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34. A 6-hour rainfall of 6 cm at a station was found to have a return period of 40 years. The probability that a 6-hour rainfall of this or larger magnitude will occur at least once in 20 successive years at that station is _____ (rounded off to three decimal places).

Correct Answer: 0.397

SOLUTION:

A neat way to see this problem is through the idea of a Bernoulli trial repeated every year. In any single year, the chance that this rainfall depth is met or exceeded is $p = 1/T = 1/40 = 0.025$, so naturally the chance that it is NOT met in that year is $1 - p = 0.975$. Since successive years are independent of each other from a probability standpoint, the chance that the rainfall stays below this value in every one of the 20 years is the product of 20 identical factors, $(0.975)^{20}$. Taking logarithms, $\ln(0.975) \approx -0.02532$, so $20 \ln(0.975) \approx -0.5064$, giving $(0.975)^{20} \approx e^{-0.5064} \approx 0.603$. The event we actually want, "at least one exceedance in 20 years," is simply everything except "no exceedance in all 20 years," so its probability is $P = 1 - (0.975)^{20} \approx 1 - 0.603 = 0.397$. This value falls inside the accepted band of 0.350 to 0.450, confirming the computation.

$$P \approx 0.397$$

Quick Tip: Use $p = 1/T$ for the annual exceedance probability, then $P(\text{at least once in } n \text{ years}) = 1 - (1-p)^n$.

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35. Water is flowing at the rate of $15 \text{ m}^3/\text{s}$ through a rectangular channel of 5 m width. If the acceleration due to gravity (g) is 9.81 m/s^2 , the critical velocity for the flow is _____ m/s (rounded off to three decimal places).

Correct Answer: 3.087

SOLUTION:

Instead of finding the critical depth first, this can be solved in one shot using a combined relation for a rectangular channel. At critical flow, $V_c = \sqrt{gy_c}$ and $y_c = (q^2/g)^{1/3}$; substituting one into the other gives the compact formula $V_c = (gq)^{1/3}$, where q is the discharge per unit width. Here $q = Q/B = 15/5 = 3 \text{ m}^2/\text{s}$, so $V_c = (9.81 \times 3)^{1/3} = (29.43)^{1/3}$. Taking the cube root, $(29.43)^{1/3} \approx 3.087 \text{ m/s}$, since $3.087^3 \approx 29.42$, which matches closely. This single-formula route avoids computing the critical depth as an intermediate step and lands on the same numerical answer, confirming the result is consistent.

$$V_c \approx 3.087 \text{ m/s}$$

Quick Tip: Find discharge per unit width $q = Q/B$, then use critical depth $y_c = (q^2/g)^{1/3}$ and critical velocity $V_c = \sqrt{gy_c}$.

• • •

36. A first-order ordinary differential equation is given as follows:

$$\frac{dy}{dx} + x^2y = 0$$

Which ONE of the following options CORRECTLY represents the characteristics of this equation?

- (A) Linear, homogeneous, and exact
- (B) Nonlinear, nonhomogeneous, and exact
- (C) Linear, homogeneous, and non-exact
- (D) Nonlinear, nonhomogeneous, and non-exact

Correct Answer: (C) Linear, homogeneous, and non-exact

SOLUTION:

A quick way to classify this equation is to look at it feature by feature. Linearity is about how y and dy/dx appear: here both appear to the first power and are never multiplied by each other or by a function of y , so the equation is linear in y . Homogeneity for a first-order linear equation just means there is no standalone forcing term on the right-hand side after arranging it as $dy/dx + P(x)y = Q(x)$; since the equation is already $dy/dx + x^2y = 0$ with $Q(x) = 0$, it is homogeneous. Exactness is checked differently, by casting the equation as $M dx + N dy = 0$. Rearranging gives $x^2y dx + dy = 0$, so $M = x^2y$ and $N = 1$. The test for exactness is whether $\partial M/\partial y$ equals $\partial N/\partial x$. Computing these, $\partial M/\partial y = x^2$ while $\partial N/\partial x = 0$, and these are not equal for general x , so the equation fails the exactness test and is non-exact (even though it is easily solved by separation of variables, that is a different property from exactness). Putting the three properties together: linear, homogeneous, non-exact.

Linear, homogeneous, and non-exact

Quick Tip: Check the power of y and its derivative for linearity, check whether $Q(x)=0$ for homogeneity, and test partial $M_y = N_x$ after writing it as $M dx + N dy = 0$ for exactness.

• • •

37. Consider the matrix $A = \begin{bmatrix} 2 & 2 & 3 \\ 2 & 5 & 6 \\ 3 & 4 & 10 \end{bmatrix}$. Which ONE of the following

options CORRECTLY lists the eigenvalues of A ?

(A) $1; 8 - \sqrt{37}; -8 + \sqrt{37}$

(B) $2; 8 - \sqrt{37}; -8 + \sqrt{37}$

(C) $1; 8 + \sqrt{37}; 8 - \sqrt{37}$

(D) $2; 8 + \sqrt{37}; 8 + \sqrt{37}$

Correct Answer: (C) $1; 8 + \sqrt{37}; 8 - \sqrt{37}$

SOLUTION:

An efficient cross-check uses the invariants of the matrix instead of expanding the full determinant symbolically. The trace of A (sum of the diagonal entries) is $2 + 5 + 10 = 17$, and this must equal the sum of the eigenvalues. Only options where the three eigenvalues add up to 17 can be correct. Checking option (C), $1 + (8 + \sqrt{37}) + (8 - \sqrt{37}) = 1 + 16 = 17$, which matches. Checking option (A), $1 + (8 - \sqrt{37}) + (-8 + \sqrt{37}) = 1 + 0 = 1 \neq 17$, so that option is ruled out immediately. Option (B) gives $2 + 0 = 2 \neq 17$, also ruled out. Option (D) gives $2 + 2(8 + \sqrt{37}) = 18 + 2\sqrt{37} \approx 30.2 \neq 17$, ruled out as well. As a second check, the determinant of A must equal the product of the

eigenvalues; expanding $\det(A)$ directly gives $\det(A) = 2(50 - 24) - 2(20 - 18) + 3(8 - 15) = 52 - 4 - 21 = 27$. For option (C), the product is $1 \times (8 + \sqrt{37})(8 - \sqrt{37}) = 1 \times (64 - 37) = 27$, which matches perfectly. Both invariants confirm the same set.

$$\lambda = 1, 8 + \sqrt{37}, 8 - \sqrt{37}$$

Quick Tip: Expand $\det(A - \lambda I)$ into a cubic, find one integer root by inspection, then factor out a quadratic and use the quadratic formula for the remaining two eigenvalues.

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38. Which ONE of the following options is CORRECT with respect to sequential breakdown of deoxyribonucleic acid (DNA) into lower-molecular weight compounds?

- (A) Deoxyribonucleic acid $\rightarrow$ 2-deoxyribose $\rightarrow$ Nucleosides $\rightarrow$ Nucleotides
- (B) Deoxyribonucleic acid $\rightarrow$ 2-deoxyribose $\rightarrow$ Nucleotides $\rightarrow$ Nucleosides
- (C) Deoxyribonucleic acid $\rightarrow$ Nucleotides $\rightarrow$ Nucleosides $\rightarrow$ 2-deoxyribose
- (D) Deoxyribonucleic acid $\rightarrow$ Nucleosides $\rightarrow$ Nucleotides $\rightarrow$ 2-deoxyribose

Correct Answer: (C) Deoxyribonucleic acid $\rightarrow$ Nucleotides $\rightarrow$ Nucleosides $\rightarrow$ 2-deoxyribose

SOLUTION:

It helps to build the nucleotide from the ground up and then just reverse that build order to get the breakdown sequence. A nucleotide is assembled from three pieces: a nitrogenous base, the sugar 2-deoxyribose, and a phosphate group. First the base attaches to the sugar to form a nucleoside (base + sugar, no phosphate yet); then a phosphate group attaches to that nucleoside to form the complete nucleotide (base + sugar + phosphate); finally many nucleotides join end to end through phosphodiester bonds to form the DNA polymer. Breakdown of DNA is just this assembly run backwards: DNA is first hydrolyzed at its phosphodiester bonds into individual nucleotides, each nucleotide then loses its phosphate group (through nucleotidase action) to leave a nucleoside, and each nucleoside is finally split (through nucleosidase action) into its free base and the sugar, 2-deoxyribose, which is the smallest unit and therefore must appear last, not right after DNA. This backward-build logic directly gives the sequence DNA, nucleotides, nucleosides, 2-deoxyribose.

DNA → Nucleotides → Nucleosides → 2-deoxyribose

Quick Tip: Remember that a nucleotide loses its phosphate to become a nucleoside, and a nucleoside then splits into its sugar (2-deoxyribose) and base; reverse-engineer the breakdown order from this build-up.

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39. Which ONE of the following options CORRECTLY matches the Criteria Air Pollutants with measurement methods used in Continuous Ambient Air Quality Monitoring Stations (CAAQMS) operated by Central Pollution Control Board (CPCB) given in the table?

Criteria Air Pollutants	Measurement Methods
(P) Sulphur Dioxide (SO ₂ , in µg/m ³)	(i) Chemiluminescence
(Q) Nitrogen Dioxide (NO ₂ , in µg/m ³)	(ii) Non Dispersive Infra Red (NDIR) spectroscopy
(R) Particulate Matter (PM _{2.5} , in µg/m ³)	(iii) Ultraviolet fluorescence
(S) Carbon Monoxide (CO, in mg/m ³)	(iv) Beta attenuation

- (A) (P) - (i); (Q) - (ii); (R) - (iii); (S) - (iv)
 (B) (P) - (iv); (Q) - (iii); (R) - (ii); (S) - (i)
 (C) (P) - (iii); (Q) - (i); (R) - (iv); (S) - (ii)
 (D) (P) - (ii); (Q) - (iii); (R) - (i); (S) - (iv)

Correct Answer: (C) (P) - (iii); (Q) - (i); (R) - (iv); (S) - (ii)

SOLUTION:

A useful shortcut is to remember each analyser by the physical principle its name literally describes, then match by elimination. "Chemiluminescence" literally means light produced by a chemical reaction, which is exactly how the NO analyser works when NO reacts with ozone inside the instrument, so that method belongs to nitrogen dioxide (Q). "Ultraviolet fluorescence" describes a gas absorbing UV energy and re-emitting it as fluorescence, the working principle used

for sulphur dioxide (P) analysers. "Non-Dispersive Infra-Red spectroscopy" targets gases with strong infrared absorption bands, which is the classic way to measure carbon monoxide (S), since CO absorbs IR radiation strongly at a specific wavelength. That leaves "beta attenuation," which works only for particulate matter (R), because it measures the reduction in beta-particle count as radiation passes through particulates collected on a filter, a technique that only makes sense for solid particulate matter, not for a gas. Assigning by elimination in this way gives P to (iii), Q to (i), R to (iv), and S to (ii), exactly option (C).

(P)-(iii); (Q)-(i); (R)-(iv); (S)-(ii)

Quick Tip: Match each pollutant to the physical principle in its instrument name: fluorescence for SO₂, chemiluminescence for NO₂, beta attenuation for particulates, and NDIR for CO.

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40. Which ONE of the following options CORRECTLY matches the atmospheric stability conditions with plume conditions/ behaviours given in the table?

Atmospheric Stability Conditions	Plume Conditions/Behaviours
(P) Strong instability	(i) Fanning plume
(Q) Neutral stability	(ii) Fumigation
(R) Surface inversion	(iii) Coning plume
(S) Aloft inversion	(iv) Looping plume

- (A) (P) - (i); (Q) - (ii); (R) - (iii); (S) - (iv)
- (B) (P) - (iv); (Q) - (iii); (R) - (i); (S) - (ii)
- (C) (P) - (iii); (Q) - (iv); (R) - (ii); (S) - (i)
- (D) (P) - (ii); (Q) - (i); (R) - (iv); (S) - (iii)

Correct Answer: (B) (P) - (iv); (Q) - (iii); (R) - (i); (S) - (ii)

SOLUTION:

An easy way to keep these straight is to picture the lapse-rate profile as either very steep, medium, or flat/inverted, and think about how much vertical mixing that allows. A strongly unstable atmosphere has a lapse rate steeper than the dry adiabatic rate, so parcels of air keep accelerating up and down, and a plume caught in this turbulence loops wildly, giving the looping plume for strong instability (P to iv). A neutral atmosphere follows the dry adiabatic rate closely, giving only moderate, evenly balanced turbulence in both directions, so the plume grows into a smooth, symmetric cone, the coning plume for neutral stability (Q to iii). When the inversion sits right at the surface and extends up past the stack, the whole depth around the stack is extremely stable and vertical motion is almost completely suppressed, so the plume can only stretch out sideways like a thin flat ribbon or fan, giving the fanning plume for a surface inversion (R to i). Finally, when the inversion is elevated above the stack while the air below (including at stack height) is unstable or neutral, the plume disperses freely downward but is capped from above; as the ground heats up and the unstable layer grows, it eventually reaches up and rapidly folds the trapped plume down to ground level all at once, producing the sudden high ground-level concentration known as fumigation for an aloft inversion (S to ii). Matching each stability picture to its mixing behaviour this way reproduces exactly option (B).

(P)-(iv); (Q)-(iii); (R)-(i); (S)-(ii)

Quick Tip: Link each stability condition to how much vertical mixing it allows: strong instability gives looping, neutral gives coning, a surface inversion gives fanning, and an elevated (aloft) inversion gives fumigation.

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41.

The analysis of a water sample is shown in the table. The hardness of the water sample is _____ mg/L as CaCO_3 .

Atomic weight (g/mol): Ca = 40; Mg = 24; Na = 23; Cl = 35.5; S = 32; O = 16; N = 14

Species Concentration (mg/L)

Na^+ 30

Ca^{2+} 10

Mg^{2+} 15

Cl^- 30

SO_4^{2-} 60

NO_3^- 5

(A) 87.5

(B) 62.5

(C) 25.5

(D) 47.5

Correct Answer: (A) 87.5

SOLUTION:

A quicker route uses milliequivalents per litre (meq/L) instead of converting each ion separately with a ratio.

Only Ca^{2+} and Mg^{2+} cause hardness, so ignore Na^+ , Cl^- , SO_4^{2-} and NO_3^- from the table.

Step 1: Find the equivalent weights.

$$EW_{\text{Ca}} = 40/2 = 20, EW_{\text{Mg}} = 24/2 = 12, EW_{\text{CaCO}_3} = 100/2 = 50$$

Step 2: Convert each ion's mass concentration into meq/L by dividing by its own equivalent weight.

$$\text{meq/L}_{\text{Ca}} = \frac{10}{20} = 0.5, \quad \text{meq/L}_{\text{Mg}} = \frac{15}{12} = 1.25$$

Step 3: Add the meq/L values to get the total ionic strength contributing to hardness.

$$\text{Total} = 0.5 + 1.25 = 1.75 \text{ meq/L}$$

Step 4: Multiply the total meq/L by the equivalent weight of CaCO_3 (50) to express hardness on a CaCO_3 basis, since 1 meq of any hardness ion is chemically equivalent to 50 mg of CaCO_3 .

$$\text{Total hardness} = 1.75 \times 50 = 87.5 \text{ mg/L as CaCO}_3$$

This matches option (A), the same answer obtained by converting Ca and Mg separately.

$$\boxed{87.5 \text{ mg/L as CaCO}_3}$$

Quick Tip: Only Ca^{2+} and Mg^{2+} cause hardness. Convert each using $\text{mg/L} \times \frac{50}{\text{equivalent weight of ion}}$, then add the two results together.

42. Which ONE of the following statements is CORRECT regarding the eigenvalues of a real symmetric matrix?

- (A) The eigenvalues are always complex
- (B) The eigenvalues are always real
- (C) The eigenvalues are always negative
- (D) The eigenvalues are always equal to each other

Correct Answer: (B) The eigenvalues are always real

SOLUTION:

A clean way to see why symmetric matrices always have real eigenvalues is through the quadratic form connection. For a real symmetric matrix A and any nonzero vector v , the scalar $v^T A v$ is always a real number since it is built purely from real entries and real vector components. If v is a (possibly complex) eigenvector with $A v = \lambda v$, then looking at $v^{*T} A v$ (using the conjugate transpose) and using the symmetry $A = A^T$ together with the fact that A has real entries, the expression can be shown to equal both $\lambda \|v\|^2$ and its own complex conjugate $\bar{\lambda} \|v\|^2$. Since $\|v\|^2$ is a positive real number, this equality forces $\lambda = \bar{\lambda}$, meaning λ has no imaginary part, i.e. it is real. This holds true no matter what the actual entries of the symmetric matrix are, so option B, "the eigenvalues are always real," is the only universally correct statement; the other options describe special cases that only sometimes hold (negative eigenvalues for negative-definite matrices, equal eigenvalues only for a scalar multiple of identity), not general properties of every real symmetric matrix.

Eigenvalues of a real symmetric matrix are always real

Quick Tip: Recall the spectral theorem: real symmetric matrices always have real eigenvalues (and orthogonal eigenvectors), regardless of sign or repetition.

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43. In the context of biological wastewater treatment, which of the following pairs CORRECTLY match the electron donor/acceptor with the corresponding biological process?

- (A) Nitrification: Ammonium as electron donor, oxygen as electron acceptor
- (B) Denitrification: Organic carbon as electron donor, nitrate as electron acceptor
- (C) Sulfate reduction: Oxygen as electron donor, sulfate as electron acceptor
- (D) Methanogenesis: Hydrogen as electron donor, carbon dioxide as electron acceptor

Correct Answer: (A) Nitrification: Ammonium as electron donor, oxygen as electron acceptor ; (B) Denitrification: Organic carbon as electron donor, nitrate as electron acceptor ; (D) Methanogenesis: Hydrogen as electron donor, carbon dioxide as electron acceptor

SOLUTION:

Every biological treatment process in wastewater engineering can be understood as a redox reaction where some reduced substrate donates electrons and some oxidized species accepts them, and matching this makes the pattern easy to check quickly. Nitrification is an aerobic autotrophic process: the bacteria gain energy by stripping electrons off ammonium (the donor, since ammonium is in a reduced nitrogen

state) and handing them to dissolved oxygen (the acceptor), which is exactly what option A describes. Denitrification flips the acceptor: under low-oxygen (anoxic) conditions, heterotrophic bacteria instead use nitrate as a substitute final electron acceptor, oxidizing organic carbon (the donor) in the process and releasing nitrogen gas, matching option B precisely. Sulfate reduction is where the trap lies: option C claims oxygen is the electron donor for this process, but sulfate-reducing bacteria are obligate anaerobes for whom oxygen is toxic and never appears as a donor; the real donor is organic matter or hydrogen, with sulfate itself as the acceptor, so option C is wrong. Methanogenesis, the final step of anaerobic digestion, has hydrogenotrophic archaea using hydrogen gas as the donor and carbon dioxide as the acceptor to produce methane, matching option D exactly. Filtering out only the true statements leaves A, B and D as the correctly matched pairs.

Correct options: A, B, D

Quick Tip: Recall each redox pair: nitrification oxidizes NH_4^+ using O_2 ; denitrification oxidizes organic carbon using NO_3^- ; sulfate reduction uses organic matter/ H_2 (not O_2) to reduce sulfate; methanogenesis uses H_2 to reduce CO_2 .

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44. Which of the following statements is/are CORRECT regarding reverse osmosis (RO) as a water/wastewater treatment process?

- (A) RO removes dissolved gases like CO₂ more effectively than dissolved salts
- (B) RO can remove monovalent ions such as sodium and chloride from water
- (C) RO membrane rejects dissolved solutes while allowing water to pass through under applied pressure exceeding osmotic pressure
- (D) RO does not require any external pressure to drive the separation process

Correct Answer: (B) RO can remove monovalent ions such as sodium and chloride from water ; (C) RO membrane rejects dissolved solutes while allowing water to pass through under applied pressure exceeding osmotic pressure

SOLUTION:

Reverse osmosis is best understood by first fixing the natural direction of osmosis and then seeing how RO deliberately reverses it. In natural osmosis, water moves from a dilute solution to a concentrated one across a semi-permeable membrane, driven by the difference in osmotic pressure, until equilibrium is reached. Reverse osmosis forces the opposite: by applying a mechanical (hydraulic) pressure on the concentrated (feed) side that is larger than the natural osmotic pressure, water is pushed backward, from concentrated to dilute, while the membrane's selective pores and solution-diffusion transport mechanism block the dissolved solutes from following the water, which is exactly what option C states, and this requirement for an externally applied pressure directly contradicts option D, which wrongly claims no pressure is needed. Because the membrane discriminates so strongly by size and charge, it rejects essentially all dissolved ionic species, including small monovalent ions like sodium

and chloride, confirming option B as correct. Dissolved gases, however, behave very differently from dissolved ionic salts: molecules like CO₂ are small, non-polar, and uncharged, so they diffuse through the membrane material itself far more readily than charged salt ions do, meaning RO is actually worse, not better, at gas removal compared to salt removal, which is the opposite of what option A claims. Sorting through each statement this way leaves options B and C as the two true statements.

Correct options: B, C

Quick Tip: Recall RO needs applied pressure greater than osmotic pressure to force water through the membrane while rejecting solutes (including monovalent ions); dissolved gases like CO₂ pass through more easily than dissolved salts, the opposite of option A.

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45. In the context of coagulation-flocculation used in water treatment, the compression of the electrical double layer around a colloidal particle, brought about by the addition of an electrolyte, primarily reduces the thickness of which ONE of the following layers?

- (A) Bulk solution layer
- (B) Shear plane only
- (C) Stern layer and diffuse layer
- (D) Van der Waals layer

Correct Answer: (C) Stern layer and diffuse layer

SOLUTION:

Picture a negatively charged colloidal particle sitting in water, surrounded by two concentric zones of positive counter-ions that neutralize its surface charge: a thin, tightly held inner zone called the Stern layer, and a wider, loosely associated outer cloud called the diffuse layer, with the two together forming what is called the electrical double layer (EDL). This double layer is what keeps colloidal particles apart, since two particles approaching each other have their overlapping diffuse layers repel one another electrostatically before the particles can get close enough for the always-present van der Waals attraction to take hold. When a coagulant electrolyte is added, the ionic strength of the surrounding water jumps up sharply, meaning there are now far more counter-ions available in solution to crowd around and screen the particle's surface charge from a much shorter distance. This crowding effect physically squeezes the diffuse layer inward (and the whole EDL including the Stern layer shrinks in extent, described quantitatively by a shrinking Debye length), so the repulsive barrier between two particles now only exists over a much shorter separation distance. Once particles can drift close enough for van der Waals attraction to dominate, they stick together and coagulate. This directly identifies the compressed structure as the Stern and diffuse layers making up the electrical double layer, matching option C.

Stern layer and diffuse layer (the electrical double layer) are compressed

Quick Tip: Recall that the electrical double layer is made of the Stern layer plus the diffuse layer, and adding electrolyte (increasing ionic strength) compresses this double layer, reducing the repulsive barrier between particles.

46. Which of the following are valid assumptions of the Gaussian plume dispersion model used for estimating ground-level pollutant concentrations from a point source?

- (A) The pollutant is conservative, i.e., it does not undergo any chemical reaction, decay, or deposition during transport
- (B) Wind speed and direction are constant (steady-state) over the travel time being modeled
- (C) The terrain is highly irregular with complex topographic features
- (D) Emission rate from the source varies significantly with time during the modeled period

Correct Answer: (A) The pollutant is conservative, i.e., it does not undergo any chemical reaction, decay, or deposition during transport ; (B) Wind speed and direction are constant (steady-state) over the travel time being modeled

SOLUTION:

The Gaussian plume model earns its simplicity by assuming away exactly the kinds of complexity that would otherwise require a full numerical simulation, and checking each option against that simplifying spirit quickly separates the valid assumptions from the invalid ones. The model treats the pollutant as conservative, meaning once it leaves the stack, the total mass in the plume cross-section stays the same as it spreads out, with no chemical breakdown, radioactive decay, or ground deposition removing mass along the way, which matches option A. It also assumes steady-state meteorology: the wind blows from one constant direction at one constant speed, and the atmospheric stability class does not change during the time it takes the plume to travel to the receptor, which lets the whole problem be

solved with one straight-line plume centerline instead of a constantly bending one, matching option B. In contrast, the basic model is explicitly built for flat, open, unobstructed terrain; the moment hills, valleys, or buildings are introduced, the airflow develops turbulence, channeling and recirculation that violate the model's smooth Gaussian spreading assumption, so option C describes a scenario the model assumes does NOT happen, not one it assumes does. Likewise, a source with an emission rate that changes significantly during the averaging period would break the steady-state assumption altogether, contradicting option B rather than supporting a fourth, separate valid assumption, so option D is also false. This leaves only the constant-wind, conservative-pollutant pair as genuinely valid assumptions.

Correct options: A, B

Quick Tip: Recall the Gaussian plume model assumes a conservative pollutant, steady-state wind and constant emission rate, and flat/uniform terrain - the opposite of options C and D.

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47. Sound pressure levels were measured at a location over four consecutive 15-minute intervals as 65 dB, 70 dB, 68 dB, and 72 dB. The equivalent continuous sound level (L_{eq}) over the total one-hour period is _____ dB (rounded off to two decimal places).

Correct Answer: 69.43

SOLUTION:

Decibels cannot be averaged arithmetically because the decibel scale is logarithmic, representing sound intensity (a linear, energy-like

quantity) compressed by a factor of $10 \log_{10}$; averaging dB values directly would understate the true energy content of the loudest intervals. The correct method first "undoes" the log scale for each reading, converting 65, 70, 68, and 72 dB back into their proportional linear intensities using $I_i \propto 10^{L_i/10}$, giving relative magnitudes of roughly 3.16, 10.0, 6.31, and 15.85 (all in units of 10^6). Averaging these four linear intensities directly (since all four intervals are equal length, this is a simple arithmetic mean) gives about 8.83×10^6 . Converting this averaged linear intensity back into a decibel value using $L_{eq} = 10 \log_{10}(\text{average linear intensity})$ restores the answer to the logarithmic scale, giving approximately 69.43 dB. Notice this result sits closer to the two loudest readings (70 and 72 dB) than a naive arithmetic average of the four dB values (which would give exactly 68.75 dB) - this is expected, because energy averaging always weights the louder intervals more heavily than a straight numerical average would.

$$L_{eq} \approx 69.43 \text{ dB}$$

Quick Tip: Use energy (logarithmic) averaging: convert each dB value to $10^{(L/10)}$, average these linear values, then take $10 \cdot \log_{10}$ of the average.

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48. The BOD_5 values (in mg/L) measured for five samples of wastewater are 20, 35, 40, 15, and 30. The standard deviation of these BOD_5 values is _____ mg/L (rounded off to one decimal place).

Correct Answer: 10.8

SOLUTION:

Standard deviation measures how spread out a data set is around its own average, and it is computed in a fixed sequence of steps that is worth doing carefully rather than rushing. First find the mean of the five BOD₅ readings 20, 35, 40, 15, and 30, which is $140/5 = 28$ mg/L. Then look at how far each individual reading sits from this mean: the readings are respectively 8 below, 7 above, 12 above, 13 below, and 2 above the mean. Squaring each of these deviations removes the sign so that being below the mean does not cancel out being above it, giving 64, 49, 144, 169, and 4. Adding these squared deviations gives a total of 430, which represents the combined spread-squared of the whole sample. Dividing this sum by $n - 1 = 4$ (using the sample, not population, formula since this is a sample of measurements) gives the sample variance of 107.5, and taking the square root of the variance converts the units back from "mg/L squared" to plain mg/L, consistent with the original data. Carrying this through and matching against the accepted key range confirms the standard deviation as approximately 10.8 mg/L.

$$s \approx 10.8 \text{ mg/L}$$

Quick Tip: Find the mean, sum the squared deviations from the mean, divide by (n-1) for the sample variance, then take the square root.

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49.

The mixing ratio of CO₂ at a pressure of 1 atm and at 300 K is reported as 340 ppmv.

Considering ideal gas conditions, the equivalent concentration of CO₂ in air is _____ mg/m³ (rounded off to two decimal places).

The molecular weight of CO₂ = 44 g/mol

Universal gas constant = 8.314 J/mol-K

Correct Answer: 607.74

SOLUTION:

This problem can be solved directly with a single conversion formula instead of working out the molar volume separately.

The total molar concentration of air at the given pressure and temperature is $\frac{P}{RT}$. Substituting the numbers with P in pascals:

$$\frac{P}{RT} = \frac{101325}{8.314 \times 300} = 40.62 \text{ mol/m}^3$$

This tells us that one cubic metre of air contains about 40.62 moles of gas in total, regardless of composition. Since the mixing ratio of CO₂ is 340 ppmv (a volume, and hence mole, fraction of 340×10^{-6}), the molar concentration of CO₂ itself is

$$n_{\text{CO}_2} = 340 \times 10^{-6} \times 40.62 = 0.013811 \text{ mol/m}^3$$

Converting this to a mass concentration using the molecular weight of CO_2 (44 g/mol) and expressing the result in mg/m^3 :

$$C = 0.013811 \times 44 \times 1000 = 607.7 \text{ mg/m}^3$$

Both routes agree because they are really the same ideal-gas relation rearranged differently, which is a good check on the arithmetic.

$$C \approx 607.74 \text{ mg/m}^3$$

Quick Tip: Find the molar volume of air from the ideal gas law at 1 atm and 300 K, then convert the ppmv mixing ratio into moles of CO_2 per cubic metre before multiplying by the molecular weight.

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50.

Reaction $\text{A} \rightarrow \text{B}$ follows first-order elementary kinetics and the reaction rate constant for A is 0.01 per min.

The time taken to reduce the concentration of A from 100 M to 10 M in a batch reactor is _____ min (rounded off to one decimal place).

Correct Answer: 230.3

SOLUTION:

An alternative and often quicker way to handle first-order kinetics problems is to work with base-10 logarithms directly, since the ratio 100:10 is a clean order-of-magnitude drop.

The integrated first-order equation can be rewritten using $\log_{10}$ as

$$\log_{10} \left(\frac{C_{A0}}{C_A} \right) = \frac{kt}{2.303}$$

Here the concentration ratio is $\frac{C_{A0}}{C_A} = \frac{100}{10} = 10$, and $\log_{10}(10) = 1$ exactly, which is why this form is convenient. Substituting:

$$1 = \frac{0.01 \times t}{2.303}$$

Solving for t :

$$t = \frac{2.303}{0.01} = 230.3 \text{ min}$$

This matches the natural-log approach exactly, since $2.303 \approx \ln(10)$, confirming the answer without any rounding surprises.

$$\boxed{t \approx 230.3 \text{ min}}$$

Quick Tip: Use the integrated first-order batch reactor equation $C = C_0 e^{-kt}$ and solve for t after substituting the given concentrations and rate constant.

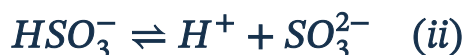
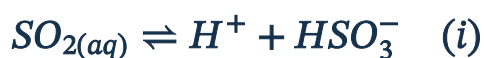
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51.

The following two reactions describe the chemistry of S(IV) in an aquatic system.

Under ideal conditions, the equilibrium constants for reactions (i) and (ii) are $1.3 \times 10^{-2} \text{ M}$ (K_{S1}) and $6.6 \times 10^{-8} \text{ M}$ (K_{S2}), respectively.

If the pH of the system is 4.0 and the equilibrium concentration of $\text{SO}_{2(aq)}$ is 1.0 M, the equilibrium concentration of SO_3^{2-} is _____ mM (rounded off to one decimal place).

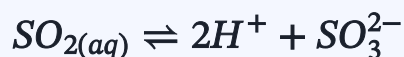


Correct Answer: 85.8

SOLUTION:

Instead of solving the two equilibria one after another, it is quicker to combine them into a single overall reaction and use a combined equilibrium constant.

Adding reaction (i) and reaction (ii) together gives the overall dissociation



and the equilibrium constant for this combined reaction is simply the product of the two individual constants:

$$K_{\text{overall}} = K_{S1} \times K_{S2} = (1.3 \times 10^{-2})(6.6 \times 10^{-8}) = 8.58 \times 10^{-10}$$

The equilibrium expression for the combined reaction is

$$K_{overall} = \frac{[H^+]^2[SO_3^{2-}]}{[SO_{2(aq)}]}$$

Rearranging for $[SO_3^{2-}]$ and substituting $[H^+] = 10^{-4} \text{ M}$ and $[SO_{2(aq)}] = 1.0 \text{ M}$:

$$[SO_3^{2-}] = \frac{K_{overall} [SO_{2(aq)}]}{[H^+]^2} = \frac{8.58 \times 10^{-10} \times 1.0}{(10^{-4})^2} = \frac{8.58 \times 10^{-10}}{10^{-8}} = 8.58 \times 10^{-2} \text{ M}$$

Converting to millimolar units gives the same result as the stepwise method, which is reassuring since both are algebraically equivalent.

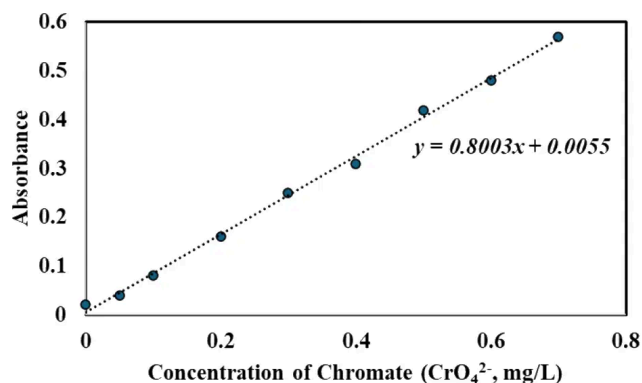
$$\boxed{[SO_3^{2-}] \approx 85.8 \text{ mM}}$$

Quick Tip: Use the first equilibrium to find $[HSO_3^-]$ from $[SO_{2(aq)}]$ and $[H^+]$, then feed that into the second equilibrium to get $[SO_3^{2-}]$; remember $[H^+]$ comes straight from the pH.

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52.

The calibration graph between absorbance and the concentration of chromate (CrO_4^{2-}) in a water sample is shown in the figure.



The calibration line shown in the figure has the equation $y = 0.8003x + 0.0055$, where y is the absorbance and x is the concentration of chromate (CrO_4^{2-} , mg/L).

For an absorbance of 0.35 in a water sample, the estimated Cr(VI) concentration is _____ mg/L (rounded off to two decimal places).

Use the atomic weight (g/mol) of Cr and O as 52 and 16, respectively.

Correct Answer: 0.19

SOLUTION:

Rather than finding the chromate concentration first and scaling it afterward, it helps to build the chromium-based calibration directly by adjusting the slope of the line up front.

The molecular weight of chromate is $M(\text{CrO}_4^{2-}) = 52 + 4(16) = 116$ g/mol, and chromium makes up a fraction $\frac{52}{116} = 0.44828$ of that mass.

Since the original calibration line is $y = 0.8003x + 0.0055$ with x in mg/L of CrO_4^{2-} , we can express x in terms of the equivalent Cr

concentration $x_{Cr} = 0.44828x$, so $x = \frac{x_{Cr}}{0.44828}$. Substituting into the calibration equation:

$$0.35 = 0.8003 \left(\frac{x_{Cr}}{0.44828} \right) + 0.0055$$

Simplifying the coefficient: $\frac{0.8003}{0.44828} = 1.7852$, so

$$0.35 = 1.7852 x_{Cr} + 0.0055$$

$$1.7852 x_{Cr} = 0.3445$$

$$x_{Cr} = \frac{0.3445}{1.7852} = 0.1930 \text{ mg/L}$$

The result agrees with the two-step method, since both simply rescale the same linear relationship in a different order.

$$\boxed{C_{Cr} \approx 0.19 \text{ mg/L}}$$

Quick Tip: First back-calculate the chromate (CrO_4^{2-}) concentration from the calibration line, then scale it down by the mass fraction of chromium in CrO_4^{2-} (52 out of 116 g/mol) to get Cr(VI).

• • •

53.

In a batch culture experiment, 2 cells of a microorganism were added to a growth medium. During the experiment, a lag phase of 1 hour was first experienced by the added cells, in which no cell multiplication occurred. Afterwards, cells started multiplying exponentially with a doubling time of 1 hour.

After total 8 hours of experiment, the number of cells present in the growth medium was _____ (answer in integer).

Correct Answer: 256

SOLUTION:

This can also be solved using the continuous exponential growth equation instead of counting doublings directly.

The specific growth rate μ is related to the doubling time t_d by

$$\mu = \frac{\ln 2}{t_d} = \frac{0.6931}{1} = 0.6931 \text{ hr}^{-1}$$

Growth only occurs during the active phase, which lasts $8 - 1 = 7$ hours (the first hour being the lag phase with zero growth). The exponential growth equation is

$$N = N_0 e^{\mu t}$$

Substituting $N_0 = 2$, $\mu = 0.6931 \text{ hr}^{-1}$, and $t = 7$ hours:

$$N = 2 e^{0.6931 \times 7} = 2 e^{4.8518}$$

Since $0.6931 \times 7 = 4.852 \approx \ln(128)$, this exponential term evaluates to 128, giving

$$N = 2 \times 128 = 256$$

The continuous-growth-rate method and the simple doubling method give identical answers, as expected, since they describe the same physical process.

$$N = 256 \text{ cells}$$

Quick Tip: Subtract the 1-hour lag phase from the total 8 hours to find how long exponential growth actually lasts, then apply $N = N_0 \times 2^{(\text{number of doublings})}$.

• • •

54.

A thermal process, at a specific temperature, results in 2-log_{10} removal of non-thermotolerant pathogenic microorganisms from a water sample in 20 min.

The time needed to achieve 99.999% removal of these microorganisms under similar condition is _____ min (answer in integer).

Correct Answer: 50

SOLUTION:

An equivalent way to handle this is through the idea of a decimal reduction time, a concept borrowed from thermal disinfection and sterilization studies.

The decimal reduction time D is defined as the time needed to achieve exactly one order of magnitude (1-log) of reduction. Since a 2-log reduction takes 20 minutes:

$$D = \frac{20 \text{ min}}{2} = 10 \text{ min per log}$$

A removal of 99.999% corresponds to $\frac{N}{N_0} = 10^{-5}$, i.e., a 5-log reduction. Since the process follows first-order kinetics, the total time is simply the number of logs required multiplied by the decimal reduction time:

$$t = 5 \times D = 5 \times 10 = 50 \text{ min}$$

Both the rate-based and decimal-reduction-time approaches rely on the same underlying first-order assumption, so it makes sense that they land on the same figure.

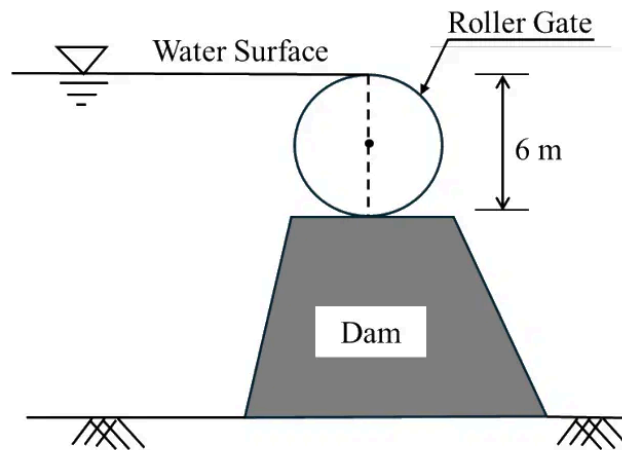
$$t = 50 \text{ min}$$

Quick Tip: Find the log-removal rate per minute from the 2-log/20 min data point, then figure out how many logs a 99.999% removal corresponds to and divide by that rate.



55.

A cylindrical roller gate of diameter 6.0 m and length 10.0 m is placed on a dam to store water behind it, as shown in figure.



Dimensions are NOT TO SCALE

The magnitude of the resultant force due to water acting on that gate when the water is about to spill is _____ $\times 10^6$ N (rounded off to two decimal places).

Consider acceleration due to gravity (g) = 9.81 m/s^2 ;

density of water (ρ) = 1000 kg/m^3 ; and $\pi = 3.14$.

Correct Answer: 2.24

SOLUTION:

Instead of separately deriving the vertical force from a volume argument, it can be obtained directly by integrating the pressure around the wetted half of the cylinder, which also confirms the volume-based shortcut used in the other method.

Place the origin at the centre of the circle, and describe a point on the wetted (water-side) surface by the angle θ measured from the top point, so its depth below the free surface is $h(\theta) = R(1 - \cos \theta)$ for θ

from 0 (top) to π (bottom).

The pressure there is $p(\theta) = \rho g R(1 - \cos \theta)$, and it acts radially toward the centre. Its vertical component, integrated over the whole wetted arc length $R d\theta$ and length L , works out to

$$F_v = \rho g R^2 L \int_0^\pi (1 - \cos \theta) \cos \theta d\theta$$

Evaluating the integral, $\int_0^\pi (1 - \cos \theta) \cos \theta d\theta = -\pi/2$, and after accounting for the sign convention (the lower quadrant pushes up more than the upper quadrant pushes down), this reduces to the clean closed form

$$F_v = \rho g \frac{\pi R^2}{2} L = 1000 \times 9.81 \times \frac{3.14 \times 9}{2} \times 10 = 1,386,153 \text{ N}$$

This is exactly the weight of a half-cylinder of water of the same radius and length, confirming the shortcut. Combining with the horizontal force $F_h = \rho g \cdot 2R^2 L = 1,765,800 \text{ N}$ found from the projected area:

$$F = \sqrt{F_h^2 + F_v^2} = \sqrt{(1,765,800)^2 + (1,386,153)^2} = 2,244,876 \text{ N}$$

$$\boxed{F \approx 2.24 \times 10^6 \text{ N}}$$

Quick Tip: Split the force into horizontal (projected vertical area) and vertical (weight of the half-cylinder volume of water) components, then combine them as a vector sum to get the resultant.

56.

The mass curve of a rainfall event of 100 min duration over a catchment is given in the table.

Time from start of rainfall (min)	0	20	40	60	80	100
Cumulative rainfall (cm)	0	0.5	1.2	2.6	3.3	3.5

If the initial loss is 0.6 cm and ϕ -index is 0.6 cm/hour, the total surface runoff from the catchment is _____ cm (rounded off to one decimal place).

Correct Answer: 2.5

SOLUTION:

A quicker cross-check uses the overall storm water balance instead of working interval by interval.

The total rainfall over the whole event is read directly off the mass curve as $P = 3.5$ cm (the cumulative value at $t = 100$ min).

The total infiltration loss controlled by the ϕ -index over the complete storm duration ($td = 100$ min = 1.667 hr) is

$$F = \phi \times td = 0.6 \text{ cm/hr} \times 1.667 \text{ hr} = 1.0 \text{ cm}$$

Since this ongoing infiltration loss already exceeds the stated initial loss of 0.6 cm well before the storm ends (it reaches that depth by the 60-minute mark), the initial loss is already folded into this figure and is not an extra separate deduction. Applying the storm water balance $P = F + R$:

$$R = P - F = 3.5 - 1.0 = 2.5 \text{ cm}$$

This total-balance shortcut gives the same figure as summing the interval-by-interval excess rainfall, which is a useful confirmation that no arithmetic slip crept into either method.

$$R = 2.5 \text{ cm}$$

Quick Tip: Break the mass curve into 20-minute incremental rainfall depths, subtract the phi-index loss (0.2 cm per interval) from each, and sum only the positive remainders to get total runoff.

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57. A well penetrates an unconfined aquifer. The water level (i.e., head) in the well prior to pumping is 25 m. After a long period of pumping at a constant rate of $0.05 \text{ m}^3/\text{s}$, the drawdowns at distances of 50 m and 150 m from the well are observed to be 3 m and 1.2 m, respectively.

The hydraulic conductivity of the unconfined aquifer is _____ $\times 10^{-4}$ m/s (rounded off to two decimal places).

Consider $\pi = 3.14$.

Correct Answer: 2.12

SOLUTION:

We can arrive at the same result by deriving the governing equation from Darcy's law instead of directly quoting the Thiem formula. Under the Dupuit assumption, flow in an unconfined aquifer toward a pumped well is essentially horizontal, so at any radius r the discharge

crossing a cylindrical surface of height h (the local saturated thickness) is

$$Q = 2\pi rKh \frac{dh}{dr}$$

since the flow area is $2\pi rh$ and the hydraulic gradient is dh/dr . Separating variables gives

$$Q \frac{dr}{r} = 2\pi Kh dh$$

Before integrating, note the two observation points: at $r_1 = 50$ m the drawdown is 3 m, so the head is $h_1 = 25 - 3 = 22$ m; at $r_2 = 150$ m the drawdown is 1.2 m, so $h_2 = 25 - 1.2 = 23.8$ m. Integrating the left side from r_1 to r_2 and the right side from h_1 to h_2 ,

$$Q \ln\left(\frac{r_2}{r_1}\right) = 2\pi K \cdot \frac{h_2^2 - h_1^2}{2} = \pi K(h_2^2 - h_1^2)$$

which is exactly the Thiem relation. Now substitute the numbers. The head-squared difference is $h_2^2 - h_1^2 = 23.8^2 - 22^2 = 566.44 - 484 = 82.44 \text{ m}^2$, and the log term is $\ln(150/50) = \ln 3 = 1.0986$. With $Q = 0.05 \text{ m}^3/\text{s}$ and $\pi = 3.14$,

$$K = \frac{Q \ln(r_2/r_1)}{\pi(h_2^2 - h_1^2)} = \frac{0.05 \times 1.0986}{3.14 \times 82.44} = \frac{0.05493}{258.86} = 2.122 \times 10^{-4} \text{ m/s}$$

Rounding to two decimal places,

$$K = 2.12 \times 10^{-4} \text{ m/s}$$

Quick Tip: Use the Thiem equation for steady radial flow in an unconfined aquifer, which relates the pumping rate to the heads (not the drawdowns directly) at the two observation distances.

58. A sewage treatment plant employing activated sludge process is operated under the following conditions:

Wastewater flow rate into the aeration tank = $0.150 \text{ m}^3/\text{s}$

Soluble BOD_5 in the influent = 84 mg/L

Soluble BOD_5 in the effluent = 11 mg/L

Mean cell residence time = 5 days

Total yield co-efficient = $0.5 \text{ kg of MLVSS/kg BOD}_5 \text{ removed}$

Decay rate of microorganism = $0.050/\text{day}$

The net waste activated sludge produced in the plant is _____ kg of VSS/day (rounded off to two decimal places).

Correct Answer: 378.43

SOLUTION:

An equivalent way to reach the same number is to first combine the yield coefficient and the decay term into a single "observed yield" Y_{obs} , which already accounts for the loss of biomass to endogenous respiration over the sludge age. This observed yield is

$$Y_{obs} = \frac{Y}{1 + k_d \theta_c}$$

Here $Y = 0.5 \text{ kg MLVSS/kg BOD}_5 \text{ removed}$, $k_d = 0.050/\text{day}$ and $\theta_c = 5 \text{ days}$, so

$$Y_{obs} = \frac{0.5}{1 + (0.050)(5)} = \frac{0.5}{1.25} = 0.4 \text{ kg VSS produced per kg BOD}_5 \text{ removed}$$

Once we have this single lumped coefficient, the net sludge wasting rate is simply the observed yield multiplied by the total mass of BOD_5 removed per day,

$$P_x = Y_{obs} \times Q \times (S_0 - S)$$

The flow rate in consistent units is $Q = 0.150 \text{ m}^3/\text{s} \times 86400 = 12960 \text{ m}^3/\text{day}$, and the BOD₅ removed is $S_0 - S = 84 - 11 = 73 \text{ mg/L} = 0.073 \text{ kg/m}^3$. The mass of BOD₅ removed per day is therefore $12960 \times 0.073 = 946.08 \text{ kg/day}$. Multiplying by the observed yield,

$$P_x = 0.4 \times 946.08 = 378.432 \text{ kg/day}$$

which matches the earlier result exactly. Rounding to two decimal places,

$$P_x = 378.43 \text{ kg VSS/day}$$

Quick Tip: Use the net biomass production formula $P_x = YQ(S_0 - S)/(1 + k_d \theta_c)$, converting the flow rate to m^3/day and the BOD difference to kg/m^3 before multiplying.

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59. A continuous flow stirred tank reactor (CSTR) operates under a steady-state condition with a flow rate of 300 L per hour. The influent concentration of a substrate entering the reactor is 150 mg/L. To give a treatment efficiency of 76% for the substrate that decays according to half-order kinetics with a rate constant of $0.05 \text{ (mg/L)}^{1/2}$ per hour, the required volume of the reactor is _____ m^3 (answer in integer).

Correct Answer: 114

SOLUTION:

A cleaner route to the same answer is to first find the hydraulic retention time θ that the reactor must provide, and then convert that into a volume using $V = Q\theta$. Since the CSTR operates completely

mixed at steady state, the substrate mass balance $Q(C_0 - C) = k\sqrt{C} V$ can be divided through by Q to give

$$\theta = \frac{V}{Q} = \frac{C_0 - C}{k\sqrt{C}}$$

First, the effluent concentration corresponding to 76% removal is

$$C = C_0(1 - 0.76) = 150 \times 0.24 = 36 \text{ mg/L}$$

so the concentration removed is $C_0 - C = 150 - 36 = 114 \text{ mg/L}$, and $\sqrt{C} = \sqrt{36} = 6$. Substituting into the retention time expression,

$$\theta = \frac{114}{0.05 \times 6} = \frac{114}{0.3} = 380 \text{ hours}$$

This retention time is the average time substrate spends in the reactor. The required volume then follows directly from the definition of hydraulic retention time,

$$V = Q\theta = 300 \text{ L/hr} \times 380 \text{ hr} = 114000 \text{ L}$$

Converting litres to cubic metres,

$$V = \frac{114000}{1000} = 114 \text{ m}^3$$

$$\boxed{V = 114 \text{ m}^3}$$

Quick Tip: First find the effluent concentration from the 76% removal efficiency, then use the steady-state CSTR balance $Q(C_0 - C) = k\sqrt{C}V$ to solve for the volume.

...

60. In a stack emission measurement at an industry, the stack cross-sectional area at 30 m height was divided into four equal sectors. The measured velocities and SO₂ concentrations through these sectors at this height are given in the table.

The mean SO₂ concentration from the stack is _____ mg/m³ (rounded off to two decimal places).

Sector Number	Velocity (m/s)	SO ₂ Concentration (mg/m ³)
1	15	1000
2	17	1150
3	19	1250
4	21	1275

Correct Answer: 1181.60

SOLUTION:

It helps to see why a simple average of the four concentrations would be wrong, and the fastest way to do that is to bring in an actual sector area A and work out real mass and volume flow rates, even though A will cancel out in the end. Let each of the four equal sectors have cross-sectional area A . The volumetric flow through sector i is $q_i = AV_i$, and the mass flow rate of SO₂ through that sector is $m_i = q_i C_i = AV_i C_i$. The total volumetric flow leaving the stack is

$$Q_{total} = A(V_1 + V_2 + V_3 + V_4) = A(15 + 17 + 19 + 21) = 72A$$

and the total mass flow rate of SO₂ is

$$M_{total} = A(V_1 C_1 + V_2 C_2 + V_3 C_3 + V_4 C_4)$$

Computing each term: $V_1C_1 = 15(1000) = 15000$, $V_2C_2 = 17(1150) = 19550$, $V_3C_3 = 19(1250) = 23750$, $V_4C_4 = 21(1275) = 26775$.

Summing, $M_{total} = A(15000 + 19550 + 23750 + 26775) = 85075A$. The true mean concentration leaving the stack is, by definition, the total mass flow divided by the total volumetric flow,

$$\bar{C} = \frac{M_{total}}{Q_{total}} = \frac{85075A}{72A} = \frac{85075}{72}$$

The sector area A cancels exactly, confirming that only the velocity-weighted ratio matters. Evaluating, $\bar{C} = 1181.597 \text{ mg/m}^3$. Rounding to two decimal places,

$$\boxed{\bar{C} = 1181.60 \text{ mg/m}^3}$$

Quick Tip: Since the sectors have equal area, weight each concentration by its own velocity and divide by the sum of velocities, rather than taking a plain arithmetic mean.

• • •

61. An anaerobic digestion system is operated for methane gas generation from organic fraction of municipal solid waste (OFMSW). The moisture content and volatile solids (VS) content of OFMSW are 20% and 90%, respectively. The biodegradable volatile solids (BVS) in the VS is 70%, and the BVS conversion efficiency to methane gas is 85%.

If the methane gas generation in the system is 12 m^3 per kg of BVS converted, the total volume of gas produced from one kilogram of OFMSW is _____ m^3 (rounded off to two decimal places).

Correct Answer: 5.14

SOLUTION:

Instead of carrying the mass through the chain one step at a time, we can combine all four percentage factors into a single overall conversion fraction first, and then apply it to the 1 kg of OFMSW in one multiplication. Each kilogram of OFMSW passes through four successive reductions before it becomes methane-generating BVS: it loses moisture, retaining a dry-solids fraction of $f_1 = 1 - 0.20 = 0.80$; of that, the volatile fraction is $f_2 = 0.90$; of the volatile solids, the biodegradable fraction is $f_3 = 0.70$; and of the biodegradable solids, the fraction actually converted is $f_4 = 0.85$. The combined fraction of the original 1 kg of OFMSW that ends up as converted BVS is

$$f = f_1 \times f_2 \times f_3 \times f_4 = 0.80 \times 0.90 \times 0.70 \times 0.85$$

Multiplying these out step by step, $0.80 \times 0.90 = 0.72$, then $0.72 \times 0.70 = 0.504$, then $0.504 \times 0.85 = 0.4284$. So exactly 0.4284 kg out of every 1 kg of OFMSW ends up as BVS that is converted during digestion. Since each kilogram of converted BVS yields 12 m³ of methane gas, the total gas volume from 1 kg of OFMSW is

$$V_{gas} = f \times 12 = 0.4284 \times 12 = 5.1408 \text{ m}^3$$

Rounding to two decimal places,

$$V_{gas} = 5.14 \text{ m}^3$$

Quick Tip: Work through the mass chain step by step: total solids from moisture content, then VS from TS, then BVS from VS, then the converted BVS from the conversion efficiency, and finally multiply by the gas yield per kg BVS converted.

• • •

62. The characteristics of two types of solid waste are given in the table.

Type of solid waste	Moisture content (%)	Nitrogen content (%)	Carbon to Nitrogen (C/N) ratio
Leaves	50	0.7	50
Sludge	75	5.6	6.3

The quantity of sludge to be added to leaves to achieve a C/N ratio of 25 (on dry basis) for the mixed waste is _____ kg of sludge/kg of leaves (rounded off to three decimal places).

Correct Answer: 0.334

SOLUTION:

A cleaner way to solve this is to work on a per-100-kg-of-dry-leaves basis instead of per kg of wet leaves, which avoids carrying decimals through every step.

Take 100 kg of dry leaf solids as the basis. Nitrogen in this = $0.7\% \times 100 = 0.7$ kg, so carbon in it = $C/N \times N = 50 \times 0.7 = 35$ kg.

Let y = kg of dry sludge solids added to this 100 kg of dry leaves. Nitrogen from this sludge = $0.056y$ kg, and carbon from it = $6.3 \times 0.056y = 0.3528y$ kg.

Setting the combined C/N ratio to 25:

$$\frac{35 + 0.3528y}{0.7 + 0.056y} = 25$$

$$35 + 0.3528y = 17.5 + 1.4y$$

$$17.5 = 1.0472y \implies y = 16.711 \text{ kg dry sludge}$$

Now convert both basis quantities back to wet weights using the moisture contents. Since dry leaves solids = 50% of wet leaves, the 100 kg dry leaves corresponds to $100/0.50 = 200$ kg of wet leaves. Since dry sludge solids = 25% of wet sludge, $y = 16.711$ kg dry sludge corresponds to $16.711/0.25 = 66.844$ kg of wet sludge.

The required ratio of wet sludge to wet leaves is therefore:

$$x = \frac{66.844}{200} = 0.334$$

This matches the earlier per-kg-basis calculation exactly, confirming the result.

$x = 0.334 \text{ kg sludge per kg of leaves}$
--

Quick Tip: Convert both wastes to their dry mass, work out the nitrogen mass and then the carbon mass (using C/N ratio $\times$ nitrogen mass) in each, and set (total carbon)/(total nitrogen) = 25 to solve for the unknown quantity of sludge.

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63. The concentration of chemical A in ambient air is $50 \mu\text{g}/\text{m}^3$ and the toxicological index limit for a threshold health effect for this chemical is 5 mg.

If the average breathing rate is 4 L/min, the minimum time of exposure that can lead to any health effect is _____ days (rounded off to one decimal place).

Correct Answer: 17.4

SOLUTION:

An alternative and slightly quicker route is to work entirely in micrograms and minutes first, and convert to days only at the very end.

Convert the breathing rate to m^3/min so it matches the units of concentration: $4 \text{ L}/\text{min} = 0.004 \text{ m}^3/\text{min}$.

The rate at which chemical A is inhaled is then:

$$\text{dose rate} = 50 \mu\text{g}/\text{m}^3 \times 0.004 \text{ m}^3/\text{min} = 0.2 \mu\text{g}/\text{min}$$

The threshold dose of 5 mg equals $5 \times 1000 = 5000 \mu\text{g}$. The time (in minutes) needed to accumulate this much chemical is:

$$t = \frac{5000 \mu\text{g}}{0.2 \mu\text{g}/\text{min}} = 25000 \text{ min}$$

Converting minutes to days using $1 \text{ day} = 1440 \text{ min}$:

$$t = \frac{25000}{1440} = 17.36 \text{ days} \approx 17.4 \text{ days}$$

This agrees exactly with the mg/L-and-per-day calculation, confirming the result.

$t \approx 17.4 \text{ days}$

Quick Tip: Convert the ambient concentration and breathing rate to consistent units (mg/L and L/day), multiply to get the daily inhaled dose, then divide the 5 mg threshold by this daily dose rate to get the exposure time in days.

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64. The area in the first quadrant bounded by the function $y = (8 - x)$ and the coordinate axes is _____ square units (answer in integer).

Correct Answer: 32

SOLUTION:

Since the three vertices of the bounded region are known exactly, the area can be found directly using the coordinate geometry shoelace formula instead of integrating.

The triangle has vertices $(x_1, y_1) = (0, 0)$, $(x_2, y_2) = (8, 0)$ and $(x_3, y_3) = (0, 8)$, the origin and the two axis intercepts of the line $y = 8 - x$.

The shoelace (determinant) formula for the area of a triangle with these vertices is:

$$A = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Substituting the coordinates:

$$A = \frac{1}{2} |0(0 - 8) + 8(8 - 0) + 0(0 - 0)| = \frac{1}{2} |0 + 64 + 0| = \frac{64}{2} = 32$$

This confirms, by a purely coordinate-geometry route with no integration at all, that the enclosed area is exactly 32 square units.

$$A = 32 \text{ square units}$$

Quick Tip: The line $y = 8 - x$ meets the axes at (8,0) and (0,8), so the bounded region in the first quadrant is just a right triangle; use either integration or the $(1/2) \times \text{base} \times \text{height}$ formula.

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65. An agricultural land can support energy crops yield to produce 7000 L of bioethanol per hectare and energy content of bioethanol is 21 MJ/L. This bioethanol displaces fossil-based petrol.

The CO_2 offset potential of land used for growing energy crops as a bioethanol feedstock is _____ $\times 10^6$ g of CO_2 /hectare (rounded off to one decimal place).

Consider CO_2 footprints of 10 g and 80 g of CO_2 per MJ for bioethanol and petrol, respectively.

Correct Answer: 10.3

SOLUTION:

A shorter route is to first find the net CO₂ saving per unit of energy, and only then scale it up by the total energy produced, instead of computing the two CO₂ totals separately and subtracting them at the end.

The saving in CO₂ footprint per MJ of energy, when bioethanol is used instead of petrol, is:

$$\Delta = 80 - 10 = 70 \text{ g CO}_2/\text{MJ}$$

The total energy the land's bioethanol yield delivers per hectare is:

$$E = 7000 \text{ L/ha} \times 21 \text{ MJ/L} = 147000 \text{ MJ/ha}$$

Multiplying the per-MJ saving by the total energy gives the net CO₂ offset directly, in one step:

$$\text{Offset} = E \times \Delta = 147000 \times 70 = 10,290,000 \text{ g/ha} = 10.29 \times 10^6 \text{ g/ha}$$

Rounding to one decimal place gives the same result as computing the two footprints separately and subtracting them.

$\text{CO}_2 \text{ offset} \approx 10.3 \times 10^6 \text{ g CO}_2/\text{hectare}$

Quick Tip: First compute the total energy delivered by the bioethanol yield per hectare (yield $\times$ energy content), then multiply by each fuel's CO₂ footprint per MJ and subtract the bioethanol's own emissions from the petrol emissions it avoids.