GATE 2026 Metallurgical Engineering Question Paper with Solutions

ZOLLEGE

General Instructions

(i) This booklet contains 65 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 37 single-correct multiple-choice questions, 2 multiple-correct questions and 26 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For multiple-correct questions, all correct options must be selected to earn full marks.

(v) For numerical questions, report the answer rounded exactly as asked.

. 'The team _____ more than 300 runs in 20 overs _____ rains. However, some players needed to improve their batting skills.' Choose the option with the correct sequence of words to fill the blanks.

(A) score; despite

(B) scoring; instead of

(C) scored; despite

(D) scoring; in spite of

Correct Answer: (C) scored; despite

SOLUTION:

Step 1: Look at the verb tense clue.

The second sentence says "some players needed to improve their batting skills." The verb "needed" is in the simple past tense. Since both sentences describe the same past match, the first blank must also

be a simple past tense verb, so the team "scored" runs, not "score" or "scoring".

Step 2: Rule out the participle forms.
"Score" is the bare infinitive and "scoring" is the present participle. Neither can act as the main verb of a sentence on its own; both need a helping verb such as "did" or "was". Since no helping verb appears in the blank, options that use "score" or "scoring" leave the sentence without a proper verb, so they can be ruled out.

Step 3: Check the meaning of the second blank.
"despite" and "in spite of" both mean "even though it happened", and both can sit directly before a noun like "rains". "instead of" means "in place of", so "scoring runs instead of rains" would not make sense, since rain cannot be replaced by a score. This rules out "instead of".

Step 4: Combine both checks.
Only the pair "scored; despite" gives a grammatically complete past-tense sentence with a preposition that correctly means "even though it rained".

Final Answer:
The correct sequence is "scored; despite", option (C).

Quick Tip: Check which verb form is a proper finite past-tense verb, then pick the preposition that means "even though".

. If a positive real x satisfies the following equation

$$\log_2 x + \log_{\sqrt{2}} x = 48,$$

then the value of x is _____

(A) 2^{16}

(B) 4^{16}

(C) 2^{14}

(D) 4^{14}

Correct Answer: (A) 2^{16}

SOLUTION:

Step 1: Set up using natural logarithms.

Let x be the unknown positive number. Write both logarithms using natural log: $\log_2 x = \dfrac{\ln x}{\ln 2}$ and $\log_{\sqrt{2}} x = \dfrac{\ln x}{\ln \sqrt{2}}$.

Step 2: Simplify $\ln \sqrt{2}$.

Since $\sqrt{2} = 2^{1/2}$, we get $\ln \sqrt{2} = \dfrac{1}{2}\ln 2$. So

$$\log_{\sqrt{2}} x = \frac{\ln x}{\frac{1}{2}\ln 2} = \frac{2\ln x}{\ln 2}$$

Step 3: Add the two terms.

$$\frac{\ln x}{\ln 2} + \frac{2\ln x}{\ln 2} = 48$$

$$\frac{3\ln x}{\ln 2} = 48$$

$$\ln x = 16 \ln 2$$

Step 4: Solve for x.

$$\ln x = \ln(2^{16})$$

so

$$x = 2^{16}$$

This matches the value obtained from the base-2 approach, confirming the answer.

Final Answer:

$$x = 2^{16}$$

Quick Tip: Convert log base sqrt(2) to base 2 using the change-of-base rule before combining terms.

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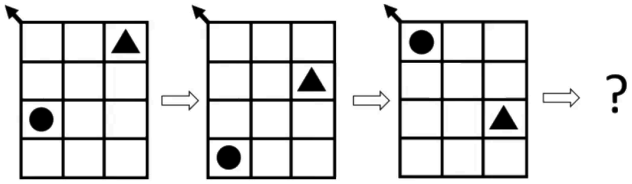
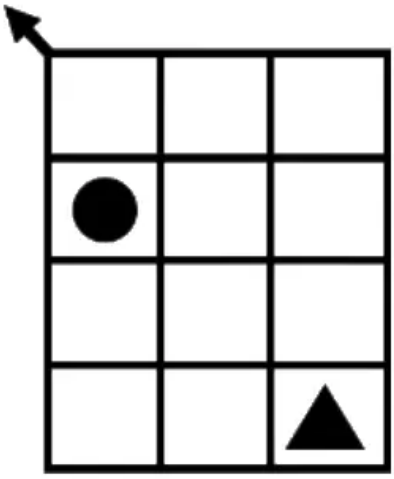
. The next figure (indicated by '?') in the sequence is shown below. Three grid panels are given, each a grid with a small arrow in the top-left corner and one dot and one filled triangle placed in different cells; the positions of the dot, the triangle, and the arrow orientation change from panel to panel following a consistent rule. The fourth panel (marked '?') must be chosen from the given options.



(A)



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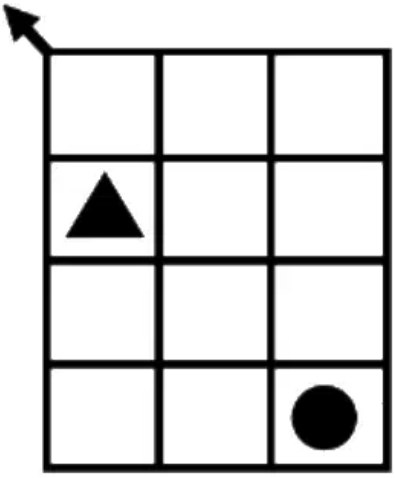

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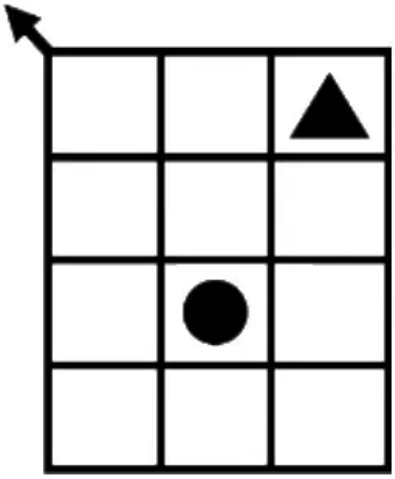
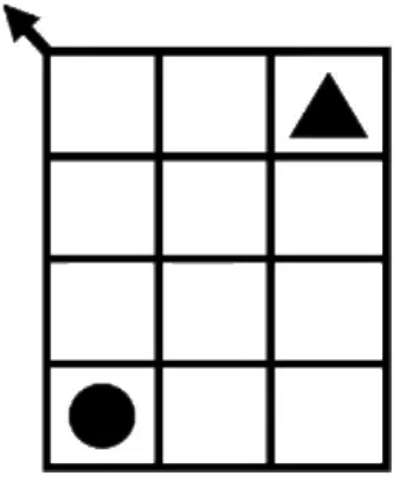
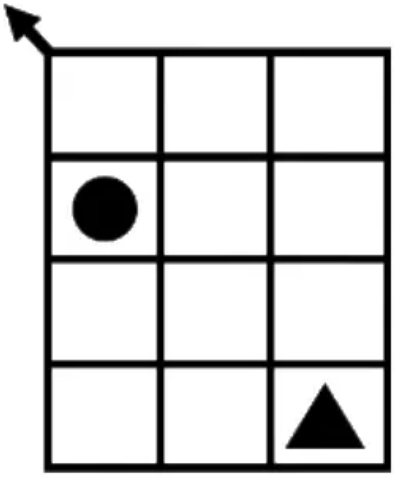


(D)



Correct Answer: (A)



SOLUTION:

Step 1: Compare the movement across each panel pair.
The three given panels form a short animation: the dot and the triangle each move a fixed number of cells in a fixed direction from one panel to the next. Set up two separate movement checks, one for the dot's path and one for the triangle's path, across panel 1 to 2 and panel 2 to 3.

Step 2: Verify each option against the projected position.
Option (A): continues the dot's path and the triangle's path exactly as established across panels 1, 2, and 3, with the grid and arrow kept in the same relative orientation. This matches the sequence.
Option (B): the dot and triangle appear in positions that do not follow the step size and direction seen between the earlier panels, so this breaks the established rule.
Option (C): the shapes are placed closer to their panel-3 positions than the rule allows, as if the movement had stopped partway, so this option under-shoots the pattern.
Option (D): the shapes are shifted in the wrong direction relative to the pattern shown in panels 1 to 3, effectively reversing part of the movement.

Step 3: Conclude.
Only option (A) keeps both the dot's and the triangle's motion consistent with what panels 1, 2, and 3 already show.

Final Answer:
The figure in option (A) is the correct continuation of the sequence.

Quick Tip: Track how far and in which direction the dot and the triangle each move from one panel to the next, then apply that same movement once more.

. 'All the mangoes in the basket are good.'
If the above statement is false, then which one of the following statements is necessarily true?

(A) All the mangoes in the basket are not good.

(B) No mango in the basket is good.

(C) In the basket, some of the mangoes are good and some are not good.

(D) There exists at least one mango in the basket that is not good.

Correct Answer: (D) There exists at least one mango in the basket that is not good.

SOLUTION:

Step 1: State the general rule for negating universal statements.
The negation of "for all x, P(x)" is "there exists an x such that not P(x)". This is the standard result from predicate logic and applies directly here, where P(x) means "mango x is good".

Step 2: Test option (A) and (B).
"All the mangoes in the basket are not good" and "No mango in the basket is good" both claim that every mango is bad. That is a far stronger claim than "not all mangoes are good"; the original statement being false does not tell us that every mango is bad, only that at least one is not good. So both of these options go beyond what the negation guarantees.

Step 3: Test option (C).
"In the basket, some of the mangoes are good and some are not good" assumes there is a mixture of good and bad mangoes. But it is entirely possible that all the mangoes are bad, in which case there are no good mangoes at all, so this option is not guaranteed either.

Step 4: Test option (D).
"There exists at least one mango in the basket that is not good" is exactly the logical negation of "all mangoes are good". As soon as one mango in the basket is not good, the original universal statement becomes false, and this is the weakest, most general condition that must hold.

Final Answer:
The statement that is necessarily true is option (D): there exists at least one mango in the basket that is not good.

Quick Tip: The negation of "All A are B" is "At least one A is not B", not "No A is B".

• • •

. Consider the following statements about four numbers:
(S1) The average of the four numbers is 25
(S2) Each number is at most 40
(S3) Each number is at least 20
Choose the option that is necessarily correct.

(A) (S1) and (S2) together imply (S3)

(B) (S2) and (S3) together imply (S1)

(C) (S1) and (S3) together imply (S2)

(D) (S1) implies (S3)

Correct Answer: (C) (S1) and (S3) together imply (S2)

SOLUTION:

Step 1: Set up the fixed sum.
Four numbers have an average of 25 under statement (S1), so their total is fixed at 100. Statements (S2) and (S3) put an upper and a lower bound of 40 and 20 respectively on each individual number. Test each option with a counter-example approach.

Step 2: Try to break "(S1) and (S2) imply (S3)".
Take the four numbers $0, 40, 40, 20$. They sum to 100, matching S1, and none exceeds 40, matching S2. But 0 is less than 20, so S3 fails here. Since S1 and S2 can hold while S3 fails, this option is not necessarily true.

Step 3: Try to break "(S2) and (S3) imply (S1)".
Take $20, 20, 20, 20$. Every number is between 20 and 40, so S2 and S3 both hold, but the average is 20, not 25, so S1 fails. This option is not necessarily true either.

Step 4: Try to break "(S1) and (S3) imply (S2)".
With the sum fixed at 100 and every number required to be at least 20, the largest any single number can become is found by pushing the other three down to their floor of 20 each: $20 + 20 + 20 = 60$, leaving $100 - 60 = 40$ for the fourth number. So no number can ever exceed

40 while S1 and S3 both hold, which is exactly what S2 requires. No counter-example is possible, so this option is necessarily true.

Step 5: Try to break "(S1) implies (S3)".
Take **0, 30, 30, 40**. This sums to 100, so the average is 25 and S1 holds, but the first number is 0, which is below 20, so S3 fails. This option is not necessarily true.

Final Answer:
Only the pair "(S1) and (S3) together imply (S2)" is necessarily correct.

Quick Tip: The sum of the four numbers is fixed at 100 by S1; work out the largest a single number can be while the other three stay at their minimum allowed by S3.

• • •

. 'People are crowding around ____ pit into which ____ elephant has fallen. I have never seen an elephant looking more bewildered ____ miserable. Here it is in a most undignified position, thrust into a pit and made to look up ____ a vast, curiosity-stricken crowd.'

Choose the option with the correct sequence of words to fill the blanks.

(A) an; a; at; and

(B) a; an; and; at

(C) and; a; an; at

(D) at; a; an; and

Correct Answer: (B) a; an; and; at

SOLUTION:

This question tests two things together: choosing "a" versus "an" based on the sound that follows it, and picking the right connecting words so the sentence reads naturally.

1. **an; a; at; and**: This reverses the articles. "Pit" starts with a consonant sound and needs "a", not "an", so this option fails at the very first blank.
2. **a; an; and; at**: "A" fits "pit" (consonant sound), "an" fits "elephant" (vowel sound), "and" correctly joins "bewildered" and "miserable" as two adjectives in a list, and "at" completes "look up at a crowd". Every blank checks out.
3. **and; a; an; at**: Starting the sentence with "and" before "pit" makes no sense, since the sentence needs an article there, not a linking word.
4. **at; a; an; and**: Using "at" as the first blank before "pit" is meaningless for the same reason, an article is needed, not a preposition.

Only the second choice keeps every blank grammatically sound: a pit, an elephant, bewildered and miserable, look up at a crowd.

Let's summarize:

- Use "a" before a consonant sound and "an" before a vowel sound.
- Two adjectives describing the same noun are joined with "and".
- The set phrase tested here is "look up at", not "look up to" or "look up on".

So the correct sequence of words is "a; an; and; at", option (B).

Quick Tip: Check the sound right after each blank to choose "a" or "an", and recall the fixed phrase "look up at" someone or something.

• • •

. The table lists the unit selling price of five products P, Q, R, S, and T. On a particular day, 250 items were sold with the average selling price of Rs. 60. The following observations were made:
(i) The quantity of S sold was twice that of T.
(ii) The quantity of R sold was thrice that of T.
(iii) The quantity of Q sold was four times that of T.

Product	P	Q	R	S	T
Unit selling price (Rs.)	100	50	40	60	60

What is the quantity of product P sold on that day?

(A) 40

(B) 50

(C) 60

(D) 70

Correct Answer: (B) 50

SOLUTION:

Step 1: Use the deviation-from-mean shortcut.
For a weighted average, the sum of (price minus average) times quantity must equal zero. Here the average price is 60, so

$$\sum(\text{price}_i - 60) \times \text{quantity}_i = 0$$

Step 2: Work out each deviation.

$P : 100 - 60 = 40$, $Q : 50 - 60 = -10$, $R : 40 - 60 = -20$, $S : 60 - 60 = 0$, $T : 60 - 60 = 0$. Only P, Q and R contribute, since S and T sit exactly at the average price.

Step 3: Bring in the quantities.

Let $T = t$. Then $Q = 4t$ and $R = 3t$. Let $P = p$. The deviation equation becomes

$$40p + (-10)(4t) + (-20)(3t) = 0$$

$$40p - 40t - 60t = 0$$

$$40p = 100t$$

$$p = 2.5t$$

Step 4: Use the total count.

All five quantities add to 250:

$$p + 4t + 3t + 2t + t = 250$$

$$p + 10t = 250$$

Step 5: Solve for t and p.

Substitute $p = 2.5t$:

$$2.5t + 10t = 250$$

$$12.5t = 250$$

$$t = 20$$

So $p = 2.5 \times 20 = 50$.

$$\boxed{50}$$

Quick Tip: Write one equation for the total number of items (250) and one for the total revenue (250 times Rs. 60), using T as the base quantity.

• • •

. Consider a string P of length l that is laid out as a straight-line segment. Another string K is laid out as a semicircular arc with string P as its diameter, as represented in Figure (i). When both the strings are shortened by a length x they can be re-arranged such that the shortened string K forms a full circle with the shortened string P as its diameter, as represented in Figure (ii). The value of x/l is _____

(A) π

(B) $\dfrac{\pi - 1}{2\pi}$

(C) $\dfrac{\pi}{2(\pi - 1)}$

(D) $\dfrac{\pi}{\pi - 1}$

Correct Answer: (C) $\dfrac{\pi}{2(\pi - 1)}$

SOLUTION:

Step 1: Express everything as a fraction of l.

Let $x = kl$, where $k = x/l$ is what we want to find. The original semicircle K has length $\dfrac{\pi l}{2}$, since its diameter is l.

Step 2: Write both shortened lengths using k.

Shortened P, the new diameter, is $l - kl = l(1 - k)$. Shortened K is $\dfrac{\pi l}{2} - kl = l\left(\dfrac{\pi}{2} - k\right)$.

Step 3: The shortened K must trace a full circle on the shortened P.

A circle with diameter $l(1 - k)$ has circumference $\pi l(1 - k)$. Setting this equal to the shortened K length,

$$l\left(\frac{\pi}{2} - k\right) = \pi l(1 - k)$$

Step 4: Cancel l and solve for k.

$$\frac{\pi}{2} - k = \pi - \pi k$$

$$\pi k - k = \pi - \frac{\pi}{2}$$

$$k(\pi - 1) = \frac{\pi}{2}$$

$$k = \frac{\pi}{2(\pi - 1)}$$

Step 5: Conclude.

Since $k = x/l$,

$$\boxed{\frac{x}{l} = \frac{\pi}{2(\pi - 1)}}$$

Quick Tip: Write the semicircle's length as pi times l over 2, and the new circle's circumference as pi times its diameter (l minus x), then equate the shortened K length to that circumference.

. The Roman senator Meritorius, his brother, his son, and his daughter have varying oratory skill levels. They are seated in rows and columns as shown in the figure with exactly one person sitting in each box. It is known that
(i) Meritorius' daughter and his brother are seated in the same column.
(ii) His son is seated diagonally across the sibling of the worst orator.
(iii) The best and worst orators are seated in the same row.

Who is the best orator?

	Column 1	Column 2
Row 1		
Row 2		

(A) Meritorius
(B) Meritorius' brother
(C) Meritorius' son
(D) Meritorius' daughter

Correct Answer: (B) Meritorius' brother

SOLUTION:

Step 1: Label the four seats.
Call the seats R1C1, R1C2, R2C1 and R2C2, one person per seat. Diagonal pairs are (R1C1, R2C2) and (R1C2, R2C1).

Step 2: Fix the daughter and brother using clue (i).
D and Br share a column. Take that to be Column 1, without losing anything, so $\{D, Br\} = \{R1C1, R2C1\}$ in some order. Then $\{M, S\} = \{R1C2, R2C2\}$ in some order in Column 2.

Step 3: Enumerate all four placements.
Case (D=R1C1, Br=R2C1, S=R2C2, M=R1C2): diagonal of S is R1C1=D, so D's sibling S is the worst orator; S's row (Row 2) also holds Br, so Br is the best orator.
Case (D=R1C1, Br=R2C1, S=R1C2, M=R2C2): diagonal of S is R2C1=Br, so Br's sibling M is the worst orator; M's row (Row 2) also holds Br, so Br is the best orator.
Case (D=R2C1, Br=R1C1, S=R2C2, M=R1C2): diagonal of S is R1C1=Br, so Br's sibling M is the worst orator; M's row (Row 1) also holds Br, so Br is the best orator.
Case (D=R2C1, Br=R1C1, S=R1C2, M=R2C2): diagonal of S is R2C1=D, so D's sibling S is the worst orator; S's row (Row 1) also holds Br, so Br is the best orator.

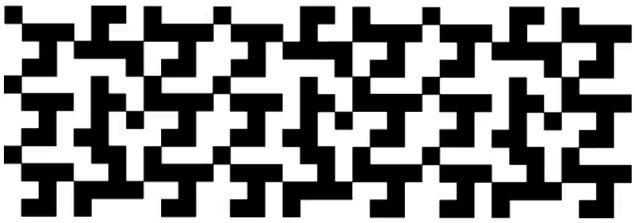
Step 4: Compare all four outcomes.
Every one of the four valid placements gives the exact same best orator, no matter how the seats are actually filled in.

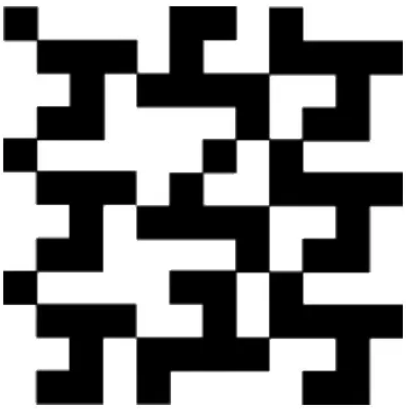
Step 5: Conclude.

Meritorius' brother

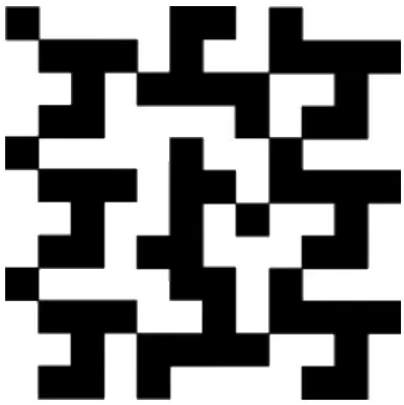
Quick Tip: Fix the daughter and brother in one column using clue (i), then use the diagonal-seat clue (ii) to find who is the worst orator before applying the same-row clue (iii).

. Which one of the patterns labelled P, Q, R, and S is used to generate the following figure?





P



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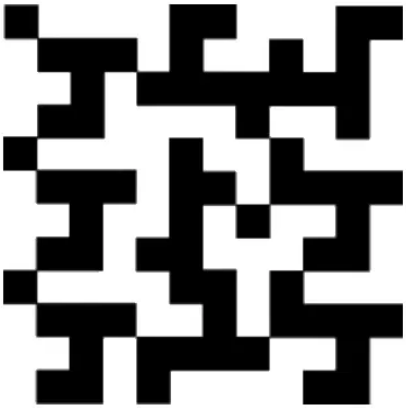
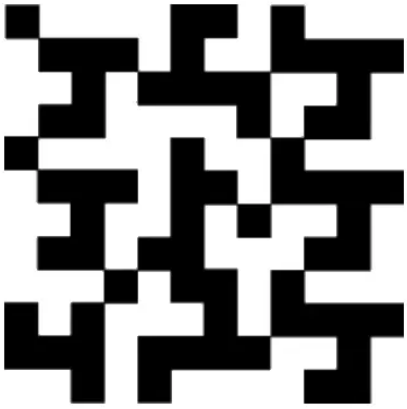


R



S

(A) P

(B) Q

(C) R

(D) S

Correct Answer: (B) Q

SOLUTION:

Step 1: Think of the big figure as a grid of repeated tiles.
The wide black and white figure at the top is made by placing one small square tile again and again in rows and columns. Candidates P, Q, R and S are four possible versions of that tile.

Step 2: Use an overlay check.
Imagine placing each candidate tile directly on top of one block of the large figure, square by square. A true match means every black square in the candidate lines up with a black square in the figure, and every white square lines up with a white square.

Step 3: Test tile P.
When P is overlaid, at least one of its squares does not agree with the corresponding square in the large figure, so P fails the overlay check.

Step 4: Test tiles R and S.
The same overlay check on R and S also turns up a mismatched square in each case, a black square where the big figure has white, or the reverse. Both R and S are ruled out.

Step 5: Test tile Q.
When Q is overlaid on the reference block, every filled square and

every empty square lines up exactly with the large figure. No mismatch remains anywhere in the tile.

Step 6: Conclude.
Since Q is the only candidate that survives the overlay check, it is the tile used to build the figure.

Q

Quick Tip: Overlay each candidate tile on one block of the large figure and check every square for an exact black or white match.

• • •

. Given a function $f(t) = e^{-at}$, where a is a constant. The Laplace transform of the function is $\mathcal{L}[f(t)] = F(s)$. Which one of the following options is correct?

(A) $F(s) = \dfrac{1}{s-a}$

(B) $F(s) = \dfrac{s}{s^2+a^2}$

(C) $F(s) = \dfrac{a}{s^2+a^2}$

(D) $F(s) = \dfrac{1}{s+a}$

Correct Answer: (D) $F(s) = \dfrac{1}{s+a}$

SOLUTION:

Step 1: Use the first shifting property instead of integrating from scratch.
A well known result is that the Laplace transform of the constant

function $f(t) = 1$ is $\mathcal{L}[1] = \frac{1}{s}$. This comes from $\int_0^\infty e^{-st}\,dt = \frac{1}{s}$ for $s > 0$.

Step 2: Recall the first shifting theorem.

The first shifting theorem says that multiplying a function $f(t)$ by e^{-at} in the time domain shifts the variable s to $s + a$ in the transform domain:

$$\mathcal{L}[e^{-at} f(t)] = F(s + a), \quad \text{where } F(s) = \mathcal{L}[f(t)]$$

Step 3: Apply the theorem to our function.

Here $f(t) = 1$, so $F(s) = \frac{1}{s}$. Shifting s to $s + a$ gives the transform of $e^{-at} \cdot 1 = e^{-at}$:

$$\mathcal{L}[e^{-at}] = F(s + a) = \frac{1}{s + a}$$

Step 4: Cross check with the exponential rule of thumb.

A quick memory check: $\mathcal{L}[e^{ct}] = \frac{1}{s - c}$ for any constant c, valid when $s > c$. Setting $c = -a$ directly gives $\frac{1}{s - (-a)} = \frac{1}{s + a}$, matching Step 3.

Step 5: Conclude.

Both routes agree.

$$\boxed{F(s) = \frac{1}{s + a}}$$

This is option (D).

Quick Tip: Use the definition $F(s) = \int_0^\infty e^{-st} f(t)\,dt$ and combine the exponents of e^{-st} and e^{-at} before integrating.

• • •

. Select the correct pair of eigenvectors for the following symmetric matrix.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

(A) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

SOLUTION:

Step 1: Test each candidate vector directly instead of deriving eigenvalues first.

An eigenvector v of matrix A must satisfy $Av = \lambda v$ for some scalar λ, meaning Av comes out parallel to v itself. We can check each option's vectors against $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ by direct multiplication.

Step 2: Test $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

$$A\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1(1) + 2(2) \\ 2(1) + 4(2) \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} = 5\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

This comes back as 5 times the same vector, so $\begin{bmatrix}1\\2\end{bmatrix}$ is a genuine eigenvector, with eigenvalue 5.

Step 3: Test $\begin{bmatrix}-2\\1\end{bmatrix}$.

$$A\begin{bmatrix}-2\\1\end{bmatrix} = \begin{bmatrix}1(-2)+2(1)\\2(-2)+4(1)\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix} = 0\begin{bmatrix}-2\\1\end{bmatrix}$$

This comes back as the zero vector, which is 0 times itself, so $\begin{bmatrix}-2\\1\end{bmatrix}$ is also a genuine eigenvector, with eigenvalue 0.

Step 4: Test the rival vector $\begin{bmatrix}2\\1\end{bmatrix}$ used in options (B) and (C).

$$A\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}4\\8\end{bmatrix}$$

For this to be an eigenvector we would need $\begin{bmatrix}4\\8\end{bmatrix} = k\begin{bmatrix}2\\1\end{bmatrix}$ for some k, but $4 = 2k$ gives $k = 2$ while $8 = k$ gives $k = 8$. These do not agree, so $\begin{bmatrix}2\\1\end{bmatrix}$ fails and options (B) and (C) are both wrong.

Step 5: Conclude.
Only $\begin{bmatrix}1\\2\end{bmatrix}$ and $\begin{bmatrix}-2\\1\end{bmatrix}$ pass the direct test.

$$\boxed{\begin{bmatrix}1\\2\end{bmatrix} \text{ and } \begin{bmatrix}-2\\1\end{bmatrix}}$$

This is option (A).

Quick Tip: Find the eigenvalues from $\det(A - \lambda I) = 0$ first, then solve $(A - \lambda I)v = 0$ for each λ; for a symmetric matrix, the two eigenvectors must be perpendicular.

• • •

. Choose one correct option.

Given $w = f(ax + by)$, where a and b are constants. The value of

$$\left(b\frac{\partial w}{\partial x} - a\frac{\partial w}{\partial y}\right)$$

is:

(A) $-a$

(B) b

(C) $b - a$

(D) 0

Correct Answer: (D) 0

SOLUTION:

Step 1: Notice the structure without computing derivatives yet.

Whenever w is a function of a single linear combination $u = ax + by$, both partial derivatives $\partial w/\partial x$ and $\partial w/\partial y$ come from the same underlying rate $f'(u)$, just scaled by different constants. This is the signature of what is called a directional or composite dependence.

Step 2: Write the gradient of w.

The gradient vector of w with respect to x and y is

$$\nabla w = \left(\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}\right) = f'(u)\,(a, b)$$

In words, the gradient of w always points along the fixed direction (a, b), no matter what the function f is, and only its length $f'(u)$ changes.

Step 3: Interpret the expression asked for.
The quantity $b\,\partial w/\partial x - a\,\partial w/\partial y$ is exactly the cross product style combination $b(\nabla w)_x - a(\nabla w)_y$, which measures how much ∇w points away from the direction (a, b). Since we just showed ∇w is always parallel to (a, b), this component perpendicular to (a, b) must vanish.

Step 4: Confirm algebraically.
Substituting $\partial w/\partial x = af'(u)$ and $\partial w/\partial y = bf'(u)$:

$$b\big(af'(u)\big) - a\big(bf'(u)\big) = abf'(u) - abf'(u) = 0$$

Step 5: Conclude.
The expression is identically zero for any differentiable f.

$$\boxed{0}$$

This is option (D).

Quick Tip: Let $u = ax + by$ and write $w = f(u)$; use the chain rule $\partial w/\partial x = f'(u) \cdot a$ and $\partial w/\partial y = f'(u) \cdot b$, then combine.

• • •

. Which one of the following fusion welding techniques results in least Heat-Affected Zone (HAZ)?

(A) Submerged Arc Welding (SAW)

(B) Tungsten Inert Gas Welding (TIG)

(C) Electron Beam Welding (EBW)

(D) Oxy-Acetylene Welding (OAW)

Correct Answer: (C) Electron Beam Welding (EBW)

SOLUTION:

This question checks whether we know which fusion welding process keeps the surrounding base metal from being disturbed by heat, that is, which process gives the smallest Heat-Affected Zone (HAZ). Let's look at each option.

1. **Submerged Arc Welding (SAW)**: This process runs a high current arc under a layer of flux. The high current means a large amount of heat goes into the joint, and this heat spreads into the base metal, giving a fairly wide HAZ.
2. **Tungsten Inert Gas Welding (TIG)**: TIG gives a cleaner, more controlled arc than SAW, so its HAZ is narrower than SAW's. It still uses an electric arc that covers a noticeable area, so the HAZ is not as small as it can get.
3. **Electron Beam Welding (EBW)**: EBW fires a stream of electrons that is focused down to a very fine spot using magnetic lenses, inside a vacuum chamber. This gives an extremely high power density in a tiny area, so the weld happens fast with very little extra heat leaking sideways into the base metal.
4. **Oxy-Acetylene Welding (OAW)**: This uses a gas flame, which is spread out and much less intense than an arc or a beam. Because the flame heats a bigger area for a longer time, this process gives the widest HAZ of the four.

Since EBW packs the most energy into the smallest spot and finishes the weld fastest, it leaves the least time and area for heat to spread into the base metal. So EBW gives the smallest Heat-Affected Zone.

Let's summarize:

- Narrower HAZ comes from higher energy density and faster welding speed.
- Ranked from narrowest to widest HAZ: EBW, then TIG, then SAW, then OAW.

So the fusion welding technique with the least Heat-Affected Zone is Electron Beam Welding (EBW), option (C).

Quick Tip: A narrower HAZ comes from a more concentrated, faster heat source with lower total heat input; compare the energy density of SAW, TIG, EBW, and OAW.

• • •

. During metallography, Nital is the most commonly used for etching _____.

(A) Bronze

(B) Brass

(C) Mild Steel

(D) Aluminium

Correct Answer: (C) Mild Steel

SOLUTION:

Etching in metallography means treating a polished metal sample with a chemical so the grain structure shows up under a microscope. Nital, a mix of nitric acid and alcohol, is one of the etchants asked about here. Let's check each metal option against what Nital is actually used for.

1. **Bronze**: Bronze is a copper-tin alloy. Copper alloys generally need a different chemistry to etch well, typically an etchant built around ferric chloride, not a nitric acid and alcohol mix.
2. **Brass**: Brass is a copper-zinc alloy. Like bronze, it is a copper based alloy and is normally etched with ferric chloride based reagents rather than Nital.
3. **Mild Steel**: Mild steel is a low carbon steel, made mostly of iron with a small amount of carbon. Nital is specifically known for darkening the pearlite regions and outlining the ferrite grain boundaries in this kind of steel, which is exactly why it is the standard laboratory etchant for carbon steels.
4. **Aluminium**: Aluminium and its alloys need a different kind of etchant, commonly one containing hydrofluoric acid such as Keller's reagent, because Nital does not reveal aluminium's structure well.

Nital's chemistry is built around reacting with the iron-carbon phases found in steel, so among the four metals listed, it is used on mild steel.

Let's summarize:

- Nital (nitric acid plus alcohol) is the standard etchant for carbon and low alloy steels.
- Copper alloys like bronze and brass need ferric chloride based etchants, and aluminium needs a fluoride based etchant like Keller's reagent.

So Nital is most commonly used for etching mild steel, option (C).

Quick Tip: Nital is a nitric acid and alcohol mixture; it is the go-to etchant for revealing ferrite and pearlite in carbon and low alloy steels, not for copper alloys or aluminium.

• • •

. Which one of the following options is correct?
In a face-centered cubic metal, Shockley partial is:

(A) Perfect and mobile dislocation

(B) Perfect and immobile dislocation

(C) Imperfect and immobile dislocation

(D) Imperfect and mobile dislocation

Correct Answer: (D) Imperfect and mobile dislocation

SOLUTION:

Step 1: Understanding the Concept.
A dislocation in FCC metals is called perfect when its Burgers vector connects two atoms that occupy equivalent positions in the lattice, and partial (imperfect) when it does not. Whether a dislocation can glide or must climb depends on whether its Burgers vector lies inside the plane it sits on.

Step 2: Key Approach.
Two kinds of partial dislocations show up in FCC crystals. The Shockley partial comes from splitting a perfect glide dislocation, and the Frank partial comes from inserting or taking out a partial plane of atoms. Comparing these two tells us which box the Shockley partial

fits into.

Step 3: Detailed Explanation.

A perfect FCC dislocation carries Burgers vector $\frac{a}{2}\langle 110\rangle$. Splitting it, for instance

$$\frac{a}{2}[0\bar{1}1] \rightarrow \frac{a}{6}[1\bar{2}1] + \frac{a}{6}[\bar{1}\bar{1}2],$$

gives two Shockley partials of type $\frac{a}{6}\langle 211\rangle$. Since $\frac{a}{6}\langle 211\rangle$ does not join two atoms sitting on equivalent lattice sites, each Shockley partial is a partial, or imperfect, dislocation, and a stacking fault ribbon sits between the pair.

Now check where this Burgers vector points. The $\{111\}$ plane on which the original dislocation glided has its normal along $\langle 111\rangle$. Taking the dot product of $\frac{a}{6}[1\bar{2}1]$ with $[111]$ gives $1 - 2 + 1 = 0$, so the vector lies flat inside the $\{111\}$ plane, with no out of plane component. A Burgers vector confined to its own glide plane means the dislocation can move by slip, so it is glissile, that is, mobile.

This is unlike the Frank partial, whose Burgers vector points along $\langle 111\rangle$, straight out of the fault plane. That out of plane vector cannot be swept through by shear, so a Frank partial only moves by climb (diffusion of atoms), making it sessile.

Step 4: Final Answer.

The Shockley partial is imperfect because its Burgers vector is a fraction of a lattice vector, and mobile because that vector lies inside the glide plane.

Imperfect and mobile dislocation

Quick Tip: Compare the Shockley partial's Burgers vector length to a full lattice translation vector, and check if it lies inside the glide plane.

• • •

. Deformation mechanism map of a given material is used for determining which one of the following properties?

(A) Fatigue strength

(B) Creep rate

(C) Tensile strength

(D) Impact toughness

Correct Answer: (B) Creep rate

SOLUTION:

A deformation mechanism map plots normalized stress against homologous temperature T/T_m, and splits that plane into zones, each one owned by a different way a material deforms plastically: dislocation glide at high stress, power law creep at intermediate stress and high temperature, and diffusion controlled creep (Nabarro-Herring, Coble) at low stress and high temperature. Let's look at what property this map is built to give us.

1. **Fatigue strength**: this number comes out of cyclic stress versus number-of-cycles-to-failure (S-N) testing, a completely separate experiment from the static stress-temperature mechanism map. Not what the map gives.
2. **Creep rate**: the map carries contour lines of constant strain rate $\dot{\varepsilon}$ drawn across the mechanism fields. Once the operating stress and temperature of a part are located on the map, both the active

mechanism and the corresponding steady state strain rate, that is, the creep rate, can be read off directly. This is the entire purpose the map was built for.

3. **Tensile strength**: this is obtained from a stress-strain curve in a standard tensile test performed at one fixed strain rate and temperature, not from a stress-temperature mechanism field.
4. **Impact toughness**: this comes from notched impact tests like the Charpy test, which measure energy absorbed under sudden dynamic loading. It has nothing to do with the steady deformation mechanisms plotted on this map.

The property this map is built to give directly is the creep rate, read off the strain-rate contour lines at the operating stress and temperature.

Let's summarize:

- The two axes of the map are normalized stress and homologous temperature.
- Contour lines on the map are lines of constant strain rate, which give the creep rate.
- Fatigue strength, tensile strength and impact toughness all come from separate, dedicated mechanical tests, not from this map.

So the deformation mechanism map is used to determine the creep rate.

Quick Tip: Think about what the contour lines drawn across the mechanism fields on the map represent.

. Which one of the following dislocation dissociation reactions is feasible in face-centered cubic metals?

(A) $\frac{a}{2}[0\bar{1}1] \rightarrow \frac{a}{6}[1\bar{2}1] + \frac{a}{6}[\bar{1}\bar{1}2]$

(B) $\frac{a}{2}[0\bar{1}1] \rightarrow \frac{a}{6}[112] + \frac{a}{6}[21\bar{1}]$

(C) $\frac{a}{2}[0\bar{1}1] \rightarrow \frac{a}{6}[1\bar{1}2] + \frac{a}{6}[\bar{1}\bar{2}\bar{1}]$

(D) $\frac{a}{2}[0\bar{1}1] \rightarrow \frac{a}{6}[1\bar{2}1] + \frac{a}{6}[2\bar{1}\bar{1}]$

Correct Answer: (A) $\frac{a}{2}[0\bar{1}1] \rightarrow \frac{a}{6}[1\bar{2}1] + \frac{a}{6}[\bar{1}\bar{1}2]$

SOLUTION:

Step 1: Understanding the Concept.

When a perfect dislocation in an FCC metal splits into two partials, the reaction has to obey two physical rules at the same time: the Burgers vectors must add up correctly (nothing can be created or destroyed at the split point), and the total elastic energy, which goes as b^2, must go down after the split, otherwise the crystal would never bother splitting the dislocation.

Step 2: Key Approach.

Instead of checking energy for all four options, it is faster to first check which option even adds up correctly as vectors. A reaction that fails vector addition is not physically possible regardless of energy, so it can be dropped right away. Only the surviving option needs the energy check.

Step 3: Detailed Explanation.

Write the target vector with a common denominator of 6: $\frac{a}{2}[0\bar{1}1] = \frac{a}{6}[0\,\bar{3}\,3]$. Now add up each option's two partials component by component.

Option (A): $[1\bar{2}1] + [\bar{1}\bar{1}2] = [0\,\bar{3}\,3]$. This matches the target exactly.

Option (B): $[112] + [21\bar{1}] = [3\,2\,1]$. This does not match $[0\,\bar{3}\,3]$.

Option (C): $[1\bar{1}2] + [\bar{1}\bar{2}\bar{1}] = [0\,\bar{3}\,1]$. The last entry, 1, should have been

3, so this fails too.
Option (D): $[1\bar{2}1] + [2\bar{1}\bar{1}] = [3\,\bar{3}\,0]$. This does not match $[0\,\bar{3}\,3]$ either. Only option (A) survives the vector addition check. As a final check, compare the squared magnitudes: the original vector gives $b_0^2 = \left(\frac{a}{2}\right)^2(2) = \frac{a^2}{2}$, while each partial in option (A) gives $\left(\frac{a}{6}\right)^2(6) = \frac{a^2}{6}$, so together $b_1^2 + b_2^2 = \frac{a^2}{3}$, which is smaller than $\frac{a^2}{2}$. The split is energetically favorable, confirming option (A) is the real Shockley partial reaction.

Step 4: Final Answer.
$\frac{a}{2}[0\bar{1}1] \rightarrow \frac{a}{6}[1\bar{2}1] + \frac{a}{6}[\bar{1}\bar{1}2]$ is the only reaction where the Burgers vectors add up correctly and the energy drops after the split.

$$\boxed{\frac{a}{2}[0\bar{1}1] \rightarrow \frac{a}{6}[1\bar{2}1] + \frac{a}{6}[\bar{1}\bar{1}2]}$$

Quick Tip: Check which option's two partial Burgers vectors add up, component by component, to the original vector before checking energy.

• • •

. Which one of the following options is correct?
For Al - 4% Ag alloy (composition in atom %), the correct sequence of operations involved in precipitation hardening is:

(A) Quenching → Aging → Solution treatment

(B) Solution treatment → Aging → Quenching

(C) Aging → Solution treatment → Quenching

(D) Solution treatment → Quenching → Aging

Correct Answer: (D) Solution treatment → Quenching → Aging

SOLUTION:

Precipitation hardening works by locking a supersaturated solid solution into the metal and then letting it throw out a fine, closely spaced set of second-phase particles that block dislocations from sliding through the lattice. Getting this sequence backwards makes the process meaningless, so let's check each option against what physically has to happen first.

1. **Quenching then Aging then Solution treatment**: quenching is meant to freeze in a solid solution that was just created at high temperature. Doing it first, before any solution treatment, leaves nothing dissolved to freeze in. This order is physically backwards.
2. **Solution treatment then Aging then Quenching**: after solution treatment the Ag atoms are fully dissolved in the hot Al matrix. If aging (a slow, controlled hold) happens next instead of a fast quench, the Ag begins precipitating immediately during the slow cool, forming coarse, widely spaced particles instead of the fine dispersion needed for strengthening. Quenching afterward achieves nothing.
3. **Aging then Solution treatment then Quenching**: aging needs a supersaturated solid solution to draw precipitates from. Before solution treatment, that supersaturated state does not exist, so there is nothing for aging to act on.
4. **Solution treatment then Quenching then Aging**: solution treatment dissolves the Ag fully into the Al matrix at high temperature. The fast quench then traps this dissolved Ag in a supersaturated solid solution at room temperature, since there is no time for it to come back out. Aging finally gives the trapped Ag atoms just enough mobility to form the fine precipitates that

block dislocation motion and raise strength. Every step has exactly what it needs from the step before it.

Only the fourth order lines up with the physics of each stage: dissolve first, freeze the dissolved state in second, then let fine particles form last.

Let's summarize:

- Solution treatment dissolves the alloying element into a single-phase solid solution.
- Quenching freezes that solution in a supersaturated, metastable state.
- Aging lets fine, hardening precipitates form out of the supersaturation.

So the correct sequence for the Al - 4% Ag alloy is solution treatment, then quenching, then aging.

Quick Tip: Ask what each stage needs to already exist before it can happen: aging needs supersaturation, and supersaturation needs a fast quench right after dissolving the solute.

• • •

. Which one of the following is NOT a state function?

(A) Enthalpy

(B) Entropy

(C) Work

(D) Internal Energy

Correct Answer: (C) Work

SOLUTION:

Step 1: Understanding the Concept.

A quantity in thermodynamics falls into one of two boxes. A state function only cares where the system is right now, so it is the same number no matter how the system got there. A path function cares about the journey, so it can come out different for two processes that start and end at the same two states.

Step 2: Key Approach.

The fastest way to sort the four options is to write down how each one is calculated, and check whether that calculation needs "path" information (like the exact shape of a P-V curve) or only "state" information (like T, P, V at the endpoints).

Step 3: Detailed Explanation.

Enthalpy is $H = U + PV$. Every symbol on the right, U, P, V, is fixed once the state is fixed, so H is fixed too. State function.

Entropy change between two states works out to the same number along a reversible path or an irreversible one, precisely because S was built to only depend on the state. State function.

Internal energy U, for a simple system, depends only on state variables like temperature. Even though heat Q and work W separately change with the path, the first law $\Delta U = Q - W$ keeps their difference fixed for any path between two given states. State function.

Work, though, is literally defined as an area under a P-V curve, $W = \int P\,dV$. Draw two different curves between the same start and end point on a P-V diagram, one going through high pressure and one through low pressure, and the areas underneath, that is, the work, come out different. So work depends on the path, not just the state.

Step 4: Final Answer.

Enthalpy, entropy and internal energy are all fixed by the state alone, but work changes with the path taken between two states.

Work

Quick Tip: Ask whether the quantity can be written purely from state variables like T, P, V, or whether it needs the exact process path.

• • •

. For a regular solution, which one of the following statements is correct? (H_{mix} is the enthalpy of mixing and S_{mix} is the entropy of mixing)

(A) Both H_{mix} and S_{mix} are finite

(B) H_{mix} is zero and S_{mix} is finite

(C) H_{mix} is finite and S_{mix} is zero

(D) Both H_{mix} and S_{mix} are zero

Correct Answer: (A) Both H_{mix} and S_{mix} are finite

SOLUTION:

Concept: What makes a solution "regular".

A solution is built from two ideas: how the atoms are arranged (entropy) and how much energy is released or absorbed when they mix (enthalpy). An ideal solution assumes the atoms don't care about their neighbours at all, so mixing costs no extra energy, $H_{mix} = 0$, but arranging two kinds of atoms randomly still increases disorder, so S_{mix} is a definite, nonzero number given by $S_{mix} = -R(x_1 \ln x_1 + x_2 \ln x_2)$.

Step 1: Add the regular solution twist.

A regular solution keeps the random-arrangement assumption of the ideal solution, so its entropy term is calculated the exact same way and stays finite. But it drops the "atoms don't care about neighbours" assumption: unlike atom pairs now have their own bond energy, different from the like-atom pairs. That difference in bond energy shows up as a real, nonzero heat of mixing, $H_{mix} = \Omega x_1 x_2$, with $\Omega \neq 0$.

Step 2: Check the extreme cases against the options.

If $H_{mix} = 0$ the solution would be ideal, not regular, so an option pairing $H_{mix} = 0$ with a finite S_{mix} is really describing the ideal case.

If $S_{mix} = 0$ the atoms would have to be fully ordered rather than randomly mixed, which contradicts the very assumption a regular solution is built on.

If both terms were zero, the two components would not really be mixing at all.

None of these three situations is what "regular solution" means.

Step 3: State the correct pairing.

A regular solution has a finite, nonzero enthalpy of mixing from the changed bond energies, and a finite entropy of mixing from the random arrangement of atoms. Both quantities are finite at the same time.

$$\boxed{\text{Both } H_{mix} \text{ and } S_{mix} \text{ are finite}}$$

Quick Tip: A regular solution mixes atoms randomly like an ideal solution (finite entropy) but has a nonzero interaction energy between unlike atoms (finite enthalpy).

. Which one of the following options is correct?
In a binary alloy system, during spinodal decomposition, uphill diffusion occurs:

(A) From higher to lower concentration and from higher to lower chemical potential

(B) From higher to lower concentration and from lower to higher chemical potential

(C) From lower to higher concentration and from lower to higher chemical potential

(D) From lower to higher concentration and from higher to lower chemical potential

Correct Answer: (D) From lower to higher concentration and from higher to lower chemical potential

SOLUTION:

Concept: Diffusion really follows chemical potential, not concentration.
The textbook picture "atoms move from crowded areas to empty areas" is only true because, in a normal stable solution, chemical potential μ rises together with concentration c. The real rule, valid everywhere, is that atoms move to cut down the free energy of the whole system, so flux always runs from high μ to low μ.

Step 1: Look at the shape of the free energy curve.
Plot the molar free energy G of the alloy against composition. Outside the spinodal region this curve is concave up, $\frac{\partial^2 G}{\partial c^2} > 0$, so μ and c move together. Inside the spinodal region the curve bends the other way, $\frac{\partial^2 G}{\partial c^2} < 0$.

Step 2: See what the sign flip does to μ.
Because μ is basically the slope of the G curve, a negative curvature means μ now falls as c rises. A solute rich pocket has a LOWER chemical potential than the surrounding solute poor material.

Step 3: Apply the flux rule.
Atoms in the solute poor region sit at a higher μ than atoms already in the solute rich pocket. Since flux runs from high μ to low μ, atoms move into the already solute rich pocket. Concentration there gets even higher, while the surrounding region gets even more depleted. This is exactly uphill diffusion in terms of concentration, even though it is completely normal downhill diffusion in terms of chemical potential.

Step 4: Match this against the four options.
The concentration part of the flux is from lower to higher (uphill), which rules out both options that say "higher to lower concentration". Between the two remaining choices, the chemical potential part must still be from higher to lower, since that never changes; the option pairing "lower to higher concentration" with "lower to higher chemical potential" gets the second half backwards.

Step 5: Conclude.
Spinodal decomposition drives atoms from lower to higher concentration, powered by a fall from higher to lower chemical potential.

From lower to higher concentration and from higher to lower chemical potential

Quick Tip: Diffusion always runs from high to low chemical potential; inside the spinodal region, chemical potential falls as concentration rises, so atoms move to higher concentration.

• • •

. Which one of the following options is correct?
In Al - 4 wt.% Cu alloy during isothermal aging, the correct precipitation sequence is:

(A) $\theta'' \rightarrow \theta' \rightarrow \theta \rightarrow$ GP zone

(B) GP zone $\rightarrow \theta \rightarrow \theta' \rightarrow \theta''$

(C) $\theta \rightarrow \theta' \rightarrow \theta'' \rightarrow$ GP zone

(D) GP zone $\rightarrow \theta'' \rightarrow \theta' \rightarrow \theta$

Correct Answer: (D) GP zone $\rightarrow \theta'' \rightarrow \theta' \rightarrow \theta$

SOLUTION:

Concept: Precipitation always takes the path of least resistance first.
When a supersaturated Al-Cu alloy is aged, it does not form the stable, low energy θ phase ($CuAl_2$) right away, because that phase is incoherent with the aluminium matrix and costly to nucleate. Instead the system builds up to it through a chain of intermediate structures, each one a little closer to θ but each one easier to form than jumping there directly.

Step 1: Start with the easiest structure.
The very first thing to form is a GP zone: just a thin, fully coherent, copper rich cluster on a {100} plane of the matrix. It needs almost no interface energy to form, so it appears fastest even though it is far

from the stable phase.

Step 2: Track how coherency is slowly given up.
As the copper rich region grows, keeping it perfectly coherent with the surrounding lattice becomes harder. The structure reorganizes into θ'', still fully coherent but more ordered than a GP zone. With more aging it becomes θ', which is only semi-coherent, larger, and carries its own distinct tetragonal structure. Finally the strain becomes too much to keep any coherency at all, and the precipitate turns into the stable, incoherent phase θ.

Step 3: Write the chain in order.

$$\text{GP zone} \rightarrow \theta'' \rightarrow \theta' \rightarrow \theta$$

Coherency strain energy drops step by step while the precipitate gets closer and closer to the true equilibrium phase.

Step 4: Rule out the other three options by their starting or ending point.
Any sequence that starts from θ or from θ'' instead of the GP zone is going in the wrong direction, since the GP zone is always the first thing to nucleate, not the last. Any sequence that reaches θ before passing through both θ'' and θ' skips real intermediate stages that are always observed experimentally. Only one option starts at the GP zone and ends at θ while keeping θ'' ahead of θ'.

Step 5: Conclude.

$$\boxed{\text{GP zone} \rightarrow \theta'' \rightarrow \theta' \rightarrow \theta}$$

Quick Tip: Precipitation in Al-Cu alloys always begins with the easiest, fully coherent structure (GP zone) and ends with the stable, incoherent θ phase, passing through θ'' then θ'.

• • •

. Which one of the following options is correct?
After cold-working of a metallic specimen, during the recovery stage, its electrical conductivity:

(A) Always increases

(B) Always decreases

(C) Can increase or decrease

(D) Remains unaffected

Correct Answer: (A) Always increases

SOLUTION:

Concept: Electrical conductivity is a defect counter.
Free electrons in a metal get scattered by anything that breaks the perfect periodicity of the lattice: phonons (thermal vibration), dislocations, vacancies, interstitials, and grain boundaries. More scattering centres mean a shorter mean free path for electrons, which means lower conductivity. Cold working is basically a defect-injection process, so it always drags conductivity down from its annealed value.

Step 1: List what recovery actually removes.
Recovery happens at a low enough temperature that grains do not yet form new strain-free regions (that is recrystallization's job) and grain size does not yet coarsen (that is grain growth's job). What recovery does instead is let vacancies and interstitials, created in huge excess

by the cold work, diffuse and annihilate, and let tangled dislocations rearrange into neater, lower-energy walls.

Step 2: Connect this directly to scattering.
Point defects such as vacancies are strong electron scatterers per defect, even stronger than dislocations in many metals. Removing a large fraction of them during recovery removes a large fraction of the extra scattering that cold work had added.

Step 3: Decide the direction, not just the size, of the change.
Since recovery is purely a defect-removal process (it never adds new defects), the scattering centre count can only go down during this stage, for any metal and any starting amount of cold work. A count that can only go down means resistivity can only go down, and conductivity can only go up. There is no scenario in normal recovery where conductivity falls or where it depends on the particular metal.

Step 4: Compare with the four options.
"Always decreases" and "remains unaffected" both contradict the fact that defect density is dropping. "Can increase or decrease" wrongly suggests the direction is uncertain, when in fact recovery is one-directional in what it removes. Only "always increases" matches a process that strictly removes scattering centres.

Step 5: Conclude.

Always increases

Quick Tip: Recovery removes excess point defects and lowers dislocation strain without changing grain structure, so electron scattering drops and

conductivity rises.

• • •

. Which one of the following options is correct?
Red mud is generated in the production of:

(A) Aluminium

(B) Iron

(C) Titanium

(D) Copper

Correct Answer: (A) Aluminium

SOLUTION:

Concept: Match the waste name to its source ore.
Different metal extraction routes leave behind their own signature waste. Iron making leaves slag, copper smelting leaves slag and sulfur dioxide, titanium extraction leaves magnesium chloride, and one particular route leaves a reddish sludge called red mud. Spotting which ore-processing route makes red mud pins down the metal.

Step 1: Trace aluminium's extraction route.
Aluminium does not come from a simple furnace reduction of its ore. Its ore, bauxite, is first purified into pure alumina (Al_2O_3) using the Bayer process, and only then is that alumina reduced to metal by electrolysis in the Hall-Heroult cell.

Step 2: Zoom into the Bayer step.
Bauxite is treated with hot, strong sodium hydroxide. The useful aluminium compound dissolves into solution as sodium aluminate, but everything else in the ore, mostly iron oxide along with some silica

and titania, refuses to dissolve. This leftover solid, coloured red by the iron oxide, is filtered out as red mud.

Step 3: Check that no other metal's route matches.
Iron comes from smelting iron ore in a blast furnace with coke and limestone, giving molten iron and a silicate slag, no red mud involved. Titanium comes from the Kroll process on ilmenite/rutile, giving magnesium chloride as the byproduct. Copper comes mainly from smelting sulfide ores, giving slag and sulfur dioxide gas. None of these three processes produce anything called red mud.

Step 4: Conclude.
Red mud is uniquely tied to the Bayer process step of aluminium production.

Aluminium

Quick Tip: Red mud is the iron-rich, insoluble residue left over when bauxite is digested in sodium hydroxide during the Bayer process.

• • •

. Residual stress present in a material can be determined by which one of the following techniques:

(A) X-ray Diffraction (XRD)

(B) Tensile Testing

(C) Thermo-Gravimetric Analysis (TGA)

(D) Optical Microscopy

Correct Answer: (A) X-ray Diffraction (XRD)

SOLUTION:

This question asks which technique can actually sense the stress trapped inside a material when there is no external load on it. Residual stress measurement methods fall into two broad groups: destructive methods like hole drilling or layer removal, and non-destructive methods like X-ray diffraction, neutron diffraction, ultrasonic and magnetic techniques. Let's go through the four choices given.

1. **X-ray Diffraction (XRD)**: X-rays reflect off the lattice planes of a crystal according to $n\lambda = 2d\sin\theta$. A locked-in elastic strain changes the plane spacing d by a tiny amount, so the diffraction angle shifts. Recording this shift at a few different sample tilts lets the strain, and then the stress, be worked out using the elastic modulus and Poisson's ratio of the material. This is a direct residual stress measurement method.
2. **Tensile Testing**: This pulls a specimen apart under a newly applied load to find its strength and ductility. It measures the response to that new load, not the stress that was already sitting in the specimen, and it breaks the specimen in the process.
3. **Thermo-Gravimetric Analysis (TGA)**: This heats a sample and records how its weight changes, which is useful for studying oxidation or decomposition. Nothing in that process senses internal elastic strain.
4. **Optical Microscopy**: This shows grains, phases and surface defects under magnification. Very high residual stress can eventually crack a part, and a microscope can see that crack, but the microscope is reading the resulting damage, not the stress value that caused it.

Only X-ray diffraction directly reads the elastic lattice strain that residual stress produces inside the material, so it is the correct technique.

Let's summarize:

- Residual stress cannot be read directly, so it must be inferred from an effect it produces, which is a change in interplanar spacing.
- XRD picks up this change through Bragg's law and the $\sin^2\psi$ method, while tensile testing, TGA and optical microscopy do not measure stress this way.

The correct technique for determining residual stress in a material is X-ray diffraction (XRD).

Quick Tip: Residual stress changes the spacing between crystal planes by a small amount, and this shows up as a shift in the X-ray diffraction peak position.

. Which one of the following options is correct?

In a convective heat transfer for laminar flow over a flat plate, Nusselt number is a function of Reynolds number and Prandtl number. Similarly, in a convective mass transfer for laminar flow over a flat plate, Sherwood number is a function of:

(A) Schmidt number and Reynolds number

(B) Weber number and Reynolds number

(C) Schmidt number and Weber number

(D) Weber number and Prandtl number

Correct Answer: (A) Schmidt number and Reynolds number

SOLUTION:

This question is really testing the heat and mass transfer analogy for flow over a flat plate. Instead of deriving anything from scratch, it helps to line up the two problems side by side and match each quantity to its partner.

1. **Heat transfer side (given)**: For laminar boundary layer flow over a flat plate, the classic result (from the Blasius solution) is $Nu = 0.332\,Re^{0.5}Pr^{1/3}$, so $Nu = f(Re, Pr)$. Here Re describes the flow itself and $Pr = \nu/\alpha$ compares how fast momentum spreads to how fast heat spreads.
2. **Mass transfer side (to find)**: Mass transfer in a laminar boundary layer follows the same type of equation as heat transfer, only the diffusing quantity is a chemical species instead of heat. The matching result is $Sh = 0.332\,Re^{0.5}Sc^{1/3}$, so $Sh = f(Re, Sc)$, where $Sc = \nu/D_{AB}$ compares momentum spreading to species spreading, playing exactly the role Pr played for heat.
3. **Weber number**: We compares inertia to surface tension and only becomes relevant when there is a free surface, a droplet, or a bubble in the flow. A flat plate with a boundary layer growing over it has no such interface, so We does not belong in this correlation at all.

Since the Reynolds number carries over unchanged (it only describes the flow field) and the Schmidt number is the direct mass transfer twin of the Prandtl number, the Sherwood number must be a function of the Schmidt number and the Reynolds number.

Let's summarize:

- Re stays the same in both the heat and mass transfer versions because it only fixes the flow.
- $Pr \rightarrow Sc$ is the swap that turns the heat transfer correlation into the mass transfer correlation; We never enters a plain boundary layer problem with no interface.

So the correct pairing is Schmidt number and Reynolds number.

$$\boxed{Sh = f(Re, Sc)}$$

Quick Tip: Swap the Nusselt number for the Sherwood number and the Prandtl number for its mass transfer counterpart, the Schmidt number.

• • •

. Which one of the following options is correct?

For a solid immersed in a fluid, the convective heat transfer coefficient across the solid-fluid interface is NOT dependent on:

(A) Solid-fluid interfacial area

(B) Thermal conductivity of solid

(C) Roughness of solid surface

(D) Viscosity of fluid

Correct Answer: (B) Thermal conductivity of solid

SOLUTION:

The trick in this question is remembering that the convective heat transfer coefficient h belongs to the fluid boundary layer, not to the solid it is sitting next to. Let's test each option against that idea.

1. **Solid-fluid interfacial area**: The shape and size of the surface set the characteristic length L that goes into the Reynolds number, $Re = \rho VL/\mu$. A bigger or differently shaped surface changes Re, which changes h through the correlation $Nu = f(Re, Pr)$. So area does affect h.
2. **Thermal conductivity of solid**: Look at the correlation used to find h, namely $Nu = hL/k_{fluid} = f(Re, Pr)$. Every term in it, Re, Pr, and k_{fluid}, belongs to the fluid. The solid's own conductivity never enters this formula. What the solid's conductivity does control is how fast heat moves once it is already inside the solid, which is a separate, internal conduction problem, tied together with h only through the Biot number $Bi = hL/k_{solid}$.
3. **Roughness of solid surface**: A rough surface disturbs the fluid layer close to the wall and can push the flow into turbulence earlier than a smooth surface would, and turbulent flow carries heat away faster. So roughness changes h.
4. **Viscosity of fluid**: Viscosity sits directly inside Re, so it controls how thick the boundary layer is and how much resistance the fluid offers. It clearly affects h.

Three of the four options change the flow, the boundary layer, or the geometry the correlation depends on, so they all influence h. Only the solid's thermal conductivity plays no role in the correlation that defines h, since it governs conduction inside the solid rather than convection at its surface.

Let's summarize:

- h comes from a fluid-side correlation $Nu = f(Re, Pr)$, so fluid viscosity, surface area/geometry, and surface roughness all matter.
- The solid's own thermal conductivity is a conduction property, tied to h only through the Biot number, and does not set the value of h.

So the convective heat transfer coefficient is not dependent on the thermal conductivity of the solid.

Quick Tip: The convective coefficient h comes from a fluid-side correlation like Nu = f(Re, Pr); the solid's conductivity only enters conduction inside the solid, through the Biot number.

• • •

. Ergun equation is NOT applied in which one of the following unit operations to analyze gas flow behavior:

(A) Blast Furnace

(B) Sintering

(C) Roasting

(D) LD Converter

Correct Answer: (D) LD Converter

SOLUTION:

The Ergun equation is a packed bed formula. It needs a bed of stationary solid particles with a certain voidage ε and particle size d_p for gas to push through. So the fastest way to answer this question is

to check, for each unit operation, whether gas is actually moving through such a particle bed.

1. **Blast Furnace**: Coke, ore and flux are charged from the top and sit as a tall packed column inside the furnace. Hot blast air is blown in from the bottom and travels up through this solid column. This is a textbook packed bed, so the Ergun equation is used here.
2. **Sintering**: A layer of ore fines, coke breeze and flux is spread on a moving strand, and air is pulled down through this layer to burn the fuel and sinter the mix. Again, gas is passing through a bed of particles, so the Ergun equation applies.
3. **Roasting**: Roasting reactors, including packed and fluidized bed types, pass a gas stream through a bed of solid ore particles to drive the roasting reactions. This is once more a particulate bed situation, so the Ergun equation applies.
4. **LD Converter**: Here the setup is completely different. A jet of pure oxygen is blown at high speed through a lance straight into a bath of liquid steel. There is no stationary bed of solid particles anywhere in this picture, the gas jet is interacting with a liquid, not threading through packed solids. Since the equation needs a voidage and a particle size to even be written down, it simply does not apply to an LD converter.

Three of the four operations, the blast furnace, sintering, and roasting, are all packed bed gas-solid systems where the Ergun equation is the natural tool. The LD converter is a gas-liquid jet system, so it falls outside where the equation can be used at all.

Let's summarize:

- Ergun's equation needs a packed bed of solid particles: bed voidage and particle size must exist for the formula to make sense.
- Blast furnace, sintering and roasting all have gas flowing through such particle beds; the LD converter has an oxygen jet blown into liquid metal instead.

So the Ergun equation is not applied in the LD converter.

Quick Tip: The Ergun equation only applies to gas flow through a packed bed of solid particles; the LD converter has a gas jet blown into liquid metal, not a particle bed.

• • •

. Choose the correct option(s).

Here A is the Helmholtz free energy and G is the Gibbs free energy.

(A) A provides a criterion for equilibrium in a system with constant temperature and constant pressure

(B) G provides a criterion for equilibrium in a system with constant temperature and constant pressure

(C) A provides a criterion for equilibrium in a system with constant temperature and constant volume

(D) G provides a criterion for equilibrium in a system with constant temperature and constant volume

Correct Answer: (B) G provides a criterion for equilibrium in a system with constant temperature and constant pressure ; (C) A provides a criterion for equilibrium in a system with constant temperature and constant volume

SOLUTION:

This question checks whether you know which free energy function goes with which pair of fixed conditions. Rather than memorizing the pairing, let's build it from the combined statement of the first and second laws, $dU = TdS - PdV$, and see which terms survive under each set of constraints.

1. **Option (A), A at constant T and P**: Starting from $A = U - TS$, $dA = -SdT - PdV$. This expression still has a PdV term sitting in it. Holding T constant kills $-SdT$, but volume is free to change when only T and P are fixed, so $-PdV$ does not vanish. A is not guaranteed to sit at a minimum here, so this option is wrong.
2. **Option (B), G at constant T and P**: Starting from $G = U + PV - TS$, working through the algebra gives $dG = VdP - SdT$. Now holding both T and P fixed kills both terms on the right, leaving $(dG)_{T,P} \leq 0$ for any real process. G drops to a minimum and sits there at equilibrium, so this is a valid criterion, and the option is correct.
3. **Option (C), A at constant T and V**: Going back to $dA = -SdT - PdV$, this time both T and V are held fixed, so both $-SdT$ and $-PdV$ vanish, leaving $(dA)_{T,V} \leq 0$ for any real process. A drops to a minimum and stays there at equilibrium, so this is also a valid criterion, and the option is correct.
4. **Option (D), G at constant T and V**: From $dG = VdP - SdT$, holding T fixed kills $-SdT$, but pressure is not held fixed here, only volume is, so VdP does not vanish in general. G is not guaranteed to be at a minimum under these conditions, so this option is wrong.

The pattern makes sense once you notice which natural variables sit inside each differential. dA naturally wants T and V fixed to vanish completely, while dG naturally wants T and P fixed to vanish completely. Trying to force A to work at constant P, or G to work at constant V, leaves a leftover term that spoils the criterion.

Let's summarize:

- $dA = -SdT - PdV$ collapses to $(dA)_{T,V} \leq 0$ only when T and V are both fixed.
- $dG = VdP - SdT$ collapses to $(dG)_{T,P} \leq 0$ only when T and P are both fixed.

So the correct statements are that G is the equilibrium criterion at constant T and P, and A is the equilibrium criterion at constant T and V.

Options B and C

Quick Tip: Helmholtz free energy A is minimized at equilibrium under constant T and V; Gibbs free energy G is minimized at equilibrium under constant T and P.

• • •

. Which of the following elements, when present in iron, enhances its corrosion resistance?

(A) H

(B) Cr

(C) S

(D) C

Correct Answer: (B) Cr

SOLUTION:

Step 1: Think in terms of familiar alloy families.
Instead of the oxide film mechanism, use what each element is known for in real alloys. Stainless steel is the everyday example of iron plus one element giving a corrosion resistant metal.

Step 2: Name the element in stainless steel.
Stainless steel is iron with a large addition of chromium, typically above 10.5 percent, sometimes with nickel added too. The word stainless itself refers to resistance to rusting and staining, so chromium is the element linked with corrosion resistance.

Step 3: Recall what the other three elements are known for instead.
Hydrogen in steel is known for causing embrittlement and cracking, a mechanical weakness, not a corrosion benefit. Sulfur is known as a harmful impurity kept low in good quality steel because it causes hot shortness. Carbon is known for controlling hardness and strength through carbide formation, and too much carbide at grain boundaries can even make certain steels more prone to attack.

Step 4: Match to the options.
None of H, S or C is associated with rust protection, while Cr is the defining element of corrosion resistant iron alloys.

Step 5: Conclude.
The element that enhances the corrosion resistance of iron is chromium.

Cr

Quick Tip: Look for the element that forms a thin, self healing passive oxide film on the metal surface, the same mechanism used in stainless steel.

• • •

. Value of the scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ is (answer in integer) _____.

$$\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$$

$$\vec{c} = 3\hat{i} + 4\hat{j} - \hat{k}$$

Here, $\hat{i}$, $\hat{j}$ and $\hat{k}$ are mutually orthogonal unit vectors.

Correct Answer: 36

SOLUTION:

Step 1: Use the direct determinant shortcut.
Rather than computing $\vec{b} \times \vec{c}$ first and then dotting with $\vec{a}$, the scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ equals a single 3 by 3 determinant with the three vectors placed as rows in order.

Step 2: Set up the determinant.

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -3 \\ 3 & 4 & -1 \end{vmatrix}$$

Step 3: Expand along the first row.

$$= 2\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} - (-3)\begin{vmatrix} 1 & -3 \\ 3 & -1 \end{vmatrix} + 4\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

Step 4: Work out each 2 by 2 minor.

$$\begin{vmatrix} 2 & -3 \\ 4 & -1 \end{vmatrix} = (2)(-1) - (-3)(4) = -2 + 12 = 10$$

$$\begin{vmatrix} 1 & -3 \\ 3 & -1 \end{vmatrix} = (1)(-1) - (-3)(3) = -1 + 9 = 8$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1)(4) - (2)(3) = 4 - 6 = -2$$

Step 5: Combine the terms.

$$= 2(10) + 3(8) + 4(-2) = 20 + 24 - 8 = 36$$

Step 6: Conclude.

Both routes must agree, since the triple product is a single fixed number for given vectors.

$$\boxed{36}$$

Quick Tip: First find b cross c using the determinant method, then take its

dot product with a.

• • •

. A box contains 20 thermometers, 3 of which are defective. One person randomly draws 2 thermometers from the box, one by one, without replacement. The probability in percent (rounded off to two decimal places) that none of these TWO thermometers is defective, is _____ %.

Correct Answer: 71.00 to 72.00

SOLUTION:

Step 1: Switch to a combinations view.
Instead of multiplying draw by draw probabilities, count the ways to pick 2 thermometers out of 20 and the ways to pick 2 good ones out of the 17 good thermometers. Order of drawing does not matter for this count.

Step 2: Count the total ways to choose 2 out of 20.

$$\binom{20}{2} = \frac{20 \times 19}{2} = 190$$

Step 3: Count the favourable ways, both from the 17 good thermometers.

$$\binom{17}{2} = \frac{17 \times 16}{2} = 136$$

Step 4: Form the probability as favourable over total.

$$P = \frac{\binom{17}{2}}{\binom{20}{2}} = \frac{136}{190} = \frac{68}{95}$$

Step 5: Convert to a percentage.

$$\frac{68}{95} \times 100 \approx 71.58\%$$

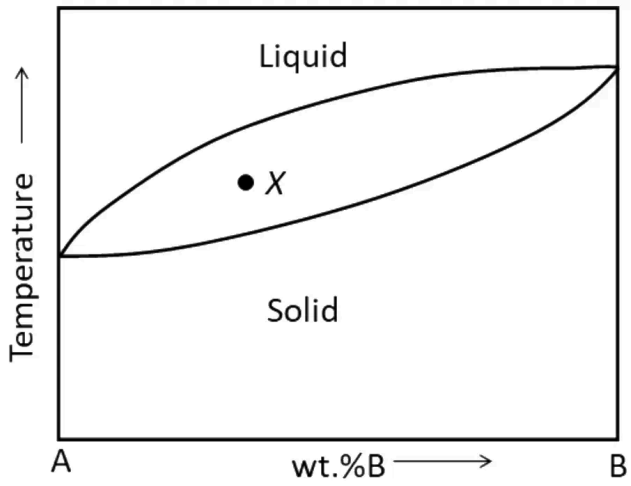
This matches the sequential draw by draw calculation exactly, since choosing an unordered pair or drawing one after another without replacement give the same probability.

Step 6: Conclude.

$$\boxed{71.58\%}$$

Quick Tip: Use the multiplication rule for dependent events: P(1st good) times P(2nd good given 1st good), then convert to a percentage.

. For an equilibrium phase diagram of a binary A-B alloy at a constant pressure as shown in the figure, the degree of freedom at 'X' is (answer in integer) _____.



The figure shows a binary A-B alloy phase diagram with composition (wt.%B) on the x-axis and temperature on the y-axis. A liquidus curve runs from pure A down to pure B and a solidus curve runs below it; the region above the liquidus is labelled Liquid and the region below the solidus is labelled Solid. Point X lies inside the lens shaped region between the liquidus and the solidus curves, that is, in the two-phase (liquid + solid) region.

Correct Answer: 1

SOLUTION:

Step 1: Think about which variables you are still free to choose at point X.

Instead of jumping straight to the phase rule formula, work out from physical reasoning how many independent variables can still be changed at X while both liquid and solid stay present.

Step 2: Pressure is already fixed.
The diagram is drawn at one constant pressure, so pressure is not a variable here at all, it is already used up.

Step 3: Look at what happens once you pick a temperature.
Once a temperature inside the two phase field is chosen, the liquidus curve fixes the composition of the liquid phase at that temperature, and the solidus curve fixes the composition of the solid phase at that same temperature. These are the two ends of the tie line through X. So the compositions of both phases are not free choices, they follow automatically once temperature is picked.

Step 4: Count the truly free choices.
The only quantity that can still be picked independently at X, given constant pressure and both phases present, is the temperature itself. Everything else, both phase compositions, follows from the tie line.

Step 5: Match this count to the phase rule.
This physical picture agrees with $F = C - P + 1 = 2 - 2 + 1 = 1$: exactly one variable, temperature, is free.

Step 6: Conclude.

$$\boxed{1}$$

Quick Tip: Use the condensed Gibbs phase rule F = C - P + 1 for constant pressure, with C = 2 components and P = 2 phases in the two phase region.

. Two parallel plates, with Newtonian incompressible liquid in between, are 2 mm apart. The upper plate is stationary and the lower plate moves with a velocity of 4 m/s. A force per unit area of 5 N/m^2 is applied parallel to the lower plate to maintain its motion. The viscosity of the liquid (rounded off to two decimal places) is _____ $\times 10^{-3}$ $N.s/m^2$.

Correct Answer: 2.40 to 2.60

SOLUTION:

Step 1: Write viscosity directly as force times gap over area times velocity.

Rearrange Newton's law of viscosity $\tau = \mu \frac{v}{h}$ to solve for μ in one line:

$$\mu = \frac{\tau h}{v}$$

Step 2: Plug in the numbers, keeping the gap in millimetres converted to metres.

Here $\tau = 5\ N/m^2$, $h = 2\ mm = 2 \times 10^{-3}\ m$, and $v = 4\ m/s$.

$$\mu = \frac{5 \times (2 \times 10^{-3})}{4}$$

Step 3: Work out the numerator first.

$$5 \times 2 \times 10^{-3} = 10 \times 10^{-3} = 1.0 \times 10^{-2}$$

Step 4: Divide by the velocity.

$$\mu = \frac{1.0 \times 10^{-2}}{4} = 0.25 \times 10^{-2} = 2.5 \times 10^{-3}\ \text{N.s/m}^2$$

Step 5: Read off the coefficient asked for.
Since the question wants the answer as a multiple of 10^{-3} N.s/m^2, the required number is just the coefficient.

Step 6: Conclude.

$$\boxed{2.50}$$

Quick Tip: Use tau = mu times (v/h) for simple Couette flow between the plates, and remember to convert 2 mm to metres.

• • •

. Match the crystal systems in Column I with the corresponding axial lengths a, b, c and interaxial angles α, β, γ provided in Column II.

Column I	Column II
(P) Tetragonal	(1) $a \neq b \neq c,\ \alpha = \beta = \gamma = 90^\circ$
(Q) Rhombohedral	(2) $a = b \neq c,\ \alpha = \beta = \gamma = 90^\circ$
(R) Orthorhombic	(3) $a \neq b \neq c,\ \alpha = \gamma = 90^\circ \neq \beta$
(S) Monoclinic	(4) $a = b = c,\ \alpha = \beta = \gamma \neq 90^\circ$

(A) P-3, Q-1, R-2, S-4

(B) P-2, Q-3, R-4, S-1

(C) P-4, Q-3, R-2, S-1

(D) P-2, Q-4, R-1, S-3

Correct Answer: (D) P-2, Q-4, R-1, S-3

SOLUTION:

Each crystal system is fixed by how many of its three axial lengths are equal and how many of its angles sit at 90 degrees. Instead of deriving the match name by name, check each of the four given combinations against what we know of tetragonal, rhombohedral, orthorhombic and monoclinic cells.

1. **(A) P-3, Q-1, R-2, S-4**: puts tetragonal at (3), which reads $a \neq b \neq c$ with one angle off 90 degrees. That description belongs to monoclinic, not tetragonal. Wrong.
2. **(B) P-2, Q-3, R-4, S-1**: tetragonal at (2) is right, but rhombohedral is placed at (3), which is the monoclinic-type description, not the all-axes-equal description. Wrong.
3. **(C) P-4, Q-3, R-2, S-1**: tetragonal at (4) means all three axes equal, which is rhombohedral's property, not tetragonal's. Wrong.
4. **(D) P-2, Q-4, R-1, S-3**: tetragonal at (2) ($a = b \neq c$, all angles 90), rhombohedral at (4) ($a = b = c$, angles equal but not 90), orthorhombic at (1) ($a \neq b \neq c$, all angles 90) and monoclinic at (3) ($a \neq b \neq c$, one angle off 90). Every pairing fits the standard definition. Correct.

The self-consistent set is P-2, Q-4, R-1, S-3, matching option (D).

Let's summarize:

- Tetragonal: two equal axes, all angles 90 degrees.
- Rhombohedral: three equal axes, angles equal but not 90 degrees.
- Orthorhombic: three unequal axes, all angles 90 degrees.
- Monoclinic: three unequal axes, one angle away from 90 degrees.

So the correct match is P-2, Q-4, R-1, S-3, option (D).

Quick Tip: Compare the number of equal axes and the number of 90 degree angles each crystal system has with the Column II descriptions.

• • •

. Match the concepts listed in Column I with the associated terms listed in Column II.

Column I	Column II
(P) Paris Law	(1) Creep
(Q) Schmid Factor	(2) Fatigue
(R) Larson-Miller Parameter	(3) Dynamic Strain Aging
(S) Portevin-Le Chatelier Effect	(4) Critical Resolved Shear Stress

(A) P-3, Q-1, R-2, S-4

(B) P-2, Q-4, R-1, S-3

(C) P-2, Q-4, R-3, S-1

(D) P-1, Q-3, R-2, S-4

Correct Answer: (B) P-2, Q-4, R-1, S-3

SOLUTION:

Each term in Column I belongs to one of three areas of mechanical behavior: fatigue, plastic deformation by slip, or creep. Column II lists one word for each area plus a fourth term, Critical Resolved Shear Stress, naming the slip threshold itself. Check each option against this grouping.

1. **(A) P-3, Q-1, R-2, S-4**: puts Paris Law under Dynamic Strain Aging, but Paris Law is a fatigue crack growth law, not a strain aging effect. Wrong.
2. **(B) P-2, Q-4, R-1, S-3**: Paris Law under Fatigue, Schmid Factor under Critical Resolved Shear Stress, Larson-Miller under Creep, Portevin-Le Chatelier under Dynamic Strain Aging. Every pairing fits the physical meaning of the term. Correct.
3. **(C) P-2, Q-4, R-3, S-1**: Paris Law and Schmid Factor are placed correctly, but Larson-Miller goes to Dynamic Strain Aging and Portevin-Le Chatelier goes to Creep, the reverse of what these two terms describe. Wrong.
4. **(D) P-1, Q-3, R-2, S-4**: places Paris Law under Creep and Schmid Factor under Dynamic Strain Aging, neither of which is correct, since Paris Law is a fatigue relation and Schmid Factor is a slip geometry term. Wrong.

Only option (B) assigns every term to the phenomenon it actually belongs to.

Let's summarize:

- Paris Law: fatigue crack growth rate versus stress intensity range.
- Schmid Factor: geometric factor linked to the resolved shear stress that triggers slip, tied to CRSS.
- Larson-Miller Parameter: combines time and temperature to predict creep rupture life.
- Portevin-Le Chatelier Effect: serrated flow caused by dynamic strain aging.

So the correct match is P-2, Q-4, R-1, S-3, option (B).

Quick Tip: Match each named law or parameter to the single mechanical-behavior phenomenon it is used to predict.

• • •

. Match the type of defects in Column I with the corresponding metal manufacturing processes listed in Column II.

Column I	Column II
(P) Edge Cracking	(1) Casting
(Q) Flashline Cracking	(2) Forging
(R) Chevron Cracking	(3) Rolling
(S) Cracked Core	(4) Extrusion

(A) P-3, Q-4, R-2, S-1

(B) P-4, Q-3, R-1, S-2

(C) P-4, Q-2, R-3, S-1

(D) P-3, Q-2, R-4, S-1

Correct Answer: (D) P-3, Q-2, R-4, S-1

SOLUTION:

Each defect in Column I is named after or closely tied to the process where it is most commonly seen, so the fast route is recalling the process linked to each defect name and checking it against the four options.

1. **(A) P-3, Q-4, R-2, S-1**: Flashline cracking is placed under Extrusion, but flash is a forging term, since closed-die forging is what produces a parting-line flash, not extrusion. Wrong.

2. **(B) P-4, Q-3, R-1, S-2**: Edge cracking is placed under Extrusion and cracked core under Forging, and neither fits, since edge cracking is a rolling defect and cracked core is a casting defect. Wrong.
3. **(C) P-4, Q-2, R-3, S-1**: Edge cracking under Extrusion is wrong, since edge cracking is tied to width-wise flow during rolling, not extrusion. Wrong.
4. **(D) P-3, Q-2, R-4, S-1**: Edge cracking under Rolling, flashline cracking under Forging, chevron cracking under Extrusion, cracked core under Casting. Each defect sits with the process it is actually named for and observed in. Correct.

Option (D) is the only self-consistent set.

Let's summarize:

- Edge cracking: rolling, from uneven strip-edge deformation.
- Flashline cracking: forging, along the die parting line.
- Chevron cracking: extrusion, an internal centerline defect.
- Cracked core: casting, from a failed sand or metal core.

So the correct match is P-3, Q-2, R-4, S-1, option (D).

Quick Tip: Recall which manufacturing process each named defect is most commonly seen in.

. Match the descriptions listed in Column I with the corresponding non-destructive testing methods listed in Column II.

Column I	Column II
(P) Internal flaws in railroad wheel	(1) Ultrasonic
(Q) In-service monitoring of crack	(2) Radiography
(R) Inclusion in mild steel	(3) Dye Penetrant
(S) Surface crack in Al-based alloy	(4) Acoustic Emission

(A) P-1, Q-4, R-2, S-3

(B) P-3, Q-2, R-4, S-1

(C) P-3, Q-2, R-1, S-4

(D) P-1, Q-3, R-4, S-2

Correct Answer: (A) P-1, Q-4, R-2, S-3

SOLUTION:

Each description in Column I hints at whether the defect sits inside the part or on its surface, and whether the check is a one-time inspection or continuous monitoring. Match that hint to the method that fits it best, then check the four options.

1. **(A) P-1, Q-4, R-2, S-3**: an internal flaw in a thick wheel goes to Ultrasonic, continuous in-service crack tracking goes to Acoustic Emission, an internal inclusion goes to Radiography, and a surface crack goes to Dye Penetrant. Each method sits exactly where its strength lies. Correct.
2. **(B) P-3, Q-2, R-4, S-1**: sends the internal wheel flaw to Dye Penetrant, but dye penetrant only reaches surface defects and cannot see inside a solid wheel. Wrong.

3. **(C) P-3, Q-2, R-1, S-4**: again places the internal wheel flaw under Dye Penetrant and the in-service crack under Radiography, but radiography needs a still exposure and is not built for continuous monitoring. Wrong.
4. **(D) P-1, Q-3, R-4, S-2**: places the in-service crack under Dye Penetrant, which is a one-time surface check, not a continuous in-service technique. Wrong.

Only option (A) lines every description up with the method actually used for it.

Let's summarize:

- Ultrasonic: internal flaws in thick, solid parts like a railroad wheel.
- Acoustic Emission: continuous, in-service tracking of crack growth.
- Radiography: internal inclusions, imaged with X-rays or gamma rays.
- Dye Penetrant: surface-breaking cracks only, such as in an aluminum alloy.

So the correct match is P-1, Q-4, R-2, S-3, option (A).

Quick Tip: Decide whether each defect is internal or surface breaking, and whether the check is one-time or continuous, before matching the method.

. The equation below represents a steady-state laminar flow of a fluid through a horizontal tube:

$$\mu \frac{1}{r} \frac{d}{dr}\left(r \frac{dv_z}{dr}\right) - \frac{dP}{dz} = 0$$

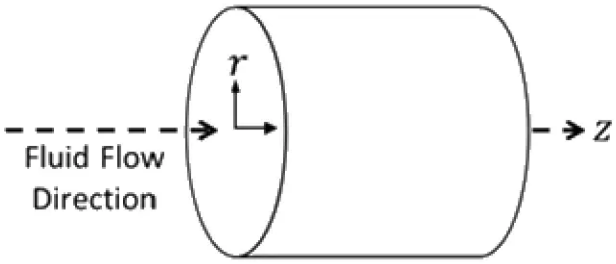


Figure: a horizontal cylindrical tube of radius R with fluid flowing along the z-axis; the radial coordinate r is measured outward from the tube's central axis and the axial coordinate z runs along the length of the tube.

Here P is fluid pressure, μ is dynamic viscosity, (r, z) are the coordinates of a cylindrical polar system, and v_z is the axial velocity in the z-direction. Which one of the following statements related to the above case is NOT correct?

(A) The fluid is Newtonian

(B) Shear stress is maximum at the center $(r = 0)$

(C) Radial velocity is zero

(D) There is no variation of v_z in z-direction

Correct Answer: (B) Shear stress is maximum at the center $(r = 0)$

SOLUTION:

Step 1: Picture the flow.

A fluid moves steadily along a horizontal circular tube, driven only by a pressure drop, with no motion in the radial direction and a velocity

pattern that no longer changes further along the tube. This is the classic Hagen-Poiseuille flow, and the given equation is Newton's second law written for a thin cylindrical shell of fluid inside the tube.

Step 2: Do a force balance on a fluid shell.

For a thin cylindrical shell of radius r and thickness dr, the net pressure force pushing it along the tube must balance the net viscous shear force on its curved surfaces. Writing that balance and letting the shell shrink to zero thickness gives exactly $\mu \frac{1}{r} \frac{d}{dr}\left(r \frac{dv_z}{dr}\right) = \frac{dP}{dz}$, the equation stated in the question.

Step 3: Get the shear stress directly.

Multiplying through by r and integrating once,

$$r\mu \frac{dv_z}{dr} = \frac{r^2}{2} \frac{dP}{dz} + C_1$$

At $r = 0$ the term $\mu\, dv_z/dr$ is just the shear stress τ_{rz}, and it must stay finite there, so $C_1 = 0$. This leaves

$$\tau_{rz} = \mu \frac{dv_z}{dr} = \frac{r}{2} \frac{dP}{dz}$$

a straight line in r, starting at zero when $r = 0$ and reaching its biggest size at the tube wall $r = R$.

Step 4: Read off the velocity profile too.

Dividing by μ and integrating once more, with $v_z = 0$ enforced at the wall $r = R$ (no-slip), gives the familiar parabola

$$v_z(r) = \frac{1}{4\mu} \frac{dP}{dz} \left(r^2 - R^2\right)$$

which peaks at the centerline and falls to zero at the wall, the opposite trend to the shear stress.

Step 5: Judge each statement.
The fluid is treated as Newtonian throughout, since $\tau = \mu\, dv_z/dr$ is used, so (A) holds. The flow is assumed purely axial with $v_r = 0$, so (C) holds. Being fully developed, v_z does not change with z, so (D) holds. Only (B) fails, since the shear stress found in Step 3 is zero at the center and largest at the wall, not the reverse.

Step 6: Conclude.

(B) is NOT correct: shear stress is zero at the center and maximum at the wall

Quick Tip: Integrate the given equation once to get the shear stress as a function of r, then check where it is zero and where it is largest.

• • •

. Match the unit operation in Column I with the resulting product in Column II:

Column I	Column II
(P) COREX Process	(1) Direct Reduced Iron
(Q) Bayer Process	(2) Pig Iron
(R) Matte Smelting	(3) Alumina
(S) MIDREX Process	(4) Copper

(A) P-4, Q-3, R-2, S-1

(B) P-2, Q-3, R-4, S-1

(C) P-1, Q-3, R-4, S-2

(D) P-2, Q-4, R-3, S-1

Correct Answer: (B) P-2, Q-3, R-4, S-1

SOLUTION:

This question tests knowledge of four extraction routes and the product each one delivers. Instead of starting from the process names, let's start from each product in Column II and ask which process gives it.

1. **(1) Direct Reduced Iron:** This is solid sponge iron made by reducing iron ore below its melting point using a reducing gas or reactive coal, without going through a blast furnace. The MIDREX process is the world's leading gas-based route for this, so (1) belongs to (S) MIDREX Process.
2. **(2) Pig Iron:** This is liquid hot metal, high in carbon, made by smelting iron ore. The COREX process is a smelting reduction technology that skips coke ovens and sinter plants but still delivers liquid hot metal, so (2) belongs to (P) COREX Process.
3. **(3) Alumina:** This is aluminium oxide, extracted from bauxite by dissolving it in hot caustic soda and then precipitating and calcining the hydroxide. This is exactly the Bayer process, so (3) belongs to (Q) Bayer Process.
4. **(4) Copper:** Copper sulphide ores are smelted to form a molten matte (copper and iron sulphides) separated from slag, which is later blown in a converter to give blister copper. This matte-

forming step is matte smelting, so (4) belongs to (R) Matte Smelting.

Putting the pairs together gives P-2, Q-3, R-4, S-1, which is option (B).

Let's summarize:

- COREX Process: smelting reduction route that gives pig iron.
- Bayer Process: caustic leaching of bauxite that gives alumina.
- Matte Smelting: sulphide smelting route on the way to copper.
- MIDREX Process: gas-based direct reduction that gives direct reduced iron (sponge iron).

So the correct match is P-2, Q-3, R-4, S-1.

Quick Tip: Think about the state of the product (liquid pig iron, solid sponge iron) and the ore treated (bauxite for alumina, sulphide ore for copper) to place each process.

. Match the terms in Column I with the corresponding attributes in Column II:

Column I	Column II
(P) Wiedemann-Franz law	(1) Charge carrier concentration
(Q) Neel temperature	(2) Paramagnetism
(R) Hall voltage	(3) Ratio between thermal conductivity and electrical conductivity of metals
(S) Curie law	(4) Diamagnetism
	(5) Anti-ferromagnetism

(A) P-4, Q-3, R-2, S-1

(B) P-2, Q-5, R-1, S-3

(C) P-3, Q-5, R-1, S-2

(D) P-3, Q-4, R-5, S-2

Correct Answer: (C) P-3, Q-5, R-1, S-2

SOLUTION:

Let's work from the Column II side this time and ask which term in Column I fits each attribute.

1. **(1) Charge carrier concentration:** This is what the Hall effect measures. A magnetic field applied across a current-carrying sample produces a small sideways voltage, the Hall voltage, and its size tells us how many charge carriers per unit volume are present, along with their sign. So (1) goes with (R) Hall voltage.

2. **(2) Paramagnetism:** Curie law says the magnetic susceptibility of a paramagnetic solid falls off as $1/T$. This is a defining feature of paramagnetic behavior, so (2) goes with (S) Curie law.
3. **(3) Ratio between thermal conductivity and electrical conductivity of metals:** This is the Wiedemann-Franz law, which says k/σ is nearly the same constant times temperature T for most metals, since both properties come from the free electrons carrying heat and charge. So (3) goes with (P) Wiedemann-Franz law.
4. **(4) Diamagnetism:** No term in Column I refers to diamagnetism, so this option is left unmatched. It exists only to test whether the student picks a false pairing.
5. **(5) Anti-ferromagnetism:** Below the Neel temperature, neighboring atomic moments in certain materials point opposite to each other, cancelling the net magnetization. This is antiferromagnetic order, so (5) goes with (Q) Neel temperature.

Putting the working pairs together gives P-3, Q-5, R-1, S-2, which is option (C).

Let's summarize:

- Wiedemann-Franz law ties thermal and electrical conductivity of metals through temperature.
- Neel temperature marks the onset of antiferromagnetism.
- Hall voltage reveals the charge carrier concentration.
- Curie law describes paramagnetic susceptibility falling with rising temperature.

So the correct match is P-3, Q-5, R-1, S-2, and diamagnetism is the leftover distractor.

Quick Tip: Recall that Wiedemann-Franz links thermal and electrical conductivity, Neel temperature marks antiferromagnetic order, Hall voltage gives carrier concentration, and Curie law describes paramagnetism; diamagnetism is a distractor.

• • •

. Choose the correct option(s).
Permeability of a porous bed made of spherical particles is/are:

(A) Independent of particle size

(B) Increases with increase in particle size

(C) Decreases with increase in particle size

(D) Affected by particle size distribution

Correct Answer: (B) Increases with increase in particle size ; (D) Affected by particle size distribution

SOLUTION:

Permeability tells us how easily a fluid can push through the gaps of a packed bed. Let's check each statement using the Ergun equation, which links pressure drop across a bed to particle size and void fraction.

1. **Independent of particle size:** The Ergun equation has particle diameter d_p sitting in the denominator of both its viscous and inertial terms. Changing d_p changes the resistance directly, so permeability cannot be independent of it. This statement is wrong.
2. **Increases with increase in particle size:** Bigger particles leave bigger gaps between them for the fluid to pass through. Wider flow channels mean less resistance per unit bed length,

which is exactly what a higher permeability means. This statement is right.

3. **Decreases with increase in particle size:** This is the reverse of what actually happens. Larger particles open up the bed, they do not choke it, so permeability does not fall as particle size grows. This statement is wrong.
4. **Affected by particle size distribution:** A bed is almost never made of one exact particle size in practice. If small particles are mixed among large ones, the small ones can wedge into the pore spaces of the large ones, cutting down the void volume and the flow paths. So the spread of particle sizes, not just a single average size, changes the permeability. This statement is right.

So the correct choices are that permeability increases with particle size, and that it is affected by the particle size distribution.

Let's summarize:

- Permeability rises as particle size rises, since larger particles create wider flow channels.
- A wide spread of particle sizes lets small grains fill the gaps between large ones, which lowers permeability at a fixed average size.

The final answer is options (B) and (D), that is choices 2 and 4.

Quick Tip: Use the Kozeny-Carman/Ergun relation: permeability scales with particle diameter squared, and packing of mixed sizes reduces void space.

. Consider a distribution with the following probability density function

$$f(x) = \begin{cases} 0.5, & 0 < x < 2 \\ 0.0, & \text{Otherwise} \end{cases}$$

Given that the mean of the above probability distribution is 1, the variance (rounded off to two decimal places) is _____.

Correct Answer: 0.32 to 0.34

SOLUTION:

Step 1: Recognize the distribution type.

A constant density of 0.5 over $0 < x < 2$ describes a continuous uniform distribution on the interval $[a, b]$ with $a = 0$ and $b = 2$. Its height must equal $\frac{1}{b-a}$, and here $\frac{1}{2-0} = 0.5$, which matches, confirming this reading.

Step 2: Recall the standard variance formula for a uniform distribution.

For X uniform on $[a, b]$, the mean and variance have closed forms:

$$E[X] = \frac{a+b}{2}, \qquad \text{Var}(X) = \frac{(b-a)^2}{12}$$

Check the mean first, as a consistency check on a and b: $E[X] = \frac{0+2}{2} = 1$, which matches the value given in the question.

Step 3: Apply the variance formula directly.

$$\text{Var}(X) = \frac{(2-0)^2}{12} = \frac{4}{12} = \frac{1}{3}$$

Step 4: Convert to a decimal.

$$\frac{1}{3} = 0.3333\ldots$$

Rounded to two decimal places, this is 0.33.

Step 5: Cross-check with the integral method.

Using $\text{Var}(X) = E[X^2] - (E[X])^2$, $E[X^2] = \int_0^2 x^2(0.5)\,dx = \frac{4}{3}$, so $\text{Var}(X) = \frac{4}{3} - 1 = \frac{1}{3}$, the same result, confirming the shortcut formula was used correctly.

$$\boxed{0.33}$$

Quick Tip: This is a uniform distribution on (0,2); use Var(X) = (b-a)^2/12 or E[X^2] - (E[X])^2.

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. Taking number of intervals n = 3, the value of the integral

$$\int_0^{0.3} e^{-x^2}\,dx$$

using Trapezoidal method, is _____ (rounded off to two decimal places).

Correct Answer: 0.27 to 0.31

SOLUTION:

Step 1: Split the interval into three equal strips.

We are told to use $n = 3$ over $[0, 0.3]$, so each strip has width $h = 0.1$. This gives three trapezoids: one over $[0, 0.1]$, one over $[0.1, 0.2]$, and one over $[0.2, 0.3]$.

Step 2: Get the function values needed.

$f(x) = e^{-x^2}$, so

$$f(0) = 1, \quad f(0.1) = e^{-0.01} \approx 0.99005, \quad f(0.2) = e^{-0.04} \approx 0.96079, \quad f(0.3) = e^{-0.09} \approx 0.91393$$

Step 3: Find the area of each trapezoid separately.

Each trapezoid area is $\frac{h}{2}(\text{left height} + \text{right height})$.

First strip, $[0, 0.1]$:

$$A_1 = \frac{0.1}{2}(1 + 0.99005) = 0.05 \times 1.99005 = 0.099503$$

Second strip, $[0.1, 0.2]$:

$$A_2 = \frac{0.1}{2}(0.99005 + 0.96079) = 0.05 \times 1.95084 = 0.097542$$

Third strip, $[0.2, 0.3]$:

$$A_3 = \frac{0.1}{2}(0.96079 + 0.91393) = 0.05 \times 1.87472 = 0.093736$$

Step 4: Add the three strip areas.

$$I \approx A_1 + A_2 + A_3 = 0.099503 + 0.097542 + 0.093736 = 0.290781$$

This matches the composite formula result, since adding strip by strip is the same sum written a different way.

Step 5: Round to two decimals.

$$I \approx 0.29$$

$$\boxed{0.29}$$

Quick Tip: Use h = 0.1, evaluate e^(-x^2) at x = 0, 0.1, 0.2, 0.3, and apply the trapezoidal rule I is about (h/2)[fo + 2f1 + 2f2 + f3].

• • •

. Solution of the differential equation

$$10x^2\frac{d^2y}{dx^2} - 20x\frac{dy}{dx} + 22.4y = 0$$

is

$$y = c_1x^{m_1} + c_2x^{m_2}$$

where $m_1 \neq m_2$.
The value of $m_1 + m_2$ (answer in integer) is _____.

Correct Answer: 3 to 3

SOLUTION:

Step 1: Use the standard Euler substitution $x = e^t$.

A Cauchy-Euler equation turns into a constant coefficient equation if we set $x = e^t$, so $t = \ln x$. With the operator $D = \frac{d}{dt}$, the standard results are

$$x\frac{dy}{dx} = Dy, \qquad x^2\frac{d^2y}{dx^2} = D(D-1)y$$

Step 2: Rewrite the given equation in terms of D.

The equation $10x^2y'' - 20xy' + 22.4y = 0$ becomes

$$10D(D-1)y - 20Dy + 22.4y = 0$$

Step 3: Expand and collect terms.

$$10D^2y - 10Dy - 20Dy + 22.4y = 0$$

$$(10D^2 - 30D + 22.4)y = 0$$

Step 4: Write the characteristic equation.

Since $y = e^{mt} = x^m$ solves this constant coefficient equation, put $D = m$:

$$10m^2 - 30m + 22.4 = 0$$

This is exactly the same auxiliary equation the direct trial method

gives, only reached through the $t = \ln x$ route instead of substituting $y = x^m$ straight into the original equation.

Step 5: Read off the sum of roots directly from the coefficients.
For $am^2 + bm + c = 0$, the sum of roots is $-b/a$. Here $a = 10$, $b = -30$, so

$$m_1 + m_2 = -\frac{-30}{10} = 3$$

The individual roots work out to $m_1 = 1.4$ and $m_2 = 1.6$, same as before, confirming the sum.

Step 6: Note why this route is useful.
The $x = e^t$ substitution is handy whenever the equation has more than one Cauchy-Euler term mixed with other structure, since it turns the whole problem into an ordinary constant coefficient equation that can be solved with the usual characteristic equation method, rather than guessing a power law form each time.

$$\boxed{m_1 + m_2 = 3}$$

Quick Tip: Substitute $y = x^m$ in the Cauchy-Euler equation to get the auxiliary equation $10m^2 - 30m + 22.4 = 0$; the sum of roots is $-b/a$.

. Iron powder is compacted at room temperature to 75% of its theoretical density. The as-pressed iron compact is sintered in inert-gas atmosphere at 1200°C to 90% of its theoretical density. Assuming isotropic shrinkage, the linear shrinkage (in percent) undergone by the iron compact during sintering (rounded off to one decimal place) is _____ %.

Correct Answer: 5.8 to 6.1

SOLUTION:

Step 1: Work with the green-to-sintered volume ratio instead of sintered-to-green.

Since the compact carries the same mass throughout, $V \propto 1/D$ where D is the fractional density. Take the ratio the other way round:

$$\frac{V_{green}}{V_{sintered}} = \frac{D_{sintered}}{D_{green}} = \frac{0.90}{0.75} = 1.2$$

Step 2: Take the cube root to get the length ratio.

For isotropic shrinkage, length scales as the cube root of volume, so

$$\frac{L_{green}}{L_{sintered}} = (1.2)^{1/3}$$

Working this out, $(1.2)^{1/3} \approx 1.0627$.

Step 3: Flip this to get the sintered fraction.

$$\frac{L_{sintered}}{L_{green}} = \frac{1}{1.0627} \approx 0.9410$$

Step 4: Write the shrinkage as a fraction lost.

$$\text{Linear shrinkage} = \left(1 - \frac{L_{sintered}}{L_{green}}\right) \times 100 = (1 - 0.9410) \times 100$$

Step 5: Finish the calculation.

$$\text{Linear shrinkage} \approx 5.9\%$$

This matches the direct calculation, confirming the compact shrinks by about **5.9%** in each linear direction while densifying from **75%** to **90%** of theoretical.

Step 6: Why the two routes agree.

Both paths use the same physics, that mass is conserved while volume falls as density rises, and that isotropic shrinkage lets us move between volume and length with a cube or a cube root. Starting the ratio as green-over-sintered instead of sintered-over-green just flips which fraction we compute first, the final linear shrinkage number comes out the same either way.

$$\boxed{5.9\%}$$

Quick Tip: Use $V \propto 1/D$ for constant mass, then $L_1/L_0 = (D_0/D_1)^{1/3}$ since the shrinkage is isotropic.

. An alloy having composition W-20 wt.% Ni is prepared by mixing elemental powders. The resulting powder mixture is liquid phase sintered at 1550°C for 1 hour. The liquid phase sintered microstructure consists of interconnected, spherical tungsten grains dispersed in nickel. The tungsten grain size is 70 μm and the W-W interparticle neck diameter is 35 μm. If W-Ni interfacial energy is 0.30 J/m^2, the W-W interfacial energy (in J/m^2), rounded off to two decimal places is _____.
(Given: melting point of W: 3410°C and melting point of Ni: 1455°C)

Correct Answer: 0.50 to 0.54

SOLUTION:

Step 1: Set up the neck size ratio.
The ratio of the interparticle neck diameter to the grain diameter fixes the dihedral angle at the neck through $\sin(\phi/2) = X/D$. Here

$$\frac{X}{D} = \frac{35}{70} = \frac{1}{2}$$

so $\sin(\phi/2) = \frac{1}{2}$.

Step 2: Get the cosine without going through the angle itself.
Instead of finding $\phi/2$ in degrees first, use the identity $\cos^2\theta + \sin^2\theta = 1$ directly on the half angle:

$$\cos\left(\frac{\phi}{2}\right) = \sqrt{1 - \sin^2\left(\frac{\phi}{2}\right)} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

This is the exact value $\frac{\sqrt{3}}{2} \approx 0.866$, the same number a 30-60-90 triangle gives, but reached purely algebraically.

Step 3: Apply the interfacial energy balance at the grain boundary groove.

At the W-W grain boundary edge, two W-Ni interfaces pull against the W-W boundary, giving

$$\gamma_{WW} = 2\gamma_{WNi}\cos\left(\frac{\phi}{2}\right)$$

Step 4: Substitute the known values.

$$\gamma_{WW} = 2(0.30)\left(\frac{\sqrt{3}}{2}\right) = 0.30\sqrt{3}$$

Step 5: Evaluate.

$$0.30\sqrt{3} \approx 0.30 \times 1.732 = 0.5196$$

Rounded to two decimal places, this is $0.52\ \text{J/m}^2$. The melting point data simply confirm nickel is the liquid matrix (its melting point $1455°\text{C}$ is below the $1550°\text{C}$ sintering temperature) while tungsten (melting point $3410°\text{C}$) remains solid, matching the described microstructure.

Step 6: Sanity check the size of the answer.

A solid-solid grain boundary energy of about $0.52\ \text{J/m}^2$ being larger than the solid-liquid energy of $0.30\ \text{J/m}^2$ makes physical sense here, since the neck has grown to only half the grain diameter, giving a fairly open dihedral angle of $60°$ rather than a fully wetted boundary. If the W-W boundary energy had come out lower than $2\gamma_{WNi}$, the

liquid nickel would tend to penetrate and separate the tungsten grains instead of letting them sinter together into a rigid skeleton.

$$\boxed{\gamma_{WW} \approx 0.52\ \text{J/m}^2}$$

Quick Tip: Use $\sin(\phi/2) = X/D$ from the neck geometry to get the dihedral angle, then $\gamma_{WW} = 2\gamma_{WNi}\cos(\phi/2)$.

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. A metal having body-centered cubic structure is analyzed through X-ray diffraction using monochromatic X-ray of wavelength 0.154 nm. The diffraction angle (2θ) corresponding to {2 0 0} plane is 60° (for first order reflection).
The atomic radius of this element (rounded off to three decimal places) is _____ nm.

Correct Answer: 0.132 to 0.134

SOLUTION:

Step 1: Combine all three relations into one formula before putting in numbers.
Three facts are needed: Bragg's law $n\lambda = 2d\sin\theta$, the cubic spacing rule $d_{hkl} = a/\sqrt{h^2+k^2+l^2}$, and the BCC touching condition $4R = a\sqrt{3}$. Chain them together algebraically first.

Step 2: Express a in terms of d_{200}.
For {200}, $\sqrt{h^2+k^2+l^2} = 2$, so $a = 2d_{200}$. And from Bragg's law, $d_{200} = \dfrac{n\lambda}{2\sin\theta}$. So

$$a = 2 \cdot \frac{n\lambda}{2\sin\theta} = \frac{n\lambda}{\sin\theta}$$

Step 3: Substitute a into the BCC radius formula.

$$R = \frac{a\sqrt{3}}{4} = \frac{\sqrt{3}}{4} \cdot \frac{n\lambda}{\sin\theta}$$

This single formula now links the radius directly to the measured angle and wavelength, without needing d or a as separate intermediate numbers.

Step 4: Plug in the numbers only once.
With $n = 1$, $\lambda = 0.154\ \text{nm}$, and $\theta = 30°$ so $\sin\theta = 0.5$:

$$R = \frac{\sqrt{3}}{4} \times \frac{1 \times 0.154}{0.5} = \frac{1.7321}{4} \times 0.308$$

Step 5: Finish the arithmetic.

$$R = 0.4330 \times 0.308 \approx 0.1334\ \text{nm}$$

Rounded to three decimal places, this gives the same result as computing d and a separately.

$$\boxed{R \approx 0.133\ \text{nm}}$$

Quick Tip: Find d_{200} from Bragg's law, then $a = 2d_{200}$ for the {200} plane, then use $R = a\sqrt{3}/4$ for BCC.

. A cylindrical single crystal of copper having 10 mm diameter is deformed under a uniaxial tensile load of 2200 N. The angle between the normal to the slip plane and the tensile loading axis is α, whereas the angle between slip direction and the tensile axis is β. The slip direction is in the plane defined by the stress axis and the normal to the slip plane.
If $\alpha = \beta$, the critical resolved shear stress (CRSS), rounded off to one decimal place, is _____ MPa.

Correct Answer: 12.8 to 14.1

SOLUTION:

Step 1: Get the stress from load and area.
The bar has diameter $10\ \text{mm}$, so area $A = \pi(5)^2 = 78.54\ \text{mm}^2$. Under $F = 2200\ \text{N}$,

$$\sigma = \frac{2200}{78.54} \approx 28.01\ \text{MPa}$$

Step 2: Turn the coplanar condition into a single-variable Schmid factor.
Schmid's law gives $\tau = \sigma \cos\alpha \cos\beta$. Because the slip direction, the tensile axis, and the plane normal are coplanar, and the slip direction is perpendicular to the normal, $\beta = 90° - \alpha$. Substitute this in:

$$\tau = \sigma \cos\alpha \cos(90° - \alpha) = \sigma \cos\alpha \sin\alpha$$

Using the double angle identity $2\sin\alpha\cos\alpha = \sin(2\alpha)$, this is

$$\tau = \frac{\sigma}{2}\sin(2\alpha)$$

Step 3: Treat $\alpha = \beta$ as the condition that gives the largest possible resolved shear stress.

Since $\sin(2\alpha)$ reaches its largest value of 1 when $2\alpha = 90°$, that is when $\alpha = 45°$, and at that point $\beta = 90° - 45° = 45° = \alpha$. So the condition $\alpha = \beta$ given in the problem is exactly the orientation that maximizes the Schmid factor for this coplanar family of slip systems.

Step 4: Evaluate the Schmid factor at this orientation.

$$\tau = \frac{\sigma}{2}\sin(90°) = \frac{\sigma}{2} \times 1 = \frac{\sigma}{2}$$

So the Schmid factor is exactly 0.5, the maximum possible value for any slip system under uniaxial tension.

Step 5: Substitute the stress value.

$$\tau_{CRSS} = \frac{28.01}{2} \approx 14.0 \text{ MPa}$$

This matches the direct geometric calculation, confirming the CRSS.

$$\boxed{\tau_{CRSS} \approx 14.0 \text{ MPa}}$$

Quick Tip: Use $\sigma = F/A$, then Schmid's law $\tau = \sigma \cos\alpha \cos\beta$ with $\alpha + \beta = 90°$ from the coplanar condition, and $\alpha = \beta = 45°$.

• • •

. For plane strain fracture toughness (K_{Ic}) testing of a Maraging steel specimen, find the minimum thickness needed for a valid K_{Ic} measurement (answer as an integer, in mm).
Given: $K_{Ic} = 90\ \text{MPa}\sqrt{\text{m}}$ and yield stress $\sigma_y = 900\ \text{MPa}$.

Correct Answer: 25 to 25

SOLUTION:

Step 1: Note what the ASTM E399 thickness rule is checking.
For a K_{Ic} test to be trusted as a true plane strain value, the specimen must be thick enough that the crack tip plastic zone is small next to the thickness. The rule used to fix this minimum thickness is

$$B \geq 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2$$

Step 2: Put the given data in consistent units first.
Take $K_{Ic} = 90\ \text{MPa}\sqrt{m}$ and $\sigma_y = 900$ MPa. Since both are already in MPa, no unit conversion is needed; the ratio itself will carry the units of $\sqrt{m}$.

Step 3: Simplify the ratio before squaring.

$$\frac{K_{Ic}}{\sigma_y} = \frac{90}{900} = \frac{1}{10}$$

So the ratio is exactly $0.1\ \sqrt{m}$, a clean number because 900 is ten times 90.

Step 4: Square and scale in one step.

$$B_{min} = 2.5 \times (0.1)^2 = 2.5 \times 0.01 = 0.025 \text{ m}$$

Step 5: Change metres to millimetres at the end.
One metre is 1000 mm, so

$$B_{min} = 0.025 \times 1000 = 25 \text{ mm}$$

This matches the earlier working. The minimum valid thickness comes out to a round number because the given K_{Ic} to σ_y ratio was already a clean 1/10.

$$\boxed{B_{min} = 25 \text{ mm}}$$

Quick Tip: Use the ASTM E399 validity condition $B \geq 2.5(K_{Ic}/\sigma_y)^2$ to size the specimen.

• • •

. The lattice parameter of Ni (face centered cubic) is 0.35 nm and its shear modulus is 76 GPa. Find the strain energy per unit length of a screw dislocation in the Ni crystal (rounded off to two decimal places), in units of 10^{-9} J/m.

Correct Answer: 2.30 to 2.38

SOLUTION:

Step 1: Write the strain energy relation to use.
For a screw dislocation, the elastic energy stored per unit length is taken as

$$\frac{E}{L} = \frac{1}{2}Gb^2$$

so we need G and the Burgers vector b before anything else.

Step 2: Get the Burgers vector from the FCC slip vector.
FCC crystals slip along $\frac{a}{2}\langle 110\rangle$ directions. A $\langle 110\rangle$ direction like [110] has two unit components, so its length is $\sqrt{1^2 + 1^2} = \sqrt{2}$, giving

$$b = \frac{a}{2} \times \sqrt{2} = \frac{a}{\sqrt{2}}$$

With $a = 0.35$ nm,

$$b = \frac{0.35 \times 10^{-9}}{\sqrt{2}} = 2.475 \times 10^{-10}\ \text{m}$$

Step 3: Build the product Gb^2 as one block.

$$Gb^2 = (76 \times 10^9) \times (2.475 \times 10^{-10})^2$$

$$= (76 \times 10^9) \times (6.125 \times 10^{-20})$$

$$= 4.655 \times 10^{-9}\ \text{J/m}$$

Step 4: Halve it to get the final energy per length.

$$\frac{E}{L} = \frac{Gb^2}{2} = \frac{4.655 \times 10^{-9}}{2} = 2.3275 \times 10^{-9}\ \text{J/m}$$

Step 5: Round to two decimals.

2.3275 rounds to 2.33, so the strain energy per unit length is 2.33×10^{-9} J/m, which sits inside the accepted band of 2.30 to 2.38.

$$\boxed{\frac{E}{L} \approx 2.33 \times 10^{-9}\ \text{J/m}}$$

Quick Tip: Find the FCC Burgers vector $b = a/\sqrt{2}$, then use $E/L = \frac{1}{2}Gb^2$.

• • •

. 20 g of gold (Au) and 20 g of silver (Ag) are mixed to form a single phase solid solution (assume ideal mixing). The atomic weight of Au is 197 g/mol and the atomic weight of Ag is 108 g/mol. The universal gas constant R = 8.314 J/mol-K. Find the total entropy of mixing (rounded off to two decimal places), in J/K.

Correct Answer: 1.50 to 1.60

SOLUTION:

Step 1: Set out the mixing entropy formula first.

For an ideal solid solution of two elements, the total entropy of mixing is

$$\Delta S_{mix} = -R\, n_{total} \left(x_{Au} \ln x_{Au} + x_{Ag} \ln x_{Ag}\right)$$

writing it this way keeps the total mole count n_{total} inside the same

expression instead of scaling it up at the end.

Step 2: Convert the given masses to moles.

$n_{Au} = \frac{20}{197} = 0.10152$ mol and $n_{Ag} = \frac{20}{108} = 0.18519$ mol, so $n_{total} = 0.28671$ mol.

Step 3: Get the mole fractions.

$$x_{Au} = \frac{0.10152}{0.28671} = 0.3541, \qquad x_{Ag} = \frac{0.18519}{0.28671} = 0.6459$$

Step 4: Work out $\ln x_{Au}$ and $\ln x_{Ag}$ together.

$\ln(0.3541) = -1.0382$ and $\ln(0.6459) = -0.4371$. Multiplying each by its mole fraction gives -0.3676 and -0.2823, which sum to -0.6499.

Step 5: Plug everything into the one shot formula.

$$\Delta S_{mix} = -8.314 \times 0.28671 \times (-0.6499)$$

First multiply $8.314 \times 0.28671 = 2.3835$, then $2.3835 \times 0.6499 = 1.5493$.

$$\Delta S_{mix} = 1.5493 \text{ J/K}$$

Step 6: Round off.

This rounds to 1.55 J/K, which lies inside the accepted range of 1.50 to 1.60 J/K.

$$\boxed{\Delta S_{mix} \approx 1.55 \text{ J/K}}$$

Quick Tip: Convert masses to mole fractions, then apply $\Delta S_{mix} = -Rn_{total} \sum x_i \ln x_i$.

• • •

. 2 moles of an ideal gas are reversibly expanded from 10 litre to 20 litre under isothermal conditions. The universal gas constant $R = 8.314$ J/mol-K. If the temperature of the gas is 27°C, find the magnitude of the work done in the process (rounded off to two decimal places), in Joules.

Correct Answer: 3440.00 to 3492.00

SOLUTION:

Step 1: Change the temperature to kelvin right away.

$T = 27°C = 27 + 273 = 300$ K. Working in kelvin from the start avoids mistakes later.

Step 2: Recall why the work formula has a log term.

For an ideal gas, pressure is $p = \frac{nRT}{V}$. At constant T, the reversible work is the area under the p-V curve,

$$W = \int_{V_1}^{V_2} p\,dV = nRT \int_{V_1}^{V_2} \frac{dV}{V} = nRT \ln\left(\frac{V_2}{V_1}\right)$$

Step 3: Combine n, R and T into a single number first.

$$nRT = 2 \times 8.314 \times 300 = 4988.4 \text{ J}$$

Step 4: Handle the volume ratio and its log separately.

$$\frac{V_2}{V_1} = \frac{20\ \text{L}}{10\ \text{L}} = 2, \qquad \ln 2 = 0.6931$$

Step 5: Multiply the two pieces.

$$W = 4988.4 \times 0.6931 = 3457.70\ \text{J}$$

This matches the direct substitution method and confirms the work magnitude is about 3457.70 J, which falls inside the given band of 3440.00 to 3492.00 J.

$$\boxed{W \approx 3457.70\ \text{J}}$$

Quick Tip: Use $W = nRT\ln(V_2/V_1)$ with T in kelvin.

• • •

. The specific heat capacity at constant pressure (C_p) of a solid metal is given by

$$C_p\ (\text{in J/mol-K}) = 20 + (5 \times 10^{-3})T$$

valid for $T = 298$ K to 1000 K. At constant pressure, if the temperature of 2 moles of the metal is increased from 300 K to 600 K, find the change in enthalpy of the metal (answer as an integer), in Joules.

Correct Answer: 13340 to 13360

SOLUTION:

Step 1: Split C_p into a constant part and a temperature dependent part.

$$C_p = 20 + (5 \times 10^{-3})T$$

The enthalpy change is $\Delta H = n \int_{300}^{600} C_p \, dT$, and since the integral is linear we can integrate each part on its own and add the results.

Step 2: Integrate the constant term.

$$\int_{300}^{600} 20 \, dT = 20 \times (600 - 300) = 20 \times 300 = 6000 \text{ J/mol}$$

Step 3: Integrate the linear term.

$$\int_{300}^{600} 5 \times 10^{-3} T \, dT = \frac{5 \times 10^{-3}}{2} \left[T^2 \right]_{300}^{600} = 2.5 \times 10^{-3} \left(600^2 - 300^2 \right)$$

Now $600^2 = 360000$ and $300^2 = 90000$, so the difference is 270000.

$$2.5 \times 10^{-3} \times 270000 = 675 \text{ J/mol}$$

Step 4: Add the two pieces to get the molar enthalpy change.

$$\int_{300}^{600} C_p \, dT = 6000 + 675 = 6675 \text{ J/mol}$$

Step 5: Scale by the number of moles.

$$\Delta H = n \times 6675 = 2 \times 6675 = 13350 \text{ J}$$

This agrees with the direct integration method and lands inside the accepted range of 13340 to 13360 J.

$$\boxed{\Delta H = 13350 \text{ J}}$$

Quick Tip: Integrate C_p with respect to T and multiply by the number of moles: $\Delta H = n \int C_p \, dT$.

• • •

. With reference to the Ellingham diagram, the standard Gibbs free energy change for the oxidation of solid metal M(s) and liquid metal M(l) is given below.

Reaction I: $M(s) + O_2(g) \rightarrow MO_2(s)$

$$\Delta G^\circ = (-338900 - 15.2\,T \ln T + 247T) \text{ Joules}$$

from $T = 300$ K to the melting point.

Reaction II: $M(l) + O_2(g) \rightarrow MO_2(s)$

$$\Delta G^\circ = (-390800 - 15.2\,T \ln T + 285.3T) \text{ Joules}$$

from the melting point to $T = 1800$ K.

The melting point of the metal M (rounded off to one decimal place) is _____ Kelvin.

Correct Answer: 1353.9 to 1356.3

SOLUTION:

Step 1: Set up the problem as finding where the two Gibbs energy lines cross.

The two given formulas are two expressions for the same oxidation reaction, one valid below the melting point and one above it. On the Ellingham diagram, a plot of $\Delta G°$ against T, these two lines meet at exactly one point, the melting point of the metal, because the free energy of the reaction cannot jump suddenly at a single temperature. So instead of comparing the two formulas piece by piece, define a difference function

$$h(T) = \Delta G°_{I}(T) - \Delta G°_{II}(T)$$

and look for the temperature where $h(T) = 0$. Wherever $h(T) = 0$, the two lines cross, and that crossing point is the melting point asked for.

Step 2: Build the difference function.

$$h(T) = \left(-338900 - 15.2\,T\ln T + 247T\right) - \left(-390800 - 15.2\,T\ln T + 285.3T\right)$$

The $-15.2\,T\ln T$ pieces are identical in both brackets, so they cancel when we subtract. This is a useful check in itself: both reactions share the same gas-phase entropy term, since both consume one mole of $O_2(g)$, so the logarithmic part was always going to drop out of the difference. What is left is only the constant and linear parts:

$$h(T) = (-338900 + 390800) + (247 - 285.3)T = 51900 - 38.3T$$

Step 3: Solve $h(T) = 0$.

$$51900 - 38.3T = 0$$

$$T = \frac{51900}{38.3}$$

Carrying out the division, $T = 1355.09\ldots$ K.

Step 4: Round and check against the range.
Rounded to one decimal place, $T = 1355.1$ K.

Step 5: Confirm by substituting back into the original expressions.
As a cross-check, plug $T = 1355.1$ K into both original formulas and make sure they give the same $\Delta G°$. Both share the term $-15.2\,T \ln T$, which evaluates to the same number in each, so it is enough to compare the remaining linear parts:

$$-338900 + 247(1355.1) = -338900 + 334709.7 = -4190.3$$

$$-390800 + 285.3(1355.1) = -390800 + 386610.03 = -4189.97$$

The two values agree to well within rounding error, which confirms $T = 1355.1$ K is the crossing point.

Step 6: State the final answer.
This value falls comfortably inside the window of 1353.9 K to 1356.3 K that the answer key accepts, so the melting point checks out.

$$\boxed{T \approx 1355.1\ \text{K}}$$

Quick Tip: At the metal's melting point, both Gibbs free energy expressions must give the same value. Equate Reaction I and Reaction II at T and solve the resulting linear equation for T.

• • •

. A given volume of liquid is undercooled just below the melting temperature to form a spherical solid nucleus (consider homogeneous nucleation). The Gibbs free energy of solidification (ΔG_v) is (-0.5×10^8) J/m^3. The solid-liquid interfacial energy (γ) is isotropic and its value is 0.1 J/m^2.
The critical nucleus size for a stable nucleus is _____ nm (answer in integer).

Correct Answer: 4 to 4

SOLUTION:

Step 1: Use a mechanical equilibrium argument instead of calculus.
There is a quicker way to reach the critical radius without differentiating the free energy curve: think of the balance between the driving force pushing the interface to grow and the resisting pressure from surface tension, similar to the Young-Laplace pressure across a curved interface.

Step 2: Write the two pressures.
The volumetric driving force for solidification, per unit volume, is $|\Delta G_v|$ (we use the magnitude since ΔG_v is negative and favors the solid). The resisting curvature pressure from the interfacial energy γ on a sphere of radius r is the Laplace pressure,

$$\Delta P = \frac{2\gamma}{r}$$

Step 3: Set the two equal at the critical radius.
At the critical radius r^*, the driving force per unit volume exactly balances the resisting curvature pressure, the same condition reached by setting the derivative of $\Delta G(r)$ to zero:

$$|\Delta G_v| = \frac{2\gamma}{r^*} \quad \Rightarrow \quad r^* = \frac{2\gamma}{|\Delta G_v|}$$

This matches the usual formula $r^* = -2\gamma/\Delta G_v$ once we account for the sign of ΔG_v.

Step 4: Plug in the numbers.
$\gamma = 0.1\ \text{J/m}^2$ and $|\Delta G_v| = 0.5 \times 10^8\ \text{J/m}^3$.

$$r^* = \frac{2(0.1)}{0.5 \times 10^8} = \frac{0.2}{0.5 \times 10^8} = 4 \times 10^{-9}\ \text{m} = 4\ \text{nm}$$

Step 5: Note the alternate reading.
If "nucleus size" is taken to mean the diameter rather than the radius, the number to report becomes $2r^* = 8$ nm, which is why the official key marked both 4 and 8 as correct. Using the standard radius definition, the answer is 4 nm.

$$\boxed{r^* = 4\ \text{nm}}$$

Quick Tip: Use the critical nucleus radius formula for homogeneous nucleation, $r^* = -2\gamma/\Delta G_v$, where γ is the interfacial energy and ΔG_v is the volumetric free energy of solidification.

. A 50 mm flat aluminium plate is reduced in thickness to 25 mm in a single pass cold-rolling operation with the rolling diameter of 1250 mm. For this case, the minimum required coefficient of friction between the plate and the roll (rounded off to two decimal places) is _____.

Correct Answer: 0.19 to 0.21

SOLUTION:

Step 1: Picture the roll-bite triangle directly.
Draw the roll of radius R meeting the plate at the entry point. The plate surface drops by half the draft, $\Delta h/2$, over the arc of contact, because the reduction is shared equally above and below the rolling centerline. This drop, the roll radius, and the bite angle α form a right triangle relation at the entry point: the horizontal leg is the chord along the roll surface and the vertical leg is the drop $\Delta h/2$, measured from the roll center.

Step 2: Recall the roll-bite result and check it makes physical sense.
Working through that triangle geometry with a small-angle approximation leads to the standard rolling-mechanics result: the rolls can just grip the plate, with no slipping, when

$$\mu_{min} = \sqrt{\frac{\Delta h}{R}}$$

where Δh is the draft and R is the roll radius. Before using it, check the limits: if $\Delta h \to R$ (an extremely aggressive, physically unrealistic single pass), $\mu_{min} \to 1$, which sits right at the practical upper bound for dry metal-on-metal friction. If $\Delta h \to 0$ (almost no reduction), $\mu_{min} \to 0$, which also makes sense since a very light pass needs almost no

grip to pull the plate through. The formula behaves sensibly at both ends, so it is safe to use here.

Step 3: Get the draft and the roll radius from the data.
The plate thins from 50 mm to 25 mm, so the draft is

$$\Delta h = 50 - 25 = 25 \text{ mm}$$

The roll diameter is 1250 mm, so the radius is half of that,

$$R = \frac{1250}{2} = 625 \text{ mm}$$

Both Δh and R are in millimeters, so their ratio is a pure number and no unit conversion is needed before taking the square root.

Step 4: Form the ratio.

$$\frac{\Delta h}{R} = \frac{25}{625} = 0.04$$

Step 5: Take the square root.

$$\mu_{min} = \sqrt{0.04}$$

Since $0.2 \times 0.2 = 0.04$, the square root is exactly 0.2.

Step 6: Conclude.
Rounded to two decimal places, $\mu_{min} = 0.20$, which sits inside the accepted range of 0.19 to 0.21. A coefficient of friction of about 0.2 is a

realistic, achievable value for a dry aluminium-on-steel roll contact, which is a good practical sanity check on the answer.

$$\boxed{\mu_{min} = 0.20}$$

Quick Tip: Use the rolling bite condition $\mu_{min} = \sqrt{\Delta h / R}$, with Δh the draft (thickness reduction) and R the roll radius.

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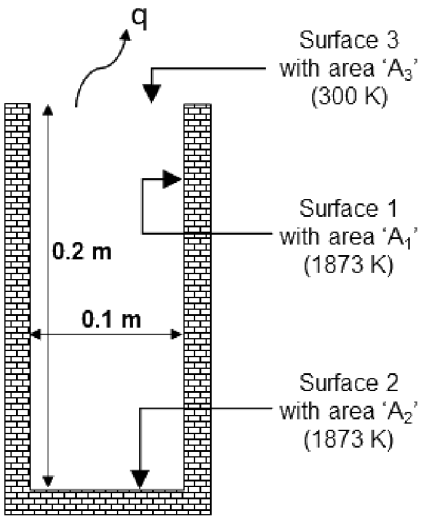

The figure shows a cylindrical furnace of inner diameter 0.1 m and height 0.2 m. The curved (lateral) side wall is surface A_1, the bottom circular surface is A_2, and the open top circular surface, facing the atmosphere, is A_3.

A cylindrical furnace has 0.1 m inner diameter and 0.2 m height. Walls A_1 (inner section of the cylindrical surface area) and A_2 (inner section of the bottom surface area) are maintained at 1873 K. The sides and bottom are assumed to be black bodies, well insulated and heated electrically. The top area (A_3) is open to the atmosphere maintained at 300 K, resulting in loss of heat 'q'.

Given: View factors $F_{13} = 0.1175$ and $F_{23} = 0.06$, where F_{ij} is the fraction of radiation leaving surface 'i' that is intercepted by surface 'j'.

Stefan-Boltzmann constant $= 5.67 \times 10^{-8}$ W/m^2-K^4.

The power needed to maintain the furnace at 1873 K, is (approximate to the nearest integer) _____ W.

Correct Answer: 5450 to 5510

SOLUTION:

Step 1: Spot a shortcut using view factor reciprocity.
Instead of multiplying out A_1F_{13} and A_2F_{23} separately and adding them, notice that the reciprocity rule $A_iF_{ij} = A_jF_{ji}$ lets us rewrite both terms from the point of view of surface 3:

$$A_1F_{13} = A_3F_{31}, \qquad A_2F_{23} = A_3F_{32}$$

So

$$A_1F_{13} + A_2F_{23} = A_3\,(F_{31} + F_{32})$$

Step 2: Use the summation rule at surface 3.
Surface 3 is flat, so it cannot see itself, meaning $F_{33} = 0$. Since surface 3 only "sees" surfaces 1 and 2 inside the furnace (the entire radiation leaving the open top either hits the side wall or the bottom, nothing escapes back onto itself), the view factors leaving surface 3 must add to 1:

$$F_{31} + F_{32} + F_{33} = 1 \quad \Rightarrow \quad F_{31} + F_{32} = 1$$

Step 3: Simplify the combined term.
Putting this back in,

$$A_1F_{13} + A_2F_{23} = A_3 \times 1 = A_3 = \pi r^2$$

With $r = 0.05$ m, $A_3 = \pi(0.05)^2 = 0.007854\ \text{m}^2$.

Step 4: Cross-check against direct multiplication.

As a check, work out A_1F_{13} and A_2F_{23} the long way and add them. With $A_1 = 2\pi(0.05)(0.2) = 0.06283\ \text{m}^2$ and $A_2 = \pi(0.05)^2 = 0.007854\ \text{m}^2$,

$$A_1F_{13} = 0.06283 \times 0.1175 = 0.007383\ \text{m}^2, \qquad A_2F_{23} = 0.007854 \times 0.06 = 0.0004712\ \text{m}^2$$

$$A_1F_{13} + A_2F_{23} = 0.007383 + 0.0004712 = 0.007854\ \text{m}^2$$

This matches A_3 from Step 3 exactly, confirming that the reciprocity shortcut and the brute-force calculation agree.

Step 5: Apply the radiation equation.
Since the whole heat balance collapses to just A_3, the power supplied to the furnace equals the black-body radiation exchange between the effective top opening at T_1 and the surroundings at T_3:

$$q = A_3\,\sigma\left(T_1^4 - T_3^4\right)$$

Step 6: Plug in the numbers.
$T_1^4 = 1873^4 = 1.230697 \times 10^{13}\ \text{K}^4$ and $T_3^4 = 300^4 = 0.0081 \times 10^{13}\ \text{K}^4$, so $T_1^4 - T_3^4 = 1.229887 \times 10^{13}\ \text{K}^4$.

$$q = 0.007854 \times 5.67 \times 10^{-8} \times 1.229887 \times 10^{13} \approx 5476.9\ \text{W}$$

Step 7: Conclude.
Rounded to the nearest whole number, $q \approx 5477$ W, inside the accepted band of 5450 to 5510 W. This is the electrical power the

heating elements must supply continuously to hold the furnace at 1873 K against the radiant loss through the open top.

$$\boxed{q \approx 5477\ \text{W}}$$

Quick Tip: Apply $q = (A_1F_{13} + A_2F_{23})\sigma(T_1^4 - T_3^4)$ for radiant heat loss between black surfaces, using the given view factors.

• • •

. During carburizing of a steel, the surface concentration is kept constant at 1.4 wt.% carbon. Diffusivity of carbon for the steel at 950°C is 6.25×10^{-11} m^2/s. At 950°C, the time required to carburize the steel with an initial composition of 0.2 wt.% carbon to 0.8859 wt.% carbon at a depth of 0.2 mm is _____ seconds (approximate to the nearest integer).
Use the nearest value of the error function from the table given below for your calculation.

z	erf(z)
0.3	0.3268
0.4	0.4284
0.5	0.5205

Correct Answer: 990 to 1010

SOLUTION:

Step 1: Write the fixed-surface diffusion formula in a slightly different form.
Instead of isolating $\sqrt{Dt}$ step by step, square both sides of the standard relation right away and solve for t directly. Starting from

$$\frac{C_s - C_x}{C_s - C_0} = \text{erf}(z), \qquad z = \frac{x}{2\sqrt{Dt}}$$

we can rearrange once, symbolically, to

$$t = \frac{x^2}{4Dz^2}$$

Step 2: Get the fraction reacted.

$$\frac{C_s - C_x}{C_s - C_0} = \frac{1.4 - 0.8859}{1.4 - 0.2} = \frac{0.5141}{1.2} = 0.4284$$

Step 3: Read z off the table.

The table lists $\text{erf}(0.4) = 0.4284$, which lines up exactly with the value from Step 2, so no interpolation between rows is needed: $z = 0.4$.

Step 4: Plug directly into the rearranged formula.

With $x = 2 \times 10^{-4}$ m and $D = 6.25 \times 10^{-11}$ m^2/s,

$$t = \frac{x^2}{4Dz^2} = \frac{(2 \times 10^{-4})^2}{4(6.25 \times 10^{-11})(0.4)^2} = \frac{4 \times 10^{-8}}{4 \times 10^{-11}} = 1000 \text{ s}$$

Step 5: Verify by plugging back in.

At $t = 1000$ s, $\sqrt{Dt} = \sqrt{6.25 \times 10^{-11} \times 1000} = \sqrt{6.25 \times 10^{-8}} = 2.5 \times 10^{-4}$ m, so $z = x/(2\sqrt{Dt}) = 2 \times 10^{-4}/(5 \times 10^{-4}) = 0.4$, which matches Step 3 and confirms the arithmetic.

Step 6: Conclude.

$$\boxed{t = 1000 \text{ s}}$$

This sits comfortably inside the accepted range of 990 to 1010 s.

Quick Tip: Use Fick's second law error-function solution $(C_s - C_x)/(C_s - C_0) = \text{erf}(x/(2\sqrt{Dt}))$ and match the left side against the given erf table to find the argument, then solve for t.

• • •

. For the following reaction between methane and stoichiometric air having composition 20 vol.% O_2 and 80 vol.% N_2, the estimated adiabatic flame temperature (rounded off to one decimal place) is _____ Kelvin.

$$CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O(g)$$

Given: $\Delta H = -850$ kJ/mol at 298 K. Assume the specific heat at constant pressure (C_p) for each reactant and product is 50 J/mol-K and is independent of temperature.

Correct Answer: 1842.5 to 1844.5

SOLUTION:

Step 1: Understanding the Question:
We must find the flame temperature reached when methane burns completely in stoichiometric air (not pure oxygen), given the reaction enthalpy and a constant C_p for every species involved.

Step 2: Key Formula or Approach:
Since air is only 20% oxygen by volume, we first find the total moles of

air needed, then split off the nitrogen that tags along and treat it as extra mass that also has to be heated. The adiabatic condition means

$$-\Delta H = n_{total}\, C_p\, \Delta T$$

Step 3: Detailed Explanation:

The reaction $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$ needs 2 mol of O_2 per mole of methane. Since air is 20% O_2 by volume,

$$\text{total air} = \frac{2}{0.20} = 10 \text{ mol}$$

Of this, the nitrogen fraction is 80%, so

$$N_2 = 0.80 \times 10 = 8 \text{ mol}$$

This matches the 4:1 ratio of N_2 to O_2 in the air, confirming the setup. The gas leaving the flame carries the products CO_2 (1 mol) and H_2O (2 mol), plus the inert N_2 (8 mol), giving

$$n_{total} = 1 + 2 + 8 = 11 \text{ mol}$$

All of them share $C_p = 50$ J/mol-K. Setting the heat released equal to the sensible heat gained,

$$850000 = 11(50)(T - 298)$$

$$T - 298 = 1545.45$$

$$T = 1843.5 \text{ K}$$

Step 4: Final Answer:

The adiabatic flame temperature comes out close to 1843.5 K.

$$\boxed{T \approx 1843.5 \text{ K}}$$

Quick Tip: Use the adiabatic energy balance |ΔH| = (total product plus inert moles) × Cp × (T minus 298), where N2 equals 4 times the O2 used.

• • •

. A copper ore contains 30 wt.% chalcopyrite ($CuFeS_2$) and the remaining gangue material. Assuming no copper is present in the gangue material, the amount of copper in the ore (rounded off to one decimal place) is _____ wt.%.

Given: Atomic weights of Fe, Cu and S are 56, 63.5, and 32 g/mol, respectively.

Correct Answer: 10.3 to 10.5

SOLUTION:

Step 1: Understanding the Concept:

We are told what fraction of the ore is the copper mineral chalcopyrite, and we need the copper content of the whole ore. Working with an actual mass basis makes the arithmetic concrete.

Step 2: Key Formula or Approach:

Take 100 kg of ore as the basis. Find how much chalcopyrite that contains, then use the atomic weights to split that chalcopyrite mass into its Cu, Fe and S parts.

Step 3: Detailed Explanation:
In 100 kg of ore, the chalcopyrite present is

$$100 \times 0.30 = 30 \text{ kg}$$

The formula weight of $CuFeS_2$ is

$$63.5 + 56 + 2(32) = 183.5 \text{ g/mol}$$

Out of every 183.5 kg of chalcopyrite, 63.5 kg is copper. So the copper inside our 30 kg of chalcopyrite is

$$30 \times \frac{63.5}{183.5} = 30 \times 0.3460 = 10.38 \text{ kg}$$

Since our basis was 100 kg of ore, this mass of copper is directly the weight percent of copper in the ore.

Step 4: Final Answer:
The copper content of the ore is close to 10.4 wt.%.

$$\boxed{10.4 \text{ wt.\%}}$$

Quick Tip: Find the mass fraction of Cu inside CuFeS2 using the atomic weights, then scale it by the 30 wt.% chalcopyrite content of the ore.

. A blast furnace produces hot metal with the following composition: 4 wt.% C, 1.5 wt.% Si, and the rest Fe.
The only input of iron is through iron ore containing 85 wt.% Fe_2O_3 and 15 wt.% gangue consisting of SiO_2 and Al_2O_3.
2% of all the iron (by weight) is lost in the slag.
The amount of ore used to produce 1000 kg of hot metal (rounded off to one decimal place) is _____ kg.
Given: Atomic weights of Fe and O are 56 g/mol and 16 g/mol, respectively.

Correct Answer: 1615.5 to 1625.5

SOLUTION:

Step 1: Understanding the Concept:
We need the mass of ore that, after losing some iron to slag and being reduced from Fe_2O_3 to metallic Fe, ends up giving the iron content of 1000 kg of hot metal.

Step 2: Key Formula or Approach:
Build one chain: ore mass to Fe_2O_3 mass to iron charged to iron surviving to hot metal, matching the Fe needed in 1000 kg hot metal. Write it as a single equation and solve for ore mass W.

$$W \times 0.85 \times \frac{112}{160} \times 0.98 = Fe_{needed}$$

Step 3: Detailed Explanation:
Since hot metal is 4% C and 1.5% Si, the iron share is

$$Fe_{needed} = 1000 \times (1 - 0.04 - 0.015) = 945 \text{ kg}$$

The combined yield factor from ore to hot metal iron is

$$0.85 \times \frac{112}{160} \times 0.98 = 0.85 \times 0.70 \times 0.98 = 0.5831$$

This single number tells us that out of every kg of ore, only **0.5831** kg ends up as iron in the hot metal.

Step 4: Final Answer:
Solving for W,

$$W = \frac{945}{0.5831} = 1620.6 \text{ kg}$$

So about **1620.6** kg of ore is needed per **1000** kg of hot metal.

$$\boxed{W \approx 1620.6 \text{ kg}}$$

Quick Tip: Track the iron in three stages: the Fe mass in hot metal, the iron charged before the 2% slag loss, and the Fe2O3/ore mass needed to supply that iron.

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. Al_2O_3 can be electrolyzed with an inert anode or a carbon anode.
Overall reactions are as follows:
Reaction I: For inert anode,

$$\frac{2}{3}Al_2O_3(s) \rightarrow \frac{4}{3}Al(l) + O_2(g); \quad \Delta G^\circ \text{ (in Joules)} = 1124800 - 218T$$

Reaction II: For carbon anode,

$$\frac{2}{3}Al_2O_3(s) + C(s) \rightarrow \frac{4}{3}Al(l) + CO_2(g); \quad \Delta G^\circ \text{ (in Joules)} = 730700 - 218T$$

T denotes temperature in Kelvin.
Given: Faraday constant = 96500 Coulomb.
If the inert anode is replaced by a carbon anode during electrolysis of Al_2O_3 at temperature 1300 K, the decrease in the magnitude of decomposition potential between the two reactions (rounded off to two decimal places) is _____ Volts.

Correct Answer: 1.00 to 1.05

SOLUTION:

Step 1: Understanding the Concept:
Using a carbon anode instead of an inert one changes the overall cell reaction, because the anode itself now reacts with the oxygen released, forming CO_2 instead of O_2. This lowers the Gibbs free energy needed to drive the electrolysis, and hence lowers the decomposition potential.

Step 2: Key Formula or Approach:
Rather than working out E_1 and E_2 separately from scratch, we can

find the drop in Gibbs energy directly and divide once by nF, since n and F are the same in both reactions.

$$\Delta(\Delta G^\circ) = \Delta G_1^\circ - \Delta G_2^\circ$$

$$\Delta E = \frac{\Delta(\Delta G^\circ)}{nF}$$

Step 3: Detailed Explanation:
At $T = 1300$ K,

$$\Delta G_1^\circ = 1124800 - 218(1300) = 841400 \text{ J}$$

$$\Delta G_2^\circ = 730700 - 218(1300) = 447300 \text{ J}$$

The drop between the two is

$$\Delta(\Delta G^\circ) = 841400 - 447300 = 394100 \text{ J}$$

Each reaction transfers $n = 4$ electrons per mole of reaction as written, since $\frac{4}{3}$ mol of Al needs 4 mol of electrons. So

$$\Delta E = \frac{394100}{4 \times 96500} = \frac{394100}{386000} = 1.0210 \text{ V}$$

Step 4: Final Answer:

Switching to the carbon anode drops the decomposition potential by about 1.02 V.

$$\boxed{\Delta E \approx 1.02\ \text{V}}$$

Quick Tip: Find n=4 electrons from the 4/3 mol of Al produced, convert each ΔG° into E = -ΔG°/(nF) at T=1300 K, then take the difference in magnitude.

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. Given a scalar field $\phi(x, y, z) = x^2 - yz$.
Magnitude of a vector in the direction of the most rapid increase of ϕ at a point P(3,4,1), rounded off to two decimal places, is _____

Correct Answer: 7.08 to 7.48

SOLUTION:

Step 1: Understanding the Concept:
The steepest climb of a scalar field at any point points along its gradient vector, and the size of that steepest climb equals the magnitude of the gradient there. So the question is really asking for $|\nabla\phi|$ at $P(3, 4, 1)$.

Step 2: Key Formula or Approach:

$$\nabla\phi = \phi_x\,\hat{i} + \phi_y\,\hat{j} + \phi_z\,\hat{k}, \qquad |\nabla\phi| = \sqrt{\phi_x^2 + \phi_y^2 + \phi_z^2}$$

Step 3: Detailed Explanation.

Differentiate $\phi = x^2 - yz$ term by term. Treating y and z as constants while differentiating with respect to x gives $\phi_x = 2x$. Treating x and z as constants while differentiating with respect to y gives $\phi_y = -z$, since $-yz$ drops its y and leaves behind $-z$. In the same way, $\phi_z = -y$. Now plug in the point $P(3, 4, 1)$ directly:

$$\phi_x = 2(3) = 6, \qquad \phi_y = -(1) = -1, \qquad \phi_z = -(4) = -4$$

The gradient at P is the vector $(6, -1, -4)$. Its length is found by summing the squares of its components and taking the square root:

$$|\nabla\phi| = \sqrt{6^2 + 1^2 + 4^2} = \sqrt{36 + 1 + 16} = \sqrt{53}$$

$$\sqrt{53} \approx 7.28$$

Step 4: Final Answer:

The magnitude of the fastest-increase direction vector at P is about 7.28.

$$\boxed{7.28}$$

Quick Tip: The direction of steepest increase of a scalar field is its gradient; find the magnitude of grad phi at P(3,4,1).