JEE Main Mathematics

Sample Paper – 3

Duration: 60 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **30 Questions** divided into two sections, modelled on the Mathematics section of **JEE Main**.
- **Section A (Q1–Q20):** Single Correct MCQs. Each correct answer carries **+4 marks**; each wrong answer carries $-$**1 mark**. Unattempted questions carry 0 marks.
- **Section B (Q21–Q30):** Integer Answer Type. Each correct answer carries **+4 marks**. There is **no negative marking**. Attempt any **5** of 10 questions.
- Syllabus level: **Class 11–12 (NCERT) — JEE Main Mathematics syllabus.**
- Personal calculators, log tables, mobile phones and electronic gadgets are strictly prohibited. Rough sheets will be provided at the exam centre.

SECTION A — Single Correct MCQs (Q1–Q20) [+4 / −1]

Q1. Let R be the relation on $\{1, 2, 3, 4\}$ defined by $R = \{(a, b) : a \text{ divides } b\}$. Which of the following correctly describes R?

(A) Reflexive and Symmetric

(B) An Equivalence Relation

(C) Reflexive but not Symmetric

(D) Symmetric only

Q2. If the equation $2x^2 + kx + 3 = 0$ has two equal roots, then the value of k is:

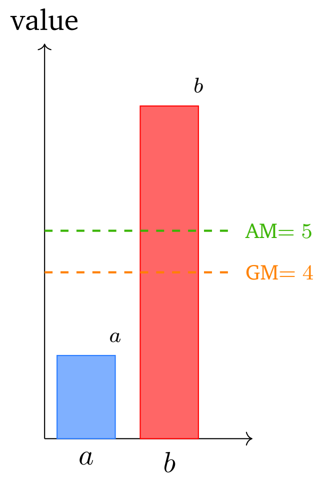
(A) $\pm\sqrt{6}$

(B) ± 12

(C) ± 3

(D) $\pm 2\sqrt{6}$

Q3. If the A.M. of two positive numbers is 5 and their G.M. is 4, the two numbers are:



(A) 2 and 8

(B) 4 and 6

(C) 1 and 9

(D) 3 and 7

Q4. Let ω be a complex cube root of unity ($\omega \neq 1$, $\omega^3 = 1$). The value of $1 + \omega^4 + \omega^8$ is:

(A) 1

(B) 0

(C) -1

(D) 3

Q5. The number of ways to arrange 4 identical red balls and 3 identical blue balls in a row is:

(A) 12

(B) 21

(C) 35

(D) 70

Q6. The greatest binomial coefficient in the expansion of $(1+x)^7$ is:

(A) 21

(B) 28

(C) 7

(D) 35

Q7. The trace of the matrix $A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$ (sum of diagonal elements) is:

diagonal

3	1
2	4

(A) 7

(B) 8

(C) 6

(D) 5

Q8. The values of x satisfying $\begin{vmatrix} x & 2 \\ 3 & x \end{vmatrix} = 0$ are:

(A) $\pm\sqrt{3}$

(B) $\pm\sqrt{6}$

(C) ± 2

(D) ± 3

Q9. The value of $\lim_{x \to 1} \dfrac{x^5 - 1}{x - 1}$ is:

(A) 1

(B) 3

(C) 5

(D) ∞

Q10. For $f(x) = x^2 - 3x + 2$ on the interval $[1, 2]$, the value of c guaranteed by Rolle's Theorem is:

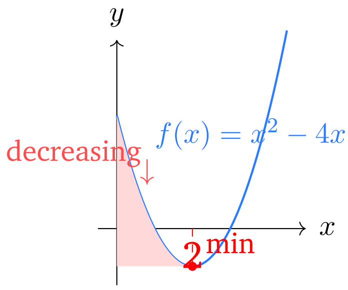
(A) 1

(B) 2

(C) $\frac{1}{3}$

(D) $\frac{3}{2}$

Q11. The function $f(x) = x^2 - 4x + 3$ is strictly decreasing on:



(A) $(-\infty, 2)$

(B) $(2, \infty)$

(C) $(0, 4)$

(D) $(1, 3)$

Q12. $\int \frac{1}{\sqrt{1-x^2}} dx$ equals:

(A) $\cos^{-1} x + C$

(B) $\sin^{-1} x + C$

(C) $\tan^{-1} x + C$

(D) $-\sin^{-1} x + C$

Q13. The value of $\int_0^{\pi/2} \cos^2 x \, dx$ is:

(A) $\frac{\pi}{2}$

(B) 1

(C) $\dfrac{\pi}{4}$

(D) 0

Q14. The degree of the differential equation $\left(\dfrac{dy}{dx}\right)^2 + y = 0$ is:

(A) 1

(B) 3

(C) 0

(D) 2

Q15. The perpendicular distance from the point $(2, 3)$ to the line $3x - 4y + 5 = 0$ is:

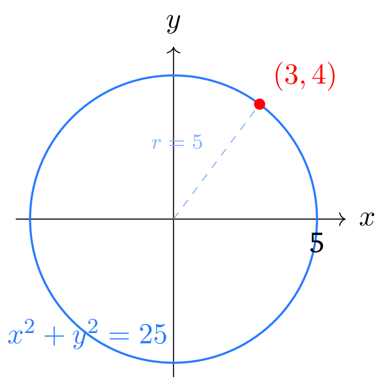
(A) $\dfrac{1}{5}$

(B) $\dfrac{2}{5}$

(C) 5

(D) 1

Q16. The point $(3, 4)$ with respect to the circle $x^2 + y^2 = 25$ lies:



(A) Inside the circle

(B) On the circle

(C) Outside the circle

(D) At the centre

Q17. The length of the latus rectum of the parabola $y^2 = 8x$ is:

(A) 4

(B) 2

(C) 8

(D) 16

Q18. The eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is:

(A) $\frac{3}{4}$

(B) $\frac{\sqrt{7}}{3}$

(C) $\frac{\sqrt{3}}{4}$

(D) $\frac{\sqrt{7}}{4}$

Q19. The value of $\sin 75^\circ$ is:

(A) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(B) $\frac{\sqrt{6}-\sqrt{2}}{4}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{\sqrt{2}+1}{4}$

Q20. The value of $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$ is:

(A) $\frac{7\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{3}$

SECTION B — Integer Answer Type (Q21–Q30) [+4 each, no negative marking; attempt any 5]

Q21. Let $\vec{a} = (1, 2, 2)$ and $\vec{b} = (2, 1, -2)$. Find the magnitude $|\vec{a} \times \vec{b}|$.

Answer: ________

Q22. Find the perpendicular distance from the origin to the plane $2x + y - 2z = 9$.

Answer: ________

Q23. From a pack of 52 cards, one card is drawn at random. Find the value of $13 \times P(\text{king or queen})$.

Answer: ________

Q24. Find the mean of the data set $\{3, 5, 7, 9, 11\}$.

Answer: ________

Q25. Find the sum $1 + 3 + 5 + 7 + \cdots + 15$ (sum of first 8 odd numbers).

Answer: ________

Q26. The volume of a cube is increasing at 6 cm^3/s. At the instant when the edge length $x = 3$ cm, find $9 \times \dfrac{dx}{dt}$ (in cm/s).

Answer: ________

Q27. Find the value of $\displaystyle\int_0^2 3x^2\,dx$.

Answer: ________

Q28. Find the coefficient of the middle term in the expansion of $(x + 1)^6$.

Answer: ________

Q29. If $z = 1 + i$, find $|z^2|$.

Answer: ________

Q30. Find the sum of all elements of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

Answer: ________

End of Question Paper — Detailed Solutions on the following pages

Detailed Solutions

Sample Paper – 3 | Mathematics

SECTION A Solutions (Q1–Q20)

Q1.

Solution

Concept — Relations and their Properties: A relation R on a set A is reflexive if $(a, a) \in R$ for all $a \in A$; symmetric if $(a, b) \in R \Rightarrow (b, a) \in R$; transitive if $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.

Step 1 — Check Reflexivity: For every element $a \in \{1, 2, 3, 4\}$, we need $a|a$ (i.e., a divides a). Since every integer divides itself, $(1, 1), (2, 2), (3, 3), (4, 4) \in R$. Hence R is **reflexive**.

Step 2 — Check Symmetry: Consider $(1, 2) \in R$ since $1|2$. For symmetry we need $(2, 1) \in R$, i.e., $2|1$. But 2 does not divide 1. Hence R is **not symmetric**.

Step 3 — Conclusion on Equivalence: Since R is not symmetric, it cannot be an equivalence relation (which requires reflexivity, symmetry, and transitivity all together).

Why other options are wrong:

- (A) Claims symmetric — disproved above.
- (B) Equivalence requires symmetry — not satisfied.
- (D) Not symmetric only; R is also reflexive.

Final Answer: R is reflexive but not symmetric $\Rightarrow \boxed{(C)}$

Answer: (C) Go Back to Q1

Q2.

Solution

Concept — Equal Roots of Quadratic (Discriminant): A quadratic $ax^2+bx+c=0$ has two equal roots if and only if the discriminant $\Delta = b^2 - 4ac = 0$.

Step 1 — Identify Coefficients: Given $2x^2 + kx + 3 = 0$: $a = 2$, $b = k$, $c = 3$.

Step 2 — Set Discriminant to Zero:

$$k^2 - 4(2)(3) = 0 \implies k^2 = 24 \implies k = \pm\sqrt{24} = \pm 2\sqrt{6}.$$

Why other options are wrong:

- (A) $\pm\sqrt{6}$: gives $k^2 = 6 \neq 24$.
- (B) ± 12: gives $k^2 = 144 \neq 24$.
- (C) ± 3: gives $k^2 = 9 \neq 24$.

Final Answer: $k = \pm 2\sqrt{6} \Rightarrow \boxed{(D)}$

Answer: (D) Go Back to Q2

Q3.

Solution

Concept — AM–GM Relationship: If two positive numbers are a and b, then $\text{AM} = \frac{a+b}{2}$ and $\text{GM} = \sqrt{ab}$.

Step 1 — Set Up Equations:

$$\frac{a+b}{2} = 5 \implies a + b = 10, \qquad \sqrt{ab} = 4 \implies ab = 16.$$

Step 2 — Form Quadratic: The numbers a and b are roots of $t^2 - (a+b)t + ab = 0$:

$$t^2 - 10t + 16 = 0.$$

Step 3 — Solve:

$$t = \frac{10 \pm \sqrt{100 - 64}}{2} = \frac{10 \pm \sqrt{36}}{2} = \frac{10 \pm 6}{2}.$$

So $t = 8$ or $t = 2$. The two numbers are **2 and 8**.

Verification: $\text{AM} = \frac{2+8}{2} = 5 \checkmark$; $\text{GM} = \sqrt{2 \cdot 8} = \sqrt{16} = 4 \checkmark$.

Why other options are wrong:

- (B) $4, 6$: $\text{AM} = 5 \checkmark$ but $\text{GM} = \sqrt{24} \neq 4 \times$.
- (C) $1, 9$: $\text{AM} = 5 \checkmark$ but $\text{GM} = 3 \neq 4 \times$.
- (D) $3, 7$: $\text{GM} = \sqrt{21} \neq 4 \times$.

Final Answer: The two numbers are 2 and 8 $\Rightarrow \boxed{(A)}$

Answer: (A) Go Back to Q3

Q4.

Solution

Concept — Cube Roots of Unity: The complex cube roots of unity satisfy $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$.

Step 1 — Reduce Powers: Since $\omega^3 = 1$:

$$\omega^4 = \omega^{3+1} = \omega^3 \cdot \omega = 1 \cdot \omega = \omega,$$

$$\omega^8 = \omega^{6+2} = (\omega^3)^2 \cdot \omega^2 = 1 \cdot \omega^2 = \omega^2.$$

Step 2 — Evaluate Expression:

$$1 + \omega^4 + \omega^8 = 1 + \omega + \omega^2 = 0.$$

This uses the fundamental identity for cube roots of unity.

Why other options are wrong:

- (A) 1: confuses $1 + \omega^3 + \omega^6 = 1 + 1 + 1 = 3$ (different expression).
- (C) -1: the identity $1 + \omega + \omega^2 = 0$, not -1.
- (D) 3: only if all powers were $\omega^0 = 1$.

Final Answer: $1 + \omega^4 + \omega^8 = 0 \Rightarrow \boxed{(B)}$

Answer: (B) Go Back to Q4

Q5.

Solution

Concept — Arrangements with Identical Objects: The number of ways to arrange n objects where p are identical of one type and q are identical of another type is $\dfrac{n!}{p!\,q!}$.

Step 1 — Count Total Objects: Total balls $= 4$ (red) $+3$ (blue) $= 7$.

Step 2 — Apply Formula:

$$\text{Number of arrangements} = \frac{7!}{4!\,3!} = \frac{5040}{24 \times 6} = \frac{5040}{144} = 35.$$

Step 3 — Verify: $\frac{7!}{4!\,3!} = \binom{7}{4} = \binom{7}{3} = 35$. This counts the ways to choose 4 positions out of 7 for the red balls (the rest go to blue).

Why other options are wrong:

- (A) 12: incorrect; this might come from $4! \div 2 = 12$.
- (B) 21: $\binom{7}{2} = 21$, wrong combination.
- (D) 70: $\binom{8}{4} = 70$, wrong n.

Final Answer: 35 arrangements $\Rightarrow \boxed{(C)}$

Answer: (C) **Go Back to Q5**

Q6.

Solution

Concept — Greatest Binomial Coefficient: In the expansion of $(1+x)^n$, the binomial coefficients are $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$. When n is odd, the two middle terms $\binom{n}{\frac{n-1}{2}}$ and $\binom{n}{\frac{n+1}{2}}$ are equal and greatest.

Step 1 — Identify n: Here $n = 7$ (odd). The middle terms are T_4 and T_5.

Step 2 — Compute the Greatest Coefficient:

$$\binom{7}{3} = \frac{7!}{3!\,4!} = \frac{7 \times 6 \times 5}{6} = 35.$$

Similarly $\binom{7}{4} = 35$. So the greatest binomial coefficient is **35**.

Why other options are wrong:

- (A) 21: $\binom{7}{2} = 21$ — not the maximum.
- (B) 28: $\binom{8}{2} = 28$ — wrong n.
- (C) 7: $\binom{7}{1} = 7$ — the smallest non-trivial coefficient.

Final Answer: Greatest binomial coefficient $= 35 \Rightarrow \boxed{(D)}$

Answer: (D) **Go Back to Q6**

Q7.

Solution

Concept — Trace of a Matrix: The trace of a square matrix is the sum of its main diagonal elements: $\text{tr}(A) = \sum_i a_{ii}$.

Step 1 — Identify Diagonal Elements: For $A = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$, the diagonal elements are $a_{11} = 3$ (highlighted in blue in the diagram) and $a_{22} = 4$.

Step 2 — Compute Trace:

$$\text{tr}(A) = 3 + 4 = 7.$$

Why other options are wrong:

- (B) 8: sum of anti-diagonal elements $1+4+2+1$? No; $1+4 = 5$ or $3+1+2+4 = 10$; none give 8 logically.
- (C) 6: might confuse off-diagonal sum $1 + 2 + 1 + 2 = 6$? Irrelevant.
- (D) 5: equals $a_{21} + a_{12} = 2 + 1 = 3$ plus something; not the diagonal sum.

Final Answer: $\text{tr}(A) = 3 + 4 = 7 \Rightarrow \boxed{(A)}$

Answer: (A) Go Back to Q7

Q8.

Solution

Concept — Determinant of 2×2 Matrix: $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$

Step 1 — Compute Determinant:

$$\begin{vmatrix} x & 2 \\ 3 & x \end{vmatrix} = x \cdot x - 2 \cdot 3 = x^2 - 6.$$

Step 2 — Set Equal to Zero and Solve:

$$x^2 - 6 = 0 \implies x^2 = 6 \implies x = \pm\sqrt{6}.$$

Why other options are wrong:

- (A) $\pm\sqrt{3}$: implies $x^2 = 3$, which means the determinant was $x^2 - 3$, incorrect.
- (C) ± 2: implies $x^2 = 4$, i.e., $2 \cdot 3 = 4$, which is wrong.

- (D) ± 3: implies $x^2 = 9$; the off-diagonal product is $2 \times 3 = 6 \neq 9$.

Final Answer: $x = \pm\sqrt{6} \Rightarrow \boxed{(B)}$

Answer: (B) Go Back to Q8

Q9.

Solution

Concept — Standard Limit Formula:

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n\, a^{n-1}.$$

Step 1 — Identify and Apply Formula: Here $a = 1$, $n = 5$:

$$\lim_{x \to 1} \frac{x^5 - 1}{x - 1} = 5 \cdot 1^{5-1} = 5 \cdot 1 = 5.$$

Alternative — Factor and Cancel: $x^5 - 1 = (x-1)(x^4 + x^3 + x^2 + x + 1)$, so

$$\lim_{x \to 1}(x^4 + x^3 + x^2 + x + 1) = 1 + 1 + 1 + 1 + 1 = 5.$$

Why other options are wrong:

- (A) 1: only if $n = 1$.
- (B) 3: only if $n = 3$.
- (D) ∞: the $x - 1$ factor cancels, so no blow-up at $x = 1$.

Final Answer: Limit $= 5 \Rightarrow \boxed{(C)}$

Answer: (C) Go Back to Q9

Q10.

Solution

Concept — Rolle's Theorem: If f is continuous on $[a, b]$, differentiable on (a, b), and $f(a) = f(b)$, then there exists $c \in (a, b)$ such that $f'(c) = 0$.

Step 1 — Verify Conditions: $f(x) = x^2 - 3x + 2$ is a polynomial, so continuous and differentiable everywhere.

$$f(1) = 1 - 3 + 2 = 0, \qquad f(2) = 4 - 6 + 2 = 0.$$

$f(1) = f(2) = 0$ ✓. Rolle's Theorem applies.

Step 2 — Find c:

$$f'(x) = 2x - 3 = 0 \implies x = \frac{3}{2}.$$

Since $\frac{3}{2} \in (1, 2)$, the value is $c = \dfrac{3}{2}$.

Why other options are wrong:

- (A) $c = 1$: endpoint, not interior; $f'(1) = 2(1) - 3 = -1 \neq 0$.
- (B) $c = 2$: endpoint; $f'(2) = 2(2) - 3 = 1 \neq 0$.
- (C) $c = \frac{1}{3}$: not in $(1, 2)$; $f'(\frac{1}{3}) = \frac{2}{3} - 3 \neq 0$.

Final Answer: $c = \dfrac{3}{2} \Rightarrow \boxed{(D)}$

Answer: (D) Go Back to Q10

Q11.

Solution

Concept — Monotonicity via First Derivative: f is strictly decreasing on an interval I if $f'(x) < 0$ for all $x \in I$.

Step 1 — Differentiate: $f(x) = x^2 - 4x + 3 \implies f'(x) = 2x - 4 = 2(x - 2)$.

Step 2 — Find Where $f'(x) < 0$:

$$2(x - 2) < 0 \implies x < 2 \implies x \in (-\infty, 2).$$

The graph confirms: the parabola has vertex at $x = 2$ (minimum); it decreases to the left of $x = 2$ (shaded red region).

Step 3 — Increasing Region: For $x > 2$: $f'(x) > 0$, so f is increasing on $(2, \infty)$.

Why other options are wrong:

- (B) $(2, \infty)$: this is the *increasing* region.
- (C) $(0, 4)$: f decreases on $(0, 2)$ but increases on $(2, 4)$; not entirely decreasing.
- (D) $(1, 3)$: same issue — crosses the vertex at $x = 2$.

Final Answer: Strictly decreasing on $(-\infty, 2) \Rightarrow \boxed{(A)}$

Answer: (A) Go Back to Q11

Q12.

Solution

Concept — Standard Integral (Inverse Trig): The standard result is $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$.

Step 1 — Verify by Differentiating: $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ ✓.

Step 2 — Distinguish from $\cos^{-1} x$: $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$ (note the minus sign).

Why other options are wrong:

- (A) $\cos^{-1} x + C$: derivative is $-\frac{1}{\sqrt{1-x^2}}$, which has the wrong sign.
- (C) $\tan^{-1} x + C$: derivative is $\frac{1}{1+x^2}$, different denominator.
- (D) $-\sin^{-1} x + C$: derivative is $-\frac{1}{\sqrt{1-x^2}}$, wrong sign.

Final Answer: $\sin^{-1} x + C \Rightarrow \boxed{(B)}$

Answer: (B) Go Back to Q12

Q13.

Solution

Concept — Definite Integral of $\cos^2 x$: Use the identity $\cos^2 x = \frac{1+\cos 2x}{2}$.

Step 1 — Rewrite Integrand:

$$\int_0^{\pi/2} \cos^2 x\,dx = \int_0^{\pi/2} \frac{1+\cos 2x}{2}\,dx.$$

Step 2 — Integrate Term by Term:

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi/2} = \frac{1}{2}\left[\left(\frac{\pi}{2} + \frac{\sin \pi}{2}\right) - \left(0 + \frac{\sin 0}{2}\right)\right].$$

Step 3 — Evaluate: $\sin\pi = 0$ and $\sin 0 = 0$, so:

$$= \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4}.$$

Why other options are wrong:

- (A) $\pi/2$: this is $\int_0^{\pi/2} 1\,dx$, integrating 1 not $\cos^2 x$.
- (B) 1: ignores the π from integration limits.
- (D) 0: $\int_0^{\pi/2} \cos x\,dx = 1 \neq 0$; $\cos^2 x \geq 0$ so integral > 0.

Final Answer: $\frac{\pi}{4} \Rightarrow \boxed{(C)}$

Answer: (C) Go Back to Q13

Q14.

Solution

Concept — Order and Degree of a Differential Equation:

- *Order*: Order of the highest derivative present.
- *Degree*: Power of the highest-order derivative after the equation is free of radicals and fractions in derivatives.

Step 1 — Identify Highest Derivative: The equation $\left(\frac{dy}{dx}\right)^2 + y = 0$ involves $\frac{dy}{dx}$, which is a first-order derivative.

Step 2 — Find the Degree: The highest-order derivative here is $\frac{dy}{dx}$ (order $= 1$), and it appears raised to the **power 2**. The equation is already polynomial in the derivative. Hence degree $= 2$.

Why other options are wrong:

- (A) Degree 1: that would require $(dy/dx)^1$, but here the exponent is 2.
- (B) Degree 3: no cube of any derivative appears.
- (C) Degree 0: a degree of 0 is undefined/meaningless for DEs.

Final Answer: Degree $= 2 \Rightarrow \boxed{(D)}$

Answer: (D) Go Back to Q14

Q15.

Solution

Concept — Perpendicular Distance from Point to Line: The perpendicular distance from (x_1, y_1) to the line $ax + by + c = 0$ is

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Step 1 — Identify Parameters: Line: $3x - 4y + 5 = 0$ so $a = 3$, $b = -4$, $c = 5$. Point: $(x_1, y_1) = (2, 3)$.

Step 2 — Substitute:

$$d = \frac{|3(2) + (-4)(3) + 5|}{\sqrt{3^2 + (-4)^2}} = \frac{|6 - 12 + 5|}{\sqrt{9 + 16}} = \frac{|-1|}{\sqrt{25}} = \frac{1}{5}.$$

Why other options are wrong:

- (B) $\frac{2}{5}$: numerator would need to be 2; but $|6 - 12 + 5| = 1 \neq 2$.
- (C) 5: this is the denominator $\sqrt{25}$, not the distance.
- (D) 1: numerator without dividing by 5.

Final Answer: $d = \frac{1}{5} \Rightarrow \boxed{(A)}$

Answer: (A) Go Back to Q15

Q16.

Solution

Concept — Position of a Point w.r.t. a Circle: For circle $x^2 + y^2 = r^2$ and point $P = (x_1, y_1)$:

- $x_1^2 + y_1^2 < r^2 \Rightarrow$ inside
- $x_1^2 + y_1^2 = r^2 \Rightarrow$ on the circle
- $x_1^2 + y_1^2 > r^2 \Rightarrow$ outside

Step 1 — Compute $x_1^2 + y_1^2$:

$$3^2 + 4^2 = 9 + 16 = 25.$$

Step 2 — Compare with r^2: $r^2 = 25$. Since $x_1^2 + y_1^2 = 25 = r^2$, the point $(3, 4)$ lies

on the circle. The TikZ diagram shows the point on the circumference with radius $r = 5$.

Note: This is the classic 3-4-5 right triangle: $3^2 + 4^2 = 5^2 = 25$.

Why other options are wrong:

- (A) Inside: requires $3^2 + 4^2 < 25$; but $25 \not< 25$.
- (C) Outside: requires $3^2 + 4^2 > 25$; but $25 \not> 25$.
- (D) Centre: the centre is $(0, 0)$, not $(3, 4)$.

Final Answer: $(3, 4)$ lies on the circle $\Rightarrow \boxed{(B)}$

Answer: (B) Go Back to Q16

Q17.

Solution

Concept — Latus Rectum of a Parabola: For the standard parabola $y^2 = 4ax$, the length of the latus rectum is $4a$.

Step 1 — Compare with Standard Form: Given $y^2 = 8x$. Comparing with $y^2 = 4ax$:

$$4a = 8 \implies a = 2.$$

Step 2 — Compute Latus Rectum: Length of latus rectum $= 4a = 4 \times 2 = 8$.

Recall: The latus rectum is the chord through the focus $(a, 0) = (2, 0)$ perpendicular to the axis of the parabola. Its endpoints are $(2, 4)$ and $(2, -4)$, giving length $4 - (-4) = 8$.

Why other options are wrong:

- (A) 4: equals $2a$, half the latus rectum (semi-latus rectum).
- (B) 2: equals a, the focal distance only.
- (D) 16: equals $2 \times (4a)$, double the correct answer.

Final Answer: Length of latus rectum $= 8 \Rightarrow \boxed{(C)}$

Answer: (C) Go Back to Q17

Q18.

Solution

Concept — Eccentricity of an Ellipse: For $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with $a>b>0$: $c^2=a^2-b^2$, $e=\frac{c}{a}$.

Step 1 — Identify a^2 and b^2: $a^2=16$ (larger denominator), $b^2=9$.

Step 2 — Compute c:

$$c^2=a^2-b^2=16-9=7 \implies c=\sqrt{7}.$$

Step 3 — Compute Eccentricity:

$$e=\frac{c}{a}=\frac{\sqrt{7}}{\sqrt{16}}=\frac{\sqrt{7}}{4}.$$

Why other options are wrong:

- (A) $\frac{3}{4}$: this is $b/a=3/4$, not the eccentricity.
- (B) $\frac{\sqrt{7}}{3}$: uses $b=3$ in denominator instead of $a=4$.
- (C) $\frac{\sqrt{3}}{4}$: computes $c^2=a^2-b^2$ incorrectly as 3.

Final Answer: $e=\frac{\sqrt{7}}{4} \Rightarrow \boxed{(D)}$

Answer: (D) Go Back to Q18

Q19.

Solution

Concept — Compound Angle Formula: $\sin(A+B)=\sin A\cos B+\cos A\sin B$.

Step 1 — Decompose $75°$: $75°=45°+30°$.

Step 2 — Apply Formula:

$$\sin 75° = \sin 45° \cos 30° + \cos 45° \sin 30°.$$

Step 3 — Substitute Known Values:

$$=\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\cdot\frac{1}{2}=\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}=\frac{\sqrt{6}+\sqrt{2}}{4}.$$

Why other options are wrong:

- (B) $\frac{\sqrt{6}-\sqrt{2}}{4}$: this is $\sin 15^\circ = \sin(45^\circ - 30^\circ)$, not 75°.
- (C) $\frac{\sqrt{3}}{2}$: this is $\sin 60^\circ$, not 75°.
- (D) $\frac{\sqrt{2}+1}{4}$: not a standard value for 75°.

Final Answer: $\sin 75^\circ = \dfrac{\sqrt{6}+\sqrt{2}}{4} \Rightarrow \boxed{(A)}$

Answer: (A) Go Back to Q19

Q20.

Solution

Concept — Principal Value of $\cos^{-1}$: The range of $\cos^{-1}$ is $[0, \pi]$. We must express the cosine value in terms of an angle in $[0, \pi]$.

Step 1 — Find $\cos\frac{7\pi}{6}$: $\frac{7\pi}{6} = \pi + \frac{\pi}{6}$, so:

$$\cos\frac{7\pi}{6} = \cos\left(\pi + \frac{\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}.$$

Step 2 — Find $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$: We need $\theta \in [0, \pi]$ such that $\cos\theta = -\dfrac{\sqrt{3}}{2}$.

$$\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}.$$

So $\cos^{-1}\left(\cos\dfrac{7\pi}{6}\right) = \dfrac{5\pi}{6}$.

Step 3 — Verify $\frac{5\pi}{6} \in [0, \pi]$: $\frac{5\pi}{6} \approx 2.618 \in [0, \pi]$ ✓.

Why other options are wrong:

- (A) $\frac{7\pi}{6}$: outside the range $[0, \pi]$; $\cos^{-1}$ cannot return this.
- (C) $\frac{\pi}{6}$: $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} > 0$, but our value is negative.
- (D) $\frac{\pi}{3}$: $\cos\frac{\pi}{3} = \frac{1}{2} \neq -\frac{\sqrt{3}}{2}$.

Final Answer: $\cos^{-1}\left(\cos\dfrac{7\pi}{6}\right) = \dfrac{5\pi}{6} \Rightarrow \boxed{(B)}$

Answer: (B) Go Back to Q20

SECTION B Solutions — Integer Answer Type (Q21–Q30)

Q21.

Solution

Concept — Cross Product and Its Magnitude: $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Step 1 — Compute the Cross Product: $\vec{a} = (1, 2, 2)$, $\vec{b} = (2, 1, -2)$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 1 & -2 \end{vmatrix} = \hat{i}(2 \cdot (-2) - 2 \cdot 1) - \hat{j}(1 \cdot (-2) - 2 \cdot 2) + \hat{k}(1 \cdot 1 - 2 \cdot 2).$$

Step 2 — Evaluate Each Component:

$$\begin{aligned} \hat{i}\text{-component} &= (-4 - 2) = -6, \\ \hat{j}\text{-component} &= -(-2 - 4) = -(-6) = 6, \\ \hat{k}\text{-component} &= (1 - 4) = -3. \end{aligned}$$

So $\vec{a} \times \vec{b} = (-6,\ 6,\ -3)$.

Step 3 — Find the Magnitude:

$$|\vec{a} \times \vec{b}| = \sqrt{(-6)^2 + 6^2 + (-3)^2} = \sqrt{36 + 36 + 9} = \sqrt{81} = 9.$$

Final Answer: $|\vec{a} \times \vec{b}| = \boxed{9}$

Answer: (9) Go Back to Q21

Q22.

Solution

Concept — Distance from Origin to Plane: Distance from origin to $ax+by+cz = d$ is $\dfrac{|d|}{\sqrt{a^2 + b^2 + c^2}}$.

Step 1 — Identify Coefficients: Plane: $2x + y - 2z = 9$. So $a = 2$, $b = 1$, $c = -2$, $d = 9$.

Step 2 — Compute Distance:

$$\text{Distance} = \frac{|9|}{\sqrt{2^2 + 1^2 + (-2)^2}} = \frac{9}{\sqrt{4 + 1 + 4}} = \frac{9}{\sqrt{9}} = \frac{9}{3} = 3.$$

Final Answer: Distance $= \boxed{3}$

Answer: (3) Go Back to Q22

Q23.

Solution

Concept — Probability of Union (Mutually Exclusive Events): $P(A \cup B) = P(A) + P(B)$ when A and B are mutually exclusive.

Step 1 — Count Favourable Outcomes: Number of kings $= 4$. Number of queens $= 4$. (No card is both a king and a queen.) Total favourable $= 4 + 4 = 8$.

Step 2 — Compute Probability:

$$P(\text{king or queen}) = \frac{8}{52} = \frac{2}{13}.$$

Step 3 — Compute Required Value:

$$13 \times P(\text{king or queen}) = 13 \times \frac{2}{13} = 2.$$

Final Answer: $13 \times P(\text{king or queen}) = \boxed{2}$

Answer: (2) Go Back to Q23

Q24.

Solution

Concept — Arithmetic Mean: $\bar{x} = \dfrac{\text{Sum of observations}}{\text{Number of observations}}$.

Step 1 — Compute Sum: $3 + 5 + 7 + 9 + 11 = 35$.

Step 2 — Compute Mean:

$$\bar{x} = \frac{35}{5} = 7.$$

Alternate Observation: The data $\{3, 5, 7, 9, 11\}$ is an AP with first term 3, common difference 2, and 5 terms. The mean of an AP equals its middle term $= 7$. **Final Answer:** Mean $= \boxed{7}$

Answer: (7) Go Back to Q24

Q25.

Solution

Concept — Sum of First n Odd Numbers: The sum of the first n odd numbers is n^2.

Step 1 — Identify n: The sequence is $1, 3, 5, 7, 9, 11, 13, 15$. The k-th odd number is $2k - 1$. Here $2k - 1 = 15 \Rightarrow k = 8$. So $n = 8$.

Step 2 — Apply Formula:

$$S = 1 + 3 + 5 + \cdots + 15 = 8^2 = 64.$$

Verification by Direct Summation:

$$(1 + 3) + (5 + 7) + (9 + 11) + (13 + 15) = 4 + 12 + 20 + 28 = 64\checkmark.$$

Final Answer: Sum $= \boxed{64}$

Answer: (64) Go Back to Q25

Q26.

Solution

Concept — Related Rates (Chain Rule): Differentiate $V = x^3$ with respect to t: $\frac{dV}{dt} = 3x^2\frac{dx}{dt}$.

Step 1 — Set Up: $\frac{dV}{dt} = 6$ cm^3/s, $x = 3$ cm.

Step 2 — Solve for $\frac{dx}{dt}$:

$$6 = 3(3)^2\frac{dx}{dt} = 3 \times 9 \times \frac{dx}{dt} = 27\frac{dx}{dt}.$$

$$\frac{dx}{dt} = \frac{6}{27} = \frac{2}{9} \text{ cm/s}.$$

Step 3 — Compute Required Value:

$$9 \times \frac{dx}{dt} = 9 \times \frac{2}{9} = 2.$$

Final Answer: $9 \times \frac{dx}{dt} = \boxed{2}$

Answer: (2) Go Back to Q26

Q27.

Solution

Concept — Definite Integration using Power Rule: $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C.$

Step 1 — Integrate:

$$\int_0^2 3x^2\,dx = 3 \cdot \left[\frac{x^3}{3}\right]_0^2 = \left[x^3\right]_0^2.$$

Step 2 — Apply Limits:

$$= 2^3 - 0^3 = 8 - 0 = 8.$$

Final Answer: $\int_0^2 3x^2\,dx = \boxed{8}$

Answer: (8) Go Back to Q27

Q28.

Solution

Concept — Middle Term in Binomial Expansion: In $(x+1)^n$, the general term is $T_{r+1} = \binom{n}{r} x^{n-r}$. For $n = 6$ (even), the middle term is the $(n/2+1)$-th term $= T_4$.

Step 1 — Find Middle Term: $n = 6$: total terms $= 7$ (odd count), middle term is T_4 (4th term), corresponding to $r = 3$.

Step 2 — Compute T_4:

$$T_4 = \binom{6}{3} x^{6-3} \cdot 1^3 = \binom{6}{3} x^3 = 20x^3.$$

Step 3 — Extract Coefficient: The coefficient of the middle term $= \binom{6}{3} = \dfrac{6!}{3!\,3!} = \dfrac{720}{36} = 20$. **Final Answer:** Coefficient of middle term $= \boxed{20}$

Answer: (20) Go Back to Q28

Q29.

Solution

Concept — Modulus of Power of Complex Number: $|z^n| = |z|^n$ for any complex z.

Step 1 — Compute $|z|$: $z = 1 + i$, so $|z| = \sqrt{1^2 + 1^2} = \sqrt{2}$.

Step 2 — Compute $|z^2|$:

$$|z^2| = |z|^2 = (\sqrt{2})^2 = 2.$$

Verification by Direct Calculation: $z^2 = (1+i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$. So $|z^2| = |2i| = 2$ ✓. **Final Answer:** $|z^2| = \boxed{2}$

Answer: (2) Go Back to Q29

Q30.

Solution

Concept — Sum of All Elements of a Matrix: Sum of all elements $= \sum_{i,j} a_{ij}$.

Step 1 — List All Elements: $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Elements: $a_{11} = 1$, $a_{12} = 2$, $a_{21} = 3$, $a_{22} = 4$.

Step 2 — Add All Elements:

$$1 + 2 + 3 + 4 = 10.$$

Note: $\mathrm{tr}(A) = a_{11} + a_{22} = 1 + 4 = 5$ (trace uses only diagonal). Sum of all elements is different: it includes a_{12} and a_{21} as well. **Final Answer:** Sum of all elements $= \boxed{10}$

Answer: (10) Go Back to Q30

Answer Key

Sample Paper – 3 | Mathematics

Section A — Single Correct MCQs

1	C	2	D	3	A	4	B	5	C
6	D	7	A	8	B	9	C	10	D
11	A	12	B	13	C	14	D	15	A
16	B	17	C	18	D	19	A	20	B

Section B — Integer Answer Type

21	9	22	3
23	2	24	7
25	64	26	2
27	8	28	20
29	2	30	10

Click on any answer to jump to its detailed solution.

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