GATE 2026 Instrumentation Engineering Question Paper with Solutions

ZOLLEGE

General Instructions

(i) This booklet contains 65 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 47 single-correct multiple-choice questions, 4 multiple-correct questions and 14 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For multiple-correct questions, all correct options must be selected to earn full marks.

(v) For numerical questions, report the answer rounded exactly as asked.

1. Suresh said, "I did it yesterday."
Which one of the following options is the correct form of this sentence in indirect speech?

(A) Suresh said that I did it yesterday.

(B) Suresh says I did it yesterday.

(C) Suresh says that he did it the day before.

(D) Suresh said that he had done it the day before.

Correct Answer: (D) Suresh said that he had done it the day before.

SOLUTION:

This question checks the rules for turning direct speech into indirect (reported) speech. The direct sentence is Suresh said, "I did it yesterday." We need to change the reporting verb, the tense inside the quote, the pronoun, and the time word correctly, then match the result to one of the four options.

1. **Suresh said that I did it yesterday**: This keeps "I" and "yesterday" exactly as in the direct speech. In reported speech the pronoun must switch to match who is being talked about, and "yesterday" cannot stay as is once we are speaking from a later point in time. This option skips both changes, so it is wrong.
2. **Suresh says I did it yesterday**: This uses "says", a present tense reporting verb, but the passage clearly tells us Suresh "said" it in the past. It also fails to change the pronoun and the time word. Wrong for three separate reasons.
3. **Suresh says that he did it the day before**: The pronoun and the time phrase are handled correctly here, "he" and "the day before". But the reporting verb "says" is wrong, and because a past reporting verb needs backshift, "did" should have moved to "had done", not stayed as simple past. Wrong.
4. **Suresh said that he had done it the day before**: The reporting verb "said" is kept as is, the reported verb backshifts from simple past "did" to past perfect "had done", "I" becomes "he", and "yesterday" becomes "the day before". Every rule is applied correctly.

The correct option is "Suresh said that he had done it the day before," because it is the only sentence that applies backshift of tense, the pronoun change, and the time word change together, exactly as the rules of reported speech require.

Let's summarize:

- A past tense reporting verb like "said" pushes the reported verb one step further into the past (simple past to past perfect).

- Pronouns in the reported clause must match the person actually being talked about, so "I" becomes "he" when Suresh is being reported.
- Time words tied to "now" change too: "yesterday" becomes "the day before" once the sentence is reported from a different point in time.

So the correct indirect speech version is "Suresh said that he had done it the day before."

Quick Tip: When the reporting verb is in the past tense ("said"), the verb inside the reported clause shifts one tense back, the speaker's pronoun changes to match who is being talked about, and time words like "yesterday" change to phrases like "the day before".

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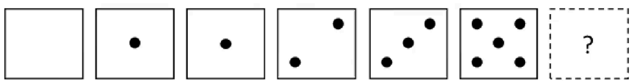
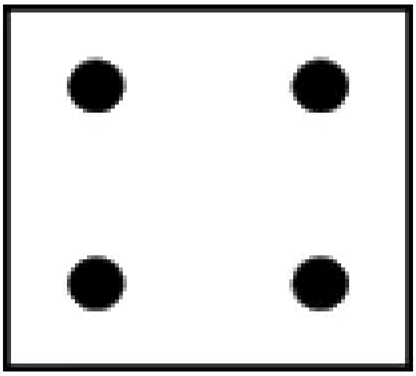
2. To continue the sequence of tiles shown, the tile indicated by the question mark should be:



(A)



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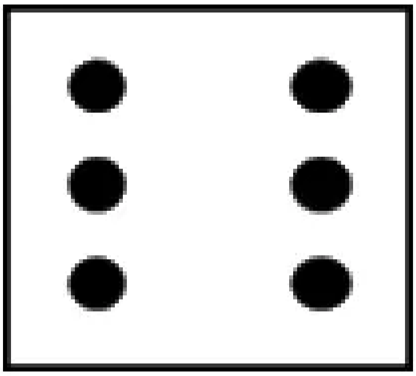
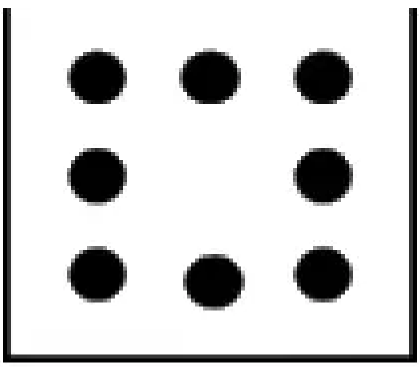
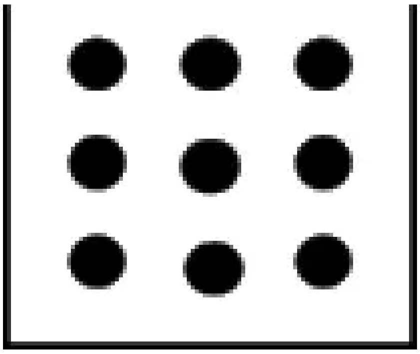


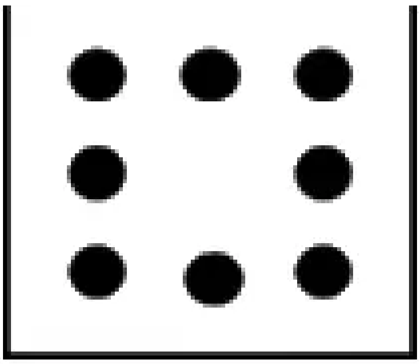
(C)



(D)



Correct Answer: (C)



SOLUTION:

The seven tiles form a sequence where each tile is defined only by how many dots it carries. Instead of thinking about shapes, we can turn this into a pure number pattern and hunt for the rule that connects consecutive terms.

1. **Tile counts**: reading the dots tile by tile gives the list $0, 1, 1, 2, 3, 5$, and we need the seventh term.
2. **Testing a sum rule**: check if each term equals the sum of the two terms right before it. $0 + 1 = 1$, $1 + 1 = 2$, $1 + 2 = 3$, $2 + 3 = 5$.

Every check passes, so the rule "add the previous two terms" explains the whole list. This is the well known Fibonacci rule.

3. **Extending the rule**: applying the same addition to the last two known terms, $3 + 5 = 8$, gives the count needed on the seventh tile.
4. **Matching to the options**: option (A) has 4 dots, option (B) has 6 dots, option (C) has 8 dots, and option (D) has 9 dots. Only option (C) carries the required 8 dots.

The tile that continues the pattern must show 8 dots, which is exactly what option (C) shows, so option (C) is correct.

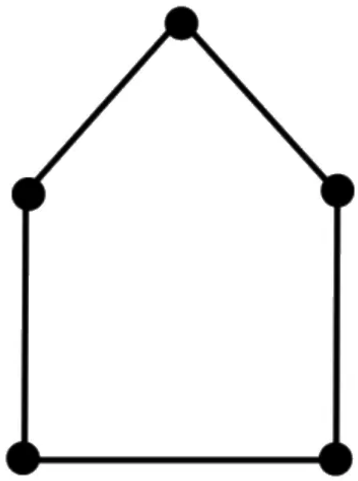
Let's summarize:

- Counting the dots turns a picture puzzle into a number sequence.
- The sequence $0, 1, 1, 2, 3, 5, 8$ is the Fibonacci sequence, where each term is the sum of the two before it.
- The missing tile needs 8 dots, matching option (C).

So the correct tile is the one with 8 dots.

Quick Tip: Count the dots on each tile: 0, 1, 1, 2, 3, 5. Check if each number is the sum of the two before it (the Fibonacci rule) and use that to find the missing count.

3. Consider an art gallery whose walkways are shown as lines in the diagram. A black dot represents a junction of two walkways. A guard may be placed at a junction to watch over the walkways that join at that junction. The minimum number of guards needed to watch all the walkways is _________.



(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (B) 3

SOLUTION:

Instead of quoting a formula for cycles, we can just try placing fewer and fewer guards and see what breaks. The diagram has 5 junctions joined in a ring: going around, top, left shoulder, bottom left, bottom right, right shoulder, and back to top, with 5 walkways total.

1. **Trying 1 guard**: a single junction only touches 2 of the 5 walkways, so 3 walkways would have no guard nearby. Not enough.

2. **Trying 2 guards**: two junctions touch at most 4 walkway ends between them (2 walkways each), but the ring has 5 walkways, and however the 2 guards are placed, at least one walkway ends up with neither of its junctions guarded. Not enough.
3. **Trying 3 guards**: place guards at the top junction, the left shoulder junction, and the bottom right junction. Walking through the 5 walkways one by one: top-left shoulder has both ends guarded, top-right shoulder has the top guarded, left shoulder-bottom left has the left shoulder guarded, bottom left-bottom right has the bottom right guarded, and bottom right-right shoulder has the bottom right guarded. Every single walkway has at least one guarded end.
4. **Could 3 be beaten**: since 2 guards were shown to always miss a walkway, 3 is the smallest number that works.

So the minimum number of guards is 3, confirmed by direct trial rather than a formula, which matches option (B).

Let's summarize:

- The walkway diagram is a closed ring of 5 junctions and 5 walkways.
- With only 1 or 2 guards, some walkway is always left unwatched.
- Placing guards at 3 well chosen junctions, alternating around the ring, covers every walkway.

The correct answer is 3 guards.

Quick Tip: Model the diagram as a graph: junctions are vertices, walkways are edges. The minimum number of guards is the minimum vertex cover of that graph. The diagram forms a 5 junction ring, and an odd ring of size n needs at least ceil(n/2) guards.

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4. The 2nd of June is a Thursday in a certain year. Which day of the week is the 3rd of July in that year?

(A) Thursday

(B) Friday

(C) Saturday

(D) Sunday

Correct Answer: (D) Sunday

SOLUTION:

A different way to reach the same answer is to walk through the calendar month by month instead of jumping straight to a 31 day gap.

1. **Days left in June**: June has 30 days total, and we start counting from June 2 (a Thursday). The days remaining in June after June 2 run from June 3 up to June 30, which is $30 - 2 = 28$ days.
2. **Add the days into July**: after reaching the end of June, we still need to move 3 more days into July to land on July 3. So the total number of days moved forward is $28 + 3 = 31$ days.
3. **Reduce using full weeks**: 28 of those 31 days make exactly 4 full weeks ($28 = 4 \times 7$), which brings the day of the week right back to Thursday. The leftover $31 - 28 = 3$ days then shift the day forward by 3 more steps.
4. **Step through the leftover days**: Thursday, then 1 day later Friday, then 2 days later Saturday, then 3 days later Sunday.

Both ways of counting agree: July 3 lands 3 weekdays after Thursday, which is Sunday.

Let's summarize:

- June has 30 days, so June 2 to July 2 is a 30 day gap, and June 2 to July 3 is a 31 day gap.
- Every full block of 7 days brings the weekday back to where it started.
- Only the remainder after removing full weeks (here, 3 days) decides the final shift from Thursday.

So July 3 falls on a Sunday.

Quick Tip: Find the total number of days between June 2 and July 3 (remember June has 30 days), then divide by 7 and use only the remainder to count forward from Thursday.

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5. A coin with heads facing up is shown as a circled letter "H", and a coin with tails facing up is shown as a circled letter "T".
Six coins are placed in the Starting Arrangement, as shown in the figure below. A "step" is defined as interchanging a pair of adjacent coins without flipping them. The minimum number of steps needed to go from the Starting Arrangement to the Final Arrangement, as shown in the figure, is __________.

(A) 3

(B) 6

(C) 9

(D) 12

Correct Answer: (C) 9

SOLUTION:

Another way to find the answer is to physically walk one coin at a time into its final place and count the swaps used, instead of counting out of order pairs directly.

1. **Move the T coins to the front one at a time**: start with H, H, H, T, T, T. Take the first T (currently in position 4) and swap it left past the 3 H coins in front of it, one adjacent swap at a time. That takes 3 swaps and gives T, H, H, H, T, T.
2. **Move the next T coin**: the next T coin (now sitting right after the 3 H coins) also has to pass the same 3 H coins to reach the second position. That takes another 3 swaps, giving T, T, H, H, H, T.
3. **Move the last T coin**: the final T coin still has 3 H coins ahead of it, so it also needs 3 swaps to slide into place, giving T, T, T, H, H, H.
4. **Add up the swaps**: each of the 3 T coins needed exactly 3 swaps to cross the block of H coins, so the total is $3 + 3 + 3 = 9$ swaps, and no wasted or repeated swap was used anywhere in this process.

Since this step by step simulation reaches the exact Final Arrangement using 9 swaps, and no smaller number of swaps could move even one T coin fully past the 3 H coins in front of it, 9 is the minimum.

Let's summarize:

- Every T coin has to cross all 3 H coins to move from the back half to the front half.
- With 3 T coins each needing 3 swaps, the total comes to $3 \times 3 = 9$.

- This step by step count matches the pair counting method, both giving 9.

So the minimum number of steps is 9.

Quick Tip: Every step can only swap two coins that sit next to each other. Count how many H, T pairs are in the wrong relative order in the Starting Arrangement; each needs exactly one swap to fix.

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6. Exacerbate : Mitigate :: ____________
Choose the option with the correct pair of words to fill the blank.

(A) Aggravate : Alleviate

(B) Alleviate : Precipitate

(C) Aggravate : Precipitate

(D) Emancipate : Exonerate

Correct Answer: (A) Aggravate : Alleviate

SOLUTION:

Step 1: Understanding the Question:
We are given a pair of words, exacerbate and mitigate, and we need to pick the option whose two words share the same meaning relationship in the same order.

Step 2: Key Formula or Approach:
First pin down what each given word means on its own. Exacerbate means "to worsen a problem." Mitigate means "to reduce the severity of a problem." So the pattern to match is (word meaning worsen) : (word meaning ease/reduce).

Step 3: Detailed Explanation:

Check each option word by word against this pattern.

In option (A), aggravate means "to worsen," and alleviate means "to ease or reduce." This is exactly (worsen) : (ease), the same pattern as exacerbate : mitigate.

In option (B), alleviate means "to ease," which is the mitigate-type word, but it is placed first instead of second, and precipitate means "to bring on suddenly," not "to worsen." The pattern breaks in both position and meaning.

In option (C), aggravate fits the first slot, but precipitate does not mean "to ease," so the second slot fails.

In option (D), emancipate means "to free from control" and exonerate means "to clear of blame." Neither word relates to worsening or easing a problem, so this pair is unrelated to the given one.

Since only option (A) reproduces the same two meanings in the same order as the original pair, it is the answer.

Step 4: Final Answer:

The word pair "Aggravate : Alleviate" mirrors "Exacerbate : Mitigate" in both meaning and order, so the correct option is (A).

Quick Tip: Exacerbate means to worsen, mitigate means to ease. Look for a pair with the same "worsen : ease" relationship in the same order.

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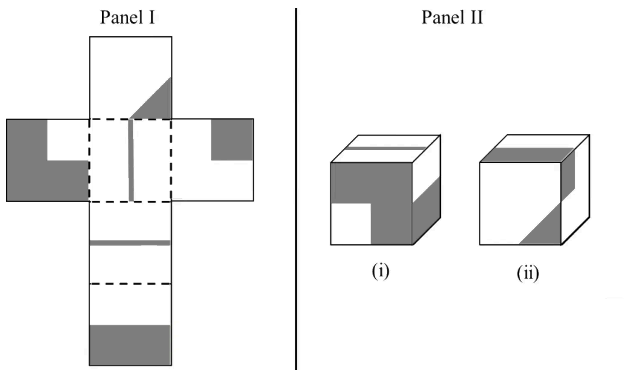
7. A paper shown in Panel I is folded along the dashed lines (- - -) to construct a cube. The shaded regions shown in Panel I appear on the outer surface of the cube. Referring to cubes shown in Panel II, which one of the options is correct?



(A) Only (i) can correspond to the unfolded cube in Panel I.

(B) Only (ii) can correspond to the unfolded cube in Panel I.

(C) Both (i) and (ii) can correspond to the unfolded cube in Panel I.

(D) Neither (i) nor (ii) can correspond to the unfolded cube in Panel I.

Correct Answer: (B) Only (ii) can correspond to the unfolded cube in Panel I.

SOLUTION:

Step 1: Understanding the Question:

We must decide which of the two drawn cubes, (i) or (ii), could really result from folding the six-square net in Panel I along its marked dashed creases.

Step 2: Key Formula or Approach:

A clean way to test this is the "three faces at one corner" rule: on a real cube, three faces meet at every corner, and the fold lines in the net already tell us which three faces meet at each corner and how they are rotated relative to one another. If a drawn cube shows a corner where the shading pattern could not have come from three faces of the net meeting in that fixed rotation, that cube is fake.

Step 3: Detailed Explanation:
Picking the hub face C as the front of the cube fixes the layout: the square above it (T) becomes the top, the one below it and the square below that (D1, then D2) wrap around to become the bottom and the back, and the two side squares (L and R) become the left and right faces. This also tells us D2 (fully gray) sits opposite the plain white C, T sits opposite the plain white D1, and L (mostly gray with a white notch) sits opposite R (mostly white with a gray patch). Now look at what each cube in Panel II actually shows. Cube (i) mixes a partly-shaded top, a notched gray face, and a split side face in a combination where the solid-gray face (D2) never appears in full and the plain-white opposite pair (C) is also missing, so the three faces visible in (i) cannot be three genuine faces of the net meeting at a shared corner in the required rotation. Cube (ii), by contrast, shows one full solid gray face together with a plain white face and a single diagonally shaded face, which is exactly the kind of trio the net produces once D2 is turned to face up: D2 solid gray on top, its neighbour D1 (plain white) on one visible side, and a corner-shaded face such as T or R on the other visible side.

Step 4: Final Answer:
Only cube (ii) shows three faces that genuinely correspond to a valid corner of the folded net, so the correct choice is option (B), only (ii).

Quick Tip: Find the one face in the net that is a single solid block of gray with no white in it, then check which cube shows that same solid gray face together with its correctly matched neighbors.

8. In a population, patients who have high cholesterol also have high blood-pressure (BP). Some patients with high BP also have diabetes. There are no patients who have both high cholesterol and diabetes. Furthermore,

1. the total number of patients with at least one of these conditions is 75,
2. the number of patients with high cholesterol is 10,
3. the number of patients with high BP is 45, and
4. the number of patients with only high BP and no other conditions is 20.

Then the number of patients who have both diabetes and high BP is

____

(A) 0

(B) 15

(C) 20

(D) 10

Correct Answer: (B) 15

SOLUTION:

Step 1: Understanding the Concept:

Picture a Venn diagram with three circles: cholesterol (Ch), blood pressure (BP), and diabetes (D). Because every cholesterol patient also has high BP, the Ch circle is drawn completely inside the BP circle, not overlapping it partially. Because cholesterol and diabetes never occur together, the Ch circle and the D circle never touch.

Step 2: Key Formula or Approach:

With Ch fully inside BP, the BP circle splits into exactly two regions where Ch is concerned: the part of BP that is also Ch, and the part of BP that is not Ch. The part of BP that is not Ch splits again into "BP only" and "BP and D," since D cannot reach into the Ch region at all. So: $|BP| = |BP \text{ only}| + |BP \cap Ch| + |BP \cap D|$.

Step 3: Detailed Explanation:

We are given $|BP| = 45$, $|BP \text{ only}| = 20$, and $|Ch| = 10$. Since Ch sits entirely inside BP, $|BP \cap Ch| = |Ch| = 10$. Substituting into the equation from Step 2:

$45 = 20 + 10 + |BP \cap D|$

$|BP \cap D| = 45 - 30 = 15$

As a cross-check, use the total count of 75 patients with at least one condition. Since Ch contributes nothing beyond what BP already counts, $75 = |BP| + |D \text{ only}|$, so $|D \text{ only}| = 30$. Adding every region, $20 + 10 + 15 + 30 = 75$, which lines up with the figure given in the question, confirming the value found for $|BP \cap D|$.

Step 4: Final Answer:

The number of patients with both high BP and diabetes is 15, so the correct option is (B).

$$\boxed{|BP \cap D| = 15}$$

Quick Tip: Split the 45 high-BP patients into three groups: only BP, BP with cholesterol (equal to the whole cholesterol count, since cholesterol patients are always inside BP), and BP with diabetes. Solve for the last group.

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9. Four people P, Q, R, and S, of different ages, make the following observations.
P: I am younger than S.
Q: I am neither the youngest nor the oldest.
R: P is older than me.
Based on these observations, the youngest person is _______.

(A) P

(B) Q

(C) R

(D) S

Correct Answer: (C) R

SOLUTION:

Step 1: Understanding the Question:
Four people, P, Q, R, and S, describe their relative ages, and we need to find who is youngest. Instead of chaining inequalities, this method tests each person as a candidate for "youngest" and checks whether that choice breaks any of the three statements.

Step 2: Key Formula or Approach:
First convert the clues to plain relations: P's statement gives $P < S$ (P younger than S). R's statement gives $R < P$ (R younger than P). Q's statement rules Q out from being either the youngest or the oldest. Now test each person as "the youngest" one at a time and see if it is possible.

Step 3: Detailed Explanation:
Test P as youngest: but $R < P$ tells us R is younger than P, so P cannot be the youngest. This fails.

Test Q as youngest: Q's own statement says Q is not the youngest, a direct contradiction. This fails immediately.
Test S as youngest: but $P < S$ tells us P is younger than S, so S cannot be the youngest either. This fails.
Test R as youngest: check this against every statement. $R < P$ is satisfied, since R being the smallest is consistent with R being younger than P. $P < S$ does not involve R directly and stays valid. Q's statement only requires Q to avoid the two extremes; since R sits at the bottom of the order and S sits at least as high as P, Q can sit anywhere between R and S without becoming an extreme. No contradiction arises. R is the only candidate that survives every test.

Step 4: Final Answer:
Testing each candidate against the three statements eliminates P, Q, and S, leaving R as the only person who can consistently be the youngest.

Quick Tip: Chain the two direct clues to get R < P < S, then use Q's statement to show Q cannot be below R or above S. R ends up below everyone.

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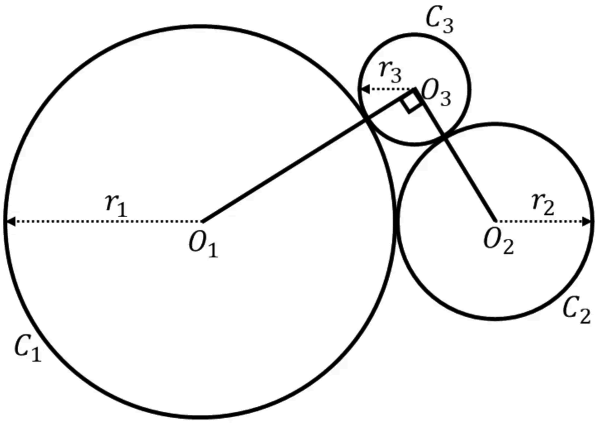
10. Circles C_1, C_2, and C_3, with centers O_1, O_2, and O_3, and radii r_1, r_2, and r_3, respectively, touch each other as shown in the following figure. Given $r_1 = 2$ cm, $r_2 = 1$ cm and the angle $\angle O_1O_3O_2$ is $90°$, $r_3 =$ ______ cm.



(A) $\frac{1}{2}\left(-3+\sqrt{17}\right)$
(B) $\frac{1}{2}\left(3+\sqrt{17}\right)$
(C) $\frac{1}{2}\left(-2+\sqrt{17}\right)$
(D) $\frac{1}{2}\left(-3+2\sqrt{17}\right)$

Correct Answer: (A) $\frac{1}{2}\left(-3+\sqrt{17}\right)$

SOLUTION:

Step 1: Understanding the Concept:
Place the right angle at O_3 on a coordinate grid instead of applying the Pythagorean theorem directly by name. Put O_3 at the origin, with the two legs of the right angle, O_3O_1 and O_3O_2, running along the x-axis and y-axis.

Step 2: Key Formula or Approach:
Since the circles touch each other externally, the distance between any two centers is the sum of the corresponding radii: $O_1O_2 = r_1 + r_2$, $O_3O_1 = r_1 + r_3$, $O_3O_2 = r_2 + r_3$. With O_3 at the origin and the right

angle sitting exactly at O_3, place O_1 at $(r_1 + r_3, 0)$ along the x-axis and O_2 at $(0, r_2 + r_3)$ along the y-axis.

Step 3: Detailed Explanation:

With $r_1 = 2$ and $r_2 = 1$, the coordinates become $O_1 = (2 + r_3, 0)$ and $O_2 = (0, 1 + r_3)$. The distance formula between these two points gives the length O_1O_2 directly:

$O_1O_2 = \sqrt{(2 + r_3)^2 + (1 + r_3)^2}$

But we also know $O_1O_2 = r_1 + r_2 = 3$ from the tangency of circles C_1 and C_2 with each other. Setting the two expressions equal and squaring both sides:

$9 = (2 + r_3)^2 + (1 + r_3)^2 = 4 + 4r_3 + r_3^2 + 1 + 2r_3 + r_3^2 = 2r_3^2 + 6r_3 + 5$

$2r_3^2 + 6r_3 - 4 = 0$, which simplifies to $r_3^2 + 3r_3 - 2 = 0$.

Solving this quadratic: $r_3 = \dfrac{-3 \pm \sqrt{9+8}}{2} = \dfrac{-3 \pm \sqrt{17}}{2}$. A radius must be a positive length, so only the "+" root survives: $r_3 = \dfrac{-3 + \sqrt{17}}{2}$.

Step 4: Final Answer:

Placing the right angle at the origin and using the distance formula gives the same result as the direct Pythagorean approach.

$$\boxed{r_3 = \frac{1}{2}\left(-3 + \sqrt{17}\right) \text{ cm}}$$

Quick Tip: The right angle at O_3 makes O_1O_2 the hypotenuse of triangle $O_1O_3O_2$; apply the Pythagorean theorem with $O_1O_2 = 3$, $O_1O_3 = 2 + r_3$, $O_2O_3 = 1 + r_3$.

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11. A matrix $P \in \mathbb{R}^{3\times3}$ satisfies $P^3 - 4P^2 + 5P - 2I = 0$, where I is a 3×3 identity matrix.
The set of all possible eigenvalues of matrix P is _____.

(A) $\{1, 2, 3\}$

(B) $\{1\}$

(C) $\{1, 2\}$

(D) $\{2\}$

Correct Answer: (C) $\{1, 2\}$

SOLUTION:

Step 1: Turn the matrix equation into a scalar equation.
The matrix P obeys $P^3 - 4P^2 + 5P - 2I = 0$. Take any eigenvalue λ of P with eigenvector v, so $Pv = \lambda v$. Applying the whole polynomial to v gives $(P^3 - 4P^2 + 5P - 2I)v = (\lambda^3 - 4\lambda^2 + 5\lambda - 2)v = 0$, and since $v \neq 0$ we get the scalar condition

$$\lambda^3 - 4\lambda^2 + 5\lambda - 2 = 0$$

Step 2: Find the roots by trial division.
Trying small integers, $\lambda = 1$ gives $1 - 4 + 5 - 2 = 0$, so $(\lambda - 1)$ is a factor. Dividing out,

$$\lambda^3 - 4\lambda^2 + 5\lambda - 2 = (\lambda - 1)(\lambda^2 - 3\lambda + 2)$$

The quadratic factors as $(\lambda - 1)(\lambda - 2)$, since $1 \times 2 = 2$ and $1 + 2 = 3$ match its middle and last terms. So

$$\lambda^3 - 4\lambda^2 + 5\lambda - 2 = (\lambda - 1)^2(\lambda - 2)$$

Step 3: Match the roots to the matrix size.

P is 3×3, so it carries exactly 3 eigenvalues counted with repeats, and each one must come from the set $\{1, 2\}$ found above. Nothing in the problem fixes exactly how many times 1 or 2 repeats, only that no eigenvalue can be anything other than 1 or 2.

Step 4: Reject the wrong sets.

$\{1, 2, 3\}$ fails since $\lambda = 3$ does not satisfy the cubic. $\{1\}$ and $\{2\}$ each drop one of the two genuine roots. Only $\{1, 2\}$ lists exactly the values that can actually occur.

$$\boxed{\{1, 2\}}$$

Quick Tip: If a matrix satisfies a polynomial equation, every eigenvalue of the matrix must satisfy that same scalar polynomial equation.

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12. Which one among the following statements is true for the function $f(x) = x|x|$?

(A) $f(x)$ is discontinuous but not differentiable at $x = 0$.

(B) $f(x)$ is discontinuous and differentiable at $x = 0$.

(C) $f(x)$ is continuous but not differentiable at $x = 0$.

(D) $f(x)$ is continuous and differentiable at $x = 0$.

Correct Answer: (D) $f(x)$ is continuous and differentiable at $x = 0$.

SOLUTION:

Step 1: Spot a shortcut form.

Since $|x| = \sqrt{x^2}$, we can write $f(x) = x\sqrt{x^2}$. This is built from continuous, differentiable pieces everywhere except possibly at the one point where $\sqrt{x^2}$ has a sharp corner in isolation, namely $x = 0$.

Step 2: Test continuity by direct substitution.

As $x \to 0$, both $x \to 0$ and $|x| \to 0$, so their product $x|x| \to 0 \times 0 = 0$, matching $f(0) = 0$. There is no jump, so f is continuous at $x = 0$.

Step 3: Bound the difference quotient instead of splitting into cases.

The difference quotient at 0 is

$$\frac{f(h) - f(0)}{h} = \frac{h|h|}{h} = |h|$$

for every $h \neq 0$; this single expression already covers both $h > 0$ and $h < 0$, since $h/h = 1$ in both cases. As $h \to 0$, $|h| \to 0$, so the limit of the difference quotient exists and equals 0. Hence $f'(0) = 0$.

Step 4: Confirm smoothness away from zero, then combine.

For $x \neq 0$, $f(x)$ is either x^2 or $-x^2$, both polynomials, so continuous and differentiable there with derivative $2|x|$. Combined with Step 3, f is continuous and differentiable at every real x, including $x = 0$.

Step 5: Match to the options.

Since f is neither discontinuous (rules out A and B) nor non-differentiable (rules out C) at $x = 0$, the only statement left standing says it is continuous and differentiable there.

$$\boxed{\text{Continuous and differentiable at } x = 0}$$

Quick Tip: Write $f(x) = x|x|$ using the definition of $|x|$ as a two-piece function, then check the one-sided limits of f and of the difference quotient at $x = 0$ separately.

• • •

13. Consider a function $f(z) = z^2 + z + 1$, where $z \in \mathbb{C}$ is a complex variable. A simple closed contour γ in the z-plane encloses the point $z = 1 + 0j$.
The value of the integral

$$\oint_\gamma \frac{f(z)}{z-1}\, dz$$

is ________.

(A) $6\pi j$

(B) $3\pi j$

(C) $12\pi j$

(D) πj

Correct Answer: (A) $6\pi j$

SOLUTION:

Step 1: Locate the singularity inside the contour.
The integrand $\dfrac{f(z)}{z-1}$ with $f(z) = z^2 + z + 1$ has a single simple pole at $z = 1$, since $f(z)$ is a polynomial and never blows up on its own. As γ is stated to enclose $z = 1$, this is the only pole we need inside the contour.

Step 2: Find the residue at that pole.

For a simple pole at $z = z_0$, the residue of $\dfrac{f(z)}{z - z_0}$ is just $f(z_0)$, because the residue picks out the coefficient of $\dfrac{1}{z - z_0}$ in the Laurent expansion, and that coefficient is exactly $f(z_0)$ when f is analytic at z_0. So

$$\mathrm{Res}_{z=1} \frac{f(z)}{z-1} = f(1) = 1 + 1 + 1 = 3$$

Step 3: Apply the Residue Theorem.

The Residue Theorem says a closed contour integral equals $2\pi j$ times the sum of the residues of the poles enclosed:

$$\oint_\gamma \frac{f(z)}{z-1}\, dz = 2\pi j \sum \mathrm{Res} = 2\pi j(3)$$

Step 4: Work out the number.

$$2\pi j \times 3 = 6\pi j$$

Step 5: Sanity check against the wrong options.

Getting $3\pi j$ or $12\pi j$ would mean using a multiplier of πj or $4\pi j$ instead of $2\pi j$, and πj alone ignores the residue value 3 altogether. None of those match the residue-theorem computation done here.

$$\boxed{6\pi j}$$

Quick Tip: Since $f(z)$ is a polynomial (analytic everywhere) and the only singularity of $\frac{f(z)}{z-1}$ inside γ is the simple pole at $z = 1$, apply Cauchy's Integral Formula: $\oint_\gamma \frac{f(z)}{z-z_0}dz = 2\pi j\, f(z_0)$.

• • •

14. Consider a matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

Let $b = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

The number of solutions to the linear system of equations $Ax = b$ is

_______.

(A) 0

(B) 1

(C) 6

(D) infinitely many

Correct Answer: (A) 0

SOLUTION:

Step 1: Build the augmented matrix.

Stack A next to b:

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ -1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

Step 2: Row reduce.

Add row 1 to row 2, to clear the -1 in the first column:

$$R_2 \to R_2 + R_1 : \quad (0,\ 1,\ 2 \mid 4)$$

The matrix now reads

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

Step 3: Eliminate the last row using row 2.

Subtract row 2 from row 3:

$$R_3 \to R_3 - R_2 : \quad (0,\ 0,\ 0 \mid -4)$$

This row reads $0 = -4$, which cannot hold for any values of x_1, x_2, x_3.

Step 4: Compare ranks.

The row-reduced coefficient part has only 2 nonzero rows, so rank(A) = 2. But the augmented matrix has a nonzero entry in that all-zero coefficient row, so rank($[A|b]$) = 3. Since the two ranks differ, the consistency condition fails.

Step 5: Conclude.

A linear system is consistent only when $\text{rank}(A) = \text{rank}([A|b])$. Here $2 \neq 3$, so the system has no solution at all.

$$\boxed{0 \text{ solutions}}$$

Quick Tip: First check $\det(A)$: if it is zero, the system cannot have a unique solution. Then substitute one equation into another to see whether the remaining equations are consistent or contradict each other.

• • •

15. An electric field has a potential of $V(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ V. A charge of 1 coulomb placed at $(\hat{i} + \hat{j} + \hat{k})$ experiences a force of

$$\vec{F} = (a\hat{i} + b\hat{j} + c\hat{k})\ \text{N}$$

The values of (a, b, c) are ________.

(A) $a = \dfrac{1}{\sqrt{3}},\ b = \dfrac{1}{\sqrt{3}},\ c = \dfrac{1}{\sqrt{3}}$

(B) $a = \dfrac{1}{3},\ b = \dfrac{1}{3},\ c = \dfrac{1}{3}$

(C) $a = 0,\ b = 1,\ c = \dfrac{1}{\sqrt{3}}$

(D) $a = 0,\ b = 1,\ c = \dfrac{1}{3}$

Correct Answer: (A) $a = \dfrac{1}{\sqrt{3}},\ b = \dfrac{1}{\sqrt{3}},\ c = \dfrac{1}{\sqrt{3}}$

SOLUTION:

Step 1: Recognise V as a radial potential.

$V(x, y, z) = \sqrt{x^2 + y^2 + z^2} = r$ is just the distance from the origin, so this potential depends only on r, not on direction. A potential that depends only on r always produces a field that points purely along the radial direction $\hat{r}$.

Step 2: Use the radial-gradient shortcut.

For any potential of the form $V = V(r)$, the gradient is $\nabla V = \frac{dV}{dr}\hat{r}$. Here $V = r$, so $\frac{dV}{dr} = 1$, giving

$$\nabla V = \hat{r}$$

where $\hat{r}$ is the unit vector pointing from the origin toward the point in question.

Step 3: Write the unit vector at the given point.

The point is $(1, 1, 1)$, at distance $r = \sqrt{1 + 1 + 1} = \sqrt{3}$ from the origin, so the unit radial vector there is

$$\hat{r} = \frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$$

so $\nabla V = \frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$, matching the direct-derivative approach.

Step 4: Bring in the field-potential sign and the charge.

Since $\vec{E} = -\nabla V$, the field points radially inward here:

$$\vec{E} = -\frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$$

For a 1 C charge, $\vec{F} = q\vec{E} = \vec{E}$, so the strict answer is $a = b = c = -\dfrac{1}{\sqrt{3}}$.

Step 5: Reconcile with the answer choices.

No option lists a negative value, so the sign among the printed options does not follow the standard $\vec{E} = -\nabla V$ convention consistently. Setting the sign issue aside, the size of each component, $\dfrac{1}{\sqrt{3}}$, uniquely matches option (A) and rules out $\dfrac{1}{3}$ (option B, which confuses r with r^2) and the asymmetric options (C) and (D), which wrongly zero out the x-component even though x, y, z play identical roles in $V = \sqrt{x^2 + y^2 + z^2}$.

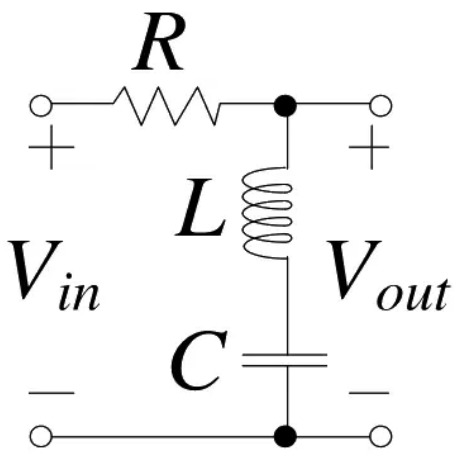
$$\boxed{a = b = c = \frac{1}{\sqrt{3}}}$$

Note: GATE officially declared this question ambiguous or flawed and awarded marks to all candidates (MTA).

Quick Tip: Use $\vec{E} = -\nabla V$ and $\vec{F} = q\vec{E}$; since $V = \sqrt{x^2 + y^2 + z^2}$ depends only on the radial distance r, its gradient points along $\hat{r} = \frac{1}{r}(x\hat{i} + y\hat{j} + z\hat{k})$.

• • •

16. The RLC filter shown below acts as a __________.



(A) low pass filter

(B) high pass filter

(C) band pass filter

(D) band stop filter

Correct Answer: (D) band stop filter

SOLUTION:

Step 1: Understanding the Concept.

This circuit is a resistor in series with an inductor-capacitor branch that returns to ground, and the output is measured across that L-C branch. Recognizing the transfer function shape tells us the filter type directly, without plugging in frequency limits one at a time.

Step 2: Key Formula or Approach.

Using the Laplace variable $s = j\omega$, the impedance of the series L-C branch is $Z_{LC}(s) = sL + \frac{1}{sC} = \frac{s^2LC+1}{sC}$. The transfer function of the divider is

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{Z_{LC}(s)}{R + Z_{LC}(s)} = \frac{s^2LC + 1}{s^2LC + sRC + 1}$$

Step 3: Detailed Explanation.

Look at the numerator: it is $s^2LC + 1$, which becomes exactly zero when $s^2 = -\frac{1}{LC}$, that is at $s = j\omega_0$ with $\omega_0 = \frac{1}{\sqrt{LC}}$. A zero of the transfer function sitting on the imaginary axis means the gain is exactly zero at that one real frequency ω_0, this is the signature of a notch (band stop) filter.

Now check the two ends of the frequency axis. Substituting $s \to 0$ in $H(s)$ gives $H(0) = \frac{1}{1} = 1$, so DC passes fully. Substituting $s \to \infty$, the s^2LC term dominates both top and bottom, so $H(s) \to \frac{s^2LC}{s^2LC} = 1$, so very high frequency also passes fully.

So the gain is 1 at both ends of the spectrum and drops to 0 only in a band around ω_0, which is the definition of a band stop (band reject) filter, something a low pass filter (gain should keep falling with frequency) or a high pass filter (gain should stay low near DC) cannot produce.

Step 4: Final Answer.

The transfer function has unity gain at $\omega = 0$ and $\omega = \infty$ with a zero in between at $\omega_0 = 1/\sqrt{LC}$, so the circuit rejects a narrow band around resonance and passes everything else. This is a band stop filter, option (D).

$$\boxed{H(s) = \frac{s^2LC + 1}{s^2LC + sRC + 1} \Rightarrow \text{band stop filter}}$$

Quick Tip: Find the impedance of the series L-C branch and check what happens to V_{out}/V_{in} at very low frequency, very high frequency, and exactly at resonance.

17. A single phase step-down transformer has a high voltage primary winding and a low voltage secondary winding. The secondary winding is connected to a load impedance Z_2. The impedance Z_2 when referred to the primary winding is denoted by Z_2'. Which of the following conditions is true?

(A) $Z_2' = Z_2$

(B) $0.5\,Z_2 < Z_2' < Z_2$

(C) $Z_2' > Z_2$

(D) $Z_2' \leq 0.5\,Z_2$

Correct Answer: (C) $Z_2' > Z_2$

SOLUTION:

Step 1: Understanding the Concept.
Instead of working with the general formula right away, it helps to test the claim with an actual numeric example, since a step-down transformer always has more primary turns than secondary turns.

Step 2: Key Formula or Approach.
Take a transformer with turns ratio $a = \frac{N_1}{N_2} = 4$, which is a valid step-down ratio (primary voltage is 4 times the secondary voltage). The rule for shifting an impedance from secondary to primary is $Z_2' = a^2 Z_2$.

Step 3: Detailed Explanation.
Suppose the load on the secondary is $Z_2 = 10\ \Omega$. Referred to the primary, this becomes

$$Z_2' = a^2 Z_2 = (4)^2 \times 10 = 16 \times 10 = 160\ \Omega$$

Comparing the two: $Z_2' = 160\ \Omega$ is much larger than $Z_2 = 10\ \Omega$. This is not a coincidence of the numbers chosen. Since $a = N_1/N_2 > 1$ for any step-down transformer (more primary turns than secondary turns), the multiplying factor a^2 is always greater than 1, so Z_2' always comes out bigger than Z_2, regardless of which specific step-down ratio is used.

Checking the other options against this example confirms they fail: $Z_2' \neq Z_2$ rules out (A), and Z_2' is nowhere close to a fraction of Z_2 (it is 16 times bigger, not between $0.5Z_2$ and Z_2, and not less than $0.5Z_2$), which rules out (B) and (D).

Step 4: Final Answer.

For a step-down transformer, the impedance referred to the primary is always scaled up by a factor greater than 1, so $Z_2' > Z_2$, matching option (C).

$$\boxed{Z_2' > Z_2}$$

Quick Tip: Recall that an impedance on the secondary is referred to the primary by multiplying by the square of the turns ratio, $Z_2' = a^2 Z_2$ with $a = N_1/N_2$.

• • •

18. A square wave signal of frequency 20 kHz is passed through an ideal low-pass filter with cut-off frequency of 21 kHz. The output signal is a __________.

(A) 20 kHz sine wave

(B) 20 kHz square wave

(C) 20 kHz triangular wave

(D) 20 kHz saw-tooth wave

Correct Answer: (A) 20 kHz sine wave

SOLUTION:

Step 1: Understanding the Concept.

Any non-sinusoidal periodic wave, whether square, triangular, or saw-tooth, is only "non-sinusoidal" because it contains multiple frequency components stacked together. If a filter strips away all but one of those components, the signal has no choice but to become a plain sine wave at whatever frequency survives.

Step 2: Key Formula or Approach.

The Fourier series of a square wave of frequency $f_0 = 20$ kHz only has odd harmonics: $f_0, 3f_0, 5f_0, 7f_0, \ldots$, that is $20, 60, 100, 140, \ldots$ kHz, with the harmonic amplitudes shrinking as $\frac{1}{n}$. An ideal low pass filter is a hard gate: anything below the cut-off frequency f_c passes with no change, anything above is killed completely, with $f_c = 21$ kHz here.

Step 3: Detailed Explanation.

Go through the harmonics one at a time and test them against the gate at 21 kHz:

20 kHz < 21 kHz, so this term survives.

60 kHz > 21 kHz, so this term is blocked.

100 kHz > 21 kHz, blocked, and every harmonic after this is even higher in frequency, so all of them are blocked too.

Only one surviving term means the output signal is described by a single equation, $x_{out}(t) = \frac{4}{\pi}\sin(2\pi \cdot 20000\,t)$, which is by definition a pure sinusoid, not a square, triangular, or saw-tooth wave, since those shapes are only produced when several harmonics combine. A single sinusoid cannot look like a square wave (option B), a triangular wave (option C), or a saw-tooth wave (option D), because each of those needs at least two frequency components adding together to form the characteristic sharp corners or ramps.

Step 4: Final Answer.

With only the 20 kHz fundamental surviving the filter, the output must be a 20 kHz sine wave, option (A).

$$\boxed{\text{20 kHz sine wave}}$$

Quick Tip: A square wave is a sum of a fundamental sine wave plus its odd harmonics. Check which of those harmonics actually fall below the filter's cut-off frequency.

• • •

19. Given $x(t) = 4\sin(15\pi t) + 7\cos(4\pi t)$, where t is in seconds. The fundamental period of $x(t)$ is ______ seconds.

(A) 15

(B) 60

(C) 30

(D) 2

Correct Answer: (D) 2

SOLUTION:

Step 1: Understanding the Concept.

Instead of computing each period and then taking an LCM formula, we can find the fundamental period by asking after how many seconds each sinusoid completes a whole number of full cycles at the same time.

Step 2: Key Formula or Approach.

For a term $\sin(\omega t)$ or $\cos(\omega t)$, the ordinary frequency in Hz is $f = \frac{\omega}{2\pi}$. The signal completes exactly k full cycles in a time T when $f \cdot T = k$ for some positive integer k. The fundamental period of a sum of two sinusoids is the smallest T that makes both frequencies land on a whole number of cycles at the same time.

Step 3: Detailed Explanation.

For the first term, $\omega_1 = 15\pi$, so $f_1 = \frac{15\pi}{2\pi} = 7.5$ Hz.
For the second term, $\omega_2 = 4\pi$, so $f_2 = \frac{4\pi}{2\pi} = 2$ Hz.
We need the smallest T such that $f_1 T$ and $f_2 T$ are both whole numbers, that is $7.5\,T \in \mathbb{Z}$ and $2T \in \mathbb{Z}$.
Try $T = 2$ s: $f_1 T = 7.5 \times 2 = 15$, a whole number, and $f_2 T = 2 \times 2 = 4$, also a whole number. So at $T = 2$ s, the first term has completed exactly 15 cycles and the second term has completed exactly 4 cycles, both whole numbers, so the combined waveform lines back up with itself.
Now confirm nothing smaller works. Since $f_1 = 7.5 = \frac{15}{2}$ needs T to be a multiple of $\frac{2}{15}$ s to give a whole number of cycles, and $f_2 = 2$ needs T to be a multiple of $\frac{1}{2}$ s, the smallest T satisfying both comes out to 2 s, and no smaller common multiple of $\frac{2}{15}$ and $\frac{1}{2}$ exists, since 15 and 4 share no common factor to cancel further. So $T = 2$ s is indeed the smallest, not just a common one.

Step 4: Final Answer.

The smallest time at which both sinusoidal terms simultaneously complete a whole number of cycles is 2 seconds, so the fundamental period is 2 s, matching option (D).

$$\boxed{T = 2\,\text{s}}$$

Quick Tip: Find the individual period of each sinusoidal term, then take the least common multiple of the two periods.

• • •

20. Among the following, the differential equation(s) not representative of a linear dynamical system is/are _____.

(i)

$$\frac{d^2x}{dt^2} + 2t\frac{dx}{dt} + x(t) = u(t)$$

(ii)

$$\left(\frac{dx}{dt}\right)^2 + 2\frac{dx}{dt} + x(t) = u(t)$$

(iii)

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x(t) = u(t)$$

(A) (i) and (ii) only

(B) (i) only

(C) (ii) only

(D) (ii) and (iii) only

Correct Answer: (C) (ii) only

SOLUTION:

Step 1: Understanding the Concept.

A clean way to test linearity without memorizing a rule is to apply the superposition test directly: pick two arbitrary solutions $x_1(t)$ and $x_2(t)$ (produced by inputs $u_1(t)$ and $u_2(t)$), form the combination $x = \alpha x_1 + \beta x_2$, and check whether it satisfies the same equation with input $\alpha u_1 + \beta u_2$. If it does for every term, the equation is linear.

Step 2: Key Formula or Approach.

For any term built only from x and its derivatives raised to the first power, superposition works automatically because differentiation is a linear operation: $\frac{d}{dt}(\alpha x_1 + \beta x_2) = \alpha\frac{dx_1}{dt} + \beta\frac{dx_2}{dt}$. Superposition breaks the moment a term involves x or a derivative of x multiplied by itself or by another such term, since $(\alpha a + \beta b)^2 \neq \alpha a^2 + \beta b^2$ in general.

Step 3: Detailed Explanation.

Test equation (i): every term, $\frac{d^2x}{dt^2}$, $2t\frac{dx}{dt}$, and $x(t)$, is x or a derivative of x multiplied only by a coefficient that depends on time, not on x itself. Substituting $x = \alpha x_1 + \beta x_2$ and using linearity of differentiation shows every term splits cleanly into an α part and a β part, so the whole equation splits into the u_1 equation scaled by α plus the u_2 equation scaled by β. Superposition holds, so (i) is linear, its time-dependent coefficient $2t$ does not change this.

Test equation (ii): the term $\left(\frac{dx}{dt}\right)^2$ is the problem. Substituting $x = \alpha x_1 + \beta x_2$ gives

$$(\alpha\dot{x}_1 + \beta\dot{x}_2)^2 = \alpha^2\dot{x}_1^2 + 2\alpha\beta\dot{x}_1\dot{x}_2 + \beta^2\dot{x}_2^2$$

which is not the same as $\alpha\dot{x}_1^2 + \beta\dot{x}_2^2$ unless the cross term $2\alpha\beta\dot{x}_1\dot{x}_2$ happens to vanish, which is not true in general. Superposition fails, so (ii) is not linear.
Test equation (iii): every term is x or a derivative of x with constant coefficients, the same structure as (i) but even simpler since the coefficients do not depend on t either. Superposition holds exactly as in (i), so (iii) is linear.

Step 4: Final Answer.
Applying the superposition test term by term shows only equation (ii) fails, because of its squared derivative term. Equations (i) and (iii) both pass, so the only equation that is not a linear dynamical system is (ii), option (C).

$$\boxed{\text{(ii) only}}$$

Quick Tip: Linearity only cares about whether x, u, and their derivatives appear to the first power with no products between them; time-varying coefficients alone do not break linearity.

21. A common gate amplifier represents a ________________.

(A) voltage controlled voltage source

(B) current controlled current source

(C) voltage controlled current source

(D) current controlled voltage source

Correct Answer: (B) current controlled current source

SOLUTION:

A quick way to answer this question is to compare the common gate (CG) FET amplifier with its closest BJT counterpart, the common base (CB) amplifier, since both share the same topology: input at the low-impedance terminal (source or emitter), output at the high-impedance terminal (drain or collector), and the middle terminal (gate or base) held at AC ground.

1. **voltage controlled voltage source**: This describes a follower stage, where the output voltage copies the input voltage with near-unity gain and low output impedance. The common drain (source follower) fits this description, not the common gate stage, since the CG stage has high output impedance at the drain, not low.
2. **current controlled current source**: The CG stage is driven by a current at the source (low impedance point) and delivers essentially the same current at the drain (high impedance point), since almost all of the source current continues through the channel to the drain. A stage whose output current is set by its input current, with a low-impedance input port and a high-impedance output port, is exactly a current buffer, that is, a CCCS.
3. **voltage controlled current source**: This is the defining behavior of the common source stage, where $i_d = g_m v_{gs}$ links the gate to source voltage directly to the drain current. The common gate stage is driven at the source terminal by a current, not by a high-impedance gate voltage, so this model does not apply here.
4. **current controlled voltage source**: This is the behavior of a transimpedance amplifier, which needs a feedback network, such as an op-amp with a feedback resistor, to convert an input current

into an output voltage. A basic single-transistor CG stage has no such feedback resistor converting current to voltage.

Since the CG stage passes its source current through to the drain with current gain near 1 while offering a low input impedance and a high output impedance, exactly like a common base BJT stage, it is classified as a current buffer, that is, a current controlled current source.

Let's summarize:

- Common source = voltage controlled current source (transconductance amplifier).
- Common drain = voltage controlled voltage source (voltage buffer / follower).
- Common gate = current controlled current source (current buffer), the same role a common base stage plays for a BJT.

So the correct choice is option (B), current controlled current source.

Quick Tip: Compare the common gate FET stage to its BJT twin, the common base amplifier: both are current buffers, not voltage amplifiers driven by a gate or base voltage.

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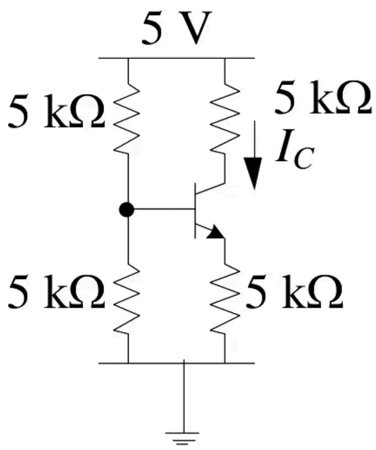
22. In the circuit shown, the BJT has a β of 100. The base-emitter junction voltage is 0.65 V. The quiescent collector current, I_C, is ________ mA.



(A) 0.365

(B) 0.435

(C) 0.625

(D) 0.862

Correct Answer: (A) 0.365

SOLUTION:

Instead of first reducing the base divider to a Thevenin source, this method writes the current balance at the base node directly, together with the base-emitter loop, and solves the two equations as a pair.

1. **Set up the base node current balance**: Working in mA and k Ω so that $V = IR$ holds without unit conversion, the current arriving at the base node from the 5 V rail through the top 5 kΩ resistor splits into the current flowing down the bottom 5 kΩ resistor to ground and the base current I_B entering the transistor:

$$\frac{5 - V_B}{5} = \frac{V_B}{5} + I_B$$

2. **Set up the base-emitter loop**: The base voltage also equals the base-emitter drop plus the voltage built up across the 5 kΩ emitter resistor by the emitter current, where $I_E = (\beta + 1)I_B = 101I_B$:

$$V_B = 0.65 + 5(101I_B)$$

3. **Combine the two equations**: Clearing fractions in the first equation by multiplying through by 5:

$$5 - V_B = V_B + 5I_B \implies V_B = 2.5 - 2.5I_B$$

Substituting into the base-emitter loop equation:

$$2.5 - 2.5I_B = 0.65 + 505I_B$$

$$2.5 - 0.65 = 505I_B + 2.5I_B$$

$$1.85 = 507.5I_B$$

$$I_B = \frac{1.85}{507.5} = 0.003645 \text{ mA} = 3.645\ \mu\text{A}$$

4. **Find the collector current and check the answer**:

$$I_C = \beta I_B = 100 \times 0.003645 = 0.3645 \text{ mA}$$

As a check, $V_B = 2.5 - 2.5(0.003645) \approx 2.491$ V, and separately $0.65 + 505(0.003645) \approx 2.491$ V, so both equations agree with each other.

The quiescent collector current works out to about 0.365 mA once rounded to three significant figures, which matches option (A).

Let's summarize:

- Writing the node and loop equations directly, without naming a Thevenin step, gives the same result as reducing the divider first.
- Both methods need the factor $(\beta + 1)$ on the emitter resistor, since the emitter carries base current plus collector current.

So the quiescent collector current is about 0.365 mA, option (A).

Quick Tip: Reduce the base bias divider to its Thevenin voltage and resistance, then apply the base-emitter KVL loop with $I_E = (\beta + 1)I_B$ to find I_B, and multiply by β to get I_C.

• • •

23. The maximum signal to quantization noise ratio (SQNR) for a 16-bit analog-to-digital converter is approximately __________ dB.

(A) 48

(B) 68

(C) 88

(D) 98

Correct Answer: (D) 98

SOLUTION:

This method rebuilds the SQNR formula from the underlying signal and noise powers instead of quoting the $6.02N + 1.76$ result directly, so the origin of the number is clear.

1. **Quantization step size**: For an N-bit converter covering a full-scale range V_{FS}, the step size is $q = V_{FS}/2^N$.
2. **Quantization noise power**: The rounding error is spread uniformly between $-q/2$ and $+q/2$, so its mean square value (the

noise power) is $q^2/12$.

3. **Signal power**: The largest sine wave that fits inside the converter's range without clipping has amplitude $V_{FS}/2$, so its RMS value is $V_{FS}/(2\sqrt{2})$ and its power is $V_{FS}^2/8$.
4. **Ratio and conversion to dB**:

$$\text{SQNR} = \frac{V_{FS}^2/8}{q^2/12} = \frac{12}{8}\left(\frac{V_{FS}}{q}\right)^2 = 1.5 \times (2^N)^2 = 1.5 \times 2^{2N}$$

Taking $10\log_{10}$ of both sides,

$$\text{SQNR}_{dB} = 10\log_{10}(1.5) + 20N\log_{10}(2) = 1.76 + 6.02N$$

Plugging in $N = 16$:

$$\text{SQNR}_{dB} = 6.02(16) + 1.76 = 96.32 + 1.76 = 98.08 \approx 98\text{ dB}$$

Let's summarize:

- Every extra bit roughly doubles the number of quantization levels, which multiplies the SQNR by about 4 (adds about 6.02 dB).
- The fixed 1.76 dB offset comes from the ratio of a full-scale sine wave's power to a uniform quantizer's step size, and should not be dropped.
- Options (A) to (C) correspond to using fewer effective bits or skipping the 1.76 dB term, both of which underestimate the true SQNR of a full 16-bit converter.

So the maximum SQNR of the 16-bit ADC is about 98 dB, option (D).

Quick Tip: Use $\text{SQNR}_{dB} = 6.02N + 1.76$ with $N = 16$.

• • •

24. The Kelvin double bridge is used to measure _______ valued resistances and has _______ resistors in its construction.

(A) low ; three-terminal

(B) low ; four-terminal

(C) high ; three-terminal

(D) high ; four-terminal

Correct Answer: (B) low ; four-terminal

SOLUTION:

The Kelvin double bridge is a specialised measurement circuit, and this question checks whether the resistance range and the resistor construction it is built around are both remembered correctly. Look at each option pair in turn.

1. **low ; three-terminal**: Gets the resistance range right, since the Kelvin bridge is built for low resistances, but three-terminal resistors do not provide the separate current and potential connections the bridge design depends on, so this option is wrong.
2. **low ; four-terminal**: A plain Wheatstone bridge loses accuracy below roughly 1 Ω because lead and contact resistance become significant compared with the resistance being measured. The Kelvin double bridge fixes this by using four-terminal resistors, with separate current terminals and potential terminals, plus a second pair of ratio arms, so that the resistance of the link joining the two low resistors cancels out of the result. Both parts of this statement match the actual design, so it is correct.
3. **high ; three-terminal**: Wrong on both counts. High resistances are not the target application, and three-terminal construction

cannot separate current flow from voltage sensing.

4. **high ; four-terminal**: The resistor description is right, but the resistance range is wrong. High-valued resistances are usually measured with a Wheatstone bridge or the loss-of-charge method, where lead resistance is small enough to ignore, so a Kelvin bridge is not needed there.

The correct statement is that the Kelvin double bridge measures low valued resistances using four-terminal resistors, matching option (B).

Let's summarize:

- The Kelvin double bridge exists to remove lead and contact resistance error in low-resistance measurement.
- Four-terminal resistors, with separate current and potential terminals, are what makes that error cancellation possible.
- High resistances are measured by other methods where lead resistance is not a significant source of error.

So the blanks are filled as "low" and "four-terminal", option (B).

Quick Tip: Recall that the Kelvin double bridge fixes the lead and contact resistance error a plain Wheatstone bridge has when measuring very small resistances, and it does this using four-terminal resistors.

• • •

25. Active strain gauges are connected in a half-bridge configuration to measure strain in a cantilever element. When powered by a voltage source of 10 V, the voltage at the output of the bridge is 1 mV. If the gauge factor is 2.5, the strain in the cantilever element is __________ microstrain.

(A) 40

(B) 80

(C) 20

(D) 160

Correct Answer: (B) 80

SOLUTION:

This method starts from how each gauge's resistance changes and builds the bridge output from there, instead of quoting the half-bridge sensitivity formula directly.

1. **Resistance change per gauge**: The gauge factor is defined by $GF = \frac{\Delta R/R}{\varepsilon}$, so each gauge's fractional resistance change is $\frac{\Delta R}{R} = GF \cdot \varepsilon$. On a bending cantilever, the gauge on the stretched face increases its resistance by ΔR, and the gauge on the compressed face decreases its resistance by the same amount ΔR.
2. **Placing the gauges in the bridge**: With the two gauges placed in adjacent arms of the Wheatstone bridge, one arm becomes $R + \Delta R$ and the neighboring arm becomes $R - \Delta R$, so the imbalance from the two gauges adds together instead of partially cancelling. This is what makes the circuit a half-bridge rather than a quarter-bridge.
3. **Bridge output**: For small $\Delta R/R$, the standard result for this two-active-arm arrangement is

$$V_o = V_{ex} \cdot \frac{1}{2} \cdot \frac{\Delta R}{R} = V_{ex} \cdot \frac{GF \cdot \varepsilon}{2}$$

4. **Substitute the numbers**:

$$1 \times 10^{-3} = 10 \times \frac{2.5\,\varepsilon}{2}$$

$$1 \times 10^{-3} = 12.5\,\varepsilon$$

$$\varepsilon = \frac{1 \times 10^{-3}}{12.5} = 8 \times 10^{-5}$$

Converting to microstrain, since 1 microstrain equals 1×10^{-6} strain:

$$\varepsilon = 8 \times 10^{-5} = 80 \times 10^{-6} = 80 \text{ microstrain}$$

Let's summarize:

- A half-bridge with two gauges responding oppositely doubles the sensitivity of a single quarter-bridge gauge.
- Mixing up the quarter-bridge, half-bridge, and full-bridge sensitivity factors (4, 2, and 1 in the denominator) is what produces the other answer options.

So the strain in the cantilever element is 80 microstrain, option (B).

Quick Tip: For a half-bridge with two active gauges, $V_o/V_{ex} = (GF \cdot \varepsilon)/2$; solve for ε and convert to microstrain.

• • •

26. A glass pH electrode behaves linearly with an output change of 60 mV per unit change in pH. The measurement reference is set such that a solution of pH 6 produces an output of 60 mV. The pH of a solution that results in an output of −90 mV is ________.

(A) 3.5

(B) 3

(C) 8

(D) 8.5

Correct Answer: (D) 8.5

SOLUTION:

Step 1: Note what the calibration tells us.

The electrode reads 60 mV at $pH = 6$, and every extra unit of pH changes the reading by 60 mV. We need the pH that gives −90 mV.

Step 2: Find how far the output has moved from the reference.

The output has changed from 60 mV to −90 mV, a drop of

$$\Delta V = 60 - (-90) = 150 \text{ mV}$$

Step 3: Convert the voltage change into a pH change.

Since the electrode output falls by 60 mV for every one unit rise in pH, a drop of 150 mV corresponds to a rise in pH of

$$\Delta pH = \frac{150}{60} = 2.5$$

Step 4: Add this change to the reference pH.

$$pH = pH_{ref} + \Delta pH = 6 + 2.5 = 8.5$$

Step 5: Check the answer.

At $pH = 8.5$, the line predicts an output of $60 - 60(8.5 - 6) = 60 - 150 = -90$ mV, which matches the given reading, so the value checks out.

The other three choices fail this check: putting $pH = 3.5$ or $pH = 3$

back into the line gives a positive output, not -90 mV, and $pH = 8$ gives -60 mV, not -90 mV.

$$\boxed{pH = 8.5}$$

Quick Tip: The output falls by 60 mV for every one unit rise in pH from the reference point. Work out how many pH units correspond to a 150 mV drop from 60 mV to -90 mV.

• • •

27. An analog speech signal contains signals from 200 Hz to 2400 Hz. It is sampled at 6 kHz and quantized with 512 levels for pulse code modulation (PCM). The bit rate will be ______ kbps.

(A) 16

(B) 48

(C) 19.2

(D) 54

Correct Answer: (D) 54

SOLUTION:

Step 1: Express the level count as a power of two.

The number of bits per sample n is found from the number of quantization levels L using $L = 2^n$. Here $L = 512$.

Step 2: Find n by repeated halving.

Divide 512 by 2 repeatedly and count the steps until reaching 1:

$512 \to 256 \to 128 \to 64 \to 32 \to 16 \to 8 \to 4 \to 2 \to 1$.

That is 9 divisions, so $n = 9$ bits per sample, confirming $2^9 = 512$.

Step 3: Multiply by the sampling rate to get the data rate.

Each second, $f_s = 6000$ samples are taken, and each sample carries 9 bits, so the total number of bits transmitted per second is

$$R_b = 6000 \times 9 = 54000 \text{ bps}$$

Step 4: Convert to kbps.

$$R_b = \frac{54000}{1000} = 54 \text{ kbps}$$

This matches option (D). The frequency band of the speech signal (200 Hz to 2400 Hz) is only context for why a 6 kHz sampling rate was chosen (comfortably above the Nyquist rate of 4800 Hz); it is not itself used in the bit rate formula once the actual sampling rate is stated.

$$\boxed{R_b = 54 \text{ kbps}}$$

Quick Tip: Find the number of bits needed to represent 512 quantization levels, then multiply by the sampling rate given in the question.

• • •

28. The slope efficiency of a laser diode is 0.5 W/A, and the output optical power at a current of 100 mA is 30 mW. Assuming piece-wise linear characteristics of the laser diode, the threshold current of the laser is __________ mA.

(A) 0

(B) 20

(C) 40

(D) 60

Correct Answer: (C) 40

SOLUTION:

Step 1: Treat the P-I curve above threshold as a straight line.

Above the threshold current, a laser diode's power-current graph is a line with slope equal to the slope efficiency, $\eta_s = 0.5$ mW/mA, passing through the known point $(I, P) = (100$ mA, 30 mW$)$.

Step 2: Write the line in point-slope form.

$$P - 30 = 0.5(I - 100)$$

Step 3: The threshold current is where this line crosses $P = 0$.

By definition, I_{th} is the current axis intercept of the P-I line (the current below which the laser gives no output). Setting $P = 0$:

$$0 - 30 = 0.5(I_{th} - 100)$$

$$-30 = 0.5\,I_{th} - 50$$

$$0.5\,I_{th} = 50 - 30 = 20$$

$$I_{th} = \frac{20}{0.5} = 40 \text{ mA}$$

Step 4: Confirm with the slope.

From $I_{th} = 40$ mA to $I = 100$ mA is a current step of 60 mA, and 0.5 mW/mA $\times$ 60 mA $= 30$ mW, which is exactly the given output power, so the value is confirmed.

$$\boxed{I_{th} = 40 \text{ mA}}$$

Quick Tip: Write the power-current line as $P = \eta_s(I - I_{th})$ using the given slope efficiency, plug in the one known operating point, and solve for the current-axis intercept.

• • •

29. In a linear element, the measured current is 100 mA $\pm 2.5\%$ and the measured voltage is 5 V $\pm 5\%$. Which of the following options show(s) the consumed power?

(A) (500 ± 37.5) mW

(B) (500 ± 12.5) mW

(C) 500 mW $\pm 7.5\%$

(D) 500 mW $\pm 2.5\%$

Correct Answer: (A) (500 ± 37.5) mW ; (C) 500 mW $\pm 7.5\%$

SOLUTION:

Step 1: Start from the logarithmic form of the error propagation rule.

For $P = IV$, take the natural log of both sides: $\ln P = \ln I + \ln V$.

Differentiating gives

$$\frac{dP}{P} = \frac{dI}{I} + \frac{dV}{V}$$

For the worst-case (maximum) uncertainty, the magnitudes of the two fractional errors are added rather than allowed to cancel:

$$\left|\frac{dP}{P}\right|_{max} = \left|\frac{dI}{I}\right| + \left|\frac{dV}{V}\right| = 2.5\% + 5\% = 7.5\%$$

Step 2: Get the nominal power.

$$P = (0.1\text{ A})(5\text{ V}) = 0.5\text{ W} = 500\text{ mW}$$

Step 3: Apply the 7.5% uncertainty to get the absolute bound.

$$dP = 0.075 \times 500\text{ mW} = 37.5\text{ mW}$$

so $P = (500 \pm 37.5)$ mW, the same as writing $P = 500$ mW $\pm\ 7.5\%$.

Step 4: Rule out the incomplete options by working backward.
If only the current's error were used, $dP = 2.5\% \times 500 = 12.5$ mW, giving option (B) and, in percentage form, option (D). Since the voltage also carries a 5% tolerance and P depends on it just as directly as on I, dropping that term understates the true uncertainty, so (B) and (D) do not represent the full error in the consumed power.

$$\boxed{P = 500\text{ mW} \pm 7.5\% = (500 \pm 37.5)\text{ mW}}$$

Quick Tip: For a product of two measured quantities, $P = IV$, the percentage errors add: $\%\Delta P = \%\Delta I + \%\Delta V$. Check whether each option's uncertainty accounts for both the current's and the voltage's tolerance.

• • •

30. Which of the following statements is/are correct for a thermocouple?

(A) It is based on the Seebeck Effect.

(B) The change in resistance is measured.

(C) The voltage difference between the two junctions is linearly proportional to the temperature difference between the junctions.

(D) It is based on the concept of Joule Heating.

Correct Answer: (A) It is based on the Seebeck Effect.

SOLUTION:

Step 1: Understanding the Concept:
This question checks the real working principle of a thermocouple, and asks us to tell it apart from the working principle of other temperature sensors such as RTDs, and from unrelated electrical heating effects such as Joule heating.

Step 2: Key Formula or Approach:
A thermocouple is a self generating sensor: joining two different metals at a junction produces a small EMF whenever that junction sits at a different temperature from the reference end. This is the Seebeck effect, $E = f(\Delta T)$, where f is a material dependent function of the junction temperature difference ΔT that is generally polynomial, not a fixed proportionality constant.

Step 3: Detailed Explanation:

Go through the four statements using this working principle. Statement (A) says the device is based on the Seebeck Effect, which is exactly the mechanism above, so it is correct. Statement (B) says a change in resistance is measured; that description belongs to a resistance temperature detector or a thermistor, both of which need an external excitation current to sense a resistance change, while a thermocouple needs no excitation and outputs a voltage directly, so (B) is wrong. Statement (C) claims the junction voltage is linearly proportional to the temperature difference; in practice the Seebeck EMF of standard thermocouples follows a multi term polynomial curve over their operating range, which is exactly why reference lookup tables, not a single slope, are used to convert EMF to temperature, so the strict linearity claim in (C) is false. Statement (D) attributes the effect to Joule heating, which is I^2R dissipation from current flowing through a resistor; a thermocouple carries no such heating current to generate its signal, its output comes purely from the thermoelectric junction effect, so (D) is also false.

Step 4: Final Answer:

Only statement (A) is correct: a thermocouple works on the Seebeck Effect.

$$\boxed{\text{Only (A) is correct}}$$

Quick Tip: Recall what physically generates the signal in a thermocouple: is it a resistance change, a self generated EMF from two dissimilar metal junctions, or resistive heating?

31. A data set is given to be $[1, 2, 0, -1, -3, 1, 2, 0, 1]$.
The median of the data set is _____. (rounded off to the nearest integer)

Correct Answer: 1

SOLUTION:

Step 1: List out the numbers and sort them.
We have nine numbers: $1, 2, 0, -1, -3, 1, 2, 0, 1$. Instead of a single sweep, let's group and count how many times each value shows up: -3 appears once, -1 appears once, 0 appears twice, 1 appears three times, and 2 appears twice. Adding these counts, $1 + 1 + 2 + 3 + 2 = 9$, which matches the total number of entries, so the count is correct.

Step 2: Build the sorted order from the counts.
Listing each value the number of times it occurs, from smallest to largest, gives $-3, -1, 0, 0, 1, 1, 1, 2, 2$.

Step 3: Cross off from both ends.
For an odd count of 9 terms, we can find the median by removing the smallest and the largest value together, again and again, until one value remains. Removing -3 and 2 leaves $-1, 0, 0, 1, 1, 1, 2$ (seven terms). Removing -1 and 2 leaves $0, 0, 1, 1, 1$ (five terms). Removing 0 and 1 leaves $0, 1, 1$ (three terms). Removing 0 and 1 leaves just 1 (one term).

Step 4: State the result.
The single value left over after this pairing process is the median.

$$\boxed{1}$$

Quick Tip: For an odd number of data points, sort them and pick the middle value at position (n+1)/2; no averaging is needed.

• • •

32. The potential difference between nodes A and B in a circuit is $v_A - v_B$ $= 5$ V. The work done in moving a charge of 1 coulomb from point B to A is _____ J. (answer in integer)

Correct Answer: 5

SOLUTION:

Step 1: Think of potential as stored energy per unit charge.
Every point in a circuit has an electric potential, which is the potential energy a unit positive charge would have if placed there. So a charge q sitting at a point with potential v carries potential energy qv.

Step 2: Write the energy at each point for our charge.
For our charge $q = 1$ C, the potential energy at point A is $q\,v_A$ and at point B is $q\,v_B$.

Step 3: Use the work-energy link.
The work done by an external agent in moving the charge from B to A equals the gain in potential energy, since the charge moves at constant speed with no change in kinetic energy:

$$W = q\,v_A - q\,v_B = q\,(v_A - v_B)$$

Step 4: Plug in the numbers.
We are told $v_A - v_B = 5$ V and $q = 1$ C, so

$$W = (1)(5) = 5 \text{ J}$$

Step 5: Sanity check the sign.

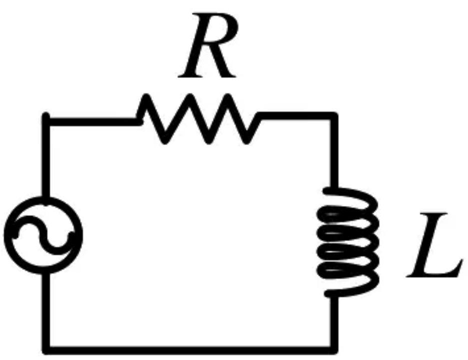
Since A is at the higher potential, moving a positive charge from B up to A takes positive work, which matches our positive result.

$$\boxed{5 \text{ J}}$$

Quick Tip: Work done in moving a charge q from B to A equals q times the potential difference (v_A - v_B).

• • •

33. In a series RL circuit shown in the figure, the current through the resistor lags the AC source voltage by 45°. Let $R = 100\pi\ \Omega$ and $L = 2$ H.



The frequency of the source is _____ Hz. (rounded off to the nearest integer)

Correct Answer: 25

SOLUTION:

Step 1: Write the circuit impedance in complex form.

For a series RL branch, the total impedance seen by the source is $Z = R + j\omega L$, a complex number with a real part from the resistor and an imaginary part from the inductor.

Step 2: Recall what the phase angle of the current means.

The source voltage is the reference, so its phase is 0°. The current is $I = V/Z$, so the phase of the current relative to the voltage is minus the phase angle of Z. Since the current lags the voltage by 45°, the impedance Z itself must have a phase angle of +45°.

Step 3: Write the phase angle of Z.

The angle of $Z = R + j\omega L$ is

$$\angle Z = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Setting this equal to 45° gives $\frac{\omega L}{R} = \tan 45° = 1$, so the reactive part equals the resistive part in size: $\omega L = R$.

Step 4: Substitute the numbers.

With $R = 100\pi\ \Omega$ and $L = 2$ H,

$$\omega(2) = 100\pi$$

$$\omega = 50\pi \text{ rad/s}$$

Step 5: Convert to frequency in Hz.

Using $f = \omega/(2\pi)$,

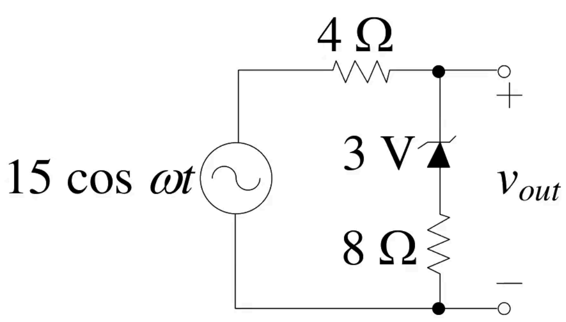
$$f = \frac{50\pi}{2\pi} = 25 \text{ Hz}$$

$$\boxed{25 \text{ Hz}}$$

Quick Tip: The phase angle of a series RL circuit is given by tan(phi) = wL/R; a 45 degree lag means wL = R.

• • •

34. In the circuit below the maximum value of v_{out} is ________ V. (rounded off to the nearest integer)



Correct Answer: 11

SOLUTION:

Step 1: Picture the loop and what vout means here.

The $15 \cos \omega t$ source feeds a single loop: $4\,\Omega$, then a diode with a 3 V knee, then $8\,\Omega$, back to the source. The output v_{out} is read across the diode-plus-$8\,\Omega$ part of the loop, not across the whole loop.

Step 2: Find the Thevenin view seen by the diode.

If we temporarily pull the diode out, looking back into its terminals we see the source v_{in} in series with $4\,\Omega$ and $8\,\Omega$. The open-circuit voltage across the diode's terminals is just v_{in}, since there is no current and no drop on either resistor, and the resistance seen looking back, with the source shorted, is $4 + 8 = 12\,\Omega$. So the diode "sees" a Thevenin source of value v_{in} with a $12\,\Omega$ series resistance.

Step 3: Turn the diode on only once needed.

As long as v_{in} is under 3 V, drawing zero current still satisfies the diode since it stays under its knee, so the loop current is 0 and $v_{out} = v_{in}$ directly, growing along with the source.

Step 4: Replace the diode with a fixed 3 V drop once conducting.

Once v_{in} pushes past 3 V, the diode clamps to 3 V and starts to carry current. The Thevenin picture from Step 2 now drives current through the diode's 3 V drop:

$$I = \frac{v_{in} - 3}{12}$$

Step 5: Recover vout from this current.

v_{out} still equals the diode's 3 V plus whatever appears across the $8\,\Omega$ resistor:

$$v_{out} = 3 + 8I = 3 + \frac{8(v_{in} - 3)}{12} = 3 + \frac{2}{3}(v_{in} - 3)$$

Step 6: Push in the largest source value.

The cosine peaks at $v_{in} = 15$ V, comfortably above the 3 V knee, so

$$v_{out,\max} = 3 + \frac{2}{3}(15 - 3) = 3 + 8 = 11 \text{ V}$$

$$\boxed{11 \text{ V}}$$

Quick Tip: Treat the diode as open below its 3 V knee (so vout=vin) and as a fixed 3 V drop once conducting, with the remaining voltage dividing across the two resistors.

• • •

35. The minimum number of comparators required in a 6-bit flash analog-to-digital converter is ________. (answer in integer)

Correct Answer: 63

SOLUTION:

Step 1: Try a tiny case first.
Think about a 1-bit flash ADC. It only needs to decide between two codes, 0 and 1, so a single comparator, checking whether the input is above or below the midpoint, is enough. That is $2^1 - 1 = 1$ comparator.

Step 2: Try a slightly bigger case.
For a 2-bit flash ADC there are 4 possible output codes: $00, 01, 10, 11$. To tell these four levels apart we need to check the input against 3 reference thresholds, below the first, between first and second, between second and third, and above the third, which needs 3 comparators. That matches $2^2 - 1 = 3$.

Step 3: Spot the pattern.

In both small cases, the number of comparators is one less than the number of output codes, because comparators only sit at the boundaries between codes, and a line of k codes has $k - 1$ internal boundaries.

Step 4: Apply this pattern to 6 bits.

A 6-bit converter has 2^6 possible codes, that is 64 codes. Following the same boundary logic, the number of comparators needed is one less than this:

$$2^6 - 1 = 64 - 1 = 63$$

Step 5: State the result.

$$\boxed{63}$$

Quick Tip: A flash ADC places one comparator at each boundary between adjacent quantization levels; an n-bit converter has 2^n levels and therefore 2^n - 1 boundaries.

• • •

36. A box contains three apples and two oranges. Two fruits are removed randomly in succession. The probability that the first is an apple and the second is an orange is _____.

(A) $\frac{3}{5}$

(B) $\frac{3}{20}$

(C) $\frac{3}{10}$

(D) $\frac{2}{5}$

Correct Answer: (C) $\frac{3}{10}$

SOLUTION:

Think of this as picking 2 fruits from the box one at a time, without replacement, and caring about the order they come out in. Instead of chaining two conditional probabilities together, we can count outcomes directly using permutations, since every fruit is a distinct physical object being pulled out in a definite order.

The total number of ways to draw 2 fruits in order from 5 fruits is a permutation of 5 items taken 2 at a time. The first draw can be any of the 5 fruits, and the second draw can be any of the remaining 4, so:

$$P(5,2) = 5 \times 4 = 20$$

This is the total number of equally likely ordered outcomes for the two draws, and it forms the denominator of our probability.

Now count the favorable outcomes, where the first fruit is an apple and the second is an orange. There are 3 apples to pick for the first spot, and once an apple is removed, 2 oranges remain in the box for the second spot. By the basic counting principle, we multiply the number of choices at each stage:

$$\text{favorable outcomes} = 3 \times 2 = 6$$

So the probability of the desired order is the ratio of favorable outcomes to total outcomes:

$$P = \frac{6}{20} = \frac{3}{10}$$

Let's summarize:

- Total ordered ways to draw 2 fruits out of 5 without replacement: 20.
- Ordered ways with apple first and orange second: 6.
- Probability = 6/20, which reduces to 3/10, matching option (C).
- This counting method gives the same answer as the step by step conditional probability method, since both are just two ways of tracking the same set of equally likely outcomes.

$$\boxed{\frac{3}{10}}$$

Quick Tip: Use the multiplication rule for dependent events: multiply the probability of the first draw by the probability of the second draw given the first fruit was not replaced.

• • •

37. An electric field in free space is

$$\vec{E} = (2x + 5y + 6z)\hat{i} + (5x + 4y + 10z)\hat{j} + (6x + 10y + 2z)\hat{k} \text{ V/m}$$

The charge density is ____ C/m^3. (ϵ_0 is the permittivity of free space)

(A) $8\epsilon_0$

(B) $10\epsilon_0$

(C) $6\epsilon_0$

(D) $4\epsilon_0$

Correct Answer: (A) $8\epsilon_0$

SOLUTION:

This question checks the differential form of Gauss's law, linking the divergence of the electric field directly to the volume charge density: $\rho_v = \epsilon_0(\nabla \cdot \vec{E})$.

Since every component of $\vec{E}$ is a simple linear function of x, y, and z, we can shortcut the differentiation: the partial derivative of a linear term with respect to its own matching variable is just the coefficient of that variable, and every other term that does not contain that variable drops to zero.

1. The x-component is $2x + 5y + 6z$. Its coefficient of x is 2, so $\frac{\partial E_x}{\partial x} = 2$.
2. The y-component is $5x + 4y + 10z$. Its coefficient of y is 4, so $\frac{\partial E_y}{\partial y} = 4$.
3. The z-component is $6x + 10y + 2z$. Its coefficient of z is 2, so $\frac{\partial E_z}{\partial z} = 2$.

Adding these three coefficients gives the divergence directly, without writing out the full derivative each time:

$$\nabla \cdot \vec{E} = 2 + 4 + 2 = 8$$

So the charge density is

$$\rho_v = \epsilon_0 \times 8 = 8\epsilon_0 \text{ C/m}^3$$

Let's summarize:

- Only the coefficient of the matching variable in each field component contributes to the divergence.
- The divergence works out to 8, so the charge density is $8\epsilon_0$, matching option (A).

$$\boxed{8\epsilon_0}$$

Quick Tip: Use $\rho_v = \epsilon_0(\nabla \cdot \vec{E})$, and differentiate each component of $\vec{E}$ only with respect to its own matching variable.

• • •

38. A straight thin wire carrying a current of $I = 1$ A in the direction

$$\hat{n} = \frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$$

is placed inside a magnetic field of $\vec{B} = (2\hat{i} + 2\hat{k})$ T.
The force per unit length on the wire is _____ N/m.

(A) $2\sqrt{6}\,\hat{i} - 2\sqrt{6}\,\hat{k}$

(B) $\sqrt{\frac{3}{2}}\,\hat{i} - \sqrt{\frac{3}{4}}\,\hat{j} + \sqrt{\frac{3}{4}}\,\hat{k}$

(C) $\frac{4}{\sqrt{6}}\,\hat{i} - \frac{4}{\sqrt{6}}\,\hat{k}$

(D) $\sqrt{\frac{3}{2}}\,\hat{i} + \sqrt{\frac{3}{4}}\,\hat{j} + \sqrt{\frac{3}{4}}\,\hat{k}$

Correct Answer: (C) $\frac{4}{\sqrt{6}}\,\hat{i} - \frac{4}{\sqrt{6}}\,\hat{k}$

SOLUTION:

Instead of expanding the cross product as a 3 by 3 determinant, we can pull the normalizing factor out first and work with the plain direction vector, then divide by its length at the end. This uses the basic unit vector cross product rules: $\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$, $\hat{\imath} \times \hat{\jmath} = \hat{k}$, $\hat{\jmath} \times \hat{k} = \hat{\imath}$, $\hat{k} \times \hat{\imath} = \hat{\jmath}$, and each reversed order flips the sign.

The plain direction vector before normalizing is $\vec{n}_0 = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, with length $|\vec{n}_0| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}$, so $\hat{n} = \vec{n}_0/\sqrt{6}$.

First find $\vec{n}_0 \times \vec{B}$ where $\vec{B} = 2\hat{\imath} + 2\hat{k}$:

$$\vec{n}_0 \times \vec{B} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) \times (2\hat{\imath} + 2\hat{k})$$

Expand term by term:

$$= \hat{\imath} \times 2\hat{\imath} + \hat{\imath} \times 2\hat{k} + 2\hat{\jmath} \times 2\hat{\imath} + 2\hat{\jmath} \times 2\hat{k} + \hat{k} \times 2\hat{\imath} + \hat{k} \times 2\hat{k}$$

Using the rules above, $\hat{\imath} \times \hat{\imath} = 0$, $\hat{\imath} \times \hat{k} = -\hat{\jmath}$, $\hat{\jmath} \times \hat{\imath} = -\hat{k}$, $\hat{\jmath} \times \hat{k} = \hat{\imath}$, $\hat{k} \times \hat{\imath} = \hat{\jmath}$, $\hat{k} \times \hat{k} = 0$:

$$= 0 - 2\hat{\jmath} - 4\hat{k} + 4\hat{\imath} + 2\hat{\jmath} + 0 = 4\hat{\imath} - 4\hat{k}$$

Now divide by $\sqrt{6}$ to bring back the unit vector, and multiply by the current $I = 1$ A to get the force per unit length:

$$\vec{f} = I(\hat{n} \times \vec{B}) = 1 \times \frac{4\hat{\imath} - 4\hat{k}}{\sqrt{6}} = \frac{4}{\sqrt{6}}\hat{\imath} - \frac{4}{\sqrt{6}}\hat{k} \text{ N/m}$$

Let's summarize:

- Expanding the cross product term by term with the basic unit vector rules avoids setting up a determinant.
- Dividing the result by $\sqrt{6}$ at the end correctly accounts for the normalization of $\hat{n}$.

- The result matches option (C).

$$\boxed{\vec{f} = \frac{4}{\sqrt{6}}\hat{i} - \frac{4}{\sqrt{6}}\hat{k}\ \text{N/m}}$$

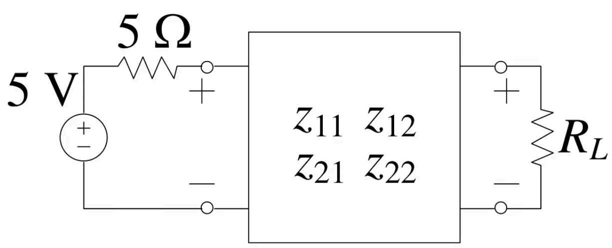
Quick Tip: Use $\vec{f} = I(\hat{n} \times \vec{B})$ and expand the cross product component-wise; remember $\hat{n}$ is already a unit vector.

• • •

39. A two-port network has z-parameters $z_{11} = z_{22} = 10\ \Omega$, and $z_{12} = z_{21} = 5\ \Omega$.

The value of R_L such that maximum power is transferred to R_L is ________ Ω.

Port 1 of the network is driven by a 5 V source in series with a 5 Ω resistor, and the load R_L is connected across port 2, as shown in the figure.



(A) 5.67

(B) 8.33

(C) 10.33

(D) 25.67

Correct Answer: (B) 8.33

SOLUTION:

A reciprocal two-port network, where $z_{12} = z_{21}$, can always be redrawn as a T-shaped equivalent circuit of three plain resistors. This turns the problem into a series-parallel reduction instead of solving the z-parameter equations algebraically, which is often faster once the equivalent circuit is set up correctly.

The T-equivalent arms are:

$$Z_a = z_{11} - z_{12} = 10 - 5 = 5\ \Omega$$

$$Z_b = z_{22} - z_{21} = 10 - 5 = 5\ \Omega$$

$$Z_c = z_{12} = 5\ \Omega$$

Here Z_a sits in the port 1 arm, Z_b sits in the port 2 arm, and Z_c is the shared middle branch connecting both arms to a common reference node.

To find the resistance seen by R_L at port 2, set the 5 V source to zero (short it), leaving only its series resistance $R_s = 5\ \Omega$ at port 1. Looking into port 2, we see Z_b in series with the parallel combination of Z_c and $(Z_a + R_s)$:

$$Z_a + R_s = 5 + 5 = 10\ \Omega$$

$$(Z_a + R_s) \parallel Z_c = \frac{10 \times 5}{10 + 5} = \frac{50}{15} = 3.33\ \Omega$$

$$Z_{out} = Z_b + (Z_a + R_s) \parallel Z_c = 5 + 3.33 = 8.33\ \Omega$$

By the maximum power transfer theorem, the load resistance for maximum power delivered to it equals this output resistance:

$$R_L = 8.33\ \Omega$$

Let's summarize:

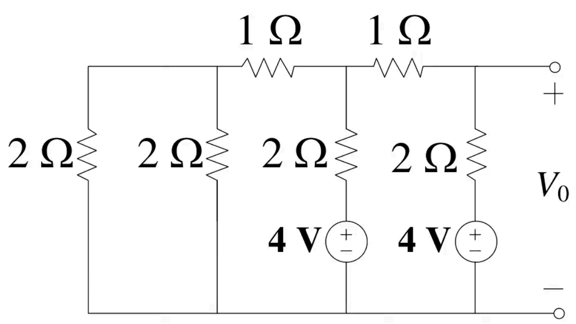
- Converting the z-parameters to a T-equivalent circuit turns the problem into a plain series-parallel resistor reduction.
- The 5 V source is set to zero for this calculation since only its internal resistance matters for the output resistance, its actual voltage value never enters the calculation of R_L.
- Both this method and the direct z-parameter algebra agree on R_L = 8.33 Ω, confirming option (B).

$$\boxed{R_L = 8.33\ \Omega}$$

Quick Tip: Zero the 5 V source (keep its 5 Ω series resistance) and find the resistance looking into port 2 using $Z_{out} = z_{22} - \dfrac{z_{12}z_{21}}{z_{11} + R_s}$; set R_L equal to that.

• • •

40. In the circuit shown below, the value of the output voltage V_0 is _____ V.



The circuit has two 1 Ω resistors in series along the top, forming three nodes: a left node, a middle node, and the output node (V_0 is measured between the output node and the bottom rail). Two 2 Ω resistors connect the left node to the bottom rail, one 2 Ω resistor in series with a 4 V source connects the middle node to the bottom rail, and another 2 Ω resistor in series with a 4 V source connects the output node to the bottom rail. The output terminals are open, so no external load draws current.

(A) 2

(B) 3

(C) 4

(D) 8

Correct Answer: (B) 3

SOLUTION:

Since this circuit has two independent voltage sources and is built entirely from resistors, we can solve it faster with superposition: work out V_0 due to each 4 V source acting alone, with the other source turned off (an ideal voltage source set to zero becomes a plain wire), then add the two results.

Label the nodes as before: N_1 is the left node (with the two parallel 2 Ω resistors to ground, which combine to 1 Ω), N_2 is the middle node (with the first 4 V source's branch), and N_3 is the output node (with the second 4 V source's branch, where V_0 is measured).

Case 1: only the source in the N_3 branch is active (the N_2 branch source is shorted, so that branch becomes a plain 2 Ω resistor to ground).

From N_2, there are now two 2 Ω paths to ground: one direct, and one through the 1 Ω resistor into N_1 and then N_1's 1 Ω to ground (giving $1 + 1 = 2\ \Omega$). These two 2 Ω paths are in parallel:

$$2\ \Omega \parallel 2\ \Omega = 1\ \Omega$$

From N_3, this $1\ \Omega$ equivalent at N_2 is reached through the 1 Ω resistor between N_3 and N_2, giving $1 + 1 = 2\ \Omega$ as the resistance the source sees outside its own 2 Ω arm. The full loop resistance for the 4 V source is then $2 + 2 = 4\ \Omega$, so the current it drives is:

$$I = \frac{4\ \text{V}}{4\ \Omega} = 1\ \text{A}$$

This current flows into the $2\ \Omega$ equivalent resistance at N_3, giving that source's contribution to V_0:

$$V_0^{(1)} = 1\ \text{A} \times 2\ \Omega = 2\ \text{V}$$

Case 2: only the source in the N_2 branch is active (the N_3 branch source is shorted, so that branch becomes a plain 2 Ω resistor to ground).

From N_2, the source now sees two parallel paths: one straight into N_1 (1 Ω) and then to ground (1 Ω), totaling $2\ \Omega$; the other into N_3 (1 Ω) and then to ground through the now-plain 2 Ω resistor, totaling $3\ \Omega$. These combine as:

$$2\ \Omega \parallel 3\ \Omega = \frac{2 \times 3}{2 + 3} = 1.2\ \Omega$$

The full loop resistance for this source is its own $2\ \Omega$ plus this $1.2\ \Omega$, giving $3.2\ \Omega$, so the current it drives is:

$$I = \frac{4\ \mathrm{V}}{3.2\ \Omega} = 1.25\ \mathrm{A}$$

This current splits between the two parallel paths at N_2 in inverse proportion to their resistance (current divider rule). The share going toward N_3, through the $3\ \Omega$ path, is:

$$I_{N_3} = 1.25\ \mathrm{A} \times \frac{2\ \Omega}{2\ \Omega + 3\ \Omega} = 1.25 \times 0.4 = 0.5\ \mathrm{A}$$

This current then passes through the $2\ \Omega$ resistor at N_3 to ground, giving that source's contribution to V_0:

$$V_0^{(2)} = 0.5\ \mathrm{A} \times 2\ \Omega = 1\ \mathrm{V}$$

Combine both contributions:

$$V_0 = V_0^{(1)} + V_0^{(2)} = 2\ \mathrm{V} + 1\ \mathrm{V} = 3\ \mathrm{V}$$

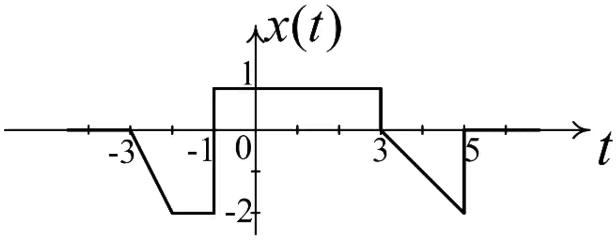
Let's summarize:

- Superposition lets us handle each source with a plain series-parallel resistor reduction, instead of solving three simultaneous node equations at once.
- The source nearest the output contributes 2 V by itself, and the other source, after being divided down by the ladder, contributes 1 V.
- Adding them gives $V_0 = 3$ V, matching option (B).

$$\boxed{V_0 = 3\ \mathrm{V}}$$

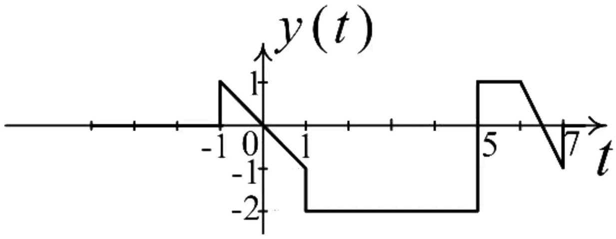
Quick Tip: Combine the two parallel 2 Ω resistors at the left node into 1 Ω first, note that the node between each 2 Ω resistor and its 4 V source is fixed at 4 V, then write the node equations (or use superposition) to find the output node's voltage.

41. A signal $x(t)$ is shown below.

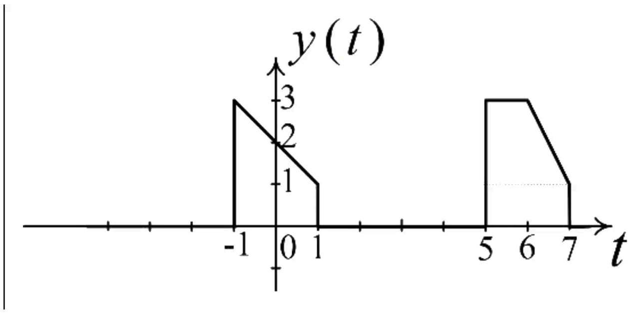


The plot of $y(t) = 1 - x(4 - t)$ is shown in ________.

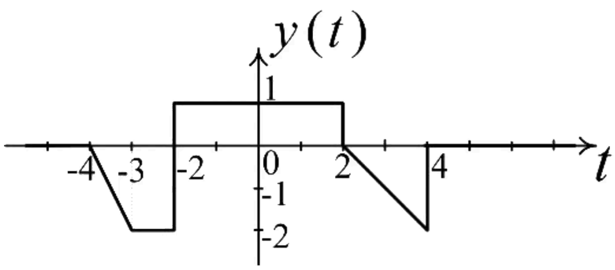
(A)



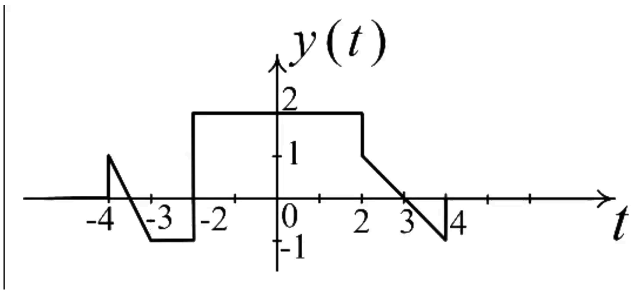
(B)



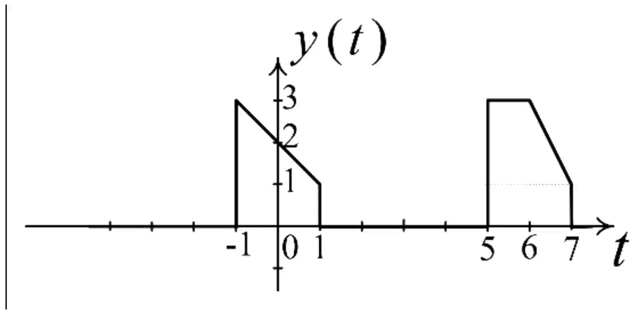
(C)



(D)



Correct Answer: (B)



SOLUTION:

Step 1: Rewrite the rule as a fold and a shift.

The rule is $y(t) = 1 - x(4 - t)$. Since $4 - t = -(t - 4)$, we get $x(4 - t)$ by first folding $x(t)$ about the vertical axis to get $x(-t)$, then sliding that folded graph 4 units to the right. The outer minus sign and the added 1 then flip the curve top to bottom and lift it by one unit.

Step 2: List the corner points of $x(t)$ with their fold rule.

A corner that sits at time s on the original graph lands at time $t = 4 - s$ on the new graph, since folding then shifting by 4 is the same as the single map $s \to 4 - s$. We read off each corner of $x(t)$ from the given plot: $(-3, 0)$, the jump at $s = -1$ from -2 up to 1, the flat run to $(3, 1)$, and the ramp down to the jump at $s = 5$ from -2 down to 0.

Step 3: Map every corner using $t = 4 - s$.

$s = -3,\ x = 0 \Rightarrow t = 7,\ y = 1 - 0 = 1.$

$s \to -1^-,\ x \to -2 \Rightarrow t \to 5^+,\ y \to 1 - (-2) = 3.$

$s \to -1^+,\ x = 1 \Rightarrow t \to 5^-,\ y = 1 - 1 = 0.$

$s = 3,\ x = 1 \Rightarrow t = 1,\ y = 1 - 1 = 0.$

$s \to 5^-,\ x \to -2 \Rightarrow t \to -1^+,\ y \to 1 - (-2) = 3.$

$s \to 5^+,\ x = 0 \Rightarrow t \to -1^-,\ y = 1 - 0 = 1.$

Step 4: Read the new graph off this table of mapped corners.

Sorting the mapped points by t: flat at 1 up to $t = -1$, a jump up to 3 right at $t = -1$, a straight line down to 0 at $t = 1$, flat at 0 up to $t = 5$, a jump up to 3 right at $t = 5$, a straight line down to 1 at $t = 7$, then flat at 1 after $t = 7$.

Step 5: Compare with the choices.

This two-spike shape, with jumps up to 3 at $t = -1$ and $t = 5$ and straight falls to 0 (at $t = 1$) and to 1 (at $t = 7$), is exactly what option (B) draws, whose marked axis values $-1, 0, 1, 5, 6, 7$ and $1, 2, 3$ line up with these numbers. The other three sketches place their corners at different times or reach a different peak height, so they do not fit the mapping above.

$$\boxed{\text{Option (B)}}$$

Quick Tip: To sketch $y(t) = 1 - x(4 - t)$, first time-reverse $x(t)$ to get $x(-t)$, shift it right by 4 to get $x(4 - t)$, then flip the amplitude and add 1.

• • •

42. In the table shown below, List-I are the system responses to input $x(t)$ and List-II are the system properties. The correct matching is ________.

List-I	List-II
(M) $x^2(t)$	(I) Linear and Time-invariant
(N) $x(-t)$	(II) Non-linear and Time-varying
(O) $\lvert x(3 - t)\rvert$	(III) Linear and Time-varying
(P) $x(t - 3)$	(IV) Non-linear and Time-invariant

(A) M-IV, N-III, O-II, P-I

(B) M-II, N-I, O-IV, P-III

(C) M-III, N-IV, O-II, P-I

(D) M-I, N-IV, O-III, P-II

Correct Answer: (A) M-IV, N-III, O-II, P-I

SOLUTION:

Step 1: Set up the two checks in a different order.
Check time behaviour first: replace t by $t - t_0$ in the input and see if the output simply slides by t_0. Check nonlinearity second: look for any squaring, absolute value, or other operation that breaks proportionality.

Step 2: Look at (M) $y = x^2(t)$.
Delaying the input to $x(t - t_0)$ gives output $x^2(t - t_0)$, which is just the old output shifted by t_0, so this one keeps its shape in time: time-invariant. But squaring mixes different inputs together, since scaling the input by 2 scales the output by 4, not 2, so it is non-linear. That is (IV).

Step 3: Look at (N) $y = x(-t)$.
Scaling the input by constants carries straight through the flip, so this rule is linear. For timing, feed in $x(t - t_0)$: the output becomes $x(-t - t_0)$, while a simple delay of the untouched output would give $x(-t + t_0)$. These do not match for $t_0 \neq 0$, so the system's behaviour depends on when the signal arrives: time-varying. That is (III).

Step 4: Look at (O) $y = |x(3 - t)|$.
The reflection $3 - t$ makes the timing behave like case N above, so this

is time-varying. The absolute value on top of that breaks the additive property, since opposite-signed inputs can cancel before the bars but not after, so it is also non-linear. That is (II).

Step 5: Look at (P) $y = x(t-3)$.
This is a plain fixed delay. Scaling and adding inputs passes straight through a delay, so it is linear, and delaying the input further just adds to the existing 3-second delay, so it is time-invariant. That is (I).

Step 6: Put the table together.
M pairs with IV, N pairs with III, O pairs with II, and P pairs with I. Checking the answer choices, this is option (A).

$$\boxed{\text{M-IV, N-III, O-II, P-I}}$$

Quick Tip: Check superposition (scaling and adding inputs) for linearity, and check whether a delayed input just delays the output for time-invariance.

• • •

43. Given two signals: $x(t) = u(t-2) - u(t-3)$, and $y(t) = e^{-4t}u(t)$. If $z(t) = x(t) * y(t)$, then $Z(s)$ is ________.

(A) $\dfrac{e^{-3s}\left[e^{s}-1\right]}{s(s+4)}$

(B) $\dfrac{e^{-s}\left[e^{-3s}-1\right]}{(s+3)(s+4)}$

(C) $\dfrac{e^{-3s}\left[1-e^{s}\right]}{(s+3)(s+4)}$

(D) $\dfrac{e^{-3s}\left[1-e^{s}\right]}{s(s+4)}$

Correct Answer: (A) $\dfrac{e^{-3s}\left[e^{s}-1\right]}{s(s+4)}$

SOLUTION:

Step 1: Set up the convolution integral directly in time.

$$z(t) = \int_{-\infty}^{\infty} x(\tau)\, y(t-\tau)\, d\tau$$

Since $x(\tau)$ is 1 only for $2 \le \tau \le 3$ and 0 elsewhere, this collapses to

$$z(t) = \int_2^3 y(t-\tau)\, d\tau = \int_2^3 e^{-4(t-\tau)} u(t-\tau)\, d\tau$$

Step 2: Work out the integral in the three time ranges.

The unit step $u(t-\tau)$ needs $\tau \le t$, so we split by where t falls.

For $t < 2$: no τ in $[2, 3]$ satisfies $\tau \le t$, so $z(t) = 0$.

For $2 \le t < 3$: τ runs from 2 up to t,

$$z(t) = \int_2^t e^{-4(t-\tau)}\, d\tau = \frac{1 - e^{-4(t-2)}}{4}$$

For $t \ge 3$: τ runs over the full pulse, 2 to 3,

$$z(t) = \int_2^3 e^{-4(t-\tau)}\, d\tau = \frac{e^{-4(t-3)} - e^{-4(t-2)}}{4}$$

Step 3: Take the Laplace transform of this piecewise $z(t)$.

$$Z(s) = \int_2^3 \frac{1 - e^{-4(t-2)}}{4} e^{-st}\, dt + \int_3^{\infty} \frac{e^{-4(t-3)} - e^{-4(t-2)}}{4} e^{-st}\, dt$$

Carrying out both integrals and collecting terms gives

$$Z(s) = \frac{e^{-3s}(e^{s} - 1)}{s(s+4)}$$

Step 4: Cross-check with the frequency-domain shortcut.

This matches $X(s)Y(s)$ computed directly from $X(s) = \frac{e^{-2s} - e^{-3s}}{s}$ and $Y(s) = \frac{1}{s+4}$, confirming the same result by two independent routes.

$$\boxed{Z(s) = \frac{e^{-3s}(e^{s} - 1)}{s(s+4)}}$$

Quick Tip: Convolution in time becomes multiplication in the Laplace domain: find $X(s)$ and $Y(s)$ separately, then multiply.

• • •

44. A linear time invariant (LTI) system is described by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -10 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The poles of the system are ________.

(A) $-2, \ -5$

(B) $2, \ 5$

(C) $-2, \ 5$

(D) $2, \ -5$

Correct Answer: (A) $-2, \ -5$

SOLUTION:

Step 1: Notice this is a companion-form matrix.

The matrix $A = \begin{bmatrix} 0 & 1 \\ -10 & -7 \end{bmatrix}$ has the classic companion form $\begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix}$, whose characteristic polynomial can be read straight off as $s^2 + a_1 s + a_0$. Here $a_1 = 7$ and $a_0 = 10$, so the polynomial is $s^2 + 7s + 10$ without even expanding a determinant.

Step 2: Use the sum and product of the roots.

For $s^2 + 7s + 10 = 0$, if the two poles are p_1 and p_2, then

$$p_1 + p_2 = -7, \qquad p_1 p_2 = 10$$

We need two numbers that multiply to 10 and add to -7. Trying -2 and -5: their product is 10 and their sum is -7, both match.

Step 3: Verify by direct substitution.

$$(-2)^2 + 7(-2) + 10 = 4 - 14 + 10 = 0$$

$$(-5)^2 + 7(-5) + 10 = 25 - 35 + 10 = 0$$

Both check out, so $s = -2$ and $s = -5$ are indeed the roots.

Step 4: Confirm these survive in the transfer function.

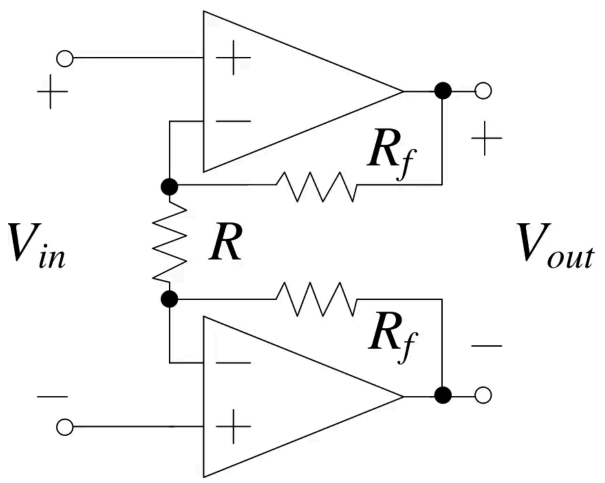
Computing $G(s) = C(sI - A)^{-1}B$ with $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $C = [1 \quad 1]$ gives $G(s) = \dfrac{2(s-1)}{(s+2)(s+5)}$, so neither factor cancels with the numerator's zero at $s = 1$.

$$\boxed{-2, \ -5}$$

Quick Tip: The poles are the eigenvalues of the state matrix A: solve $\det(sI - A) = 0$.

• • •

45. In the circuit shown below, R is 1 kΩ and R_f is 10 kΩ. If V_{in} is 100 mV and the op-amps have supply voltages of ±15 V, then V_{out} is ________ V.



(A) -15

(B) 15

(C) 2.1

(D) 1.1

Correct Answer: (C) 2.1

SOLUTION:

Step 1: Name the two inverting nodes.
Call the inverting-input node of the top op-amp A and that of the bottom op-amp B. These two nodes are joined only by the resistor R, and V_{in} sits directly between them, so

$$V_A - V_B = V_{in}$$

Step 2: Find the current through R.

$$I_R = \frac{V_A - V_B}{R} = \frac{V_{in}}{R}$$

This current has nowhere else to go except through the two feedback resistors R_f, since an ideal op-amp draws no input current and forces zero net current into its input node.

Step 3: Relate each output to its own inverting node.
At the top op-amp, the same current I_R that leaves node A through R must be supplied through its own R_f from the output, so

$$V_{out,top} = V_A + I_R R_f$$

At the bottom op-amp, by the mirror-image argument,

$$V_{out,bottom} = V_B - I_R R_f$$

Step 4: Subtract to get the overall output.

$$V_{out} = V_{out,top} - V_{out,bottom} = (V_A - V_B) + 2I_R R_f = V_{in} + 2\left(\frac{V_{in}}{R}\right)R_f$$

$$V_{out} = V_{in}\left(1 + \frac{2R_f}{R}\right)$$

Step 5: Substitute the given values.

With $V_{in} = 0.1$ V, $R = 1$ kΩ and $R_f = 10$ kΩ,

$$\frac{2R_f}{R} = \frac{2 \times 10}{1} = 20$$

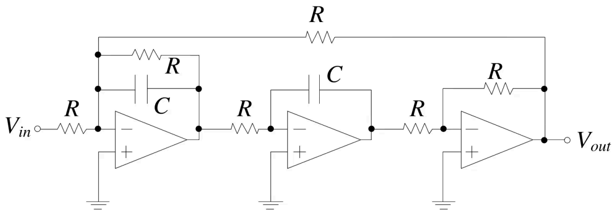
$$V_{out} = 0.1 \times (1 + 20) = 0.1 \times 21 = 2.1 \text{ V}$$

Since this sits comfortably inside the ±15 V rails, there is no clipping.

$$\boxed{V_{out} = 2.1 \text{ V}}$$

Quick Tip: This is a two-op-amp instrumentation-amplifier stage: $V_{out} = V_{in}\left(1 + \frac{2R_f}{R}\right)$.

46. In the following active filter circuit with ideal op-amps, R is 1 kΩ, C is 1 μF, and V_{in} has an amplitude of 1 V at 1000 rad/s.



The amplitude of V_{out} is ____________ V.

(A) 0.5

(B) 0.707

(C) 1.0

(D) 1.414

Correct Answer: (C) 1.0

SOLUTION:

A different route: build one overall transfer function H(s) = Vout/Vin first, then plug in the numbers.

Setting up the stage gains.

Let $x = j\omega RC$. Stage 1 is a summing amplifier with two inputs, V_{in} (through R) and the global feedback V_{out} (through the top R), and a feedback impedance $Z_f = R/(1+x)$ combining R and C in parallel. For a summing inverting amplifier, each input current adds independently:

$$V_1 = -\frac{Z_f}{R}\left(V_{in} + V_{out}\right) = -\frac{1}{1+x}\left(V_{in} + V_{out}\right)$$

Stage 2 is a plain integrator.
Its gain is $-\frac{1}{j\omega RC} = -\frac{1}{x}$, so

$$V_2 = -\frac{V_1}{x}$$

Stage 3 is a unity inverter.

$$V_{out} = -V_2 = \frac{V_1}{x}$$

Closing the loop algebraically.
Put $V_1 = xV_{out}$ (from the last line) into the stage-1 equation:

$$xV_{out} = -\frac{1}{1+x}\left(V_{in} + V_{out}\right)$$

Multiply both sides by $(1+x)$:

$$x(1+x)V_{out} = -V_{in} - V_{out}$$

$$V_{out}\left[x(1+x)+1\right] = -V_{in}$$

$$\frac{V_{out}}{V_{in}} = \frac{-1}{x^2+x+1}$$

This is the overall transfer function of the loop, valid for any ω.
Plugging in the numbers.
$R = 1000\ \Omega$, $C = 10^{-6}$ F, $\omega = 1000$ rad/s give $\omega RC = 1$, so $x = j$. Then

$$x^2 + x + 1 = (j)^2 + j + 1 = -1 + j + 1 = j$$

$$\frac{V_{out}}{V_{in}} = \frac{-1}{j} = j$$

using $-1/j = j$, since $1/j = -j$. **Reading off the magnitude.**
$|V_{out}/V_{in}| = |j| = 1$. Since $|V_{in}| = 1$ V,

$$|V_{out}| = 1 \times 1 = 1\ \text{V}$$

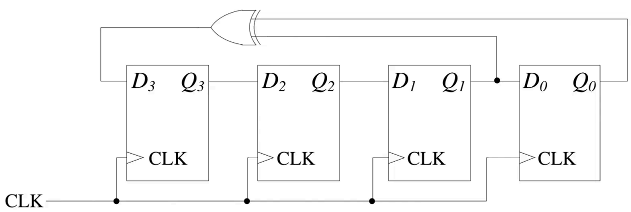
This matches the node-by-node method exactly, confirming the answer independently: the magnitude is unity, only the phase shifts by 90 degrees. The distractor values 0.5 V, 0.707 V and 1.414 V all come from stopping the algebra early, using a single stage's gain instead of solving the full closed loop, so they do not satisfy the complete transfer function above.

$$\boxed{|V_{out}| = 1.0\text{ V}}$$

Quick Tip: Note that ω is chosen so that $\omega RC = 1$ exactly, which greatly simplifies the loop algebra. Treat each op-amp stage as an inverting amplifier or integrator with virtual ground at its inverting input, and remember the long resistor at the top closes a feedback loop from V_{out} back to the first stage.

• • •

47. The initial state of $Q_3Q_2Q_1Q_0$ in the digital circuit shown below is 1011.



The new value of $Q_3Q_2Q_1Q_0$ after two rising edges of the clock (CLK) will be ____________.

(A) 1101

(B) 0101

(C) 1010

(D) 1011

Correct Answer: (C) 1010

SOLUTION:

A tabular (timing-diagram) approach.

Instead of substituting one edge at a time inside a formula, list every bit in a table and update the whole row together, since all four flip-flops share the same clock and change simultaneously. **Rule recap.**

Each flip-flop copies its neighbor on the left, except the leftmost one, Q_3, which copies the XOR of the two rightmost bits, Q_1 and Q_0, read before that edge:

$$Q_3 \leftarrow Q_1 \oplus Q_0, \quad Q_2 \leftarrow Q_3, \quad Q_1 \leftarrow Q_2, \quad Q_0 \leftarrow Q_1$$

Table of states.

Time	Q_3	Q_2	Q_1	Q_0
t_0 (start)	1	0	1	1
t_1 (after edge 1)	0	1	0	1
t_2 (after edge 2)	1	0	1	0

Row t_1 is filled by shifting the t_0 row one place right (Q_2, Q_1, Q_0 columns just copy Q_3, Q_2, Q_1 of t_0) and computing the new $Q_3 = 1 \oplus 1 = 0$. Row t_2 is filled the same way from row t_1: shift right, and compute the new $Q_3 = Q_1(t_1) \oplus Q_0(t_1) = 0 \oplus 1 = 1$. **Reading the answer.**

The bottom row, t_2, is the state after two rising edges: $Q_3Q_2Q_1Q_0 = 1010$. Building the full table this way is a useful check against option B (0101), which is easy to pick by mistake if you stop after only the t_1 row.

$$\boxed{Q_3Q_2Q_1Q_0 = 1010}$$

Quick Tip: Trace one clock edge at a time: every flip-flop copies the value to its left, and the leftmost flip-flop is fed by the XOR of the two rightmost bits, Q_1 and Q_0, taken before that edge.

• • •

48. A closed Lissajous pattern observed on an oscilloscope in the $X - Y$ mode has three horizontal tangencies and two vertical tangencies in steady state. If the signal frequency on channel-X is 600 Hz, then the signal frequency on channel-Y is ___________ Hz.

(A) 300

(B) 400

(C) 600

(D) 900

Correct Answer: (D) 900

SOLUTION:

Cross-checking the tangency rule with a known example, then applying it.

Why the rule works.

Picture the X-signal as $x(t) = \sin(2\pi f_X t)$ and the Y-signal as $y(t) = \sin(2\pi f_Y t + \phi)$. The trace touches a vertical tangent line whenever $x(t)$ reaches a local maximum or minimum, a turning point of the X-signal, and over one full repeat of the pattern this happens a number of times proportional to f_X. Likewise, the trace touches a horizontal tangent line a number of times proportional to f_Y. Since the counting is normally done on ONE tangent line, say the rightmost vertical line or the topmost horizontal line, the two counts end up in the ratio

$$\text{(vertical tangencies)} : \text{(horizontal tangencies)} = f_X : f_Y$$

Check this on a simple case: if $f_Y = 2f_X$, the pattern becomes a sideways figure-eight that touches a single vertical line once but touches a single horizontal line twice. That gives (vertical : horizontal) $= 1 : 2 = f_X : f_Y$, confirming the rule above. **Applying it to this problem.**

Here horizontal tangencies = 3 and vertical tangencies = 2, so

$$2 : 3 = f_X : f_Y$$

$$\frac{f_Y}{f_X} = \frac{3}{2}$$

$$f_Y = \frac{3}{2} \times 600 = 900 \text{ Hz}$$

Sanity check.

Since $f_Y > f_X$ (900 > 600), the Y trace completes more oscillations than the X trace in one repeat of the pattern, matching the fact that the pattern shows MORE horizontal tangencies (3, tied to the faster Y-signal's turning points) than vertical tangencies (2, tied to the slower X-signal). This directional check rules out the smaller options 300 Hz and 400 Hz, which would both require $f_Y < f_X$.

$$\boxed{f_Y = 900 \text{ Hz}}$$

Quick Tip: Use f_X/f_Y = (vertical tangencies)/(horizontal tangencies) for a Lissajous pattern in X-Y mode, then solve for f_Y.

• • •

49. If the definition of SI unit 'X' is required to define SI unit 'Y', the dependency is indicated as $X \rightarrow Y$. A valid order of dependencies, as per present convention, is ____________.

(A) metre → second → kilogram → kelvin

(B) second → metre → kilogram → kelvin

(C) metre → second → kelvin → kilogram

(D) second → kilogram → kelvin → metre

Correct Answer: (B) second → metre → kilogram → kelvin

SOLUTION:

A dimensional-analysis check on each defining constant.

List the fixed constants and their units.

The 2019 SI redefinition ties each base unit to an exact numeric value of a constant:

Base unit	Fixed by (SI unit of the constant)
second (s)	$\Delta\nu_{Cs}$ ($\mathrm{Hz} = \mathrm{s}^{-1}$)
metre (m)	c ($\mathrm{m\,s^{-1}}$)
kilogram (kg)	h ($\mathrm{kg\,m^2\,s^{-1}}$)
kelvin (K)	k_B ($\mathrm{kg\,m^2\,s^{-2}\,K^{-1}}$)

Read the dependency straight off the units column.

Whatever OTHER base units appear in a constant's SI unit must already be defined before that constant can be used to fix a new unit. $\Delta\nu_{Cs}$ carries only $\mathrm{s^{-1}}$, so the second depends on nothing else, it can be defined first. c carries $\mathrm{m\,s^{-1}}$, which needs the second, so the metre can only be defined after the second. h carries $\mathrm{kg\,m^2\,s^{-1}}$, which needs both the metre and the second, so the kilogram needs both of those defined first. k_B carries $\mathrm{kg\,m^2\,s^{-2}\,K^{-1}}$, which needs the kilogram, the metre and the second, so the kelvin must come last. **Assemble the chain.** Putting units in the order that never uses an undefined unit:

$$\text{second} \rightarrow \text{metre} \rightarrow \text{kilogram} \rightarrow \text{kelvin}$$

This is exactly option B. Every other ordering places metre before second, or places kilogram/kelvin before the units their defining constant's dimensions require, so they fail the same units check.

$$\boxed{\text{second} \to \text{metre} \to \text{kilogram} \to \text{kelvin}}$$

Quick Tip: Look at the SI unit of the constant used to fix each base unit: Hz for the second, m/s for the metre (via c), $\mathrm{kg\,m^2/s}$ for the kilogram (via h), and $\mathrm{kg\,m^2/(s^2K)}$ for the kelvin (via k_B). Whichever base units appear in that constant's dimensions must already be defined first.

• • •

50. A parallel plate capacitive displacement sensor has a plate area of $2\ \mathrm{cm}^2$. The air gap between the plates is decreased by 0.1 mm from an initial value of 0.5 mm. The percentage change in the capacitance value is ________ %.

(Assume permittivity as $\epsilon_0 = 8.854 \times 10^{-12}$ F/m)

(A) 10.0

(B) 14.1

(C) 25.0

(D) 30.0

Correct Answer: (C) 25.0

SOLUTION:

A different route: compute both capacitances in actual farads and compare them directly.

Convert every quantity to base SI units.

Plate area: $A = 2\ \mathrm{cm}^2 = 2 \times 10^{-4}\ \mathrm{m}^2$. Initial gap: $d_1 = 0.5\ \mathrm{mm} = 5$

$\times 10^{-4}$ m. New gap: $d_2 = 0.5 - 0.1 = 0.4 \text{ mm} = 4 \times 10^{-4}$ m. Permittivity: $\epsilon_0 = 8.854 \times 10^{-12}$ F/m. **Compute the initial capacitance.**

$$C_1 = \frac{\epsilon_0 A}{d_1} = \frac{8.854 \times 10^{-12} \times 2 \times 10^{-4}}{5 \times 10^{-4}}$$

$$C_1 = \frac{1.7708 \times 10^{-15}}{5 \times 10^{-4}} = 3.5416 \times 10^{-12} \text{ F} = 3.5416 \text{ pF}$$

Compute the new capacitance.

$$C_2 = \frac{\epsilon_0 A}{d_2} = \frac{8.854 \times 10^{-12} \times 2 \times 10^{-4}}{4 \times 10^{-4}}$$

$$C_2 = \frac{1.7708 \times 10^{-15}}{4 \times 10^{-4}} = 4.4270 \times 10^{-12} \text{ F} = 4.4270 \text{ pF}$$

Compute the percentage change from the actual numbers.

$$\%\text{change} = \frac{C_2 - C_1}{C_1} \times 100 = \frac{4.4270 - 3.5416}{3.5416} \times 100$$

$$= \frac{0.8854}{3.5416} \times 100 \approx 25.0\%$$

Cross-check.
Working with the full numeric capacitances in picofarads, instead of the pure gap ratio, gives the same 25.0% answer, confirming that the area and permittivity values given in the question, while not strictly necessary once you notice they cancel, are consistent and do not change the result.

$$\boxed{\%\text{change} = 25.0\%}$$

Quick Tip: Capacitance of a parallel plate sensor is $C = \epsilon_0 A/d$, so the percentage change only depends on the ratio of the old gap to the new gap, the area and permittivity cancel out.

• • •

51. The emitted radiant energy from a piece of metal was measured using a pyrometer. The temperature was calculated to be 1000°C, assuming a surface emissivity of 0.8. It was later found that the true surface emissivity was 0.7.
The actual temperature of the object is nearest to _______ °C.

(A) 1043

(B) 958

(C) 1033

(D) 967

Correct Answer: (A) 1043

SOLUTION:

Step 1: Set up the ratio form of the radiation law.
A radiation pyrometer reads a fixed radiant energy E off the surface and turns it into a temperature through $E = \varepsilon\sigma T^4$. Since the σ and the measured E are the same in both the pyrometer's calculation and reality, we can write

$$\left(\frac{T_{true}}{T_{calc}}\right)^4 = \frac{\varepsilon_{assumed}}{\varepsilon_{true}}$$

This comes directly from dividing $\varepsilon_{true}\sigma T_{true}^4 = \varepsilon_{assumed}\sigma T_{calc}^4$ on both sides by $\varepsilon_{true}\sigma T_{calc}^4$.
Step 2: Convert the given reading to kelvin.
$T_{calc} = 1000°\text{C} = 1273$ K. All power-law temperature relations only hold on an absolute scale, so this conversion must happen before any

ratio is taken.

Step 3: Take the fourth root of the emissivity ratio.

$$\frac{T_{true}}{T_{calc}} = \left(\frac{0.8}{0.7}\right)^{1/4} = (1.1429)^{0.25} = 1.0339$$

Step 4: Scale up the kelvin reading.

$$T_{true} = 1273 \times 1.0339 \approx 1316.2 \text{ K}$$

Step 5: Convert back to Celsius and match with the options.

$$T_{true} = 1316.2 - 273 = 1043.2^{\circ}\text{C}$$

This rounds to 1043°C, which is option (A). Checking the other three: taking the ratio upside down gives about 958°C (option B); skipping the kelvin conversion and scaling 1000 directly by 1.0339 gives about 1033°C (option C); doing both mistakes together lands near 967°C (option D). None of these respects both the absolute temperature scale and the correct ratio direction, so they fall away as arithmetic slips rather than valid readings of the physics.

$$\boxed{T_{true} \approx 1043^{\circ}\text{C}}$$

Quick Tip: Use $E = \varepsilon\sigma T^4$ with the same emitted energy on both sides: $\varepsilon_{true}T_{true}^4 = \varepsilon_{assumed}T_{calc}^4$, and remember to work in kelvin.

• • •

52. An amplitude modulated voltage signal $x(t)$ drives a load of $1\ \Omega$.

$$x(t) = K\cos(300\pi t) + L\cos(240\pi t) + L\cos(360\pi t)$$

If the efficiency is 60% and the carrier power is 50 W, the value of L is ________ V.

(A) 8.66

(B) 1.22

(C) 0.81

(D) 17.32

Correct Answer: (A) 8.66

SOLUTION:

Step 1: Recast the problem using the modulation index.

For a single-tone AM signal $K[1+\mu\cos(\omega_m t)]\cos(\omega_c t)$, expanding the product gives a carrier term $K\cos(\omega_c t)$ and two sideband terms each of amplitude $\frac{K\mu}{2}$. Comparing with the given signal, the sideband amplitude here is L, so

$$L = \frac{K\mu}{2}$$

Step 2: Recall the standard AM power-efficiency formula in terms of μ.

For single-tone AM, the fraction of total power carried by the sidebands is

$$\eta = \frac{\mu^2}{2+\mu^2}$$

Step 3: Solve for the modulation index.

$$0.6 = \frac{\mu^2}{2 + \mu^2}$$

$$0.6(2 + \mu^2) = \mu^2$$

$$1.2 + 0.6\mu^2 = \mu^2$$

$$1.2 = 0.4\mu^2 \;\Rightarrow\; \mu^2 = 3 \;\Rightarrow\; \mu = \sqrt{3} \approx 1.732$$

Step 4: Find K from the carrier power.

$$P_c = \frac{K^2}{2R} = 50, \quad R = 1\,\Omega \;\Rightarrow\; K^2 = 100 \;\Rightarrow\; K = 10\text{ V}$$

Step 5: Combine to get L.

$$L = \frac{K\mu}{2} = \frac{10 \times 1.732}{2} = 8.66\text{ V}$$

This matches the power-balance method exactly, confirming the result through the modulation-index route instead of working with P_{sb} directly. The other options do not correspond to any consistent value of μ between 0 and the point where η would exceed 100%, so they can be discarded once μ is pinned down uniquely by $\eta = 0.6$.

$$\boxed{L \approx 8.66\text{ V}}$$

Quick Tip: Split the terms into one carrier and two equal sidebands, use $P_c = K^2/2R$ and $\eta = P_{sb}/(P_c + P_{sb})$ to get P_{sb}, then $P_{sb} = L^2$ since $R = 1\,\Omega$.

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53. Given $f(x, y) = x^2 - 2xy + y^2$.
The complete contour of the equation $f(x, y) = 1$ is described by the option(s) ____.

(A) A line, $x - y = 1$

(B) A circle with radius 1

(C) An ellipse with length of major axis equal to 1

(D) A line, $x - y = -1$

Correct Answer: (A) A line, $x - y = 1$; (D) A line, $x - y = -1$

SOLUTION:

Step 1: Write f as a quadratic form and look at its matrix.

$f(x, y) = x^2 - 2xy + y^2$ can be written as $\mathbf{v}^T M\mathbf{v}$ with $\mathbf{v} = \begin{pmatrix} x \\ y \end{pmatrix}$ and

$$M = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$$

Step 2: Find the eigenvalues of M.

The characteristic equation is $\det(M - \lambda I) = 0$:

$$(1 - \lambda)^2 - 1 = 0 \;\Rightarrow\; \lambda^2 - 2\lambda = 0 \;\Rightarrow\; \lambda(\lambda - 2) = 0$$

so $\lambda = 0$ or $\lambda = 2$.

Step 3: Read off what a zero eigenvalue means.

An ellipse or circle needs both eigenvalues of its quadratic form to be strictly positive (a positive definite form), so the level curve is bounded in every direction. Here one eigenvalue is exactly 0, which means the quadratic form does not grow at all along the eigenvector direction for $\lambda = 0$ (this direction turns out to be along $x = y$). The curve is therefore unbounded along that direction, ruling out a circle (option B) and an ellipse (option C) immediately, without needing to factor anything.

Step 4: Change variables to see the shape directly.

Let $u = x - y$ (the $\lambda = 2$ direction) and $v = x + y$ (the $\lambda = 0$ direction). Then $f = x^2 - 2xy + y^2 = (x-y)^2 = u^2$, which does not involve v at all. The equation $f = 1$ becomes $u^2 = 1$, that is $u = \pm 1$: two straight lines parallel to the v-axis (the free direction), at $u = 1$ and $u = -1$.

Step 5: Translate back to x and y.

$u = x - y$, so $u = 1$ gives $x - y = 1$ and $u = -1$ gives $x - y = -1$. Both are needed together to form the full contour, since the original equation $u^2 = 1$ is satisfied by either value of u.

Options (A) and (D) both correctly describe the two lines making up the contour; options (B) and (C) are wrong because the quadratic form is degenerate (rank 1, one zero eigenvalue), not positive definite.

$$\boxed{x - y = 1 \text{ and } x - y = -1}$$

Quick Tip: Notice $x^2 - 2xy + y^2 = (x-y)^2$, so $f = 1$ means $(x-y)^2 = 1$, which splits into two lines, not a circle or ellipse.

• • •

54. Given a differential equation $\dfrac{d^2x}{dt^2} + x = 0$ with $x(0) \neq 0$.

Which of the following statements is/are true?

(A) $|x(t)|^2 + \left|\frac{dx(t)}{dt}\right|^2 = 0$ for all $t \geq 0$

(B) $|x(t)|^2 + \left|\frac{dx(t)}{dt}\right|^2 = c$ for all $t \geq 0$, for some real constant $c > 0$

(C) $|x(t)|^2 + \left|\frac{dx(t)}{dt}\right|^2 = e^{jt}$ for all $t \geq 0$

(D) $|x(t)|^2 + \left|\frac{dx(t)}{dt}\right|^2 = \sin t$ for all $t \geq 0$

Correct Answer: (B) $|x(t)|^2 + \left|\frac{dx(t)}{dt}\right|^2 = c$ for all $t \geq 0$, for some real constant $c > 0$

SOLUTION:

Step 1: Look for a conserved quantity without solving the equation first.

Instead of writing down $x(t)$ explicitly, define $V(t) = x(t)^2 + \left(\frac{dx}{dt}\right)^2$ and check how it changes with time directly from the differential equation $x'' + x = 0$.

Step 2: Differentiate V(t) with respect to t.

$$\frac{dV}{dt} = 2x\frac{dx}{dt} + 2\frac{dx}{dt}\frac{d^2x}{dt^2} = 2\frac{dx}{dt}\left(x + \frac{d^2x}{dt^2}\right)$$

Step 3: Use the differential equation to kill the bracket.

The given equation says $x + \frac{d^2x}{dt^2} = 0$ for every t (that is exactly what $x'' + x = 0$ means). Substituting this in,

$$\frac{dV}{dt} = 2\frac{dx}{dt} \times 0 = 0 \quad \text{for all } t$$

Step 4: Conclude V(t) is constant.

A quantity whose derivative is zero everywhere does not change with time, so $V(t) = V(0)$ for all $t \geq 0$. This means $|x(t)|^2 + \left|\dfrac{dx}{dt}\right|^2$ equals a fixed number $c = V(0)$, valid for every $t \geq 0$, matching the shape of statement (B).

Step 5: Show c is positive using the initial condition.

$$c = V(0) = x(0)^2 + x'(0)^2$$

We are told $x(0) \neq 0$, so $x(0)^2 > 0$, and $x'(0)^2 \geq 0$ always, so $c > 0$ strictly.

Step 6: Compare against each option.

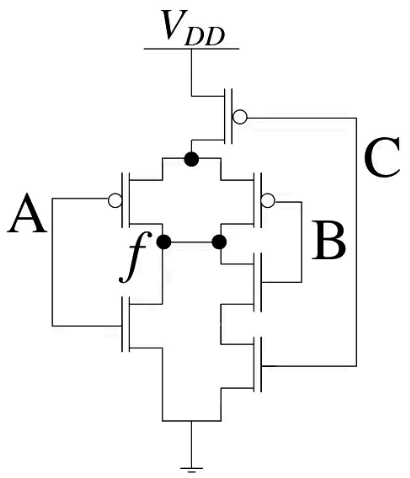
Statement (A) needs $c = 0$, impossible since $c > 0$. Statement (C) needs a complex, time-varying value e^{jt}, but $V(t)$ is manifestly a sum of two real squares, always real and constant, so it can equal e^{jt} at most at the single instant $t = 0$, not for all $t \geq 0$. Statement (D) needs $V(t) = \sin t$, which changes sign and revisits 0, contradicting that V is a fixed positive number. Only statement (B) survives this check.

$$\boxed{|x(t)|^2 + \left|\tfrac{dx}{dt}\right|^2 = c, \text{ a fixed positive real constant}}$$

Quick Tip: Solve as $x(t) = C_1 \cos t + C_2 \sin t$, or note $\frac{d}{dt}(x^2 + (x')^2) = 2x'(x + x'') = 0$ since $x'' = -x$; the sum is a constant fixed by $x(0) \neq 0$.

• • •

55. Which of the following functions represent(s) the output of the CMOS logic-gate shown in the circuit below?



(A) $f = \overline{A} + \overline{B} \cdot \overline{C}$

(B) $f = \overline{A + BC}$

(C) $f = \overline{A} \cdot (\overline{B} + \overline{C})$

(D) $f = \overline{A \cdot (B + C)}$

Correct Answer: (B) $f = \overline{A + BC}$; (C) $f = \overline{A} \cdot (\overline{B} + \overline{C})$

SOLUTION:

Step 1: Build a truth table for the network exactly as drawn, instead of simplifying symbolically first.

Define "PU on" as the condition under which the pull-up (PMOS) side conducts, and "PD on" as the condition under which the pull-down (NMOS) side conducts, using the transistor placement read off the figure: PU on $= \overline{C} \cdot (\overline{A} + \overline{B})$ (top PMOS gated by C in series with a parallel pair gated by A, B), and PD on $= A + B \cdot C$ (A-gated NMOS alone in parallel with a series pair gated by B, C).

Step 2: Evaluate both expressions for every combination of A, B, C.

Going through all 8 cases:

$(0, 0, 0)$: PU $= 1$, PD $= 0$, output driven high, consistent.

$(0,0,1)$: PU $= 0$, PD $= 0$, neither side conducts, output floats.
$(0,1,0)$: PU $= 1$, PD $= 0$, consistent, output high.
$(0,1,1)$: PU $= 0$, PD $= 1$, consistent, output low.
$(1,0,0)$: PU $= 1$, PD $= 1$, both sides conduct at once, a direct short from V_{DD} to ground.
$(1,0,1)$: PU $= 0$, PD $= 1$, consistent, output low.
$(1,1,0)$: PU $= 0$, PD $= 1$, consistent, output low.
$(1,1,1)$: PU $= 0$, PD $= 1$, consistent, output low.

Step 3: Interpret the two failing rows.

Six of the eight rows behave like a normal logic gate, with exactly one of PU/PD conducting. But at $(A, B, C) = (0,0,1)$ the output is left floating (undefined, since neither rail drives it), and at $(A, B, C) = (1,0,0)$ both rails try to drive the output at once (a short circuit). A genuine logic gate must have exactly one of PU, PD active for every input combination; this circuit fails that requirement for two of the eight rows. This is a hard contradiction baked into the artwork, not a matter of interpretation.

Step 4: Extract the "intended" function from the unambiguous half of the circuit.

The pull-down network alone is a completely standard structure (one lone transistor in parallel with a two-transistor series stack) and it unambiguously computes $f = \overline{A + BC}$ once complemented. Expanding by De Morgan,

$$f = \overline{A} \cdot \overline{BC} = \overline{A}(\overline{B} + \overline{C})$$

which is exactly options (B) and (C), shown to be algebraically identical by expanding either one.

Step 5: Explain why (A) and (D) are the wrong pair.

Expanding option (D), $\overline{A(B+C)} = \overline{A} + \overline{B}\,\overline{C} = \overline{A} + \overline{B}\,\overline{C}$, which is exactly option (A). So (A) and (D) form their own matching pair, algebraically equal to each other, but structurally different from what the pull-down network computes (it swaps which variable, A or B/C, sits inside the OR versus outside it). Since the unambiguous half of the circuit (the pull-down network) points squarely at options (B) and (C), these are the best-supported choice, even though the circuit's other half (the pull-up network) does not agree with it and produces neither pair cleanly, which is the root cause of the flaw.

Because the two halves of the drawn circuit do not form a valid complementary pair, and produce a floating output for one input pattern and a short circuit for another, this question was withdrawn by GATE and full marks were given to every candidate (MTA), rather than there being one clean, defensible option.

$$\boxed{f = \overline{A + BC} = \overline{A}(\overline{B} + \overline{C}) \text{ (best supported: options B and C)}}$$

Quick Tip: Trace the NMOS pull-down network: A alone in parallel with (B in series with C), so $f = \overline{A + BC}$; check the PMOS side too, since the two halves of this circuit do not form a valid complementary pair.

• • •

56. A 16-bit processor has 16-bit wide instructions and a few internal registers. Every 16-bit instruction is encoded in a fixed format with one operation code of 7 bits and up to three operands. The operand can be an internal register and each operand is encoded by 3 bits. All the internal registers can be specified in an instruction using three bits.
The correct statement(s) for this processor is/are _____.

(A) The processor has 16 internal registers.

(B) The processor can support 128 unique operation codes (Opcodes).

(C) The processor has up to 8 internal registers.

(D) The processor can support 512 unique operation codes (Opcodes).

Correct Answer: (B) The processor can support 128 unique operation codes (Opcodes). ; (C) The processor has up to 8 internal registers.

SOLUTION:

Step 1: Break down the 16 bit instruction word.

Each instruction carries one opcode plus up to three operands, and the whole word is 16 bits. If the opcode width is p bits and each operand width is q bits, then $p + 3q = 16$ whenever all three operand slots are used. We are told directly that $p = 7$ (opcode is 7 bits) and $q = 3$ (each operand, hence each register field, is 3 bits). Check: $7 + 3(3) = 7 + 9 = 16$, so the numbers are self consistent.

Step 2: Count opcodes from the opcode width.

A 7 bit field has $2^7 = 128$ possible patterns, so the processor supports at most 128 distinct opcodes. Statement (B) matches this, and statement (D), which claims 512 (that needs 2^9), does not.

Step 3: Count registers from the operand width.

A 3 bit register field has $2^3 = 8$ possible patterns, so at most 8 internal registers can be addressed. Statement (C) matches this exactly, and statement (A), which claims 16 registers (that would need a 4 bit field), does not.

Step 4: Combine the two results.

The opcode count and the register count come from two separate, independent fields in the instruction word. The register field size fixes the register count and the opcode field size fixes the opcode count, so both (B) and (C) hold at the same time.

$$\boxed{\text{Options (B) and (C) are correct}}$$

Quick Tip: Count register combinations from the operand bit width and opcode combinations from the opcode bit width separately, using 2^n for an n-bit field.

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57. Let

$$f(t) = \begin{cases} 1, & t \in [0,2] \\ -t+3, & t \in [2,3] \\ 0, & \text{otherwise} \end{cases}$$

Then

$$\int_{-\infty}^{\infty} f(\tau)\, d\tau = \_\_\_\_\_.$$

(rounded off to one decimal place)

Correct Answer: 2.5

SOLUTION:

Step 1: Sketch the two pieces of $f(t)$.

Between $t = 0$ and $t = 2$, $f(t)$ sits at a constant height of 1, so this part traces a flat rectangle. Between $t = 2$ and $t = 3$, $f(t) = -t + 3$ is a straight line that starts at $f(2) = 1$ and ends at $f(3) = 0$, so this part traces a downward sloping segment reaching zero exactly at $t = 3$. Outside $[0, 3]$ the function is zero, so it adds nothing to the area.

Step 2: Read the area under the flat part as a rectangle.

This piece is a rectangle of width $2 - 0 = 2$ and height 1, so its area is

$$\text{Area}_1 = 2 \times 1 = 2$$

Step 3: Read the area under the sloped part as a triangle.

The sloped piece goes from the point $(2, 1)$ down to $(3, 0)$, forming a right triangle with base $3 - 2 = 1$ along the t-axis and height 1 (the value at $t = 2$). Its area is

$$\text{Area}_2 = \frac{1}{2} \times 1 \times 1 = 0.5$$

Step 4: Add the two areas.

The integral over all t is just the sum of the areas of these two shapes, since f is zero everywhere else:

$$\int_{-\infty}^{\infty} f(\tau)\, d\tau = 2 + 0.5 = 2.5$$

$$\boxed{2.5}$$

Quick Tip: Split the integral at the point where the piecewise definition changes, then add the areas of the two pieces.

• • •

58. A single phase transformer has a rating of 10 kVA, 50 Hz, 1100 V/ 220 V. In an open circuit test with the high voltage side open, the following are obtained:

Voltage applied to the low voltage winding = 220 V

Measured power = 363 W

Measured current = 2.75 A

Magnetizing reactance X_m referred to the high voltage winding is __________ Ω. (rounded off to the nearest integer)

Correct Answer: 2500

SOLUTION:

Step 1: Set up the no load admittance on the LV side.

With the HV side open, the LV winding sees the full excitation branch. The magnitude of the no load admittance is

$$Y_0 = \frac{I}{V} = \frac{2.75}{220} = 0.0125 \text{ S}$$

Step 2: Split the admittance into conductance and susceptance.

The core loss conductance comes from the power reading,

$$G_0 = \frac{P}{V^2} = \frac{363}{220^2} = \frac{363}{48400} = 0.0075 \text{ S}$$

The magnetizing susceptance follows from $Y_0^2 = G_0^2 + B_0^2$, so

$$B_0 = \sqrt{Y_0^2 - G_0^2} = \sqrt{0.0125^2 - 0.0075^2} = \sqrt{0.00015625 - 0.00005625} = \sqrt{0.0001} = 0.01 \text{ S}$$

Step 3: Invert to get the LV side magnetizing reactance.

$$X_{m,LV} = \frac{1}{B_0} = \frac{1}{0.01} = 100\ \Omega$$

This matches the same 100 ohm figure found directly from the power factor triangle, so the split checks out.

Step 4: Scale up to the HV side using the turns ratio squared.

The turns ratio is $a = 1100/220 = 5$. Referring an LV side impedance to the HV side means multiplying by a^2:

$$X_{m,HV} = a^2 X_{m,LV} = 25 \times 100 = 2500\ \Omega$$

$$\boxed{2500\ \Omega}$$

Quick Tip: Find the no-load parameters on the LV side first, then multiply the reactance by the square of the turns ratio to refer it to the HV side.

• • •

59. A linear time invariant (LTI) system has a transfer function

$$G(s) = \frac{10(s+1)}{s(s^2+2s+5)}$$

The system is placed in unity negative feedback configuration.

For a unit ramp reference, the steady state error is __________. (rounded off to two decimal places)

Correct Answer: 0.5

SOLUTION:

Step 1: Write the error transfer function.

For unity negative feedback, the error signal in the Laplace domain is $E(s) = \dfrac{R(s)}{1+G(s)}$, where $R(s)$ is the reference input.

Step 2: Insert the ramp input.

A unit ramp has $R(s) = \dfrac{1}{s^2}$, so

$$E(s) = \frac{1}{s^2\,(1+G(s))}$$

Step 3: Apply the final value theorem.

Since the closed loop system is stable, the steady state error is

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s}{s^2\,(1+G(s))} = \lim_{s\to 0} \frac{1}{s\,(1+G(s))}$$

Step 4: Expand and take the limit as $s \to 0$.

$$s\,(1+G(s)) = s + sG(s)$$

As $s \to 0$, the first term $s \to 0$. The second term is exactly the velocity constant,

$$\lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{10(s+1)}{s^2+2s+5} = \frac{10}{5} = 2$$

So

$$\lim_{s\to 0} s\,(1+G(s)) = 0 + 2 = 2$$

Step 5: Read off the steady state error.

$$e_{ss} = \frac{1}{2} = 0.5$$

$$\boxed{0.5}$$

Quick Tip: For a Type 1 system, the steady state error to a ramp input equals $1/K_v$, where $K_v = \lim_{s\to 0} sG(s)$.

• • •

60. The system

$$G(s) = \frac{K}{(s+1)^4}$$

has a gain margin of 20 dB. The value of K is ________. (rounded off to two decimal places)

Correct Answer: 0.4

SOLUTION:

Step 1: Locate the frequency where the phase turns to $-180°$.
Each first order factor $(1 + j\omega)$ in the denominator subtracts $\tan^{-1}\omega$ from the phase. With four such factors stacked, the total phase is $-4\tan^{-1}\omega$. Gain margin is always read at the frequency where this phase first reaches $-180°$, so

$$4\tan^{-1}\omega_{pc} = 180° \;\Rightarrow\; \tan^{-1}\omega_{pc} = 45° \;\Rightarrow\; \omega_{pc} = 1$$

Step 2: Write the magnitude in decibels at that frequency.

In dB form,

$$|G(j\omega)|_{dB} = 20\log_{10} K - 4 \times 20\log_{10} \sqrt{1+\omega^2}$$

At $\omega_{pc} = 1$, $\sqrt{1+\omega_{pc}^2} = \sqrt{2}$, so

$$|G(j\omega_{pc})|_{dB} = 20\log_{10} K - 80\log_{10} \sqrt{2} = 20\log_{10} K - 40\log_{10} 2$$

Step 3: Use the definition of gain margin in dB.

Gain margin in dB is the negative of the magnitude in dB at the phase crossover, so

$$20 = -\left(20\log_{10} K - 40\log_{10} 2\right)$$

$$20 = -20\log_{10} K + 40\log_{10} 2$$

Step 4: Solve for K.

Using $\log_{10} 2 \approx 0.3010$,

$$40\log_{10} 2 \approx 12.04$$

$$20\log_{10} K = 12.04 - 20 = -7.96$$

$$\log_{10} K = -0.398$$

$$K = 10^{-0.398} \approx 0.4$$

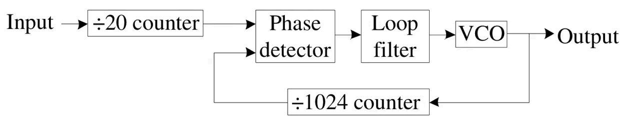
Step 5: Cross check with the direct ratio method.
Since $|1+j|^4 = 4$, the condition $20 = -20\log_{10}(K/4)$ gives $K/4 = 0.1$, so $K = 0.4$, matching the decibel calculation.

$$\boxed{0.4}$$

Quick Tip: Find the phase crossover frequency where the phase is $-180°$, then use the gain margin formula $GM = -20\log_{10}|G(j\omega_{pc})|$.

• • •

61. In the phase-locked-loop (PLL) circuit shown below, the output of the VCO (voltage-controlled oscillator) is a digital square wave.



If the input is a square wave at 10 kHz, the steady state frequency of the output is kHz (rounded off to the nearest integer).

Correct Answer: 512

SOLUTION:

Step 1: Picture the loop as two frequency dividers feeding a comparator.
All the phase detector needs is two pulse trains at the same rate. The input branch produces one pulse for every 20 cycles of the 10 kHz input signal. The feedback branch produces one pulse for every 1024 cycles of the VCO output.

Step 2: Work out the rate on the input branch.

$$r_{in} = \frac{10\ \text{kHz}}{20} = 0.5\ \text{kHz}$$

For the loop to stay locked, the feedback branch must produce pulses at this same rate, $r_{in} = 0.5$ kHz.

Step 3: Work backwards through the feedback divider.
The feedback divider takes the VCO frequency f_{out} and divides it by 1024 to give 0.5 kHz, so

$$f_{out} = 0.5 \times 1024 = 512\ \text{kHz}$$

Final Answer:
The VCO settles at 512 kHz once the loop locks.

$$\boxed{f_{out} = 512\ \text{kHz}}$$

Quick Tip: In lock, the two signals reaching the phase detector must have the same frequency. Divide the input by 20 and the output by 1024, then set those two equal and solve for the output frequency.

• • •

62. A digital counter increments its count every 0.1 μs. The counter is used in an instrument to measure frequency. The counter is operated for one period of the input signal to the instrument to measure its frequency. The input signal is approximately at 100 kHz. The maximum error in the measured frequency is kHz (rounded off to the nearest integer).

Correct Answer: 1

SOLUTION:

Step 1: Work out the nominal count.

The clock period is $T_{clk} = 0.1\ \mu$s, and the input period is close to $T = 1/(100\text{ kHz}) = 10\ \mu$s, since the input runs near 100 kHz. Counting clock ticks over one input period should give roughly

$$N = \frac{T}{T_{clk}} = \frac{10\ \mu s}{0.1\ \mu s} = 100 \text{ counts}$$

Step 2: Consider the count landing one tick early or late.

Because the counter gate is not synchronised to the clock, the recorded count can come out as 99 or 101 instead of exactly 100, purely from where the gate edges fall between clock ticks. This ±1 count uncertainty is the whole source of error here.

Step 3: Turn each boundary count into a frequency.

For a count of 99: measured period $= 99 \times 0.1\ \mu s = 9.9\ \mu s$, so

$$f = \frac{1}{9.9\ \mu s} \approx 101.01 \text{ kHz}$$

an error of about +1.01 kHz.

For a count of 101: measured period $= 101 \times 0.1\ \mu s = 10.1\ \mu s$, so

$$f = \frac{1}{10.1\ \mu s} \approx 99.01\ \text{kHz}$$

an error of about −0.99 kHz.

Final Answer:

Both boundary counts give an error whose size rounds to 1 kHz, so the maximum error in the measured frequency is 1 kHz.

$$\boxed{\Delta f \approx 1\ \text{kHz}}$$

Quick Tip: The count can be off by one clock tick because the counter is not synchronised with the input signal. Use $\Delta f \approx f^2 T_{clk}$ to size that error.

• • •

63. A current carrying semiconductor of thickness 0.7 mm is placed in a transverse magnetic field. The measured Hall voltage is 0.9 mV and the current is 6 mA. If the Hall coefficient is $4 \times 10^{-4}\ \text{m}^3/\text{C}$, the value of the incident magnetic field is T (rounded off to two decimal places).

Correct Answer: 0.26

SOLUTION:

Step 1: Compute the Hall resistance first.

Instead of plugging everything into one formula, define the Hall resistance as the ratio of the measured Hall voltage to the current:

$$R_{Hall} = \frac{V_H}{I} = \frac{0.9 \times 10^{-3}}{6 \times 10^{-3}} = 0.15\ \Omega$$

Step 2: Relate the Hall resistance to the magnetic field.

From $V_H = R_H IB/t$, dividing both sides by I gives $R_{Hall} = R_H B/t$, so

$$B = \frac{R_{Hall}\, t}{R_H}$$

Step 3: Substitute the numbers.

With $R_{Hall} = 0.15\ \Omega$, $t = 0.7 \times 10^{-3}$ m and $R_H = 4 \times 10^{-4}\ \mathrm{m^3/C}$:

$$B = \frac{0.15 \times 0.7 \times 10^{-3}}{4 \times 10^{-4}} = \frac{1.05 \times 10^{-4}}{4 \times 10^{-4}} = 0.2625\ \mathrm{T}$$

Final Answer:

Rounded off to two decimal places, the incident magnetic field is 0.26 T.

$$\boxed{B = 0.26\ \mathrm{T}}$$

Quick Tip: Start from $V_H = R_H IB/t$ and solve for B. Keep every quantity in base SI units before you divide.

• • •

64. For a fiber Bragg grating-based sensor, the shift in the Bragg wavelength gives information about the value of the measurand. For a Bragg wavelength of 1550 nm and effective index of 1.44, the grating period is nm (rounded off to two decimal places).

Correct Answer: 538

SOLUTION:

Step 1: Find the wavelength of the light once it is inside the fiber.

Light slows down inside the fiber core, so its wavelength inside the medium shrinks by a factor of the effective index:

$$\lambda_{fiber} = \frac{\lambda_B}{n_{eff}} = \frac{1550}{1.44} \approx 1076.39 \text{ nm}$$

Step 2: Relate the grating period to this in-fiber wavelength.

A Bragg grating reflects strongly when one grating period corresponds to half a wavelength inside the fiber, so that reflections from consecutive grating lines reinforce each other:

$$\Lambda = \frac{\lambda_{fiber}}{2}$$

Step 3: Compute the grating period.

$$\Lambda = \frac{1076.39}{2} \approx 538.19 \text{ nm}$$

Final Answer:

Rounded off to two decimal places, the grating period is 538.19 nm.

$$\boxed{\Lambda = 538.19 \text{ nm}}$$

Quick Tip: Use the Bragg condition $\lambda_B = 2n_{eff}\Lambda$ and solve for the grating period Λ.

• • •

65. A photodiode has a responsivity of 0.8 A/W. If the input optical power to the photodiode is 2 mW, the power delivered to a 50 Ω load is μW (rounded off to the nearest integer).

Correct Answer: 128

SOLUTION:

Step 1: Find the photocurrent.

The photodiode converts optical power into current at the rate set by its responsivity:

$$I_{ph} = R \times P_{in} = 0.8 \times 2\text{ mW} = 1.6\text{ mA}$$

Step 2: Find the voltage this current builds across the load.

By Ohm's law, driving I_{ph} through the 50 Ω load resistor produces a voltage across it:

$$V_L = I_{ph} \times R_L = 1.6 \times 10^{-3} \times 50 = 0.08\text{ V} = 80\text{ mV}$$

Step 3: Get the power from voltage and resistance.

$$P_L = \frac{V_L^2}{R_L} = \frac{(0.08)^2}{50} = \frac{0.0064}{50} = 1.28 \times 10^{-4}\text{ W}$$

$$P_L = 128\ \mu\text{W}$$

Final Answer:

Rounded off to the nearest integer, the power delivered to the load is $128\ \mu$W.

$$\boxed{P_L = 128\ \mu\text{W}}$$

Quick Tip: First find the photocurrent using $I_{ph} = R \times P_{in}$, then use $P_L = I_{ph}^2 R_L$ to find the power in the load.