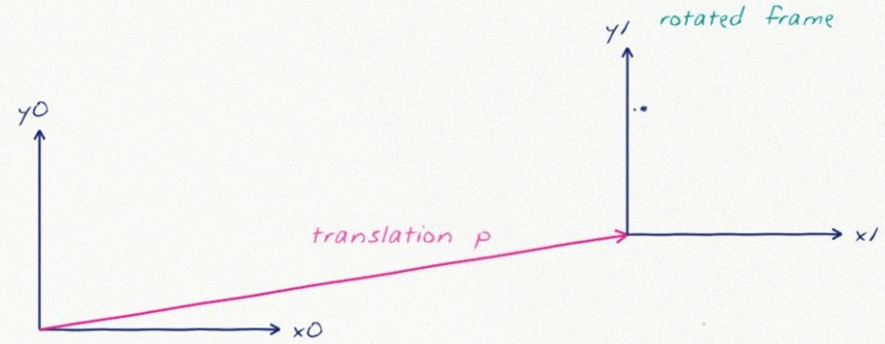
GATE Robotics Revision Notes

Complete Robotics Formula Book Shortcuts and GATE Checks

SYLLABUS MAP

Purpose :-

This final notebook consolidates the equations, solution routes, dimension checks and common traps used across robotics mathematics, mechanics, control, sensing and autonomy.



Coverage route :-

→ rotation checks
→ transform chains
→ planar arm
→ jacobian route
→ dynamic equation

How to revise :-

Read the physical idea first. Draw the model, write symbols, select the governing relation and finish with a limiting-case check.

ROTATION CHECKS

Definition :-

A valid rotation is orthogonal and has determinant positive one.

Concept :-

Use transpose for the inverse and inspect column norms before trusting numerical data.

$$R^T R = I, \quad \det R = 1$$

GATE check :-

A determinant of minus one indicates a reflection.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model
→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: TRANSFORM CHAINS.

Frame letters should cancel through the multiplication chain.

TRANSFORM CHAINS

Definition :-

Homogeneous transforms combine rotation and translation across named frames.

Concept :-

Frame letters should cancel through the multiplication chain.

$$T_{AC} = T_{AB}\, T_{BC}$$

GATE check :-

The inverse translation is minus R transpose p, not only minus p.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: PLANAR ARM.

Use the cosine rule for inverse kinematics and retain both elbow branches.

PLANAR ARM

Definition :-

Two-link geometry produces position by adding rotated link vectors.

Concept :-

Use the cosine rule for inverse kinematics and retain both elbow branches.

$$x = l_1 \cos q_1 + l_2 \cos(q_1 + q_2)$$

GATE check :-

Verify every inverse solution with forward kinematics.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: JACOBIAN ROUTE.

Its transpose maps task wrench to joint torque.

JACOBIAN ROUTE

Definition :-

The Jacobian maps differential joint motion to task motion.

Concept :-

Its transpose maps task wrench to joint torque.

$$\dot{x} = J\dot{q}, \quad \tau = J^T F$$

GATE check :-

At singularity, ordinary inverse formulas fail.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: DYNAMIC EQUATION.

Separate terms before substituting values so missing physics is visible.

Linked relation :-

$$M\ddot{q} + C\dot{q} + g = \tau + J^T F$$

DYNAMIC EQUATION

Definition :-

Manipulator torque balances inertia, velocity coupling, gravity and contact.

Concept :-

Separate terms before substituting values so missing physics is visible.

$$M\ \ddot{q} + C\ \dot{q} + g = \tau + J^T F$$

GATE check :-

M must have joint-inertia units and compatible dimensions.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: TRAJECTORY SCALING.

Longer duration reduces required speed and acceleration.

TRAJECTORY SCALING

Definition :-

Polynomial or trapezoidal profiles set smooth joint motion between boundary states.

Concept :-

Longer duration reduces required speed and acceleration.

$$q(t) = \sum a_i t^i$$

GATE check :-

Check extrema inside the interval, not only endpoints.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model
→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: FEEDBACK LOOP.

Use the characteristic equation for stability and transient analysis.

FEEDBACK LOOP

Definition :-

Closed-loop response depends on plant, controller and feedback path.

Concept :-

Use the characteristic equation for stability and transient analysis.

$$T=G/(1+GH)$$

GATE check :-

Confirm the feedback sign before writing the denominator.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: SECOND ORDER SHORTCUTS.

Overshoot falls as damping rises, while speed mainly scales with natural frequency.

SECOND ORDER SHORTCUTS

Definition :-

Natural frequency and damping ratio summarize dominant second-order response.

Concept :-

Overshoot falls as damping rises, while speed mainly scales with natural frequency.

> Ts approximately 4/(zetaomegan)

GATE check :-

The approximation assumes an underdamped dominant pair.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model
→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: PID CHECKS.

Integral removes bias but needs anti-windup under actuator saturation.

PID CHECKS

Definition :-

PID combines present, accumulated and predicted error effects.

Concept :-

Integral removes bias but needs anti-windup under actuator saturation.

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u=Kp et+Ki integral et+Kd e_dot
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GATE check :-

Derivative should be filtered when measurement noise is significant.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: STATE SPACE.

Controllability and observability rank tests precede pole placement.

STATE SPACE

Definition :-

State-space models connect internal state, input and measured output.

Concept :-

Controllability and observability rank tests precede pole placement.

$$\dot{x} = Ax + Bu,\quad y = Cx + Du$$

GATE check :-

Matrix dimensions reveal many setup errors immediately.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: MOTOR EQUATIONS.

Electrical and mechanical equations meet through these constants.

MOTOR EQUATIONS

Definition :-

Motor current creates torque and speed creates back emf.

Concept :-

Electrical and mechanical equations meet through these constants.

$T = K_t i$, $e = K_e \omega$

GATE check :-

Continuous torque is usually limited by RMS current and heating.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: GEAR SHORTCUTS.

Reflected load inertia decreases with ratio squared at the motor.

GEAR SHORTCUTS

Definition :-

Ideal reduction increases torque and lowers speed.

Concept :-

Reflected load inertia decreases with ratio squared at the motor.

$$\omega_{out} = \omega_{in}/N, \quad J_{ref} = J_L/N^2$$

GATE check :-

Include efficiency and define the direction of ratio.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: SAMPLING AND FILTERING.

Practical systems also use an analog anti-alias filter and generous rate margin.

SAMPLING AND FILTERING

Definition :-

Sampling must exceed twice the highest retained frequency.

Concept :-

Practical systems also use an analog anti-alias filter and generous rate margin.

$$f_s > 2 f_{max}$$

GATE check :-

A digital filter cannot remove aliasing that already occurred.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: KALMAN UPDATE.

Lower measurement covariance gives the sensor more influence.

Linked relation :-

$$K = P H^T (H P H^T + R)^{-1}$$

KALMAN UPDATE

Definition :-

Kalman gain weights prediction and measurement using their uncertainty.

Concept :-

Lower measurement covariance gives the sensor more influence.

$$K = P H^T (H P H^T + R)^{-1}$$

GATE check :-

Covariance units must match the associated state and measurement.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model
→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: MOBILE KINEMATICS.

Differential wheel speeds map to forward and angular velocity.

MOBILE KINEMATICS

Definition :-

A unicycle moves along its heading and turns by yaw rate.

Concept :-

Differential wheel speeds map to forward and angular velocity.

$$\dot{x} = v\cos\theta,\quad \dot{y} = v\sin\theta$$

GATE check :-

Wrap heading error after subtraction.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model
→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: STEREO DEPTH.

Larger baseline and focal length improve depth sensitivity within visibility limits.

STEREO DEPTH

Definition :-

Depth is inversely proportional to stereo disparity.

Concept :-

Larger baseline and focal length improve depth sensitivity within visibility limits.

$$z = fB/d$$

GATE check :-

Near-zero disparity creates very uncertain depth.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: GRAPH SEARCH.

Admissible consistent heuristics preserve optimality and reduce expansion.

GRAPH SEARCH

Definition :-

A star combines exact path cost and heuristic remaining cost.

Concept :-

Admissible consistent heuristics preserve optimality and reduce expansion.

fzgth

GATE check :-

Negative edge costs invalidate Dijkstra-style finalization.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model
→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: PROBABILITY CHECKS.

Normalization makes the final probability sum or integrate to one.

PROBABILITY CHECKS

Definition :-

Bayes rule combines likelihood and prior into a posterior belief.

Concept :-

Normalization makes the final probability sum or integrate to one.

$$P(x \mid z) \propto P(z \mid x)P(x)$$

GATE check :-

Conditional probabilities cannot be reversed without Bayes rule.

Question route :-

Mark the known state and the required output. Use the model assumptions before substituting any number.

→ draw the frame, graph or free-body model

→ keep signs, units and coordinates consistent

Answer check :-

Test one simple or limiting case. The result must match the physical trend and remain inside the stated assumptions.

→ reject options with impossible units or direction

Connection forward :-

Next topic: ROTATION CHECKS.

Use transpose for the inverse and inspect column norms before trusting numerical data.

QUICK FORMULA MAP

Core relations :-

Rotation Checks : $R^T R = I$, $\det R = 1$

Transform Chains : $T_{AC} = T_{AB}\ T_{BC}$

Planar Arm : $x = l_1 \cos q_1 + l_2 \cos(q_1 + q_2)$

Jacobian Route : $\dot{x} = J\dot{q}$, $\tau = J^T F$

Dynamic Equation : $M\ddot{q} + C\dot{q} + g = \tau + J^T F$

Trajectory Scaling : $q(t) = \sum a_i t^i$

Feedback Loop : $T = G/(1+GH)$

Second Order Shortcuts : $T_s \approx 4/(\zeta\omega_n)$

Pid Checks : $u = K_p e + K_i \int e + K_d \dot{e}$

State Space : $\dot{x} = Ax + Bu$, $y = Cx + Du$

Motor Equations : $\tau = K_t i$, $e = K_e \omega$

Recall ladder :-

→ state the model and coordinate frame

→ write symbols before numbers

→ test units and limiting behaviour

GATE TIPS & TRAPS

High-return checks :-

A determinant of minus one indicates a reflection.

The inverse translation is minus R transpose p, not only minus p.

Verify every inverse solution with forward kinematics.

At singularity, ordinary inverse formulas fail.

M must have joint-inertia units and compatible dimensions.

Common mistakes :-

Derivative should be filtered when measurement noise is significant.

Matrix dimensions reveal many setup errors immediately.

Continuous torque is usually limited by RMS current and heating.

Include efficiency and define the direction of ratio.

Final recall :-

Sketch first, state the frame, keep one sign convention and verify whether the answer is physically reachable.

Rapid recall checks :-

SAMPLING AND FILTERING: A digital filter cannot remove aliasing that already occurred.

GATE PROBLEM DRILL 1

Focus pair :-

Connect rotation checks with trajectory scaling. Many robotics questions hide this connection inside a diagram or data table.

$$R^T R = I,\ \det R = 1$$

Question route :-

Translate the wording into a state, input and output. Decide whether the relation is geometric, dynamic, probabilistic or algorithmic.

- → label every frame, axis and time index
- → separate measured data from estimated state
- → apply constraints before solving
- → use the simplest valid model first

$$q(t) = \sum a_i t^i$$

Fast elimination :-

A determinant of minus one indicates a reflection. Check extrema inside the interval, not only endpoints.

Answer check :-

Verify dimensions, range, sign and limiting behaviour. For a matrix answer, also check its shape and frame order.

GATE PROBLEM DRILL 2

Focus pair :-

Connect planar arm with second order shortcuts. Many robotics questions hide this connection inside a diagram or data table.

x=l1 cos q1+l2 cos(q1+q2)

Question route :-

Translate the wording into a state, input and output. Decide whether the relation is geometric, dynamic, probabilistic or algorithmic.

- → label every frame, axis and time index
- → separate measured data from estimated state
- → apply constraints before solving
- → use the simplest valid model first

∴ Ts approximately 4/(zetaomegan)

Fast elimination :-

Verify every inverse solution with forward kinematics. The approximation assumes an underdamped dominant pair.

Answer check :-

Verify dimensions, range, sign and limiting behaviour. For a matrix answer, also check its shape and frame order.

GATE PROBLEM DRILL 3

Focus pair :-

Connect dynamic equation with state space. Many robotics questions hide this connection inside a diagram or data table.

$$M\ q\_ddot + C\ q\_dot + g = T + J\ T\ F$$

Question route :-

Translate the wording into a state, input and output. Decide whether the relation is geometric, dynamic, probabilistic or algorithmic.

- → label every frame, axis and time index
- → separate measured data from estimated state
- → apply constraints before solving
- → use the simplest valid model first

$$x\_dot = A\ x + B\ u,\ y = C\ x + D\ u$$

Fast elimination :-

M must have joint-inertia units and compatible dimensions. Matrix dimensions reveal many setup errors immediately.

Answer check :-

Verify dimensions, range, sign and limiting behaviour. For a matrix answer, also check its shape and frame order.