General Instructions

(i) This booklet contains 65 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 37 single-correct multiple-choice questions, 1 multiple-correct question and 27 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For multiple-correct questions, all correct options must be selected to earn full marks.

(v) For numerical questions, report the answer rounded exactly as asked.

. Suresh said, "I did it yesterday."
Which one of the following options is the correct form of this sentence in indirect speech?

(A) Suresh said that I did it yesterday.

(B) Suresh says I did it yesterday.

(C) Suresh says that he did it the day before.

(D) Suresh said that he had done it the day before.

Correct Answer: (D) Suresh said that he had done it the day before.

SOLUTION:

This question tests the rules for turning direct speech into indirect (reported) speech. The original sentence is: Suresh said, "I did it yesterday." Three changes are needed here: the tense inside the quote must shift back one step, the pronoun "I" must switch to match the subject "Suresh", and the time word "yesterday" must change since we are reporting it later. Let's check each option against these three rules.

1. **Suresh said that I did it yesterday.**: The pronoun "I" is left unchanged even though Suresh is being talked about in the third person, the verb "did" is not shifted back to "had done", and "yesterday" is left as is. None of the three required changes are made.
2. **Suresh says I did it yesterday.**: The reporting verb is wrongly changed from "said" to "says", turning it into the present tense, and the tense inside the quote and the time word are also left unchanged.
3. **Suresh says that he did it the day before.**: The pronoun and the time word are handled correctly here, "he" and "the day before" are both right, but the reporting verb

is still wrongly written as "says" instead of "said", and "did" is not shifted to "had done".

4. **Suresh said that he had done it the day before.**: The reporting verb "said" is kept in the past tense as in the original, "did" correctly shifts back to "had done", the pronoun becomes "he", and "yesterday" becomes "the day before". All three rules are satisfied.

Since the original reporting verb "said" is already in the past tense, it stays "said" in the indirect version too, it is only the verb reported inside the quote that shifts further back to the past perfect. That single distinction is what separates the correct option from the three incorrect ones.

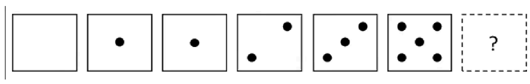
Let's summarize:

- A past-tense reporting verb like "said" does not change to "says" or "said" plus "to" in reported speech, it stays "said".
- The verb inside the quote shifts one tense back: simple past becomes past perfect.
- Pronouns and time words like "yesterday" change to fit the new speaker and the new time of reporting.

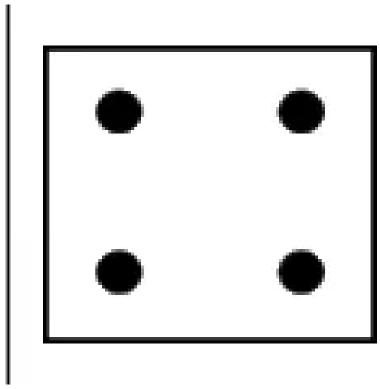
The correct indirect form is "Suresh said that he had done it the day before", which is option (D).

Quick Tip: Direct-to-indirect speech needs three checks: does the reporting verb "said" stay as is, does the verb inside the quote shift one tense back, and does the time word "yesterday" change to match the new point of view.

. To continue the sequence of tiles shown, the tile indicated by the question mark should be:



(A)



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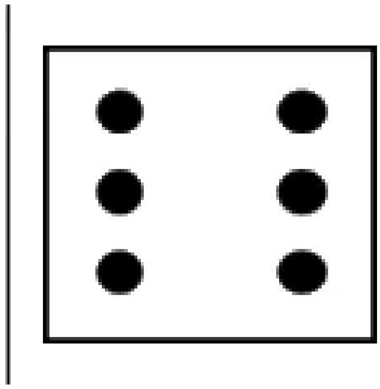
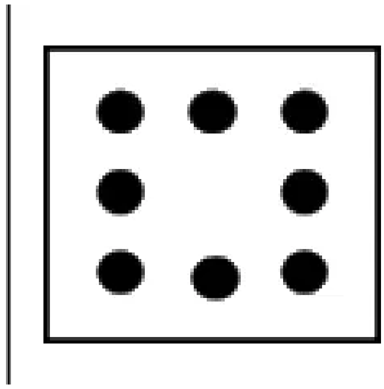
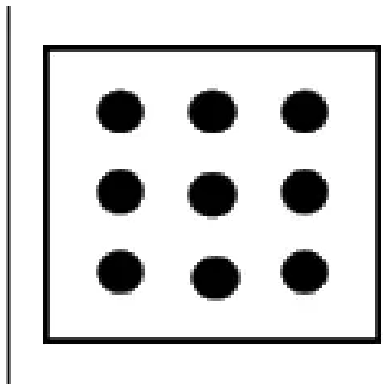


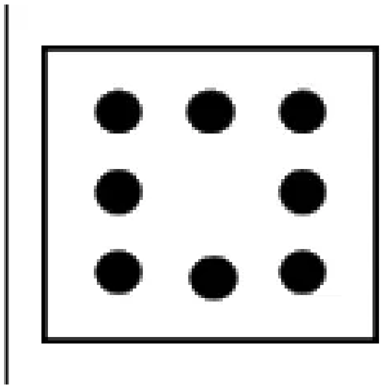
(C)



(D)



Correct Answer: (C)



SOLUTION:

This is a number pattern hidden inside a picture. Instead of reading the tiles as pictures, count the dots in each one: 0, 1, 1, 2, 3, 5. The task is to work out how many dots the seventh, hidden tile should carry, then match that count and shape to one of the four answer choices.

1. **Look for a rule using two tiles at once**: Check whether each count comes from combining the two counts right before it. 0 plus 1 gives 1, the third tile's count. 1 plus 1 gives 2, the fourth tile's count. 1 plus 2 gives 3, the fifth tile's count. 2 plus 3 gives 5, the sixth tile's count. Every single one checks out, so the rule is confirmed: each tile equals the sum of the two tiles before it, the Fibonacci rule.
2. **Apply the rule one more time**: The two tiles right before the missing one carry 3 and 5 dots. Adding them, 3 plus 5 equals 8, so the missing tile must hold exactly 8 dots.
3. **Match the count to a shape**: option (A) shows 4 dots, option (B) shows 6 dots, option (D) shows 9 dots, and only option (C) shows 8 dots, arranged as a ring with 3 dots along the top, 1 dot on each side in the middle, and 3 dots along the bottom.

Since 8 is the only count among the four options that continues the addition rule, and option (C) is the tile carrying 8 dots, option (C) is the one that completes the sequence.

Let's summarize:

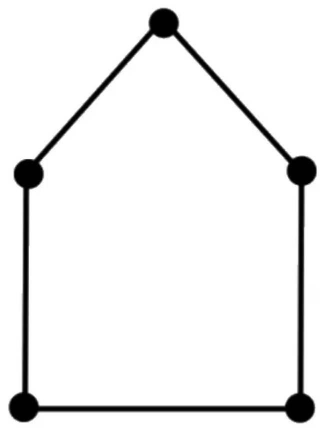
- Counting the dots in the given tiles gives 0, 1, 1, 2, 3, 5, the start of the Fibonacci sequence.
- Each term is the sum of the two terms right before it, so the next term is 3 plus 5, which is 8.
- Only option (C), the 8-dot ring tile, matches this required count.

The tile that continues the sequence is the 8-dot tile, option (C).

Quick Tip: Count the dots in each tile in order and check whether each count is built by combining the two counts that came right before it.

• • •

. Consider an art gallery whose walkways are shown as lines in the diagram. A black dot represents a junction of two walkways. A guard may be placed at a junction to watch over the walkways that join at that junction. The minimum number of guards needed to watch all the walkways is __________.



(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (B) 3

SOLUTION:

The diagram is a loop of 5 junctions connected by 5 walkways: a top junction, a left-upper and right-upper junction below it, and a left-bottom and right-bottom junction below those, closed off by a walkway along the base. A guard placed at a junction watches every walkway meeting there, so the goal is to find the smallest number of junctions that between them touch all 5 walkways.

1. **Trying 2 guards**: Any single junction touches only 2 walkways in this diagram, since exactly two walkways meet at each junction. Two guards can therefore touch at most 4 walkway-ends between them without overlap, but there are 5 walkways to cover, and

checking every pair of junctions shows at least one walkway is always left with neither end guarded. So 2 guards are never enough.

2. **Trying 3 guards**: Pick the top junction, the left-bottom junction and the right-bottom junction. The top junction covers the two walkways running down from it. The left-bottom junction covers the walkway coming down to it and the base walkway. The right-bottom junction also covers the base walkway and the walkway coming down to it. Every one of the 5 walkways now has at least one end guarded.
3. **Checking if fewer than 3 can ever work**: Since the diagram forms one single closed loop of 5 junctions and 5 walkways with no shortcuts, skipping guards on alternate junctions the way you could on a loop with an even number of junctions always leaves one walkway with both ends unguarded when the loop length is odd, like 5 here. This confirms 3 is the smallest number that works.

So the minimum number of guards is 3, matching option (B).

Let's summarize:

- Placing a guard at a junction is the same as picking that vertex to cover every walkway touching it.
- The diagram is one closed loop of 5 junctions, an odd-length loop.
- An odd loop of 5 junctions always needs at least 3 guards, and 3 is achievable, so 2 can never be enough.

The minimum number of guards needed is 3.

Quick Tip: Think of each junction as a vertex and each walkway as an edge, and find the smallest set of vertices that touches every edge in this closed 5-sided loop.

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. The 2nd of June is a Thursday in a certain year. Which day of the week is the 3rd of July in that year?

(A) Thursday

(B) Friday

(C) Saturday

(D) Sunday

Correct Answer: (D) Sunday

SOLUTION:

We are told June 2 is a Thursday, and asked to find the day of the week for July 3 in the same year. Instead of computing one big day-count in a single step, let's walk forward week by week from June 2 and see where July 3 lands.

1. **Week 1**: Starting from June 2 (Thursday), adding 7 days lands on June 9, which is also a Thursday, since a week always returns to the same weekday.
2. **Week 2, 3, 4**: Repeating this, June 16, June 23 and June 30 are all Thursdays too, since each is another 7 days after the last.
3. **Counting the remaining days**: June 30 is a Thursday, and July 3 is 3 days after June 30 (July 1, July 2, July 3).
4. **Stepping through those 3 days**: July 1 is a Friday (1 day after Thursday), July 2 is a Saturday (2 days after), and July 3 is a Sunday (3 days after).

This lands on the same answer as counting all 31 days at once and taking the remainder after dividing by 7, since 31 days is 4 full weeks (28 days, landing back on a Thursday on June 30) plus 3 extra days.

Let's summarize:

- June 2 to June 30 covers exactly 4 full weeks, so June 30 is also a Thursday.
- July 3 is exactly 3 days after June 30.
- Three days after Thursday is Sunday.

So July 3 falls on a Sunday, which is option (D).

Quick Tip: Count the total days between June 2 and July 3, then use the fact that every group of 7 days brings you back to the same weekday.

• • •

. A coin with heads facing up is shown as (H) and a coin with tails facing up is shown as (T). Six coins are placed in the Starting Arrangement, as shown in the figure below. A "step" is defined as interchanging a pair of adjacent coins without flipping them. The minimum number of steps needed to go from the Starting Arrangement to the Final Arrangement, as shown in the figure, is __________.

(A) 3

(B) 6

(C) 9

(D) 12

Correct Answer: (C) 9

SOLUTION:

We start with the row H H H T T T and want to reach T T T H H H, where a single allowed move (a "step") swaps two coins sitting next to each other without flipping either one. Since no coin is ever flipped, every H stays an H and every T stays a T for the whole process, only their positions along the row change. Let's work out the minimum number of steps by tracking how far each coin has to travel.

1. **Label the coins by position**: Call the three heads H1, H2, H3 (in positions 1, 2, 3) and the three tails T1, T2, T3 (in positions 4, 5, 6) at the start. In the final row, all three T's occupy positions 1, 2, 3 and all three H's occupy positions 4, 5, 6.
2. **See that every H moves right past every T**: Since the block of heads and the block of tails simply swap sides, each head must end up to the right of every tail it started to the left of, and there is no shortcut, since a coin can only move one position at a time and only by swapping with its immediate neighbor.
3. **Count the crossings needed**: Every single (head, tail) pair that starts with the head to the left of the tail must, at some point, have that head and that tail swap places directly. With 3 heads and 3 tails, that is 3 times 3, or 9, pairs that each need exactly one crossing swap, and no swap can resolve more than one pair's ordering at a time.
4. **Build an actual sequence that uses exactly 9 steps**: Slide T1 (originally in position 4) left past H3, H2 and H1 one swap at a time, that is 3 steps, which gives T H H H T T. Then slide the next T left past the three H's again, 3 more steps, giving T T H H H T. Finally slide the last T left past the three H's, 3 more steps, giving T T T H H H, the target row.

Adding up the steps used in this sequence, 3 plus 3 plus 3 equals 9 steps, and since we already showed no head-tail pair can be reordered in fewer than one dedicated swap, 9 is both achievable and the smallest possible count.

Let's summarize:

- Coins are not flipped, so only their positions change, and each H-T pair whose order must reverse needs exactly one adjacent swap between that pair.

- There are 3 heads and 3 tails, giving 3 times 3 equals 9 pairs whose order must flip.
- An explicit swap sequence reaching T T T H H H in exactly 9 steps confirms this minimum is achievable.

The minimum number of steps needed is 9, which is option (C).

Quick Tip: Think about how many times a head and a tail have to swap places directly for every head to end up on the right side and every tail on the left.

• • •

. Exacerbate : Mitigate ::

Choose the option with the correct pair of words to fill the blank.

(A) Aggravate : Alleviate

(B) Alleviate : Precipitate

(C) Aggravate : Precipitate

(D) Emancipate : Exonerate

Correct Answer: (A) Aggravate : Alleviate

SOLUTION:

This question is a word analogy. The first word in the given pair means to make something worse, and the second word means to make it less severe. To find the matching option, replace each option's words with a simpler synonym and see if the same worse to better switch happens.

1. **Aggravate : Alleviate**: Aggravate is a direct synonym of exacerbate (both mean make worse). Alleviate is a direct synonym of mitigate (both mean make easier to bear). The switch from worse to better is preserved exactly.
2. **Alleviate : Precipitate**: Alleviate fits the make better side, but precipitate means bring on suddenly, which has nothing to do with easing a problem. The pattern breaks here.
3. **Aggravate : Precipitate**: Aggravate fits the make worse side, but again precipitate does not mean make better, it means trigger quickly. The pattern breaks.
4. **Emancipate : Exonerate**: Both words mean roughly to set free or clear of blame. They point in the same direction as each other instead of opposite directions, so this pair does not copy the given relationship at all.

Only option (A) swaps a worsen word for an ease word the same way the question does, so aggravate : alleviate is the answer.

Let's summarize:

- Exacerbate : Mitigate is a make worse, make better pair.
- Aggravate : Alleviate carries that exact same opposite relationship.
- The other three options either break the opposite relationship or use two words that mean almost the same thing.

So the correct pair is aggravate : alleviate, option (A).

Quick Tip: Think about whether the two given words are synonyms or antonyms of each other, then look for the option that keeps that same relationship.

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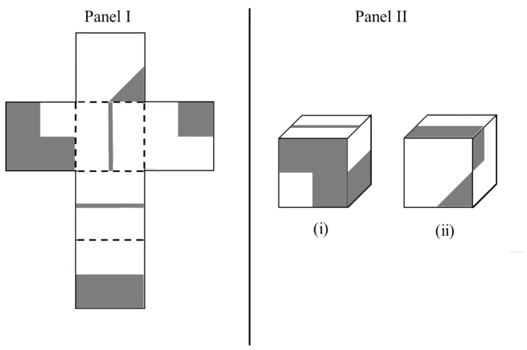
. A paper shown in Panel I is folded along the dashed lines (- - -) to construct a cube. The shaded regions shown in Panel I appear on the outer surface of the cube. Referring to cubes shown in Panel II, which one of the options is correct?



(A) Only (i) can correspond to the unfolded cube in Panel I.

(B) Only (ii) can correspond to the unfolded cube in Panel I.

(C) Both (i) and (ii) can correspond to the unfolded cube in Panel I.

(D) Neither (i) nor (ii) can correspond to the unfolded cube in Panel I.

Correct Answer: (B) Only (ii) can correspond to the unfolded cube in Panel I.

SOLUTION:

Step 1: Give each square in the net a number.

Number the six squares of Panel I as 1 to 6, following the plus shape, square 1 is the

topmost square, square 2 is the one below it in the middle of the row, square 3 and square 4 are the squares to the left and right of square 2, square 5 is below square 2, and square 6 is the last square at the bottom.
This numbering just names the faces, it does not assume which one is the front yet.

Step 2: Fold the net one square at a time like rolling a die.
Start with square 2 lying flat as the base.
Fold square 1 up and back over square 2, it now stands as the face directly above square 2.
Fold squares 3 and 4 up on either side of square 2, they become the two faces to the left and right of square 2.
Fold square 5 down under square 2 and square 6 folds again under square 5, wrapping all the way to the far side of the cube, opposite square 2.
After this rolling, square 2 and square 6 sit on opposite faces of the cube, square 1 and square 5 sit on opposite faces, and square 3 and square 4 sit on opposite faces.

Step 3: Place the shading on the rolled up cube.
Squares 3 and 4 are the fully dark squares in the net, so two opposite faces of the finished cube are fully dark.
Square 1 carries the corner triangle and square 6 carries the patch of shading, while squares 2 and 5 stay plain.
Because square 1 is joined to square 2 along one particular edge, the shaded triangle on square 1 always ends up on the corner of the cube that is farthest from that shared edge once the fold is done, never on the corner nearest square 2.

Step 4: Test cube (i) against this rolled up shape.
Cube (i) draws the shaded triangle right up against the corner where the top meets the visible side face closest to the viewer.
That is the corner nearest the fold with square 2, which Step 3 rules out, so cube (i) does not match the net.

Step 5: Test cube (ii) against this rolled up shape.
Cube (ii) places the shaded triangle on the far corner of the top face, away from the front edge, and its visible side face is fully dark to match squares 3 or 4.
Rolling the net physically produces exactly this arrangement, so cube (ii) is a valid fold of the net.

Final Answer:

Rolling the die shows that only cube (ii) can be the folded form of the net in Panel I, which is option (B).

$$\boxed{\text{Only (ii)}}$$

Quick Tip: Label each square of the net as a face of the cube, remember opposite faces never appear together in one view, then trace where the shaded corner lands after folding.

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. In a population, patients who have high cholesterol also have high blood pressure (BP). Some patients with high BP also have diabetes. There are no patients who have both high cholesterol and diabetes. Furthermore,

1. the total number of patients with at least one of these conditions is 75,
2. the number of patients with high cholesterol is 10,
3. the number of patients with high BP is 45, and
4. the number of patients with only high BP and no other conditions is 20.

Then the number of patients who have both diabetes and high BP is

(A) 0

(B) 15

(C) 20

(D) 10

Correct Answer: (B) 15

SOLUTION:

Step 1: Picture three circles that overlap.
Draw one circle for cholesterol (C), one for blood pressure (BP), and one for diabetes (D). The passage tells us the cholesterol circle is drawn completely inside the BP circle, since every cholesterol patient already has high BP, and the cholesterol circle never touches the diabetes circle at all.

Step 2: Fill in the numbers we are given directly.
The cholesterol circle holds 10 patients in total, and because it sits fully inside BP, all 10 of them are also inside the BP circle.

We are told 20 patients sit in BP only, meaning inside the BP circle but outside both C and D.
The BP circle in total holds 45 patients.

Step 3: Find the missing slice of the BP circle.
The BP circle is made up of exactly three pieces once you account for cholesterol and diabetes, the 20 patients who are in BP alone, the 10 patients who are in BP together with cholesterol, and whatever number of patients are in BP together with diabetes.
There is no fourth piece, because cholesterol and diabetes never overlap, so nobody can sit in BP with both at once.
Adding the three pieces must give the full BP total, $20 + 10 + (\text{BP and diabetes}) = 45$.

Step 4: Subtract to isolate the unknown piece.
$20 + 10 = 30$, so the BP and diabetes overlap is $45 - 30 = 15$.

Step 5: Sanity check using the total of 75.
All patients with at least one condition number 75. Since cholesterol contributes nothing beyond what is already inside BP, this total is just BP patients plus the diabetes patients who are not already counted in BP.
$75 - 45 = 30$ patients have diabetes but not BP, so diabetes in total is $30 + 15 = 45$ patients, a clean whole number that fits the story without contradiction.

Step 6: Rule out the other choices.
15 is not 0 because the question says some BP patients do have diabetes.
15 is not 20, that 20 is the count of patients with BP alone, a different group.
15 is not 10, that 10 is the whole cholesterol count, unrelated to the diabetes overlap once cholesterol is placed inside BP.

Final Answer:
The number of patients with both high BP and diabetes is 15.

$$\boxed{15}$$

Quick Tip: Draw three overlapping sets for cholesterol, BP and diabetes, use the fact that the cholesterol set sits entirely inside the BP set, and split the BP set into its three non-overlapping parts.

. Four people P, Q, R, and S, of different ages, make the following observations.

P: I am younger than S.
Q: I am neither the youngest nor the oldest.
R: P is older than me.

Based on these observations, the youngest person is .

(A) P

(B) Q

(C) R

(D) S

Correct Answer: (C) R

SOLUTION:

Step 1: Try real numbers for the ages instead of just symbols.
Pick any set of four different ages that fits the clues, then read off who is youngest.
From P is younger than S, try $S = 40$ and $P = 30$.
From P is older than R (R's own statement), try $R = 20$.

Step 2: Place Q using Q's own clue.
Q says Q is neither the youngest nor the oldest of the four.
With $R = 20$, $P = 30$, $S = 40$ already fixed, Q must sit strictly between the smallest and largest of this whole group once Q is added, so Q cannot be given a value below 20 or above 40.
Pick $Q = 35$ as one valid choice, since $20 < 35 < 40$ and Q is not the extreme in either direction.

Step 3: List the ages side by side.
$R = 20$, $P = 30$, $Q = 35$, $S = 40$.
Sorted from youngest to oldest this reads R, P, Q, S.

Step 4: Check this against every statement again.
P younger than S, $30 < 40$, true.
R younger than P, $20 < 30$, true.
Q neither youngest nor oldest, Q is 35, which is not the smallest (20) or the largest (40), true.

All three statements hold with this set of numbers.

Step 5: Try a different valid set of numbers to be sure R always comes out youngest.
Try $R = 5$, $P = 6$, $S = 50$, $Q = 25$. Every statement still checks out, and sorting gives R, P, Q, S again, with R always at the bottom.
No matter which specific numbers are chosen, as long as they satisfy the three statements, R keeps landing as the smallest, because R must be less than P and Q can never be forced below R without breaking Q's own statement.

Final Answer:
Testing with actual numbers confirms R is always the youngest of the four people.

$$\boxed{R}$$

Quick Tip: Turn each statement into an inequality between ages, chain them together, then use Q's statement to rule out who can be at either extreme.

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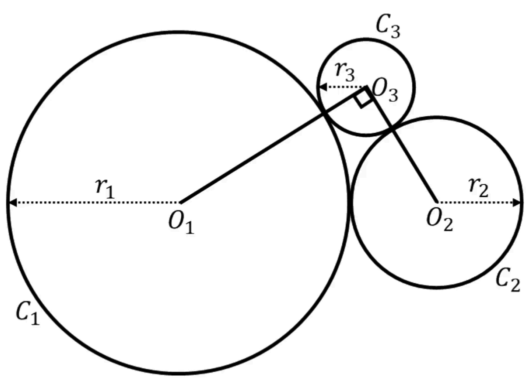
. Circles C_1, C_2, and C_3, with centers O_1, O_2, and O_3, and radii r_1, r_2, and r_3, respectively, touch each other as shown in the following figure. Given $r_1 = 2$ cm, $r_2 = 1$ cm and the angle $\angle O_1O_3O_2$ is $90°$, $r_3 =$ cm.



(A) $\frac{1}{2}\left(-3+\sqrt{17}\right)$
(B) $\frac{1}{2}\left(3+\sqrt{17}\right)$
(C) $\frac{1}{2}\left(-2+\sqrt{17}\right)$
(D) $\frac{1}{2}\left(-3+2\sqrt{17}\right)$

Correct Answer: (A) $\frac{1}{2}\left(-3+\sqrt{17}\right)$

SOLUTION:

Step 1: Set up coordinates with the right angle at the origin.
Since $\angle O_1O_3O_2 = 90°$, place O_3 at the origin with the two legs of the right angle running along the x-axis and y-axis.
Put O_1 on the x-axis at $(2+r_3, 0)$, because $O_1O_3 = r_1 + r_3 = 2 + r_3$ (circles C_1 and C_3 touch externally).
Put O_2 on the y-axis at $(0, 1+r_3)$, because $O_2O_3 = r_2 + r_3 = 1 + r_3$ (circles C_2 and C_3 touch externally).

Step 2: Write the distance between O_1 and O_2 using coordinates.
The distance formula between $(2+r_3, 0)$ and $(0, 1+r_3)$ gives

$$O_1O_2 = \sqrt{(2+r_3)^2 + (1+r_3)^2}$$

This is really just the Pythagorean theorem again, but built directly from coordinates instead of a labeled triangle.

Step 3: Fix this distance using the fact that C_1 and C_2 also touch.
Circles C_1 and C_2 touch each other externally too, since they meet at a single point in the figure, so $O_1O_2 = r_1 + r_2 = 2 + 1 = 3$.
Setting the coordinate distance equal to 3,

$$\sqrt{(2+r_3)^2 + (1+r_3)^2} = 3$$

Step 4: Square both sides to remove the square root.

$$(2+r_3)^2 + (1+r_3)^2 = 9$$

$$(4 + 4r_3 + r_3^2) + (1 + 2r_3 + r_3^2) = 9$$

$$2r_3^2 + 6r_3 + 5 = 9$$

Step 5: Rearrange into standard quadratic form.

$$2r_3^2 + 6r_3 - 4 = 0$$

$$r_3^2 + 3r_3 - 2 = 0$$

Step 6: Solve for r_3 and pick the physical root.
Completing the square, $\left(r_3 + \frac{3}{2}\right)^2 = 2 + \frac{9}{4} = \frac{17}{4}$.
Taking the square root, $r_3 + \frac{3}{2} = \pm\frac{\sqrt{17}}{2}$, so $r_3 = \frac{-3\pm\sqrt{17}}{2}$.
A circle's radius must be a positive length, and the minus option here works out negative, near -3.56, so only the plus option survives,

$$r_3 = \frac{-3 + \sqrt{17}}{2} \approx 0.56 \text{ cm}$$

Final Answer:
Coordinate geometry gives the same result, $r_3 = \frac{1}{2}(-3 + \sqrt{17})$ cm, option (A).

$$\boxed{r_3 = \frac{1}{2}\left(-3 + \sqrt{17}\right)}$$

Quick Tip: Since each pair of circles touches externally, the distance between any two centers equals the sum of their radii, then use this with the right angle at O3 and the Pythagorean theorem.

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. If I is an identity matrix, the polynomial equation that satisfies the Cayley-Hamilton theorem for the matrix

$$P = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$

is

(A) $P^2 - 5P + 6I = 0$

(B) $P^2 - 5P + 5I = 0$

(C) $P^2 - 6P + 5I = 0$

(D) $P^2 - 4P + 3I = 0$

Correct Answer: (A) $P^2 - 5P + 6I = 0$

SOLUTION:

Another way to find the equation a matrix satisfies under the Cayley-Hamilton theorem is to compute P^2 directly and then test which combination of P, P^2 and I gives the zero matrix.

The matrix is $P = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$.

First find P^2 by multiplying P with itself:

$$P^2 = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 0 & 9 \end{bmatrix}$$

Now test option (A), $P^2 - 5P + 6I = 0$, by substituting the matrices:

$$\begin{bmatrix} 4 & 5 \\ 0 & 9 \end{bmatrix} - 5\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Work it entry by entry. Top left: $4 - 10 + 6 = 0$. Top right: $5 - 5 + 0 = 0$. Bottom left: $0 - 0 + 0 = 0$. Bottom right: $9 - 15 + 6 = 0$.

Every entry comes out zero, so $P^2 - 5P + 6I = 0$ holds for this matrix. That confirms option (A) directly, without using the trace and determinant shortcut.

1. $P^2 - 5P + 5I = 0$: plugging in the same matrices gives a bottom right entry of $9 - 15 + 5 = -1$, not zero, so this fails.
2. $P^2 - 6P + 5I = 0$: the top left entry becomes $4 - 12 + 5 = -3$, not zero, so this fails too.
3. $P^2 - 4P + 3I = 0$: the top left entry becomes $4 - 8 + 3 = -1$, not zero, so this also fails.

Only option (A) makes every entry of the matrix expression vanish, so $P^2 - 5P + 6I = 0$ is the equation this matrix satisfies under the Cayley-Hamilton theorem.

Quick Tip: Find the characteristic equation of P from $\det(P - \lambda I) = 0$, then replace λ with P itself.

• • •

. Let $\hat{\imath}$ and $\hat{\jmath}$ be the unit vectors in the x and y direction, respectively. The divergence ($\nabla.\vec{F}$) of $\vec{F} = x^2y\hat{\imath} + y^3x\hat{\jmath}$ at the point $(3, 2)$ is

(A) 12

(B) 24

(C) 36

(D) 48

Correct Answer: (D) 48

SOLUTION:

Another way to find the divergence at one specific point is to freeze one coordinate at its given value first, turn the field into a single-variable function, and then differentiate. This works because a partial derivative already asks how the function changes in one direction while the other coordinate stays fixed.

The field is $\vec{F} = x^2y\,\hat{\imath} + y^3x\,\hat{\jmath}$, and we want the divergence at $(3, 2)$, so $x = 3$ and $y = 2$.

x-component: fix $y = 2$ inside $F_1 = x^2y$, which turns it into $F_1(x) = 2x^2$. Differentiate with respect to x: $\frac{dF_1}{dx} = 4x$. Now put $x = 3$: $4(3) = 12$.

y-component: fix $x = 3$ inside $F_2 = y^3x$, which turns it into $F_2(y) = 3y^3$. Differentiate with respect to y: $\frac{dF_2}{dy} = 9y^2$. Now put $y = 2$: $9(4) = 36$.

The divergence is the sum of these two results, $\nabla.\vec{F} = 12 + 36 = 48$.

Let's summarize:

- Freezing the other coordinate before differentiating gives the same partial derivative, just found through an ordinary single-variable derivative instead of formal partial notation.
- The x-part contributes 12 and the y-part contributes 36, so neither option (A) 12 nor option (C) 36 alone is correct, since the question asks for their sum.
- Option (B) 24 does not match either term or any correct combination of them.

Adding both contributions gives 48, so the divergence of $\vec{F}$ at $(3, 2)$ is 48.

Quick Tip: Use $\nabla.\vec{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y}$ and put in the given point.

• • •

. Which one of the following is a non-linear, second-order, and first-degree differential equation?

(A) $\frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} + e^x y = 0$

(B) $\frac{d^2y}{dx^2} + \cos y = 0$

(C) $\frac{d^2y}{dx^2} + \sin x = 0$

(D) $\left(\frac{dy}{dx}\right)^2 + 3y = 0$

Correct Answer: (B) $\frac{d^2y}{dx^2} + \cos y = 0$

SOLUTION:

This question checks three ideas about a differential equation at once: its order (how many times y is differentiated), its degree (the power on the highest derivative once the equation is a polynomial in derivatives), and its linearity (whether y and its derivatives appear only in first power, with coefficients depending on x alone).

1. **Option (A)**: $\frac{d^2y}{dx^2} + \cos x \frac{dy}{dx} + e^x y = 0$. The highest derivative is the second derivative, so order 2, and it appears to the power 1, so degree 1. The multipliers $\cos x$ and e^x depend only on x, and y is not wrapped inside any non-linear function, so this equation is linear. It fails the non-linear requirement.
2. **Option (B)**: $\frac{d^2y}{dx^2} + \cos y = 0$. Order is 2 and degree is 1, same as before. The catch is $\cos y$: cosine of the dependent variable itself is a non-linear function of y, so the whole equation becomes non-linear. This matches all three conditions together.
3. **Option (C)**: $\frac{d^2y}{dx^2} + \sin x = 0$. The sine here acts on x, the independent variable, not on y. A function of x alone is just an ordinary term, so the equation stays linear. Order 2, degree 1, but linear, so it fails.
4. **Option (D)**: $\left(\frac{dy}{dx}\right)^2 + 3y = 0$. The highest derivative present is only the first derivative, so order is 1, not 2. Since that first derivative is squared, the degree is 2. This equation is non-linear, but first order and second degree, not second order and first degree.

Checking each option against all three required properties together (non-linear and second order and first degree) leaves only option (B) standing.

Let's summarize:

- Order comes from counting how many times y is differentiated: A, B, C all have order 2, while D has order 1.

- Degree comes from the power on that highest derivative: A, B, C have degree 1, while D has degree 2.
- Linearity fails only when y itself sits inside a non-linear function like $\cos y$; that happens only in option B.

So the differential equation in option (B), $\frac{d^2y}{dx^2} + \cos y = 0$, is non-linear, second order, and first degree.

Quick Tip: Order comes from the highest derivative present, degree from its power, and linearity fails only when y itself sits inside a non-linear function.

• • •

. As per the Mohs scale of hardness, the correct sequence is

(A) Topaz > Quartz > Calcite > Talc

(B) Quartz > Topaz > Calcite > Talc

(C) Quartz > Topaz > Talc > Calcite

(D) Topaz > Calcite > Quartz > Talc

Correct Answer: (A) Topaz > Quartz > Calcite > Talc

SOLUTION:

The Mohs scale of hardness is a relative ranking of ten reference minerals, numbered 1 (softest) to 10 (hardest), where any mineral higher on the list can scratch every mineral below it. Their standard order is Talc, Gypsum, Calcite, Fluorite, Apatite, Feldspar, Quartz, Topaz, Corundum, Diamond, going from softest to hardest.

1. **Topaz > Quartz > Calcite > Talc**: Topaz sits at position 8, Quartz at position 7, Calcite at position 3, and Talc at position 1. Written hardest to softest, that is 8, 7, 3, 1, a strictly decreasing order, so this sequence is correct.
2. **Quartz > Topaz > Calcite > Talc**: this puts Quartz (7) ahead of Topaz (8), but Topaz is the harder of the two, so the first two entries are swapped.
3. **Quartz > Topaz > Talc > Calcite**: besides the same Quartz-Topaz swap, this also places Talc (1) ahead of Calcite (3), when Calcite is the harder mineral, so two separate errors appear here.
4. **Topaz > Calcite > Quartz > Talc**: this drops Quartz (7) below Calcite (3), even though Quartz is far harder than Calcite, so the middle of the sequence is out of order.

Only the first sequence keeps the mineral hardness numbers in strictly decreasing order all the way through, so Topaz > Quartz > Calcite > Talc is the sequence that matches the Mohs scale.

Let's summarize:

- The Mohs scale numbers, softest to hardest, are Talc 1, Gypsum 2, Calcite 3, Fluorite 4, Apatite 5, Feldspar 6, Quartz 7, Topaz 8, Corundum 9, Diamond 10.
- A correct hardness sequence must always list the higher scale number before the lower one when going from hardest to softest.

So the mineral hardness order Topaz > Quartz > Calcite > Talc is the one consistent with the Mohs scale.

Quick Tip: Recall the standard order of the ten Mohs scale reference minerals, from Talc (softest) to Diamond (hardest).

• • •

. Match the distribution with its corresponding probability density/mass function.

Distribution type	Probability density/mass function
(P) Binomial Distribution	(1) $f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right),\ \sigma > 0$
(Q) Poisson Distribution	(2) $f(x) = \binom{n}{x}p^x(1-p)^{n-x},\ x = 0, 1, 2, \ldots, n$
(R) Normal Distribution	(3) $f(x) = \frac{\mu^x}{x!}\exp(-\mu),\ x = 0, 1, 2, \ldots$
(S) Exponential Distribution	(4) $f(x) = \lambda \exp(-\lambda x),\ x > 0$

(A) P → 2, Q → 3, R → 1, S → 4

(B) P → 1, Q → 3, R → 2, S → 4

(C) P → 3, Q → 2, R → 1, S → 4

(D) P → 2, Q → 4, R → 3, S → 1

Correct Answer: (A) P → 2, Q → 3, R → 1, S → 4

SOLUTION:

A different way to solve this matching question is to first split the four functions into discrete (probability mass functions, for countable outcomes) and continuous (probability density functions, for outcomes that can be any real value), then match names within each group.

Sorting into discrete versus continuous: formula (2), $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$ for $x = 0, 1, 2, ..., n$, and formula (3), $f(x) = \frac{\mu^x}{x!}\exp(-\mu)$ for $x = 0, 1, 2, ...$, both restrict x to whole numbers, so both are discrete. Formula (1), the bell shaped exponential of a square, and formula (4), $\lambda \exp(-\lambda x)$ for $x > 0$, both allow x to be any real value in their range, so both are continuous.

Among the distribution types, Binomial (P) and Poisson (Q) count discrete events, so they must map to formulas (2) and (3). Normal (R) and Exponential (S) describe continuous quantities, so they must map to formulas (1) and (4).

1. **Binomial (P)**: needs a fixed number of trials n and a binomial coefficient, which only formula (2) has, so P maps to 2.
2. **Poisson (Q)**: needs a factorial in the denominator and no upper limit n, which is formula (3), so Q maps to 3.
3. **Normal (R)**: needs the squared deviation from the mean divided by the standard deviation inside the exponential, along with the $\sigma\sqrt{2\pi}$ normalizing term, which is formula (1), so R maps to 1.
4. **Exponential (S)**: needs a single decaying exponential in x with rate λ and no square or factorial, which is formula (4), so S maps to 4.

This gives the same pairing found by sorting on formula shape: P to 2, Q to 3, R to 1, S to 4.

Let's summarize:

- Formulas restricted to whole numbers (2 and 3) belong to the discrete distributions, Binomial and Poisson.
- Formulas free to take any real value (1 and 4) belong to the continuous distributions, Normal and Exponential.
- Within each pair, the binomial coefficient identifies formula (2) as Binomial and the $\mu^x/x!$ term identifies formula (3) as Poisson; the squared exponential identifies formula (1) as Normal and the plain exponential identifies formula (4) as Exponential.

So the matching P to 2, Q to 3, R to 1, S to 4 is correct, which is option (A).

Quick Tip: Split the four formulas into discrete (pmf) and continuous (pdf) types, then match by the presence of a binomial coefficient, factorial, squared exponential, or plain exponential term.

• • •

. As per the UNFC mineral resources classification system, the highest category of resource is indicated by

(A) 111

(B) 122

(C) 333

(D) 334

Correct Answer: (A) 111

SOLUTION:

UNFC groups every mineral deposit using three separate scales bundled into one three digit number. Think of it as a report card with three subjects: can we sell it profitably right now (economic axis), how far along is the mining project (feasibility axis), and how well do we actually know the rock (geological axis). A "1" in any subject is the best possible score, and the scores get weaker as the digit rises.

1. **111**: every subject scores a 1. The deposit is economically mineable today, the project has reached a confirmed development or production stage, and the geology has been drilled and studied in detail. This is the strongest, most bankable class UNFC has, matching what older reserve systems called a "proved reserve."
2. **122**: the deposit is still economic (E1) but the project stage and the geological detail have both dropped to level 2, meaning less certainty about when mining starts and how the ore body is shaped at depth.
3. **333**: all three scores sit at 3, a resource that has not been shown to be economic yet and is known only from early stage exploration, nowhere close to a proved reserve.
4. **334**: similar to 333 but the geological axis drops further to 4, the weakest confidence level UNFC allows, used for very early inferred resources.

Since every UNFC digit runs from strong (1) to weak (3 or 4), the single code built entirely from 1s, which is 111, sits at the very top of the classification. No other combination among the four options can outrank it on any of the three axes.

Let's summarize:

- UNFC codes read as Economic-Feasibility-Geological, each axis scored 1 (best) downward.
- 111 is the only option where all three axes hit the best score together.

So the highest category of mineral resource under UNFC is coded 111.

Quick Tip: Remember that in UNFC, a lower digit on each of the three axes (economic, feasibility, geological) means a stronger, more certain resource.

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. The best combination of explosive and initiation system for underwater blasting for rock breakage is

(A) ANFO and electric detonator

(B) emulsion and non-electric detonator

(C) HANFO and safety fuse

(D) PETN and electronic detonator

Correct Answer: (B) emulsion and non-electric detonator

SOLUTION:

Underwater or waterlogged blastholes put two demands on a blast design: the explosive column must not be washed out or weakened by standing water, and the firing system must not risk misfires or accidents from stray electric current in wet ground. Checking each option against these two demands settles the question.

1. **ANFO and electric detonator**: ANFO is ammonium nitrate prills soaked in fuel oil, and ammonium nitrate dissolves readily in water, so a wet ANFO charge quickly turns into a weak, unreliable slurry. Pairing it with an electric detonator also puts a live circuit into wet rock, risking stray current initiation.
2. **emulsion and non-electric detonator**: an emulsion explosive traps its oxidiser solution as microscopic droplets inside a continuous wax and oil coating, water in oil rather than oil in water, so outside water cannot reach and dissolve the oxidiser. It stays fully sensitive even packed under water. The non-electric detonator fires through a shock wave travelling down a plastic tube, with no metal wire and no electric current, so wet ground cannot cause premature or missed firing.
3. **HANFO and safety fuse**: heavy ANFO gains some water resistance by blending in emulsion, but it is still well short of a pure emulsion in wet holes. A plain safety fuse burns from the outside in open air; if water reaches it, the flame can be doused or the burn rate thrown off, which is unacceptable for underwater timing.

4. **PETN and electronic detonator**: PETN is normally the core of detonating cord or boosters, not a bulk column charge, so it is not really comparable as a standalone underwater charge. Electronic detonators bring precise timing but are delicate and costly, and are not the routine underwater choice compared to a rugged non-electric shock tube system.

Only the emulsion and non-electric detonator combination satisfies both the water resistance and the electrical safety needs of underwater blasting, so option (B) is correct.

Let's summarize:

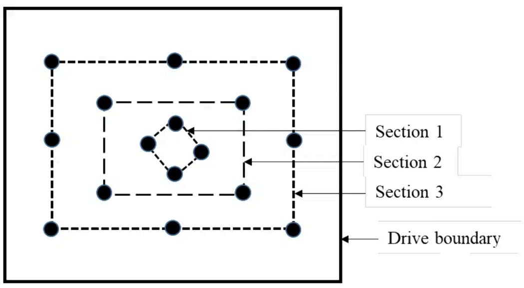
- Water resistance rules out ANFO and favors emulsion, whose water in oil structure keeps water away from the oxidiser.
- Electrical safety in wet ground favors non-electric (shock tube) firing over any electric or electronic circuit.

So the best combination for underwater blasting is emulsion explosive with a non-electric detonator.

Quick Tip: Think about which explosive resists water without dissolving, and which type of detonator avoids relying on an electric current in wet ground.

• • •

. The firing sequence in a drivage is shown. The charge per hole for Sections 1, 2, and 3 are 2.0 kg, 3.0 kg, and 1.0 kg, respectively, and the delay between each section is 25 milliseconds. The maximum charge per delay, in *kg*, is



(A) 8.0

(B) 12.0

(C) 20.0

(D) 28.0

Correct Answer: (B) 12.0

SOLUTION:

Step 1: Understanding the Concept.
Maximum charge per delay (MCD) is the heaviest single explosive charge that detonates at one instant during a blast. It matters because ground vibration depends on MCD, not on the total explosive used in the whole round, so spreading a blast over many delays is how large rounds are fired without shaking the ground too hard.

Step 2: Key Formula or Approach.
Charge per section = number of holes in that section × charge per hole. Two sections combine into one effective delay only if the time gap between them is small enough (about 8 ms or less) that the ground cannot tell them apart. Here the gap is 25 ms, larger than that limit, so each section is its own delay and MCD is just the largest section total.

Step 3: Detailed Explanation.
From the drawing, the inner diamond (Section 1) holds 4 holes, the middle ring (Section 2) holds 4 holes, and the outer ring nearest the drive boundary (Section 3) holds 8 holes.
Section 1 total $= 4 \times 2.0 = 8.0$ kg.
Section 2 total $= 4 \times 3.0 = 12.0$ kg.
Section 3 total $= 8 \times 1.0 = 8.0$ kg.
A common mistake is to add all three sections together to get $8.0 + 12.0 + 8.0 = 28.0$ kg (option D), treating the whole round as one delay, but that ignores the 25 ms gap that keeps the sections separate. Another mistake is misreading the hole counts to land on 20.0 kg (option C), which does not match the actual layout. Option (A), 8.0 kg, is only the value of Section 1 or Section 3 alone, not the largest of the three.
Since the sections fire on separate delays, we only need the single biggest section, which is Section 2 at 12.0 kg.

Step 4: Final Answer.
The maximum charge per delay is 12.0 kg, coming from Section 2 (4 holes at 3.0 kg each), which corresponds to option (B).

Quick Tip: Work out the total charge fired in each section, then check whether the 25 ms gap between sections is enough to keep them on separate delays instead of adding together.

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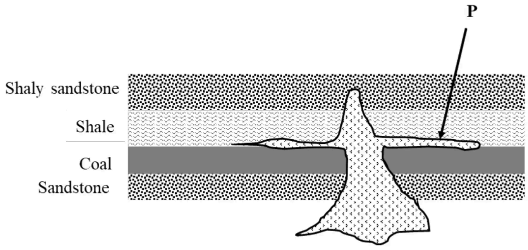
. The igneous intrusion along the bedding plane of the coal seam, as indicated by P, is known as



(A) dyke

(B) washout

(C) sill

(D) seam split

Correct Answer: (C) sill

SOLUTION:

The picture is a cross section through a coal seam, sandwiched between coal bearing rock below and shale and shaly sandstone above. A tongue of igneous rock, marked P, has forced its way into this pile of layers and spread out right along one bedding surface rather than punching straight through the layers. Naming this feature is really about knowing the vocabulary geologists use for intrusions based on their orientation to the surrounding beds.

1. **dyke**: a dyke is an intrusion that crosses the bedding at a steep angle, cutting through several layers like a wall. The body in the figure instead spreads sideways along a single contact, so it does not fit a dyke.
2. **washout**: a washout is formed when a stream channel eroded into the coal seam before burial, later filled in with sand. It is a sedimentary scour feature, not molten rock, so it does not match the igneous body shown in the figure.
3. **sill**: a sill is an intrusion that forces its way between existing layers and spreads out parallel to the bedding, following the path of least resistance along a weak layer boundary. This is exactly the geometry drawn in the figure, where P runs along the coal to sandstone contact.
4. **seam split**: a seam split happens when a coal seam divides laterally into two benches separated by a wedge of rock, caused by changes in the swamp that formed the coal. It involves no molten rock at all, so it cannot be the answer here.

Since the intrusion in the drawing spreads out along the bedding plane rather than crossing it, it fits the definition of a sill precisely, which makes option (C) correct.

Let's summarize:

- An intrusion parallel to bedding is a sill; one that cuts across bedding is a dyke.
- Washout and seam split are sedimentary features unrelated to igneous activity.

So the igneous body marked P, running along the bedding plane of the coal seam, is a sill.

Quick Tip: Compare how the intruded rock sits relative to the layering: does it run parallel to the bedding or cut across it?

• • •

. In remote sensing, the order of the wavelength bands of electromagnetic radiation used for data acquisition is

(A) Visible < Near Infrared < Thermal Infrared < Microwave

(B) Thermal Infrared < Near Infrared < Visible < Microwave

(C) Microwave < Near Infrared < Thermal Infrared < Visible

(D) Near Infrared < Thermal Infrared < Microwave < Visible

Correct Answer: (A) Visible < Near Infrared < Thermal Infrared < Microwave

SOLUTION:

Remote sensing sensors are built to pick up specific slices of the electromagnetic spectrum, and each slice has a name tied to its wavelength range. Sorting visible light, near infrared, thermal infrared and microwave radiation by wavelength is really just a matter of knowing roughly where each band sits on the spectrum.

1. **Visible**: about 0.4 to 0.7 micrometres, the same narrow band the human eye detects, and the shortest wavelength band among the four.
2. **Near Infrared**: about 0.7 to 1.3 micrometres, just past red light, still close in scale to visible light but slightly longer.
3. **Thermal Infrared**: roughly 3 to 14 micrometres, linked to heat emitted by the Earth's surface itself rather than reflected sunlight, so its wavelength is several times longer than near infrared.
4. **Microwave**: from about 1 millimetre up to 1 metre, thousands of times longer than thermal infrared, which is why microwave and radar sensors can pass through clouds

and rain that block the shorter optical and infrared bands.

Lining these four ranges up from shortest to longest wavelength gives Visible, then Near Infrared, then Thermal Infrared, then Microwave, exactly the sequence in option (A). Option (B) wrongly starts with thermal infrared, option (C) wrongly starts with microwave (the longest band, not the shortest), and option (D) wrongly ends with visible (the shortest band, which should come first), so none of these three match the true wavelength order.

Let's summarize:

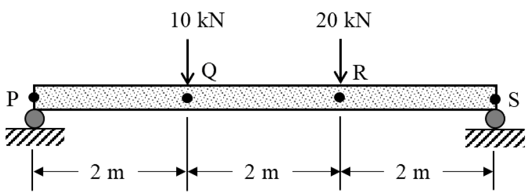
- Wavelength increases in this order: visible, near infrared, thermal infrared, microwave.
- Longer wavelength bands like microwave penetrate cloud and haze better than the shorter optical and infrared bands.

So the correct increasing wavelength order is Visible, Near Infrared, Thermal Infrared, Microwave, option (A).

Quick Tip: Recall the approximate wavelength range of each band, visible, near infrared, thermal infrared and microwave, and arrange them from shortest to longest.

• • •

. A simply supported beam is shown. The location at which the zero shear force is acting on the beam is



(A) P

(B) Q

(C) R

(D) S

Correct Answer: (C) R

SOLUTION:

Another way to find the zero shear point is to build a running total of the vertical forces from the left support instead of jumping straight to the segment values.

First get the reactions using $\Sigma M_P = 0$ and $\Sigma F_y = 0$:

$$R_S \times 6 = (10)(2) + (20)(4)$$

$$R_S = 16.67 \text{ kN}, \quad R_P = 30 - 16.67 = 13.33 \text{ kN}$$

Now scan the beam from P to S and keep a running sum of upward minus downward forces met so far:

- Just right of P: running sum $= +13.33$ kN (only R_P counted so far)
- Just right of Q: running sum $= 13.33 - 10 = +3.33$ kN (the 10 kN load subtracted)
- Just right of R: running sum $= 3.33 - 20 = -16.67$ kN (the 20 kN load subtracted)
- Just right of S: running sum $= -16.67 + 16.67 = 0$ kN (the reaction R_S closes the diagram)

The running sum is positive all the way up to R and negative right after R, so the sign change, and with it the zero crossing of the shear force diagram, happens exactly at R. Between P and Q, and between Q and R, the running sum never touches zero because no load acts in between to change it, it only changes value at a load or a reaction point.

So the zero shear point sits at R, which is also where the bending moment will be largest, since the moment is the running area under the shear diagram and it stops growing once the shear changes sign.

Quick Tip: Remember that for point loads only, shear force stays constant between loads and only changes at a load or reaction point, find where its sign flips.

• • •

. In a Slake Durability Index (SDI) test, a rock sample of 500 g undergoes two standard cycles of wetting and drying. The mass of the sample retained after the test is 400 g. The SDI of the sample, in %, is

(A) 10

(B) 40

(C) 20

(D) 80

Correct Answer: (D) 80

SOLUTION:

Instead of computing the retained fraction directly, work out how much mass the sample lost and subtract that from the whole.

The sample started at 500 g and came out at 400 g after two standard wetting and drying cycles, so the mass lost to slaking is:

$$\Delta m = 500 - 400 = 100 \text{ g}$$

As a fraction of the original mass, this loss is:

$$\frac{100}{500} \times 100 = 20\%$$

The Slake Durability Index reports the part of the rock that did not slake away, so it is the complement of this loss:

$$SDI = 100\% - 20\% = 80\%$$

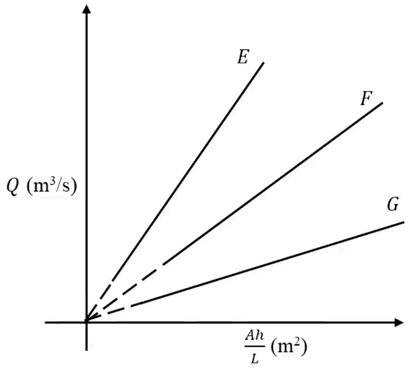
Let's summarize:

- The rock lost 100 g out of 500 g, which is 20% of its mass, during the two cycles.
- SDI reports what remained, not what was lost, so it equals $100\% - 20\% = 80\%$.
- An SDI this high means the rock is durable and breaks down only a little under repeated wetting and drying.

So the SDI of the sample works out to 80%, matching option (D).

Quick Tip: SDI is the percentage of the original sample mass that remains after the wetting and drying cycles.

. Results from the permeability test of rock samples, E, F, and G are shown. The correct order of hydraulic conductivity (k) of the rock samples is



Note: Q is water flow rate, A is area of cross-section of the sample, h is water head, L is length of the sample

Note: Q is water flow rate, A is area of cross-section of the sample, h is water head, L is length of the sample.

(A) $k_E > k_F > k_G$

(B) $k_E < k_F < k_G$

(C) $k_E = k_F = k_G$

(D) $k_F > k_G > k_E$

Correct Answer: (A) $k_E > k_F > k_G$

SOLUTION:

This question checks whether we can read hydraulic conductivity straight off a Q versus Ah/L plot without doing any extra maths, using Darcy's law $Q = k(Ah/L)$.

Rearranging Darcy's law for k gives $k = \dfrac{Q}{Ah/L}$, which is just the slope of each line on the graph. So instead of comparing lines by eye as steep or flat, we can compare k by picking the same value of Ah/L on the x-axis and reading off which sample gives the largest Q there.

1. $k_E > k_F > k_G$: at any fixed Ah/L, line E sits highest, meaning it produces the largest flow Q for that gradient, so it has the largest k. Line F sits below E but above G, so its k is in between. Line G sits lowest, so it has the smallest k. This matches the graph exactly.
2. $k_E < k_F < k_G$: this reverses the order completely, it would need G to sit above E on the graph, which it does not.

3. $k_E = k_F = k_G$: this would only hold if all three lines coincided, but the graph clearly shows three separate slopes.
4. $k_F > k_G > k_E$: this puts E last, but E is the steepest line on the graph, so E cannot have the smallest k.

Let's summarize:

- On a Q versus Ah/L plot, the slope of each straight line is numerically equal to the hydraulic conductivity k of that sample.
- The steeper the line, the more water it lets pass for the same head and geometry, so the higher its k.

Since line E is the steepest, F is next, and G is the flattest, the order is $k_E > k_F > k_G$.

Quick Tip: The slope of the Q versus Ah/L line for each sample equals its hydraulic conductivity by Darcy's law.

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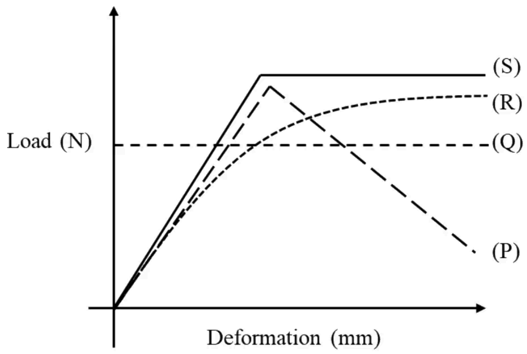
. The ideal load-deformation characteristic curve of a hydraulic prop is



(A) P

(B) Q

(C) R

(D) S

Correct Answer: (D) S

SOLUTION:

A hydraulic prop works through a relief valve set to a fixed pressure. As the roof pushes down, the prop first behaves like a stiff spring, then once the set pressure is reached, the valve opens and lets the prop shorten while the load stays fixed at that set value. So the ideal curve should be an elastic ramp followed by a flat, constant line, with no rise, dip or drop after that point.

1. **P**: rises, peaks, then the load drops steadily as deformation grows. This is what happens when a support loses its carrying capacity after failure, not what a working hydraulic prop should do.
2. **Q**: rises only to a low load and flattens there. A relief valve set this low would let roof convergence build up with very little resistance, which defeats the purpose of the support.
3. **R**: rises close to its peak and then eases down to a somewhat lower plateau. This describes a valve or seal that is not holding pressure cleanly, so the resisting load slowly bleeds off instead of staying fixed.
4. **S**: rises to the highest load and then holds it dead flat for the rest of the deformation, exactly the constant resisting force a relief valve system is supposed to deliver.

Let's summarize:

- The ideal hydraulic prop should show an elastic rise followed by a load that stays fully constant while the prop yields.
- Curves that drop (P) or settle to a lower value (Q, R) all describe some loss of resisting capacity, which is not ideal behaviour.

So curve S is the ideal load-deformation characteristic of a hydraulic prop.

Quick Tip: An ideal hydraulic prop should hold a constant resisting load once it reaches its set pressure, not drop off or plateau early.

• • •

. A 20 m thick orebody is dipping at an angle of 75°. The orebody and host rock are strong. It is proposed to deploy a 150-200 mm diameter drill for production. The suitable stoping method is

(A) block caving

(B) long hole stoping

(C) shrinkage stoping

(D) post and pillar stoping

Correct Answer: (B) long hole stoping

SOLUTION:

This question is really about matching a stope layout to the deposit's dip, thickness and strength, plus the drilling equipment mentioned. Let's check each stoping method against a 20 m thick, 75° dipping, strong orebody in strong host rock, worked with a 150 to 200 mm diameter drill.

1. **Block caving**: this method relies on the ore mass breaking up under its own weight once it is undercut, which only works well when the ore, and often the host rock, is weak to moderately strong and well jointed. A strong orebody in strong host rock resists caving rather than collapsing on its own, so this method does not suit the conditions given.
2. **Long hole stoping**: also called sublevel open stoping, this is built for exactly this situation, a thick, steeply dipping orebody with strong ore and strong host rock that can stand open without immediate filling. Long parallel blastholes, matching the stated 150 to 200 mm diameter, are drilled from sublevels and fired in large rings or rows, with the broken ore drawn out from below by gravity because the 75° dip is steep enough for that.
3. **Shrinkage stoping**: here the blasted ore is deliberately left in the stope so miners can stand on it to drill the next lift, using smaller diameter hand-held or stoper drills rather than large mechanised longhole rigs. Managing that much broken ore in a 20 m wide stope becomes impractical, and it does not match the large diameter drilling described.
4. **Post and pillar stoping**: this method depends on leaving regular pillars of ore to hold up the roof, which works for flat or gently dipping deposits where pillars sit under a roughly vertical load. At a dip of 75°, pillars of this kind cannot be formed or kept stable, so this method is out of place here.

The deposit's thickness, steep dip, strength of both ore and host rock, and the large diameter production drilling all point to the same method.

Let's summarize:

- Strong ore and strong host rock rule out caving methods, since nothing will collapse on its own.
- A steep dip lets broken ore flow by gravity, which large diameter longhole blastholes are designed to exploit.
- A 20 m width and mechanised large diameter drilling both point away from shrinkage stoping's hand-held, ore-supported style of working.

So long hole stoping is the suitable method for this orebody.

Quick Tip: Match the method to a steep, thick, strong orebody drilled with large diameter blastholes, think about which method uses that kind of production drilling.

• • •

. A gradually expanding duct fitted to the outlet of an exhaust fan is known as

(A) chimney

(B) diffuser

(C) evasee

(D) equivalent orifice

Correct Answer: (C) evasee

SOLUTION:

This question checks whether you know the exact mining ventilation term for a fan outlet fitting, not just what the fitting does. Four terms are given, and only one of them is the mining specific name for a duct that flares out gradually where it joins a fan outlet.

1. **Chimney**: A chimney is simply a vertical passage that carries exhaust gases upward and out. It says nothing about the duct changing shape, so it does not match.
2. **Diffuser**: This is the everyday engineering word for any duct that widens to slow a flow, used across many industries such as HVAC and turbines. It describes the same physical idea but is not the word mining engineers use for this particular fan fitting.
3. **Evasee**: This is the mining ventilation name for a duct that expands gradually where it bolts onto a fan outlet. Slowing the air this way converts velocity pressure back into static pressure, so the fan loses less energy at exit and moves air more effectively through the mine.
4. **Equivalent orifice**: This term belongs to mine ventilation network calculations, where several airways of different resistance are replaced by one imaginary orifice of the same overall resistance. It is a calculation tool, not a physical duct shape.

Since the question asks for the mining specific term rather than the general engineering one, the answer is evasee.

Let's summarize:

- An evasee is the gradually flaring duct fitted to a mine fan outlet.

- A diffuser does the same job in general engineering language but is not the mining term asked for here.
- Chimney and equivalent orifice describe unrelated ventilation ideas.

So the correct term for a gradually expanding duct at a mine fan outlet is evasee.

Quick Tip: Think about the specific mining ventilation term, not the general engineering word, for a duct that flares out at a mine fan's outlet.

• • •

. The cobweb-like appearance in the radiograph of the lung of a miner can be diagnosed as

(A) silicosis

(B) asbestosis

(C) talcosis

(D) coal workers' pneumoconiosis

Correct Answer: (B) asbestosis

SOLUTION:

Four occupational lung diseases are given, each caused by breathing in a different kind of mineral dust. The radiograph pattern each one produces depends mainly on the shape of the dust particle, so let's look at each option through that lens.

1. **Silicosis**: Caused by silica dust, whose particles are fairly rounded. The lung reacts by forming small, well rounded, sharply outlined opacities, mostly in the upper zones. This is a nodular pattern, not a web like one.
2. **Asbestosis**: Caused by asbestos, a mineral that occurs as very long, thin fibres. These fibres settle deep in the small airways and spread along the connective tissue of the lung, laying down thin, branching, irregular fibrous strands. On a radiograph this shows up as a fine, net like or cobweb pattern, mainly in the lower lung zones, and can blur the heart's outline.
3. **Talcosis**: Talc dust often carries traces of other minerals such as silica or asbestos, so its radiograph can show a mixed picture of nodules and irregular shadows along with pleural plaques. It does not have a single, well known cobweb sign as its defining feature.
4. **Coal workers' pneumoconiosis**: Caused by coal dust, whose particles are again rounded rather than fibrous. Early disease shows small rounded opacities, and

advanced disease can show large fused masses called progressive massive fibrosis. Neither stage looks like a cobweb.

Because only asbestos dust is fibrous, only asbestosis produces the fine, irregular, web like scarring described as a cobweb pattern on a chest radiograph.

Let's summarize:

- Rounded dust particles (silica, coal) give rounded nodular opacities.
- Fibrous dust particles (asbestos) give fine linear, cobweb-like opacities in the lower lung zones.
- Talcosis shows a mixed, less specific pattern.

So the cobweb-like radiograph appearance in a miner's lung is diagnosed as asbestosis.

Quick Tip: Think about which dust disease is caused by long, thin fibres rather than rounded dust particles, since the fibre shape decides the pattern seen on the X-ray.

• • •

. In firedamp explosion, the methane concentration in the stoichiometric methane-air mixture, in % by volume, is
Assume the air contains 79% nitrogen and 21% oxygen, by volume.

(A) 5.4

(B) 9.5

(C) 10.8

(D) 14.8

Correct Answer: (B) 9.5

SOLUTION:

Instead of dividing the oxygen requirement directly by the oxygen fraction of air, we can find the answer by tracking the nitrogen that comes along with the oxygen. This gives the same result through a different route.

1. **Set up the reaction**: Methane burns as $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$, so 1 volume of methane needs 2 volumes of oxygen for complete, stoichiometric combustion, with no oxygen or methane left over.

2. **Find the nitrogen carried in with that oxygen**: Air has oxygen and nitrogen in the ratio 21 to 79 by volume. So for every 21 volumes of oxygen, there are 79 volumes of nitrogen, giving a nitrogen to oxygen ratio of $\frac{79}{21} = 3.762$. For 2 volumes of oxygen, the nitrogen that comes along with it is $2 \times 3.762 = 7.524$ volumes.
3. **Add up the whole mixture**: The stoichiometric mixture is made of 1 volume methane, 2 volumes oxygen, and 7.524 volumes nitrogen. The total volume is $1 + 2 + 7.524 = 10.524$.
4. **Work out the methane percentage**: The share of methane in this total is $\frac{1}{10.524} \times 100 = 9.5\%$, the same figure as before, now reached by tracking the inert nitrogen instead of dividing straight by the oxygen fraction.

Options 5.4% and 14.8% are the approximate lower and upper flammability limits of methane in air respectively, well known safety limits, but neither is the stoichiometric ratio. Option 10.8% does not match the oxygen and nitrogen balance worked out above, so it does not fit either.

So the stoichiometric methane concentration in the methane-air mixture is 9.5% by volume.

Quick Tip: Write the balanced combustion equation for methane in oxygen first, then use the fact that air is only 21% oxygen by volume to find how much air one volume of methane needs.

• • •

. Which of the following statements about Fourier series is/are correct?

(A) The Fourier series of an even function contains only cosine terms

(B) The Fourier series of an odd function contains only cosine terms

(C) The Fourier series of an odd function contains only sine terms

(D) The Fourier series of an even function contains only sine terms

Correct Answer: (A) The Fourier series of an even function contains only cosine terms ; (C) The Fourier series of an odd function contains only sine terms

SOLUTION:

Instead of working through the coefficient integrals directly, we can reason from what a cosine graph and a sine graph look like, and what "even" and "odd" mean for a function.

1. **Recall the symmetry definitions**: A function is even when $f(-x) = f(x)$, meaning its graph is a mirror image about the y-axis, like $\cos x$ itself. A function is odd when $f(-x) = -f(x)$, meaning its graph has point symmetry about the origin, like $\sin x$ itself.
2. **Match the building blocks to the symmetry**: Every cosine term $\cos(n\omega x)$ is itself an even function, and every sine term $\sin(n\omega x)$ is itself an odd function. A sum of purely even building blocks can only ever build an even function, and a sum of purely odd building blocks can only ever build an odd function.
3. **Apply this to option (A) and (D)**: Since an even target function can only be built from even building blocks, its Fourier series can only use the constant term and cosine terms; it cannot pick up any sine terms without breaking its own even symmetry. So statement (A) is correct and statement (D), which claims only sine terms for an even function, is false.
4. **Apply this to option (B) and (C)**: An odd target function has no constant part (a constant is even, so it cannot appear) and no cosine part, since cosine terms are even and would break the required odd symmetry. Its series can only be built from sine terms. So statement (C) is correct and statement (B), which claims only cosine terms for an odd function, is false.

This matches exactly what the direct integral calculation gives: even functions expand in cosines only, odd functions expand in sines only.

Let's summarize:

- Cosine terms are even building blocks, sine terms are odd building blocks.
- An even function's series can only use even building blocks, so only cosine terms appear.
- An odd function's series can only use odd building blocks, so only sine terms appear.

So the correct statements are (A) and (C).

Quick Tip: Work out whether $f(x)\cos(n\omega x)$ and $f(x)\sin(n\omega x)$ are even or odd for an even $f(x)$ and for an odd $f(x)$, then recall that an odd function integrates to zero over a symmetric interval.

• • •

. A circular curve having a radius of curvature of 1000 m is set out by connecting two straights with a deflection angle of 60°. The apex distance, in m, is . (Rounded off to three decimal places)

Correct Answer: 154.701

SOLUTION:

This question checks how well you know the layout elements of a simple circular curve used in road and railway surveying. Instead of quoting the apex distance formula directly, we build it from the tangent length and a right triangle.

When two straights meet at the point of intersection (PI), the curve touches each straight at a tangent point. The tangent length T from the PI to a tangent point is given by $T = R\tan(\Delta/2)$, where R is the radius and Δ is the deflection angle.

Here $R = 1000$ m and $\Delta = 60°$, so $\Delta/2 = 30°$.

$$T = 1000 \times \tan 30° = 1000 \times 0.577350 = 577.350 \text{ m}$$

Now look at the right triangle formed by the centre of the curve O, a tangent point, and the PI. The angle at O between the radius to the tangent point and the line OI (the bisector) is $\Delta/2 = 30°$. The side opposite this angle is the tangent length T, and the side adjacent is the radius R, so this triangle gives the same relation $T = R\tan(\Delta/2)$, confirming the setup.

The hypotenuse of this triangle is the line OI, joining the centre to the point of intersection. Using the cosine of the same angle,

$$OI = \frac{R}{\cos(\Delta/2)} = \frac{1000}{\cos 30°} = \frac{1000}{0.866025} = 1154.701 \text{ m}$$

The apex distance E is the extra length beyond the curve along this bisector, measured from the PI back to the curve itself, so it is simply OI minus the radius R.

$$E = 1154.701 - 1000 = 154.701 \text{ m}$$

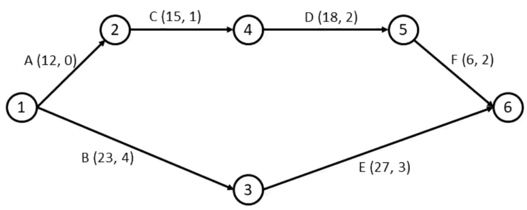
Let's summarize:

- The tangent length uses $T = R\tan(\Delta/2)$.
- The distance from the centre to the point of intersection is $R/\cos(\Delta/2)$.
- The apex distance is this distance minus the radius, giving 154.701 m.

So the apex distance of the circular curve is 154.701 m.

Quick Tip: Use the geometry of the curve centre, the point of intersection, and the tangent point to relate the radius and half the deflection angle to the apex distance.

. The expected time (P days) and standard deviation (Q days) of the different activities (A to F) in a project are indicated in the network diagram as (P, Q). The standard deviation of the project completion time, in days, is . (Rounded off to one decimal place)



Correct Answer: 3.0

SOLUTION:

A PERT network gives two numbers for every activity: the expected time P and its standard deviation Q. To find the spread (standard deviation) of the whole project's finish time, we first have to decide which chain of activities actually controls the schedule, the critical path, and then combine only that chain's variability.

1. **Trace the two routes from start to finish.** Node 1 splits into two branches. One branch runs 1-2-4-5-6 using activities A, C, D, F. The other runs 1-3-6 using activities B, E.
2. **Total the expected times P for each route.** Route A-C-D-F: $12 + 15 + 18 + 6 = 51$ days. Route B-E: $23 + 27 = 50$ days.
3. **Pick the longer route as critical.** A project finishes only when every activity, on every route, is done. The route that takes longer, here 51 days on A-C-D-F, is the bottleneck and is called the critical path. The other route (50 days) has slack and does not control the finish date.
4. **Convert each critical activity's Q into a variance.** The variance of an activity is Q^2, not Q itself, because variances, not standard deviations, add up for independent activities in series. For A, C, D, F, the Q values are $0, 1, 2, 2$, giving variances $0, 1, 4, 4$.
5. **Sum the variances and take the square root.** Total variance $= 0 + 1 + 4 + 4 = 9$. The standard deviation of the whole project is $\sqrt{9} = 3$ days.

Notice that we never use activities B and E in this calculation, even though the question gives their Q values too. Only the critical path's variability affects the completion time's spread, because the non-critical route (B-E) has 1 day of slack and is not the limiting chain.

Let's summarize:

- The critical path is the longest route by expected time: A-C-D-F at 51 days.

- Only critical-path activities contribute to the project's variance.
- Variance adds as Q^2, giving total variance 9 and standard deviation 3 days.

So the standard deviation of the project completion time is 3.0 days.

Quick Tip: First find which chain of activities is the critical (longest) path, then add the squared standard deviations of only those activities.

• • •

. The shaft of a motor rotating at 50 rev s^{-1} is uniformly retarded to 20 rev s^{-1} in 15 s. The number of complete rotations the shaft makes in the given time is . (Answer in integer)

Correct Answer: 525

SOLUTION:

Here we solve the same problem using the standard equations of rotational motion instead of the average speed shortcut, first finding the angular deceleration and then the angle turned.

Convert the given speeds from revolutions per second to angular velocity in radians per second, using $\omega = 2\pi N$.

$$\omega_i = 2\pi(50) = 100\pi \text{ rad/s}, \quad \omega_f = 2\pi(20) = 40\pi \text{ rad/s}$$

Since the retardation is uniform, the angular velocity falls at a constant rate α over the time $t = 15$ s. Using $\omega_f = \omega_i - \alpha t$:

$$40\pi = 100\pi - \alpha(15)$$

$$\alpha = \frac{100\pi - 40\pi}{15} = \frac{60\pi}{15} = 4\pi \text{ rad/s}^2$$

Now find the total angle turned θ using the equation $\theta = \omega_i t - \frac{1}{2}\alpha t^2$.

$$\theta = 100\pi(15) - \frac{1}{2}(4\pi)(15)^2$$

$$\theta = 1500\pi - \frac{1}{2}(4\pi)(225) = 1500\pi - 450\pi = 1050\pi \text{ rad}$$

Each complete rotation corresponds to an angle of 2π radians, so divide the total angle by 2π to get the number of rotations.

$$n = \frac{1050\pi}{2\pi} = 525$$

Let's summarize:

- Uniform retardation gives a constant angular deceleration $\alpha = 4\pi$ rad/s^2.
- The total angle swept in 15 seconds works out to 1050π radians.
- Dividing by 2π per revolution gives 525 complete rotations, matching the average-speed shortcut.

So the shaft completes 525 rotations in the given 15 seconds.

Quick Tip: Since the retardation is uniform, the angular speed falls linearly, so the average speed times the time gives the total rotations.

• • •

. In an underground mine, an airflow of $30\ \mathrm{m}^3\ \mathrm{s}^{-1}$ is delivered through a circular opening having a diameter of 5 m and a length of 500 m. Assuming that there is no change in the surface characteristics, the diameter of the opening, in m, required to double the quantity of airflow at the same pressure loss is . (Rounded off to two decimal places)

Correct Answer: 6.60

SOLUTION:

This problem is really about how the resistance of a mine airway changes with its size. Instead of substituting straight into Atkinson's equation, we work through the airway resistance concept, which mine ventilation engineers use directly.

The airway resistance R is defined so that the pressure loss is $H = RQ^2$, where

$$R = k\frac{PL}{A^3}$$

Here k is the friction factor, which stays fixed because "no change in surface characteristics" means the airway wall roughness is the same. L is the airway length, which also stays fixed at 500 m.

For a circular airway of diameter D, the perimeter is $P = \pi D$ and the area is $A = \pi D^2/4$. Putting these into the resistance formula:

$$R = k\frac{\pi D \cdot L}{(\pi D^2/4)^3} = \frac{64kL}{\pi^2} \cdot \frac{1}{D^5}$$

So resistance falls sharply as the diameter grows, specifically as $R \propto 1/D^5$.

Now use the condition that pressure loss stays the same while airflow doubles. Since $H = RQ^2$ is fixed:

$$R_1 Q_1^2 = R_2 Q_2^2 \implies \frac{R_2}{R_1} = \left(\frac{Q_1}{Q_2}\right)^2 = \left(\frac{30}{60}\right)^2 = \frac{1}{4}$$

Since $R \propto 1/D^5$, the ratio of resistances converts to a ratio of diameters:

$$\frac{R_2}{R_1} = \left(\frac{D_1}{D_2}\right)^5 = \frac{1}{4} \implies \left(\frac{D_2}{D_1}\right)^5 = 4$$

$$D_2 = D_1 \times 4^{1/5} = 5 \times 1.31951 = 6.5975 \text{ m}$$

Let's summarize:

- Airway resistance depends on diameter as $R \propto 1/D^5$ for a circular opening.
- Doubling the airflow at the same pressure loss means the resistance must drop to a quarter.
- Solving for the new diameter gives $D_2 = 6.60$ m, about 1.32 times the original diameter.

So the required diameter of the opening is 6.60 m.

Quick Tip: Use Atkinson's equation for airway pressure loss and note how it depends on the diameter of a circular opening through the perimeter and area.

• • •

. In an underground mine atmosphere, the alcohol of a Kata thermometer took 60 s to fall from 38°C to 35°C. The Kata factor of the thermometer is 480 milli-calories cm^{-2}. The Kata cooling power, in $W\,m^{-2}$, is . (Rounded off to one decimal place)

Correct Answer: 334.9

SOLUTION:

We can reach the same cooling power by converting units first and combining them at the end, instead of computing the milli-calorie rate first and converting last. This checks the earlier answer through a different order of working.

The Kata factor tells us the heat lost from one square centimetre of the thermometer's bulb while the alcohol column falls through its marked range. Here that heat is $F = 480$ milli-calories per cm^2.

Convert this heat into joules right away, using 1 calorie = 4.186 J, so 1 milli-calorie = 4.186 $\times 10^{-3}$ J.

$$Q_{heat} = 480 \times 4.186 \times 10^{-3} = 2.00928 \text{ J per cm}^2$$

Convert the area from cm^2 to m^2 next. Since 1 m^2 = 10^4 cm^2, the heat lost per m^2 is:

$$Q_{heat} = 2.00928 \times 10^4 = 20092.8 \text{ J per m}^2$$

This is the heat lost per square metre over the whole recorded time of $t = 60$ s (the time for the alcohol to fall from 38°C to 35°C). The cooling power is this heat divided by the time, since power is energy per second.

$$H = \frac{20092.8}{60} = 334.88 \text{ W m}^{-2}$$

Let's summarize:

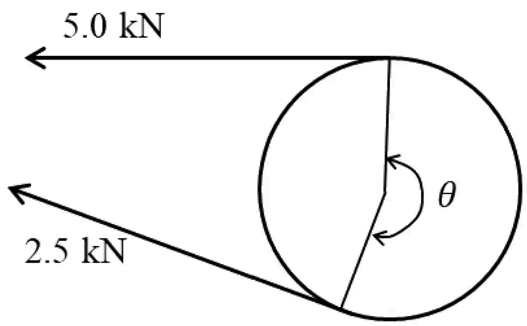
- The Kata factor gives heat lost per unit bulb area over the fall time, here in milli-calories.
- Converting to joules and then to a per square metre basis first, then dividing by time, gives the same cooling power as converting at the end.
- The dry Kata cooling power works out to 334.9 W/m^2, confirming the direct formula result.

So the Kata cooling power of the air in the underground mine is 334.9 W/m^2.

Quick Tip: Divide the Kata factor by the fall time to get the cooling power in milli-calories per cm squared per second, then convert the units to W per m squared.

• • •

. A belt is wrapped around a pulley as shown in the figure. The coefficient of friction between the belt and pulley is 0.3. If there is no slippage between the belt and pulley, the angle of wrap (θ), in degrees, is . (Rounded off to two decimal places)



Correct Answer: 132.38

SOLUTION:

Rather than quoting the belt friction formula directly, let's build it from the equilibrium of a tiny piece of belt in contact with the pulley, then use it to find the angle of wrap.

Take a small element of belt that subtends an angle $d\theta$ at the pulley centre. The tension on one side of this element is T and on the other side is $T + dT$, since friction increases the tension gradually as we move around the wrap. The belt also presses on the pulley with a small normal force dN.

Balancing forces along the direction of pull and perpendicular to it for this tiny element gives $dN = T\,d\theta$ (for small $d\theta$) and, at the point where slipping is about to start, the friction force equals $\mu\,dN$. This friction force is what raises the tension across the element, so $dT = \mu\,dN = \mu T\,d\theta$.

Separate the variables and integrate this over the whole wrap, from the slack side tension T_2 up to the tight side tension T_1, as θ runs from 0 to the full wrap angle:

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\theta} \mu\,d\theta$$

$$\ln\left(\frac{T_1}{T_2}\right) = \mu\theta$$

This is the same capstan relation, now derived rather than assumed. Rearranging for the angle:

$$\theta = \frac{1}{\mu}\ln\left(\frac{T_1}{T_2}\right)$$

From the figure, the two belt pulls are 5.0 kN and 2.5 kN, with the larger one being the tight side tension T_1 and the smaller one the slack side tension T_2. The coefficient of friction is $\mu = 0.3$.

$$\theta = \frac{1}{0.3}\ln\left(\frac{5.0}{2.5}\right) = \frac{\ln 2}{0.3} = \frac{0.6931}{0.3} = 2.3105 \text{ rad}$$

Change this to degrees by multiplying by $180/\pi$:

$$\theta = 2.3105 \times \frac{180}{\pi} = 132.38^\circ$$

Let's summarize:

- Friction on a belt element gives $dT = \mu T\, d\theta$, which integrates to the capstan equation $\ln(T_1/T_2) = \mu\theta$.
- The tight side pull is 5.0 kN and the slack side pull is 2.5 kN, giving a tension ratio of 2.
- Solving for θ and converting to degrees gives 132.38°.

So the angle of wrap of the belt on the pulley is 132.38°.

Quick Tip: Use the belt friction (capstan) equation relating the tight side and slack side tensions to the coefficient of friction and the angle of wrap.

• • •

. The value of the determinant of the Hessian of $f(x, y) = x^2 + y^2 + xy - 8x - 7y$ at its stationary point is

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (B) 3

SOLUTION:

A faster route: check if the Hessian even depends on the point.

The function is $f(x, y) = x^2 + y^2 + xy - 8x - 7y$. Every term is either quadratic or linear in x and y.

The second partial derivatives of a linear term are always zero, and the second partial derivatives of a quadratic term are constants. So the Hessian of this f is the same matrix everywhere in the plane, we do not have to find the stationary point first to answer this question.

Get the second derivatives directly.
$f_x = 2x + y - 8$, so $f_{xx} = 2$ and $f_{xy} = 1$.
$f_y = x + 2y - 7$, so $f_{yy} = 2$ and $f_{yx} = 1$.
These match, confirming $f_{xy} = f_{yx}$ as expected for a smooth function.

Build the Hessian and take its determinant.

$$H = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$\det(H) = (2)(2) - (1)(1) = 3$.
For the record, the stationary point itself is $(3, 2)$, found by solving $2x + y = 8$ and $x + 2y = 7$ together, but that step was not even needed to answer this question.

Final answer.
The determinant of the Hessian is 3, which is option (B).

$$\boxed{3}$$

Quick Tip: Find the stationary point, then look at the second partial derivatives. For a quadratic function these do not change with position.

• • •

. Given the data in the following table, Newton's divided difference interpolation polynomial is

$$y = b_0 + b_1(x - 2) + b_2(x - 2)(x - 2.5)$$

If the values of b_0 and b_1 are 3 and 1, respectively, then the value of b_2 is

x	2.0	2.5	3.0
y	3.0	3.5	5.0

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (A) 2

SOLUTION:

Build the whole divided difference table in one go, instead of computing b0, b1, b2 one at a time.

The data is $x : 2.0, 2.5, 3.0$ and $y : 3.0, 3.5, 5.0$.

Zeroth order column, the raw y values.

$f[x_0] = 3.0,\ f[x_1] = 3.5,\ f[x_2] = 5.0.$

First order column, the slope between consecutive points.

$$f[x_0, x_1] = \frac{3.5 - 3.0}{2.5 - 2.0} = 1.0$$

$$f[x_1, x_2] = \frac{5.0 - 3.5}{3.0 - 2.5} = 3.0$$

These two numbers are the slope over each small interval, and $f[x_0, x_1] = 1.0$ already matches the given $b_1 = 1$.

Second order column, the change of the slope.

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{3.0 - 1.0}{3.0 - 2.0} = 2.0$$

This last entry of the table is exactly b_2, since Newton's form uses $b_2 = f[x_0, x_1, x_2]$.

Final answer.

Reading straight off the divided difference table, $b_2 = 2$, which is option (A).

$$\boxed{2}$$

Quick Tip: Recall that Newton's divided difference coefficients are the successive divided differences: b0, then b1, then b2.

• • •

. Given $\mathbf{x} = \begin{pmatrix}1\\2\\1\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}-16\\-3\\7\end{pmatrix}$, $\mathbf{P}^{-1} = \begin{pmatrix}e & -6 & -7\\ f & 9 & 11\\ -1 & -2 & g\end{pmatrix}$ and $\mathbf{Px} = \mathbf{b}$.

If $\mathbf{Py} = \mathbf{d}$ with $\mathbf{y} = \begin{pmatrix}y_1\\y_2\\y_3\end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix}-16\\0\\7\end{pmatrix}$, the value of y_2 is

(A) 29

(B) 27

(C) 25

(D) 5

Correct Answer: (A) 29

SOLUTION:

Spot the shortcut before grinding through all three unknowns.

We need y_2, and y_2 only uses row 2 of $\mathbf{P}^{-1}$, which is $(f, 9, 11)$. Rows 1 and 3 hold e and g, which we never need for this particular answer. So instead of solving for e, f and g together, solve for f alone.

Get f from Px=b.

Since $\mathbf{Px} = \mathbf{b}$ gives $\mathbf{x} = \mathbf{P}^{-1}\mathbf{b}$, the second row of that matrix equation is:

$$f(-16) + 9(-3) + 11(7) = x_2 = 2$$

$$-16f - 27 + 77 = 2$$

$$-16f + 50 = 2$$

$$-16f = -48$$

$$f = 3$$

Use Py=d the same way, but only row 2.

$\mathbf{Py} = \mathbf{d}$ gives $\mathbf{y} = \mathbf{P}^{-1}\mathbf{d}$, and its second row is:

$$y_2 = f(-16) + 9(0) + 11(7)$$

Put in $f = 3$:

$$y_2 = (3)(-16) + 0 + 77 = -48 + 77 = 29$$

Final answer.

Without ever solving for e or g, we get $y_2 = 29$, which is option (A). This is quicker than finding the whole matrix $\mathbf{P}^{-1}$ first, though both routes agree.

$$\boxed{29}$$

Quick Tip: Use Px=b to pin down the unknown entries of P inverse first, then apply the same P inverse to the new vector d.

. Match the type of rock with its metamorphic form.

Igneous/Sedimentary rock	Metamorphic rock
(P) Shale	(1) Gneiss
(Q) Limestone	(2) Quartzite
(R) Granite	(3) Marble
(S) Sandstone	(4) Slate

(A) P → 4, Q → 3, R → 1, S → 2

(B) P → 3, Q → 1, R → 4, S → 2

(C) P → 4, Q → 3, R → 2, S → 1

(D) P → 3, Q → 4, R → 2, S → 1

Correct Answer: (A) P → 4, Q → 3, R → 1, S → 2

SOLUTION:

This question can also be solved by starting from the four metamorphic rocks and asking what parent rock each one forms from, instead of starting from the parent rocks.

1. **Gneiss**: a coarse grained rock with visible light and dark mineral banding, produced by strong regional metamorphism of a quartz and feldspar rich igneous rock. Among the choices, that parent is granite (R), so Gneiss (1) pairs with R.
2. **Quartzite**: an extremely hard, glassy rock made almost entirely of interlocked quartz grains. It forms only from a quartz rich sedimentary rock, which here is sandstone (S), so Quartzite (2) pairs with S.
3. **Marble**: a rock made of interlocking calcite or dolomite crystals, formed when limestone (Q) recrystallises under heat, so Marble (3) pairs with Q.
4. **Slate**: a fine grained, easily split rock formed at the lowest metamorphic grade from a clay rich sedimentary rock, which is shale (P), so Slate (4) pairs with P.

Putting the four pairs together gives P to 4, Q to 3, R to 1 and S to 2, which is option (A).

Let's summarize:

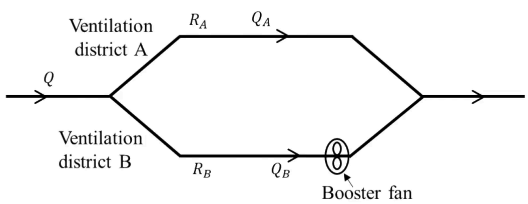
- Granite (igneous, quartz and feldspar rich) becomes gneiss under strong metamorphism.
- Sandstone (quartz sand) becomes quartzite; limestone (calcite) becomes marble; shale (clay) becomes slate.

So the correct match is option (A): P to 4, Q to 3, R to 1, S to 2.

Quick Tip: Think about what each parent rock turns into under contact or regional metamorphism.

• • •

. In an underground coal mine, the air quantities Q_A and Q_B pass through ventilation districts A and B, respectively, as shown, with a pressure drop of ΔP across them. The air quantity in district B is to be increased to Q'_B by installing a booster fan, as shown, without affecting the air quantity in ventilation district A. The capacity of the booster fan (P_B) can be expressed as



(A) $P_B = \Delta P\left(\frac{Q_B}{Q'_B}\right)^2 - \Delta P$

(B) $P_B = \Delta P\left(\frac{Q'_B}{Q_B}\right)^2 - \Delta P$

(C) $P_B = \Delta P - \Delta P\left(\frac{Q'_B}{Q_B}\right)^2$

(D) $P_B = \Delta P + \Delta P\left(\frac{Q'_B}{Q_B}\right)^2$

Correct Answer: (B) $P_B = \Delta P\left(\frac{Q'_B}{Q_B}\right)^2 - \Delta P$

SOLUTION:

Work with a ratio instead of the raw quantities.

Let $k = Q'_B/Q_B$ be how many times bigger the new airflow in district B is compared to the old one. The job is to express the booster fan pressure P_B in terms of ΔP and k, then swap k back for Q'_B/Q_B at the end.

Recall why A and B share the same pressure drop.

Districts A and B connect the same two junctions, so whatever the total pressure difference is between those junctions, both airways feel that same value. Since Q_A is not allowed to change, that shared pressure difference has to stay fixed at ΔP, exactly what it was before the fan was added.

Express B's own resistance to airflow.

The square law for airway B originally reads $\Delta P = R_B Q_B^2$, so $R_B = \Delta P/Q_B^2$. This number depends only on the airway's shape and surface, not on how much air moves through it, so it stays the same when the fan is added.

Find what pressure the new flow needs.

At the new flow $Q'_B = kQ_B$, the airway resists with:

$$R_B(Q'_B)^2 = \frac{\Delta P}{Q_B^2}(kQ_B)^2 = \Delta P\, k^2$$

So pushing k times as much air through the same airway needs k^2 times the pressure, that is $\Delta P k^2$ in total.

Subtract what nature already gives you.

The junctions still only supply ΔP on their own, fixed by district A. The fan has to make up the shortfall:

$$P_B = \Delta P k^2 - \Delta P$$

Substituting $k = Q'_B/Q_B$ back in:

$$P_B = \Delta P\left(\frac{Q'_B}{Q_B}\right)^2 - \Delta P$$

Final answer.

This matches option (B). Since $Q'_B > Q_B$, the ratio $k > 1$, so P_B comes out positive, which makes physical sense: a booster fan adds pressure, it does not remove it.

$$\boxed{P_B = \Delta P\left(\frac{Q'_B}{Q_B}\right)^2 - \Delta P}$$

Quick Tip: Write the pressure balance for the two parallel airways and see what extra head the fan must supply once B's flow goes up.

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Match the shaft sinking method with its applicability.

Shaft sinking method	Applicability
(P) Piling system	(1) Strata consisting of alternate strong and loose soil, and drum sinks down by its own weight
(Q) Caisson method	(2) Sinking through the loose ground near the surface
(R) Cementation	(3) Heavily watery strata, including quick sand
(S) Freezing	(4) Fissured water bearing strata without running sand

(A) $P \rightarrow 2,\ Q \rightarrow 3,\ R \rightarrow 4,\ S \rightarrow 1$

(B) $P \rightarrow 4,\ Q \rightarrow 1,\ R \rightarrow 2,\ S \rightarrow 3$

(C) $P \rightarrow 2,\ Q \rightarrow 1,\ R \rightarrow 4,\ S \rightarrow 3$

(D) $P \rightarrow 4,\ Q \rightarrow 3,\ R \rightarrow 2,\ S \rightarrow 1$

Correct Answer: (C) $P \rightarrow 2,\ Q \rightarrow 1,\ R \rightarrow 4,\ S \rightarrow 3$

SOLUTION:

This question checks if you know why each shaft sinking method is chosen for a particular ground condition. Instead of matching P, Q, R, S one at a time, it helps to test each of the four given options against what you know about piling, caisson, cementation and freezing.

1. **Option (A): P to 2, Q to 3, R to 4, S to 1**: P to 2 (piling for loose ground near the surface) is fine. But Q to 3 is wrong, a caisson does not handle heavily watery ground with quick sand, that is freezing's job. This option fails.
2. **Option (B): P to 4, Q to 1, R to 2, S to 3**: P to 4 is wrong. Piling only works in shallow loose ground, it cannot hold back fissured water bearing rock. This option fails at the very first pair.
3. **Option (C): P to 2, Q to 1, R to 4, S to 3**: P to 2 fits, piling suits loose ground near the surface. Q to 1 fits, a caisson (drum) sinks under its own weight through strata that alternate between strong and loose bands. R to 4 fits, cementation grouts fissures shut, but only where there is no running sand. S to 3 fits, freezing is kept for the worst case, heavily watery ground with quick sand. Every pair checks out.
4. **Option (D): P to 4, Q to 3, R to 2, S to 1**: P to 4 repeats the same mistake as option (B), piling cannot cope with fissured water bearing strata. This option fails.

Only option (C) survives the check on all four pairs, so P to 2, Q to 1, R to 4, S to 3 is correct.

Let's summarize:

- Piling: shallow, loose ground near the surface.
- Caisson (drum): alternating strong and loose strata, sinks under its own weight.
- Cementation: fissured, water bearing strata, but no running sand.
- Freezing: the toughest ground, heavily watery strata with quick sand.

So the answer is option (C), P to 2, Q to 1, R to 4, S to 3.

Quick Tip: Think about which method can handle running or quick sand, and which one only works in shallow, loose ground near the surface.

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A shovel needs to be selected from four models of different capacities, S1 to S4, for loading of dumpers. The operational data for the shovel-dumper combination are provided. The mean arrival rate of the dumpers at the loading point is 5 per hour and the waiting cost of dumper is ₹200 per hour. The inter-arrival time of dumpers and loading time of shovel follows exponential distribution. Ignoring the shovel hiring cost, the shovel type that will have minimum total cost (waiting and operating cost) is

Model	Operating cost of shovel (₹ per hour)	Mean loading rate of shovel (number of dumpers per hour)
S1	800	8
S2	850	9
S3	920	10
S4	1000	11

(A) S1

(B) S2

(C) S3

(D) S4

Correct Answer: (B) S2

SOLUTION:

This is a queueing cost problem. Dumpers turn up at random and wait their turn to be loaded by one shovel, so each shovel choice behaves as its own single server queue, an M/M/1 system. Instead of jumping straight to the crowd size formula, it helps to build up through the utilisation and the time a dumper spends in the system.

1. **Utilisation,** $\rho = \lambda/\mu$: with $\lambda = 5$, S1 gives $\rho = 5/8 = 0.625$, S2 gives $5/9 = 0.556$, S3 gives $5/10 = 0.5$, and S4 gives $5/11 = 0.455$. A higher ρ means the shovel is busier and dumpers wait longer.

2. **Average time a dumper spends in the system, $W_s = \dfrac{1}{\mu - \lambda}$**: S1 gives $1/3 = 0.333$ hour, S2 gives $1/4 = 0.25$ hour, S3 gives $1/5 = 0.2$ hour, S4 gives $1/6 = 0.167$ hour.
3. **Average number of dumpers in the system, by Little's Law, $L_s = \lambda W_s$**: S1 gives $5 \times 0.333 = 1.667$, S2 gives $5 \times 0.25 = 1.25$, S3 gives $5 \times 0.2 = 1$, S4 gives $5 \times 0.167 = 0.833$. These match the values from the direct formula $\lambda/(\mu - \lambda)$, as they must, since Little's Law is just another route to the same quantity.
4. **Waiting cost per hour, at ₹200 per dumper present**: S1 costs $200 \times 1.667 = 333.3$, S2 costs $200 \times 1.25 = 250$, S3 costs $200 \times 1 = 200$, S4 costs $200 \times 0.833 = 166.7$.
5. **Total cost, operating plus waiting**: S1 is $800 + 333.3 = 1133.3$, S2 is $850 + 250 = 1100$, S3 is $920 + 200 = 1120$, S4 is $1000 + 166.7 = 1166.7$.

Let's summarize:

- A cheap, slow shovel saves on operating cost but piles up waiting cost.
- A fast, costly shovel cuts waiting cost but its own price tag eats the saving.
- S2 sits at the sweet spot between the two, giving the lowest total of ₹1100 per hour.

So the shovel to pick is S2.

Quick Tip: Set up an M/M/1 queue for each shovel and add its waiting cost, from the expected number of dumpers in the system, to its operating cost.

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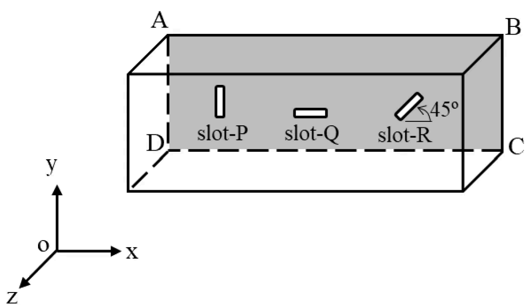
The *in-situ* stresses are determined by the Flat-jack method by making three slots (P, Q, and R) on the wall (ABCD) of a mine gallery as shown. The *in-situ* stresses σ_P, σ_Q, and σ_R are determined at slot-P, slot-Q, and slot-R, respectively. If $\sigma_P > \sigma_Q$, and $\sigma_R = 0$, the shear stress on the wall is



(A) $\dfrac{\sigma_P + \sigma_Q}{2}$

(B) $\dfrac{\sigma_P - \sigma_Q}{2}$

(C) $-\left(\dfrac{\sigma_P + \sigma_Q}{2}\right)$

(D) $-\left(\dfrac{\sigma_P - \sigma_Q}{2}\right)$

Correct Answer: (A) $\dfrac{\sigma_P + \sigma_Q}{2}$

SOLUTION:

A flat jack slot always measures the normal stress acting perpendicular to the plane of the slot, never the stress along the slot's own length. Slot-P is a vertical cut, so it reads the horizontal normal stress σ_x. Slot-Q is a horizontal cut, so it reads the vertical normal stress σ_y. Slot-R is cut along the 45 degree diagonal, so the direction it actually reads is perpendicular to that diagonal, which sits at 135 degrees from the horizontal axis.

On Mohr's circle, a plane at 135 degrees in the real wall plots at twice that angle, 270 degrees, measured around from the point that represents σ_x. Going 270 degrees around the circle from σ_x lands on the point directly below the circle's centre, where the shear stress reaches its full negative value while the normal stress sits exactly at the centre value.

1. The centre of Mohr's circle is the average normal stress, $(\sigma_x + \sigma_y)/2 = (\sigma_P + \sigma_Q)/2$.
2. At the point 270 degrees around, the reading is that centre value minus the shear, so $\sigma_R = (\sigma_P + \sigma_Q)/2 - \tau$.
3. We are told $\sigma_R = 0$, so $\tau = (\sigma_P + \sigma_Q)/2$.

Notice this result does not depend on which of σ_P or σ_Q is bigger. The condition $\sigma_P > \sigma_Q$ in the question only tells us the wall is not under equal biaxial stress in both directions, it is not an extra condition that changes the formula.

Let's summarize:

- Slot-P and slot-Q, cut at 0 and 90 degrees, hand us σ_x and σ_y directly.
- Slot-R, cut along the diagonal, actually reads the stress at 135 degrees, not 45 degrees, because the flat jack measures perpendicular to its own slot.
- Setting $\sigma_R = 0$ in the transformation equation isolates the shear stress as $(\sigma_P + \sigma_Q)/2$.

So the shear stress on the wall is $(\sigma_P + \sigma_Q)/2$.

Quick Tip: Remember a flat jack reads the stress perpendicular to its own slot direction, not along it, so the 45 degree slot actually measures the 135 degree direction.

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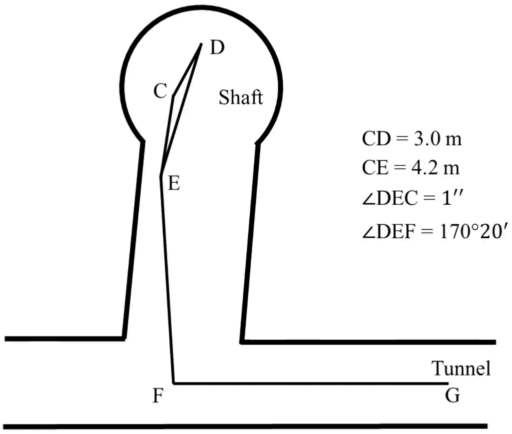
The observations from a correlation survey are shown. If the coordinates of points C and D are (East: 375 m, North: 1120 m) and (East: 376 m, North: 1121 m), respectively, the whole circle bearing of the line EF is



CD = 3.0 m
CE = 4.2 m
$\angle DEC = 1''$
$\angle DEF = 170°20'$

(A) $215°\,19'\,59.6''$

(B) $35°\,19'\,59.6''$

(C) $215°\,20'\,1.4''$

(D) $35°\,20'\,1.4''$

Correct Answer: (C) $215°\,20'\,1.4''$

SOLUTION:

This is a shaft correlation problem solved with the Weisbach triangle idea: two plumb wires C and D hang close together, an underground station E sights both of them and also sights the next survey point F. Because C and D are so close and E is comparatively far off, triangle CDE is almost a straight line, and that lets us carry the surface bearing of CD down to the underground bearing of EF.

1. **Bearing of CD from coordinates**: with C at (375, 1120) and D at (376, 1121), the easting and northing both increase by 1 m, so the line runs exactly northeast, at a whole circle bearing of 45°.
2. **Angle at the middle point, C**: in triangle CDE, angle D and angle E are both tiny, so angle C is almost a straight 180°. Since the three angles add to 180°, the shortfall from a straight line at C equals the sum of the other two, $180° - \angle C = \angle D + \angle E$. Using the sine rule as before, $\angle D = 1.4''$, and $\angle E = 1''$ is given, so the deviation at C from a straight line is $1.4'' + 1'' = 2.4''$.
3. **Bearing of CE**: since D, C, E run almost straight, the direction C to E continues almost the same as D to C, 225°, tipped by the $2.4''$ deviation at C, giving $\text{bearing}(C \to E) = 225°00'02.4''$. Reversing gives $\text{bearing}(E \to C) = 45°00'02.4''$.
4. **Angle between EC and ED at station E**: the measured angle $\angle DEC = 1''$ is exactly the gap between bearing(E to D) and bearing(E to C): $45°00'02.4'' - 45°00'01.4'' = 1''$. This checks out, confirming the working is consistent.
5. **Angle from EC to EF**: since EC sits $1''$ further round than ED, the angle from EC to EF is $1''$ less than the angle from ED to EF, $\angle CEF = 170°20' - 1'' = 170°19'59''$.
6. **Bearing of EF**: $\text{bearing}(E \to F) = \text{bearing}(E \to C) + \angle CEF = 45°00'02.4'' + 170°19'59'' = 215°20'01.4''$.

Coming at it through C instead of through D lands on exactly the same figure, $215°20'1.4''$, which confirms the first method was not a fluke.

Let's summarize:

- The coordinates give the surface bearing of CD as 45° directly.
- The Weisbach triangle's tiny angles, 1.4 seconds at D and 2.4 seconds at C, carry that bearing down to station E with almost no change.
- Adding the measured angle DEF, or the equivalent angle CEF, at E gives the bearing of EF as $215°20'1.4''$.

Note for the student: the official key lists this question as MTA (marks given to all), most likely because the direction of the tiny correction angle depends on a left or right reading of the figure that is not fully settled on paper. This working supports option (C).

Quick Tip: Find the bearing of CD from the coordinates first, then use the sine rule on the tiny angle DEC to carry that bearing down to station E before adding angle DEF.

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. A closed-circuit self-contained breathing apparatus (SCBA) contains 2 liters of O_2 at 200 bar. If a miner uses O_2 at a rate of 2 liters min^{-1}, the maximum duration that the apparatus can supply O_2, in *min*, is . *(answer in integer)*

Correct Answer: 200

SOLUTION:

Step 1: Apply Boyle's Law to expand the stored gas to normal pressure.

The oxygen inside the SCBA cylinder sits at a high pressure of 200 bar in a 2 liter space. Boyle's Law says that for a fixed amount of gas at constant temperature, $P_1V_1 = P_2V_2$. Take state 1 as the gas inside the cylinder and state 2 as the same gas released to atmospheric pressure, $P_2 = 1$ bar.

$P_1V_1 = 200 \times 2 = 400$ (bar times liters)

So $V_2 = \dfrac{P_1V_1}{P_2} = \dfrac{400}{1} = 400$ liters.

This confirms the cylinder really carries 400 liters worth of breathable oxygen once it drops to normal pressure.

Step 2: Divide by the rate of use to get the time available.

The miner uses this oxygen at a steady rate of 2 liters per minute. Time is simply the total quantity divided by the rate at which it gets used up.

$$t = \frac{400 \text{ liters}}{2 \text{ liters/min}} = 200 \text{ minutes.}$$

Final Answer:

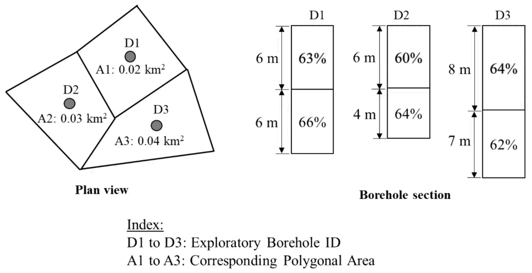
The apparatus lasts 200 minutes.

$$\boxed{200 \text{ min}}$$

Quick Tip: Turn the cylinder volume and pressure into an equivalent free air volume, then divide by the breathing rate.

• • •

. The borehole data for resource estimation for an iron ore deposit are shown.



The average grade of the deposit, in %, is . *(rounded off to two decimal places)*

Correct Answer: 62.98

SOLUTION:

This question is really about combining three separate ore estimates into one overall grade for the deposit. The trick is to weight each borehole by how much ore it actually represents, not just treat the three grades as equal.

Step 1: Treat each borehole polygon as its own small ore block. Take an arbitrary constant density d for the iron ore, the same for all three blocks since it is one deposit. The tonnage each block represents is area times thickness times density.

Block D1: area $= 0.02$ km2, thickness $= 12$ m, so tonnage $\propto 0.02 \times 12 = 0.24$ units.

Block D2: area $= 0.03$ km2, thickness $= 10$ m, so tonnage $\propto 0.03 \times 10 = 0.30$ units.

Block D3: area $= 0.04$ km2, thickness $= 15$ m, so tonnage $\propto 0.04 \times 15 = 0.60$ units.

Step 2: Find the grade inside each block first. Each borehole passes through two bands, so average them by thickness.

D1: $(6 \times 63 + 6 \times 66)/12 = 64.50\%$

D2: $(6 \times 60 + 4 \times 64)/10 = 61.60\%$

D3: $(8 \times 64 + 7 \times 62)/15 = 63.07\%$

Step 3: Find the metal content carried by each block, then add them up. Metal content is tonnage multiplied by grade.

D1: $0.24 \times 64.50 = 15.48$

D2: $0.30 \times 61.60 = 18.48$

D3: $0.60 \times 63.07 = 37.84$

Total metal $= 15.48 + 18.48 + 37.84 = 71.80$, and total tonnage $= 0.24 + 0.30 + 0.60 = 1.14$.

The deposit's overall grade is the total metal divided by the total tonnage: 71.80/1.14 = 62.98%. Grade is never simply averaged across boreholes, it is always tonnage weighted, the same way a mine planner blends ore coming from different faces.

Let's summarize:

- Average each borehole's own grade by its intercept thickness first.
- Weight the three boreholes by tonnage (area times thickness), not by area alone.
- The deposit's average grade works out to 62.98%.

So the average grade of the iron ore deposit is 62.98%.

Quick Tip: First average the grade inside each borehole by intercept thickness, then combine the three boreholes using area times thickness as the weight.

• • •

. A mine workshop needs to assign 4 jobs to 4 service engineers. The cost for performing a job by an individual service engineer is given. A typical job can be assigned to only one service engineer. If the service engineer S1 cannot perform the job J3, and S3 cannot perform the job J4, the optimal cost for completion of jobs is . *(answer in integer)*

Service Engineer	J1	J2	J3	J4
S1	50	50	---	20
S2	70	40	20	70
S3	90	30	50	---
S4	70	20	60	30

Correct Answer: 130

SOLUTION:

With only 4 jobs and 4 engineers, and just two combinations ruled out, it is faster here to reason directly about the cheapest way to cover all four jobs than to run the full Hungarian tableau. The goal is one engineer per job, every job covered, lowest total cost.

1. **Look for the standout cheap cells.** Scanning the table, J2 is cheapest for S4 (20) and for S3 (30). J1 is cheapest for S1 (50). J3 is cheapest for S2 (20). J4 is cheapest for S1 (20) and S4 (30).

2. **Give S2 to J3.** S2's cost of 20 on J3 is the lowest value S2 offers anywhere, and the only engineer who could beat it on J3, S1, is blocked from that job. So S2-J3 = 20 is locked in.
3. **Check the branch where S4 takes J2.** S4-J2 = 20 is the single lowest number left in the table once J3 is taken. But S3 cannot do J4, so with J2 gone S3 must take J1 at 90, leaving S1 to take J4 at 20. Total: 20+20+90+20 = 150.
4. **Check the branch where S3 takes J2 instead.** S3-J2 = 30. That frees up J4 for S4 at 30, and leaves S1 to take J1 at 50. Total: 30+30+50+20 = 130.
5. **Compare the two branches.** 130 is lower than 150, so S3 taking J2 (not S4) is the better choice.

Checking every other feasible pairing given the two forbidden cells, none beats 130. The winning combination is S1-J1 (50), S2-J3 (20), S3-J2 (30), S4-J4 (30).

Let's summarize:

- S2 must take J3, since that is by far its cheapest option and S1 cannot use J3 to compete for it.
- Testing S1, S3 and S4 against J1, J2 and J4 shows S1-J1, S3-J2, S4-J4 beats every other pairing.
- The minimum total cost works out to 130.

So the optimal cost for completing all four jobs is 130.

Quick Tip: Use the Hungarian assignment method, blocking S1-J3 and S3-J4 with a very high cost so they never get picked.

• • •

. Using linear regression (least squares), the best-fit line for the given dataset is $y = 2.2x + 2.3$. Given the residual sum of squares is 25.8, the coefficient of determination (r^2) is . *(rounded off to two decimal places)*

x 2 3 4 5

y 8 9 7 16

Correct Answer: 0.48

SOLUTION:

Another way to reach the same r^2 is to work out how much of the variation the regression line actually explains, rather than starting from what it fails to explain.

Step 1: Find the total variation in y (SST). The y values are 8, 9, 7 and 16, with mean $\bar{y} = 40/4 = 10$. Squaring each deviation from this mean and adding them gives $SST = 4 + 1 + 9 + 36 = 50$.

Step 2: Find the explained variation (SSR) instead of starting from the error. The regression splits total variation into two parts: the part the line explains (SSR) and the part it misses (SSE, the residual sum of squares), so $SST = SSR + SSE$. Since $SSE = 25.8$: $SSR = SST - SSE = 50 - 25.8 = 24.2$.

Step 3: The coefficient of determination is the explained share of the total variation. $r^2 = SSR/SST = 24.2/50 = 0.484$.

Both routes must agree, because $1 - SSE/SST$ and SSR/SST are just two ways of writing the same split of total variation into explained and unexplained parts.

Let's summarize:

- SST measures the total spread of y around its own mean, here 50.
- SSR = SST - SSE = 24.2 is the part of that spread the fitted line accounts for.
- $r^2 = SSR/SST = 0.48$ rounded to two decimal places.

So the coefficient of determination is 0.48.

Quick Tip: Find SST from the spread of the y values about their mean, then use $r^2 = 1 - SSE/SST$.

• • •

. A plug dam is to be constructed in a mine gallery of size 3.5 m × 2.5 m. The interfacial shear strength between the surrounding rock and plug dam is 1.0 MPa. The minimum thickness of the dam, in *m*, to withstand a water pressure of 10.0 MPa, is . *(rounded off to two decimal places)*

Correct Answer: 7.29

SOLUTION:

The same result can be reached by thinking in terms of forces per meter of dam perimeter instead of total forces, which is how a mine engineer would size the plug on site.

Step 1: Work out the water pressure force per meter of perimeter. The gallery face is 3.5 m by 2.5 m, so its area is 8.75 m2 and its perimeter is $2(3.5 + 2.5) = 12$ m. The total water force is $P \times A = 10.0 \times 8.75 = 87.5$ MN, spread around a perimeter of 12 m. So the force each meter of perimeter must resist is $87.5/12 = 7.29$ MN per meter.

Step 2: Work out how much resisting force one meter of thickness supplies per meter of perimeter. The interface shear strength is $\tau = 1.0$ MPa $= 1.0$ MN/m2. For a 1 m strip of perimeter and thickness t, the contact area is $1 \times t$ m2, so the resisting force it supplies is $1.0 \times t$ MN, meaning each meter of thickness gives 1.0 MN/m of resistance.

Step 3: Match supply to demand. Each meter of thickness gives 1.0 MN/m of resistance, and we need 7.29 MN/m to balance the water. So the required thickness is $t = 7.29/1.0 = 7.29$ m.

Let's summarize:

- The water load per meter of gallery perimeter is 7.29 MN/m.
- Each meter of dam thickness supplies 1.0 MN/m of shear resistance at this interface strength.
- Matching the two gives a minimum thickness of 7.29 m.

So the minimum thickness of the plug dam is 7.29 m.

Quick Tip: Balance the total water force on the dam face against the shear resistance developed around its perimeter.

• • •

. A shovel costing (P) Rs. 20 crores has a salvage value (S) of Rs. 2 crores with a useful life (n) of 10 years. The annual depreciation is evaluated using the declining balance method as given by

$$\text{Depreciation rate} = 1 - (S/P)^{1/n}$$

The depreciated cost of the shovel, in *crores*, in its 2nd year is Rs. . *(rounded off to two decimal places)*

Correct Answer: 12.62

SOLUTION:

Instead of first computing the depreciation rate as a percentage and reapplying it, it's quicker to build the two-year multiplier directly from the ratio S/P using indices.

Step 1: Write the book value formula symbolically. Under the declining balance method, after k years the book value is $BV_k = P(1-r)^k$, where $r = 1-(S/P)^{1/n}$. Since $(1-r) = (S/P)^{1/n}$, this means $BV_k = P \times (S/P)^{k/n}$. This is a shortcut: we never actually need r on its own, since $(S/P)^{1/n}$ raised to the kth power is exactly $(S/P)^{k/n}$.

Step 2: Plug in the numbers for year 2. Here $P = 20$, $S = 2$, $n = 10$ and $k = 2$, so $S/P = 2/20 = 0.1$ and $k/n = 2/10 = 0.2$.

$BV_2 = 20 \times (0.1)^{0.2}$

$(0.1)^{0.2} = 0.6310$, so $BV_2 = 20 \times 0.6310 = 12.62$ crores.

Step 3: Cross check against the year-by-year method. Working year by year gives $r = 1 - (0.1)^{0.1} = 0.2057$, $BV_1 = 20(0.7943) = 15.89$ crores, and $BV_2 = 15.89(0.7943) = 12.62$ crores. Both routes land on the same number, since $(1-r)^2 = ((S/P)^{1/n})^2 = (S/P)^{2/n}$.

It's worth being clear about what "depreciated cost in its 2nd year" means here. Taken as the book value remaining at the end of year 2, the answer is 12.62 crores. Taken instead as just the depreciation charged during year 2 alone (the drop from BV_1 to BV_2), the answer would be $15.89 - 12.62 = 3.27$ crores. This solution answers the first reading, the remaining depreciated value of the asset.

Let's summarize:

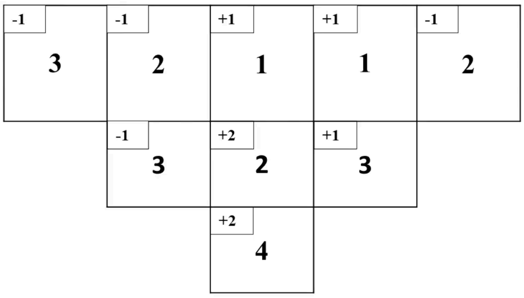
- The two-year book value multiplier is $(S/P)^{2/n} = (0.1)^{0.2} = 0.6310$.
- $BV_2 = P \times 0.6310 = 12.62$ crores.
- This matches the step-by-step declining balance calculation exactly.

So the depreciated cost of the shovel at the end of year 2 is Rs. 12.62 crores.

Quick Tip: Find the yearly declining-balance depreciation rate from S, P and n, then apply it twice to shrink the book value down to the end of year 2.

• • •

. An economic block model along with extraction sequence is shown. The numbers given at the centre of the block represent the year of extraction, and that at the corners represent the block economic value in Rs. (lakhs). If the discount rate is 10%, the net present value (NPV), in *lakhs*, is Rs. . *(rounded off to two decimal place)*



Correct Answer: 2.43

SOLUTION:

Instead of discounting each of the nine blocks one at a time, it's cleaner to first group the blocks by which year they get mined, add up the values within each year, and only then discount each year's total once.

Step 1: Group the blocks by extraction year. Reading the model, year 1 has two blocks worth +1 and +1. Year 2 has three blocks worth -1, -1 and +2. Year 3 has three blocks worth -1, -1 and +1. Year 4 has one block worth +2.

Step 2: Add up the value scheduled for each year.

Year 1 total: $1 + 1 = 2$ lakhs.

Year 2 total: $-1 - 1 + 2 = 0$ lakhs.

Year 3 total: $-1 - 1 + 1 = -1$ lakh.

Year 4 total: $+2$ lakhs.

Step 3: Discount each year's total back to the present at 10% and add. The discount factor for year t is $1/(1.1)^t$.

Year 1: $2/(1.1)^1 = 2 \times 0.9091 = 1.8182$

Year 2: $0/(1.1)^2 = 0$

Year 3: $-1/(1.1)^3 = -1 \times 0.7513 = -0.7513$

Year 4: $2/(1.1)^4 = 2 \times 0.6830 = 1.3660$

$NPV = 1.8182 + 0 - 0.7513 + 1.3660 = 2.4329$, which rounds to 2.43 lakhs. This matches discounting all nine blocks separately, since addition and discounting can be reordered freely, grouping by year first is just a shortcut that avoids repeating the same discount factor many times.

Let's summarize:

- Grouping blocks by year gives net cash flows of +2, 0, -1 and +2 lakhs for years 1 through 4.
- Discounting each year's net flow at 10% and adding gives the total NPV.
- The result is Rs. 2.43 lakhs.

So the net present value of the block extraction sequence is Rs. 2.43 lakhs.

Quick Tip: Discount each block's economic value back to the present using its own extraction year, then add all nine discounted values.

• • •

. A main mechanical ventilator installed for an underground mine develops a pressure of 25 mm wg. A natural ventilation pressure (NVP) of 15 mm wg acting in the mine aids the ventilator, and a total 500 $\mathrm{m}^3\ \mathrm{min}^{-1}$ of air is circulated in the mine. Considering the same NVP aiding the ventilator, the pressure required, in mm wg, to be generated by the ventilator to circulate 1000 $\mathrm{m}^3\ \mathrm{min}^{-1}$ of air, is . (answer in integer)

Correct Answer: 145

SOLUTION:

Step 1: Understanding the Question:
A fan and a natural ventilation pressure (NVP) push air through the same set of mine airways together. We know the fan pressure and NVP at one airflow, and we need the new fan pressure at double that airflow, with the NVP staying the same.

Step 2: Key Formula or Approach:
Airway resistance stays fixed when only the quantity of air changes, so pressure lost to resistance grows with the square of the quantity, $P \propto Q^2$. Because the NVP is aiding the fan, the total driving pressure at any flow is $P_{total} = P_{fan} + NVP$, and it is this total pressure that obeys the square law, not the fan pressure alone.

Step 3: Detailed Explanation:
At the first flow, $Q_1 = 500\ \mathrm{m}^3/\mathrm{min}$, the total pressure is $P_{total,1} = 25 + 15 = 40$ mm wg.
The new flow $Q_2 = 1000\ \mathrm{m}^3/\mathrm{min}$ is exactly double Q_1, so the flow ratio is 2.

Squaring this ratio gives the new total pressure: $P_{total,2} = 40 \times 2^2 = 40 \times 4 = 160$ mm wg.
The NVP has not changed, so it still contributes 15 mm wg toward this 160 mm wg total.
Whatever is left is what the fan itself must generate: $P_{fan,2} = 160 - 15 = 145$ mm wg.

Step 4: Final Answer:
The ventilator has to develop 145 mm wg to move 1000 m3/min of air, with the 15 mm wg NVP still helping it along.

Quick Tip: Total pressure overcoming a fixed set of airways follows a square law with airflow, and NVP simply adds to the ventilator pressure when it is aiding.

• • •

. Dust particles from a blasting operation rise up in the atmosphere to a height of 500 m. The viscosity of air is $1.80 \times 10^{-5}\ \mathrm{kg\,m^{-1}\,s^{-1}}$. Assume the particles have quiescent settling (no turbulence) in atmosphere. Neglecting air density, the settling time, in days, of a dust particle of 2.5 μm diameter and 3600 $\mathrm{kg\,m^{-3}}$ density is . (rounded off to two decimal places)

Correct Answer: 8.49

SOLUTION:

Step 1: Understanding the Concept:
A dust particle thrown up by a blast falls back down through calm air. Because it is so small and light, it very quickly reaches a steady falling speed and keeps that speed the rest of the way down. That steady speed is the Stokes' law terminal velocity.

Step 2: Key Formula or Approach:
Stokes' law for a sphere settling in a viscous fluid, with the fluid (air) density ignored as instructed, gives $v_t = \dfrac{\rho_p g d^2}{18\mu}$. Once v_t is known, the settling time is simply the drop height divided by this constant speed, $t = h/v_t$.

Step 3: Detailed Explanation:
Work in convenient sub units first. The diameter is $d = 2.5\ \mu m$, which is $2.5 \times 10^{-3}\ mm$ $= 2.5 \times 10^{-6}\ m$, so $d^2 = 6.25 \times 10^{-12}\ m^2$.
The numerator of the terminal velocity formula is $\rho_p g d^2 = 3600 \times 9.81 \times 6.25 \times 10^{-12}$ $= 2.207 \times 10^{-7}$ (SI units).
The denominator is $18\mu = 18 \times 1.80 \times 10^{-5} = 3.24 \times 10^{-4}$.
Dividing gives $v_t = 2.207 \times 10^{-7}/3.24 \times 10^{-4} = 6.81 \times 10^{-4}\ m/s$, which is only about 0.68

mm every second, showing how slowly a fine dust particle falls.
The fall height is 500 m, so the time taken is $t = 500/(6.81 \times 10^{-4}) = 733945\ s$.
There are $24 \times 3600 = 86400$ seconds in a day, so $t = 733945/86400 = 8.49$ days.

Step 4: Final Answer:
The dust particle needs close to 8.49 days to settle out of the atmosphere and reach the ground.

Quick Tip: A dust-sized particle in still air settles at a constant Stokes' law terminal velocity; divide the fall height by that velocity to get the time.

• • •

. In a biochemical oxygen demand (BOD) test, 15 ml of wastewater sample was diluted with distilled water to completely fill a 300 ml BOD bottle and incubated at 20 °C for 5 days. The dissolved oxygen (DO) level before and after incubation are 9.2 mg liter^{-1} and 4.4 mg liter^{-1}, respectively. The BOD of the sample, in mg liter^{-1}, is . (answer in integer)

Correct Answer: 96

SOLUTION:

Step 1: Understanding the Concept:
The sample was too concentrated to test directly, so a small part of it, 15 ml, was diluted with clean water to fill the whole 300 ml bottle. This means the sample only makes up a fraction of the bottle's contents.

Step 2: Key Formula or Approach:
The fraction of the bottle that is actual sample is $f = 15/300 = 1/20$. The oxygen drop measured in the bottle was caused only by this fraction of sample, so the BOD of the full strength sample is the measured drop divided by that fraction: $BOD = \Delta DO/f$.

Step 3: Detailed Explanation:
The dissolved oxygen fell from 9.2 mg/L before incubation to 4.4 mg/L after 5 days, a drop of $\Delta DO = 9.2 - 4.4 = 4.8$ mg/L.
The sample fraction is $f = 15/300 = 0.05$.
Dividing the observed drop by that fraction, $BOD = 4.8/0.05 = 96$ mg/L.
This matches scaling up by the reciprocal of the fraction, $1/0.05 = 20$, which is the same dilution factor used the other way around.

Step 4: Final Answer:
The five day BOD of the wastewater sample works out to 96 mg per liter.

Quick Tip: Scale the observed drop in dissolved oxygen up by the dilution factor of the BOD bottle to get the BOD of the original sample.

• • •

. A rake of wagons, with inter-wagon gap of 100 cm, is moving at a speed of 0.4 km per hour under a silo loading system. The dimensions of the wagon are 8 m (L) × 3 m (W) × 3 m (H). The silo stops discharging the material between the wagons. Considering the fill factor of the wagon as 0.95 and the bulk density of coal as 1.2 tonne per cubic meter, the loading rate of the silo, in tonne per hour, is . (rounded off to nearest integer)

Correct Answer: 3648

SOLUTION:

Step 1: Understanding the Question:
A silo drops coal into wagons as they roll past underneath. It cannot load anything while the gap between two wagons is passing, so we need the rate at which mass is delivered averaged over a full wagon plus gap cycle.

Step 2: Key Formula or Approach:
Find how many wagon-plus-gap lengths pass the silo in one hour, then multiply by the mass carried in each wagon. This gives $\text{Rate} = (\text{wagons passing per hour}) \times (\text{mass per wagon})$.

Step 3: Detailed Explanation:
One wagon plus its trailing gap covers $8\ m + 1\ m = 9\ m$ of track (the gap of 100 cm equals 1 m).
The rake speed is $0.4\ km/h = 400\ m/h$.
So the number of such 9 m units passing the silo in one hour is $400/9 = 44.44$ per hour.
Each wagon carries a coal volume of $8 \times 3 \times 3 \times 0.95 = 68.4\ m^3$, filled to only 0.95 of the box, and at a bulk density of 1.2 tonne/m3 this is a mass of $68.4 \times 1.2 = 82.08$ tonne.
Multiplying the wagons per hour by the mass each one carries, $\text{Rate} = 44.44 \times 82.08 = 3648$ tonne/hour.

Step 4: Final Answer:

The silo delivers coal into the rake at about 3648 tonnes per hour.

Quick Tip: Work out the coal mass one wagon carries, then the time one wagon length plus the inter-wagon gap takes to pass the silo at the given speed.

• • •

. A turbine pump is designed to generate water head of 60 m. The impeller speed of the pump is 2000 rpm and the manometric efficiency is 70%. Neglecting the impeller blade curvature, the impeller diameter of the pump, in cm, is . (rounded off to two decimal places)

Correct Answer: 27.69

SOLUTION:

Step 1: Understanding the Concept:

A centrifugal (turbine) pump spins water outward and this spinning motion, not friction, is what raises the water's pressure head. Ignoring blade curvature means we treat the whole tip speed of the impeller as doing useful work on the water, which is the simplest form of Euler's pump theory.

Step 2: Key Formula or Approach:

Under this assumption the manometric head links directly to the tip speed as $H_m = \eta_{man}$ u_2^2/g. Once u_2 is found, convert it to a diameter through the angular speed of rotation, ω $= 2\pi N/60$, using $u_2 = \omega r$ where r is the impeller radius.

Step 3: Detailed Explanation:

Rearranging the head formula for tip speed: $u_2 = \sqrt{H_m g/\eta_{man}} = \sqrt{(60)(9.81)/0.70}$ $= \sqrt{840.86} = 29.00\ m/s$.

The angular speed at 2000 rpm is $\omega = 2\pi(2000)/60 = 209.44\ rad/s$.

The impeller radius is then $r = u_2/\omega = 29.00/209.44 = 0.1385\ m$.

Doubling for the diameter, $D = 2r = 0.2769\ m$, which is $27.69\ cm$.

Step 4: Final Answer:

Working through the angular speed and radius gives the same impeller diameter, about 27.69 cm.

Quick Tip: Neglecting blade curvature means the exit whirl velocity equals the impeller tip speed, so the ideal head is u2 squared over g; link that to the manometric efficiency and the diameter formula.

• • •

. The information for a dragline operation is given:
Bucket capacity: 20 m^3
Bucket fill factor: 0.9
Digging and filling time: 20 s
Swinging (to and fro) time: 32 s
Dumping time: 10 s
Dragline utilization: 90%
The output of the dragline, in $m^3\,h^{-1}$, is . (rounded off to one decimal place)

Correct Answer: 940.6

SOLUTION:

Step 1: Understanding the Question:
A dragline repeats a dig, swing, dump cycle over and over, but it is not actually working every second of the hour since there are stoppages and delays. We need its real, practical output per hour after accounting for both the bucket not filling completely and the machine not running the whole hour.

Step 2: Key Formula or Approach:
Instead of scaling the final output by the utilization, scale the working time itself: out of every hour, the dragline is actually productive for only 3600×0.90 seconds. Multiplying the number of cycles that fit into that productive time by the effective bucket load gives the same answer from a different direction.

Step 3: Detailed Explanation:
The effective bucket load, after the 0.9 fill factor, is $20 \times 0.9 = 18\ m^3$ per cycle.
One cycle takes digging (20 s) plus the full to and fro swing (32 s) plus dumping (10 s), so $t_{cycle} = 62\ s$.
The productive seconds available in an hour, after the 90% utilization, are $3600 \times 0.90 = 3240\ s$.
The number of cycles that fit into this productive time is $3240/62 = 52.26$ cycles.
Multiplying by the load per cycle, $Q = 52.26 \times 18 = 940.6\ m^3/h$.

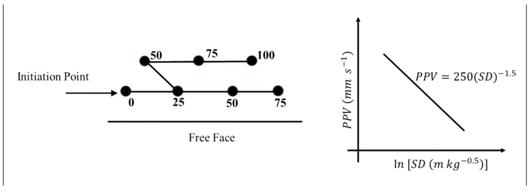
Step 4: Final Answer:

Scaling the working time instead of the output gives the same practical dragline output, about 940.6 cubic metres per hour.

Quick Tip: Add up the digging, swinging and dumping times for one full cycle, find cycles per hour, then multiply by the effective bucket load and the utilization factor.

• • •

. In a bench blasting, as shown, 49 kg of explosive is put in each hole. The detonation time of holes, in millisecond, and the associated scale distance (SD) vs. peak particle velocity (PPV) plot for the given blast pattern is shown.



The PPV at a distance of 100 m, in $\mathrm{mm\,s^{-1}}$, is . (rounded off to two decimal places)

Correct Answer: 7.79

SOLUTION:

Step 1: Understanding the Concept:

In production blasting, holes are not all fired at once, they are fired in a timed sequence (millisecond delays) so that each hole breaks toward a free face created by the hole before it. Vibration at a distant point depends only on the charge that fires at any one instant, not on the total charge in the whole blast, because delayed charges act as separate, smaller events.

Step 2: Key Formula or Approach:

The scaled distance approach used by USBM style vibration laws is $SD = D/\sqrt{W}$, where W is the biggest single-instant (per delay) charge weight, and D is the distance to the point of interest. The site specific attenuation curve then gives $PPV = 250(SD)^{-1.5}$.

Step 3: Detailed Explanation:

From the timing diagram, the front row holes fire at 0, 25, 50 and 75 ms, and the back row holes fire at 50, 75 and 100 ms. Two delay times, 50 ms and 75 ms, are shared between a front row hole and a back row hole, meaning a pair of holes goes off together at each of those instants.

Since every hole carries 49 kg, the charge firing simultaneously at these shared instants is $W = 49 + 49 = 98\ kg$, and this is larger than any single hole's 49 kg, so it is the value that controls vibration.

The scaled distance at 100 m is $SD = 100/\sqrt{98} = 100/9.8995 = 10.10\ m\ kg^{-0.5}$.

Raising this to the power 1.5 gives $10.10^{1.5} = 32.11$, so $PPV = 250/32.11 = 7.79\ mm/s$.

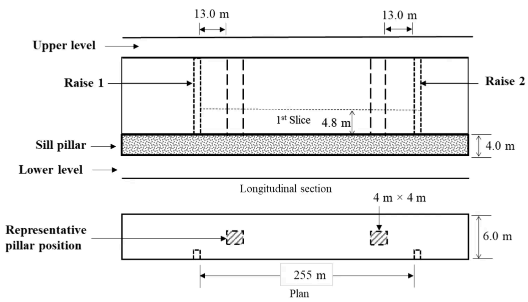
Step 4: Final Answer:

Because a pair of holes coincides at 50 ms and again at 75 ms, the governing per delay charge is 98 kg, giving a predicted PPV of about 7.79 mm/s at 100 m.

Quick Tip: Find the largest charge weight that detonates in the same delay from the firing time diagram, use it to scale the distance, then plug into the given PPV versus scaled distance equation.

• • •

. A regular shaped cut and fill stope is shown. Pillars of size $4\ \text{m} \times 4\ \text{m}$ are left at an interval of $13\ \text{m}$ along the length of the stope. If the density of the mined ore is $2.5\ \text{tonne/m}^3$, and slices are extracted to the full stope width, the total tonnage of ore recovered from the 1st slice of the stope is $\times 10^3$. (rounded off to one decimal place)



Correct Answer: 15.5

SOLUTION:

Here is a second way to reach the same tonnage, working through the recovery percentage instead of subtracting a pillar volume directly.

First fix the pillar spacing. The pillars are $4\ \text{m} \times 4\ \text{m}$ and the drawing marks a $13\ \text{m}$ gap between the faces of neighbouring pillars, so pillar centres sit $4 + 13 = 17\ \text{m}$ apart. Over the $255\ \text{m}$ length of the stope this gives $255/17 = 15$ pillars.

Now work out what fraction of the plan area is lost to pillars. The full plan area of the slice is $255 \times 6.0 = 1530\ \mathrm{m}^2$. The pillars take up $15 \times (4 \times 4) = 240\ \mathrm{m}^2$ of that. So the fraction of area lost is $240/1530 = 0.1569$, meaning 15.69% of the slice area is left as pillars and 84.31% is actually mined out.

Next find the gross tonnage as if there were no pillars at all. Gross volume $= 255 \times 6.0 \times 4.8 = 7344\ \mathrm{m}^3$, and at a density of $2.5\ \mathrm{tonne/m}^3$ this is $7344 \times 2.5 = 18360$ tonnes.

Apply the recovery fraction to this gross tonnage: $18360 \times 0.8431 = 15479.7$ tonnes, which is the same 15.5×10^3 tonnes as before.

Let's summarize:

- Pillar centre spacing is 17 m (4 m pillar plus 13 m gap), giving 15 pillars over 255 m.
- Pillars remove about 15.7% of the slice's plan area.
- Applying that loss to the gross slice tonnage gives the same answer as subtracting pillar volume directly.

So the 1st slice yields about 15.5×10^3 tonnes of ore.

Quick Tip: Work out how many 4 m pillars actually fit along the stope length once you account for the 13 m gap between them, then think about what volume that removes from the slice you are costing out.

• • •

. A coal seam of $2.4\ \mathrm{m}$ thickness is mined with a DERD shearer at a cutting speed of $0.6\ \mathrm{km/h}$. The web depth of the cut is $0.5\ \mathrm{m}$. The average cross-sectional area of coal on the AFC during transportation is $0.24\ \mathrm{m}^2$. For the evacuation of cut coal from the face with 10% spillage, the required minimum velocity of the AFC, in $m\ s^{-1}$, is . (rounded off to two decimal places)

Correct Answer: 0.75

SOLUTION:

Let's work this out on a per-minute basis instead of straight into seconds, which keeps the numbers a little rounder before the final conversion.

The cutting speed is $0.6\ \mathrm{km/h}$. Converting to metres per minute: $0.6\ \mathrm{km/h} = 600\ \mathrm{m/h} = 10\ \mathrm{m/min}$.

The shearer removes a slice of coal whose cross-section is the seam thickness times the web depth of cut, so the volume cut per minute is $2.4 \times 0.5 \times 10 = 12\ \mathrm{m}^3/\mathrm{min}$.

Now bring in the spillage. Only 90% of what is cut ever makes it onto the AFC, the rest falls off the face before it is picked up, so the conveyor only needs to move $0.9 \times 12 = 10.8\ \text{m}^3/\text{min}$.

Convert this to a per second basis to match the units asked for: $10.8\ \text{m}^3/\text{min} = 10.8/60 = 0.18\ \text{m}^3/\text{s}$.

The conveyor's carrying capacity is its cross-sectional area times its belt speed, so with an area of $0.24\ \text{m}^2$: $v = 0.18/0.24 = 0.75\ \text{m/s}$.

Let's summarize:

- The shearer cuts coal at a steady volumetric rate set by seam thickness, web depth and cutting speed.
- Spillage of 10% means the AFC only has to move 90% of that cut volume.
- Dividing the required volumetric flow by the AFC's cross-section gives the minimum belt speed.

So the AFC must run at a minimum of $0.75\ \text{m/s}$.

Quick Tip: Find the volume of coal the shearer cuts each second from the seam thickness, web depth and cutting speed, then think about how much of that actually has to be carried away once some of it spills off the face.

• • •

. A bord and pillar panel has 24 developed pillars of size $40\ \text{m} \times 40\ \text{m}$ (centre to centre) under extraction. The following information is given:
Gallery width = 4 m
Extraction height = 3 m
Extraction ratio during depillaring = 80%
Specific gravity of coal = 1.4
Incubation period = 6 months
Average working days per month = 25
In order to extract the panel within the incubation period, the minimum rate of production, in tonne per day, is . (rounded off to one decimal place)

Correct Answer: 696.7

SOLUTION:

Let's approach this by first working out the tonnage locked up in a single pillar, then scaling up to the whole panel, rather than totalling the volume first.

Each pillar's centre to centre spacing is 40 m, but 4 m of that on each pillar's boundary belongs to the surrounding gallery, so the actual coal pillar left standing is $40 - 4 = 36$ m on a side.

Volume of coal in one pillar $= 36 \times 36 \times 3 = 3888 \text{ m}^3$, using the 3 m extraction height.

During depillaring only 80% of a pillar's coal is actually recovered, so the recoverable volume per pillar is $0.8 \times 3888 = 3110.4 \text{ m}^3$.

At a specific gravity of 1.4, the mass recoverable from one pillar is $3110.4 \times 1.4 = 4354.56$ tonnes.

With 24 pillars in the panel, the total coal to be extracted is $24 \times 4354.56 = 104509.44$ tonnes.

The incubation period gives the time limit: 6 months at 25 working days a month is 150 days. So the panel must be cleared at a rate of at least $104509.44/150 = 696.7296$ tonne per day.

Let's summarize:

- Subtracting the gallery width from the centre to centre spacing gives the true pillar size of 36 m by 36 m.
- Only 80% of a pillar's coal is won during depillaring.
- Scaling one pillar's recoverable tonnage up to 24 pillars and dividing by the 150 available days gives the same minimum rate.

So the panel needs a minimum production rate of about 696.7 tonne per day.

Quick Tip: Work out the true size of each pillar once the gallery width is taken out of the centre to centre spacing, then think about how much coal that leaves to recover within the given incubation period.

• • •

. Following observations were taken using a Tacheometer with staff held vertical. The additive and multiplying constants of the instrument are 0 and 100, respectively. The reduced level (RL) of the staff station R, in *m*, is . (rounded off to two decimal places)

Instrument station	Staff station	Staff reading (m)	Vertical angle	Remarks
P	Q	1.4, 2.7, 3.9	7°	RL of Q = 102 m
P	R	2.1, 2.8, 3.6	5°	

Correct Answer: 84.68

SOLUTION:

Instead of first isolating the elevation of the instrument's line of sight, let's work directly with the difference in RL between R and Q, since both sightings share the same instrument and the same line of collimation.

For any staff station, $\mathrm{RL} = H + V - \text{middle reading}$, where H is the (unknown but common) elevation of the line of sight. Since H is the same for both Q and R taken from station P, it cancels out when we subtract: $\mathrm{RL}_R - \mathrm{RL}_Q = (V_R - m_R) - (V_Q - m_Q)$, where m_Q and m_R are the middle hair readings.

For Q: staff intercept $s_Q = 3.9 - 1.4 = 2.5$ m, and $V_Q = \frac{100 \times 2.5}{2} \sin(14°) = 125 \times 0.2419$ $= 30.24$ m. Middle reading $m_Q = 2.7$ m, so $V_Q - m_Q = 30.24 - 2.7 = 27.54$ m.

For R: staff intercept $s_R = 3.6 - 2.1 = 1.5$ m, and $V_R = \frac{100 \times 1.5}{2} \sin(10°) = 75 \times 0.17365$ $= 13.02$ m. Middle reading $m_R = 2.8$ m, so $V_R - m_R = 13.02 - 2.8 = 10.22$ m.

Now take the difference: $\mathrm{RL}_R - \mathrm{RL}_Q = 10.22 - 27.54 = -17.32$ m.

Since RL of Q is given as 102 m, $\mathrm{RL}_R = 102 - 17.32 = 84.68$ m.

Let's summarize:

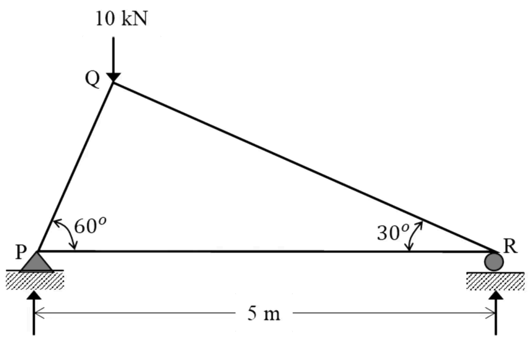
- The elevation of the line of sight is common to both sightings taken from P, so it cancels out when comparing R and Q directly.
- Each sighting's own vertical intercept minus its middle hair reading gives that station's height relative to the line of sight.
- Subtracting these two relative heights gives the RL difference between R and Q directly.

So the reduced level of station R works out to about 84.68 m.

Quick Tip: Use the vertical intercept formula on the Q sighting to pin down the elevation of the instrument's line of sight, then apply that same elevation to the R sighting.

• • •

. A truss PQR carries a vertical load of 10 kN at Q as shown. The force in member PR, in kN, is . (rounded off to two decimal places)



Correct Answer: 4.33

SOLUTION:

Let's find the same force by starting the method of joints at P instead of R, which is an equally valid check on member PR.

The triangle's third angle, at Q, is $180 - 60 - 30 = 90°$. With $PR = 5$ m, the sine rule gives $PQ = 5\sin(30°)/\sin(90°) = 2.5$ m and $QR = 5\sin(60°)/\sin(90°) = 4.33$ m.

Taking moments about P to find the roller reaction at R: the horizontal distance from P to Q is $PQ\cos 60° = 1.25$ m, so $R_R \times 5 = 10 \times 1.25$, giving $R_R = 2.5$ kN upward. By vertical equilibrium of the truss, $R_P = 10 - 2.5 = 7.5$ kN upward, with no horizontal reaction needed at P since there is no horizontal load anywhere on the truss.

Now isolate joint P. Two members meet there: PQ, rising toward Q at 60° above the horizontal, and PR, running horizontally toward R. The support reaction $R_P = 7.5$ kN also acts upward at this joint.

Vertical equilibrium at P: $R_P + F_{PQ}\sin 60° = 0 \Rightarrow F_{PQ} = -7.5/0.8660 = -8.66$ kN, so PQ is in compression.

Horizontal equilibrium at P: $F_{PQ}\cos 60° + F_{PR} = 0 \Rightarrow F_{PR} = -F_{PQ}\cos 60° = 8.66 \times 0.5 = 4.33$ kN.

Let's summarize:

- The truss's third angle works out to 90 degrees, fixing the geometry through the sine rule.

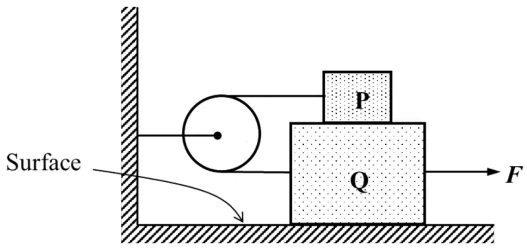
- Moments about P give the roller reaction at R, and overall vertical balance gives the reaction at P.
- Resolving forces at joint P instead of joint R gives the same tension of 4.33 kN in member PR.

So member PR carries a tensile force of about 4.33 kN.

Quick Tip: Work out the missing angle of the triangle to place joint Q, find the support reactions first, then apply the method of joints at one of the supports.

• • •

. Two blocks P and Q weighing 100 kN and 200 kN, respectively, are connected by a string passing through a massless and frictionless pulley as shown. The friction coefficient between blocks P and Q is 0.4, and that between block Q and surface is 0.3. The minimum force, F, in kN, required to pull the block Q, is . (answer in integer)



Correct Answer: 170

SOLUTION:

Let's set this up by listing every horizontal force acting on block Q directly, rather than analysing block P first.

Four horizontal forces act on Q: the applied force F pulling it forward, friction from the ground resisting that motion, friction from block P sitting on top of it, and the pull of the string, which loops from P over the fixed wall pulley and back to Q, so it also drags Q toward the wall.

Ground friction first. The ground carries the full weight resting on it, which is the weight of Q plus the weight of P transmitted down through it: $100 + 200 = 300$ kN. With a friction coefficient of 0.3 between Q and the surface, this gives $0.3 \times 300 = 90$ kN of resistance.

Now consider what block P costs Q. Block P cannot move, the inextensible string holds it fixed against the wall. As Q is dragged out from under it, kinetic friction develops between P and Q equal to $0.4 \times 100 = 40$ kN (using the normal force of 100 kN, P's own weight).

This entire 40 kN must be supplied by the string tension holding P still, so the tension in the string is also 40 kN.

Because the string loops back to Q through the same pulley, that same 40 kN tension pulls Q backward too, on top of the 40 kN friction reaction Q already feels from P. So block P effectively costs Q $40 + 40 = 80$ kN of resistance, not just 40 kN.

Adding the ground's contribution: $F = 90 + 80 = 170$ kN.

Let's summarize:

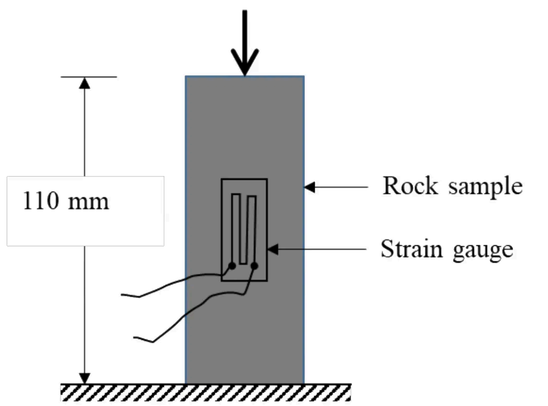
- Ground friction on Q depends on the full weight of both blocks, giving 90 kN.
- The friction between P and Q is 40 kN, and because the same string that resists this friction also loops back to Q, that 40 kN is felt twice.
- Adding these gives the minimum pulling force.

So the minimum force needed to pull block Q is 170 kN.

Quick Tip: Notice the same string ties both blocks to the fixed pulley on the wall, so think about how the tension needed to hold block P also acts back on block Q.

• • •

. In a uniaxial compressive strength test, a $120\ \Omega$ strain gauge of gauge factor 2.0 is pasted on the rock sample as shown. At the end of the test, the change in resistance of the strain gauge is $0.5\ \Omega$. The longitudinal deformation of the sample, in *mm*, is . (rounded off to two decimal places)



Correct Answer: 0.23

SOLUTION:

Here is the same result worked through percentages instead of decimal fractions, which some students find easier to follow.

The strain gauge starts at $120\ \Omega$ and its resistance changes by $0.5\ \Omega$ during the test. As a percentage, that change is $\frac{0.5}{120} \times 100 = 0.4167\%$.

The gauge factor tells us how many times bigger the percentage resistance change is compared to the percentage strain: $GF = \frac{\%\Delta R}{\%\varepsilon}$. Rearranging, the percentage strain is $\%\varepsilon = \frac{\%\Delta R}{GF} = \frac{0.4167}{2.0} = 0.2083\%$.

This percentage strain means the sample shortens by 0.2083% of its original length. The figure gives the original gauge length as $110\ \mathrm{mm}$, so the deformation is $\Delta L = \frac{0.2083}{100} \times 110 = 0.2292\ \mathrm{mm}$, which rounds to $0.23\ \mathrm{mm}$.

Let's summarize:

- Convert the resistance change to a percentage change first.
- Divide by the gauge factor to get the percentage strain.
- Apply that percentage directly to the sample's original length to find the deformation.

So the sample deforms by about 0.23 mm under the compressive load.

Quick Tip: Use the gauge factor definition to turn the measured change in resistance into a strain value, then apply that strain to the sample's original length.