JEE Main Mathematics

Sample Paper – 4

Duration: 60 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **30 Questions** divided into two sections, modelled on the Mathematics section of **JEE Main**.
- **Section A (Q1–Q20):** Single Correct MCQs. Each correct answer carries **+4 marks**; each wrong answer carries $-$**1 mark**. Unattempted questions carry 0 marks.
- **Section B (Q21–Q30):** Integer Answer Type. Each correct answer carries **+4 marks**. There is **no negative marking**. Attempt any **5** of 10 questions.
- Syllabus level: **Class 11–12 (NCERT) — JEE Main Mathematics syllabus.**
- Personal calculators, log tables, mobile phones and electronic gadgets are strictly prohibited. Rough sheets will be provided at the exam centre.

SECTION A — Single Correct MCQs (Q1–Q20) [+4 / −1]

Q1. The range of the function $f(x) = x^2 + 1$ for $x \in \mathbb{R}$ is:

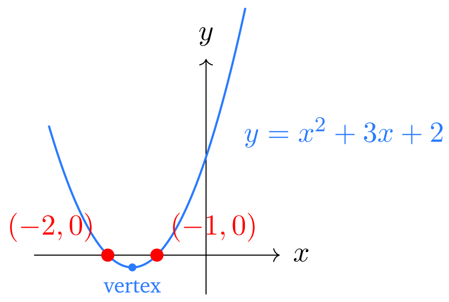
(A) $\mathbb{R}$

(B) $[0, \infty)$

(C) $(1, \infty)$

(D) $[1, \infty)$

Q2. The roots of the equation $x^2 + 3x + 2 = 0$ are:



(A) -1 and -2

(B) 1 and 2

(C) -1 and 2

(D) 1 and -2

Q3. The sum of the first 10 natural numbers is:

(A) 45

(B) 55

(C) 50

(D) 100

Q4. If $z = \cos\theta + i\sin\theta$, then $|z|$ equals:

(A) $\cos\theta$

(B) $\sin\theta$

(C) 1

(D) 2

Q5. The number of ways to arrange 5 different students in a row is:

(A) 12

(B) 20

(C) 60

(D) 120

Q6. The sum of all binomial coefficients $\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4}$ equals:

(A) 16

(B) 8

(C) 12

(D) 4

Q7. If A is a 3×3 matrix with $|A| = 5$, then $|2A|$ equals:

(A) 10

(B) 40

(C) 15

(D) 20

Q8. The cofactor of the element $a_{11} = 1$ in the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is:

(A) -4

(B) 3

(C) 4

(D) 2

Q9. The value of $\lim_{x \to 0} \frac{e^x - 1}{x}$ is:

(A) 0

(B) e

(C) ∞

(D) 1

Q10. For $f(x) = x^2$ on $[1, 4]$, the value of c in the Mean Value Theorem satisfies:

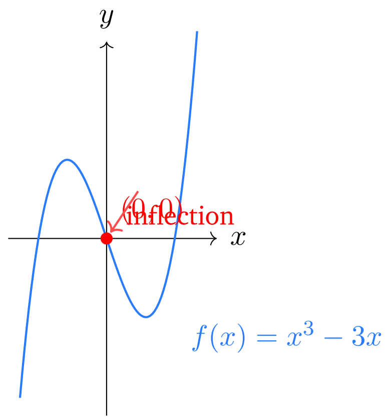
(A) $\frac{5}{2}$

(B) 3

(C) 2

(D) 4

Q11. The point of inflection of the function $f(x) = x^3 - 3x$ is:



(A) $(1, -2)$

(B) $(0, 0)$

(C) $(-1, 2)$

(D) $(2, 2)$

Q12. $\int \cos x \, dx$ equals:

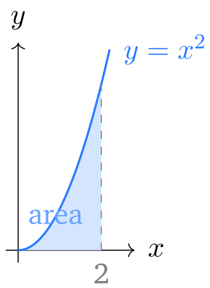
(A) $-\sin x + C$

(B) $\sec x + C$

(C) $\sin x + C$

(D) $\cos x + C$

Q13. The value of $\int_0^2 x^2 \, dx$ is:



(A) 2

(B) 4

(C) $\frac{2}{3}$

(D) $\frac{8}{3}$

Q14. The integrating factor of the differential equation $\dfrac{dy}{dx} + \dfrac{1}{x}y = x^2$ is:

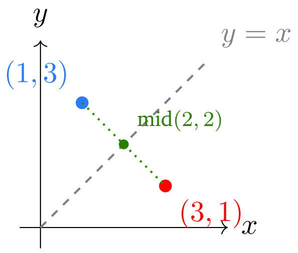
(A) x

(B) $\dfrac{1}{x}$

(C) e^x

(D) $\ln x$

Q15. The reflection of the point $(1, 3)$ in the line $y = x$ is:



(A) $(1, 3)$

(B) $(3, 1)$

(C) $(-3, -1)$

(D) $(-1, -3)$

Q16. The radical axis of the two circles $x^2 + y^2 = 4$ and $(x - 1)^2 + y^2 = 4$ is:

(A) $x = 1$

(B) $x = 2$

(C) $x = \dfrac{1}{2}$

(D) $y = \dfrac{1}{2}$

Q17. The focus of the parabola $x^2 = 4y$ is:

(A) $(0, 0)$

(B) $(1, 0)$

(C) $(-1, 0)$

(D) $(0, 1)$

Q18. The eccentricity of the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ is:

(A) $\dfrac{5}{3}$

(B) $\dfrac{3}{5}$

(C) $\dfrac{4}{3}$

(D) $\dfrac{3}{4}$

Q19. The value of $\tan(45^\circ + 30^\circ)$ is:

(A) $2 - \sqrt{3}$

(B) $2 + \sqrt{3}$

(C) $\sqrt{3} + 1$

(D) $\sqrt{3} - 1$

Q20. The principal value of $\tan^{-1}(-1)$ is:

(A) $\dfrac{\pi}{4}$

(B) $\dfrac{3\pi}{4}$

(C) $-\dfrac{\pi}{4}$

(D) $-\dfrac{3\pi}{4}$

SECTION B — Integer Answer Type (Q21–Q30)
[+4, No Negative Marking] Attempt any 5

Q21. Let $\vec{a} = (2, 3, -1)$ and $\vec{b} = (1, -1, 2)$. The value of $|\vec{a} \cdot \vec{b}|$ is ______.

Q22. The angle (in degrees) between the planes $x + y = 0$ and $x - y = 0$ is ______.

Q23. Two dice are rolled. The value of $36 \times P(\text{sum} \geq 11)$ is ______.

Q24. The variance of the data $\{1, 2, 3, 4, 5\}$ is ______.

Q25. The sum of the first 4 terms of the G.P. $1, 2, 4, 8, \ldots$ is ______.

Q26. Water drains from a cylindrical tank at the rate of 2π cm^3/s. When the radius is 2 cm, the value of $|dh/dt| \times 2$ (where h is the height of water) equals ______.

Q27. $\int_0^{\pi} \sin x \, dx$ equals ______.

Q28. The number of terms in the expansion of $(x + y + z)^3$ is ______.

Q29. The value of $|(1 + i)^8|$ is ______.

Q30. The value of $|A|$ for $A = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$ is ______.

— End of Question Paper —

Detailed Solutions

Sample Paper – 4 | Section A (Q1–Q20)

Q1.

Solution

Concept — Functions and Range: The range of a function is the set of all output values. For $f(x) = x^2 + 1$, since $x^2 \geq 0$ for all real x, the minimum of $f(x)$ is $0 + 1 = 1$, achieved at $x = 0$, and $f(x)$ can be made arbitrarily large.

Step 1 — Lower bound: Since $x^2 \geq 0$ for all $x \in \mathbb{R}$, we have $f(x) = x^2 + 1 \geq 0 + 1 = 1$.

Step 2 — Attainability and upper bound: At $x = 0$, $f(0) = 1$ (minimum achieved). As $x \to \pm\infty$, $f(x) \to +\infty$. Every value $y \geq 1$ is attained since $x = \sqrt{y-1}$ is real. So range $= [1, \infty)$.

Why other options are wrong:

- Option A: $\mathbb{R}$ includes negative values; $f(x) \geq 1$ always, so negatives are impossible.
- Option B: $[0, \infty)$ includes $y \in [0, 1)$; but $f(x) = x^2 + 1 \geq 1$, so values in $[0, 1)$ are not attained.
- Option C: $(1, \infty)$ excludes $y = 1$; but $f(0) = 1$, so 1 is in the range.

Final Answer: Range of $f(x) = x^2 + 1$ is $[1, \infty) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q1

Q2.

Solution

Concept — Quadratic Equations (Factorisation): The roots of $ax^2 + bx + c = 0$ can be found by factorising into $(x - r_1)(x - r_2) = 0$.

Step 1 — Factorise: We need two numbers whose product is $+2$ and sum is $+3$. These are 1 and 2, so:

$$x^2 + 3x + 2 = (x + 1)(x + 2) = 0$$

Step 2 — Roots: Setting each factor to zero: $x + 1 = 0 \Rightarrow x = -1$; $x + 2 = 0 \Rightarrow x = -2$.

Verification: $(-1)^2+3(-1)+2 = 1-3+2 = 0\checkmark$; $(-2)^2+3(-2)+2 = 4-6+2 = 0\checkmark$.

Why other options are wrong:

- Option B: $x = 1, 2$ give $1 + 3 + 2 = 6 \neq 0$ and $4 + 6 + 2 = 12 \neq 0$.
- Option C: $x = -1$ is correct but $x = 2$ gives $4 + 6 + 2 = 12 \neq 0$.
- Option D: $x = 1$ gives $6 \neq 0$.

Final Answer: Roots are -1 and $-2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q2

Q3.

Solution

Concept — Arithmetic Series Formula: The sum of first n natural numbers is $S_n = \dfrac{n(n+1)}{2}$.

Step 1 — Apply formula: With $n = 10$:

$$S_{10} = \frac{10 \times 11}{2} = \frac{110}{2} = 55$$

Step 2 — Verification by listing: $1+2+3+4+5+6+7+8+9+10 = (1+10) + (2+9) + (3+8) + (4+7) + (5+6) = 5 \times 11 = 55\checkmark$

Why other options are wrong:

- Option A: 45 is the sum of first 9 natural numbers, $S_9 = \frac{9\times10}{2} = 45$.
- Option C: 50 does not correspond to any standard sum formula for $n = 10$.
- Option D: 100 would be $n(n + 1)/2$ for some non-integer n; not $n = 10$.

Final Answer: Sum of first 10 natural numbers $= 55 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q3

Q4.

Solution

Concept — Modulus of a Complex Number: For $z = a + ib$, $|z| = \sqrt{a^2 + b^2}$. The modulus measures the distance from the origin in the Argand plane.

Step 1 — Identify real and imaginary parts: Here $a = \cos\theta$, $b = \sin\theta$.

Step 2 — Compute modulus:

$$|z| = \sqrt{\cos^2\theta + \sin^2\theta} = \sqrt{1} = 1$$

using the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$.

Why other options are wrong:

- Option A: $|z| = \cos\theta$ is only true for specific θ (e.g., $\theta = 0$) and can be negative.
- Option B: $|z| = \sin\theta$ is similarly not generally true and can be negative.
- Option D: $|z| = 2$ would require $\cos^2\theta + \sin^2\theta = 4$, which is impossible.

Final Answer: $|z| = 1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q4

Q5.

Solution

Concept — Permutations: The number of ways to arrange n distinct objects in a row is $n!$ (n factorial).

Step 1 — Apply factorial: For 5 different students:

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Step 2 — Reasoning: First position: 5 choices; second: 4; third: 3; fourth: 2; fifth: 1. Total $= 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Why other options are wrong:

- Option A: 12 $= 4!/2$; not a standard arrangement formula here.
- Option B: 20 $= {}^5P_2$, the number of arrangements of 2 students from 5.
- Option C: 60 $= 5!/2$; this would apply if two students were identical.

Final Answer: Number of arrangements $= 5! = 120 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q5

Q6.

Solution

Concept — Binomial Theorem (Sum of Coefficients): For $(1+x)^n$, the sum of all binomial coefficients is obtained by setting $x = 1$: $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

Step 1 — Identify n: Here $n = 4$, so the sum of coefficients $= 2^4$.

Step 2 — Compute:

$$\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 1 + 4 + 6 + 4 + 1 = 16 = 2^4$$

Why other options are wrong:

- Option B: $8 = 2^3$; this is sum for $n = 3$.
- Option C: 12 is not a power of 2 and does not equal the correct sum.
- Option D: 4 is $\binom{4}{1}$ alone; not the total sum.

Final Answer: Sum of binomial coefficients $= 2^4 = 16 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q6

Q7.

Solution

Concept — Scalar Multiple of a Determinant: For an $n \times n$ matrix A and scalar k, $|kA| = k^n|A|$.

Step 1 — Identify parameters: Here $n = 3$, $k = 2$, $|A| = 5$.

Step 2 — Apply formula:

$$|2A| = 2^3 \times |A| = 8 \times 5 = 40$$

Why other options are wrong:

- Option A: $10 = 2 \times 5$; this would be $|2A|$ for a 1×1 matrix ($k^1|A|$).
- Option C: $15 = 3 \times 5$; not related to scalar multiplication of determinants.

- Option D: 20 $= 4 \times 5 = 2^2 \times 5$; this would be for a 2×2 matrix.

Final Answer: $|2A| = 2^3 \times 5 = 40 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q7

Q8.

Solution

Concept — Cofactor: The cofactor $C_{ij} = (-1)^{i+j} M_{ij}$ where M_{ij} is the minor (determinant of the submatrix obtained by deleting row i and column j).

Step 1 — Find minor M_{11}: Delete row 1 and column 1 from $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. The remaining element is 4, so $M_{11} = 4$.

Step 2 — Compute cofactor:

$$C_{11} = (-1)^{1+1} \cdot M_{11} = (+1) \cdot 4 = 4$$

Why other options are wrong:

- Option A: -4 would be $(-1)^{1+1} \times (-4)$; but $M_{11} = +4$, not -4.
- Option B: 3 is the element a_{21}, not the cofactor of a_{11}.
- Option D: 2 is the element a_{12}, not the cofactor of a_{11}.

Final Answer: Cofactor of a_{11} is $4 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q8

Q9.

Solution

Concept — Standard Exponential Limit: The limit $\lim_{x \to 0} \dfrac{e^x - 1}{x} = 1$ is a fundamental standard result, derivable from the Taylor series of e^x.

Step 1 — Taylor expansion: $e^x = 1 + x + \dfrac{x^2}{2!} + \cdots$, so $e^x - 1 = x + \dfrac{x^2}{2} + \cdots$

Step 2 — Divide and take limit:

$$\frac{e^x - 1}{x} = 1 + \frac{x}{2} + \frac{x^2}{6} + \cdots \xrightarrow{x \to 0} 1$$

Why other options are wrong:

- Option A: 0 is incorrect; both numerator and denominator go to 0 (it is $0/0$ indeterminate), and the limit resolves to 1 via L'Hopital or Taylor.
- Option B: $e \approx 2.718$; this is not the value of this limit.
- Option C: ∞; the function actually tends to a finite value 1.

Final Answer: $\lim_{x\to 0} \dfrac{e^x - 1}{x} = 1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q9

Q10.

Solution

Concept — Mean Value Theorem (MVT): If f is continuous on $[a, b]$ and differentiable on (a, b), there exists $c \in (a, b)$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Step 1 — Compute the RHS:

$$\frac{f(4) - f(1)}{4 - 1} = \frac{16 - 1}{3} = \frac{15}{3} = 5$$

Step 2 — Solve for c: $f'(x) = 2x$, so $f'(c) = 2c = 5 \Rightarrow c = \dfrac{5}{2}$.

Verification: $c = 5/2 = 2.5 \in (1, 4)\checkmark$

Why other options are wrong:

- Option B: $c = 3$ gives $f'(3) = 6 \neq 5$.
- Option C: $c = 2$ gives $f'(2) = 4 \neq 5$.
- Option D: $c = 4$ is the endpoint, not in the open interval $(1, 4)$.

Final Answer: $c = \dfrac{5}{2} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q10

Q11.

Solution

Concept — Point of Inflection: A point of inflection occurs where $f''(x) = 0$ and f'' changes sign (concavity changes from up to down or vice versa).

Step 1 — Compute derivatives: $f(x) = x^3 - 3x$; $f'(x) = 3x^2 - 3$; $f''(x) = 6x$.

Step 2 — Set $f''(x) = 0$**:** $6x = 0 \Rightarrow x = 0$.

Step 3 — Confirm sign change and find y-value: For $x < 0$, $f''(x) < 0$ (concave down); for $x > 0$, $f''(x) > 0$ (concave up). Sign changes at $x = 0 \Rightarrow$ inflection. $f(0) = 0^3 - 3(0) = 0$. Inflection point: $(0, 0)$.

Why other options are wrong:

- Option A: $(1, -2)$ is a local minimum; $f'(1) = 0$ but $f''(1) = 6 > 0$.
- Option C: $(-1, 2)$ is a local maximum; $f'(-1) = 0$ but $f''(-1) = -6 < 0$.
- Option D: $(2, 2)$: $f(2) = 8 - 6 = 2$ but $f''(2) = 12 \neq 0$; not an inflection.

Final Answer: Point of inflection is $(0, 0) \Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q11**

Q12.

Solution

Concept — Standard Integral: $\displaystyle\int \cos x\, dx = \sin x + C$. This follows from the fact that $\dfrac{d}{dx}(\sin x) = \cos x$.

Step 1 — Recall the derivative relationship: Since $\dfrac{d}{dx}(\sin x) = \cos x$, by the fundamental theorem of calculus, $\int \cos x\, dx = \sin x + C$.

Step 2 — Verify: $\dfrac{d}{dx}(\sin x + C) = \cos x + 0 = \cos x \checkmark$

Why other options are wrong:

- Option A: $-\sin x + C$; differentiating gives $-\cos x \neq \cos x$. This is $\int (-\cos x)\, dx$.
- Option B: $\sec x + C$; $\dfrac{d}{dx}(\sec x) = \sec x \tan x \neq \cos x$.
- Option D: $\cos x + C$; $\dfrac{d}{dx}(\cos x) = -\sin x \neq \cos x$.

Final Answer: $\int \cos x\,dx = \sin x + C \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q12

Q13.

Solution

Concept — Definite Integration using Power Rule: $\displaystyle\int_a^b x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$.

Step 1 — Apply power rule:

$$\int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2$$

Step 2 — Evaluate at bounds:

$$= \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3} - 0 = \frac{8}{3}$$

Why other options are wrong:

- Option A: $2 = \frac{8}{4}$; this uses exponent 4 in the denominator instead of 3.
- Option B: $4 = 2^2$; confuses the definite integral with evaluating $f(2) = 4$.
- Option C: $\frac{2}{3} = \frac{2}{3}$; this is $\frac{1}{3}[x^2]_0^2 = \frac{1}{3}(4-0)$; wrong formula.

Final Answer: $\displaystyle\int_0^2 x^2\,dx = \frac{8}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q13

Q14.

Solution

Concept — Integrating Factor (IF) for Linear ODE: For $\dfrac{dy}{dx} + P(x)y = Q(x)$, the integrating factor is IF $= e^{\int P(x)\,dx}$.

Step 1 — Identify $P(x)$: Comparing $\dfrac{dy}{dx} + \dfrac{1}{x}y = x^2$ with standard form, $P(x) = \dfrac{1}{x}$.

Step 2 — Compute IF:

$$\text{IF} = e^{\int \frac{1}{x}\,dx} = e^{\ln|x|} = |x| = x \quad (x > 0)$$

Step 3 — Verify: Multiplying the ODE by x: $x\dfrac{dy}{dx} + y = x^3$, i.e., $\dfrac{d}{dx}(xy) = x^3 \checkmark$

Why other options are wrong:

- Option B: $\frac{1}{x} = e^{-\ln x}$; this would require $P(x) = -\frac{1}{x}$.
- Option C: e^x requires $P(x) = 1$ (constant), not $\frac{1}{x}$.
- Option D: $\ln x$; for $\text{IF} = \ln x$ we'd need $P(x) = \frac{d}{dx}\ln(\ln x) = \frac{1}{x\ln x}$.

Final Answer: Integrating Factor $= x \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q14**

Q15.

Solution

Concept — Reflection over $y = x$: The reflection of a point (a, b) in the line $y = x$ is (b, a) (coordinates are swapped).

Step 1 — Swap coordinates: Point $(1, 3)$ reflected over $y = x$ gives $(3, 1)$.

Step 2 — Verify: The midpoint of $(1, 3)$ and $(3, 1)$ is $\left(\frac{1+3}{2}, \frac{3+1}{2}\right) = (2, 2)$, which lies on $y = x\checkmark$. The segment joining them has slope $\dfrac{1-3}{3-1} = \dfrac{-2}{2} = -1$, perpendicular to $y = x$ (slope 1)$\checkmark$.

Why other options are wrong:

- Option A: $(1, 3)$ is the original point itself; no reflection occurred.
- Option C: $(-3, -1)$ would be a reflection over a different axis or rotation.
- Option D: $(-1, -3)$ would be the reflection over the origin.

Final Answer: Reflection of $(1, 3)$ over $y = x$ is $(3, 1) \Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q15**

Q16.

Solution

Concept — Radical Axis: The radical axis of two circles $S_1 = 0$ and $S_2 = 0$ is the locus of points having equal power with respect to both circles, given by $S_1 - S_2 = 0$.

Step 1 — Write in standard form:

$$S_1 : x^2 + y^2 - 4 = 0$$

$$S_2 : (x-1)^2 + y^2 - 4 = 0 \implies x^2 - 2x + 1 + y^2 - 4 = 0$$

Step 2 — Subtract S_1 from S_2:

$$(x^2 - 2x + 1 + y^2 - 4) - (x^2 + y^2 - 4) = 0$$

$$-2x + 1 = 0 \implies x = \frac{1}{2}$$

Why other options are wrong:

- Option A: $x = 1$ is the location of the centre of the second circle, not the radical axis.
- Option B: $x = 2$ does not satisfy $-2x + 1 = 0$.
- Option D: $y = \frac{1}{2}$; the radical axis is vertical (depends only on x), not horizontal.

Final Answer: Radical axis is $x = \frac{1}{2} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q16

Q17.

Solution

Concept — Focus of a Parabola: For the parabola $x^2 = 4ay$, the vertex is at the origin, it opens upward, and the focus is at $(0, a)$.

Step 1 — Identify a: Comparing $x^2 = 4y$ with $x^2 = 4ay$: $4a = 4 \Rightarrow a = 1$.

Step 2 — State focus: Focus $= (0, a) = (0, 1)$.

Why other options are wrong:

- Option A: $(0, 0)$ is the vertex, not the focus.

- Option B: $(1, 0)$ lies on the x-axis; for $x^2 = 4y$, the focus is on the y-axis.
- Option C: $(-1, 0)$ also lies on the x-axis and is incorrect.

Final Answer: Focus of $x^2 = 4y$ is $(0, 1) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q17

Q18.

Solution

Concept — Eccentricity of a Hyperbola: For $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the eccentricity is $e = \sqrt{1 + \frac{b^2}{a^2}}$.

Step 1 — Read off a^2 and b^2: From $\frac{x^2}{9} - \frac{y^2}{16} = 1$: $a^2 = 9,\ b^2 = 16$.

Step 2 — Compute eccentricity:

$$e = \sqrt{1 + \frac{16}{9}} = \sqrt{\frac{9+16}{9}} = \sqrt{\frac{25}{9}} = \frac{5}{3}$$

Why other options are wrong:

- Option B: $\frac{3}{5} < 1$; eccentricity of a hyperbola is always > 1.
- Option C: $\frac{4}{3}$; this would be $e = \frac{b}{a} = \frac{4}{3}$, which is not the correct formula.
- Option D: $\frac{3}{4} < 1$; impossible for a hyperbola.

Final Answer: Eccentricity $= \frac{5}{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q18

Q19.

Solution

Concept — Tangent Addition Formula: $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

Step 1 — Substitute values: $\tan 45^\circ = 1$, $\tan 30^\circ = \dfrac{1}{\sqrt{3}}$.

$$\tan 75^\circ = \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Step 2 — Rationalise:

$$\frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2}{3-1} = \frac{3+2\sqrt{3}+1}{2} = \frac{4+2\sqrt{3}}{2} = 2+\sqrt{3}$$

Why other options are wrong:

- Option A: $2-\sqrt{3} = \tan 15^\circ = \tan(45^\circ - 30^\circ)$; wrong sign for the addition.
- Option C: $\sqrt{3}+1 \approx 2.73$; close but $2+\sqrt{3} \approx 3.73$; option C is not the exact value.
- Option D: $\sqrt{3}-1 \approx 0.73$; this is far less than $\tan 75^\circ \approx 3.73$.

Final Answer: $\tan 75^\circ = 2+\sqrt{3} \Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q19**

Q20.

Solution

Concept — Principal Value of Inverse Trigonometric Functions: The range of $\tan^{-1}$ is $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$. The principal value of $\tan^{-1}(x)$ is the unique angle in this interval whose tangent is x.

Step 1 — Find the angle: We need $\theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ such that $\tan\theta = -1$. Since $\tan\left(-\dfrac{\pi}{4}\right) = -1$ and $-\dfrac{\pi}{4} \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$, the principal value is $-\dfrac{\pi}{4}$.

Step 2 — Verify: $\tan\left(-\dfrac{\pi}{4}\right) = -\tan\left(\dfrac{\pi}{4}\right) = -1\checkmark$

Why other options are wrong:

- Option A: $\dfrac{\pi}{4}$ gives $\tan\left(\dfrac{\pi}{4}\right) = +1 \neq -1$.
- Option B: $\dfrac{3\pi}{4}$ is outside the range $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.
- Option D: $-\dfrac{3\pi}{4}$ is outside the range $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

Final Answer: $\tan^{-1}(-1) = -\dfrac{\pi}{4} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q20

Detailed Solutions

Sample Paper – 4 | Section B (Q21–Q30)

Q21.

Solution

Concept — Dot Product of Vectors: For $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$, $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

Step 1 — Compute dot product:

$$\vec{a} \cdot \vec{b} = (2)(1) + (3)(-1) + (-1)(2) = 2 - 3 - 2 = -3$$

Step 2 — Take absolute value:

$$|\vec{a} \cdot \vec{b}| = |-3| = 3$$

Final Answer: $|\vec{a} \cdot \vec{b}| = 3 \Rightarrow \boxed{3}$

Answer: (3) Go Back to Q21

Q22.

Solution

Concept — Angle Between Two Planes: The angle θ between planes with normal vectors $\vec{n_1}$ and $\vec{n_2}$ satisfies $\cos\theta = \dfrac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}||\vec{n_2}|}$.

Step 1 — Identify normal vectors: For plane $x + y = 0$: $\vec{n_1} = (1, 1, 0)$. For plane $x - y = 0$: $\vec{n_2} = (1, -1, 0)$.

Step 2 — Compute angle:

$$\vec{n_1} \cdot \vec{n_2} = (1)(1) + (1)(-1) + (0)(0) = 1 - 1 + 0 = 0$$

$$\cos\theta = \frac{|0|}{\sqrt{2}\cdot\sqrt{2}} = 0 \ \Rightarrow\ \theta = 90^\circ$$

Final Answer: Angle between planes $= 90^\circ \Rightarrow \boxed{90}$

Answer: (90) Go Back to Q22

Q23.

Solution

Concept — Classical Probability: $P(E) = \dfrac{\text{Number of favourable outcomes}}{\text{Total outcomes}}$. For two dice, total outcomes $= 36$.

Step 1 — List favourable outcomes (sum ≥ 11):

- Sum $= 11$: $(5, 6), (6, 5)$ — 2 outcomes
- Sum $= 12$: $(6, 6)$ — 1 outcome
- Total favourable: 3

Step 2 — Compute:

$$P(\text{sum} \geq 11) = \frac{3}{36} = \frac{1}{12}$$

$$36 \times P(\text{sum} \geq 11) = 36 \times \frac{1}{12} = 3$$

Final Answer: $36 \times P(\text{sum} \geq 11) = 3 \Rightarrow \boxed{3}$

Answer: (3) Go Back to Q23

Q24.

Solution

Concept — Variance: For data $x_1, \ldots, x_n$ with mean $\bar{x}$: $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$.

Step 1 — Compute mean:

$$\bar{x} = \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$$

Step 2 — Compute squared deviations and variance:

$$(1-3)^2 + (2-3)^2 + (3-3)^2 + (4-3)^2 + (5-3)^2 = 4 + 1 + 0 + 1 + 4 = 10$$

$$\sigma^2 = \frac{10}{5} = 2$$

Final Answer: Variance $= 2 \Rightarrow \boxed{2}$

Answer: (2) Go Back to Q24

Q25.

Solution

Concept — Sum of Geometric Progression: For G.P. with first term a, common ratio r, and n terms: $S_n = \dfrac{a(r^n - 1)}{r - 1}$ (for $r \neq 1$).

Step 1 — Identify parameters: $a = 1$, $r = 2$, $n = 4$.

Step 2 — Compute sum:

$$S_4 = \frac{1 \cdot (2^4 - 1)}{2 - 1} = \frac{16 - 1}{1} = 15$$

Verification: $1 + 2 + 4 + 8 = 15\checkmark$

Final Answer: Sum of first 4 terms $= 15 \Rightarrow \boxed{15}$

Answer: (15) Go Back to Q25

Q26.

Solution

Concept — Related Rates: Volume of cylinder $V = \pi r^2 h$. Differentiating w.r.t. time: $\dfrac{dV}{dt} = \pi r^2 \dfrac{dh}{dt}$ (when r is constant).

Step 1 — Set up equation: Water drains at $\dfrac{dV}{dt} = -2\pi$ cm^3/s (negative since draining). Radius $r = 2$ cm (constant at this instant).

Step 2 — Solve for dh/dt:

$$-2\pi = \pi(2)^2 \frac{dh}{dt} = 4\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-2\pi}{4\pi} = -\frac{1}{2} \text{ cm/s}$$

Step 3 — Compute required quantity:

$$\left|\frac{dh}{dt}\right| \times 2 = \frac{1}{2} \times 2 = 1$$

Final Answer: $|dh/dt| \times 2 = 1 \Rightarrow \boxed{1}$

Answer: (1) Go Back to Q26

Q27.

Solution

Concept — Definite Integral of Sine: $\displaystyle\int \sin x\, dx = -\cos x + C$.

Step 1 — Integrate:

$$\int_0^{\pi} \sin x\, dx = \Big[-\cos x\Big]_0^{\pi}$$

Step 2 — Evaluate at bounds:

$$= -\cos\pi - (-\cos 0) = -(-1) + 1 = 1 + 1 = 2$$

Geometric interpretation: This equals the area under one arch of the sine curve, which is a known result of 2 square units.

Final Answer: $\displaystyle\int_0^{\pi} \sin x\, dx = 2 \Rightarrow \boxed{2}$

Answer: (2) Go Back to Q27

Q28.

Solution

Concept — Number of Terms in Multinomial Expansion: The number of terms in $(x_1 + x_2 + \cdots + x_r)^n$ is $\dbinom{n+r-1}{r-1}$.

Step 1 — Identify n and r: For $(x+y+z)^3$: $n = 3$, $r = 3$.

Step 2 — Apply formula:

$$\binom{n+r-1}{r-1} = \binom{3+3-1}{3-1} = \binom{5}{2} = \frac{5!}{2!\,3!} = \frac{5\times 4}{2} = 10$$

Verification: Terms are $x^3, y^3, z^3, x^2y, x^2z, y^2x, y^2z, z^2x, z^2y, xyz$ — 10 distinct monomials✓

Final Answer: Number of terms $= 10 \Rightarrow \boxed{10}$

Answer: (10) Go Back to Q28

Q29.

Solution

Concept — Modulus of Power of Complex Number: $|z^n| = |z|^n$.

Step 1 — Compute $|1+i|$:

$$|1+i| = \sqrt{1^2+1^2} = \sqrt{2}$$

Step 2 — Apply power property:

$$|(1+i)^8| = |1+i|^8 = (\sqrt{2})^8 = (2^{1/2})^8 = 2^4 = 16$$

Alternative check: $(1+i)^2 = 2i$; $(2i)^4 = 2^4 \cdot i^4 = 16 \cdot 1 = 16$✓

Final Answer: $|(1+i)^8| = 16 \Rightarrow \boxed{16}$

Answer: (16) Go Back to Q29

Q30.

Solution

Concept — Determinant of a 2×2 Matrix: For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $|A| = ad - bc$.

Step 1 — Identify elements: $a = 2,\ b = 1,\ c = -1,\ d = 3$.

Step 2 — Compute:

$$|A| = (2)(3) - (1)(-1) = 6 - (-1) = 6 + 1 = 7$$

Verification: Since $|A| = 7 \neq 0$, the matrix is invertible, which is consistent with a non-degenerate system.

Final Answer: $|A| = 7 \Rightarrow \boxed{7}$

$\boxed{\text{Answer: (7)}}$ Go Back to Q30

Answer Key — Sample Paper 4

Section A (MCQ):

Click a question number to jump to its solution									
1	D	2	A	3	B	4	C	5	D
6	A	7	B	8	C	9	D	10	A
11	B	12	C	13	D	14	A	15	B
16	C	17	D	18	A	19	B	20	C
21	3	22	90	23	3	24	2	25	15
26	1	27	2	28	10	29	16	30	7

Section B (Integer):

Click a question number to jump to its solution