JEE Main Mathematics

Sample Paper – 5

Duration: 60 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **30 Questions** divided into two sections, modelled on the Mathematics section of **JEE Main**.
- **Section A (Q1–Q20):** Single Correct MCQs. Each correct answer carries **+4 marks**; each wrong answer carries −**1 mark**. Unattempted questions carry 0 marks.
- **Section B (Q21–Q30):** Integer Answer Type. Each correct answer carries **+4 marks**. There is **no negative marking**. Attempt any **5** of 10 questions.
- Syllabus level: **Class 11–12 (NCERT) — JEE Main Mathematics syllabus.**
- Personal calculators, log tables, mobile phones and electronic gadgets are strictly prohibited. Rough sheets will be provided at the exam centre.

Section A — Single Correct Answer Type (Q1–Q20)

*Each correct answer: **+4 marks**. Wrong answer: −**1 mark**. Unattempted: 0.*

Q1. If $n(A) = 15$, $n(B) = 10$, and $n(A \cap B) = 4$, then $n(A \triangle B)$ (the number of elements in the symmetric difference of A and B) is:

(A) 17

(B) 21

(C) 25

(D) 11

Q2. For the quadratic equation $x^2 + px + q = 0$, consider the following conditions:

(I) $p < 0$ (II) $q > 0$ (III) $p^2 - 4q \geq 0$

Which of the following statements is correct regarding *both roots being real and positive*?

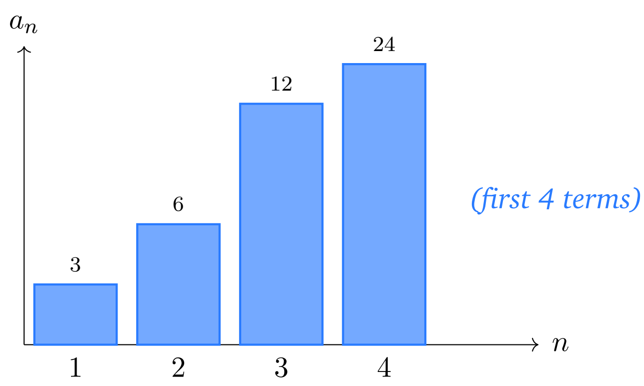
(A) Condition (I) alone is sufficient.

(B) No single condition alone is sufficient; all three together are necessary and sufficient.

(C) Condition (II) alone is sufficient.

(D) Condition (III) alone is sufficient.

Q3. A geometric progression has first term $a = 3$ and common ratio $r = 2$. The first four terms are shown in the bar chart below. The sum of the **first 6 terms** of this G.P. is:



(A) 96

(B) 100

(C) 189

(D) 192

Q4. The argument of the complex number $z = -\sqrt{3} + i$ is:

(A) $\dfrac{\pi}{6}$

(B) $\dfrac{\pi}{3}$

(C) $\dfrac{2\pi}{3}$

(D) $\dfrac{5\pi}{6}$

Q5. How many 4-digit numbers can be formed using the digits $\{1, 2, 3, 4, 5\}$ without repetition such that the number is divisible by 5?

(A) 24

(B) 60

(C) 120

(D) 48

Q6. The term independent of x in the expansion of $\left(x + \dfrac{1}{x}\right)^8$ is:

(A) 56

(B) 70

(C) 28

(D) 8

Q7. If $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \\ 2 & 1 & 0 \end{pmatrix}$, the trace of A (sum of the main diagonal elements, shown highlighted below) is:

1	2	3
0	-1	4
2	1	0

diagonal elements

(A) 3

(B) -2

(C) 0

(D) 7

Q8. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{vmatrix}$ is:

(A) -1

(B) 5

(C) -5

(D) 1

Q9. The value of $\lim_{x\to 0} \dfrac{\sqrt{1+x}-1}{x}$ is:

(A) $\dfrac{1}{2}$

(B) 1

(C) 2

(D) 0

Q10. The function $f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$ is continuous at $x = 2$. The value of k is:

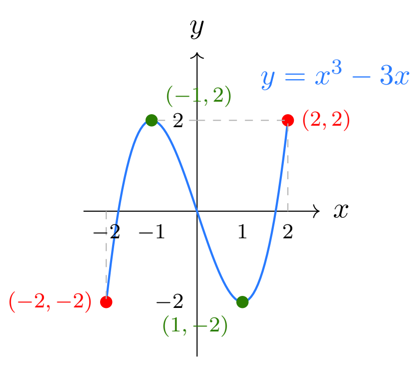
(A) 2

(B) 4

(C) 8

(D) 0

Q11. The maximum value of $f(x) = x^3 - 3x$ on the closed interval $[-2, 2]$ is:



(A) -2

(B) 0

(C) 2

(D) 4

Q12. $\int \sin^2 x\, dx$ equals:

(A) $-\cos^2 x + C$

(B) $\sin x \cdot \cos x + C$

(C) $\frac{x}{2} + \frac{\sin 2x}{4} + C$

(D) $\frac{x}{2} - \frac{\sin 2x}{4} + C$

Q13. The value of $\int_0^\pi x \sin x\, dx$ is:

(A) π

(B) 0

(C) $\frac{\pi}{2}$

(D) 2π

Q14. The degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$ is:

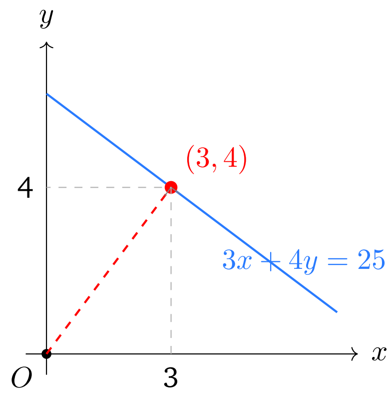
(A) 1

(B) 2

(C) 3

(D) 4

Q15. The foot of the perpendicular drawn from the origin $O(0,0)$ to the line $3x + 4y = 25$ is:



(A) $(5, 0)$

(B) $(0, 5)$

(C) $(3, 4)$

(D) $(4, 3)$

Q16. The power of the point $(5,\ 5)$ with respect to the circle $x^2 + y^2 = 9$ is:

(A) 7

(B) 25

(C) -9

(D) 41

Q17. The equation of the chord of contact drawn from the external point $(2,\ 4)$ to the parabola $y^2 = 8x$ is:

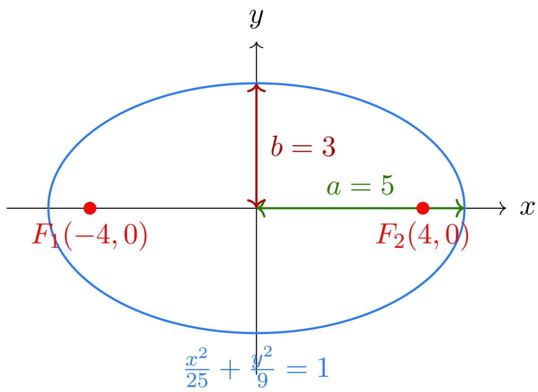
(A) $y = x + 2$

(B) $y = x - 2$

(C) $x = y + 2$

(D) $x + y = 2$

Q18. The eccentricity of the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ is:



(A) $\dfrac{3}{5}$

(B) $\dfrac{4}{5}$

(C) $\dfrac{5}{4}$

(D) $\dfrac{3}{4}$

Q19. The expression $\sin(3\theta)$ in terms of $\sin\theta$ is:

(A) $3\cos\theta - 4\cos^3\theta$

(B) $4\sin^3\theta - 3\sin\theta$

(C) $3\sin\theta - 4\sin^3\theta$

(D) $4\cos^3\theta - 3\cos\theta$

Q20. The domain of the function $f(x) = \sin^{-1}(2x - 1)$ is:

(A) $[-1,\ 1]$

(B) $\left[-\dfrac{1}{2},\ \dfrac{1}{2}\right]$

(C) $(0,\ 1)$

(D) $[0,\ 1]$

Section B — Integer Answer Type (Q21–Q30)

Each correct answer: ***+4 marks****. No negative marking. Attempt any* ***5*** *of 10 questions. Write the integer answer in the box provided.*

Q21. Vectors $\vec{a}$ and $\vec{b}$ satisfy $|\vec{a}| = 3$, $|\vec{b}| = 4$, and $|\vec{a} + \vec{b}| = 5$. Find $|\vec{a} - \vec{b}|$. ______.

Q22. Find the distance between the parallel planes $x + 2y - 2z = 3$ and $x + 2y - 2z = -6$. ______.

Q23. A bag contains 5 red and 3 blue balls. Two balls are drawn at random without replacement. If P is the probability that both balls are red, find the value of $14P$. ______.

Q24. The mean of 10 observations is 5. One observation with value 3 is replaced by 13. Find the value of $10 \times$ (new mean). ______.

Q25. The sum of the first n terms of an A.P. is given by $S_n = 3n^2 + 2n$. Find the 5th term of the A.P. ______.

Q26. A ladder 10 m long leans against a vertical wall. The foot of the ladder slides away from the wall at 2 m/s. When the foot is 6 m from the wall, the rate at which the top slides down the wall (in m/s) is $\frac{k}{4}$, where k is a positive integer. Find k. ______.

Q27. The value of $\int_0^2 (3x^2 + 2)\, dx$ is ______.

Q28. In the expansion of $(1 + x)^n$, the coefficient of x^3 is 84. Find the value of n. ______.

Q29. If $z = (1 + i)^8$, find $|z|$. ______.

Q30. If $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$, find $\det(AB)$. ______.

Detailed Solutions

Q1.

Solution

Concept: Symmetric difference of two sets: $A \triangle B = (A \setminus B) \cup (B \setminus A)$.

Step 1: Recall the formula for symmetric difference.

$$n(A \triangle B) = n(A) + n(B) - 2\,n(A \cap B)$$

Step 2: Substitute the given values.

$$n(A \triangle B) = 15 + 10 - 2 \times 4 = 25 - 8 = 17$$

Step 3: Intuitive check.

Elements only in A: $15 - 4 = 11$. Elements only in B: $10 - 4 = 6$. So $n(A \triangle B) = 11 + 6 = 17$. ✓

Why other options are wrong:

- (B) 21: Results from $n(A) + n(B) - n(A \cap B) = 21$; that is $n(A \cup B)$, not $n(A \triangle B)$.
- (C) 25: Ignores the intersection entirely: $n(A) + n(B) = 25$.
- (D) 11: This counts only the elements in A but not in B.

Answer: (A) Go Back to Q1

Q2.

Solution

Concept: For $x^2 + px + q = 0$ to have both roots real and positive, three conditions must all hold simultaneously.

Step 1: Recall Vieta's formulas.

Let α, β be the roots. Then $\alpha + \beta = -p$ and $\alpha\beta = q$.

Step 2: Analyse each condition.

- For both roots to be positive, their sum $\alpha + \beta > 0 \Rightarrow -p > 0 \Rightarrow p < 0$. **(I) is necessary.**
- Their product $\alpha\beta > 0 \Rightarrow q > 0$. **(II) is necessary.**

- For real roots, the discriminant $p^2 - 4q \geq 0$. **(III) is necessary.**

Step 3: Show no single condition suffices.

Counter-examples:

- $p = -1, q = 1$: $p < 0$ and $q > 0$, but discriminant $= 1 - 4 < 0$ (complex roots). So (I) and (II) alone are not sufficient.
- $p = 2, q = -3$: discriminant $= 4 + 12 > 0$, but roots are 1 and -3 (not both positive). So (III) alone is not sufficient.

All three conditions (I), (II), and (III) together are necessary and sufficient.

Answer: (B) Go Back to Q2

Q3.

Solution

Concept: Sum of first n terms of a G.P.: $S_n = a\dfrac{r^n - 1}{r - 1}$ for $r \neq 1$.

Step 1: Identify the parameters.

$a = 3$, $r = 2$, $n = 6$.

Step 2: Apply the sum formula.

$$S_6 = 3 \cdot \frac{2^6 - 1}{2 - 1} = 3 \cdot \frac{64 - 1}{1} = 3 \times 63 = 189$$

Step 3: Verify by listing terms.

$$3,\ 6,\ 12,\ 24,\ 48,\ 96$$

$$S_6 = 3 + 6 + 12 + 24 + 48 + 96 = 189\checkmark$$

Why other options are wrong:

- (A) 96: This is only the 6th term, not the sum.
- (B) 100: No meaningful derivation from this G.P.
- (D) 192: Results from $3 \times 2^6 = 192$, computing $3r^6$ instead of $a(r^6 - 1)/(r - 1)$.

Answer: (C) Go Back to Q3

Q4.

Solution

Concept: Argument of a complex number in the second quadrant.

Step 1: Identify the quadrant.

$z = -\sqrt{3} + i$ has $\text{Re}(z) = -\sqrt{3} < 0$ and $\text{Im}(z) = 1 > 0$, so z lies in the **second quadrant**.

Step 2: Compute the reference angle.

$$\alpha = \tan^{-1}\left(\frac{|\text{Im}(z)|}{|\text{Re}(z)|}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

Step 3: Find the argument in the second quadrant.

$$\arg(z) = \pi - \alpha = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Step 4: Verify.

$|z| = \sqrt{3+1} = 2$. In polar form: $z = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = -\sqrt{3} + i$. ✓

Why other options are wrong:

- (A) $\pi/6$: This is the reference angle only, not the argument in the second quadrant.
- (B) $\pi/3$: Argument of $z = 1 + i\sqrt{3}$ (first quadrant).
- (C) $2\pi/3$: Argument of $z = -1 + i\sqrt{3}$, where $|\text{Re}| = 1$ and $|\text{Im}| = \sqrt{3}$.

$\boxed{\text{Answer: (D)}}$ Go Back to Q4

Q5.

Solution

Concept: Counting 4-digit numbers divisible by 5, without digit repetition.

Step 1: Determine the units digit.

A number is divisible by 5 if and only if its units digit is 0 or 5. Since the available digits are $\{1, 2, 3, 4, 5\}$, the units digit **must be 5**. (There is only 1 choice.)

Step 2: Fill the remaining 3 positions.

The remaining 3 digits (thousands, hundreds, tens) are chosen from $\{1, 2, 3, 4\}$

without repetition:

$$4 \times 3 \times 2 = 24 \text{ ways}$$

Step 3: Total count.

$$\text{Total} = 24 \times 1 = 24$$

Why other options are wrong:

- (B) 60: This is ${}^5P_3 = 60$, the number of 3-digit arrangements from 5 digits; ignores the constraint.
- (C) 120: This is $5!$, the total permutations of all 5 digits.
- (D) 48: Counts as if there were 2 valid units digits.

Answer: (A) Go Back to Q5

Q6.

Solution

Concept: General term of $\left(x + \frac{1}{x}\right)^8$ and finding the term independent of x.

Step 1: Write the general term.

$$T_{r+1} = \binom{8}{r} x^{8-r} \cdot \left(\frac{1}{x}\right)^r = \binom{8}{r} x^{8-2r}$$

Step 2: Set the power of x to zero.

$$8 - 2r = 0 \implies r = 4$$

Step 3: Compute the term.

$$T_5 = \binom{8}{4} = \frac{8!}{4!\,4!} = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = \frac{1680}{24} = 70$$

Why other options are wrong:

- (A) 56: This is $\binom{8}{3} = \binom{8}{5} = 56$, the coefficient when $r = 3$ or $r = 5$.
- (C) 28: This is $\binom{8}{2} = \binom{8}{6} = 28$.
- (D) 8: This is $\binom{8}{1} = 8$, the coefficient of the second term.

Answer: (B) Go Back to Q6

Q7.

Solution

Concept: The trace of a square matrix is the sum of its main diagonal entries.

Step 1: Identify the main diagonal entries.

For $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \\ 2 & 1 & 0 \end{pmatrix}$, the main diagonal entries are $a_{11} = 1$, $a_{22} = -1$, $a_{33} = 0$.

Step 2: Compute the trace.

$$\text{tr}(A) = a_{11} + a_{22} + a_{33} = 1 + (-1) + 0 = 0$$

Why other options are wrong:

- (A) 3: This is only $a_{11} + a_{33} + a_{12} = 1 + 0 + 2$; reads wrong entries.
- (B) -2: Computes $a_{11} + a_{22} = 1 + (-1) +$ error, or reads $a_{22} = -2$ wrongly.
- (D) 7: Adds all entries in the first column or row, not the diagonal.

Answer: (C) Go Back to Q7

Q8.

Solution

Concept: Expansion of a 3×3 determinant along the first row.

Step 1: Expand along the first row.

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{vmatrix} = 1 \cdot \begin{vmatrix} 1 & 4 \\ 6 & 0 \end{vmatrix} - 2 \cdot \begin{vmatrix} 0 & 4 \\ 5 & 0 \end{vmatrix} + 3 \cdot \begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix}$$

Step 2: Evaluate each 2×2 determinant.

$$\begin{vmatrix} 1 & 4 \\ 6 & 0 \end{vmatrix} = (1)(0) - (4)(6) = 0 - 24 = -24$$

$$\begin{vmatrix} 0 & 4 \\ 5 & 0 \end{vmatrix} = (0)(0) - (4)(5) = 0 - 20 = -20$$

$$\begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix} = (0)(6) - (1)(5) = 0 - 5 = -5$$

Step 3: Combine.

$$= 1(-24) - 2(-20) + 3(-5) = -24 + 40 - 15 = 1$$

Why other options are wrong:

- (A) -1: Sign error in the expansion (e.g., wrong sign for the middle cofactor).
- (B) 5: Partial computation, stopping after the first two cofactors: $-24+40 = 16 \neq 5$.
- (C) -5: Only evaluates the third cofactor term.

Answer: (D) **Go Back to Q8**

Q9.

Solution

Concept: Evaluating a $\frac{0}{0}$ form limit by rationalisation.

Step 1: Rationalise the numerator.

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x} = \lim_{x \to 0} \frac{(\sqrt{1+x} - 1)(\sqrt{1+x} + 1)}{x(\sqrt{1+x} + 1)}$$

Step 2: Simplify the numerator using $(a-b)(a+b) = a^2 - b^2$**.**

$$= \lim_{x \to 0} \frac{(1+x) - 1}{x(\sqrt{1+x} + 1)} = \lim_{x \to 0} \frac{x}{x(\sqrt{1+x} + 1)} = \lim_{x \to 0} \frac{1}{\sqrt{1+x} + 1}$$

Step 3: Substitute $x = 0$**.**

$$= \frac{1}{\sqrt{1+0} + 1} = \frac{1}{1+1} = \frac{1}{2}$$

Alternative (L'Hôpital's Rule):

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x} \xrightarrow{L'H} \lim_{x \to 0} \frac{\frac{1}{2\sqrt{1+x}}}{1} = \frac{1}{2} \checkmark$$

Why other options are wrong:

- (B) 1: Forgets to add 1 in the denominator after rationalisation.
- (C) 2: Takes the reciprocal of the correct answer.
- (D) 0: Direct substitution gives $0/0$; the limit is not 0.

Answer: (A) Go Back to Q9

Q10.

Solution

Concept: Continuity at a point requires the limit to equal the function value.

Step 1: Find the limit as $x \to 2$ (for $x \neq 2$).

For $x \neq 2$:

$$\frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} = x+2$$

Therefore:

$$\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2}(x+2) = 4$$

Step 2: Set $f(2) = k$ equal to the limit.

$$k = 4$$

Step 3: Verify. $f(2) = 4$, and for $x \neq 2$, $f(x) = x+2 \to 4$ as $x \to 2$. ✓

Why other options are wrong:

- (A) 2: This is the value of x at which we check continuity, not the limit.
- (C) 8: Results from $(2^2-4)/(2-2)$ evaluated naively as $0/0 = 8$ (nonsensical).
- (D) 0: Direct naive substitution gives $0/0$ and mistakenly concludes 0.

Answer: (B) Go Back to Q10

Q11.

Solution

Concept: Maximum of a continuous function on a closed interval occurs at critical points or endpoints.

Step 1: Find critical points.

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 0 \implies x = \pm 1$$

Both $x = 1$ and $x = -1$ lie in $[-2, 2]$.

Step 2: Evaluate f at all critical points and endpoints.

$$\begin{aligned} f(-2) &= (-2)^3 - 3(-2) = -8 + 6 = -2 \\ f(-1) &= (-1)^3 - 3(-1) = -1 + 3 = 2 \\ f(1) &= (1)^3 - 3(1) = 1 - 3 = -2 \\ f(2) &= (2)^3 - 3(2) = 8 - 6 = 2 \end{aligned}$$

Step 3: Identify the maximum.

The largest value is **2**, attained at $x = -1$ and $x = 2$.

Why other options are wrong:

- (A) -2: This is the minimum value, not the maximum.
- (B) 0: $f(0) = 0$, which is neither the minimum nor the maximum.
- (D) 4: No value of $x \in [-2, 2]$ gives $f(x) = 4$.

Answer: (C) Go Back to Q11

Q12.

Solution

Concept: Use the double-angle identity to integrate $\sin^2 x$.

Step 1: Apply the identity $\sin^2 x = \dfrac{1 - \cos 2x}{2}$.

$$\int \sin^2 x \, dx = \int \frac{1 - \cos 2x}{2} \, dx$$

Step 2: Integrate term by term.

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos 2x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

Step 3: Verify by differentiation.

$$\frac{d}{dx}\left(\frac{x}{2} - \frac{\sin 2x}{4}\right) = \frac{1}{2} - \frac{2\cos 2x}{4} = \frac{1}{2} - \frac{\cos 2x}{2} = \frac{1-\cos 2x}{2} = \sin^2 x \checkmark$$

Why other options are wrong:

- (A) $-\cos^2 x + C$: Derivative is $2\cos x \sin x = \sin 2x \neq \sin^2 x$.
- (B) $\sin x \cos x + C$: Derivative is $\cos^2 x - \sin^2 x = \cos 2x \neq \sin^2 x$.
- (C) $\frac{x}{2} + \frac{\sin 2x}{4} + C$: Uses $+$ instead of $-$; derivative gives $\cos^2 x$, not $\sin^2 x$.

Answer: (D) Go Back to Q12

Q13.

Solution

Concept: King's property of definite integrals: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

Step 1: Let $I = \int_0^\pi x \sin x\,dx$.

Step 2: Apply King's property with $a = \pi$.

Replace x by $\pi - x$:

$$I = \int_0^\pi (\pi - x)\sin(\pi - x)\,dx = \int_0^\pi (\pi - x)\sin x\,dx$$

(using $\sin(\pi - x) = \sin x$)

Step 3: Add the two expressions for I.

$$2I = \int_0^\pi x\sin x\,dx + \int_0^\pi (\pi - x)\sin x\,dx = \pi\int_0^\pi \sin x\,dx$$

$$2I = \pi\Big[-\cos x\Big]_0^\pi = \pi\big[-\cos\pi + \cos 0\big] = \pi(1+1) = 2\pi$$

Step 4: Solve for I.

$$I = \pi$$

Why other options are wrong:

- (B) 0: Incorrect; $x \sin x \geq 0$ on $[0, \pi]$, so the integral cannot be 0.
- (C) $\pi/2$: Off by a factor of 2; results from computing $2I = \pi$ instead of $2I = 2\pi$.
- (D) 2π: This is $2I$, not I.

Answer: (A) Go Back to Q13

Q14.

Solution

Concept: The degree of a differential equation is the power of the highest-order derivative *after* clearing radicals and fractions.

Step 1: Write the given equation.

$$\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$$

Step 2: Eliminate the radical by squaring both sides.

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^3$$

Step 3: Identify the degree.

The highest-order derivative is $\frac{d^2y}{dx^2}$ (order 2), and it appears to the power **2** in the cleared equation.

$\therefore$ Degree $=$ **2**.

Why other options are wrong:

- (A) 1: The degree of d^2y/dx^2 is 1 only *before* squaring; after clearing the radical it becomes 2.
- (C) 3: This is the power of dy/dx, which relates to the *first-order* derivative, not to degree.
- (D) 4: No derivative appears to the fourth power in the cleared equation.

Answer: (B) Go Back to Q14

Q15.

Solution

Concept: Foot of the perpendicular from a point (x_1, y_1) to the line $ax+by+c=0$.

Step 1: Write the line in standard form.

$3x+4y-25=0$, so $a=3$, $b=4$, $c=-25$. Point: $(x_1, y_1)=(0,0)$.

Step 2: Apply the foot-of-perpendicular formula.

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=-\frac{ax_1+by_1+c}{a^2+b^2}$$

$$\frac{x}{3}=\frac{y}{4}=-\frac{3(0)+4(0)-25}{9+16}=-\frac{-25}{25}=1$$

Step 3: Solve for the foot.

$$x=3\times 1=3, \qquad y=4\times 1=4$$

Foot of perpendicular $=(3,4)$.

Verification: $3(3)+4(4)=9+16=25$. ✓The direction from $(0,0)$ to $(3,4)$ is $(3,4)$, which is parallel to the normal $(3,4)$ of the line. ✓

Why other options are wrong:

- (A) $(5,0)$: This lies on the line $(3\times 5+4\times 0=15\neq 25)$ — actually not on the line; it's a distractor.
- (B) $(0,5)$: $3(0)+4(5)=20\neq 25$; does not lie on the line.
- (D) $(4,3)$: $3(4)+4(3)=12+12=24\neq 25$; not on the line.

Answer: (C) Go Back to Q15

Q16.

Solution

Concept: Power of a point (h,k) with respect to a circle $x^2+y^2=r^2$ is $h^2+k^2-r^2$.

Step 1: Identify the values.

Circle: $x^2+y^2=9$, so $r^2=9$. Point: $(h,k)=(5,5)$.

Step 2: Compute the power.

$$\text{Power}=h^2+k^2-r^2=5^2+5^2-9=25+25-9=41$$

Since the power is positive $(41 > 0)$, the point $(5, 5)$ lies **outside** the circle.

Why other options are wrong:

- (A) 7: Computes $5 + 5 - 3 = 7$; uses magnitudes instead of squares.
- (B) 25: Computes only $h^2 + k^2 - 2r^2$ or forgets to subtract; $h^2 + k^2 = 50 \neq 25$.
- (C) -9: This is $-r^2$, forgetting to add $h^2 + k^2$.

Answer: (D) Go Back to Q16

Q17.

Solution

Concept: Chord of contact from an external point (h, k) to the parabola $y^2 = 4ax$ is $ky = 2a(x + h)$.

Step 1: Identify the parabola parameters.

$y^2 = 8x \Rightarrow 4a = 8 \Rightarrow a = 2$. External point: $(h, k) = (2, 4)$.

Step 2: Write the equation of the chord of contact.

$$ky = 2a(x + h) \implies 4y = 2(2)(x + 2) = 4(x + 2)$$

Step 3: Simplify.

$$4y = 4x + 8 \implies y = x + 2$$

Why other options are wrong:

- (B) $y = x - 2$: Incorrect sign for h; uses $(x - h)$ instead of $(x + h)$.
- (C) $x = y + 2$: Swaps x and y in the equation.
- (D) $x + y = 2$: An arbitrary line through $(0, 2)$ and $(2, 0)$; unrelated.

Answer: (A) Go Back to Q17

Q18.

Solution

Concept: Eccentricity of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a>b>0)$: $e=\frac{c}{a}$ where $c^2=a^2-b^2$.

Step 1: Identify a^2 and b^2.

$$\frac{x^2}{25}+\frac{y^2}{9}=1 \implies a^2=25, \quad b^2=9 \implies a=5, \quad b=3$$

Step 2: Compute c.

$$c^2=a^2-b^2=25-9=16 \implies c=4$$

Step 3: Compute eccentricity.

$$e=\frac{c}{a}=\frac{4}{5}$$

Foci are at $(\pm 4, 0)$, as shown in the figure.

Why other options are wrong:

- (A) $3/5$: Uses $b/a=3/5$ instead of c/a.
- (C) $5/4$: This is a/c, the reciprocal of the eccentricity (always >1 for an ellipse, so invalid).
- (D) $3/4$: Uses $b/c=3/4$, an incorrect ratio.

$\boxed{\text{Answer: (B)}}$ Go Back to Q18

Q19.

Solution

Concept: Triple angle formula for sine: $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$.

Derivation:

$$\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta \sin\theta$$
$$= (2\sin\theta\cos\theta)\cos\theta + (1-2\sin^2\theta)\sin\theta$$
$$= 2\sin\theta\cos^2\theta + \sin\theta - 2\sin^3\theta$$
$$= 2\sin\theta(1-\sin^2\theta) + \sin\theta - 2\sin^3\theta$$
$$= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$$

$$= 3\sin\theta - 4\sin^3\theta$$

Verification with $\theta = 30°$:

$$\sin 90° = 1; \quad 3\sin 30° - 4\sin^3 30° = 3\cdot\frac{1}{2} - 4\cdot\frac{1}{8} = \frac{3}{2} - \frac{1}{2} = 1\checkmark$$

Why other options are wrong:

- (A) $3\cos\theta - 4\cos^3\theta$: This equals $-\cos 3\theta$, the formula with cosine and wrong sign.
- (B) $4\sin^3\theta - 3\sin\theta$: This is $-\sin 3\theta$, the negative of the correct formula.
- (D) $4\cos^3\theta - 3\cos\theta$: This equals $\cos 3\theta$, the triple angle formula for cosine.

Answer: (C) Go Back to Q19

Q20.

Solution

Concept: Domain of $\sin^{-1}(u)$ requires $-1 \le u \le 1$.

Step 1: Set up the inequality for the argument.

$$-1 \le 2x - 1 \le 1$$

Step 2: Solve for x.

$$-1 + 1 \le 2x \le 1 + 1$$

$$0 \le 2x \le 2$$

$$0 \le x \le 1$$

Step 3: Write the domain.

$$\text{Domain} = [0,\ 1]$$

Verification: At $x = 0$: $\sin^{-1}(2(0) - 1) = \sin^{-1}(-1) = -\pi/2$ (defined). At $x = 1$: $\sin^{-1}(2(1) - 1) = \sin^{-1}(1) = \pi/2$ (defined). At $x = 1.1$: $\sin^{-1}(1.2)$ is undefined. ✓

Why other options are wrong:

- (A) $[-1, 1]$: This is the domain of $\sin^{-1}(x)$ itself, not of $\sin^{-1}(2x - 1)$.
- (B) $[-1/2, 1/2]$: Solves $-1 \le 2x \le 1$ (forgetting the -1 shift).
- (C) $(0, 1)$: Correct values but excludes the endpoints, which are valid for

$\sin^{-1}$.

Answer: (D) Go Back to Q20

Section B — Integer Answer Type Solutions

Q21.

Solution

Concept: Dot product from the magnitude of the sum of vectors.

Step 1: Expand $|\vec{a}+\vec{b}|^2$.

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2\,\vec{a}\cdot\vec{b} + |\vec{b}|^2$$

$$25 = 9 + 2\,\vec{a}\cdot\vec{b} + 16 = 25 + 2\,\vec{a}\cdot\vec{b}$$
$$\vec{a}\cdot\vec{b} = 0$$

So $\vec{a}$ and $\vec{b}$ are perpendicular.

Step 2: Compute $|\vec{a}-\vec{b}|^2$.

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 - 2\,\vec{a}\cdot\vec{b} + |\vec{b}|^2 = 9 - 0 + 16 = 25$$

$$|\vec{a}-\vec{b}| = 5$$

Answer: (5) Go Back to Q21

Q22.

Solution

Concept: Distance between parallel planes $ax+by+cz=d_1$ and $ax+by+cz=d_2$.

$$\text{Distance} = \frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$$

Step 1: Identify parameters.

Planes: $x+2y-2z=3$ and $x+2y-2z=-6$. So $d_1=3$, $d_2=-6$.

Step 2: Compute the distance.

$$\text{Distance} = \frac{|3-(-6)|}{\sqrt{1^2+2^2+(-2)^2}} = \frac{9}{\sqrt{1+4+4}} = \frac{9}{\sqrt{9}} = \frac{9}{3} = 3$$

Answer: (3) Go Back to Q22

Q23.

Solution

Concept: Probability of drawing 2 red balls without replacement.

Step 1: Total ways to choose 2 balls from 8.

$$\binom{8}{2} = \frac{8 \times 7}{2} = 28$$

Step 2: Favourable ways (both red, from 5 red balls).

$$\binom{5}{2} = \frac{5 \times 4}{2} = 10$$

Step 3: Compute P and then $14P$.

$$P = \frac{10}{28} = \frac{5}{14}$$

$$14P = 14 \times \frac{5}{14} = 5$$

Answer: (5) Go Back to Q23

Q24.

Solution

Concept: Effect of replacing one observation on the sum and mean.

Step 1: Find the original sum.

$$\bar{x} = 5, \quad n = 10 \implies \text{Sum} = 5 \times 10 = 50$$

Step 2: Adjust the sum for the replacement.

$$\text{New Sum} = 50 - 3 + 13 = 60$$

Step 3: Compute $10 \times$ New Mean.

$$\text{New Mean} = \frac{60}{10} = 6$$

$$10 \times \text{New Mean} = 60$$

Answer: (60) Go Back to Q24

Q25.

Solution

Concept: nth term of an A.P. from the sum formula: $a_n = S_n - S_{n-1}$.

Step 1: Find S_5 and S_4.

$$S_5 = 3(5)^2 + 2(5) = 75 + 10 = 85$$

$$S_4 = 3(4)^2 + 2(4) = 48 + 8 = 56$$

Step 2: Compute the 5th term.

$$a_5 = S_5 - S_4 = 85 - 56 = 29$$

Verification using the general formula:

$$a_n = S_n - S_{n-1} = 3n^2 + 2n - [3(n-1)^2 + 2(n-1)]$$

$$= 3n^2 + 2n - 3n^2 + 6n - 3 - 2n + 2 = 6n - 1$$

$$a_5 = 6(5) - 1 = 29\checkmark$$

Answer: (29) Go Back to Q25

Q26.

Solution

Concept: Related rates using the Pythagorean theorem.

Step 1: Set up the relation.

Let x = distance of foot from wall, y = height of top on wall.

$$x^2 + y^2 = 10^2 = 100$$

Step 2: Differentiate with respect to time t.

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \implies \frac{dy}{dt} = -\frac{x}{y} \cdot \frac{dx}{dt}$$

Step 3: Substitute $x = 6$, $dx/dt = 2$.

When $x = 6$: $y = \sqrt{100 - 36} = \sqrt{64} = 8$.

$$\frac{dy}{dt} = -\frac{6}{8} \times 2 = -\frac{3}{2} \text{ m/s}$$

The top slides *down* at $\frac{3}{2}$ m/s. This rate equals $\frac{k}{4}$:

$$\frac{k}{4} = \frac{3}{2} \implies k = 6$$

Answer: (6) Go Back to Q26

Q27.

Solution

Concept: Definite integration using the power rule and linearity.

Step 1: Integrate term by term.

$$\int_0^2 (3x^2 + 2)\, dx = \left[x^3 + 2x\right]_0^2$$

Step 2: Evaluate at the limits.

$$= \left(2^3 + 2(2)\right) - \left(0^3 + 2(0)\right) = (8 + 4) - 0 = 12$$

Answer: (12) Go Back to Q27

Q28.

Solution

Concept: Finding n given a binomial coefficient.

Step 1: Write the coefficient of x^3.

$$\binom{n}{3} = \frac{n(n-1)(n-2)}{6} = 84$$

Step 2: Solve.

$$n(n-1)(n-2) = 504$$

Step 3: Test integer values.

$$n = 9 : \quad 9 \times 8 \times 7 = 504\checkmark$$

Verification: $\binom{9}{3} = \frac{9!}{3!\,6!} = \frac{9 \times 8 \times 7}{6} = \frac{504}{6} = 84.$ ✓

Answer: (9) Go Back to Q28

Q29.

Solution

Concept: Modulus of a power of a complex number: $|z^n| = |z|^n$.

Step 1: Find $|1+i|$**.**

$$|1+i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Step 2: Apply the power rule.

$$|z| = |(1+i)^8| = |1+i|^8 = (\sqrt{2})^8 = (2^{1/2})^8 = 2^4 = 16$$

Verification using De Moivre's theorem:

$$1 + i = \sqrt{2}\, e^{i\pi/4} \implies (1+i)^8 = (\sqrt{2})^8 e^{i \cdot 8\pi/4} = 16\, e^{i2\pi} = 16 \checkmark$$

Answer: (16) Go Back to Q29

Q30.

Solution

Concept: $\det(AB) = \det(A) \cdot \det(B)$.

Step 1: Compute $\det(A)$**.**

$$\det(A) = \begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix} = (2)(4) - (3)(1) = 8 - 3 = 5$$

Step 2: Compute $\det(B)$**.**

$$\det(B) = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = (1)(1) - (0)(2) = 1$$

Step 3: Compute $\det(AB)$**.**

$$\det(AB) = \det(A) \cdot \det(B) = 5 \times 1 = 5$$

Verification by direct multiplication:

$$AB = \begin{pmatrix} 2+6 & 0+3 \\ 1+8 & 0+4 \end{pmatrix} = \begin{pmatrix} 8 & 3 \\ 9 & 4 \end{pmatrix}, \quad \det(AB) = 32 - 27 = 5\checkmark$$

$\boxed{\textbf{Answer: (5)}}$ **Go Back to Q30**

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	A
6	B	7	C	8	D	9	A	10	B
11	C	12	D	13	A	14	B	15	C
16	D	17	A	18	B	19	C	20	D
21	5	22	3	23	5	24	60	25	29
26	6	27	12	28	9	29	16	30	5