GATE 2026 Mathematics Question Paper

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General Instructions

(i) This question paper contains 65 questions. All questions are compulsory.

(ii) It comprises 32 single-correct multiple-choice questions, 14 multiple-correct questions and 19 numerical / integer-type questions.

(iii) These questions follow the GATE Mathematics examination pattern.

(iv) Questions carry 1 or 2 marks. MCQs have negative marking (1/3 or 2/3 of the marks); MSQ and numerical-answer (NAT) questions carry no negative marking.

(v) GATE is a 3-hour paper of 65 questions (100 marks).

(vi) A virtual scientific calculator is provided; physical calculators are not allowed.

(vii) Attempt every question; detailed solutions are provided in the companion solutions booklet.

(viii) For multiple-correct questions, all correct options must be selected to earn full marks.

(ix) For numerical questions, report the answer rounded exactly as asked.

1. Suresh said, "I did it yesterday." Which one of the following options is the correct form of this sentence in indirect speech?

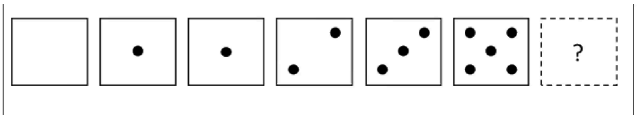
(A) Suresh said that I did it yesterday.

(B) Suresh says I did it yesterday.

(C) Suresh says that he did it the day before.

(D) Suresh said that he had done it the day before.

2. To continue the sequence of tiles shown, the tile indicated by the question mark should be:



(The tiles in the sequence contain, in order, 0 dots, 1 dot, 1 dot, 2 dots, 3 dots, and 5 dots, followed by a tile marked with a question mark.)

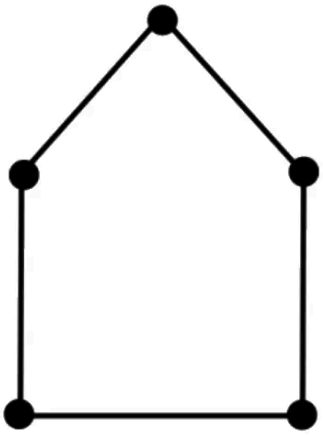
(A) A tile showing 4 dots arranged in a 2x2 square.

(B) A tile showing 6 dots.

(C) A tile showing 8 dots arranged in a ring.

(D) A tile showing 9 dots arranged in a 3x3 square.

3. Consider an art gallery whose walkways are shown as lines in the diagram. A black dot represents a junction of two walkways. A guard may be placed at a junction to watch over the walkways that join at that junction. The minimum number of guards needed to watch all the walkways is _____.



(The diagram shows five junctions joined in a closed loop shaped like a pentagon: a top junction connects down to an upper-left junction and an upper-right junction, the upper-left junction connects down to a lower-left junction, the upper-right junction connects down to a lower-right junction, and the lower-left and lower-right junctions connect to each other. This gives 5 junctions and 5 walkways in total.)

(A) 2

(B) 3

(C) 4

(D) 5

4. The 2nd of June is a Thursday in a certain year. Which day of the week is the 3rd of July in that year?

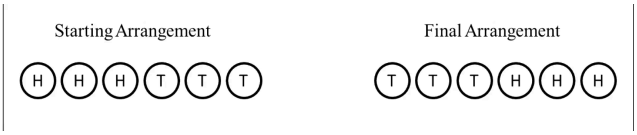
(A) Thursday

(B) Friday

(C) Saturday

(D) Sunday

5. A coin with heads facing up is shown as H and a coin with tails facing up is shown as T. Six coins are placed in the Starting Arrangement, as shown in the figure below. A "step" is defined as interchanging a pair of adjacent coins without flipping them.



Starting Arrangement: H H H T T T
Final Arrangement: T T T H H H
The minimum number of steps needed to go from the Starting Arrangement to the Final Arrangement, as shown in the figure, is _____.

(A) 3

(B) 6

(C) 9

(D) 12

6. Exacerbate : Mitigate :: _____
Choose the option with the correct pair of words to fill the blank.

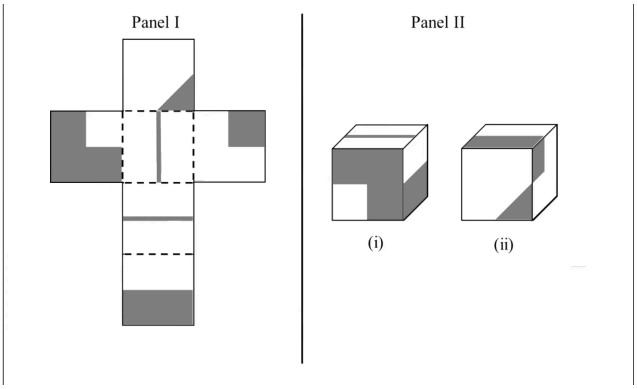
(A) Aggravate : Alleviate

(B) Alleviate : Precipitate

(C) Aggravate : Precipitate

(D) Emancipate : Exonerate

7. A paper shown in Panel I is folded along the dashed lines to construct a cube. The shaded regions shown in Panel I appear on the outer surface of the cube. Referring to the cubes shown in Panel II, which one of the options is correct?



(Panel I shows a flat cross-shaped net of six squares: a vertical strip of four squares one above another, with one extra square attached to the left of the second square in that strip and another extra square attached to its right, all joined along dashed fold lines. Some squares carry grey shaded regions and some are left plain white. Panel II shows two separate solid cubes, labelled (i) and (ii), each with grey shaded regions on some of their visible faces.)

(A) Only (i) can correspond to the unfolded cube in Panel I.

(B) Only (ii) can correspond to the unfolded cube in Panel I.

(C) Both (i) and (ii) can correspond to the unfolded cube in Panel I.

(D) Neither (i) nor (ii) can correspond to the unfolded cube in Panel I.

8. In a population, patients who have high cholesterol also have high blood-pressure (BP). Some patients with high BP also have diabetes. There are no patients who have both high cholesterol and diabetes. Furthermore,

1. the total number of patients with at least one of these conditions is 75,
2. the number of patients with high cholesterol is 10,
3. the number of patients with high BP is 45, and
4. the number of patients with only high BP and no other conditions is 20.

Then the number of patients who have both diabetes and high BP is _____

(A) 0

(B) 15

(C) 20

(D) 10

9. Four people P, Q, R, and S, of different ages, make the following observations.

P - I am younger than S.

Q - I am neither the youngest nor the oldest.

R - P is older than me.

Based on these observations, the youngest person is _____.

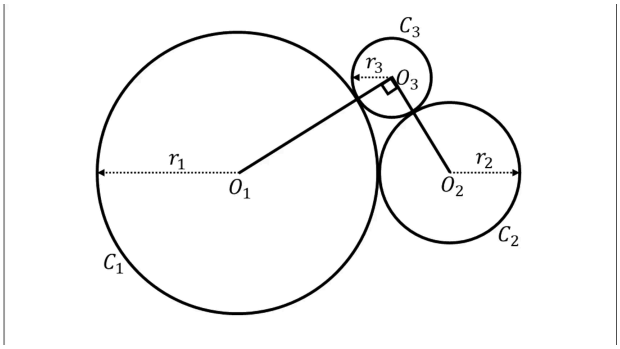
(A) P

(B) Q

(C) R

(D) S

10. Circles C_1, C_2, and C_3, with centers O_1, O_2, and O_3, and radii r_1, r_2, and r_3, respectively, touch each other as shown in the following figure.



Given $r_1 = 2$ cm, $r_2 = 1$ cm and the angle $\angle O_1O_3O_2$ is $90°$, $r_3 =$ _____ cm.

(A) $\frac{1}{2}\left(-3+\sqrt{17}\right)$

(B) $\frac{1}{2}\left(3+\sqrt{17}\right)$

(C) $\frac{1}{2}\left(-2+\sqrt{17}\right)$

(D) $\frac{1}{2}\left(-3+2\sqrt{17}\right)$

11. Let $M_3(\mathbb{R})$ be the vector space of all 3×3 real matrices over $\mathbb{R}$ under usual matrix addition and scalar multiplication. Let $T_3(\mathbb{R})$ be the set of all 3×3 real upper triangular matrices. Which one of the following is TRUE?

(A) The quotient space $M_3(\mathbb{R})/T_3(\mathbb{R})$ is isomorphic to the vector space of all 3×3 real skew symmetric matrices over $\mathbb{R}$ under usual matrix addition and scalar multiplication.

(B) The quotient space $M_3(\mathbb{R})/T_3(\mathbb{R})$ is isomorphic to the vector space of all 3×3 real symmetric matrices over $\mathbb{R}$ under usual matrix addition and scalar multiplication.

(C) The quotient space $M_3(\mathbb{R})/T_3(\mathbb{R})$ is isomorphic to the vector space of all 3×3 real lower triangular matrices over $\mathbb{R}$ under usual matrix addition and scalar multiplication.

(D) The quotient space $M_3(\mathbb{R})/T_3(\mathbb{R})$ is isomorphic to the vector space of all 3×3 real matrices with trace zero over $\mathbb{R}$ under usual matrix addition and scalar multiplication.

12. Let I be the integral defined as follows:

$$I = \int_0^1 \int_0^{\sqrt{y}} dx\,dy + \int_1^2 \int_{\sqrt{y-1}}^1 dx\,dy$$

If the order of integration is changed, then which one of the following is the correct expression for I?

(A)

$$\int_0^{1/2}\int_{-x}^{x} dy\,dx + \int_{1/2}^{1}\int_{-x^2}^{-x^2+1} dy\,dx$$

(B)

$$\int_0^{1}\int_{x^2}^{x^2+1} dy\,dx$$

(C)

$$\int_0^{1/2}\int_{x^2}^{x^2+1} dy\,dx + \int_{1/2}^{1}\int_{\sqrt{x}}^{\sqrt{x+1}} dy\,dx$$

(D)

$$\int_0^{1}\int_{x^2}^{x^2-1} dy\,dx$$

13. Let C denote the Cantor set and $f : [0,1] \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} x^{2026} & \text{for } x \in C \\ \cos(\pi x) & \text{for } x \in \left[0, \frac{1}{2}\right] \setminus C \\ \sin(\pi x) & \text{for } x \in \left[\frac{1}{2}, 1\right] \setminus C \end{cases}$$

The value of the Lebesgue integral of $f(x)$ over the interval $[0,1]$ is equal to

(A) $\frac{2}{\pi}$

(B) $\frac{1}{\pi}$

(C) $\frac{3}{\pi}$

(D) 0

14. Let G be a group of order 595. Which one of the following is TRUE?

(A) G cannot have a proper normal subgroup.

(B) G must have a proper normal subgroup.

(C) G cannot have an element of order 17.

(D) The number of Sylow 5-subgroups is 17.

15. Let $u(x,t)$ be the solution of the initial value problem for the heat equation on the real line:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \qquad -\infty < x < \infty, \qquad t > 0, \qquad k \in \mathbb{R}$$

with the initial condition

$$u(x,0) = e^{-a|x|}, \qquad a > 0.$$

If $\hat{u}(w,t) = \int_{-\infty}^{\infty} u(x,t)\, e^{iwx}\, dx$ is the Fourier transform of $u(x,t)$ with respect to x, then $\hat{u}(w,t)$ is equal to

(A)

$$\frac{2a}{a^2+w^2}\,e^{-kw^2t}$$

(B)

$$\frac{a}{a^2+w^2}\,e^{-kw^2t}$$

(C)

$$\frac{2a}{a^2+w^2}\,e^{-ka^2t}$$

(D)

$$\frac{1}{\sqrt{4\pi kt}}\,e^{\frac{-w^2}{4kt}}$$

16. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined as

$$f(x, y) = (e^x \cos y,\ e^x \sin y)$$

Which one of the following is TRUE?

(A) f is one-to-one.

(B) The Jacobian of f is negative.

(C) f is locally invertible.

(D) f is invertible on $\mathbb{R}^2$.

17. Let $f : \mathbb{C} \to \mathbb{C}$ be defined by $f(z) = |z|^2 - 5\bar{z} + 2$.
Which one of the following is TRUE?

(A) f is differentiable at $z = 5i$.

(B) f is differentiable at $z = 5$.

(C) f is differentiable at $z = -5i$.

(D) f is differentiable at $z = -5$.

18. Let $D = \{z \in \mathbb{C} \mid |z| < 1\}$ denote the unit disc in the complex plane $\mathbb{C}$. Let $f : D \to D$ be an analytic function which satisfies $f(0) = 0$. Then which one of the following is a possible value of $f'(0)$?

(A) $\frac{5}{2}i$

(B) $\frac{i}{10}$

(C) $\frac{3}{2}$

(D) $-\frac{5}{2}i$

19. Let X be the space of all continuously differentiable real valued functions on $[0, 1]$. Define the following norms on X:

$$p_1(x) = \sup\{|x(t)| : t \in [0, 1]\}$$

$$p_2(x) = \sup\left\{\left|\frac{d}{dt}x(t)\right| : t \in [0, 1]\right\}$$

and $p_3(x) = p_1(x) + p_2(x)$.
Which one of the following is TRUE?

(A) (X, p_1) is a Banach space.

(B) (X, p_2) is a Banach space.

(C) (X, p_3) is not a Banach space.

(D) (X, p_3) is a Banach space.

20. Let X be a set with at least two elements. Let d_1, d_2 and d_3 be metrics on X.

Which one of the following is NOT a metric on X?

(A) $d(x, y) := \min(d_1(x, y), 3)$ for all $x, y \in X$.

(B) $d(x, y) := \max(d_2(x, y), 3)$ for all $x, y \in X$.

(C) $d(x, y) := \dfrac{10\, d_3(x, y)}{1 + d_3(x, y)}$ for all $x, y \in X$.

(D) $d(x, y) := \dfrac{1}{3}(d_1(x, y) + d_2(x, y) + d_3(x, y))$ for all $x, y \in X$.

21. Let P be a 3×3 real symmetric positive definite matrix. Which of the following statements is/are TRUE?

(A) $Q^{\mathsf{T}}PQ$ is positive definite for all nonzero $Q \in \mathbb{R}^{3\times3}$.

(B) $Q^{\mathsf{T}}PQ$ is positive semidefinite for all $Q \in \mathbb{R}^{3\times3}$.

(C) $Q^{\mathsf{T}}PQ$ is positive definite if $Q \in \mathbb{R}^{3\times3}$ is nonsingular.

(D) $Q^{\mathsf{T}}PQ$ is not positive semidefinite for some $Q \in \mathbb{R}^{3\times3}$.

22. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defined as follows:

$$f(x,y) = \begin{cases} \dfrac{x\sqrt{x^2+y^2}}{|x|} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$$

Which of the following statements is/are TRUE?

(A) $f(x,y)$ is continuous at $(0,0)$.

(B) For any $u \in \mathbb{R}^2$, the directional derivative of f at $(0,0)$ in the direction of u exists.

(C) $\dfrac{\partial f}{\partial x}(0,0) = 1 = \dfrac{\partial f}{\partial y}(0,0)$.

(D) $f(x,y)$ is differentiable at $(0,0)$.

23. Let $\mathbb{Z}_{11}$ be the ring of integers modulo 11. Let $\mathbb{Z}_{11}[x]$ be the ring of all polynomials with coefficients in $\mathbb{Z}_{11}$. Which of the following statements is/are TRUE?

(A) The polynomial $x^2 + x + 4$ is irreducible over $\mathbb{Z}_{11}$.

(B) $\mathbb{Z}_{11}[x]/\langle x^2 + x + 4\rangle$ is a field.

(C) The ideal $\langle x^2 + x + 4\rangle$ is maximal.

(D) The ideal $\langle x^2 + x + 4\rangle$ is not a prime ideal.

24. Consider $\mathbb{R}$ with the topology

$$\tau = \{A \subseteq \mathbb{R} : A^c \text{ is finite}\} \cup \{\phi\}.$$

Which of the following statements is/are TRUE?

(A) $(\mathbb{R}, \tau)$ is a Hausdorff space.

(B) Any finite subset of $(\mathbb{R}, \tau)$ is closed.

(C) $(\mathbb{R}, \tau)$ is compact.

(D) $(\mathbb{R}, \tau)$ is connected.

25. Consider the following partial differential equation:

$$a\frac{\partial^2 f(x,y)}{\partial x^2} + b\frac{\partial^2 f(x,y)}{\partial y^2} = 8f(x,y)$$

where a and b are distinct positive real numbers.
The combination(s) of the values of the real parameters ξ and η for which $f(x,y) = e^{2\xi x+\eta y}$ is a solution of the given partial differential equation, is/are

(A) $\xi = \frac{1}{\sqrt{a}}, \eta = \frac{2}{\sqrt{b}}$

(B) $\xi = 0, \eta = 0$

(C) $\xi = \frac{1}{\sqrt{2a}}, \eta = \frac{1}{\sqrt{2b}}$

(D) $\xi = 0, \eta = 2\sqrt{\frac{2}{b}}$

26.

Consider the following assignment problem where X, Y, Z are tasks, P, Q, R are agents and the cost matrix is given by:

	X	Y	Z
P	4	2	8
Q	2	3	7
R	3	1	6

Which of the following statements is/are TRUE for an optimal assignment?

(A) The cost is 9.

(B) Agent P is assigned to task Y.

(C) Agent Q is assigned to task X.

(D) Agent R is assigned to task Y.

27.

Consider the following differential equation:

$$2x(x-2)^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + (x-2)y = 0$$

Which of the following statements is/are TRUE?

(A) The point $x = 2$ is not a singular point.

(B) The point $x = 2$ is a singular point.

(C) The point $x = 2$ is a regular singular point.

(D) The point $x = 2$ is not a regular singular point.

28.

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation which reflects every vector in $\mathbb{R}^3$ through a two-dimensional subspace of $\mathbb{R}^3$. Let $P \in \mathbb{R}^{3\times3}$ be the matrix representation of T using the basis

$$\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}\right\}.$$

Then the value of $2 \times \text{trace}(P) - 3 \times \text{determinant}(P)$ is equal to _____. (answer in integer)

29.

Let $x^3 - x + 1 \in \mathbb{Z}_3[x]$, where $\mathbb{Z}_3[x]$ is the ring of all polynomials with coefficients in $\mathbb{Z}_3$. Then the degree of the field extension

$$\mathbb{Z}_3[x]/\langle x^3 - x + 1\rangle$$

of $\mathbb{Z}_3$ is equal to _____. (answer in integer)

30.

Consider the differential equation $\frac{dy}{dx} = x + y$ with the initial condition $y(0) = 1$. Using the modified Euler's method, the second approximation to $y(h)$, where $h = 0.05$ (step size), is equal to _____. (rounded off to TWO decimal places)

31. Let

$$\alpha = \lim_{n\to\infty} \int_0^1 \frac{n^2 + (\sin e^x)^n}{7n^2 + x^8}\, dx.$$

The value of 14α is equal to _____. (Answer in integer)

32. Let $X = (\mathbb{R}^3, \|\cdot\|_1)$, where $\left\|\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right\|_1 = |x| + |y| + |z|$, and let $T : X \to X$ be the linear transformation defined by

$$T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 2 & -2 \\ 1 & 3 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

The operator norm of T is equal to _____. (Answer in integer)

33. Consider the following Linear Programming Problem (LPP):

$$\text{maximize } z = 10x_1 + 20x_2$$

subject to

$$x_1 \le 36, \quad x_2 \ge 42, \quad x_1 + x_2 \ge 48, \quad 5x_1 + x_2 \le 150, \quad x_1, x_2 \ge 0.$$

The maximum value of z in the above LPP is equal to _____. (Answer in integer)

34. The number of distinct topologies on the set $\{1, 2, 3\}$ consisting of exactly four elements is equal to _____. (Answer in integer)

35. For $a_n \in \mathbb{C}$, $n = 0, 1, 2, \ldots$, the power series

$$\sum_{n=0}^{\infty} a_n (z-2)^n$$

converges at $z = 5$ and diverges at $z = -1$. Then the radius of convergence of this power series is equal to _____. (Answer in integer)

36. The solution $y(x) = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}$ of the initial value problem

$$\frac{dy}{dx} = \begin{pmatrix} -3 & 4 \\ -2 & 3 \end{pmatrix} y, \qquad y(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

is equal to

(A) $3\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^x + \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-x}$

(B) $-3\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^x + \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-x}$

(C) $3\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^x + 2\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-x}$

(D) $3\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^x - \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-x}$

37. Let $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 16 \end{pmatrix}$. Define an inner product on $\mathbb{R}^3$ with respect to P as

$$\langle x, y \rangle_P = x^{\mathsf{T}} P y, \quad \text{for all } x, y \in \mathbb{R}^3.$$

Consider the subspace $V = \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$. Which one of the following sets is an orthonormal basis of V with respect to $\langle x, y \rangle_P$?

(A) $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$

(B) $\left\{\begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}\right\}$

(C) $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\\frac{1}{5}\\\frac{1}{5}\end{pmatrix}\right\}$

(D) $\left\{\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{pmatrix}\right\}$

38. The value of the contour integral

$$\int_{\Gamma} \frac{(1-z)(3\cos z + 5\sin z)}{1-z^{101}}\, dz,$$

where $\Gamma = \{z \in \mathbb{C} : |z| = \frac{7}{9}\}$, oriented in the counter-clockwise direction, is equal to

(A) $2\pi i$

(B) $-2\pi i$

(C) 0

(D) $4\pi i$

39. Let $f(z) = \sum_{n=1}^{\infty} 5^{-n} \cos(nz)$. On which one of the following domains, does $f(z)$ represent an analytic function?

(A) $\{z \in \mathbb{C} : |\operatorname{Im} z| < \ln 5\}$

(B) $\{z \in \mathbb{C} : |\operatorname{Im} z| > \ln 5\}$

(C) $\{z \in \mathbb{C} : |\operatorname{Re} z| < \ln 5\}$

(D) $\{z \in \mathbb{C} : |\operatorname{Re} z| > \ln 5\}$

40. Let $\delta(t)$ be the unit impulse function defined by $\delta(x - x_0) = 0$ when $x \neq x_0$ and $\int_{-\infty}^{\infty} \delta(x - x_0)\, dx = 1$.

Consider the following initial value problem

$$2\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = \delta(x - 5)$$

with $y(0) = 0$ and $\frac{dy}{dx}(0) = 0$. Which one of the following is TRUE?

(A) $y(10) = \frac{10}{\sqrt{15}} e^{-5/4} \sin\left(\frac{\sqrt{15}}{4}\right)$

(B) $y(10) = \frac{2}{\sqrt{15}} e^{-5/4} \sin\left(\frac{5\sqrt{15}}{4}\right)$

(C) $y(10) = \frac{10}{\sqrt{15}} e^{5/4} \sin\left(\frac{\sqrt{15}}{4}\right)$

(D) $y(10) = \frac{2}{\sqrt{15}} e^{5/4} \sin\left(\frac{5\sqrt{15}}{4}\right)$

41. Which one of the following statements is TRUE?

(A) The set of all 2×2 real matrices under usual matrix addition and multiplication is a Unique Factorization Domain.

(B) The set of all real valued functions under usual function addition and multiplication is a Unique Factorization Domain.

(C) The set of all polynomials with coefficients in $\mathbb{R}$ under usual polynomial addition and multiplication is a Unique Factorization Domain.

(D) The set of Riemann integrable functions in $[0, 1]$ under usual function addition and multiplication is a Unique Factorization Domain.

42. Let $\ell^\infty = \{x = (x_n)_{n\geq 1} \mid x_n \in \mathbb{R},\ \sup\{|x_n| : n = 1, 2, \ldots\} < \infty\}$ with the supremum norm. Let $T : \ell^\infty \to \ell^\infty$ be given by $T(x_1, x_2, x_3, \ldots) = \left(x_1, \frac{x_2}{2}, \frac{x_3}{3}, \ldots\right)$. Which one of the following is TRUE?

(A) T is bounded but not one-to-one.

(B) T is one-to-one but not bounded.

(C) T is bounded and the inverse (from the range of T) exists but not bounded.

(D) T is bounded and the inverse (from the range of T) is bounded.

43. Let $u(x,t)$ satisfy the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad -\infty < x < \infty,\ t > 0,\ c > 0$$

with

$$u(x,0) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\frac{\partial u}{\partial t}(x,0) = 0.$$

By using D'Alembert's formula, the maximum value of $u(0,t)$ for $t > 0$ is

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) 0

44. Consider the Laplace equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0, \qquad 0 < x < 1,\ 0 < y < 1,$$

with the boundary conditions

$$T(x,0) = x, \qquad T(0,y) = y$$

$$T(x,1) = 1 + x, \qquad T(1,y) = 1 + y.$$

Then the value of $T\left(\frac{1}{2}, \frac{1}{3}\right)$ is equal to

(A) $\frac{7}{6}$

(B) $\frac{5}{6}$

(C) $\frac{1}{6}$

(D) $\frac{1}{2}$

45. Let E_1 and E_2 be subsets of a normed linear space X, and

$$E_1 + E_2 = \{x + y : x \in E_1,\ y \in E_2\}$$

$$E_1 \times E_2 = \{(x, y) : x \in E_1,\ y \in E_2\}.$$

Which one of the following is NOT TRUE?

(A) If either of E_1 or E_2 is open, then $E_1 + E_2$ is open.

(B) If E_1 and E_2 are convex, then $E_1 + E_2$ is convex.

(C) If E_1 and E_2 are closed, then $E_1 + E_2$ is closed.

(D) If E_1 and E_2 are connected, then $E_1 \times E_2$ is connected.

46. For a transportation problem, let c_{ij} denote the unit cost of the cell (i, j). The known unit costs are shown below (a dash marks a cell whose cost is not given).

	j = 1	**j = 2**	**j = 3**
i = 1	10	12	19
i = 2	11	13	-
i = 3	-	-	-

Let α_i and β_j, $i, j = 1, 2, 3$, be the simplex multipliers associated with a basis corresponding to this unit cost table, so that $\alpha_1 = x$, $\alpha_2 = x + 1$, $\beta_1 = y$ and $\beta_2 = y + 2$. The relative cost coefficient d_{ij} is the difference between the current solution and the new improved solution.
If $x = 4$, $c_{13} = 19$ and $\beta_3 = y + 5$, then which one of the following is TRUE?

(A) $y = 6$ and $d_{13} = 7$

(B) $y = 6$ and $d_{13} = 4$

(C) $y = 5$ and $d_{13} = 7$

(D) $y = 5$ and $d_{13} = 4$

47. Let X and Y be topological spaces and $f : X \to Y$ be a continuous and bijective mapping. Which one of the following statements is TRUE?

(A) f is a homeomorphism if X and Y are compact.

(B) f is a homeomorphism if X is Hausdorff and Y is compact.

(C) f is a homeomorphism if X is compact and Y is Hausdorff.

(D) f is a homeomorphism if X and Y are Hausdorff.

48. Let P and Q be 3×3 nonzero real matrices. Assume that there exists a 3×3 real nonsingular matrix S such that $S^{-1}PS$ and $S^{-1}QS$ are both upper triangular. Which of the following statements is/are TRUE?

(A) The matrix $PQ - QP$ is invertible.

(B) The matrix $PQ - QP$ is nilpotent.

(C) The matrix $I + PQ - QP$ is invertible, where I is the 3×3 identity matrix.

(D) If $PQ - QP$ is a nonzero matrix, then $PQ - QP$ is diagonalizable.

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49. Let $S = \bigcup_{n=1}^{\infty} \left\{(x, y) \in \mathbb{R}^2 : (x - n)^2 + y^2 = \frac{1}{n^2}\right\}$ with the usual topology. Which of the following statements is/are TRUE?

(A) S is not compact.

(B) S is compact.

(C) S is connected.

(D) S is not connected.

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50. Let $f(z) = \frac{z}{1 - z}$ and $g(z) = \frac{1 + z}{1 - z}$ be two Mobius transformations defined on the unit disc $D = \{z \in \mathbb{C} : |z| < 1\}$. Consider the following statements:

S_1: $f(D) \subseteq g(D)$

S_2: $g(D) \subseteq f(D)$

Which of the following statements is/are CORRECT?

(A) S_1 is true.

(B) S_2 is true.

(C) S_2 is true and S_1 is false.

(D) Neither S_1 is true nor S_2 is true.

51. Let α and β be the roots of the indicial equation obtained in the method of finding the Frobenius series solution to the differential equation:

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1 - x^2)y = 0.$$

Which of the following statements is/are TRUE?

(A) $\alpha \neq \beta$

(B) $\alpha - \beta \in \mathbb{Z}$

(C) $\alpha - \beta \notin \mathbb{Z}$

(D) $\alpha = \beta$

52. Let $\mathbb{Q}[x]$ be the ring of all polynomials with coefficients in $\mathbb{Q}$ under the usual polynomial addition and multiplication. Let $T : \mathbb{Q}[x] \to \mathbb{Q}[x]$ be defined by

$$T(p(x)) = p(x^2), \quad \text{for all } p(x) \in \mathbb{Q}[x].$$

Which of the following statements is/are TRUE?

(A) T is a ring homomorphism.

(B) T is one-to-one.

(C) T is onto.

(D) T is a ring isomorphism.

53. Let X be any normed linear space and X' be the dual space of X. Which of the following statements is/are TRUE?

(A) If X is separable and X' is non-separable, then X must be reflexive.

(B) If X is separable and X' is non-separable, then X cannot be reflexive.

(C) If X is reflexive, then X' is reflexive.

(D) If X' is separable, then X is separable.

54. Let $(\mathbb{Z}_n, +)$ be the group of integers modulo n. Let $G = \mathbb{Z}_{30} \oplus \mathbb{Z}_{12}$ be the external direct product of $\mathbb{Z}_{30}$ and $\mathbb{Z}_{12}$. Then which of the following statements is/are TRUE?

(A) G is not a cyclic group.

(B) The order of the element $(14, 7)$ in G is 98.

(C) The order of the element $(14, 7)$ in G is 60.

(D) There is an element in G of order 360.

55. Let

$$J = \begin{pmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}.$$

Then the geometric multiplicity of the eigenvalue 2 of J is equal to _____. (Answer in integer)

56. Let $\alpha = \iint_S \vec{F} \cdot \hat{n}\, dS$, where $\vec{F} = (2x + 3z)\hat{i} + (xz - y)\hat{j} + (y^2 + 2z)\hat{k}$ and S is the sphere with centre at $(3, -1, 2)$ and radius 9. Here $\hat{n}$ is the unit normal drawn outward and $\hat{i}, \hat{j}, \hat{k}$ are unit vectors.

Then the value of $\frac{1}{36\pi}\alpha$ is equal to _____. (Answer in integer)

57. Consider the problem of maximizing

$$z = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

subject to

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 1,$$

where $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in \mathbb{R}^3$.

Then the maximum value of z is _____. (Answer in integer)

58. Let $P_3(\mathbb{R})$ be the vector space of all polynomials of degree at most three with real coefficients under usual polynomial addition and scalar multiplication. Let $T : P_3(\mathbb{R}) \to \mathbb{R}^2$ be the linear transformation defined as

$$T(p) = \big(p(1), p'(1)\big)$$

for all $p \in P_3(\mathbb{R})$, where p' is the derivative of p.

Then the nullity of T is equal to _____. (Answer in integer)

59. The number of zeros of the complex polynomial $z^6 + 5z^3 + 4z + 11$ in the annulus $\{z \in \mathbb{C} : 1 < |z| < 3\}$ is equal to _____. (Answer in integer)

60. Let $L^2[0, \pi]$ denote the space of all real valued Lebesgue square integrable functions on $[0, \pi]$. Let $T : L^2[0, \pi] \to L^2[0, \pi]$ be defined as follows:

$$T(f(x)) = \sin x \int_0^{\pi} f(t) \cos t \, dt + \cos x \int_0^{\pi} f(t) \sin t \, dt$$

Then the value of $\frac{4}{\pi}\|T\|$ is equal to _____. (Answer in integer)

61. Let $\Omega = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$ be the open unit disc and $\partial\Omega$ be its boundary. If $u(x, y)$ is the solution of the following Dirichlet problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{in } \Omega$$

$$u(x, y) = x^2 - y^2 \quad \text{on } \partial\Omega,$$

then the value of $4\left(u\left(\frac{1}{2}, 0\right) - u\left(0, \frac{1}{2}\right)\right)$ is _____.

62. Let $X = \{5, 6, 7, 8, 9, 10\}$ be equipped with the topology

$$\tau = \{\phi, X, \{5, 6, 7\}, \{8, 9, 10\}\}$$

Then the number of subsets of X which are neither open nor closed is _____.

63. Let $\alpha, \beta \in \mathbb{R}$. If $(4, 0, 2, \beta)$ is an optimal solution of the Linear Programming Problem:

$$\text{minimize} \quad x_1 + 3x_2 + 2x_3 - \alpha x_4$$

subject to

$$4x_1 + x_2 + x_3 = 18$$

$$-3x_1 + 2x_3 + x_4 = 2$$

$$x_1, x_2, x_3, x_4 \geq 0,$$

then the maximum value of $22(\alpha + \beta)$ is equal to _____.

64. Let $D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 2,\ 0 \leq y \leq 2\}$ and let $f(t)$ denote the smallest integer greater than or equal to t. Then the value of the integral

$$\iint_D f(x + y)\, dx\, dy$$

is _____.

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65. If Jacobi method is used to solve the following system of linear equations

$$\begin{pmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

with the initial guess $x^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ and $x^{(i)} = \begin{pmatrix} x_1^{(i)} \\ x_2^{(i)} \\ x_3^{(i)} \end{pmatrix}$, $i = 1, 2, 3, \ldots$, denotes the i^{th} iterate, then the value of $\left| x_1^{(2)} + x_2^{(2)} + x_3^{(2)} \right|$ is equal to _____.