GATE 2026 Physics Question Paper with Solutions

General Instructions

(i) This booklet contains 65 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 38 single-correct multiple-choice questions, 10 multiple-correct questions and 17 numerical / integer-type questions.

(iii) Attempt each question on your own before reviewing the given solution.

(iv) For multiple-correct questions, all correct options must be selected to earn full marks.

(v) For numerical questions, report the answer rounded exactly as asked.

. "He often _____ the numbers. False claims are not going to help. Honesty _____ trust", said the manager.
Choose the option with the correct order of words to fill the blanks.

(A) exaggerates; engenders

(B) excels; encourages

(C) aggravates; alleviates

(D) diminishes; eliminates

Correct Answer: (A) exaggerates; engenders

Solution:

Step 1: Read what the first blank needs to do.

The sentence says "he often _____ the numbers" and is followed by "False claims are not going to help." This tells us his habit with numbers is dishonest, he makes them look bigger or better than they really are. So the first blank needs a word close in meaning to

overstating or inflating something.

Step 2: Read what the second blank needs to do.
The manager then says "Honesty _____ trust." This is a general statement about a good quality producing a good result. The blank needs a verb meaning to create or bring about, so that "honesty" is shown as the cause of "trust."

Step 3: Check option (A).
"Exaggerates" means to make something seem larger or more important than it actually is, which fits "he often exaggerates the numbers" and matches the idea of false claims perfectly. "Engenders" means to give rise to or produce, so "honesty engenders trust" reads as a clean cause and effect. Both halves work, so (A) is correct.

Step 4: Check option (B).
"Excels" normally takes "at" or "in" something, as in "excels at his job," not a direct object like "the numbers." So "he often excels the numbers" is not natural English, even though "honesty encourages trust" sounds fine by itself. Since one half fails, (B) is wrong.

Step 5: Check option (C).
"Aggravates" means to make a problem worse, but numbers are not a problem, so "aggravates the numbers" does not fit. "Alleviates" means to reduce something unpleasant, and trust is not unpleasant, so "honesty alleviates trust" makes no sense either. Both halves fail, so (C) is wrong.

Step 6: Check option (D).
"Diminishes the numbers" would mean he makes the figures smaller, which does not connect with "false claims," since shrinking a number

is not the same as lying about it. "Honesty eliminates trust" is the opposite of what honesty actually does. Both halves fail, so (D) is wrong.

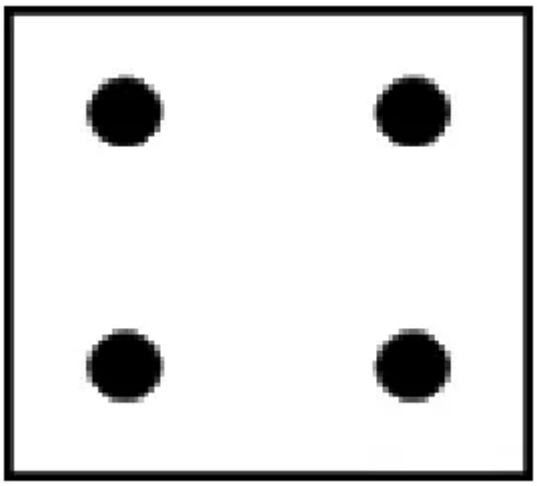
Final Answer:
Only "exaggerates; engenders" keeps both sentences logical, matching the manager's point that false claims (exaggeration) hurt trust, while honesty builds (engenders) it.

Option (A)

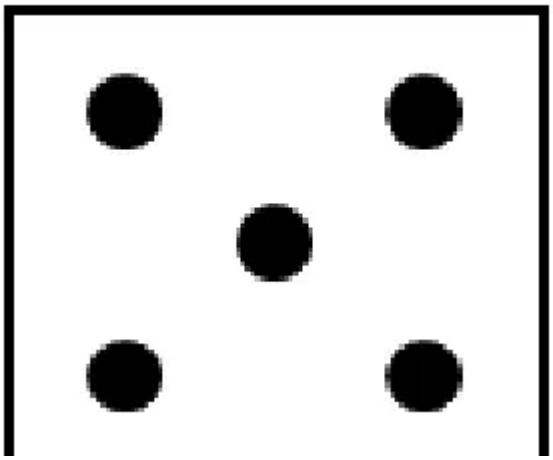
Quick Tip: **Hint:**
Look at what "false claims are not going to help" tells you about the first blank, then find a word for the second blank that shows honesty creating a good outcome.

. In the sequence of tiles shown below, the missing tile indicated by the question mark should be:

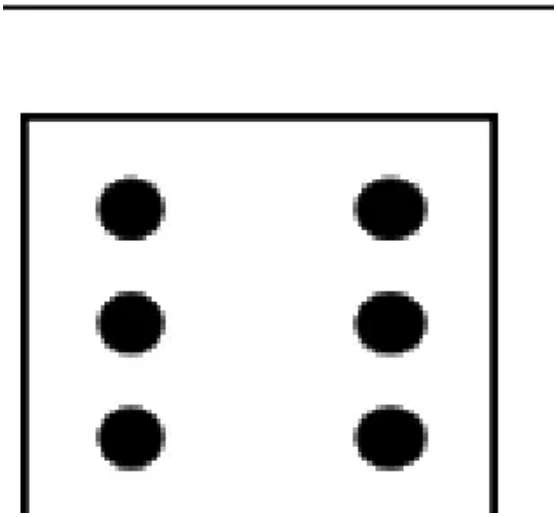
(A)



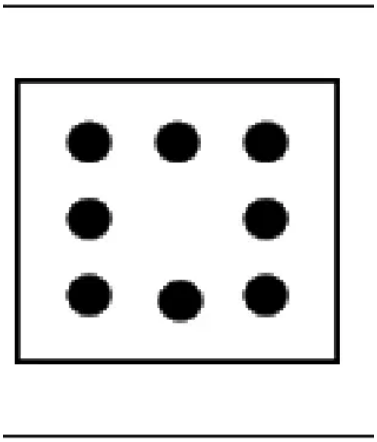
(B)



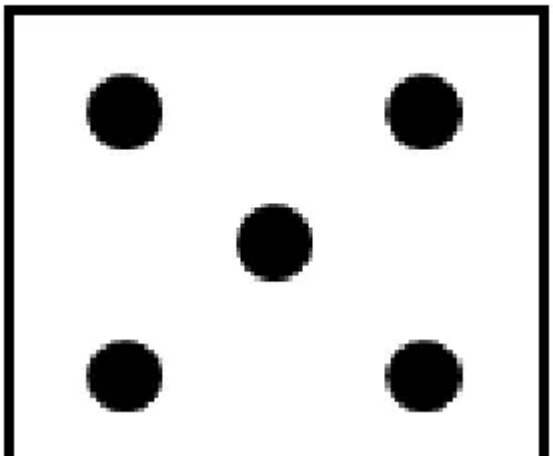
(C)



(D)



Correct Answer: (B)



Solution:

Step 1: Count the dots in each tile.

Reading the row left to right: tile 1 has 2 dots, tile 2 has 3 dots, tile 3 is missing, tile 4 has 7 dots (three across the top, one in the middle, three across the bottom), and tile 5 has 11 dots (four across the top, three in the middle row, four across the bottom). So the known totals in order are 2, 3, ?, 7, 11.

Step 2: Recognize the sequence.

2, 3, 7, and 11 are each prime numbers, numbers with no divisors other than 1 and themselves. Reading them as consecutive primes: 2 is the 1st prime, 3 is the 2nd prime, 5 is the 3rd prime, 7 is the 4th prime, 11 is the 5th prime. Tile 3 sits in the 3rd position, so it should carry the 3rd prime number, which is 5.

Step 3: Match this to the dot counts shown in the options.

Option (A) shows 4 dots, option (B) shows 5 dots arranged as 2 dots on top, 1 dot in the middle, and 2 dots on the bottom, option (C) shows 6 dots, and option (D) shows 8 dots. Only option (B) carries 5 dots, matching the missing prime.

Step 4: Rule out the remaining options.

4 is not prime, since it splits evenly as 2 times 2. 6 is not prime, since it splits evenly as 2 times 3. 8 is not prime, since it splits evenly as 2 times 4. None of these fit the prime-number rule that ties tiles 1, 2, 4, and 5 together, so only the 5-dot tile completes the sequence.

Final Answer:

The dot counts across the row are the first five prime numbers, 2, 3, 5, 7, 11, so the missing tile has 5 dots, matching option (B).

Option (B)

Quick Tip: **Hint:**
Count the dots in the tiles you can already see (2, 3, then a gap, then 7, 11) and check whether these numbers share a special property, such as all being prime, rather than following a simple addition pattern.

. A school has 100 students distributed among 1st to 10th standards. Based on this, which one of the following statements is always correct?

(A) There are at least 10 students who belong to the same standard.

(B) There is at least one student in each standard.

(C) There are at most 10 students in 10th standard.

(D) The total number of students from 1st to 5th standards is at least 50.

Correct Answer: (A) There are at least 10 students who belong to the same standard.

Solution:

Step 1: Understand what "always correct" means here.
The question does not tell us how the 100 students are actually spread across the 10 standards, it could be any distribution. So a statement is "always correct" only if it holds true for every possible way of splitting 100 students into 10 groups, not just for one convenient example.

Step 2: Apply the idea behind the pigeonhole principle.
The pigeonhole principle says that if you place n items into k boxes, at least one box must contain at least $\lceil n/k \rceil$ items, the smallest whole number that is greater than or equal to n/k. Here $n = 100$ students

and $k = 10$ standards.

Step 3: Compute the guaranteed minimum.

$$\frac{100}{10} = 10$$

Since 100 divides evenly by 10, the average number of students per standard is exactly 10. If every standard had strictly fewer than 10 students, the total across all 10 standards would be strictly less than $10 \times 10 = 100$, which contradicts the fact that there are exactly 100 students in total. So at least one standard must have 10 or more students. This is exactly what option (A) states, so (A) is always true.

Step 4: Check option (B) with a counterexample.
Suppose all 100 students belong to the 1st standard and none belong to standards 2 through 10. This still adds up to 100 students, so it is a valid distribution, but several standards have zero students. So (B) is not always true.

Step 5: Check option (C) with a counterexample.
Suppose all 100 students belong to the 10th standard. This is a valid distribution of 100 students, but it clearly has more than 10 students in the 10th standard. So (C) is not always true.

Step 6: Check option (D) with a counterexample.
Suppose all 100 students belong to standards 6 through 10, with none in standards 1 through 5. The total from 1st to 5th standards is then 0, which is far less than 50. So (D) is not always true.

Final Answer:

Only option (A) is forced to be true no matter how the 100 students are actually distributed among the 10 standards.

Option (A)

Quick Tip: **Hint:**
Try to build a distribution of 100 students across 10 standards that breaks each statement. The one statement you cannot break, no matter how you arrange the students, is the answer.

. How many 3-digit numbers can be formed using three distinct single digit prime numbers?

(A) 64

(B) 24

(C) 12

(D) 4

Correct Answer: (B) 24

Solution:

Step 1: List the single digit prime numbers.
The single digit numbers are 0 through 9, and the prime numbers among them are 2, 3, 5, and 7. So there are exactly 4 digits available to build the 3-digit number from.

Step 2: Understand what the question is asking for.
We need 3-digit numbers where all three digits are distinct and each digit is one of 2, 3, 5, 7. Since we must pick 3 different digits out of these 4 and then arrange them, this is a counting problem in two

parts, first choose which 3 digits to use, then decide the order they appear in.

Step 3: Choose which 3 of the 4 primes to use.
The number of ways to choose 3 digits out of 4 available digits is given by the combination formula:

$$\binom{4}{3} = \frac{4!}{3! \times 1!} = 4$$

So there are 4 different sets of 3 digits that could be used, for example {2,3,5}, {2,3,7}, {2,5,7}, and {3,5,7}.

Step 4: Arrange each chosen set of 3 digits.
Once a set of 3 distinct digits is picked, they can be arranged in any order to form a 3-digit number. The number of ways to arrange 3 distinct digits is:

$$3! = 3 \times 2 \times 1 = 6$$

Since none of 2, 3, 5, 7 is zero, every arrangement is a valid 3-digit number, there is no risk of a leading zero making the number only 2 digits long.

Step 5: Combine the choices and the arrangements.
Each of the 4 possible sets of digits can be arranged in 6 different orders, so the total count is:

$$4 \times 6 = 24$$

Step 6: Rule out the other options.

Option (A), 64, would come from allowing digits to repeat, for example choosing freely 3 times from 4 digits ($4^3 = 64$), but the question asks for three distinct digits, so repetition is not allowed here. Option (C), 12, undercounts, it misses some valid orderings. Option (D), 4, only counts the number of digit sets, without accounting for the different orders each set can be arranged in.

Final Answer:

Choosing 3 out of 4 single digit primes and arranging them gives $4 \times 6 = 24$ distinct 3-digit numbers.

$\boxed{24}$

Quick Tip: **Hint:**

First find how many single digit prime numbers exist, then work out how many ways you can pick 3 of them and arrange those 3 in order.

. In a group of students, 10 students like Mathematics, 12 students like English, 4 students like both Mathematics and English, and 6 students like neither Mathematics nor English. The number of students in the group is

_____

(A) 18

(B) 20

(C) 24

(D) 32

Correct Answer: (C) 24

Solution:

Step 1: Define the sets involved.

Let M be the set of students who like Mathematics, and E be the set of students who like English. We are given $|M| = 10$, $|E| = 12$, and $|M \cap E| = 4$, the number of students who like both subjects. We are also told 6 students like neither subject.

Step 2: Recall the formula for the size of a union of two sets.

When counting how many students like Mathematics or English (or both), we cannot just add $|M|$ and $|E|$ directly, because the students who like both would then be counted twice. The correct formula subtracts the overlap once:

$$|M \cup E| = |M| + |E| - |M \cap E|$$

Step 3: Substitute the given numbers.

$$|M \cup E| = 10 + 12 - 4 = 18$$

So 18 students like at least one of the two subjects, Mathematics or English or both.

Step 4: Add in the students who like neither subject.

The total group of students is made up of those who like at least one subject, plus those who like neither. Since 6 students fall into the "neither" category and do not overlap with the 18 students already counted:

$$\text{Total} = |M \cup E| + \text{neither} = 18 + 6 = 24$$

Step 5: Check why the other options do not fit.
Option (A), 18, is only the count of students who like at least one subject, it forgets to add the 6 students who like neither. Option (B), 20, would come from a wrong union calculation, for instance forgetting to subtract the overlap correctly. Option (D), 32, would come from wrongly adding $|M| + |E| + \text{neither}$ without subtracting the overlap at all, which double counts the 4 students who like both subjects.

Final Answer:
Adding the 18 students who like at least one subject to the 6 students who like neither gives a total group size of 24.

$$\boxed{24}$$

Quick Tip: **Hint:**
First work out how many students like at least one of the two subjects using the overlap, then remember to add on the students who like neither subject.

. Charity : P :: Retaliation : Q

Choose the appropriate pair of words P and Q that fit the analogy.

(A) P = Parsimonious; Q = Vengeful

(B) P = Altruistic; Q = Amicable

(C) P = Resentful; Q = Spiteful

(D) P = Magnanimous; Q = Vindictive

Correct Answer: (D) P = Magnanimous; Q = Vindictive

Solution:

Step 1: Understand the relationship.
The word "Charity" describes an act of giving generously to those in need. The blank P should be a single adjective that captures a person who behaves with such generosity as a personality trait.

Step 2: Find the matching adjective for Charity.
The word "Magnanimous" means very generous and forgiving, especially towards a rival or someone less powerful. This is the closest single-word match for a person defined by charity, so P = Magnanimous.

Step 3: Find the matching adjective for Retaliation.
"Retaliation" means hitting back at someone who has wronged you. The word "Vindictive" describes a person who has a strong and often unreasonable desire for revenge, which is the exact personality match for retaliation. So Q = Vindictive.

Step 4: Why the other options are wrong.
In option (A), "Parsimonious" means stingy or unwilling to spend money, which is the opposite of charitable, so P is wrong there. In option (B), "Altruistic" does mean unselfish, matching charity, but "Amicable" means friendly, which has nothing to do with retaliation, so Q fails. In option (C), "Resentful" means feeling bitter, which could loosely fit retaliation, but neither word in that pair describes generosity, so the P side of the pair breaks down.

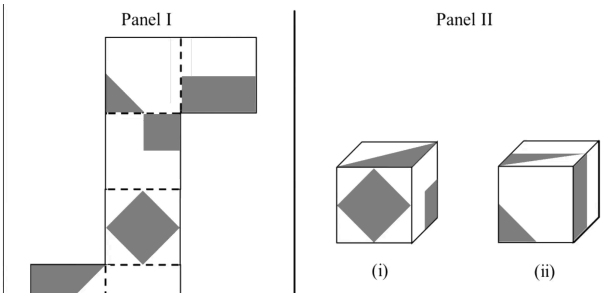
Final Answer:

Magnanimous captures a person defined by generous charity, and Vindictive captures a person defined by retaliation, so option (D) is correct.

$$\boxed{\text{P = Magnanimous, Q = Vindictive}}$$

Quick Tip: Find the single adjective that best describes extreme charity, then find its parallel single adjective for extreme retaliation, meaning a lasting character trait, not just a passing angry reaction.
Match the pair where both words are strong, single-word character traits.

. A paper shown in Panel I is folded along the dashed lines (- - -) to construct a cube. The shaded regions shown in Panel I appear on the outer surface of the cube.



Referring to cubes shown in Panel II, which one of the options is correct?

(A) Only (i) can correspond to the unfolded cube in Panel I.

(B) Only (ii) can correspond to the unfolded cube in Panel I.

(C) Both (i) and (ii) can correspond to the unfolded cube in Panel I.

(D) Neither (i) nor (ii) can correspond to the unfolded cube in Panel I.

Correct Answer: (A) Only (i) can correspond to the unfolded cube in Panel I.

Solution:

Step 1: Label the six faces of the net.
Panel I is a strip of six unit squares joined by the marked dashed fold lines: the top row has a square with a triangle shaded in its lower-left corner (face 1) joined to a fully shaded square (face 2); hanging from that seam is a smaller square carrying a small grey square patch (face 3); below face 3 sits a square with a large shaded diamond (face 4); and the bottom row has a square with a large triangle shaded from its top-left to bottom-right corner (face 5) joined to a square with a small triangle shaded near its bottom-right corner (face 6).

Step 2: Work out which faces sit next to which once folded.
Each dashed line is a hinge, so folding brings every pair of squares joined by a dashed line onto two faces of the cube that share an edge. Face 4, the diamond, sits between face 3 above it and faces 5 and 6 below it, so once folded, the diamond face becomes the front face of the cube, and the triangle patterns on faces 1, 2, 5 and 6 land on the faces that surround it, each keeping the same corner of its shaded triangle touching the same shared edge it touched in the flat net.

Step 3: Check cube (i) against this layout.
In cube (i), the diamond sits on the front face exactly as face 4 predicts. The triangle on the top face meets the front-top edge at the same corner that face 1's triangle meets the face 1 to face 4 fold line in the net, and the sliver of shading on the right face lines up with face 6's small triangle at the same shared corner. Every shaded region keeps the corner and edge contact it had in the flat net, so cube (i) is a

genuine folding of Panel I.

Step 4: Check cube (ii) against this layout.
In cube (ii), the diamond again sits correctly on the front face, but the triangle on the top face is shaded on the opposite side of its diagonal compared with face 1 in the net. A simple fold along a shared edge cannot flip a triangle to the other side of its own diagonal while keeping the diamond fixed, so this cube changes the shape that should appear on the top face. That mismatch rules out cube (ii).

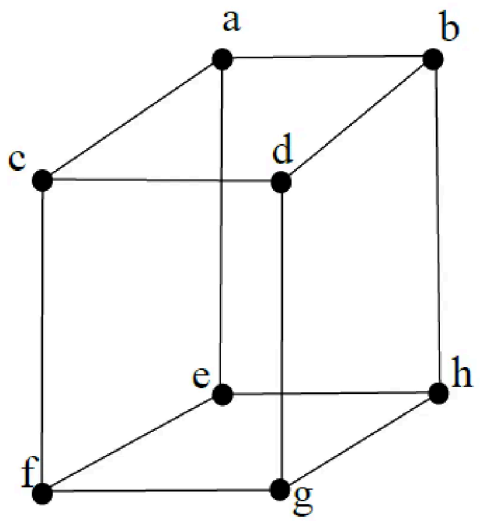
Step 5: Decide between the four options.
Since cube (i) reproduces every face of Panel I correctly and cube (ii) does not, the option that both cubes work and the option that neither works are both ruled out.

Final Answer:
Only cube (i) matches a genuine folding of the net in Panel I, so option (A) is correct.

Option (A): Only (i)

Quick Tip: Track the diamond-shaded face first, since it appears in the same position on both candidate cubes.
Then check whether the triangle-shaded faces around it keep the same corner touching the same fold line as in the flat net; a fold can never flip a shaded triangle to the other side of its diagonal.

. Consider the cube shown below with its 8 corners labelled a, b, c, d, e, f, g, and h. The figure is representative.



All corners are to be colored such that any two corners that are connected by an edge must be of different colors. The minimum number of colors required to achieve this is _____

(A) 8

(B) 4

(C) 3

(D) 2

Correct Answer: (D) 2

Solution:

Step 1: Understanding the Concept:

This is a graph coloring problem. The 8 corners of the cube are the vertices of a graph, and the 12 edges of the cube are the edges of that graph. We need the smallest number of colors so that every pair of corners joined by an edge gets different colors; this smallest number

is called the chromatic number of the graph.

Step 2: Key Formula or Approach:
Label each corner of the cube with 3 binary coordinates, one bit for each direction x, y, z, so the 8 corners become the 8 strings $000, 001, 010, 011, 100, 101, 110, 111$. Two corners are joined by a cube edge exactly when their labels differ in exactly one bit, that is, moving along one direction only.

Step 3: Detailed Explanation:
Split the 8 corners into two groups by the parity of the number of 1s in their label: group X, an even count of 1s ($000, 011, 101, 110$), and group Y, an odd count of 1s ($001, 010, 100, 111$). Since every edge changes exactly one bit, it always changes the count of 1s by exactly 1, which flips even to odd or odd to even. So every edge of the cube joins one corner from group X to one corner from group Y, and never joins two corners from the same group. Color every corner in group X with color 1 and every corner in group Y with color 2, and no edge connects two same-colored corners.

Step 4: Why fewer than 2 colors cannot work.
The cube has at least one edge, and both its ends need different colors, so 1 color is never enough for any graph that has an edge. This means 2 is the smallest possible number of colors, not just a number that happens to work.

Step 5: Why the other options are wrong.
Options (A) 8 and (B) 4 both use more colors than needed. Giving every corner its own separate color, or splitting into 4 colors, both satisfy the different-color-on-every-edge rule, since using more distinct colors than required never breaks a valid coloring, but the

question asks for the minimum. Option (C) 3 also gives a valid coloring but is still not minimal, since Step 3 already shows the rule can be met with only 2 colors.

Final Answer:
The cube's corners can be split into exactly two groups so that every edge joins the two groups, so the minimum number of colors needed is 2.

$\boxed{2}$

Quick Tip: Label each corner by how many of its 3 coordinates are 1, then color by whether that count is even or odd, like a 3D checkerboard.
Check whether any single edge of the cube ever joins two corners that share the same parity.

. Four hills H1, H2, H3, and H4 are present in an area. The following observations are made about them:

i. Neither H2 nor H3 is the easternmost hill.
ii. Neither H2 nor H3 is the westernmost hill.
iii. Neither the easternmost hill nor the westernmost hill is the southernmost hill.
iv. Two hills are located to the west of H2.
v. The southernmost hill has at least two hills to its east.

The southernmost hill is _____.

(A) H1

(B) H2

(C) H3

(D) H4

Correct Answer: (C) H3

Solution:

Step 1: Set up positions along the west-to-east line.
Since the clues talk about "easternmost", "westernmost", and hills "to the west or east" of each other, place the four hills at 4 positions along a line, numbered 1 (westernmost) to 4 (easternmost).

Step 2: Use clues (i) and (ii) to place H2 and H3.
Clue (i) says neither H2 nor H3 is at position 4, and clue (ii) says neither is at position 1. So H2 and H3 must occupy the two middle positions, 2 and 3, in some order, which forces H1 and H4 to occupy positions 1 and 4 in some order.

Step 3: Use clue (iv) to fix H2's exact position.
Clue (iv) says exactly two hills lie to the west of H2. If H2 were at position 2, only 1 hill (position 1) lies west of it, which is too few. If H2 were at position 3, exactly 2 hills (positions 1 and 2) lie to its west, matching the clue. So H2 is at position 3, which leaves position 2 for H3.

Step 4: Use clue (iii) to rule out the ends.
Clue (iii) says the southernmost hill is not the easternmost or the westernmost hill. So the southernmost hill must be H3 (position 2) or H2 (position 3), since those are the only two positions left.

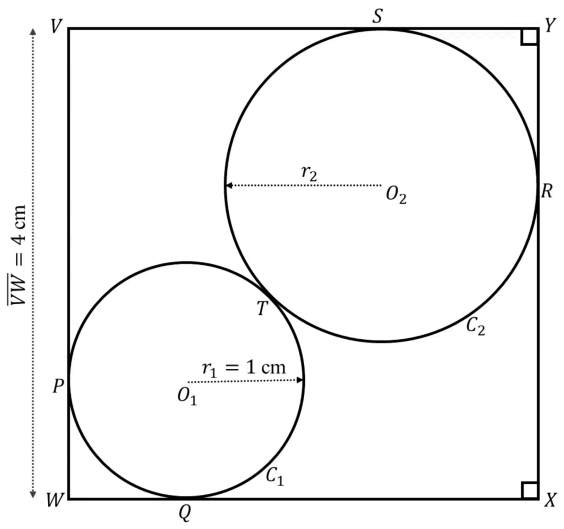
Step 5: Use clue (v) to choose between H3 and H2.
Clue (v) says the southernmost hill has at least two hills to its east. Position 2 (H3) has positions 3 and 4 to its east, which is 2 hills, so it satisfies the clue. Position 3 (H2) has only position 4 to its east, just 1 hill, which fails the at-least-two requirement. So the southernmost hill cannot be H2.

Final Answer:
Only H3 satisfies every clue, so H3 is the southernmost hill.

H3

Quick Tip: First place all four hills in a west-to-east order using clues (i), (ii) and (iv); this pins down H2 and H3 to the two middle spots and fixes H2 at position 3.
Then use clues (iii) and (v) to test which of the two middle hills can be the southernmost one.

. As shown in the figure, circle C_1 with center O_1 and radius r_1 touches the square $VWXY$ at points P and Q while circle C_2 with center O_2 and radius r_2 touches the square $VWXY$ at points R and S. The two circles touch each other at T.



Given $r_1 = 1$ cm and $\overline{VY} = \overline{VW} = 4$ cm, $r_2 =$ _____ cm.

(A) $4 - 3\sqrt{2}$

(B) $1 + 2\sqrt{2}$

(C) $7 - 4\sqrt{2}$

(D) $5 + 3\sqrt{2}$

Correct Answer: (C) $7 - 4\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

Circle C_1 touches two adjacent sides of the square, the left side and the bottom side, so its center O_1 must be at a distance r_1 from each of those two sides. In the same way, circle C_2 touches the other two adjacent sides, the top side and the right side, so its center O_2 must be

at a distance r_2 from each of those sides. Since the two circles touch each other, the distance between their centers equals the sum of their radii.

Step 2: Key Formula or Approach:

Place the square on coordinates with $W = (0,0)$, $X = (4,0)$, $V = (0,4)$, $Y = (4,4)$, since the side length is 4 cm. Then $O_1 = (r_1, r_1)$ and $O_2 = (4 - r_2, 4 - r_2)$. The circles touching each other externally at T means:

$$O_1O_2 = r_1 + r_2$$

Step 3: Detailed Explanation:

With $r_1 = 1$, $O_1 = (1,1)$. The difference between the two centers along each axis is the same: $3 - r_2$. So the squared distance between the centers is:

$$O_1O_2^2 = (3 - r_2)^2 + (3 - r_2)^2 = 2(3 - r_2)^2$$

Setting this equal to $(r_1 + r_2)^2 = (1 + r_2)^2$:

$$2(3 - r_2)^2 = (1 + r_2)^2$$

Taking the positive square root of both sides, since $r_2 < 3$ makes both $3 - r_2$ and $1 + r_2$ positive:

$$\sqrt{2}(3 - r_2) = 1 + r_2$$

Step 4: Solve for r_2.

Expand and collect the r_2 terms on one side:

$$3\sqrt{2} - \sqrt{2}r_2 = 1 + r_2$$

$$3\sqrt{2} - 1 = r_2(1 + \sqrt{2})$$

$$r_2 = \frac{3\sqrt{2} - 1}{1 + \sqrt{2}}$$

Multiply the top and bottom by $(\sqrt{2} - 1)$ to remove the surd from the bottom:

$$r_2 = \frac{(3\sqrt{2} - 1)(\sqrt{2} - 1)}{(1 + \sqrt{2})(\sqrt{2} - 1)} = \frac{6 - 3\sqrt{2} - \sqrt{2} + 1}{2 - 1} = 7 - 4\sqrt{2}$$

Step 5: Sanity check the number.

$7 - 4\sqrt{2} \approx 7 - 5.657 = 1.343$ cm. This is positive and less than 3 cm, so the value is consistent with the figure, where circle C_2 is clearly larger than circle C_1 but still fits inside the square.

Final Answer:

The radius of circle C_2 works out to $7 - 4\sqrt{2}$ cm.

$$\boxed{r_2 = 7 - 4\sqrt{2} \text{ cm}}$$

Quick Tip: Place the square on coordinates so O_1 sits at distance r_1 from two adjacent sides and O_2 sits at distance r_2 from the other two adjacent sides.
Use the fact that the circles touch, so the distance between O_1 and O_2 equals $r_1 + r_2$, and solve the resulting equation for r_2.

. In free space, an electromagnetic wave is travelling whose wavevector is $\vec{k} = 10(\hat{x} + \sqrt{3}\hat{y})$ m^{-1}. The electric field component of this electromagnetic wave is given by $\vec{E}(\vec{r}, t) = \hat{z}\,600\cos(\vec{k}\cdot\vec{r} - \omega t)$ V.m^{-1}. The speed of light in free space is $c = 3.0 \times 10^8$ m.s^{-1}. The corresponding magnetic field $\vec{B}(\vec{r}, t)$ is

(A) $\vec{B}(\vec{r}, t) = 2 \times 10^{-6}(\sqrt{3}\hat{x} - \hat{y})\cos(\vec{k}\cdot\vec{r} - \omega t)$ V.m^{-2}.s

(B) $\vec{B}(\vec{r}, t) = 10^{-6}(\sqrt{3}\hat{x} - \hat{y})\cos(\vec{k}\cdot\vec{r} - \omega t)$ V.m^{-2}.s

(C) $\vec{B}(\vec{r}, t) = 2 \times 10^{-5}(-\sqrt{3}\hat{x} + \hat{y})\cos(\vec{k}\cdot\vec{r} - \omega t)$ V.m^{-2}.s

(D) $\vec{B}(\vec{r}, t) = 10^{-5}(\sqrt{3}\hat{x} - \hat{y})\cos(\vec{k}\cdot\vec{r} - \omega t)$ V.m^{-2}.s

Correct Answer: (B) $\vec{B}(\vec{r}, t) = 10^{-6}(\sqrt{3}\hat{x} - \hat{y})\cos(\vec{k}\cdot\vec{r} - \omega t)$ V.m^{-2}.s

Solution:

Step 1: Understanding the Concept:
An electromagnetic plane wave in free space has its electric field, magnetic field, and direction of travel mutually perpendicular. Once we know the wave's direction (given by the wavevector $\vec{k}$) and the electric field $\vec{E}$, the magnetic field is fixed by the relation $\vec{B} = \frac{1}{c}\hat{k} \times \vec{E}$, where $\hat{k}$ is the unit vector along the direction of propagation.

Step 2: Key Formula or Approach:
1. Find the unit vector $\hat{k} = \vec{k}/|\vec{k}|$.
2. Use $\vec{B}(\vec{r}, t) = \frac{1}{c}\,\hat{k} \times \vec{E}(\vec{r}, t)$, since $\vec{B}$ carries the same phase $\cos(\vec{k}\cdot\vec{r}$

$-\omega t)$ as $\vec{E}$ for a wave travelling in a single direction.

Step 3: Detailed Explanation:

The wavevector is $\vec{k} = 10(\hat{x} + \sqrt{3}\hat{y})\ \text{m}^{-1}$, so its magnitude is $|\vec{k}|$ $= 10\sqrt{1^2 + (\sqrt{3})^2} = 10\sqrt{4} = 20\ \text{m}^{-1}$.

The unit vector along propagation is $\hat{k} = \frac{10(\hat{x}+\sqrt{3}\hat{y})}{20} = \frac{1}{2}(\hat{x} + \sqrt{3}\hat{y})$.

The electric field amplitude vector is $600\hat{z}\ \text{V.m}^{-1}$. Cross it with $\hat{k}$:

$$\hat{k} \times (600\hat{z}) = \frac{600}{2}\left[(\hat{x} \times \hat{z}) + \sqrt{3}(\hat{y} \times \hat{z})\right]$$

Using $\hat{x} \times \hat{z} = -\hat{y}$ and $\hat{y} \times \hat{z} = \hat{x}$:

$$\hat{k} \times (600\hat{z}) = 300\left[-\hat{y} + \sqrt{3}\hat{x}\right] = 300(\sqrt{3}\hat{x} - \hat{y})$$

Divide by $c = 3.0 \times 10^8\ \text{m.s}^{-1}$:

$$\vec{B}_0 = \frac{300(\sqrt{3}\hat{x} - \hat{y})}{3.0 \times 10^8} = 10^{-6}(\sqrt{3}\hat{x} - \hat{y})$$

Since $\vec{B}$ tracks the same phase as $\vec{E}$:

$$\vec{B}(\vec{r}, t) = 10^{-6}(\sqrt{3}\hat{x} - \hat{y})\cos(\vec{k} \cdot \vec{r} - \omega t)\ \text{V.m}^{-2}\text{.s}$$

Step 4: Why the other options are wrong.

Option (A) has the right direction $(\sqrt{3}\hat{x} - \hat{y})$ but a magnitude twice too large. Option (C) reverses the direction, using $-\sqrt{3}\hat{x} + \hat{y}$ instead, and is off by a further factor of 10 in size, the kind of mistake you get from crossing the vectors in the wrong order and misplacing a power of ten. Option (D) has the correct direction but is 10 times too large.

Final Answer:

The magnetic field is $10^{-6}(\sqrt{3}\hat{x} - \hat{y})\cos(\vec{k}\cdot\vec{r} - \omega t)$ V.m^{-2}.s, which is option (B).

$$\boxed{\vec{B}(\vec{r},t) = 10^{-6}(\sqrt{3}\hat{x} - \hat{y})\cos(\vec{k}\cdot\vec{r} - \omega t)\ \text{V.m}^{-2}\text{.s}}$$

Quick Tip: Use $\vec{B} = \frac{1}{c}\hat{k} \times \vec{E}$ with $\hat{k}$ the unit vector along $\vec{k} = 10(\hat{x} + \sqrt{3}\hat{y})$, whose magnitude is 20 m^{-1}.

. An infinitely large non-conducting thin sheet in the xy plane ($z = 0$) carries a uniform surface charge density $\sigma = 17.70 \times 10^{-12}$ C.m^{-2}. The electric field in the region $z < 0$ is $\vec{E}_2 = \hat{x} + 2\hat{y} + 3\hat{z}$. Then, the electric field $\vec{E}_1$ in the region $z > 0$ will be ($\epsilon_0 = 8.85 \times 10^{-12}$ C^2.N^{-1}.m^{-2})

(A) $\vec{E}_1 = \hat{x} + 2\hat{y} + 5\hat{z}$

(B) $\vec{E}_1 = \hat{x} + 2\hat{y} + 4\hat{z}$

(C) $\vec{E}_1 = \hat{x} + 2\hat{y} + 3\hat{z}$

(D) $\vec{E}_1 = \hat{x} + 4\hat{y} + \hat{z}$

Correct Answer: (A) $\vec{E}_1 = \hat{x} + 2\hat{y} + 5\hat{z}$

Solution:

Step 1: Understanding the Concept:

A surface charge sitting on a boundary produces a fixed jump in the electric field across that boundary. The component of $\vec{E}$ parallel to the sheet (tangential) stays the same on both sides, while the component perpendicular to the sheet (normal) jumps by σ/ϵ_0 as we cross the sheet.

Step 2: Key Formula or Approach:

For a sheet in the xy-plane with the normal direction taken as $\hat{z}$, pointing from the $z < 0$ side into the $z > 0$ side:

1. Tangential components are continuous: $E_{1x} = E_{2x}$, $E_{1y} = E_{2y}$.
2. Normal component jumps: $E_{1z} - E_{2z} = \sigma/\epsilon_0$.

Step 3: Detailed Explanation:

Read off the given field for $z < 0$: $\vec{E}_2 = \hat{x} + 2\hat{y} + 3\hat{z}$, so $E_{2x} = 1$, $E_{2y} = 2$, $E_{2z} = 3$.

Since the sheet carries no surface current, the x and y components of $\vec{E}$ do not change across it:

$$E_{1x} = 1, \qquad E_{1y} = 2$$

For the normal (z) component, compute σ/ϵ_0:

$$\frac{\sigma}{\epsilon_0} = \frac{17.70 \times 10^{-12}}{8.85 \times 10^{-12}} = 2$$

So the z-component increases by 2 on crossing from $z < 0$ to $z > 0$:

$$E_{1z} = E_{2z} + \frac{\sigma}{\epsilon_0} = 3 + 2 = 5$$

Putting the three components together:

$$\vec{E}_1 = \hat{x} + 2\hat{y} + 5\hat{z}$$

Step 4: Why the other options are wrong.

Option (B) uses $E_{1z} = 4$, as if only half the jump were added, the kind of slip that comes from mixing up this boundary-condition jump with the separate formula $\sigma/2\epsilon_0$ used for the field of an isolated sheet in open space. Option (C) keeps $\vec{E}_1$ equal to $\vec{E}_2$, ignoring the charge on the sheet altogether. Option (D) changes the tangential components instead of the normal one, which contradicts the rule that only the normal component of $\vec{E}$ can jump across a surface charge.

Final Answer:

The tangential parts of the field stay fixed and only the normal part jumps, by $\sigma/\epsilon_0 = 2$.

$$\boxed{\vec{E}_1 = \hat{x} + 2\hat{y} + 5\hat{z}}$$

Quick Tip: Tangential components of $\vec{E}$ are continuous across a charged sheet; only the normal component jumps, by σ/ϵ_0.

. Consider an operator $\hat{A}$ which is not Hermitian. Find the possible values of c and d such that the operator $(c\hat{A} - d\hat{A}^\dagger)$ is Hermitian.

(A) $c = i$ and $d = i$

(B) $c = 1$ and $d = 1$

(C) $c = -1$ and $d = i$

(D) $c = i$ and $d = -i$

Correct Answer: (A) $c = i$ and $d = i$

Solution:

Step 1: Understanding the Concept:

An operator $\hat{X}$ is Hermitian when it equals its own adjoint, $\hat{X}^\dagger = \hat{X}$. For a combination built from an operator $\hat{A}$ and its adjoint $\hat{A}^\dagger$, work out the adjoint of the whole combination and force it to match the original expression.

Step 2: Key Formula or Approach:

For a complex number c and an operator $\hat{A}$, the adjoint follows two rules: $(\hat{A}^\dagger)^\dagger = \hat{A}$, and $(c\hat{A})^\dagger = c^*\hat{A}^\dagger$, with c^* the complex conjugate of c. Since $\hat{A}$ is not Hermitian, $\hat{A}$ and $\hat{A}^\dagger$ behave as two genuinely different, independent operators, so their coefficients can be matched separately.

Step 3: Detailed Explanation:

Let $\hat{X} = c\hat{A} - d\hat{A}^\dagger$. Take its adjoint term by term:

$$\hat{X}^\dagger = c^*\hat{A}^\dagger - d^*\hat{A}$$

For $\hat{X}$ to be Hermitian, $\hat{X}^\dagger = \hat{X}$:

$$c^*\hat{A}^\dagger - d^*\hat{A} = c\hat{A} - d\hat{A}^\dagger$$

Since $\hat{A}$ and $\hat{A}^\dagger$ are independent, match the coefficient of $\hat{A}$ on both sides, and separately the coefficient of $\hat{A}^\dagger$:

$$\text{coefficient of } \hat{A}: \quad -d^* = c$$

$$\text{coefficient of } \hat{A}^\dagger: \quad c^* = -d$$

Both statements say the same thing, one is the complex conjugate of the other, so the requirement reduces to the single condition:

$$c = -d^*$$

Step 4: Check each option against $c = -d^*$.

Option (A): $c = i$, $d = i$, so $d^* = -i$ and $-d^* = i$, which equals c. This works.

Option (B): $c = 1$, $d = 1$, so $d^* = 1$ and $-d^* = -1$, which does not equal $c = 1$. This fails.

Option (C): $c = -1$, $d = i$, so $d^* = -i$ and $-d^* = i$, which does not equal $c = -1$. This fails.

Option (D): $c = i$, $d = -i$, so $d^* = i$ and $-d^* = -i$, which does not equal $c = i$. This fails.

Final Answer:

Only $c = i$ and $d = i$ satisfy $c = -d^*$, so this is the only choice that makes the operator Hermitian.

$$\boxed{c = i,\ d = i}$$

Quick Tip: Take the adjoint of $(c\hat{A} - d\hat{A}^\dagger)$ and match it back to itself; use $(c\hat{A})^\dagger = c^*\hat{A}^\dagger$.

. For a scalar field $\psi(\vec{r})$ and a vector field $\vec{A}(\vec{r})$, $\nabla \times (\vec{A}\,\psi)$ is equivalent to the expression

(A) $\psi(\nabla \times \vec{A}) - \vec{A} \times (\nabla\psi)$

(B) $\psi(\nabla \times \vec{A}) + \vec{A} \times (\nabla\psi)$

(C) Null vector

(D) $\vec{A} \times (\nabla\psi) - \psi(\nabla \times \vec{A})$

Correct Answer: (A) $\psi(\nabla \times \vec{A}) - \vec{A} \times (\nabla\psi)$

Solution:

Step 1: Understanding the Concept:

The curl of a scalar times a vector is not a basic operation you can look up on its own; it comes from applying the product rule for the curl operator, in the same spirit as the ordinary product rule for derivatives, but with an extra cross product because curl involves $\nabla\times$.

Step 2: Key Formula or Approach:

The standard vector identity for a scalar field ψ and a vector field $\vec{A}$ is:

$$\nabla \times (\psi\vec{A}) = \psi(\nabla \times \vec{A}) + (\nabla\psi) \times \vec{A}$$

This second term needs to be rewritten so it matches the form used in the answer options.

Step 3: Detailed Explanation:

Write out the identity component by component using index notation, with ϵ_{ijk} the Levi-Civita symbol and a sum over repeated indices understood:

$$[\nabla \times (\psi\vec{A})]_i = \epsilon_{ijk}\partial_j(\psi A_k) = \epsilon_{ijk}\left[(\partial_j\psi)A_k + \psi(\partial_j A_k)\right]$$

The second piece is just $\psi[\nabla \times \vec{A}]_i$. The first piece, $\epsilon_{ijk}(\partial_j\psi)A_k$, is

exactly the i-th component of $(\nabla\psi)\times\vec{A}$. So:

$$\nabla\times(\psi\vec{A})=\psi(\nabla\times\vec{A})+(\nabla\psi)\times\vec{A}$$

Now use the anti-commuting property of the cross product, $\vec{u}\times\vec{v}=-\vec{v}\times\vec{u}$, to flip the order in the second term:

$$(\nabla\psi)\times\vec{A}=-\vec{A}\times(\nabla\psi)$$

Substituting back:

$$\nabla\times(\vec{A}\psi)=\psi(\nabla\times\vec{A})-\vec{A}\times(\nabla\psi)$$

Step 4: Why the other options are wrong.
Option (B) has the wrong sign on the second term, the mistake of forgetting to flip the cross product order after moving the gradient term out front. Option (C) would need $\psi(\nabla\times\vec{A})$ and $\vec{A}\times(\nabla\psi)$ to cancel exactly for every possible ψ and $\vec{A}$, which only happens in special cases, not as a general rule. Option (D) is the correct expression with an overall sign flip, which does not match the derivation above.

Final Answer:
The identity gives $\psi(\nabla\times\vec{A})-\vec{A}\times(\nabla\psi)$, matching option (A).

$$\boxed{\nabla\times(\vec{A}\psi)=\psi(\nabla\times\vec{A})-\vec{A}\times(\nabla\psi)}$$

Quick Tip: Use the product rule $\nabla\times(\psi\vec{A})=\psi(\nabla\times\vec{A})+(\nabla\psi)\times\vec{A}$, then flip

the order of the second cross product.

. Which of the following options is correct for transformation of electric field $\vec{E}$ and magnetic field $\vec{B}$ under time reversal, i.e., $t \to -t$?

(A) $\vec{E} \to \vec{E}$ and $\vec{B} \to \vec{B}$

(B) $\vec{E} \to -\vec{E}$ and $\vec{B} \to \vec{B}$

(C) $\vec{E} \to \vec{E}$ and $\vec{B} \to -\vec{B}$

(D) $\vec{E} \to -\vec{E}$ and $\vec{B} \to -\vec{B}$

Correct Answer: (C) $\vec{E} \to \vec{E}$ and $\vec{B} \to -\vec{B}$

Solution:

Step 1: Understanding the Concept:

Time reversal means replacing t by $-t$ and asking how each physical quantity must change so the laws of physics still hold. Electric and magnetic fields are not fundamental on their own, they come from charges and currents, so first work out how charge density and current density behave under $t \to -t$, then use Maxwell's equations to see how $\vec{E}$ and $\vec{B}$ must behave.

Step 2: Key Formula or Approach:

1. Charge density ρ counts charges sitting at fixed positions, so it does not depend on which way time runs: $\rho \to \rho$ (even).
2. Current density is $\vec{J} = \rho\vec{v}$, with $\vec{v}$ a velocity. Under $t \to -t$, a trajectory $\vec{r}(t)$ traces the same path backward, so velocity $\vec{v} = d\vec{r}/dt$ flips sign: $\vec{v} \to -\vec{v}$, hence $\vec{J} \to -\vec{J}$ (odd).
3. $\vec{E}$ is sourced by ρ through Coulomb's law and Gauss's law, so it transforms the same way as ρ. $\vec{B}$ is sourced by $\vec{J}$ through the Biot-Savart law and Ampere's law, so it transforms the same way as $\vec{J}$.

Step 3: Detailed Explanation:

Since $\rho \to \rho$ is unchanged, and $\vec{E}$ is generated directly by charge distributions through $\vec{E} \propto \rho$, it follows that:

$$\vec{E} \to \vec{E}$$

Since $\vec{J} \to -\vec{J}$, and $\vec{B}$ is generated by currents through the Biot-Savart law $\vec{B} \propto \vec{J} \times \hat{r}/r^2$ integrated over the source, it follows that:

$$\vec{B} \to -\vec{B}$$

This also matches the everyday picture of a bar magnet, whose field comes from circulating electron currents. Reversing the direction of time reverses the sense of every circulating charge, which reverses the current loop's field.

Step 4: Why the other options are wrong.

Option (A) keeps both fields unchanged, missing that current, and so $\vec{B}$, depends on velocity and must flip. Option (B) flips $\vec{E}$ instead of $\vec{B}$, which would wrongly make a static charge distribution produce a different field under time reversal even though ρ itself does not change. Option (D) flips both fields, over-correcting by treating $\vec{E}$ as if it depended on velocity the way $\vec{B}$ does.

Final Answer:

$\vec{E}$ is even and $\vec{B}$ is odd under time reversal, because $\vec{E}$ traces charge, which does not change with the direction of time, while $\vec{B}$ traces current, which reverses with velocity.

$$\boxed{\vec{E} \to \vec{E},\ \vec{B} \to -\vec{B}}$$

Quick Tip: Charge density does not change under $t \to -t$, but current density flips sign since velocity flips; $\vec{E}$ follows charge, $\vec{B}$ follows current.

. On a horizontal plane, a projectile of mass m is launched from the ground with speed v_0 at an angle θ_0 with the horizontal. In addition to the gravitational force (mg), it also experiences a drag force $\vec{F}_{drag} = -\gamma\vec{v}$, where $\vec{v}$ is its velocity and γ is a constant. It hits the ground at a distance R from the point of launch with its velocity making an angle θ with the horizontal, as shown schematically in the figure below.

Then which of the following options is correct?

(A) $R = \dfrac{v_0^2 \sin 2\theta}{g}, \theta < \theta_0$

(B) $R < \dfrac{v_0^2 \sin 2\theta_0}{g}, \theta < \theta_0$

(C) $R < \dfrac{v_0^2 \sin 2\theta_0}{g}, \theta > \theta_0$

(D) $R = \dfrac{v_0^2 \sin 2\theta}{g}, \theta > \theta_0$

Correct Answer: (C) $R < \dfrac{v_0^2 \sin 2\theta_0}{g}, \theta > \theta_0$

Solution:

Step 1: Understanding the Concept:

Without any drag, a projectile launched from level ground follows a symmetric parabola. Its horizontal range is the standard formula $R_0 = \dfrac{v_0^2 \sin 2\theta_0}{g}$, and by the up-down symmetry of that parabola the ball lands at exactly the same angle it was launched at, so $\theta = \theta_0$. Here the projectile also feels a drag force $\vec{F}_{drag} = -\gamma\vec{v}$, proportional to its own velocity, so this symmetry is broken and we have to track the horizontal and vertical motion separately.

Step 2: Key Formula or Approach:

Newton's second law along each axis, with drag as the only horizontal force and gravity plus drag along the vertical, gives two independent linear equations:

$$m\frac{dv_x}{dt} = -\gamma v_x, \qquad m\frac{dv_y}{dt} = -mg - \gamma v_y$$

Solving the first with $v_x(0) = v_0 \cos\theta_0$ gives an exponential decay, and solving the second (a linear equation with a constant forcing term $-mg$) gives a term that settles down to a constant terminal value:

$$v_x(t) = v_0 \cos\theta_0\, e^{-\gamma t/m}, \qquad v_y(t) = \left(v_0 \sin\theta_0 + \frac{mg}{\gamma}\right)e^{-\gamma t/m} - \frac{mg}{\gamma}$$

Step 3: Why the landing angle is steeper ($\theta > \theta_0$):

Look at what each velocity component does as time goes on. $v_x(t)$ has nothing driving it except drag, so it just keeps decaying toward zero, no matter how long the flight lasts. $v_y(t)$, on the other hand, is pulled by gravity the whole time and only levels off at the terminal speed $-mg/\gamma$ as $t \to \infty$; it does not collapse toward zero the way v_x does. So

while both speeds shrink compared with the no-drag case, the horizontal speed shrinks away much faster (all the way to zero) than the vertical speed does (it only settles at a finite terminal value). At the moment of landing, the ratio $\tan\theta = |v_y|/v_x$ has therefore grown larger than it was at launch, $\tan\theta_0 = |v_y(0)|/v_x(0)$. That means $\theta > \theta_0$: the ball comes down steeper than it went up.

Step 4: Why the range is shorter ($R < v_0^2 \sin 2\theta_0/g$):

Drag is a dissipative force: it removes mechanical energy from the projectile at the rate γv^2 at every instant of the flight, energy that in the no-drag case would have stayed available to carry the projectile further forward. Since $v_x(t)$ is smaller at every instant than the drag-free value $v_0 \cos\theta_0$, the horizontal distance $R = \int_0^T v_x\, dt$ covered in the (also reduced) flight time T comes out smaller than the drag-free range for the same launch speed and angle. So $R < \dfrac{v_0^2 \sin 2\theta_0}{g}$.

Final Answer:

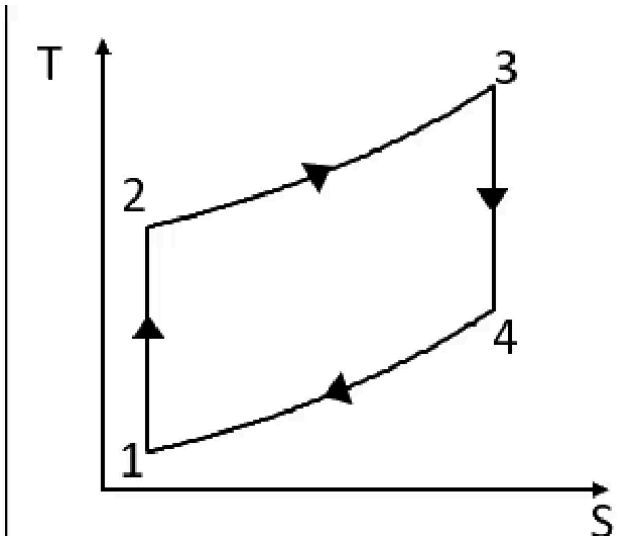
Both effects act together: the range shrinks below the ideal value and the ball lands steeper than it was thrown, which is exactly option (C).

$$\boxed{R < \frac{v_0^2 \sin 2\theta_0}{g},\ \ \theta > \theta_0 \ \ \text{(Option C)}}$$

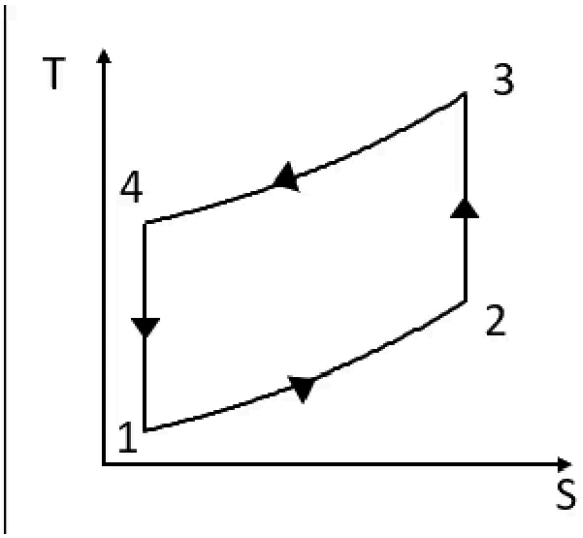
Quick Tip: Solve $m\dot{v}_x = -\gamma v_x$ and $m\dot{v}_y = -mg - \gamma v_y$ separately. v_x just decays to zero, v_y settles at a finite terminal speed, so the landing ratio $|v_y|/v_x$ grows, and the drag removes energy so the range shrinks.

. Consider the Otto cycle for an ideal gas engine consisting of two quasistatic adiabatic and two quasistatic isochoric processes. The correct temperature-entropy (T-S) phase diagram for the cycle is:

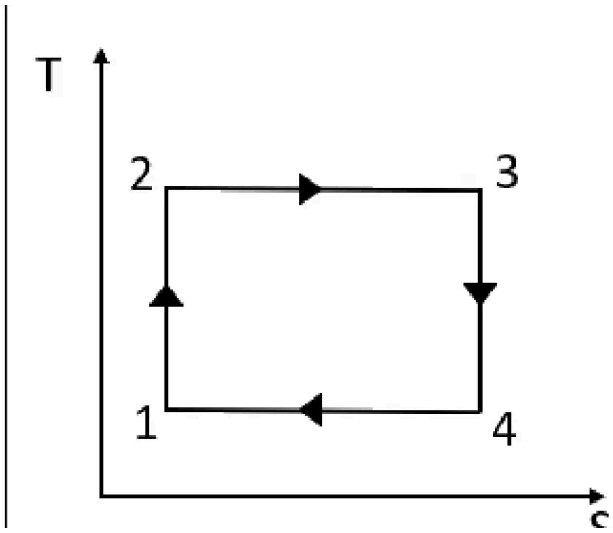
(A)



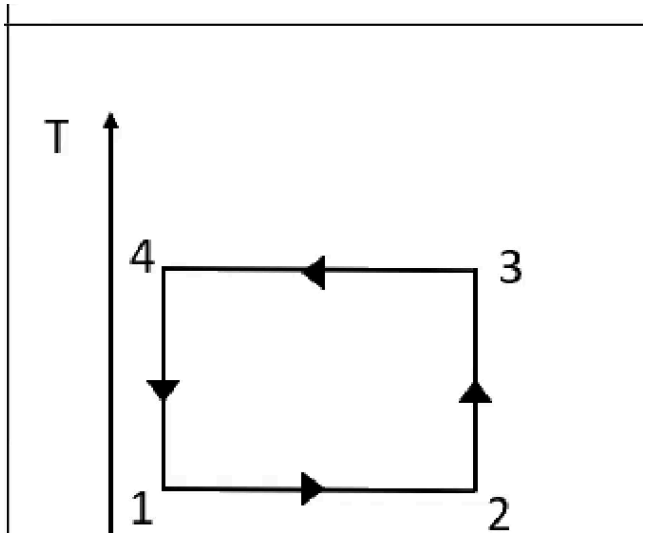
(B)



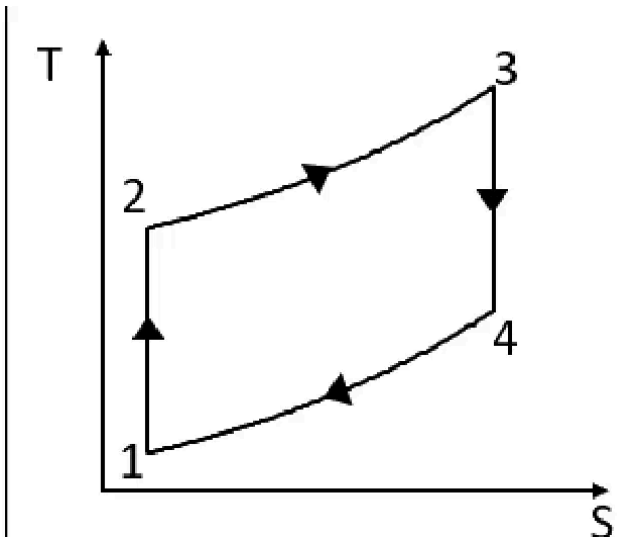
(C)



(D)



Correct Answer: (A)



Solution:

Step 1: Recall what each part of the Otto cycle does.

The Otto cycle has four quasistatic steps: 1 to 2 is an adiabatic (isentropic) compression, 2 to 3 is an isochoric (constant volume) heat addition, 3 to 4 is an adiabatic (isentropic) expansion, and 4 to 1 is an isochoric heat rejection back to the starting state. We need to know what each of these two process types looks like on a temperature versus entropy (T-S) plot.

Step 2: Key Formula or Approach:

For an ideal gas the entropy of one mole can be written as $S = C_V \ln T + R \ln V + \text{const}$. Along a quasistatic adiabatic process the process is isentropic by definition, so $dS = 0$: on a T-S plot this is a straight VERTICAL line, since T can change while S stays fixed. Along an isochoric process V is fixed but T changes, so from $S = C_V \ln T + \text{const}(V)$, which rearranges to $T = A(V)\, e^{S/C_V}$, this is an increasing, upward-curving (convex) exponential-shaped line, never a straight line.

Step 3: Work out where the two isochores sit relative to each other.

The 2-3 heat addition happens at the smaller, compressed volume V_2, and the 4-1 heat rejection happens at the larger volume $V_1 > V_2$. Since $S = C_V \ln T + R \ln V + \text{const}$, for the same entropy value a larger volume needs a smaller temperature to keep S the same (the $R \ln V$ term is larger, so $C_V \ln T$ must be smaller). So the isochore belonging to the smaller volume V_2 sits above (higher T for the same S) the isochore belonging to the larger volume V_1.

Step 4: Match this to the four panels.

We need a shape with two vertical segments (the adiabats) and two

convex upward-curving segments (the isochores), where the upper curve is the 2-3 heat addition and the lower curve is the 4-1 heat rejection, walked in the order 1 (bottom-left) up to 2, across the upper curve to 3, down to 4, and back across the lower curve to 1. This is exactly panel (A).

Step 5: Why the others fail.

Panel (B) draws the identical curve shape but swaps the positions of states 2 and 4, so walking 1 to 2 to 3 to 4 to 1 in that panel traces compression and heat addition in the wrong place relative to the curved isochores, it does not match the Otto sequence. Panels (C) and (D) draw the isochores as straight horizontal sides of a rectangle. A straight horizontal line on a T-S plot means T stays constant while S changes, that is an isothermal process, not an isochoric one, so both misrepresent the physics of the isochoric legs.

Final Answer:

Only panel (A) shows two vertical adiabats joined by two correctly-ordered convex isochores.

Option (A)

Quick Tip: An adiabatic (isentropic) step is a vertical line on a T-S plot. An isochoric step is a curved, convex line, not straight, since $S = C_V \ln T + R \ln V + \text{const.}$
The isochore at the smaller (compressed) volume sits above the one at the larger volume.

. The formula for energy E of a photon gas at temperature T in a two-dimensional box at equilibrium with $g_{2d}(\nu)$ denoting the density of states of photons is given below, where symbols ν, h and k_B have their standard meaning. The specific heat (C_V) of this photon gas obeys

$$E = \int_0^\infty d\nu\, g_{2d}(\nu) \frac{h\nu}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}$$

(A) $C_V \propto T$

(B) $C_V \propto T^2$

(C) $C_V \propto T^3$

(D) $C_V \propto T^4$

Correct Answer: (B) $C_V \propto T^2$

Solution:

Step 1: Understanding the Concept:

This is a blackbody-radiation-style problem in a lower dimension. For a photon gas confined in a box of dimension d, the number of modes per unit frequency, the density of states $g_d(\nu)$, scales as ν^{d-1}, because it comes from counting standing wave modes inside a region of radius proportional to ν in d-dimensional wavevector space, and the surface of that region scales as (radius)$^{d-1}$. Here the box is two-dimensional, so $d = 2$ and $g_{2d}(\nu) \propto \nu$.

Step 2: Key Formula or Approach:

Put $g_{2d}(\nu) = A\nu$ into the given energy integral and change to the dimensionless variable $x = h\nu/(k_B T)$:

$$E = \int_0^\infty A\nu \cdot \frac{h\nu}{\exp(h\nu/k_B T) - 1} d\nu = A \int_0^\infty \frac{h\nu^2}{\exp(h\nu/k_B T) - 1} d\nu$$

Step 3: Detailed Explanation:

With $\nu = \frac{k_B T}{h} x$ and $d\nu = \frac{k_B T}{h} dx$, every factor of ν pulls out a factor of T:

$$E = A \int_0^\infty h \left(\frac{k_B T}{h} x \right)^2 \frac{1}{e^x - 1} \cdot \frac{k_B T}{h} \, dx = A \frac{(k_B T)^3}{h^2} \int_0^\infty \frac{x^2}{e^x - 1} \, dx$$

The remaining integral over x is just a fixed pure number, it does not depend on T at all, so all the T-dependence of E sits in the prefactor:

$$E = a\,T^3, \qquad a = A \frac{k_B^3}{h^2} \int_0^\infty \frac{x^2}{e^x - 1} \, dx = \text{constant}$$

Step 4: Get the specific heat and rule out the other options.

Differentiating with respect to temperature at constant volume gives $C_V = \frac{\partial E}{\partial T} = 3aT^2$, so $C_V \propto T^2$. Option (A), $C_V \propto T$, would be the result for a one-dimensional photon gas, where $g_{1d}(\nu)$ is constant, not linear in ν. Option (C), $C_V \propto T^3$, is the familiar three-dimensional blackbody result, it is a trap for anyone who reflexively uses the 3D scaling without checking the actual dimensionality stated in the question. Option (D), $C_V \propto T^4$, does not correspond to any of these standard photon-gas dimensionalities.

Final Answer:

In a 2D photon gas, $E \propto T^3$, so $C_V \propto T^2$, option (B).

$$\boxed{C_V \propto T^2}$$

Quick Tip: In d dimensions the photon density of states goes as $g_d(\nu) \propto \nu^{d-1}$.
For $d = 2$, working through the integral gives $E \propto T^3$, so $C_V = \partial E/\partial T \propto T^2$.

. For the electric field of an electromagnetic wave given below, which of the following statements is correct?

$$\vec{E} = \hat{x}\, E_0 \cos(\omega t) + \hat{y}\, 2E_0 \cos\left(\omega t + \frac{\pi}{2}\right)$$

(A) The electric field is linearly polarised with slope 2.

(B) The electric field is circularly polarised with radius E_0.

(C) The electric field is elliptically polarised with a ratio of major to minor axis being 2.

(D) The electric field is unpolarised with the two components being phase shifted by $\pi/2$.

Correct Answer: (C) The electric field is elliptically polarised with a ratio of major to minor axis being 2.

Solution:

Step 1: Understanding the Concept:
The polarisation state of a wave is fixed by how its two transverse field components compare in amplitude and in phase. Two components in phase (or exactly out of phase by π) give linear polarisation. Two components with equal amplitude and a $\pi/2$ phase difference give circular polarisation. Two components with unequal amplitude and a $\pi/2$ phase difference give elliptical polarisation, with the ellipse's axes lined up with the coordinate axes.

Step 2: Key Formula or Approach:

Write out both components explicitly as functions of time and use $\cos(\omega t + \pi/2) = -\sin(\omega t)$:

$$E_x = E_0 \cos(\omega t), \qquad E_y = 2E_0 \cos\left(\omega t + \frac{\pi}{2}\right) = -2E_0 \sin(\omega t)$$

Step 3: Detailed Explanation:

From these, $E_x/E_0 = \cos \omega t$ and $E_y/(2E_0) = -\sin \omega t$. Squaring both and adding removes the time dependence:

$$\left(\frac{E_x}{E_0}\right)^2 + \left(\frac{E_y}{2E_0}\right)^2 = \cos^2 \omega t + \sin^2 \omega t = 1$$

This is the equation of an ellipse in the E_x-E_y plane, with semi-axis E_0 along x and semi-axis $2E_0$ along y. As time goes on, the tip of the electric field vector traces this ellipse once every period, so the wave is elliptically polarised.

Step 4: Get the axis ratio and rule out the other options.

The ratio of the major axis to the minor axis is $2E_0 : E_0 = 2$, matching option (C). Option (A) is wrong because the phase difference here is $\pi/2$, not 0 or π, so the field does not oscillate along a single fixed line. Option (B) needs the two amplitudes to be equal so the ellipse becomes a circle, but here they are E_0 and $2E_0$, not equal, so it cannot be circular. Option (D) is wrong because unpolarised describes an incoherent mix of many independent waves with randomly varying phase, not a single coherent wave like this one, which has one fixed, well defined amplitude and phase relationship at all times.

Final Answer:

The wave traces an ellipse with major to minor axis ratio 2, so the field is elliptically polarised.

$$\boxed{\text{Option (C): axis ratio } = 2}$$

Quick Tip: Write $E_y = 2E_0\cos(\omega t + \pi/2) = -2E_0\sin\omega t$ and eliminate t between E_x and E_y.
The result is an ellipse equation with semi-axes E_0 and $2E_0$.

. A gas of non-interacting ^{4}He atoms (of mass m) is in a three-dimensional trap whose energy levels can be approximated by those of a harmonic oscillator potential $V(x,y,z) = \frac{1}{2}m\omega^2(x^2+y^2+z^2)$. The chemical potential of the gas at $T = 0$ K is

(A) 0

(B) $\frac{1}{2}\hbar\omega$

(C) $\frac{3}{2}\hbar\omega$

(D) $3\hbar\omega$

Correct Answer: (C) $\frac{3}{2}\hbar\omega$

Solution:

Step 1: Understanding the Concept:
^{4}He has 2 protons, 2 neutrons and 2 electrons, an even total number of these spin-1/2 particles, so a ^{4}He atom carries integer total spin and behaves as a boson. A gas of non-interacting bosons can undergo Bose-Einstein condensation, and at exactly $T = 0$ K every single particle must sit in the lowest available single-particle energy level,

the ground state of the trap. For an ideal Bose gas at $T = 0$, the chemical potential is pinned to equal that ground-state energy, $\mu(T = 0) = \varepsilon_0$.

Step 2: Key Formula or Approach:

The trap is an isotropic 3D harmonic oscillator, $V(x, y, z) = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)$, which separates into three independent 1D oscillators along x, y and z. The energy levels of a single 1D quantum harmonic oscillator are $\left(n + \frac{1}{2}\right)\hbar\omega$, so the full 3D levels are:

$$\varepsilon(n_x, n_y, n_z) = \left(n_x + n_y + n_z + \frac{3}{2}\right)\hbar\omega, \qquad n_x, n_y, n_z = 0, 1, 2, \ldots$$

Step 3: Detailed Explanation:

The ground state has the lowest possible quantum numbers in all three directions, $n_x = n_y = n_z = 0$. Plugging these in:

$$\varepsilon_0 = \left(0 + 0 + 0 + \frac{3}{2}\right)\hbar\omega = \frac{3}{2}\hbar\omega$$

This $\frac{3}{2}\hbar\omega$ is simply the sum of three independent zero-point energies, one $\frac{1}{2}\hbar\omega$ from each of the x, y and z oscillators, which never vanish even in the ground state because of the uncertainty principle. Since $\mu(T = 0) = \varepsilon_0$, we get $\mu(T = 0) = \frac{3}{2}\hbar\omega$.

Step 4: Why the other options are wrong.

Option (A), $\mu = 0$, would only be correct for a classical ideal gas or a trap with no zero-point energy, it ignores the quantum mechanical fact that a trapped particle can never have exactly zero energy. Option (B), $\frac{1}{2}\hbar\omega$, is the zero-point energy of just one of the three 1D oscillators, not the sum over all three directions of the actual 3D trap.

Option (D), $3\hbar\omega$, would require $n_x + n_y + n_z + \frac{3}{2} = 3$, that is $n_x + n_y + n_z = \frac{3}{2}$, which is not an integer and so is not an allowed combination of quantum numbers.

Final Answer:

At $T = 0$ K, every ^{4}He atom condenses into the trap's ground state, whose energy is $\frac{3}{2}\hbar\omega$.

$$\boxed{\mu(T = 0) = \frac{3}{2}\hbar\omega}$$

Quick Tip: ^{4}He atoms are bosons, so at $T = 0$ K all of them sit in the trap's ground state.
For an isotropic 3D harmonic oscillator, the ground-state (zero-point) energy is $\frac{3}{2}\hbar\omega$, and $\mu(T = 0)$ equals that.

. Given $|v_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}$ and $|v_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}$, the tensor product $|v_1\rangle \otimes |v_2\rangle$ is

(A) $\frac{1}{2}\begin{pmatrix}1\\ -i\\ i\\ 1\end{pmatrix}$

(B) $\frac{1}{2}\begin{pmatrix}1\\ i\\ -i\\ 1\end{pmatrix}$

(C) $\frac{1}{2}\begin{pmatrix}1 & i\\ i & -1\end{pmatrix}$

(D) $\frac{1}{2}\begin{pmatrix}1 & i\\ -i & 1\end{pmatrix}$

Correct Answer: (A) $\frac{1}{2}\begin{pmatrix}1\\-i\\i\\1\end{pmatrix}$

Solution:

Step 1: Understanding the Concept:

The tensor product (also called the Kronecker product) of two column vectors builds a bigger column vector. If $|v_1\rangle$ has 2 entries and $|v_2\rangle$ has 2 entries, then $|v_1\rangle \otimes |v_2\rangle$ has $2 \times 2 = 4$ entries. Each entry of $|v_1\rangle$ gets multiplied by the whole of $|v_2\rangle$, and the results are stacked one after another.

Step 2: Key Formula or Approach:

For $|v_1\rangle = \begin{pmatrix}a\\b\end{pmatrix}$ and $|v_2\rangle = \begin{pmatrix}c\\d\end{pmatrix}$, the tensor product is

$$|v_1\rangle \otimes |v_2\rangle = \begin{pmatrix}ac\\ad\\bc\\bd\end{pmatrix}$$

Step 3: Detailed Explanation:

Here $a = 1$, $b = i$ (from $|v_1\rangle$, before the common factor $1/\sqrt{2}$), and $c = 1$, $d = -i$ (from $|v_2\rangle$). The two $1/\sqrt{2}$ factors combine to give $1/2$.

Now build each of the four entries:

$$ac = (1)(1) = 1$$

$$ad = (1)(-i) = -i$$

$$bc = (i)(1) = i$$

$$bd = (i)(-i) = -i^2 = 1$$

Stacking these in order gives

$$|v_1\rangle \otimes |v_2\rangle = \frac{1}{2}\begin{pmatrix} 1 \\ -i \\ i \\ 1 \end{pmatrix}$$

Step 4: Why the other options are wrong.

Option (B) has the middle two entries swapped in sign (i then $-i$ instead of $-i$ then i); this comes from multiplying b by c and a by d in the wrong order. Options (C) and (D) arrange the four numbers as a 2×2 matrix instead of a single stacked 4×1 column, which is not what the tensor product of two column vectors gives; a tensor product of two kets is itself a ket, not an operator.

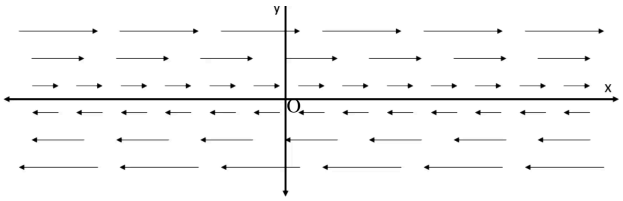
Final Answer:

The tensor product works out to a column vector with entries 1, -i, i, 1, all scaled by one half.

$$\boxed{\frac{1}{2}\begin{pmatrix} 1 \\ -i \\ i \\ 1 \end{pmatrix}}$$

Quick Tip: Tensor-product a 2-entry ket with a 2-entry ket to get a 4-entry ket: multiply each entry of the first by the whole second ket and stack the results, don't build a 2x2 matrix.

. Sketch of a two-dimensional vector field $\vec{V}$ is shown below. Here, length and arrow head of the arrows denote magnitude and direction of the vector field, respectively.



Which of the following statements is correct for $\nabla \times \vec{V}$?

(A) It is zero everywhere in the two-dimensional space.

(B) Its magnitude is non-zero and its direction is out of the two-dimensional plane.

(C) Its magnitude is non-zero and its direction is into the two-dimensional plane.

(D) It points in opposite directions above and below the x-axis.

Correct Answer: (C) Its magnitude is non-zero and its direction is into the two-dimensional plane.

Solution:

Step 1: Understanding the Concept:

Look at the picture as a rule for the field: above the x-axis the arrows point to the right (+x), and their length grows the farther up they are. Below the x-axis the arrows point to the left (-x), and their length

grows the farther down they are. This is exactly what happens if the x-component of the field grows in proportion to the y-coordinate, changing sign smoothly as y passes through zero.

Step 2: Key Formula or Approach:

Model the field as $\vec{V} = c\,y\,\hat{x}$ for some positive constant c. For $y > 0$ this gives $V_x > 0$ growing with y (arrows to the right, longer higher up); for $y < 0$ it gives $V_x < 0$ growing in size with $|y|$ (arrows to the left, longer lower down). This matches the sketch. The curl of a 2D field confined to the xy-plane, with only an x-component that depends on y, is

$$\nabla \times \vec{V} = \left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)\hat{z}$$

Since $V_y = 0$ here, this reduces to $\nabla \times \vec{V} = -\frac{\partial V_x}{\partial y}\,\hat{z}$.

Step 3: Detailed Explanation:

With $V_x = cy$, we get $\frac{\partial V_x}{\partial y} = c$, a constant. So

$$\nabla \times \vec{V} = -c\,\hat{z}$$

Because c is the same positive constant everywhere in the plane, this curl has the same fixed non-zero magnitude c at every point, and it always points along $-\hat{z}$, that is, into the page. It does not flip sign above and below the x-axis, because the slope of V_x with respect to y is the same everywhere, even though V_x itself changes sign.

Step 4: Why the other options are wrong.

Option (A) would be true only if V_x were independent of y (flat arrows

of the same length everywhere); here the length visibly changes with height, so the curl cannot be zero. Option (B) gets the sign backwards; the curl points out of the page only if V_x decreased with y, the opposite of what the sketch shows. Option (D) confuses the field itself, which does flip direction across the x-axis, with its curl, which does not; the curl depends on the rate of change of V_x with y, which stays the same constant slope on both sides of the axis.

Final Answer:
The curl has a fixed non-zero size and points into the plane everywhere.

Option (C)

Quick Tip: Model the sketch as V = c*y (x-hat) with c greater than 0: arrows flip sign at y = 0 but grow the same way in both directions, so the curl, equal to minus dVx/dy, stays one constant non-zero value into the page everywhere.

. Which one of the following is an allowed process?

(A) $\pi^- + p \to \pi^0 + n$

(B) $\pi^0 \to \gamma + \gamma + \gamma$

(C) $p + \bar{p} \to \Lambda^0 + \Lambda^0$

(D) $p + \bar{p} \to \gamma$

Correct Answer: (A) $\pi^- + p \to \pi^0 + n$

Solution:

Step 1: Understanding the Concept:
A particle reaction is allowed only if it obeys all the relevant conservation laws at once: electric charge, baryon number, strangeness (for the strong and electromagnetic interactions), and also more subtle rules like C-parity for photon decays, plus basic kinematics such as energy-momentum conservation. Checking each option against these rules tells us which one can actually happen.

Step 2: Key Formula or Approach:
Go through the four options one at a time, checking charge, baryon number, strangeness, and any special rule that applies.

Step 3: Check option (A).
$\pi^- + p \to \pi^0 + n$: charge on the left is $-1 + 1 = 0$, and on the right $0 + 0 = 0$, so charge is conserved. Baryon number is $0 + 1 = 1$ on the left and $0 + 1 = 1$ on the right, conserved. Strangeness is zero on both sides. This is the well-known pion-nucleon charge-exchange reaction, and it goes through the strong interaction with nothing forbidding it. So (A) is ALLOWED.

Step 4: Check option (B).
$\pi^0 \to \gamma + \gamma + \gamma$: the neutral pion has C-parity $C = +1$. A system of n photons has $C = (-1)^n$. For 3 photons, $C = -1$, which does not match the pion's $C = +1$. Since the electromagnetic interaction driving this decay conserves C-parity, three photons cannot come from a π^0; only the observed 2-photon decay works. So (B) is FORBIDDEN.

Step 5: Check option (C).
$p + \bar{p} \to \Lambda^0 + \Lambda^0$: baryon number on the left is $1 + (-1) = 0$. On the right, each Λ^0 carries baryon number $+1$, so two of them give $+2$. Since $0 \neq 2$, baryon number is not conserved. The correct final state

for proton-antiproton annihilation into strange baryons would need one Λ^0 and one $\bar{\Lambda}^0$, not two Λ^0. So (C) is FORBIDDEN.

Step 6: Check option (D).
$p + \bar{p} \to \gamma$: in the centre of mass frame the total momentum is zero, so a single photon in the final state would need zero momentum, but a photon with zero momentum also has zero energy, which cannot equal the rest energy of the annihilating pair. A single photon can never balance both energy and momentum for a massive system annihilating together, so annihilation into just one photon is kinematically forbidden. So (D) is FORBIDDEN.

Final Answer:
Only the pion-nucleon charge-exchange reaction respects every conservation law, so option (A) is the allowed process.

$$\boxed{\pi^- + p \to \pi^0 + n}$$

Quick Tip: Check charge, baryon number and strangeness on both sides of each reaction; also remember pi-zero to 3 photons breaks C-parity and p-pbar to a single photon breaks momentum conservation.

. Given Q is the electromagnetic charge and S is the strangeness quantum number, identify the particle(s) for which $(Q - S) = 0$ is satisfied.

(A) Σ^{*-}

(B) K^+

(C) Ω^-

(D) Δ^{++}

Correct Answer: (A) Σ^{*-} ; (B) K^+

Solution:

Step 1: Understanding the Concept:

Charge Q is the particle's electric charge in units of e, and strangeness S counts the number of strange quarks with a minus sign (an s quark carries $S = -1$, an $\bar{s}$ quark carries $S = +1$, and non-strange quarks carry $S = 0$). To test $(Q - S) = 0$ for each particle, we need its quark content, which fixes both Q and S directly.

Step 2: Check Σ^{*-} (option A).

Σ^{*-} is a spin-3/2 baryon in the decuplet with quark content dds. Its charge is the sum of the quark charges: $-\frac{1}{3} - \frac{1}{3} - \frac{1}{3} = -1$, so $Q = -1$. It contains exactly one s quark, so $S = -1$. Then $Q - S = -1 - (-1) = 0$. So (A) satisfies the condition and is TRUE.

Step 3: Check K^+ (option B).

K^+ is a meson with quark content $u\bar{s}$. Its charge is $+\frac{2}{3} + \frac{1}{3} = +1$, so $Q = +1$. It contains one $\bar{s}$ antiquark, which carries $S = +1$. Then $Q - S = 1 - 1 = 0$. So (B) satisfies the condition and is TRUE.

Step 4: Check Ω^- (option C).

Ω^- is the famous triple-strange baryon with quark content sss. Its charge is $-\frac{1}{3} \times 3 = -1$, so $Q = -1$. It has three s quarks, so $S = -3$. Then $Q - S = -1 - (-3) = 2$, which is not zero. So (C) is FALSE.

Step 5: Check Δ^{++} (option D).

Δ^{++} has quark content uuu, all up quarks with no strangeness. Its charge is $\frac{2}{3} \times 3 = +2$, so $Q = +2$, and $S = 0$. Then $Q - S = 2 - 0 = 2$, which is not zero. So (D) is FALSE.

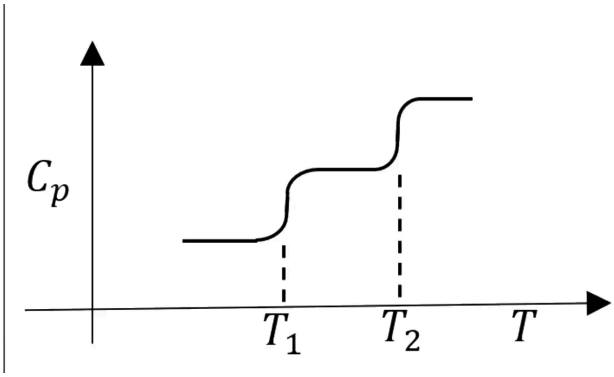
Final Answer:

Only Σ^{*-} and K^{+} satisfy $Q - S = 0$.

$$\boxed{\Sigma^{*-} \text{ and } K^{+}}$$

Quick Tip: Work out the quark content of each particle to get its charge and strangeness directly, then just subtract; only Sigma*- and K+ end up with Q equal to S.

. Schematic variation of the specific heat C_p of an ideal gas of diatomic molecules with temperature T is shown in the figure below. For rotational energy E_R and vibrational energy E_v of the molecule, which of the following options is/are correct? Here k_B is the Boltzmann constant.



(A) $E_R \cong k_B T_1$

(B) $E_R \cong k_B T_2$

(C) $E_v \cong k_B T_1$

(D) $E_v \cong k_B T_2$

Correct Answer: (A) $E_R \cong k_B T_1$; (D) $E_v \cong k_B T_2$

Solution:

Step 1: Understanding the Concept:

A diatomic molecule stores heat in translational, rotational and vibrational motion, but each type of motion only switches on once the thermal energy k_BT becomes comparable to the spacing between that motion's quantum energy levels. Translational levels are essentially continuous, so translation contributes at all temperatures. Rotational levels are closely spaced, so they switch on at a fairly low temperature. Vibrational levels are widely spaced, so they need a much higher temperature to switch on. This is exactly why the graph shows two separate step-rises in C_p rather than one smooth rise.

Step 2: Key Formula or Approach:

The rule of thumb from statistical mechanics is that a degree of freedom becomes thermally active once k_BT grows to about the size of its energy level spacing. So the temperature at which C_p starts to rise for a given motion marks $k_BT \cong$ (energy spacing for that motion).

Step 3: Assign the first step to rotation.

Since T_1 is the lower temperature (the first step-rise happens near T_1), and rotational levels are always more closely spaced than vibrational levels for a real diatomic molecule, the first rise in C_p must be rotational motion switching on. This means the rotational energy spacing satisfies $E_R \cong k_BT_1$. So (A) is TRUE, and by the same reasoning (B), which pairs E_R with the higher temperature T_2, is FALSE.

Step 4: Assign the second step to vibration.

The second, higher-temperature step-rise near T_2 must be vibrational motion switching on, since vibration needs much more thermal

energy to activate than rotation does. This means the vibrational energy spacing satisfies $E_v \cong k_B T_2$. So (D) is TRUE, and (C), which pairs E_v with the lower temperature T_1, is FALSE, since that would wrongly put vibration at a lower activation temperature than rotation.

Final Answer:

Rotational motion switches on near T_1 and vibrational motion switches on near T_2, so $E_R \cong k_B T_1$ and $E_v \cong k_B T_2$.

$$\boxed{E_R \cong k_B T_1,\ E_v \cong k_B T_2}$$

Quick Tip: The lower-temperature step in the Cp graph turns on the closely spaced rotational levels, and the higher-temperature step turns on the widely spaced vibrational levels, so ER matches T1 and Ev matches T2.

. If the perturbation $V = \lambda x^3$ is added to the Hamiltonian of a one dimensional harmonic oscillator, the matrix element $\langle m|V|0\rangle$ is/are non-zero for which of the following states? Here, the eigenstates of the harmonic oscillator are denoted by $|n\rangle$.

(A) $|m = 3\rangle$

(B) $|m = 1\rangle$

(C) $|m = 2\rangle$

(D) $|m = 5\rangle$

Correct Answer: (A) $|m = 3\rangle$; (B) $|m = 1\rangle$

Solution:

Step 1: Understanding the Concept:

The perturbation is $V = \lambda x^3$, and we want to know for which excited state $|m\rangle$ the matrix element $\langle m|V|0\rangle$ is non-zero. We write the position operator x in terms of the harmonic oscillator ladder operators a (lowering) and $a^\dagger$ (raising), since these operators tell us exactly how many quanta a given term can add or remove.

Step 2: Key Formula or Approach:

The position operator is $x = \sqrt{\frac{\hbar}{2m\omega}}(a + a^\dagger)$, so

$$x^3 = \left(\frac{\hbar}{2m\omega}\right)^{3/2}(a + a^\dagger)^3$$

To find $\langle m|x^3|0\rangle$ we act with $(a + a^\dagger)^3$ on the ground state $|0\rangle$ and see which states $|m\rangle$ appear, using $a|n\rangle = \sqrt{n}\,|n-1\rangle$ and $a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$.

Step 3: Detailed Explanation:

Apply the operator one factor at a time. First, $(a + a^\dagger)|0\rangle = |1\rangle$, since $a|0\rangle = 0$ and $a^\dagger|0\rangle = |1\rangle$.

Next, $(a + a^\dagger)|1\rangle = a|1\rangle + a^\dagger|1\rangle = |0\rangle + \sqrt{2}\,|2\rangle$, so $(a + a^\dagger)^2|0\rangle = |0\rangle + \sqrt{2}\,|2\rangle$.

Apply the operator a third time:

$$(a + a^\dagger)^3|0\rangle = (a + a^\dagger)\left(|0\rangle + \sqrt{2}\,|2\rangle\right) = a|0\rangle + a^\dagger|0\rangle + \sqrt{2}\left(a|2\rangle + a^\dagger|2\rangle\right)$$

$$= 0 + |1\rangle + \sqrt{2}\left(\sqrt{2}\,|1\rangle + \sqrt{3}\,|3\rangle\right) = |1\rangle + 2|1\rangle + \sqrt{6}\,|3\rangle$$

$$(a + a^\dagger)^3|0\rangle = 3|1\rangle + \sqrt{6}\,|3\rangle$$

So $x^3|0\rangle$ is built only from $|1\rangle$ and $|3\rangle$, each with a non-zero coefficient. This means $\langle m|x^3|0\rangle$ is non-zero only when $m = 1$ or $m = 3$, and is exactly zero for every other value of m, including $m = 2$ and $m = 5$.

Step 4: Why the other options are wrong.
Option (C), $|m = 2\rangle$, needs a net change of 2 quanta from the ground state, but three factors of a or $a^\dagger$, each changing the quantum number by exactly ± 1, can only add up to a net change of $+3, +1, -1$ or -3; a net change of 2 is impossible. Option (D), $|m = 5\rangle$, needs a net change of 5, even further out of reach of three ± 1 steps.

Final Answer:
The matrix element $\langle m|V|0\rangle$ is non-zero only for $m = 1$ and $m = 3$.

$$\boxed{|m = 3\rangle \text{ and } |m = 1\rangle}$$

Quick Tip: **Hint:**
Write x using ladder operators $a, a^\dagger$ and check which states $(a + a^\dagger)^3$ can reach starting from $|0\rangle$; three ± 1 steps can only give a net change of ± 1 or ± 3.

. For which of the following functions does the Laplacian vanish?

(A) $xe^y - ye^x$

(B) $x\cos(y) - y\cos(x)$

(C) e^{x+iy}

(D) $yx^2 - \frac{y^3}{3} - xy$

Correct Answer: (C) e^{x+iy} ; (D) $yx^2 - \frac{y^3}{3} - xy$

Solution:

Step 1: Understanding the Concept:

The Laplacian of a function $f(x, y)$ is $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$. A function with $\nabla^2 f = 0$ everywhere is called harmonic. We check each option by taking the two second partial derivatives directly and adding them.

Step 2: Check option (A).

For $f = xe^y - ye^x$: $f_x = e^y - ye^x$, so $f_{xx} = -ye^x$. Also $f_y = xe^y - e^x$, so $f_{yy} = xe^y$.

$$\nabla^2 f = xe^y - ye^x$$

This is not zero for general x, y, so (A) is not harmonic.

Step 3: Check option (B).

For $f = x\cos(y) - y\cos(x)$: $f_x = \cos(y) + y\sin(x)$, so $f_{xx} = y\cos(x)$.
Also $f_y = -x\sin(y) - \cos(x)$, so $f_{yy} = -x\cos(y)$.

$$\nabla^2 f = y\cos(x) - x\cos(y)$$

This is also not zero in general, so (B) is not harmonic.

Step 4: Check option (C).

For $f = e^{x+iy}$, differentiating twice in x brings down a factor of 1^2, and differentiating twice in y brings down a factor of $i^2 = -1$:
$f_{xx} = e^{x+iy}$ and $f_{yy} = i^2 e^{x+iy} = -e^{x+iy}$.

$$\nabla^2 f = e^{x+iy} - e^{x+iy} = 0$$

So (C) is harmonic.

Step 5: Check option (D).

For $f = yx^2 - \frac{y^3}{3} - xy$: $f_x = 2xy - y$, so $f_{xx} = 2y$. Also $f_y = x^2 - y^2 - x$, so $f_{yy} = -2y$.

$$\nabla^2 f = 2y - 2y = 0$$

So (D) is also harmonic.

Final Answer:

Options (A) and (B) give a non-zero Laplacian, while (C) and (D) give zero.

$$\boxed{e^{x+iy} \text{ and } yx^2 - \frac{y^3}{3} - xy}$$

Quick Tip: **Hint:**

Compute $f_{xx} + f_{yy}$ for each option, or recognize which ones are the real/imaginary part of an analytic function of $z = x + iy$.

. A projectile of mass m is launched from the ground with the initial speed v_0 at an angle $30°$ from the horizontal. Take the ground to be horizontal. Ignoring the drag, the magnitude of Hamilton's action $\int L\,dt$ for the particle from the beginning till it hits the ground is $f \times \left(\frac{mv_0^3}{g}\right)$. The value of f (rounded off to two decimal places) is .

Correct Answer: 0.33

Solution:

Step 1: Understanding the Concept:

Hamilton's action here is $S = \int L\,dt$, where the Lagrangian for a projectile with no drag is $L = T - U = \frac{1}{2}m(v_x^2 + v_y^2) - mgy$, taking the launch point as the reference for height. We integrate L over the whole flight, from launch ($t = 0$) to landing ($t = T_f$), for launch angle $\theta = 30°$.

Step 2: Key Formula or Approach:

With x, y measured from the launch point:

$v_x = v_0 \cos\theta$ (constant), $\quad v_y = v_0 \sin\theta - gt$, $\quad y(t) = v_0 \sin\theta\, t - \frac{1}{2}gt^2$

The particle lands when $y = 0$ again, at $T_f = \frac{2v_0 \sin\theta}{g}$. Substitute these into L, simplify, then integrate term by term over t from 0 to T_f.

Step 3: Detailed Explanation:

Substituting and expanding:

$$L = \frac{1}{2}m\left[v_0^2\cos^2\theta + (v_0\sin\theta - gt)^2\right] - mg\left(v_0\sin\theta\, t - \frac{1}{2}gt^2\right)$$

$$L = \frac{1}{2}mv_0^2 - 2mgv_0\sin\theta\, t + mg^2t^2$$

Integrate each term from 0 to $T_f = \frac{2v_0\sin\theta}{g}$:

$$\int_0^{T_f} \frac{1}{2}mv_0^2\,dt = \frac{1}{2}mv_0^2T_f = \frac{mv_0^3\sin\theta}{g}$$

$$\int_0^{T_f} \left(-2mgv_0 \sin\theta\, t\right) dt = -mgv_0 \sin\theta\, T_f^2 = -\frac{4mv_0^3 \sin^3\theta}{g}$$

$$\int_0^{T_f} mg^2 t^2\, dt = \frac{1}{3} mg^2 T_f^3 = \frac{8mv_0^3 \sin^3\theta}{g}$$

Adding the three results:

$$S = \frac{mv_0^3}{g}\left[\sin\theta - 4\sin^3\theta + \frac{8}{3}\sin^3\theta\right] = \frac{mv_0^3}{3g}\left(3\sin\theta - 4\sin^3\theta\right)$$

Since $3\sin\theta - 4\sin^3\theta = \sin(3\theta)$:

$$S = \frac{mv_0^3 \sin(3\theta)}{3g}$$

At $\theta = 30°$, $3\theta = 90°$ and $\sin(90°) = 1$, so $S = \frac{mv_0^3}{3g}$.

Step 4: Final Answer:

Comparing with $S = f \times \frac{mv_0^3}{g}$ gives $f = \frac{1}{3} = 0.3333\ldots$, which rounds to 0.33.

$$\boxed{f = 0.33}$$

Quick Tip: **Hint:**

Write $L = T - U$ using the standard projectile $x(t), y(t)$, and integrate over the whole flight time $T_f = 2v_0 \sin\theta/g$; the closed form comes out to $S = \frac{mv_0^3 \sin(3\theta)}{3g}$.

. Consider an electron in the energy eigenstate $\psi_{211}(\vec{r})$ of the hydrogen atom. Given that the radial probability distribution of the electron in such a state takes its maximum value at $r = n_0 a$, where a is the Bohr radius, and n_0 is an integer. The value of n_0 (in integer) is . The radial part of the wavefunction $\psi_{211}(\vec{r})$ is given by $R_{21}(r) = \frac{1}{\sqrt{24a^5}} re^{-r/2a}$.

Correct Answer: 4

Solution:

Step 1: Understanding the Concept:

The radial probability distribution tells us how likely we are to find the electron at a distance r from the nucleus, regardless of direction. For a state with radial wavefunction $R_{nl}(r)$, this distribution is $P(r) = r^2|R_{nl}(r)|^2$; the extra r^2 comes from the volume of a thin spherical shell of radius r. We are given $R_{21}(r) = \frac{1}{\sqrt{24a^5}} re^{-r/2a}$, and need the value of r where $P(r)$ is maximum.

Step 2: Key Formula or Approach:

$$P(r) = r^2\left[R_{21}(r)\right]^2 = r^2 \cdot \frac{1}{24a^5} r^2 e^{-r/a} = \frac{r^4 e^{-r/a}}{24a^5}$$

To maximize $P(r)$, set $\frac{dP}{dr} = 0$ and solve for r, keeping the physically meaningful maximum and ignoring the trivial $r = 0$ minimum.

Step 3: Detailed Explanation:

Differentiate $r^4 e^{-r/a}$ using the product rule:

$$\frac{d}{dr}\left(r^4 e^{-r/a}\right) = 4r^3 e^{-r/a} + r^4\left(-\frac{1}{a}\right)e^{-r/a} = r^3 e^{-r/a}\left(4 - \frac{r}{a}\right)$$

Setting this to zero: since $e^{-r/a}$ is never zero, either $r^3 = 0$ (giving $r = 0$, where the probability is zero, not a maximum) or:

$$4 - \frac{r}{a} = 0 \implies r = 4a$$

This is the physical maximum, since $P(r)$ rises from 0 at $r = 0$, reaches this peak, then decays back to 0 as $r \to \infty$.

Step 4: Final Answer:
Comparing $r = 4a$ with $r = n_0 a$, we get $n_0 = 4$.

$$\boxed{n_0 = 4}$$

Quick Tip: **Hint:**
$P(r) = r^2 R_{21}^2(r) \propto r^4 e^{-r/a}$; maximize it by setting its derivative to zero, or by maximizing $\ln[r^4 e^{-r/a}]$.

. A dielectric sphere carries a uniform polarization $P = 26\ \mu\text{C.cm}^{-2}$. The magnitude of the electric field at the center of the sphere is $E \times 10^9\ \text{N.C}^{-1}$. The value of E (rounded off to one decimal place) is . ($\epsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2.\text{N}^{-1}.\text{m}^{-2}$)

Correct Answer: 9.8

Solution:

Step 1: Understanding the Concept:
A uniformly polarized dielectric sphere produces a uniform electric field inside itself, pointing opposite to the polarization vector $\vec{P}$. This

is a standard result from the theory of bound charges: the surface bound charge on a uniformly polarized sphere creates a field that is uniform everywhere inside, including at the center.

Step 2: Key Formula or Approach:

The field inside a uniformly polarized sphere is:

$$\vec{E}_{in} = -\frac{\vec{P}}{3\epsilon_0}$$

so its magnitude is $E = \frac{P}{3\epsilon_0}$. We need P in SI units (C/m^2) and then divide by $3\epsilon_0$.

Step 3: Detailed Explanation:

Convert P from $\mu\text{C/cm}^2$ to SI units. Since $1\ \text{cm}^2 = 10^{-4}\ \text{m}^2$:

$$P = 26\ \mu\text{C/cm}^2 = 26 \times 10^{-6}\ \text{C} \times \frac{1}{10^{-4}\ \text{m}^2} = 0.26\ \text{C/m}^2$$

Now compute E:

$$E = \frac{P}{3\epsilon_0} = \frac{0.26}{3 \times 8.85 \times 10^{-12}} = \frac{0.26}{2.655 \times 10^{-11}} \approx 9.79 \times 10^9\ \text{N/C}$$

Step 4: Final Answer:

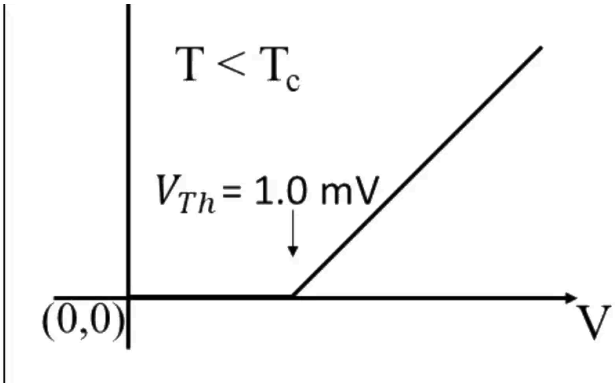
Comparing with $E \times 10^9$ N/C, we get $E \approx 9.79$, which rounds to one decimal place as 9.8.

$$\boxed{E = 9.8}$$

Quick Tip: **Hint:**

Inside a uniformly polarized sphere, $E = \frac{P}{3\epsilon_0}$; convert P to C/m^2 before plugging in.

. Consider a metal-superconductor junction connected to a dc voltage V. At $T < T_c$, where T_c is the superconductor's transition temperature, the current I versus V behavior of this junction is shown schematically in the figure below.



If the superconducting energy gap is D meV, the value of D (rounded off to one decimal place) is

Correct Answer: 2.0

Solution:

Step 1: Understanding the Concept:

In a superconductor the electrons pair up into Cooper pairs, and this pairing opens a gap in the single particle (quasiparticle) energy spectrum. No quasiparticle state exists within an energy Δ of the Fermi level, so states are empty for energies less than Δ above E_F and filled for energies less than Δ below E_F.

Step 2: Key Formula or Approach:

When this superconductor sits in a junction with a normal metal and a bias voltage V is applied, electrons from the metal at the Fermi level can only tunnel into the superconductor once they get enough extra energy to reach an allowed quasiparticle state. This condition is $eV_{Th} = \Delta$, where V_{Th} is the threshold voltage at which the current first turns on.

The forbidden window in the quasiparticle spectrum runs from $-\Delta$ to $+\Delta$ around E_F, so its full width, which is what the question means by the superconducting energy gap D, is $D = 2\Delta$.

Step 3: Detailed Explanation:

The graph shows the current staying at zero until $V_{Th} = 1.0$ mV, after which it rises in a straight line. This flat region is exactly the signature of the gap in the density of states, so the threshold read off the graph gives Δ directly:

$$\Delta = eV_{Th} = 1.0 \text{ meV}$$

Using $D = 2\Delta$:

$$D = 2 \times 1.0 = 2.0 \text{ meV}$$

Final Answer:

The superconducting energy gap read from the junction's I-V curve works out to 2.0 meV.

$$\boxed{D = 2.0 \text{ meV}}$$

Quick Tip: **Hint:**
Read the threshold voltage V_{Th} where the current starts flowing on the graph, set $eV_{Th} = \Delta$, then the full gap is $D = 2\Delta$.

. For the energy dispersion of an electron in a one-dimensional solid $E(k) = E_0 - 2\gamma\cos(ka)$, the ratio of the effective mass of the electron in the solid to the free electron mass (m_e) at $k = 0$ is R_0. Taking $\gamma = 0.5$ eV and $a = 0.5$ nm, the value of R_0 (rounded off to two decimal places) is
($\hbar = 1.054 \times 10^{-34}$ J.s, $m_e = 9.1 \times 10^{-31}$ kg, electron charge $= 1.6 \times 10^{-19}$ C)

Correct Answer: 0.31

Solution:

Step 1: Understanding the Concept:
In a solid, an electron's response to forces is set by the curvature of its energy band $E(k)$, not its free space mass. This is captured by the effective mass, defined from how fast the group velocity changes with k.

Step 2: Key Formula or Approach:
The effective mass formula is

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}$$

So the second derivative of $E(k) = E_0 - 2\gamma\cos(ka)$ with respect to k is needed first.

Step 3: Detailed Explanation:

Differentiate once:

$$\frac{dE}{dk} = 2\gamma a \sin(ka)$$

Differentiate again:

$$\frac{d^2E}{dk^2} = 2\gamma a^2 \cos(ka)$$

At $k = 0$, $\cos(0) = 1$, so:

$$\left.\frac{d^2E}{dk^2}\right|_{k=0} = 2\gamma a^2$$

This gives the effective mass:

$$m^* = \frac{\hbar^2}{2\gamma a^2}$$

Now put in the numbers, with $\gamma = 0.5$ eV $= 0.5 \times 1.6 \times 10^{-19} = 8 \times 10^{-20}$ J and $a = 0.5$ nm $= 5 \times 10^{-10}$ m, so $a^2 = 2.5 \times 10^{-19}$ m^2:

$$m^* = \frac{(1.054 \times 10^{-34})^2}{2 \times (8 \times 10^{-20}) \times (2.5 \times 10^{-19})} = \frac{1.111 \times 10^{-68}}{4 \times 10^{-38}} = 2.777 \times 10^{-31} \text{ kg}$$

The ratio to the free electron mass is:

$$R_0 = \frac{m^*}{m_e} = \frac{2.777 \times 10^{-31}}{9.1 \times 10^{-31}} = 0.305$$

Final Answer:

Rounded to two decimal places, the effective mass at the bottom of this band comes out to about 0.31 times the free electron mass.

$$\boxed{R_0 = 0.31}$$

Quick Tip: **Hint:**

The effective mass comes from the curvature of the band, $m^* = \hbar^2/(d^2E/dk^2)$, evaluated at $k = 0$.

. The specific heat $C_p(T)$ of one mole of a material as a function of temperature T is given as $C_p(T) = AT + BT^3$, where $A = 0.695$ mJ.mol^{-1}.K^{-2} and $B = 0.045$ mJ.mol^{-1}.K^{-4}. When T is changed from 1 K to 10 K at constant pressure, the change in entropy ΔS in mJ.mol^{-1}.K^{-1} (rounded off to one decimal place) is

Correct Answer: 21.2

Solution:

Step 1: Understanding the Concept:

Entropy change at constant pressure comes from integrating C_p/T over the temperature range, since $dS = \frac{C_p}{T}dT$ at constant pressure.

Step 2: Key Formula or Approach:

Divide the given $C_p(T) = AT + BT^3$ by T:

$$\frac{C_p}{T} = A + BT^2$$

Then integrate from $T_1 = 1$ K to $T_2 = 10$ K:

$$\Delta S = \int_1^{10} (A + BT^2)\, dT = A(T_2 - T_1) + \frac{B}{3}(T_2^3 - T_1^3)$$

Step 3: Detailed Explanation:

With $A = 0.695$ and $B = 0.045$ (both in the mJ.mol^{-1}.K units given), and $T_2 - T_1 = 9$, $T_2^3 - T_1^3 = 1000 - 1 = 999$:

$$\Delta S = 0.695 \times 9 + \frac{0.045}{3} \times 999$$

$$\Delta S = 6.255 + 0.015 \times 999 = 6.255 + 14.985$$

$$\Delta S = 21.24 \text{ mJ.mol}^{-1}.\text{K}^{-1}$$

Final Answer:

Rounded to one decimal place, the entropy goes up by 21.2 mJ per mole per kelvin as the sample warms from 1 K to 10 K.

$$\boxed{\Delta S = 21.2 \text{ mJ.mol}^{-1}.\text{K}^{-1}}$$

Quick Tip: **Hint:**

Divide C_p by T first, then integrate the resulting polynomial from 1 K to 10 K.

. Raman spectrum of a molecule was recorded using a source of wavelength 5000 Angstrom. The first Stokes line is observed at 5100 Angstrom. The first anti-Stokes line will appear at a wavelength L (in Angstrom). The value of L (rounded off to nearest integer) is

Correct Answer: 4904

Solution:

Step 1: Understanding the Concept:
In Raman scattering, the molecule either takes a quantum of vibrational energy from the incoming light (the Stokes line, longer wavelength, lower energy) or gives up a quantum of vibrational energy to the scattered light (the anti-Stokes line, shorter wavelength, higher energy). Both lines sit the same energy distance from the source line, just on opposite sides.

Step 2: Key Formula or Approach:
Work in wavenumbers ($1/\lambda$), since energy shifts add simply there. The vibrational wavenumber shift is:

$$\Delta\tilde{\nu} = \frac{1}{\lambda_0} - \frac{1}{\lambda_s}$$

where λ_0 is the source wavelength and λ_s is the Stokes line wavelength. The anti-Stokes line sits the same $\Delta\tilde{\nu}$ on the other side of $1/\lambda_0$:

$$\frac{1}{\lambda_a} = \frac{1}{\lambda_0} + \Delta\tilde{\nu} = \frac{2}{\lambda_0} - \frac{1}{\lambda_s}$$

Step 3: Detailed Explanation:

With $\lambda_0 = 5000$ Angstrom and $\lambda_s = 5100$ Angstrom:

$$\frac{2}{\lambda_0} = \frac{2}{5000} = 4.000 \times 10^{-4} \text{ Angstrom}^{-1}$$

$$\frac{1}{\lambda_s} = \frac{1}{5100} = 1.9608 \times 10^{-4} \text{ Angstrom}^{-1}$$

$$\frac{1}{\lambda_a} = 4.000 \times 10^{-4} - 1.9608 \times 10^{-4} = 2.0392 \times 10^{-4} \text{ Angstrom}^{-1}$$

$$\lambda_a = \frac{1}{2.0392 \times 10^{-4}} = 4903.8 \text{ Angstrom}$$

Final Answer:

Rounded to the nearest integer, the anti-Stokes line appears at 4904 Angstrom.

$$\boxed{L = 4904 \text{ Angstrom}}$$

Quick Tip: **Hint:**

The anti-Stokes line sits the same wavenumber shift above the source line that the Stokes line sits below it.

. A rocket of length 18.0 m is moving at speed $0.9c$ (where c is the speed of light) parallel to its own length, relative to the earth. The length of the rocket measured in meters by an observer on earth (rounded off to two decimal places) is

Correct Answer: 7.85

Solution:

Step 1: Understanding the Concept:

Length contraction says a moving object measured by an observer at rest looks shorter along its direction of motion than its length measured in its own rest frame. The rest frame length is called the proper length.

Step 2: Key Formula or Approach:

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

where L_0 is the proper length (18.0 m, the length in the rocket's own frame) and $v = 0.9c$ is its speed relative to the earth.

Step 3: Detailed Explanation:

$$\frac{v^2}{c^2} = (0.9)^2 = 0.81$$

$$1 - \frac{v^2}{c^2} = 1 - 0.81 = 0.19$$

$$\sqrt{0.19} = 0.4359$$

$$L = 18.0 \times 0.4359 = 7.846 \text{ m}$$

Final Answer:

Rounded to two decimal places, the observer on earth measures the rocket's length as 7.85 m.

$$\boxed{L = 7.85 \text{ m}}$$

Quick Tip: **Hint:**

Use $L = L_0\sqrt{1 - v^2/c^2}$ with $L_0 = 18.0$ m and $v = 0.9c$.

. The function $f(z)$ of the complex variable z given below,

$$f(z) = \frac{z^2 - 5z + 4}{z^3 + 4z - z^2 - 4}$$

has singular points at $z =$

(A) 1 and $(2 - i)$

(B) $2i$ and $-2i$

(C) 1 and $(2 + i)$

(D) $(2 + i)$ only

Correct Answer: (B) $2i$ and $-2i$

Solution:

Step 1: Factor the numerator.

The numerator is $z^2 - 5z + 4$. We need two numbers that multiply to 4 and add to -5, which are -1 and -4. So the numerator splits as $z^2 - 5z + 4 = (z - 1)(z - 4)$.

Step 2: Factor the denominator by grouping.

Write the denominator in standard order first: $z^3 + 4z - z^2 - 4 = z^3 - z^2 + 4z - 4$. Group the first two terms and the last two terms separately.

$$z^3 - z^2 + 4z - 4 = z^2(z - 1) + 4(z - 1)$$

Both groups share the factor $(z - 1)$, so pull it out.

$$z^3 - z^2 + 4z - 4 = (z - 1)(z^2 + 4)$$

Step 3: Simplify the function.

Put the factored numerator and denominator together.

$$f(z) = \frac{(z - 1)(z - 4)}{(z - 1)(z^2 + 4)}$$

The factor $(z - 1)$ appears in both the top and the bottom, so it cancels for every z apart from $z = 1$ itself. This means $z = 1$ is a removable singularity, not a genuine pole. Once the common factor is removed we are left with $f(z) = \frac{z - 4}{z^2 + 4}$ away from $z = 1$.

Step 4: Find the true singular points.

The remaining denominator $z^2 + 4$ vanishes when $z^2 = -4$, which

gives $z = 2i$ and $z = -2i$. At these two points the numerator $z - 4$ is not zero, so these are genuine poles of $f(z)$.

Step 5: Why the other options are wrong.
Options (A) and (C) both keep $z = 1$ as a singular point, but that factor cancels out, so $z = 1$ is not a true singularity of $f(z)$; this rules out both. Option (D) drops $-2i$ even though $z^2 + 4 = 0$ has two roots, $2i$ and $-2i$, so it is incomplete.

Final Answer:
The genuine singular points of $f(z)$ are $z = 2i$ and $z = -2i$, which is option (B).

$$\boxed{z = 2i, \ -2i}$$

Quick Tip: Factor the numerator and denominator first. Check whether any factor cancels between them before you decide which points are truly singular.

. Consider the Pauli matrices $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. The value of $\mathrm{Tr}\big(\sigma_z[\sigma_x, \sigma_y]\big)$ is

(A) $2i$

(B) i

(C) $4i$

(D) $\frac{i}{2}$

Correct Answer: (C) $4i$

Solution:

Step 1: Multiply $\sigma_x\sigma_y$.

$$\sigma_x\sigma_y = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} = \begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix}$$

Each entry comes from the usual row times column rule: top-left is $0 \cdot 0 + 1 \cdot i = i$, top-right is $0 \cdot (-i) + 1 \cdot 0 = 0$, and so on for the rest.

Step 2: Multiply $\sigma_y\sigma_x$.

$$\sigma_y\sigma_x = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix}$$

Step 3: Form the commutator $[\sigma_x, \sigma_y] = \sigma_x\sigma_y - \sigma_y\sigma_x$.

$$[\sigma_x, \sigma_y] = \begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix} - \begin{pmatrix}-i & 0\\ 0 & i\end{pmatrix} = \begin{pmatrix}2i & 0\\ 0 & -2i\end{pmatrix} = 2i\,\sigma_z$$

Step 4: Multiply by σ_z and take the trace.

$$\sigma_z[\sigma_x, \sigma_y] = \sigma_z(2i\sigma_z) = 2i\,\sigma_z^2$$

Since $\sigma_z^2 = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} = I$, we get $\sigma_z[\sigma_x, \sigma_y] = 2iI$. The trace of $2iI$ is $2i$ times the trace of the 2×2 identity, which is $2i \times 2 = 4i$.

Step 5: Why the other options are wrong.

Option (A), $2i$, is only the coefficient in $[\sigma_x, \sigma_y] = 2i\sigma_z$ before taking the trace with σ_z; it skips the last multiplication and trace step. Options (B) and (D) do not match this direct computation at all and would only appear from an arithmetic slip in the matrix multiplication.

Final Answer:

$\text{Tr}(\sigma_z[\sigma_x, \sigma_y]) = 4i$, which is option (C).

$$\boxed{4i}$$

Quick Tip: Multiply out $\sigma_x\sigma_y$ and $\sigma_y\sigma_x$ as 2×2 matrices first, then subtract to get the commutator before multiplying by σ_z.

. Which one of the following statements is true?

(A) In the decay $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$, CPT is violated.

(B) The decay $\Lambda \to p^+ + \pi^-$ is allowed and strangeness is violated.

(C) The decay $p^+ \to e^+ + \gamma$ is allowed.

(D) The decay $\Omega^- \to \Xi^0 + K^-$ is allowed.

Correct Answer: (B) The decay $\Lambda \to p^+ + \pi^-$ is allowed and strangeness is violated.

Solution:

Step 1: Check statement (A).

The decay $\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$ is the standard weak decay of the muon. It conserves charge, lepton number for each generation (an electron neutrino and a muon antineutrino appear together with the muon

disappearing), energy and momentum. CPT symmetry is a property that every local, Lorentz-invariant quantum field theory obeys automatically, and the Standard Model weak interaction that drives this decay is exactly such a theory. Nothing about this decay breaks CPT, so statement (A) is FALSE.

Step 2: Check statement (B).

The decay $\Lambda \to p^+ + \pi^-$ proceeds through the weak interaction (it is the dominant decay mode of the Λ baryon, with a relatively long lifetime typical of weak processes). The Λ carries strangeness $S = -1$, while both the proton and the pion carry $S = 0$. So strangeness changes from -1 to 0, a change of one unit. Weak interactions are allowed to change strangeness by exactly ± 1, unlike strong or electromagnetic interactions which must conserve it exactly. So this decay is allowed, and strangeness is indeed violated (changed) in the process. Statement (B) is TRUE.

Step 3: Check statement (C).

The decay $p^+ \to e^+ + \gamma$ would take a baryon (baryon number $B = 1$) to a positron and a photon, both of which have baryon number 0. Baryon number conservation is an extremely well-tested law (it is why the proton, the lightest baryon, is stable), and this decay would break it outright. This decay has never been observed and is forbidden in the Standard Model, so statement (C) is FALSE.

Step 4: Check statement (D).

For $\Omega^- \to \Xi^0 + K^-$, first check the simple conservation laws: charge is $-1 \to 0 + (-1) = -1$ (conserved), baryon number is $1 \to 1 + 0 = 1$ (conserved), and strangeness is $-3 \to -2 + (-1) = -3$ (also conserved). Even though every quantum number balances, a decay also needs enough rest-mass energy to work with. The Ω^- has a mass

of about 1672 MeV, while the Ξ^0 (about 1315 MeV) and the K^- (about 494 MeV) together add up to about 1809 MeV, more than the Ω^- itself weighs. Since the products would need more energy than the parent particle provides, this decay cannot happen. Statement (D) is FALSE.

Final Answer:
Only statement (B) correctly describes an allowed decay, since weak decays are permitted to violate strangeness by one unit.

$$\boxed{\text{(B)}}$$

Quick Tip: Check charge, baryon number and strangeness for each decay, then also check whether the parent particle actually has enough mass to produce the listed final particles.

. The Hamiltonian for a quantum particle of mass m is given below, where $\omega < \Omega$. The Schrodinger equation for this system can be solved exactly using the orthogonal transformation $x = \frac{x_1 - x_2}{\sqrt{2}}$ and $y = \frac{x_1 + x_2}{\sqrt{2}}$.

$$H = -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right] + \frac{1}{2}m\Omega^2(x^2 + y^2) + m\omega^2 xy$$

The ground state energy of this system is

(A) $\frac{\hbar}{2}\left[\sqrt{\Omega^2 - \omega^2} + \sqrt{\Omega^2 + \omega^2}\right]$

(B) $\hbar\left[\sqrt{\Omega^2 - \omega^2} + \sqrt{\Omega^2 + \omega^2}\right]$

(C) $\frac{\hbar}{2}\left[\sqrt{\Omega^2 + \Omega\omega} + \sqrt{\Omega^2 - \Omega\omega}\right]$

(D) $\hbar\left[\sqrt{\Omega^2 + \Omega\omega} - \sqrt{\Omega^2 - \Omega\omega}\right]$

Correct Answer: (A) $\frac{\hbar}{2}\left[\sqrt{\Omega^2-\omega^2}+\sqrt{\Omega^2+\omega^2}\right]$

Solution:

Step 1: Understand the coupling in the Hamiltonian.

The Hamiltonian describes a particle moving in two dimensions with an isotropic potential $\frac{1}{2}m\Omega^2(x^2+y^2)$ plus an extra cross term $m\omega^2xy$ that mixes x and y. Because of this cross term, x and y are not independent oscillators as written, so we cannot just read off two separate frequencies yet. The question tells us that rotating to a new pair of coordinates x_1, x_2 removes the mixing.

Step 2: Rewrite the potential in the new coordinates.

We are given $x=\frac{x_1-x_2}{\sqrt{2}}$ and $y=\frac{x_1+x_2}{\sqrt{2}}$. First find x^2+y^2:

$$x^2+y^2=\frac{(x_1-x_2)^2+(x_1+x_2)^2}{2}=\frac{2x_1^2+2x_2^2}{2}=x_1^2+x_2^2$$

This is expected because a rotation of coordinates never changes the total distance from the origin. Next find xy:

$$xy=\frac{(x_1-x_2)(x_1+x_2)}{2}=\frac{x_1^2-x_2^2}{2}$$

Step 3: Rewrite the kinetic term.

A rotation between two coordinate pairs is an orthogonal transformation, and the sum of second derivatives (the Laplacian) is unchanged by any orthogonal (rotation) transformation. So

$$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}$$

Step 4: Assemble the Hamiltonian in the new coordinates.

$$H = -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}\right] + \frac{1}{2}m\Omega^2(x_1^2 + x_2^2) + m\omega^2\left(\frac{x_1^2 - x_2^2}{2}\right)$$

Group the x_1 terms and the x_2 terms separately:

$$H = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} + \frac{1}{2}m(\Omega^2 + \omega^2)x_1^2\right] + \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} + \frac{1}{2}m(\Omega^2 - \omega^2)x_2^2\right]$$

This is now exactly two independent, uncoupled simple harmonic oscillators.

Step 5: Read off the two frequencies and the ground state energy.

A one-dimensional oscillator $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega_0^2x^2$ has frequency ω_0 and ground state energy $\frac{1}{2}\hbar\omega_0$. Comparing, the two frequencies here are $\omega_1 = \sqrt{\Omega^2 + \omega^2}$ (from the x_1 oscillator) and $\omega_2 = \sqrt{\Omega^2 - \omega^2}$ (from the x_2 oscillator); the condition $\omega < \Omega$ given in the question keeps ω_2 real. The total ground state energy of two independent oscillators just adds:

$$E_0 = \frac{1}{2}\hbar\omega_1 + \frac{1}{2}\hbar\omega_2 = \frac{\hbar}{2}\left[\sqrt{\Omega^2 + \omega^2} + \sqrt{\Omega^2 - \omega^2}\right]$$

Step 6: Why the other options are wrong.

Option (B) has the right square-root terms but is missing the $\frac{1}{2}$ factor that comes from adding two ground state energies of $\frac{1}{2}\hbar\omega$ each. Options (C) and (D) mix up the frequencies with $\Omega\omega$ products under the square root instead of ω^2, which does not come from this cross

term at all.

Final Answer:

The ground state energy is $\frac{\hbar}{2}\left[\sqrt{\Omega^2-\omega^2}+\sqrt{\Omega^2+\omega^2}\right]$, option (A).

$$\boxed{E_0=\frac{\hbar}{2}\left[\sqrt{\Omega^2-\omega^2}+\sqrt{\Omega^2+\omega^2}\right]}$$

Quick Tip: Rewrite x^2+y^2 and xy in terms of x_1, x_2 using the given rotation, then group terms to see two separate uncoupled oscillators.

. Consider two particles with angular momenta $j_1 = 2\hbar$ and $j_2 = \hbar/2$. If the expression

$$|j=\tfrac{5}{2}, m=\tfrac{3}{2}\rangle = c_1|j_1=2, m_1=1\rangle|j_2=\tfrac{1}{2}, m_2=\tfrac{1}{2}\rangle + c_2|j_1=2, m_1=2\rangle|j_2=\tfrac{1}{2}, m_2=-\tfrac{1}{2}\rangle$$

gives an eigenstate of the total angular momentum of the two particles, using standard notation, which of the following is true?
(Hint: $\hat{J}_\pm|j,m\rangle = \sqrt{j(j+1)-m(m\pm1)}\,|j,m\pm1\rangle$)

(A) $c_1 = \frac{2}{\sqrt{5}}, \quad c_2 = \frac{1}{\sqrt{5}}$

(B) $c_1 = \frac{1}{\sqrt{5}}, \quad c_2 = \frac{2}{\sqrt{5}}$

(C) $c_1 = \frac{1}{\sqrt{2}}, \quad c_2 = \frac{1}{\sqrt{2}}$

(D) $c_1 = 0, \quad c_2 = 1$

Correct Answer: (A) $c_1 = \frac{2}{\sqrt{5}}, \quad c_2 = \frac{1}{\sqrt{5}}$

Solution:

Step 1: Start from the fully stretched top state.

For $j_1 = 2$ and $j_2 = \frac{1}{2}$, the largest possible total is $j = j_1 + j_2 = \frac{5}{2}$, and its top state is simply the product of both particles at their own maximum m:

$$\left|\tfrac{5}{2}, \tfrac{5}{2}\right\rangle = |j_1 = 2, m_1 = 2\rangle\,|j_2 = \tfrac{1}{2}, m_2 = \tfrac{1}{2}\rangle$$

This is true because there is only one way to combine m_1 and m_2 to reach the maximum $m = \frac{5}{2}$, so no mixing coefficient is needed here.

Step 2: Lower the total state by one step using the hint formula.

Apply the total lowering operator $\hat{J}_- = \hat{J}_{1-} + \hat{J}_{2-}$ to $\left|\frac{5}{2}, \frac{5}{2}\right\rangle$. On the left side, using the hint with $j = \frac{5}{2}, m = \frac{5}{2}$:

$$\hat{J}_- \left|\tfrac{5}{2}, \tfrac{5}{2}\right\rangle = \sqrt{\tfrac{5}{2}\cdot\tfrac{7}{2} - \tfrac{5}{2}\cdot\tfrac{3}{2}}\,\left|\tfrac{5}{2}, \tfrac{3}{2}\right\rangle = \sqrt{\tfrac{35}{4} - \tfrac{15}{4}}\,\left|\tfrac{5}{2}, \tfrac{3}{2}\right\rangle = \sqrt{5}\,\left|\tfrac{5}{2}, \tfrac{3}{2}\right\rangle$$

Step 3: Apply the same lowering operator to the product state.

On the right side, $\hat{J}_{1-}$ acts only on the $j_1 = 2$ part and $\hat{J}_{2-}$ only on the $j_2 = \frac{1}{2}$ part. Using the hint for each:

$$\hat{J}_{1-}|2, 2\rangle = \sqrt{2\cdot 3 - 2\cdot 1}\,|2, 1\rangle = \sqrt{6-2}\,|2, 1\rangle = 2|2, 1\rangle$$

$$\hat{J}_{2-}\left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle = \sqrt{\tfrac{1}{2}\cdot\tfrac{3}{2} - \tfrac{1}{2}\cdot\left(-\tfrac{1}{2}\right)}\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle = \sqrt{\tfrac{3}{4} + \tfrac{1}{4}}\,\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle = 1\cdot\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

So

$$\hat{J}_-\left(|2, 2\rangle\left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle\right) = 2\,|2, 1\rangle\left|\tfrac{1}{2}, \tfrac{1}{2}\right\rangle + 1\cdot|2, 2\rangle\left|\tfrac{1}{2}, -\tfrac{1}{2}\right\rangle$$

Step 4: Match the two sides and normalise.

Both sides describe the same physical state, so

$$\sqrt{5}\left|\tfrac{5}{2},\tfrac{3}{2}\right\rangle = 2\left|2,1\right\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle + \left|2,2\right\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

Dividing through by $\sqrt{5}$:

$$\left|\tfrac{5}{2},\tfrac{3}{2}\right\rangle = \frac{2}{\sqrt{5}}\left|2,1\right\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle + \frac{1}{\sqrt{5}}\left|2,2\right\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$$

Step 5: Read off the coefficients.

Comparing term by term with the expression given in the question, $c_1 = \frac{2}{\sqrt{5}}$ (the coefficient of $|2,1\rangle|\frac{1}{2},\frac{1}{2}\rangle$) and $c_2 = \frac{1}{\sqrt{5}}$ (the coefficient of $|2,2\rangle|\frac{1}{2},-\frac{1}{2}\rangle$).

Step 6: Why the other options are wrong.

Option (B) simply swaps c_1 and c_2, which would be the answer only if the labeling of the two basis kets were reversed. Option (C) treats the two terms as equally weighted, ignoring that lowering m_1 from 2 to 1 and lowering m_2 from $\frac{1}{2}$ to $-\frac{1}{2}$ do not carry equal weight here. Option (D) would mean the state is just the unmixed product state, which is only true for the top state $m = \frac{5}{2}$, not for $m = \frac{3}{2}$.

Final Answer:

$c_1 = \frac{2}{\sqrt{5}}$ and $c_2 = \frac{1}{\sqrt{5}}$, option (A).

$$\boxed{c_1 = \frac{2}{\sqrt{5}},\ c_2 = \frac{1}{\sqrt{5}}}$$

Quick Tip: Start from the top state $|j = 5/2, m = 5/2\rangle$, which is just the simple product state, then apply the lowering operator to both sides and match coefficients.

. The energy E and degeneracy d of the second excited state of a three-dimensional, isotropic quantum harmonic oscillator with angular frequency ω are

(A) $E = \frac{7}{2}\hbar\omega,\ d = 6$

(B) $E = \frac{7}{2}\hbar\omega,\ d = 3$

(C) $E = \frac{5}{2}\hbar\omega,\ d = 3$

(D) $E = \frac{5}{2}\hbar\omega,\ d = 6$

Correct Answer: (A) $E = \frac{7}{2}\hbar\omega,\ d = 6$

Solution:

Step 1: Understanding the Concept:

A three-dimensional isotropic quantum harmonic oscillator is just three independent 1D oscillators of the same frequency ω, one along each axis. The energy of a single 1D oscillator with quantum number n_x is $(n_x + \frac{1}{2})\hbar\omega$, so the total energy only depends on the sum $n = n_x + n_y + n_z$.

Step 2: Key Formula or Approach:

The energy of the level labeled by n is $E_n = (n + \frac{3}{2})\hbar\omega$.

Its degeneracy is the number of ways three non-negative integers n_x, n_y, n_z can add up to n, which is $d(n) = \dfrac{(n+1)(n+2)}{2}$.

Step 3: Detailed Explanation:

The ground state is $n = 0$, the first excited state is $n = 1$, so the second excited state is $n = 2$.

Energy:

$$E_2 = \left(2 + \frac{3}{2}\right)\hbar\omega = \frac{7}{2}\hbar\omega$$

Degeneracy:

$$d(2) = \frac{(2+1)(2+2)}{2} = \frac{3 \times 4}{2} = 6$$

Checking this by listing every (n_x, n_y, n_z) with $n_x + n_y + n_z = 2$: $(2,0,0), (0,2,0), (0,0,2), (1,1,0), (1,0,1), (0,1,1)$, that is 6 distinct states, matching the formula.

Step 4: Why the other options are wrong.

Option (B) has the right energy but the wrong degeneracy, 3 is the degeneracy of the first excited state ($n = 1$), not the second.

Options (C) and (D) both use $E = \frac{5}{2}\hbar\omega$, which is the energy of the first excited state, not the second.

Final Answer:

The second excited state has energy $\frac{7}{2}\hbar\omega$ and degeneracy 6.

$$\boxed{E = \frac{7}{2}\hbar\omega,\ d = 6}$$

Quick Tip: Second excited state of a 3D isotropic oscillator is $n = 2$: use $E_n = (n + 3/2)\hbar\omega$ and degeneracy $d(n) = (n + 1)(n + 2)/2$.

. Two identical particles with a fixed total energy $E = 2\hbar\omega$ are in thermal equilibrium in a one-dimensional harmonic oscillator potential $\frac{1}{2}m\omega^2x^2$. Let the entropy of the particles be denoted by S_F if they are fermions with spin $\frac{1}{2}$ ($S_z = \pm\frac{\hbar}{2}$) and by S_B if they are bosons with spin 0. Then, which of the following options is correct? (k_B is the Boltzmann constant)

(A) $S_F = k_B \ln 2,\ S_B = k_B \ln 2$

(B) $S_F = 2k_B \ln 2,\ S_B = 0$

(C) $S_F = 4k_B \ln 2,\ S_B = 0$

(D) $S_F = 2k_B \ln 2,\ S_B = k_B \ln 2$

Correct Answer: (B) $S_F = 2k_B \ln 2,\ S_B = 0$

Solution:

Step 1: Understanding the Concept:

A single particle in this 1D oscillator can sit in any level $n = 0, 1, 2, \ldots$ with energy $\varepsilon_n = (n + \frac{1}{2})\hbar\omega$. With two particles the total energy is $\varepsilon_{n_1} + \varepsilon_{n_2} = (n_1 + n_2 + 1)\hbar\omega$.

Step 2: Key Formula or Approach:

Setting this equal to the given $E = 2\hbar\omega$ gives $n_1 + n_2 = 1$. Since n_1, n_2 are non-negative integers, the only way to split 1 quantum between two levels is $n_1 = 0, n_2 = 1$ (in either order); there is no solution with $n_1 = n_2$. So one particle always occupies $n = 0$ and the other occupies $n = 1$; the number of accessible microstates is found from the entropy formula $S = k_B \ln \Omega$, where Ω counts the distinct many-body quantum

states with this energy.

Step 3: Detailed Explanation:

For fermions with spin $\frac{1}{2}$, each spatial level $n = 0$ and $n = 1$ can be paired with spin up or spin down. Because the two particles sit in different spatial levels ($n = 0$ and $n = 1$), the Pauli principle places no restriction on their spins at all, so a valid antisymmetric (Slater determinant) state exists for every combination of the two spins. Number of combinations: 2 spin choices for the particle in $n = 0$ times 2 spin choices for the particle in $n = 1$ gives $\Omega_F = 2 \times 2 = 4$.

$$S_F = k_B \ln \Omega_F = k_B \ln 4 = 2k_B \ln 2$$

For bosons with spin 0, there is no spin label to combine with. The two particles still must occupy the different spatial levels $n = 0$ and $n = 1$ (there is no other way to make $n_1 + n_2 = 1$), and for identical bosons there is exactly one symmetric combination of "one particle in 0, one particle in 1", not two. So $\Omega_B = 1$.

$$S_B = k_B \ln \Omega_B = k_B \ln 1 = 0$$

Step 4: Why the other options are wrong.

Option (A) gives the bosons a nonzero entropy, but a unique, non-degenerate boson microstate has $S_B = 0$, not $k_B \ln 2$.

Option (C) overcounts the fermion microstates as if there were 16 of them ($4k_B \ln 2 = k_B \ln 16$); only 4 spin combinations are actually allowed here.

Option (D) again wrongly gives the bosons a nonzero entropy.

Final Answer:

The spin combinatorics of two spin-$\frac{1}{2}$ fermions in different orbitals gives $S_F = 2k_B \ln 2$, while the non-degenerate boson state gives $S_B = 0$.

$$\boxed{S_F = 2k_B \ln 2,\ S_B = 0}$$

Quick Tip: Total energy fixes $n_1 + n_2 = 1$, so the two particles must sit in levels 0 and 1. Count the allowed spin/label combinations for fermions and for bosons separately, then use $S = k_B \ln \Omega$.

. In the circuit shown, $V(t) = 2\sin(2000\pi t)$ Volts, where t is in seconds. The source $V(t)$ drives the non-inverting (+) input of an opamp directly, the inverting (-) input is tied to ground (0 V), and the opamp is powered from +15 V and -15 V rails with no feedback network between its output and either input. Take the opamp to be ideal. Which of the following options is correct?

(A) $V_{out}(t)$ is square wave with peak-to-peak voltage = 30 V and time period is 1 ms.

(B) $V_{out}(t)$ is a sine wave with peak-to-peak voltage = 4 V and time period is 1 ms.

(C) $V_{out}(t)$ is sine wave with peak-to-peak voltage = 30 V and time period of 1 ms.

(D) $V_{out}(t)$ is square wave with peak-to-peak voltage = 4 V and time period is 1 ms.

Correct Answer: (A) $V_{out}(t)$ is square wave with peak-to-peak voltage = 30 V and time period is 1 ms.

Solution:

Step 1: Understanding the Concept:

The opamp here has no feedback path from its output back to either input, so it is not being used as an amplifier, it is being used as an open-loop comparator. An ideal opamp with no feedback has effectively infinite gain, so the tiniest difference between its two inputs drives the output all the way to one power rail or the other.

Step 2: Key Formula or Approach:

Comparator rule: $V_{out} = +V_{sat}$ when $V_+ > V_-$, and $V_{out} = -V_{sat}$ when $V_+ < V_-$. Here $V_+ = V(t)$ and $V_- = 0$ (grounded), and the rails are ± 15 V, so $V_{sat} = 15$ V.

Step 3: Detailed Explanation:

Since $V_- = 0$, the comparison is simply whether $V(t)$ is positive or negative.

When $V(t) = 2\sin(2000\pi t) > 0$, the output snaps to $+15$ V.

When $V(t) < 0$, the output snaps to -15 V.

This makes $V_{out}(t)$ a square wave (not a scaled copy of the sine wave) that flips sign exactly when $V(t)$ crosses zero.

Peak-to-peak amplitude:

$$V_{pp} = 15 - (-15) = 30 \text{ V}$$

Time period: $V(t)$ has angular frequency $\omega = 2000\pi$ rad/s, and the output switches at the same zero crossings as $V(t)$, so it repeats with the same period as $V(t)$:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2000\pi} = \frac{1}{1000} \text{ s} = 1 \text{ ms}$$

Step 4: Why the other options are wrong.

Options (B) and (C) both say the output is a sine wave. An open-loop opamp with no feedback cannot produce a sine wave, its output can only sit at one of the two rails, so it must be a square wave.

Option (D) has the right waveform shape but the wrong amplitude, 4 V is nowhere close to the rail-to-rail swing of 30 V that an ideal saturated opamp actually produces.

Final Answer:

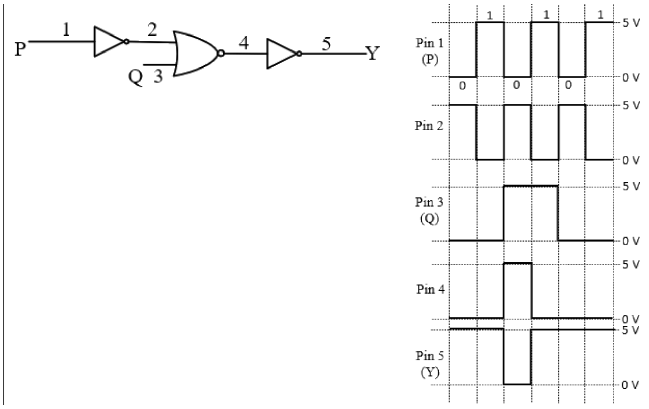
The comparator output is a 30 V peak-to-peak square wave with a 1 ms period.

$$\boxed{V_{pp} = 30\ \text{V},\ T = 1\ \text{ms}}$$

Quick Tip: With no feedback, an ideal opamp comparator output only ever sits at one of its two supply rails, comparing $V(t)$ against the grounded inverting input tells you when it flips.

•

Considering the circuit and the associated signals measured at different pins (numbered as 1, 2, 3, 4, 5) shown in the figure, the correct option is



(A) NOT gate between pins 1 and 2 is faulty.

(B) NOR gate is faulty.

(C) NOT gate between pins 4 and 5 is faulty.

(D) The NOR and output NOT gates, are both faulty.

Correct Answer: (B) NOR gate is faulty.

Solution:

Step 1: Understanding the Concept:

Write down what each gate is supposed to do if it is working correctly. Pin 2 should be P inverted (a NOT gate). Pin 4 should be the NOR of pin 2 and Q (pin 3), that is $\overline{(\text{pin 2}) + Q}$. Pin 5 ($Y$) should be pin 4 inverted (another NOT gate).

Step 2: Key Formula or Approach:

Read P, Q, and pins 2, 4, 5 off the waveform over the six marked time slots, then check each gate's output against what its own inputs demand. Whichever gate's actual output disagrees with the correct logic of its own actual inputs is the faulty one.

Step 3: Detailed Explanation:

Reading the waveform slot by slot (slots 1 to 6):

P: 0, 1, 0, 1, 0, 1.

Pin 2 (measured): 1, 0, 1, 0, 1, 0, which is exactly $\overline{P}$ in every slot. So the first NOT gate is working correctly.

Q: 0, 0, 1, 1, 0, 0.

Using the correct pin 2 values above, the NOR gate should give $\overline{(\text{pin 2}) + Q}$: 0, 1, 0, 0, 0, 1.

Pin 4 (measured): 0, 0, 1, 1, 0, 0, this is identical to Q itself, not to $\overline{(\text{pin 2}) + Q}$. The NOR gate's actual output does not match what its

own two inputs require, so the NOR gate is faulty.
Pin 5 (measured, Y): 1, 1, 0, 0, 1, 1, which is exactly the inverse of the (faulty) pin 4 values in every slot. So the output NOT gate is correctly inverting whatever it is handed, it is not at fault.

Step 4: Why the other options are wrong.
Option (A) is wrong because pin 2 matches $\overline{P}$ perfectly, that NOT gate is healthy.
Option (C) is wrong because pin 5 matches $\overline{\text{pin 4}}$ perfectly, the output NOT gate is healthy.
Option (D) is wrong because only the NOR stage disagrees with its own inputs, the output NOT gate checks out fine on the (already wrong) signal it receives.

Final Answer:
Both NOT gates behave correctly on their actual inputs; only the middle NOR gate's output fails to match $\overline{(\text{pin 2}) + Q}$.

NOR gate is faulty

Quick Tip: Compute what pin 2, pin 4 and pin 5 should be from P and Q using the stated NOT-NOR-NOT chain, then check each measured pin against that expectation to isolate the bad gate.

. A gas of N classical particles that can occupy energy levels ϵ_1 and $\epsilon_2 = \epsilon_1 + \Delta$ is in equilibrium with a reservoir at temperature T. From the schematics shown below, choose the correct dependence of the internal energy U on T.

(A) A curve that starts flat at a low value for small T, rises smoothly through an S-shaped (sigmoid) transition, and saturates to a higher flat value at large T.

(B) A curve that starts at zero and keeps rising with an ever-increasing slope as T grows, with no upper flat limit.

(C) A curve that starts high at small T and falls off smoothly to a lower flat value as T increases.

(D) A curve that starts at a small nonzero value and keeps rising with an ever-increasing slope as T grows, with no upper flat limit.

Correct Answer: (A) A curve that starts flat at a low value for small T, rises smoothly through an S-shaped (sigmoid) transition, and saturates to a higher flat value at large T.

Solution:

Step 1: Understanding the Concept:
Each of the N classical (distinguishable, non-interacting) particles can sit in either of two energy levels, ε_1 or $\varepsilon_2 = \varepsilon_1 + \Delta$. At temperature T, a single particle's average energy comes from the ordinary canonical (Boltzmann) weighting of these two levels, this is the classic "two-level system."

Step 2: Key Formula or Approach:
The single particle partition function is $z = e^{-\beta\varepsilon_1} + e^{-\beta\varepsilon_2}$, with $\beta = 1/(k_B T)$. The average energy per particle is $\langle\varepsilon\rangle = -\dfrac{\partial \ln z}{\partial \beta}$, and the total internal energy is $U(T) = N\langle\varepsilon\rangle$.

Step 3: Detailed Explanation:
Factor out $e^{-\beta\varepsilon_1}$: $z = e^{-\beta\varepsilon_1}(1 + e^{-\beta\Delta})$, so $\ln z = -\beta\varepsilon_1 + \ln(1 + e^{-\beta\Delta})$.

$$\langle \varepsilon \rangle = -\frac{\partial \ln z}{\partial \beta} = \varepsilon_1 + \frac{\Delta e^{-\beta\Delta}}{1 + e^{-\beta\Delta}} = \varepsilon_1 + \frac{\Delta}{e^{\beta\Delta} + 1}$$

$$U(T) = N\left[\varepsilon_1 + \frac{\Delta}{e^{\Delta/k_B T} + 1}\right]$$

Now look at the two extreme temperatures. As $T \to 0$, $\beta \to \infty$ so $e^{\beta\Delta} \to \infty$ and the correction term vanishes, giving $U \to N\varepsilon_1$: every particle sits frozen in the lower level.

As $T \to \infty$, $\beta \to 0$ so $e^{\beta\Delta} \to 1$ and the correction term saturates at $\Delta/2$, giving $U \to N(\varepsilon_1 + \Delta/2)$: both levels become equally populated and U stops changing.

In between, U rises smoothly from the low plateau to the high plateau as $k_B T$ becomes comparable to Δ, tracing out an S-shaped (sigmoid) curve that flattens at both ends.

Step 4: Why the other options are wrong.

Option (B) keeps rising without any upper limit, but U is physically bounded above by $N(\varepsilon_1 + \Delta/2)$ once the two levels are equally populated, it cannot increase forever.

Option (C) shows U decreasing with T, but heating a two-level system can only push more particles into the higher level, never fewer, so the internal energy cannot fall as T rises.

Option (D) also grows without bound and skips the flat low-temperature plateau altogether.

Final Answer:

$U(T)$ starts flat near $N\varepsilon_1$, rises through an S-shaped transition, and levels off at $N(\varepsilon_1 + \Delta/2)$, this is exactly the shape in graph (A).

$$\boxed{U(T):\ N\epsilon_1 \to N\left(\epsilon_1 + \frac{\Delta}{2}\right),\ \text{sigmoid shape, option (A)}}$$

Quick Tip: A two-level system's average energy per particle is bounded between ϵ_1 (all particles in the ground level) and $\epsilon_1 + \Delta/2$ (levels equally populated), and it rises smoothly between the two.

. The Lagrangian $L_0 = \frac{1}{2}m\dot{q}^2 - \frac{1}{2}m\omega^2 q^2$ with the generalized coordinate q is transformed to $L = L_0 + \alpha\frac{df(q)}{dt}$. Consider the following statements:
(i) Expression for the canonical momentum does not change.
(ii) The equation of motion does not change.
Which of the following options is correct for the above statements?

(A) Both (i) and (ii) are correct.

(B) Both (i) and (ii) are not correct.

(C) (i) is correct and (ii) is not correct.

(D) (i) is not correct and (ii) is correct.

Correct Answer: (D) (i) is not correct and (ii) is correct.

Solution:

Step 1: Understanding the Concept:
The new Lagrangian is built by adding $\alpha\frac{df(q)}{dt}$ to L_0. Since f depends only on q, the chain rule gives $\frac{df(q)}{dt} = f'(q)\dot{q}$, so $L = L_0 + \alpha f'(q)\dot{q}$. This is a term linear in $\dot{q}$ whose coefficient depends on q, which is exactly the form of a total time derivative added to a Lagrangian. We need to check what such a term does to the canonical momentum $p = \frac{\partial L}{\partial \dot{q}}$ and to the Euler-Lagrange equation of motion.

Step 2: Key Formula or Approach:

1. Canonical momentum: $p = \frac{\partial L}{\partial \dot{q}}$.

2. Euler-Lagrange equation: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$.

We work out both quantities for the given L and compare them with the ones from L_0 alone, which give simple harmonic motion.

Step 3: Detailed Explanation:

Canonical momentum from the new L:

$$p = \frac{\partial L}{\partial \dot{q}} = m\dot{q} + \alpha f'(q)$$

The momentum from L_0 alone is $m\dot{q}$, so the extra term $\alpha f'(q)$ makes the new momentum different from the old one whenever $f'(q)$ is not zero. So statement (i) is not correct.

Now the equation of motion. First,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{d}{dt}\left(m\dot{q} + \alpha f'(q)\right) = m\ddot{q} + \alpha f''(q)\dot{q}$$

since $f'(q)$ depends on time only through $q(t)$. Next,

$$\frac{\partial L}{\partial q} = -m\omega^2 q + \alpha f''(q)\dot{q}$$

because differentiating the extra term $\alpha f'(q)\dot{q}$ with respect to q (treating $\dot{q}$ as independent) gives $\alpha f''(q)\dot{q}$. Putting these into the Euler-Lagrange equation:

$$m\ddot{q} + \alpha f''(q)\dot{q} - \left(-m\omega^2 q + \alpha f''(q)\dot{q}\right) = 0$$

$$m\ddot{q} + \alpha f''(q)\dot{q} + m\omega^2 q - \alpha f''(q)\dot{q} = 0$$

The $\alpha f''(q)\dot{q}$ term cancels exactly, leaving

$$m\ddot{q} + m\omega^2 q = 0$$

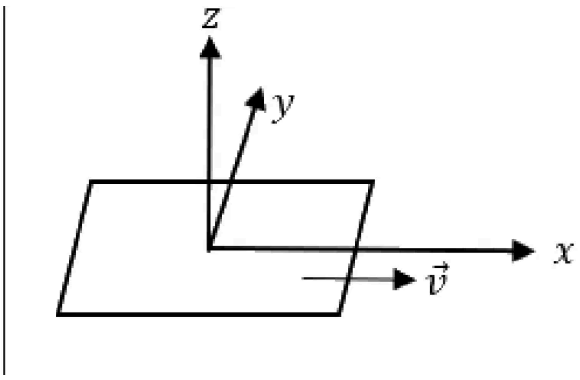
which is the same simple harmonic oscillator equation you get from L_0 by itself. So statement (ii) is correct.

Step 4: Final Answer:
Adding $\alpha\frac{df(q)}{dt}$ changes the canonical momentum but leaves the equation of motion unchanged, so (i) is not correct and (ii) is correct.

$$\boxed{\text{(D)}}$$

Quick Tip: Adding $\alpha\frac{df(q)}{dt} = \alpha f'(q)\dot{q}$ to a Lagrangian only adds a boundary term to the action, so the Euler-Lagrange equation is unchanged; but since $p = \partial L/\partial\dot{q}$ is evaluated pointwise, the momentum does shift by $\alpha f'(q)$.

. An infinitely large thin sheet in the xy-plane carries uniform positive charge density and is moving with constant velocity $\vec{v}$ in the $+x$ direction (see figure below). The direction of the corresponding Poynting vector is



(A) $+x$ for both $z < 0$ and $z > 0$

(B) $+x$ for $z < 0$ and $-x$ for $z > 0$

(C) $-x$ for $z < 0$ and $+x$ for $z > 0$

(D) $-x$ for both $z < 0$ and $z > 0$

Correct Answer: (A) $+x$ for both $z < 0$ and $z > 0$

Solution:

Step 1: Find the electric field of the sheet.

An infinite sheet with uniform positive surface charge density σ sitting in the xy-plane sets up a uniform electric field pointing straight away from the sheet on each side, of magnitude $\sigma/(2\epsilon_0)$. So

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{z} \text{ for } z > 0, \qquad \vec{E} = -\frac{\sigma}{2\epsilon_0}\hat{z} \text{ for } z < 0$$

Step 2: Find the magnetic field of the moving sheet.

A charged sheet moving at velocity $\vec{v} = v\hat{x}$ is a surface current $K = \sigma v$ flowing in the $+x$ direction. Building this field by adding up the fields of parallel line currents (each an infinite wire along x) and integrating across the sheet gives, on each side, a field along $\hat{y}$ whose sign flips across the sheet, exactly like the field of a solenoid winding:

$$\vec{B} = -\frac{\mu_0 \sigma v}{2}\hat{y} \text{ for } z > 0, \qquad \vec{B} = \frac{\mu_0 \sigma v}{2}\hat{y} \text{ for } z < 0$$

Step 3: Compute the Poynting vector for $z > 0$.

The Poynting vector is $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$. Using $\hat{z} \times \hat{y} = -\hat{x}$:

$$\vec{S} = \frac{1}{\mu_0}\left(\frac{\sigma}{2\epsilon_0}\hat{z}\right) \times \left(-\frac{\mu_0 \sigma v}{2}\hat{y}\right) = -\frac{\sigma^2 v}{4\epsilon_0}(\hat{z} \times \hat{y}) = \frac{\sigma^2 v}{4\epsilon_0}\hat{x}$$

So the Poynting vector points along $+x$ for $z > 0$.

Step 4: Compute the Poynting vector for $z < 0$.
Here both $\vec{E}$ and $\vec{B}$ flip sign compared with the $z > 0$ side, and two sign flips cancel:

$$\vec{S} = \frac{1}{\mu_0}\left(-\frac{\sigma}{2\epsilon_0}\hat{z}\right) \times \left(\frac{\mu_0 \sigma v}{2}\hat{y}\right) = -\frac{\sigma^2 v}{4\epsilon_0}(\hat{z} \times \hat{y}) = \frac{\sigma^2 v}{4\epsilon_0}\hat{x}$$

which again points along $+x$. So the same-sign flip in both E and B keeps the energy flow direction fixed on both sides of the sheet.

Step 5: Why the other options are wrong.
Options (B) and (D) both claim the Poynting vector flips sign across the sheet, but since $\vec{E}$ and $\vec{B}$ both flip sign together when crossing from $z > 0$ to $z < 0$, their cross product does not flip, so the flow direction stays the same on both sides. Option (C) claims the field points in $-x$, which is the opposite of what the cross product $\hat{z} \times (-\hat{y}) = \hat{x}$ actually gives.

Final Answer:
The Poynting vector points in the $+x$ direction on both sides of the sheet, in the same sense as the sheet's own motion.

$$\boxed{\text{(A)}}$$

Quick Tip: Use $E = \sigma/2\epsilon_0$ away from the sheet and $B = \mu_0 K/2$ (with $K = \sigma v$) circling the current sheet like a solenoid winding; both E and B flip sign together across the sheet, so $\vec{S} = \vec{E} \times \vec{B}/\mu_0$ keeps the same sign on both sides.

. A positive point charge is fixed at the origin. At some distance from it on the x axis, a point dipole is kept pointing in the $+y$ direction. The force on the dipole is

(A) 0

(B) in the $+y$ direction

(C) in the $-y$ direction

(D) in the $+x$ direction

Correct Answer: (B) in the $+y$ direction

Solution:

Step 1: Understanding the Concept:

A point dipole with dipole moment $\vec{p}$ sitting in a non-uniform external field $\vec{E}$ feels a net force even though the total charge is zero, because the field is slightly different at its two ends. The force is given by $\vec{F} = (\vec{p} \cdot \nabla)\vec{E}$, evaluated at the location of the dipole. Here the field is that of the point charge q fixed at the origin, and $\vec{p} = p\hat{y}$ sits at a point $(d, 0, 0)$ on the x-axis.

Step 2: Key Formula or Approach:

1. Field of a point charge: $\vec{E}(x, y, z) = \frac{kq}{r^3}(x, y, z)$, with $r = \sqrt{x^2 + y^2 + z^2}$ and $k = 1/4\pi\epsilon_0$.

2. Since $\vec{p}$ only has a y-component, the force reduces to $\vec{F} = p\frac{\partial \vec{E}}{\partial y}$, evaluated at $(d, 0, 0)$.

Step 3: Detailed Explanation:

Differentiate each component of $\vec{E}$ with respect to y:

$$\frac{\partial E_x}{\partial y} = kq\,\frac{\partial}{\partial y}\left(\frac{x}{r^3}\right) = -\frac{3kqxy}{r^5}$$

$$\frac{\partial E_y}{\partial y} = kq\,\frac{\partial}{\partial y}\left(\frac{y}{r^3}\right) = kq\left(\frac{1}{r^3} - \frac{3y^2}{r^5}\right)$$

$$\frac{\partial E_z}{\partial y} = kq\,\frac{\partial}{\partial y}\left(\frac{z}{r^3}\right) = -\frac{3kqzy}{r^5}$$

Now put in the dipole's location $(x, y, z) = (d, 0, 0)$, so $r = d$ and $y = z = 0$. The first and third derivatives both carry a factor of y or z, so they vanish at this point. Only the middle one survives:

$$\left.\frac{\partial E_y}{\partial y}\right|_{(d,0,0)} = \frac{kq}{d^3}$$

So the gradient of $\vec{E}$ at the dipole's location is nonzero only in the y direction, and:

$$\vec{F} = p\left(0,\ \frac{kq}{d^3},\ 0\right) = \frac{kqp}{d^3}\hat{y}$$

Since q, p and d^3 are all positive, this force is a positive multiple of $\hat{y}$.

Step 4: Why the other options are wrong.

The force is not zero (A), because the field of a point charge is genuinely non-uniform, so a dipole placed in it always feels some net pull unless its moment happens to be perpendicular to the local field gradient in just the right way, which is not the case here. It is not along $-\hat{y}$ (C), because the sign of $\partial E_y/\partial y$ at this point came out

positive, not negative. It is not along $\hat{x}$ (D), because both the x and z derivatives of $\vec{E}$ vanish exactly at this symmetric point on the axis.

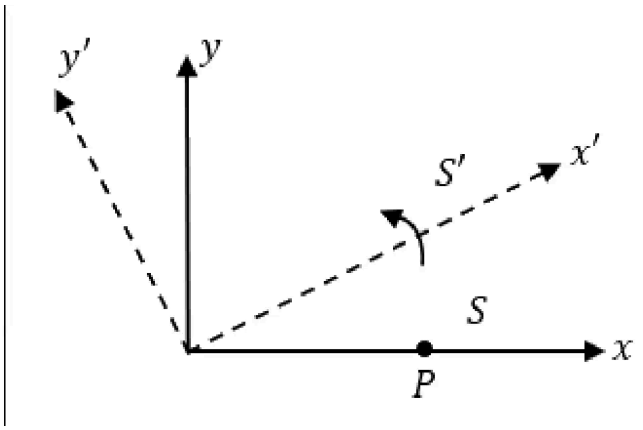
Step 5: Final Answer:

The dipole is pulled further along the y-direction it is already pointing in.

$$\boxed{\text{in the } +y \text{ direction}}$$

Quick Tip: Use $\vec{F} = (\vec{p} \cdot \nabla)\vec{E}$ for a dipole in an external field; since $\vec{p} = p\hat{y}$, only $\partial E_y/\partial y$ at the dipole's location matters, and for a point charge this comes out positive on the x-axis.

. Two frames S (solid lines) and S' (dashed lines) with common origin are shown in the figure below. Frame S is inertial while S' is rotating about the common z-axis. There is a point mass fixed at P on the x-axis of the S frame. The magnitude of the centrifugal force and the Coriolis force experienced by the mass in the S' frame is F_{cen} and F_{cor}, respectively. Which of the following options is correct for these forces?



(A) $F_{cen} = 0$ and $F_{cor} = 0$

(B) $F_{cen} \neq 0$ and $F_{cor} \neq 0$ and $F_{cen} = \frac{F_{cor}}{2}$

(C) $F_{cen} \neq 0$ and $F_{cor} \neq 0$ and $F_{cen} = 2F_{cor}$

(D) $F_{cen} \neq 0$ and $F_{cor} \neq 0$ and $F_{cen} = F_{cor}$

Correct Answer: (B) $F_{cen} \neq 0$ and $F_{cor} \neq 0$ and $F_{cen} = \frac{F_{cor}}{2}$

Solution:

Step 1: Understanding the Concept:

The mass sits still at a fixed point P on the x-axis of the inertial frame S, at distance r from the common origin, so it has zero velocity and zero acceleration in S. Frame S' rotates relative to S with angular velocity $\vec{\Omega} = \omega\hat{z}$. Because S' is non-inertial, an observer riding on S' must add pseudo-forces (centrifugal and Coriolis) to make Newton's second law work for this same mass.

Step 2: Key Formula or Approach:

1. Velocity seen in the rotating frame: $\vec{v}_S = \vec{v}_{S'} + \vec{\Omega} \times \vec{r}$.
2. Centrifugal force: $\vec{F}_{cen} = -m\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$, magnitude $m\omega^2 r$, directed away from the axis.
3. Coriolis force: $\vec{F}_{cor} = -2m\vec{\Omega} \times \vec{v}_{S'}$.

Step 3: Detailed Explanation.

Since the mass is at rest in S, $\vec{v}_S = 0$. Using the velocity relation above with $\vec{r} = r\hat{x}$:

$$0 = \vec{v}_{S'} + \omega\hat{z} \times r\hat{x} = \vec{v}_{S'} + \omega r\hat{y}$$

$$\Rightarrow \vec{v}_{S'} = -\omega r\hat{y}$$

This makes sense: since S' spins one way relative to S, a point that is actually still in S appears in S' to trace a circle of radius r at angular speed ω, in the opposite rotational sense.
Now compute the centrifugal force:

$$\vec{F}_{cen} = -m(\omega\hat{z}) \times \left[(\omega\hat{z}) \times (r\hat{x})\right] = -m\omega^2(\hat{z} \times (\hat{z} \times r\hat{x})) = -m\omega^2 r(\hat{z} \times \hat{y}) = m\omega^2 r\hat{x}$$

So $F_{cen} = m\omega^2 r$, pointing radially outward.
Now the Coriolis force, using $\vec{v}_{S'} = -\omega r\hat{y}$:

$$\vec{F}_{cor} = -2m(\omega\hat{z}) \times (-\omega r\hat{y}) = 2m\omega^2 r(\hat{z} \times \hat{y}) = -2m\omega^2 r\hat{x}$$

So $F_{cor} = 2m\omega^2 r$, pointing radially inward.
Comparing the two magnitudes:

$$F_{cen} = m\omega^2 r, \qquad F_{cor} = 2m\omega^2 r \quad \Rightarrow \quad F_{cen} = \frac{F_{cor}}{2}$$

Step 4: Why the other options are wrong.
Both forces are nonzero because P is displaced from the rotation axis and does have a nonzero apparent velocity in S', so option (A) fails. Options (C) and (D) get the ratio backwards or equal; the actual ratio is 1 : 2, not 2 : 1 or 1 : 1.

Step 5: Final Answer:
Both pseudo-forces are nonzero, and the centrifugal force is exactly half the Coriolis force.

$$\boxed{\text{(B)}}$$

Quick Tip: A mass fixed in the inertial frame S appears in the rotating frame S' to move on a circle of radius r at speed ωr; use $F_{cen} = m\omega^2 r$ and $F_{cor} = 2m\Omega v' = 2m\omega^2 r$ to get the ratio $1 : 2$.

. Which of the following operators is/are self-adjoint?

(A) $x^2\frac{d^2}{dx^2} + 3x\frac{d}{dx} + x^2$

(B) $(1 - x^2)\frac{d^2}{dx^2} - 2x\frac{d}{dx} + 3x$

(C) $(3x - 4x^3)\frac{d^2}{dx^2} + (3 - 12x^2)\frac{d}{dx} + 12$

(D) $x\frac{d^2}{dx^2} + x^2\frac{d}{dx} + \frac{5x}{3}$

Correct Answer: (B) $(1 - x^2)\frac{d^2}{dx^2} - 2x\frac{d}{dx} + 3x$; (C) $(3x - 4x^3)\frac{d^2}{dx^2} + (3 - 12x^2)\frac{d}{dx} + 12$

Solution:

Step 1: Understanding the Concept:

Each operator here has the form $L = A(x)\frac{d^2}{dx^2} + B(x)\frac{d}{dx} + C(x)$. Such an operator can always be written in the divergence, or Sturm-Liouville, form $L = \frac{d}{dx}\left(p(x)\frac{d}{dx}\right) + q(x)$ exactly when $B(x)$ equals the derivative of $A(x)$. An operator in this divergence form is formally self-adjoint under the usual real inner product, because integrating by parts twice to move both derivatives off a test function u and onto another test function w brings back the same operator with no leftover first-derivative term. So the whole question reduces to a simple check on each option.

Step 2: Key Formula or Approach:

For $L = A(x)\frac{d^2}{dx^2} + B(x)\frac{d}{dx} + C(x)$:

$$L \text{ is formally self-adjoint} \iff B(x) = A'(x)$$

We check this condition for each of the four operators.

Step 3: Check option (A).

Here $A(x) = x^2$, so $A'(x) = 2x$, while $B(x) = 3x$. Since $3x \neq 2x$ in general, the condition fails, so (A) is NOT self-adjoint.

Step 4: Check option (B).

Here $A(x) = 1 - x^2$, so $A'(x) = -2x$, and $B(x) = -2x$. These match exactly ($-2x = -2x$), so (B) IS self-adjoint. This is in fact the Legendre operator, a textbook example of a self-adjoint operator.

Step 5: Check option (C).

Here $A(x) = 3x - 4x^3$, so $A'(x) = 3 - 12x^2$, and $B(x) = 3 - 12x^2$. These also match exactly, so (C) IS self-adjoint.

Step 6: Check option (D).

Here $A(x) = x$, so $A'(x) = 1$, while $B(x) = x^2$. Since $x^2 \neq 1$ in general, the condition fails, so (D) is NOT self-adjoint.

Step 7: Final Answer:

Only the operators in (B) and (C) satisfy $B(x) = A'(x)$ and are self-adjoint.

$$\boxed{\text{(B), (C)}}$$

Quick Tip: For $L = A(x)D^2 + B(x)D + C(x)$, the operator is formally self-adjoint exactly when $B(x) = A'(x)$, which lets it be written as $\frac{d}{dx}\left(A(x)\frac{d}{dx}\right) + C(x)$. Check this coefficient match for each option.

. Consider two operators $\hat{A}$ and $\hat{B}$ which are related as $\hat{A} = \exp(i\theta\hat{B})$. If θ is a non-zero real number, which of the following statements is/are true?

(A) If $\hat{B}$ is Hermitian, then $\hat{A}$ is unitary.

(B) If $\hat{B}$ is anti-Hermitian, then $\hat{A}$ is unitary.

(C) If $\hat{B}$ is Hermitian, then $|\text{Det}(\hat{A})| = 1$.

(D) If $\hat{B}$ is anti-Hermitian, then $\hat{A}$ is Hermitian.

Correct Answer: (A) If $\hat{B}$ is Hermitian, then $\hat{A}$ is unitary. ; (C) If $\hat{B}$ is Hermitian, then $|\text{Det}(\hat{A})| = 1$. ; (D) If $\hat{B}$ is anti-Hermitian, then $\hat{A}$ is Hermitian.

Solution:

Step 1: Understand what each term means.
A Hermitian (self-adjoint) operator satisfies $\hat{B}^\dagger = \hat{B}$, so all its eigenvalues are real. An anti-Hermitian operator satisfies $\hat{B}^\dagger = -\hat{B}$, so its eigenvalues are purely imaginary. A unitary operator satisfies $\hat{A}^\dagger\hat{A} = \hat{I}$, which forces every eigenvalue of $\hat{A}$ to have magnitude 1.

Step 2: Key formula.
For any operator $\hat{X}$, the adjoint of its exponential follows the rule $(e^{\hat{X}})^\dagger = e^{\hat{X}^\dagger}$, and its determinant follows $\text{Det}(e^{\hat{X}}) = e^{\text{Tr}(\hat{X})}$. We apply both rules to $\hat{A} = e^{i\theta\hat{B}}$ with θ real.

Step 3: Check statement (A).

If $\hat{B}$ is Hermitian, $\hat{B}^\dagger = \hat{B}$, so $(i\theta\hat{B})^\dagger = -i\theta\hat{B}^\dagger = -i\theta\hat{B}$. The generator $i\theta\hat{B}$ is anti-Hermitian. Using Step 2, $\hat{A}^\dagger = e^{(i\theta\hat{B})^\dagger} = e^{-i\theta\hat{B}}$, so $\hat{A}^\dagger\hat{A} = e^{-i\theta\hat{B}}e^{i\theta\hat{B}} = \hat{I}$. So $\hat{A}$ is unitary, and (A) is TRUE.

Step 4: Check statement (B).
If $\hat{B}$ is anti-Hermitian, $\hat{B}^\dagger = -\hat{B}$, so $(i\theta\hat{B})^\dagger = -i\theta\hat{B}^\dagger = -i\theta(-\hat{B}) = i\theta\hat{B}$. The generator $i\theta\hat{B}$ is now Hermitian, not anti-Hermitian, so its exponential gives a Hermitian operator, not a unitary one. So (B) is FALSE.

Step 5: Check statement (C).
If $\hat{B}$ is Hermitian, its eigenvalues are real, so $\mathrm{Tr}(\hat{B})$ is a real number. Then $\mathrm{Tr}(i\theta\hat{B}) = i\theta\,\mathrm{Tr}(\hat{B})$ is purely imaginary. Using $\mathrm{Det}(\hat{A}) = e^{\mathrm{Tr}(i\theta\hat{B})}$, this is $e^{i(\text{real number})}$, a complex number of magnitude 1. So $|\mathrm{Det}(\hat{A})| = 1$, and (C) is TRUE.

Step 6: Check statement (D).
From Step 4, when $\hat{B}$ is anti-Hermitian, $i\theta\hat{B}$ is Hermitian. The exponential of a Hermitian operator stays Hermitian, because $(e^{\hat{H}})^\dagger = e^{\hat{H}^\dagger} = e^{\hat{H}}$ whenever $\hat{H}^\dagger = \hat{H}$. Taking $\hat{H} = i\theta\hat{B}$, we get $\hat{A}^\dagger = \hat{A}$. So $\hat{A}$ is Hermitian, and (D) is TRUE.

Final Answer:
Statements (A), (C) and (D) follow correctly from the given relation; only (B) is wrong.

$$\boxed{\text{A, C, D}}$$

Quick Tip: Test each case using $(e^{\hat{X}})^\dagger = e^{\hat{X}^\dagger}$ and $\mathrm{Det}(e^{\hat{X}}) = e^{\mathrm{Tr}(\hat{X})}$.
A Hermitian generator times i becomes anti-Hermitian, so its exponential is

unitary; an anti-Hermitian generator times i becomes Hermitian, so its exponential is Hermitian.

. Consider operators $\hat{A}$, $\hat{B}$, and $\hat{C}$ for three observables of a quantum system satisfying $[\hat{A},\hat{B}]=0$, $[\hat{B},\hat{C}]=0$, and $[\hat{A},\hat{C}]\neq 0$, with uncertainties $\Delta A, \Delta B, \Delta C$, respectively. From the options given below, which is/are implied by the commutation relations among $\hat{A}$, $\hat{B}$, and $\hat{C}$?

(A) $\Delta A\,\Delta B > 0$

(B) $\Delta A\,\Delta C > 0$

(C) $\hat{A},\hat{B}$ can be simultaneously diagonalized.

(D) $\hat{A},\hat{B},\hat{C}$ can be simultaneously diagonalized.

Correct Answer: (B) $\Delta A\,\Delta C > 0$; (C) $\hat{A},\hat{B}$ can be simultaneously diagonalized.

Solution:

Step 1: Recall the generalized uncertainty relation.
For any two Hermitian operators $\hat{X}$ and $\hat{Y}$, the Robertson uncertainty relation gives $\Delta X\,\Delta Y \geq \frac{1}{2}\left|\langle[\hat{X},\hat{Y}]\rangle\right|$. When the commutator of two operators is the zero operator, this lower bound is exactly zero for every state, so their uncertainty product is allowed to fall all the way to zero.

Step 2: Recall the simultaneous diagonalization theorem.
Two Hermitian operators can be simultaneously diagonalized, meaning they share a complete common eigenbasis, if and only if they commute. This is a standard result applied to quantum observables.

Step 3: Check statement (A).

Since $[\hat{A}, \hat{B}] = 0$, the lower bound on $\Delta A\, \Delta B$ is zero. In a common eigenstate of $\hat{A}$ and $\hat{B}$, both uncertainties vanish, so $\Delta A\, \Delta B = 0$ is possible. The relation does not force this product to stay positive in every state, so (A) is FALSE.

Step 4: Check statement (B).
Here $[\hat{A}, \hat{C}] \neq 0$, so $\hat{A}$ and $\hat{C}$ do not share a complete common eigenbasis. No state can be a simultaneous eigenstate of both, so ΔA and ΔC cannot both be driven to zero together. This is what the non-zero commutator between $\hat{A}$ and $\hat{C}$ implies, so $\Delta A\, \Delta C > 0$ and (B) is TRUE.

Step 5: Check statement (C).
Since $[\hat{A}, \hat{B}] = 0$, Step 2's theorem applies directly: $\hat{A}$ and $\hat{B}$ commute, so they can be simultaneously diagonalized. Statement (C) is TRUE.

Step 6: Check statement (D).
If $\hat{A}, \hat{B}, \hat{C}$ could all be simultaneously diagonalized, they would share one common eigenbasis, and any two operators sharing a common eigenbasis must commute, including $\hat{A}$ and $\hat{C}$. This contradicts $[\hat{A}, \hat{C}] \neq 0$. So all three cannot be simultaneously diagonalized, and (D) is FALSE.

Final Answer:
Only $\hat{A}$, $\hat{B}$ commute in a way that also blocks a full common eigenbasis for the trio; (B) and (C) are what the given commutation relations actually force.

$$\boxed{\text{B, C}}$$

Quick Tip: A zero commutator means the pair can share eigenstates, so their uncertainty product can hit zero; a non-zero commutator blocks a full common eigenbasis, so the pair's uncertainty product must stay positive. Apply this separately to each pair among $\hat{A}, \hat{B}, \hat{C}$.

. Consider the distribution of outcomes generated by N ($N \gg 1$) independent throws of (i) a coin or (ii) a six-sided dice. A coin (dice) is unbiased if both (all) its sides have equal probability to show up in a throw; it is biased otherwise. For the cases (i) and (ii) above, which of the following statements is/are true?

(A) The entropy of an unbiased coin is smaller than that of an unbiased dice.

(B) The entropy of an unbiased coin is greater than that of an unbiased dice.

(C) The entropy of a biased dice is smaller than that of an unbiased dice.

(D) The entropy of a biased coin is greater than that of an unbiased coin.

Correct Answer: (A) The entropy of an unbiased coin is smaller than that of an unbiased dice. ; (C) The entropy of a biased dice is smaller than that of an unbiased dice.

Solution:

Step 1: Recall the definition of entropy for a single throw.
For a random outcome with k possible results, each with probability p_i, the Shannon entropy per throw is $S = -\sum_{i=1}^{k} p_i \log p_i$. This number measures how spread out the outcome is: a sure outcome has zero entropy, and entropy grows as the outcomes become more evenly spread.

Step 2: Recall which distribution gives the largest entropy.
Among all probability distributions on k fixed outcomes, the entropy is maximum exactly when every outcome is equally likely, that is $p_i = 1/k$ for all i, giving $S_{max} = \log k$. Moving away from equal probabilities, biasing the coin or dice, can only lower the entropy below this maximum, never raise it.

Step 3: Compare the unbiased coin and unbiased dice.
An unbiased coin has $k = 2$ equally likely outcomes, so its entropy is $\log 2$. An unbiased dice has $k = 6$ equally likely outcomes, so its entropy is $\log 6$. Since $\log 6 > \log 2$, the dice carries more uncertainty per throw than the coin.

Step 4: Check statement (A).
From Step 3, the entropy of the unbiased coin ($\log 2$) is smaller than that of the unbiased dice ($\log 6$). So (A) is TRUE.

Step 5: Check statement (B).
Statement (B) claims the opposite ordering. Since $\log 2 < \log 6$, this is FALSE.

Step 6: Check statement (C).
By Step 2, the unbiased dice sits at the maximum possible entropy $\log 6$ among all dice distributions. Any biased dice has at least one face more likely than the others, moving it away from equal probability, so its entropy must be strictly smaller than $\log 6$. So (C) is TRUE.

Step 7: Check statement (D).
By the same reasoning, the unbiased coin already sits at the maximum entropy $\log 2$ among all coin distributions. A biased coin moves away from equal probabilities, so its entropy must be strictly

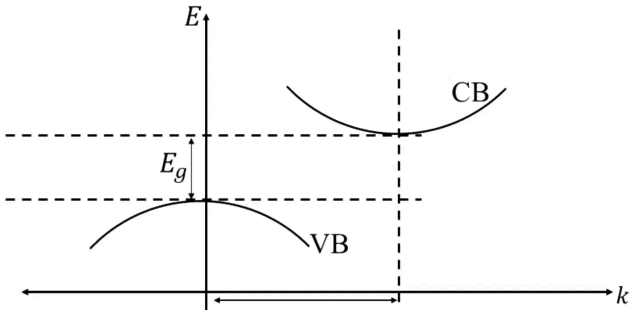
smaller, not greater, than the unbiased coin. So (D) is FALSE.

Final Answer:
Only (A) and (C) are guaranteed by the fact that uniform probabilities give the maximum entropy.

$$\boxed{\text{A, C}}$$

Quick Tip: Entropy of k equally likely outcomes is $\log k$, and this is the maximum possible entropy for that many outcomes.
Compare $\log 2$ (coin) with $\log 6$ (dice), and remember any bias away from equal probabilities can only lower the entropy.

. The dispersion ($E(k)$) of the conduction band (CB) and valence band (VB) for a semiconductor are shown schematically in the figure. Considering the possibility of an electron making a transition from the bottom of the CB to the top of the VB, which of the following options is/are correct?



(A) The transition is forbidden.

(B) A photon can be emitted with an energy exactly equal to E_g.

(C) A photon can be emitted with an energy less than E_g.

(D) A phonon can be created with a crystal momentum $\hbar q$.

Correct Answer: (C) A photon can be emitted with an energy less than E_g. ; (D) A phonon can be created with a crystal momentum $\hbar q$.

Solution:

Step 1: Read the band diagram.

The figure shows an indirect-gap semiconductor: the valence band maximum sits at $k = 0$ while the conduction band minimum sits at a different wavevector, $k = q$, an energy E_g higher. The bottom of the CB and the top of the VB are not lined up on the same vertical line in k.

Step 2: Recall what a photon can and cannot carry.

A photon carries energy $\hbar\omega$ freely, but its momentum $\hbar\omega/c$ is extremely small on the scale of a crystal's Brillouin zone, so absorbing or emitting a photon leaves the electron's crystal momentum k essentially unchanged. A transition driven by a photon alone is a vertical transition on the E vs k plot.

Step 3: Recall what a phonon can carry.

A lattice vibration (phonon) carries very little energy compared to E_g but can carry a crystal momentum comparable to q, since phonon wavevectors span the same Brillouin zone as electrons. A phonon can supply or absorb the missing momentum in a transition.

Step 4: Check statement (A).

The transition from the bottom of the CB ($k = q$) to the top of the VB ($k = 0$) needs a change of crystal momentum of $\hbar q$. This is not forbidden outright, it just cannot happen through a photon alone, it needs a phonon to carry away that momentum. So the transition is allowed as a phonon-assisted process, and (A) is FALSE.

Step 5: Check statement (B).

Energy conservation across CB bottom to VB top gives exactly E_g to give away. If this energy were carried by a photon alone with no phonon involved, momentum is not conserved, since the photon cannot supply the required $\hbar q$. A real recombination event here always needs a phonon to take some energy and momentum, so a photon carrying the full E_g alone is not the process that occurs. So (B) is FALSE.

Step 6: Check statement (C).

In the phonon-assisted process, the electron drops in energy by E_g total, splitting this between a phonon (energy E_{ph}, carrying crystal momentum $\hbar q$) and a photon. Energy conservation gives photon energy $= E_g - E_{ph}$, which is strictly less than E_g whenever a phonon is emitted alongside. So a photon of energy less than E_g can indeed be emitted, and (C) is TRUE.

Step 7: Check statement (D).

The electron's crystal momentum changes by $\hbar q$ as it drops from $k = q$ to $k = 0$. Conservation of crystal momentum requires this change to go somewhere, and a phonon with crystal momentum $\hbar q$ is exactly what supplies it. So (D) is TRUE.

Final Answer:

The recombination is an indirect, phonon-assisted transition, so (C) and (D) hold while (A) and (B) do not.

$$\boxed{\text{C, D}}$$

Quick Tip: The CB minimum and VB maximum sit at different values of k in the figure, so the transition needs a change in crystal momentum of $\hbar q$. A photon alone cannot supply that momentum change, so a phonon must be created alongside it, and the photon then carries less than E_g.

. A symmetric rigid body has moment of inertia I_1, I_2, I_3 about its principal axes 1, 2, and 3, respectively, with $I_1 = I_3 = I_\perp$ and $I_2 \neq I_\perp$. It is rotating in space with no torque on it so that its angular momentum $\vec{L}$ is constant. Let $\omega_1, \omega_2, \omega_3$ be the components of its angular velocity along the principal axes 1, 2, and 3, respectively. Which of the following quantities is/are constant during the motion of this rigid body?

(A) $\omega_1 + \omega_3$

(B) $\omega_1^2 + \omega_3^2$

(C) Angle between axis 2 and $\vec{L}$

(D) ω_2

Correct Answer: (B) $\omega_1^2 + \omega_3^2$; (C) Angle between axis 2 and $\vec{L}$; (D) ω_2

Solution:

Step 1: Write Euler's equations for the torque-free body.

For a rigid body with no external torque, Euler's equations in the body frame read

$$I_1\dot{\omega}_1 = (I_2 - I_3)\,\omega_2\omega_3$$

$$I_2\dot{\omega}_2 = (I_3 - I_1)\,\omega_3\omega_1$$

$$I_3\dot{\omega}_3 = (I_1 - I_2)\,\omega_1\omega_2$$

Step 2: Use $I_1 = I_3 = I_\perp$ to get ω_2.

The second equation carries the factor $(I_3 - I_1)$, which is zero since $I_1 = I_3$. So $I_2\dot{\omega}_2 = 0$, giving $\dot{\omega}_2 = 0$. The component ω_2 along the odd-one-out axis never changes with time, so statement (D) is TRUE.

Step 3: Reduce the remaining two equations.

With $I_1 = I_3 = I_\perp$, the first and third equations become $I_\perp\dot{\omega}_1 = (I_2 - I_\perp)\,\omega_2\omega_3$ and $I_\perp\dot{\omega}_3 = (I_\perp - I_2)\,\omega_1\omega_2$. Since ω_2 is constant, define the constant $\Omega = \dfrac{(I_2 - I_\perp)\omega_2}{I_\perp}$. The equations become $\dot{\omega}_1 = \Omega\omega_3$ and $\dot{\omega}_3 = -\Omega\omega_1$, the equations of uniform circular motion in the ω_1-ω_3 plane.

Step 4: Check statement (B).

Differentiating $\omega_1^2 + \omega_3^2$ gives $2\omega_1\dot{\omega}_1 + 2\omega_3\dot{\omega}_3 = 2\omega_1(\Omega\omega_3) + 2\omega_3(-\Omega\omega_1) = 0$. So $\omega_1^2 + \omega_3^2$ does not change with time, and statement (B) is TRUE.

Step 5: Check statement (A).

The circular-motion solution gives $\omega_1(t) = R\cos(\Omega t + \phi)$ and $\omega_3(t) = R\sin(\Omega t + \phi)$ for constants R, ϕ. The sum $\omega_1 + \omega_3$ keeps oscillating with time rather than staying fixed, so statement (A) is FALSE.

Step 6: Check statement (C).

The angle between the symmetry axis (axis 2) and the space-fixed vector $\vec{L}$ satisfies $\cos\theta = \dfrac{\vec{L}\cdot\hat{e}_2}{|\vec{L}|} = \dfrac{I_2\omega_2}{|\vec{L}|}$. Since $\vec{L}$ is fixed and conserved, $|\vec{L}|$ is constant; I_2 is a fixed body property; and ω_2 is

constant from Step 2. So $\cos\theta$ is built entirely from constants, meaning θ itself is constant. Statement (C) is TRUE, matching the well-known picture of a symmetric top's axis tracing a fixed cone around $\vec{L}$.

Final Answer:

The symmetry of the body ($I_1 = I_3$) freezes ω_2, which in turn keeps $\omega_1^2 + \omega_3^2$ and the axis-2/$\vec{L}$ angle both fixed; only the plain sum $\omega_1 + \omega_3$ keeps changing.

$$\boxed{\text{B, C, D}}$$

Quick Tip: Since $I_1 = I_3$, Euler's equation for ω_2 has a zero coefficient, so ω_2 stays fixed.
That in turn fixes $\omega_1^2 + \omega_3^2$ (via the conserved L^2 and kinetic energy) and fixes the angle between axis 2 and $\vec{L}$, while $\omega_1 + \omega_3$ alone keeps oscillating.

. An alpha particle moves towards a fixed nucleus carrying charge Ze, with initial speed v_0 and impact parameter b. Starting from a large distance from the nucleus, its distance of closest approach is r_m and its speed there is v_m. Then which of the following options is correct?

$$\left(k = \frac{1}{4\pi\epsilon_0} \text{ and } r_0 = k\frac{Ze^2}{mv_0^2}\right)$$

(A) $v_0 b = v_m r_m$

(B) $v_0 b = 2v_m r_0$

(C) For $\frac{b}{r_0} \ll 1$, $r_m = 4r_0 + \frac{b^2}{2r_0}$, ignoring higher order corrections in $\frac{b}{r_0}$

(D) For $\frac{b}{r_0} \ll 1$, $r_m = 4r_0 + \frac{b^2}{8r_0}$, ignoring higher order corrections in $\frac{b}{r_0}$

Correct Answer: (A) $v_0 b = v_m r_m$

Solution:

Step 1: Set up the two conservation laws.

The nucleus is fixed and its Coulomb force on the alpha particle always points along the line joining the two particles, so it exerts no torque about the nucleus. Angular momentum about the nucleus is conserved. The alpha particle carries charge $2e$, so its potential energy at separation r is $k(2e)(Ze)/r$, and total mechanical energy is also conserved.

Step 2: Apply angular momentum conservation.

Far from the nucleus the alpha particle moves in a straight line with speed v_0 and impact parameter b, so its angular momentum about the nucleus is $mv_0 b$. At the distance of closest approach the radial velocity is zero, so the whole speed v_m is tangential, giving angular momentum $mv_m r_m$. Conservation gives

$$v_0 b = v_m r_m$$

This is exactly option (A), and it holds at every value of b, not only for small b.

Step 3: Apply energy conservation.

Equating the energy at infinity to the energy at closest approach,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_m^2 + \frac{2kZe^2}{r_m}$$

Using $r_0 = kZe^2/(mv_0^2)$, the potential term becomes $2kZe^2/r_m = 2r_0 mv_0^2/r_m$. Dividing the whole equation by $\frac{1}{2}mv_0^2$ gives

$$1 = \left(\frac{v_m}{v_0}\right)^2 + \frac{4r_0}{r_m}$$

Step 4: Eliminate v_m using Step 2.

From Step 2, $v_m/v_0 = b/r_m$. Putting this in,

$$1 = \frac{b^2}{r_m^2} + \frac{4r_0}{r_m}$$

Multiplying through by r_m^2 gives the quadratic

$$r_m^2 - 4r_0 r_m - b^2 = 0$$

Taking the physically sensible positive root,

$$r_m = 2r_0 + \sqrt{4r_0^2 + b^2}$$

Step 5: Expand for $b/r_0 \ll 1$ and check options (B), (C), (D).

Write $\sqrt{4r_0^2 + b^2} = 2r_0\sqrt{1 + b^2/(4r_0^2)} \approx 2r_0\left(1 + \frac{b^2}{8r_0^2}\right) = 2r_0 + \frac{b^2}{4r_0}$, keeping only the leading correction. So

$$r_m \approx 4r_0 + \frac{b^2}{4r_0}$$

The coefficient of b^2 is $1/(4r_0)$, not $1/(2r_0)$ as option (C) claims and not $1/(8r_0)$ as option (D) claims, so both (C) and (D) are FALSE. For option (B): at $b = 0$, $v_0 b = 0$ and $v_m = 0$ too, so this special case does not test it; for a general b, Step 2 already fixes $v_0 b = v_m r_m$, and $2v_m r_0$

equals this only if $r_m = 2r_0$, which Step 5 shows is not true except possibly at one isolated b. So (B) is FALSE in general.

Final Answer:

Only the angular momentum relation $v_0 b = v_m r_m$ is an identity holding for every b; that is option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: **Hint:**

The Coulomb force on the alpha particle always points along the line to the nucleus, so it produces zero torque. Use that to relate v_0, b, v_m, r_m, then bring in energy conservation and expand for small b/r_0.

. Consider a particle of mass $m = 9.0 \times 10^{-5}$ kg and charge $q = 3.0 \times 10^{-4}$ C in a uniform electromagnetic field $\vec{E} = 2\,\hat{x}$ V.m^{-1}, $\vec{B} = 3\,\hat{z}$ V.m^{-2}.s. The particle is released from the coordinates $(0, 5\text{ m}, 0)$ at time $t = 0$. Starting from initial speed zero, it comes back to the y-axis for the first time at time t. The value of t in seconds (rounded off to two decimal places) is _____

Correct Answer: 0.63

Solution:

Step 1: Understand the setup.

The particle starts from rest at $(0, 5\text{ m}, 0)$, in a constant electric field $\vec{E} = E\hat{x}$, $E = 2$ V/m, and a constant magnetic field $\vec{B} = B\hat{z}$, $B = 3$ T (the unit V.m^{-2}.s is just tesla written in base SI units). Crossed $\vec{E}$ and $\vec{B}$ fields, with the particle starting at rest, always produce a cycloid: the particle keeps coming back to its starting x-position (here, the y-axis)

once every full cyclotron period.

Step 2: Write the equations of motion.

The Lorentz force is $m\dot{\vec{v}} = q(\vec{E} + \vec{v} \times \vec{B})$. With $\vec{B} = B\hat{z}$, $\vec{v} \times \vec{B} = (v_y B, -v_x B, 0)$, so

$$\dot{v}_x = \frac{qE}{m} + \omega v_y, \qquad \dot{v}_y = -\omega v_x, \qquad \omega = \frac{qB}{m}$$

Step 3: Solve the coupled equations.

Define $u = v_y + E/B$ (using $qE/m = \omega E/B$); then $\dot{v}_x = \omega u$ and $\dot{u} = -\omega v_x$, a pure rotation in the (v_x, u) plane. With $v_x(0) = 0$ and $u(0) = E/B$ (since $v_y(0) = 0$), the solution is

$$v_x(t) = \frac{E}{B}\sin(\omega t), \qquad v_y(t) = \frac{E}{B}\big(\cos(\omega t) - 1\big)$$

Step 4: Integrate to get $x(t)$.

$$x(t) = \int_0^t v_x \, dt' = \frac{E}{B\omega}\big(1 - \cos(\omega t)\big)$$

This is the coordinate away from the starting line $x = 0$ (the y-axis). The particle returns to the y-axis whenever $x(t) = 0$ again, that is $\cos(\omega t) = 1$, which first happens after $t = 0$ at $\omega t = 2\pi$.

Step 5: Compute ω and t.

$$\omega = \frac{qB}{m} = \frac{(3.0 \times 10^{-4})(3)}{9.0 \times 10^{-5}} = 10 \text{ rad/s}$$

$$t = \frac{2\pi}{\omega} = \frac{2\pi}{10} = 0.6283 \text{ s}$$

Final Answer:

Rounded to two decimal places, the particle first returns to the y-axis at

$$\boxed{t = 0.63 \text{ s}}$$

Quick Tip: **Hint:**

A particle starting from rest in crossed $\vec{E}$ and $\vec{B}$ traces a cycloid: it returns to its starting x-line once every full cyclotron period $2\pi/\omega$ with $\omega = qB/m$. Solve the coupled v_x, v_y equations, or just find ω and use this period directly.

. An atom has two energy levels with energy difference 2.2 eV between them. A gas of these atoms has 8×10^{20} atoms in the upper state and 5×10^{20} atoms in the lower state. Ignoring spontaneous emission, the maximum possible energy released by this gas of atoms by stimulated emission is E Joules. The value of E (rounded off to one decimal place) is _____ ($e = 1.6 \times 10^{-19}$ C)

Correct Answer: 52.8

Solution:

Step 1: Understand what limits stimulated emission.

For a non degenerate two level system, the Einstein coefficients for stimulated absorption and stimulated emission are equal ($B_{12} = B_{21}$).

So in a radiation field, atoms in the lower state absorb photons at a rate proportional to N_1, while atoms in the upper state emit photons (stimulated) at the same rate constant times N_2. The net number of photons produced per unit time is proportional to $N_2 - N_1$.

Step 2: See why the process has a maximum.
Every net stimulated emission event moves one atom from the upper state to the lower state: N_2 drops by 1 and N_1 rises by 1, so the difference $N_2 - N_1$ drops by 2 each time. This keeps happening as long as $N_2 > N_1$. Once $N_2 = N_1$, emission and absorption occur at equal rates and cancel exactly, so no further net energy can be released, ignoring spontaneous emission. This equalization point is what caps the total energy release.

Step 3: Find how many atoms make the downward transition.
The initial difference is

$$N_2 - N_1 = 8 \times 10^{20} - 5 \times 10^{20} = 3 \times 10^{20}$$

Since each net transition reduces this difference by 2, the number of atoms that can move down before the difference reaches zero is

$$\Delta N = \frac{N_2 - N_1}{2} = 1.5 \times 10^{20}$$

Step 4: Convert to energy.
Each downward transition releases one photon of energy 2.2 eV, so the total energy released is

$$\Delta N \times 2.2 \text{ eV} = 1.5 \times 10^{20} \times 2.2 = 3.3 \times 10^{20} \text{ eV}$$

Converting to joules using $e = 1.6 \times 10^{-19}$ C,

$$E = 3.3 \times 10^{20} \times 1.6 \times 10^{-19} = 52.8 \text{ J}$$

Final Answer:

Rounded to one decimal place,

$$\boxed{E = 52.8 \text{ J}}$$

Quick Tip: **Hint:**

Stimulated absorption and stimulated emission share the same rate constant, so the net process stops once the populations equalize. Find how many atoms must move from the upper to the lower state for that to happen, then multiply by the photon energy.

. The Hamiltonian of two interacting spin-1/2 particles is $H = \frac{A}{\hbar^2}\vec{S}_1 \cdot \vec{S}_2$, where $\vec{S}_1$ and $\vec{S}_2$ are the spin angular momenta of particles 1 and 2, respectively. Here, $A = 10.56$ eV. The energy in eV required to induce an excitation from the ground state to the excited state (rounded off to two decimal places) is _____

Correct Answer: 10.56

Solution:

Step 1: Rewrite $\vec{S}_1 \cdot \vec{S}_2$ using the total spin.

Let $\vec{S} = \vec{S}_1 + \vec{S}_2$ be the total spin. Squaring, $S^2 = S_1^2 + S_2^2 + 2\vec{S}_1 \cdot \vec{S}_2$, so

$$\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2}\left(S^2 - S_1^2 - S_2^2\right)$$

Step 2: Find the eigenvalues.

For each spin-1/2, $S_1^2 = S_2^2 = \hbar^2 s(s+1)$ with $s = 1/2$, giving $\hbar^2(3/4)$ each. Combining two spin-1/2 particles gives total spin $S = 0$ (the singlet) or $S = 1$ (the triplet), with $S^2 = \hbar^2 S(S+1)$. So

$$\vec{S}_1 \cdot \vec{S}_2 = \frac{\hbar^2}{2}\left[S(S+1) - \frac{3}{4} - \frac{3}{4}\right]$$

Step 3: Evaluate for the singlet and the triplet.

Singlet, $S = 0$: $S(S+1) = 0$, so $\vec{S}_1 \cdot \vec{S}_2 = \frac{\hbar^2}{2}\left(0 - \frac{3}{2}\right) = -\frac{3\hbar^2}{4}$.

Triplet, $S = 1$: $S(S+1) = 2$, so $\vec{S}_1 \cdot \vec{S}_2 = \frac{\hbar^2}{2}\left(2 - \frac{3}{2}\right) = \frac{\hbar^2}{4}$.

Step 4: Get the energy levels.

$$E_{singlet} = \frac{A}{\hbar^2}\left(-\frac{3\hbar^2}{4}\right) = -\frac{3A}{4}, \qquad E_{triplet} = \frac{A}{\hbar^2}\left(\frac{\hbar^2}{4}\right) = \frac{A}{4}$$

Since $A = 10.56$ eV is positive, $E_{singlet} < E_{triplet}$, so the singlet is the ground state and the triplet is the excited state.

Step 5: Compute the gap.

$$\Delta E = E_{triplet} - E_{singlet} = \frac{A}{4} - \left(-\frac{3A}{4}\right) = A$$

So the excitation energy is simply A itself:

$$\Delta E = 10.56 \text{ eV}$$

Final Answer:

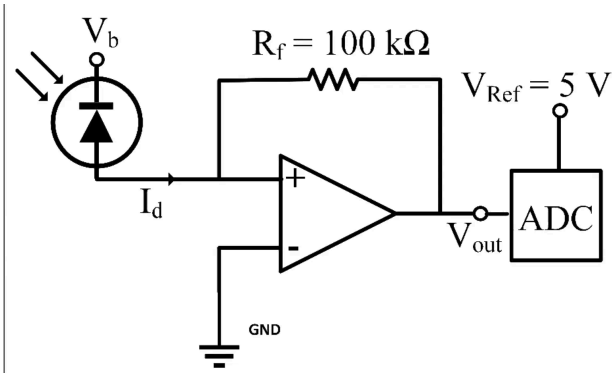
Rounded to two decimal places,

$$\boxed{\Delta E = 10.56 \text{ eV}}$$

Quick Tip: **Hint:**

Use $\vec{S}_1 \cdot \vec{S}_2 = \frac{1}{2}(S^2 - S_1^2 - S_2^2)$ with total spin $S = 0$ (singlet, ground state) or $S = 1$ (triplet, excited state), then subtract the two energies.

. The output signal (current I_d) of a reversed biased (with a voltage V_b) photodiode, on which light is incident, is fed to an amplifier (see figure). The output voltage is digitized by a 10 bit Analogue to Digital convertor (ADC) which has a reference voltage of 5 V. The smallest current which can be measured by the circuit in nano-Amperes (rounded off to one decimal place) is _____



Correct Answer: 48.8

Solution:

Step 1: Understand the circuit.

The photodiode current I_d feeds a transimpedance (current to voltage) amplifier built around an op-amp with feedback resistor R_f. The op-amp forces the whole diode current to flow through R_f, producing an output voltage

$$V_{out} = I_d R_f$$

This voltage is then digitized by the 10 bit ADC.

Step 2: Recall the ADC resolution limit.

A 10 bit ADC with reference voltage V_{Ref} divides the input range into $2^{10} = 1024$ discrete steps. The smallest voltage change it can tell apart from zero is one least significant bit (LSB),

$$V_{LSB} = \frac{V_{Ref}}{2^{10}}$$

Any current that produces less than this voltage at the amplifier output is indistinguishable from zero on the ADC, so the smallest measurable current is the one that produces exactly V_{LSB}.

Step 3: Compute V_{LSB}.

$$V_{LSB} = \frac{5\ \text{V}}{1024} = 4.8828 \times 10^{-3}\ \text{V}$$

Step 4: Convert this to the smallest measurable current.

$$I_{d,min} = \frac{V_{LSB}}{R_f} = \frac{4.8828 \times 10^{-3}}{100 \times 10^{3}} = 4.8828 \times 10^{-8}\ \text{A}$$

Converting to nano-Amperes,

$$I_{d,min} = 48.828 \text{ nA}$$

Final Answer:

Rounded to one decimal place,

$$\boxed{I_{d,min} = 48.8 \text{ nA}}$$

Quick Tip: **Hint:**

The transimpedance stage gives $V_{out} = I_d R_f$. Find the ADC least significant bit, $V_{Ref}/2^{10}$, then divide by R_f to get the smallest resolvable current.

. A capacitor is made of two circular metal plates of radius 1 m, separated by a distance $d = 1$ mm. The space between the plates is filled with a dielectric of permittivity $\epsilon_r = 5$. The capacitor is connected to a voltage $V = 10\sin(2\pi \times 10^6 \times t)$ volts, where t is in seconds. The maximum value of the magnetic field between the plates, at a radial distance $r = 0.5$ m from the centre, is $B \times 10^{-6}$ T. The value of B (rounded off to two decimal places) is _____.

(Speed of light in vacuum $c = 3 \times 10^8$ m.s^{-1})

Correct Answer: 0.87

Solution:

Step 1: Understanding the Concept:

A charging or discharging capacitor has no real current flowing

between its plates, but the changing electric field there acts as a displacement current. By the Ampere-Maxwell law, this displacement current produces a magnetic field between the plates, just as a real current would in a wire.

Step 2: Key Formula or Approach:

For a parallel plate capacitor of plate separation d, the field between the plates is $E = V/d$. Applying the Ampere-Maxwell law on a circular loop of radius r (less than the plate radius) centred on the axis:

$$B(2\pi r) = \mu_0 \epsilon \frac{dE}{dt}(\pi r^2)$$

where $\epsilon = \epsilon_r \epsilon_0$ is the permittivity of the dielectric. This gives:

$$B = \frac{\mu_0 \epsilon_r \epsilon_0 \, r}{2d} \frac{dV}{dt}$$

Since $\mu_0 \epsilon_0 = 1/c^2$, this can be written using only the given constants as:

$$B = \frac{\epsilon_r \, r}{2dc^2} \frac{dV}{dt}$$

Step 3: Detailed Explanation:

Differentiate the applied voltage: $V = 10 \sin(2\pi \times 10^6 t)$, so $dV/dt = 10(2\pi \times 10^6) \cos(2\pi \times 10^6 t)$. The maximum value of this rate is:

$$\left(\frac{dV}{dt}\right)_{max} = 10 \times 2\pi \times 10^6 = 2\pi \times 10^7 \text{ V/s}$$

Now put in the numbers, with $\epsilon_r = 5$, $r = 0.5$ m, $d = 10^{-3}$ m, $c^2 = 9$

$\times 10^{16}\ \mathrm{m^2 s^{-2}}$:

$$B_{max} = \frac{5 \times 0.5}{2 \times 10^{-3} \times 9 \times 10^{16}} \times 2\pi \times 10^{7} = \frac{2.5}{1.8 \times 10^{14}} \times 2\pi \times 10^{7}$$

$$B_{max} = 8.73 \times 10^{-7}\ \mathrm{T} = 0.87 \times 10^{-6}\ \mathrm{T}$$

Final Answer:

So $B = 0.87$, since $B_{max} = 0.87 \times 10^{-6}$ T.

$$\boxed{B = 0.87}$$

Quick Tip: **Hint:**

Use the Ampere-Maxwell law with the displacement current between the plates: $B = \frac{\epsilon_r r}{2dc^2} \frac{dV}{dt}$, and use the maximum rate of change of the given sinusoidal voltage.

. Rotational spectrum of a diatomic molecule consists of lines of equal spacing with an interval of $20.0\ \mathrm{cm^{-1}}$. Its moment of inertia is found to be $I_0 \times 10^{-47}\ \mathrm{kg.m^2}$, the value of I_0 (rounded off to one decimal place) is _____.

($h = 6.6 \times 10^{-34}$ J.s, speed of light in vacuum $c = 3 \times 10^{8}\ \mathrm{m.s^{-1}}$)

Correct Answer: 2.8

Solution:

Step 1: Understanding the Concept:

A diatomic molecule behaves like a rigid rotor. Its allowed rotational

energy levels are $E_J = B_e\, hc\, J(J+1)$, where $J = 0, 1, 2, \ldots$ is the rotational quantum number and B_e (in cm^{-1}) is the rotational constant. Transitions follow the selection rule $\Delta J = \pm 1$.

Step 2: Key Formula or Approach:

For a transition from J to $J+1$, the line appears at wavenumber $\tilde{\nu} = 2B_e(J+1)$. Consecutive lines therefore differ by a constant spacing:

$$\Delta\tilde{\nu} = 2B_e$$

The rotational constant is related to the moment of inertia I by:

$$B_e = \frac{h}{8\pi^2 cI}$$

Step 3: Detailed Explanation:

The line spacing is given as $\Delta\tilde{\nu} = 20.0\ \text{cm}^{-1}$, so:

$$B_e = \frac{\Delta\tilde{\nu}}{2} = 10.0\ \text{cm}^{-1}$$

Rearrange the formula for B_e to solve for I, remembering to use c in cm/s since B_e is in cm^{-1}: $c = 3 \times 10^8\ \text{m/s} = 3 \times 10^{10}\ \text{cm/s}$.

$$I = \frac{h}{8\pi^2 cB_e} = \frac{6.6 \times 10^{-34}}{8\pi^2 \times (3 \times 10^{10}) \times 10.0}$$

$$I = \frac{6.6 \times 10^{-34}}{2.369 \times 10^{13}} = 2.79 \times 10^{-47}\ \text{kg.m}^2$$

Final Answer:

So the moment of inertia is 2.79×10^{-47} kg.m^2, which rounds to $I_0 = 2.8$.

$$\boxed{I_0 = 2.8}$$

Quick Tip: **Hint:**

The spacing between adjacent rotational lines equals $2B_e$, and $B_e = h/(8\pi^2 cI)$. Use c in cm/s since the spacing is given in cm^{-1}.

. Copper has an electron number density of 8.3×10^{28} m^{-3}. Its Fermi energy in eV (rounded off to one decimal place) is _____.
($\hbar = 1.06 \times 10^{-34}$ J.s, mass of electron $m_e = 9.10 \times 10^{-31}$ kg, charge of electron $= 1.60 \times 10^{-19}$ C)

Correct Answer: 7.0

Solution:

Step 1: Understanding the Concept:

In the free electron model of a metal, conduction electrons fill up available momentum states from the lowest energy up, obeying the Pauli exclusion principle. At absolute zero, all states are filled up to a maximum energy called the Fermi energy, E_F.

Step 2: Key Formula or Approach:

For a free electron gas with number density n, the Fermi energy is:

$$E_F = \frac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$$

This comes from counting the allowed k-states inside a sphere of radius k_F in momentum space and setting that count equal to the total number of electrons.

Step 3: Detailed Explanation:
First evaluate the bracket:

$$3\pi^2 n = 3\pi^2 \times 8.3 \times 10^{28} = 2.458 \times 10^{30}\ \text{m}^{-3}$$

Take the two-thirds power:

$$(2.458 \times 10^{30})^{2/3} = 1.822 \times 10^{20}\ \text{m}^{-2}$$

Now put in $\hbar$ and m_e:

$$E_F = \frac{(1.06 \times 10^{-34})^2}{2 \times 9.10 \times 10^{-31}} \times 1.822 \times 10^{20}$$

$$E_F = \frac{1.1236 \times 10^{-68}}{1.82 \times 10^{-30}} \times 1.822 \times 10^{20} = 6.174 \times 10^{-39} \times 1.822 \times 10^{20}$$

$$E_F = 1.125 \times 10^{-18}\ \text{J}$$

Convert to eV by dividing by the electron charge:

$$E_F = \frac{1.125 \times 10^{-18}}{1.60 \times 10^{-19}} = 7.03\ \text{eV}$$

Final Answer:

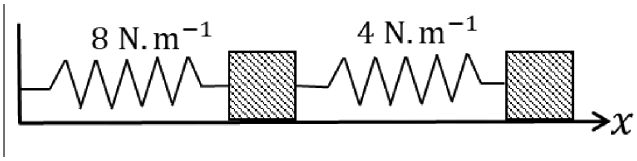
The Fermi energy of copper is about 7.0 eV.

$$\boxed{E_F = 7.0 \text{ eV}}$$

Quick Tip: **Hint:**

Use $E_F = \frac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$, then convert the answer from joules to electron volts by dividing by the electron charge.

. Two 1 kg blocks are connected to two massless springs of spring constants $8\ \text{N.m}^{-1}$ and $4\ \text{N.m}^{-1}$. The system is kept on a frictionless horizontal floor with one end of a spring attached to a wall (see figure below). They are performing oscillatory motion along the x-axis with the normal mode frequencies ω_H and ω_L ($\omega_H > \omega_L$). The ratio $\frac{\omega_H}{\omega_L}$ (rounded off to two decimal places) is _____



Correct Answer: 2.41

Solution:

Step 1: Understanding the Concept:

Set up an x-axis along the line of motion. Let x_1 and x_2 be the displacements of block 1 and block 2 from their equilibrium positions. The first spring ($k_1 = 8$ N/m) connects the wall to block 1, and the second spring ($k_2 = 4$ N/m) connects block 1 to block 2. Each block

has mass $m = 1$ kg.

Step 2: Write Newton's second law for each block.
Block 1 feels a restoring force from spring 1 (stretched by x_1) and from spring 2 (stretched by $x_2 - x_1$):

$$m\ddot{x}_1 = -k_1 x_1 + k_2(x_2 - x_1) = -(k_1 + k_2)x_1 + k_2 x_2$$

Block 2 only feels the second spring:

$$m\ddot{x}_2 = -k_2(x_2 - x_1) = k_2 x_1 - k_2 x_2$$

With $m = 1$, $k_1 = 8$, $k_2 = 4$:

$$\ddot{x}_1 = -12x_1 + 4x_2, \qquad \ddot{x}_2 = 4x_1 - 4x_2$$

Step 3: Find the normal mode frequencies.
Assume $x_1, x_2 \propto e^{i\omega t}$, so $\ddot{x}_i \to -\omega^2 x_i$. Substituting turns the equations into:

$$(12 - \omega^2)x_1 = 4x_2, \qquad 4x_1 = (4 - \omega^2)x_2$$

For a non-trivial solution the determinant of the coefficients must vanish:

$$(12 - \omega^2)(4 - \omega^2) - 16 = 0$$

$$\omega^4 - 16\omega^2 + 32 = 0$$

Step 4: Solve the quadratic in ω^2.

$$\omega^2 = \frac{16 \pm \sqrt{256 - 128}}{2} = 8 \pm 4\sqrt{2}$$

So $\omega_H^2 = 8 + 4\sqrt{2} = 13.66$ and $\omega_L^2 = 8 - 4\sqrt{2} = 2.34$, giving $\omega_H = 3.70$ rad/s and $\omega_L = 1.53$ rad/s.

Final Answer:

The ratio simplifies neatly: $\frac{\omega_H}{\omega_L} = \sqrt{\frac{8 + 4\sqrt{2}}{8 - 4\sqrt{2}}} = 1 + \sqrt{2} = 2.41.$

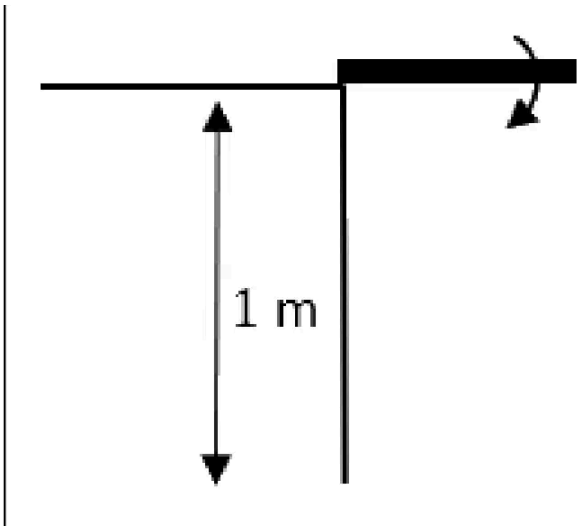
$$\boxed{\frac{\omega_H}{\omega_L} = 2.41}$$

Quick Tip: **Hint:**

Write Newton's second law for both blocks, assume $x_i \propto e^{i\omega t}$, and set the determinant of the resulting 2×2 system to zero to get $\omega^4 - 16\omega^2 + 32 = 0$.

. A 15 cm long scale is held horizontally with one of its ends on the edge of a 1 m high table and the other end resting on one's index finger. As the finger is removed (see figure below), the scale starts rotating about its end on the table. After 0.1 s, during which it has rotated by a negligibly small angle but has gained a rotational speed, it leaves the table and falls vertically towards the ground. When its centre of mass has fallen by 0.5 m, it has rotated by an angle θ. The value of θ in degrees (rounded off to one decimal place) is _____

$(g = 9.8\ \text{m.s}^{-2})$



Correct Answer: 179.4

Solution:

Step 1: Understanding the Concept:

While the scale rests on the table edge with the finger removed, it behaves as a uniform rod pivoted at one end, falling under gravity. Since the problem tells us the angle turned in the first 0.1 s is negligibly small, we can treat the torque during this stage as constant, using the horizontal orientation throughout.

Step 2: Find the angular speed gained while pivoted.

For a uniform rod of length L pivoted at one end, the moment of

inertia about that end is $I = \frac{1}{3}mL^2$. Gravity acts at the centre of mass, a distance $L/2$ from the pivot, giving torque $\tau = mg(L/2)$ (using $\cos\theta \approx 1$ since the angle stays small). So the angular acceleration is:

$$\alpha = \frac{\tau}{I} = \frac{mg(L/2)}{\frac{1}{3}mL^2} = \frac{3g}{2L}$$

With $g = 9.8\ \text{m/s}^2$ and $L = 0.15\ \text{m}$:

$$\alpha = \frac{3 \times 9.8}{2 \times 0.15} = 98\ \text{rad/s}^2$$

Starting from rest, after $t_1 = 0.1$ s the scale has gained an angular speed:

$$\omega_0 = \alpha t_1 = 98 \times 0.1 = 9.8\ \text{rad/s}$$

Step 3: Track the free-fall stage.

Once it leaves the table, no torque acts on the scale about its own centre of mass (gravity acts right at the centre of mass), so the angular speed stays fixed at $\omega_0 = 9.8$ rad/s for the rest of the fall. Since the rotation is negligible at the moment of separation, the centre of mass effectively begins this stage of falling from rest, and drops under gravity alone.

Step 4: Find the time to fall 0.5 m and the angle turned.

Using $h = \frac{1}{2}gt_2^2$ with $h = 0.5$ m:

$$t_2 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 0.5}{9.8}} = \sqrt{0.1020} = 0.319\ \text{s}$$

The scale keeps spinning at the constant rate ω_0 throughout this fall, so the angle turned is:

$$\theta = \omega_0 t_2 = 9.8 \times 0.319 = 3.130 \text{ rad}$$

Convert to degrees:

$$\theta = 3.130 \times \frac{180}{\pi} = 179.4^\circ$$

Final Answer:

The scale has turned by about 179.4° by the time its centre of mass has fallen 0.5 m.

$$\boxed{\theta = 179.4^\circ}$$

Quick Tip: **Hint:**

Find the angular speed gained while pivoted using $\alpha = 3g/(2L)$ and $\omega_0 = \alpha t_1$, then find the time to fall 0.5 m from rest, and multiply that time by ω_0 to get the angle turned.