JEE Main Mathematics

Sample Paper – 6

Duration: 60 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **30 Questions** divided into two sections, modelled on the Mathematics section of **JEE Main**.
- **Section A (Q1–Q20):** Single Correct MCQs. Each correct answer carries **+4 marks**; each wrong answer carries $-$**1 mark**. Unattempted questions carry 0 marks.
- **Section B (Q21–Q30):** Integer Answer Type. Each correct answer carries **+4 marks**. There is **no negative marking**. Attempt any **5** of 10 questions.
- Syllabus level: **Class 11–12 (NCERT) — JEE Main Mathematics syllabus.**
- Personal calculators, log tables, mobile phones and electronic gadgets are strictly prohibited. Rough sheets will be provided at the exam centre.

Section A — Single Correct Answer Type (Q1–Q20)

*Each correct answer: **+4 marks**. Wrong answer: $-$**1 mark**. Unattempted: 0.*

Q1. Which of the following functions is a **bijection** (one-one and onto) from $\mathbb{R}$ to $\mathbb{R}$?

(A) $f(x) = x^2$

(B) $f(x) = 2x + 3$

(C) $f(x) = \sin x$

(D) $f(x) = |x|$

Q2. The equations $x^2 - 3x + 2 = 0$ and $x^2 - px + 6 = 0$ have a common root. If the common root is $x = 2$, then the value of p is:

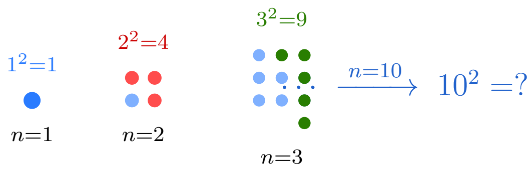
(A) 3

(B) 4

(C) 5

(D) 7

Q3. The sum of the first n odd natural numbers equals n^2. The pattern below illustrates this for small values of n. What is the sum of the first **10** odd natural numbers?



(A) 90

(B) 95

(C) 99

(D) 100

Q4. If ω is a primitive cube root of unity (i.e. $\omega \neq 1$ and $\omega^3 = 1$), then the value of $1 + \omega + \omega^2$ is:

(A) 0

(B) 1

(C) -1

(D) 3

Q5. The number of diagonals of a convex polygon with **10 sides** is:

(A) 45

(B) 35

(C) 40

(D) 30

Q6. In the expansion of $(1+x)^n$, the coefficients of x^2 and x^3 are in the ratio $1:4$. The value of n is:

(A) 10

(B) 12

(C) 14

(D) 16

Q7. A square matrix A is called **skew-symmetric** if $A^T = -A$. For any 3×3 skew-symmetric matrix, the elements on the **main diagonal** must all be:

(A) 1

(B) -1

(C) any real number

(D) 0

Q8. If a 3×3 determinant has **two identical rows**, its value is:

$$\det \begin{vmatrix} a & b & c \\ \mathrm{p} & \mathrm{q} & \mathrm{r} \\ \mathrm{p} & \mathrm{q} & \mathrm{r} \end{vmatrix} \ \text{identical rows}$$

(A) 0

(B) 1

(C) -1

(D) 2

Q9. The value of $\lim\limits_{x\to\infty} \dfrac{3x^2+5x}{2x^2-7}$ is:

(A) 0

(B) $\dfrac{3}{2}$

(C) $\dfrac{5}{7}$

(D) ∞

Q10. The function $f(x) = \frac{1}{x}$ at $x = 0$ has which type of discontinuity?

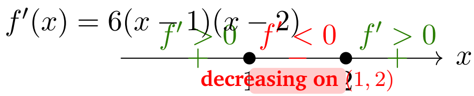
(A) Removable discontinuity

(B) Jump discontinuity

(C) Infinite discontinuity

(D) Oscillatory discontinuity

Q11. The function $f(x) = 2x^3 - 9x^2 + 12x + 4$ is **strictly decreasing** on the interval:



(A) $(0, 1)$

(B) $(2, \infty)$

(C) $(0, 2)$

(D) $(1, 2)$

Q12. $\int e^{2x}\, dx$ equals:

(A) $\frac{1}{2}e^{2x} + C$

(B) $2e^{2x} + C$

(C) $e^{2x} + C$

(D) $\frac{1}{2}e^{x} + C$

Q13. The value of $\int_1^2 \frac{x}{x^2+1}\, dx$ is:

(A) $\frac{1}{2}\ln 3$

(B) $\frac{1}{2}\ln\frac{5}{2}$

(C) $\ln 5 - \ln 2$

(D) $\frac{1}{4}\ln\frac{5}{2}$

Q14. The differential equation formed by eliminating the constant c from $y = ce^x$ is:

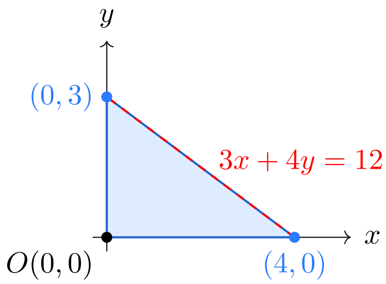
(A) $\dfrac{dy}{dx} = c$

(B) $\dfrac{dy}{dx} = e^x$

(C) $\dfrac{dy}{dx} = y$

(D) $\dfrac{dy}{dx} = \dfrac{y}{x}$

Q15. The area (in sq. units) of the triangle formed by the x-axis, the y-axis, and the line $3x + 4y = 12$ is:



(A) 12

(B) 3

(C) 4

(D) 6

Q16. The equation of the circle passing through the points $(0,0)$, $(1,0)$, and $(0,1)$ is:

(A) $x^2 + y^2 - x - y = 0$

(B) $x^2 + y^2 + x + y = 0$

(C) $x^2 + y^2 = 1$

(D) $x^2 + y^2 + x - y = 0$

Q17. The length of the latus rectum of the parabola $x^2 = 12y$ is:

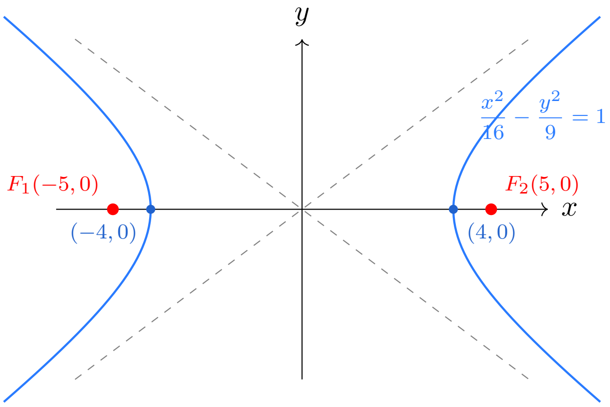
(A) 3

(B) 12

(C) 6

(D) 4

Q18. The equation of the hyperbola with foci at $(\pm 5, 0)$ and transverse axis of length 8 is:



(A) $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

(B) $\dfrac{x^2}{25} - \dfrac{y^2}{9} = 1$

(C) $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

(D) $\dfrac{x^2}{16} - \dfrac{y^2}{25} = 1$

Q19. In a triangle ABC with $A + B + C = \pi$, the value of $\sin(A + B)$ is:

(A) $\sin A$

(B) $\sin B$

(C) $\sin(A - B)$

(D) $\sin C$

Q20. The value of $\sin^{-1}\left(\dfrac{3}{5}\right) + \sin^{-1}\left(\dfrac{4}{5}\right)$ is:

(A) $\dfrac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{4}$

(D) π

Section B — Integer Answer Type (Q21–Q30)

Each correct answer: ***+4 marks****. No negative marking. Attempt any* ***5*** *of 10 questions. Write the integer answer in the box provided.*

Q21. Two vectors $\vec{a}$ and $\vec{b}$ satisfy $|\vec{a}| = 2$ and $|\vec{b}| = 3$, with the angle between them equal to $60°$. Find $\vec{a} \cdot \vec{b}$. ______.

Q22. The direction cosines of the vector $2\hat{i} - \hat{j} + 2\hat{k}$ are (l, m, n). Find the value of $9(l^2 + m^2 + n^2)$. ______.

Q23. Two cards are drawn at random from a standard deck of 52 cards without replacement. Let P be the probability that both cards are aces. Find the value of $221 \times P$. ______.

Q24. The mean of a data set is 25 and the standard deviation is 5. Find the coefficient of variation ($\text{CV} = \frac{\text{SD}}{\text{Mean}} \times 100$). ______.

Q25. The sum of the first n terms of a sequence is given by $S_n = n^2 + n$. Find the **7th term** of the sequence. ______.

Q26. The area of a circle is increasing at the rate of 4π cm^2/s. Find the rate of increase (in cm/s) of the **radius** when the radius is 2 cm. ______.

Q27. Find the value of $\displaystyle\int_0^3 (2x + 1)\, dx$. ______.

Q28. Find the sum of all coefficients in the expansion of $(2x + y)^5$ (substitute $x = y = 1$). ______.

Q29. If $z = \dfrac{1 - i}{1 + i}$, find the value of z^4. ______.

Q30. If A is a 2×2 matrix satisfying $A^2 = A$ (idempotent) and $A \neq I$, $A \neq O$, then the value of $\det(A - I)$ is ______.

Detailed Solutions

Q1.

Solution

Concept: A function $f : \mathbb{R} \to \mathbb{R}$ is a bijection if it is both one-one (injective) and onto (surjective).

Analysis of each option:

- (A) $f(x) = x^2$: Not one-one since $f(2) = f(-2) = 4$. Also not onto (no x gives $f(x) = -1$).
- **(B)** $f(x) = 2x + 3$**:** One-one: if $2x_1 + 3 = 2x_2 + 3$ then $x_1 = x_2$. Onto: for any $y \in \mathbb{R}$, $x = (y - 3)/2 \in \mathbb{R}$ satisfies $f(x) = y$. **Bijection.**
- (C) $f(x) = \sin x$: Not one-one since $\sin(0) = \sin(\pi) = 0$. Not onto since range is $[-1, 1] \neq \mathbb{R}$.
- (D) $f(x) = |x|$: Not one-one since $f(1) = f(-1) = 1$. Not onto (negative values not achieved).

Conclusion: Only the linear function $f(x) = 2x + 3$ is a bijection from $\mathbb{R}$ to $\mathbb{R}$.

Answer: (B) Go Back to Q1

Q2.

Solution

Concept: If $x = r$ is a common root of two equations, it satisfies both.

Step 1: Verify that $x = 2$ is a root of the first equation.

$$x^2 - 3x + 2 = 0 \implies (2)^2 - 3(2) + 2 = 4 - 6 + 2 = 0\checkmark$$

(Indeed, roots of $x^2 - 3x + 2 = 0$ are $x = 1$ and $x = 2$.)

Step 2: Substitute $x = 2$ into the second equation.

$$x^2 - px + 6 = 0 \implies (2)^2 - p(2) + 6 = 0$$

$$4 - 2p + 6 = 0 \implies 10 = 2p \implies p = 5$$

Step 3: Verify.

With $p = 5$: $x^2 - 5x + 6 = 0 \Rightarrow (x - 2)(x - 3) = 0$. Roots are 2 and 3. Indeed $x = 2$ is common with the first equation. ✓

Why other options are wrong:

- (A) $p = 3$: $4 - 6 + 6 = 4 \neq 0$.
- (B) $p = 4$: $4 - 8 + 6 = 2 \neq 0$.
- (D) $p = 7$: This arises if $x = 1$ is the common root: $1 - p + 6 = 0 \Rightarrow p = 7$. But the question specifies common root is $x = 2$.

Answer: (C) Go Back to Q2

Q3.

Solution

Concept: The sum of the first n odd natural numbers $= n^2$.

Step 1: Identify the first 10 odd natural numbers.

$$1,\ 3,\ 5,\ 7,\ 9,\ 11,\ 13,\ 15,\ 17,\ 19$$

Step 2: Apply the formula directly.

Sum of first $n = 10$ odd natural numbers $= n^2 = 10^2 = 100$.

Step 3: Verify by direct addition.

$$1 + 3 = 4, \quad 4 + 5 = 9, \quad 9 + 7 = 16, \quad 16 + 9 = 25, \quad 25 + 11 = 36,$$

$$36 + 13 = 49, \quad 49 + 15 = 64, \quad 64 + 17 = 81, \quad 81 + 19 = 100\checkmark$$

Why other options are wrong:

- (A) 90: This would be $1 + 3 + \cdots + 17 = 9^2 = 81$, then adding 9 instead of 19.
- (B) 95: Off by 5; no standard derivation gives this.
- (C) 99: Off by 1; confuses $10^2 - 1 = 99$ with 10^2.

Answer: (D) Go Back to Q3

Q4.

Solution

Concept: Fundamental property of cube roots of unity.

Step 1: Derive the minimal polynomial.

If $\omega^3 = 1$ and $\omega \neq 1$, then ω satisfies $x^3 - 1 = 0$, i.e. $(x-1)(x^2+x+1) = 0$.

Since $\omega \neq 1$, it must satisfy $x^2 + x + 1 = 0$, which gives $\omega^2 + \omega + 1 = 0$.

Step 2: Conclude.

$$1 + \omega + \omega^2 = 0$$

Step 3: Verify using explicit values.

$\omega = e^{2\pi i/3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i.$

$$1 + \omega + \omega^2 = 1 + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = 1 - 1 = 0\checkmark$$

Why other options are wrong:

- (B) 1: This equals $1 + \omega^0 = 2$ only if $\omega = 0$, which is wrong.
- (C) -1: Misapplies the identity $1 + \omega^2 = -\omega$ to give $-\omega$, not -1.
- (D) 3: Treats $\omega = \omega^2 = 1$, which contradicts ω being primitive.

Answer: (A) Go Back to Q4

Q5.

Solution

Concept: Number of diagonals of a convex n-gon $= \binom{n}{2} - n = \dfrac{n(n-3)}{2}$.

Step 1: Count all line segments joining any two vertices.

$$\binom{10}{2} = \frac{10 \times 9}{2} = 45$$

Step 2: Subtract the $n = 10$ sides.

$$\text{Diagonals} = 45 - 10 = 35$$

Alternatively: $\dfrac{10(10-3)}{2} = \dfrac{10 \times 7}{2} = 35.$

Why other options are wrong:

- (A) 45: This is $\binom{10}{2}$, which includes the 10 sides of the polygon.
- (C) 40: This would arise from subtracting only 5 sides instead of 10.
- (D) 30: This arises from $\binom{10}{2} - 15$; an incorrect subtraction.

Answer: (B) Go Back to Q5

Q6.

Solution

Concept: Binomial coefficients: coefficient of x^r in $(1+x)^n$ is $\binom{n}{r}$.

Step 1: Write the coefficients of x^2 and x^3.

$$\text{Coeff of } x^2 = \binom{n}{2} = \frac{n(n-1)}{2}, \qquad \text{Coeff of } x^3 = \binom{n}{3} = \frac{n(n-1)(n-2)}{6}$$

Step 2: Set up the ratio equation.

$$\frac{\binom{n}{2}}{\binom{n}{3}} = \frac{1}{4}$$

$$\frac{\frac{n(n-1)}{2}}{\frac{n(n-1)(n-2)}{6}} = \frac{1}{4} \implies \frac{6}{2(n-2)} = \frac{1}{4} \implies \frac{3}{n-2} = \frac{1}{4}$$

Step 3: Solve for n.

$$n - 2 = 12 \implies n = 14$$

Verify: $\binom{14}{2} = 91$, $\binom{14}{3} = 364 = 4 \times 91$. ✓

Why other options are wrong:

- (A) 10: $3/(10-2) = 3/8 \neq 1/4$.
- (B) 12: $3/(12-2) = 3/10 \neq 1/4$.
- (D) 16: $3/(16-2) = 3/14 \neq 1/4$.

Answer: (C) Go Back to Q6

Q7.

Solution

Concept: Skew-symmetric matrix: $A^T = -A$, so $a_{ij} = -a_{ji}$ for all i, j.

Step 1: Apply the condition to diagonal elements.

For diagonal entries, $i = j$, so:

$$a_{ii} = -a_{ii} \implies 2a_{ii} = 0 \implies a_{ii} = 0$$

Step 2: Conclude.

Every diagonal element of a skew-symmetric matrix must be **0**.

Example:

$$A = \begin{pmatrix} 0 & 2 & -3 \\ -2 & 0 & 5 \\ 3 & -5 & 0 \end{pmatrix} \implies A^T = -A\checkmark$$

The main diagonal is $(0, 0, 0)$.

Why other options are wrong:

- (A) 1: Only the identity I has all diagonal entries 1; $I^T = I \neq -I$.
- (B) -1: Diagonal entries of -1 would give $A^T = A \neq -A$.
- (C) any real number: This contradicts the derived condition $a_{ii} = 0$.

$\boxed{\text{Answer: (D)}}$ Go Back to Q7

Q8.

Solution

Concept: A fundamental property of determinants: if any two rows (or columns) of a determinant are identical, the determinant equals zero.

Proof sketch:

If rows R_i and R_j are identical, perform the row operation $R_i \to R_i - R_j$, which gives a row of zeros. A determinant with a zero row equals 0.

Alternatively: swapping two identical rows gives $\det(A) = -\det(A)$, so $2\det(A) = 0 \Rightarrow \det(A) = 0$.

Verification with 2×2 example:

$$\begin{vmatrix} p & q \\ p & q \end{vmatrix} = pq - qp = 0\checkmark$$

Why other options are wrong:

- (B) 1: Only the identity matrix has determinant 1, and it has no identical rows.
- (C) -1: Cannot result from two identical rows by any standard property.
- (D) 2: No rule for identical rows produces a value of 2.

Answer: (A) Go Back to Q8

Q9.

Solution

Concept: For limits at infinity involving rational functions, divide numerator and denominator by the highest power of x.

Step 1: Divide numerator and denominator by x^2.

$$\lim_{x\to\infty} \frac{3x^2+5x}{2x^2-7} = \lim_{x\to\infty} \frac{3+\dfrac{5}{x}}{2-\dfrac{7}{x^2}}$$

Step 2: Apply the limit.

As $x \to \infty$: $\dfrac{5}{x} \to 0$ and $\dfrac{7}{x^2} \to 0$.

$$\lim_{x\to\infty} \frac{3+\frac{5}{x}}{2-\frac{7}{x^2}} = \frac{3+0}{2-0} = \frac{3}{2}$$

Why other options are wrong:

- (A) 0: This would apply if the degree of numerator were less than the degree of denominator.
- (C) $5/7$: Results from looking at the linear and constant terms only, ignoring the dominant x^2 terms.
- (D) ∞: This would apply if the degree of numerator exceeded the degree of denominator.

Answer: (B) Go Back to Q9

Q10.

Solution

Concept: Classification of discontinuities: removable (limit exists but differs from function value), jump (left and right limits exist but are unequal), infinite (one or both one-sided limits are $\pm\infty$).

Step 1: Compute the one-sided limits of $f(x) = \frac{1}{x}$ at $x = 0$.

$$\lim_{x\to 0^+} \frac{1}{x} = +\infty, \qquad \lim_{x\to 0^-} \frac{1}{x} = -\infty$$

Step 2: Classify.

Both one-sided limits are infinite (unbounded). This is an **infinite discontinuity** (also called a non-removable discontinuity of the second kind).

Why other options are wrong:

- (A) Removable: The limit must exist (and be finite) but differ from $f(a)$. Here no finite limit exists.
- (B) Jump: Requires finite but unequal one-sided limits. Here neither limit is finite.
- (D) Oscillatory: Applies to functions like $\sin(1/x)$ at $x = 0$, where the limit does not exist due to oscillation, not infinite growth.

Answer: (C) Go Back to Q10

Q11.

Solution

Concept: A differentiable function is strictly decreasing on an interval where its derivative is strictly negative.

Step 1: Differentiate $f(x)$.

$$f(x) = 2x^3 - 9x^2 + 12x + 4$$

$$f'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x-1)(x-2)$$

Step 2: Find where $f'(x) < 0$.

Sign analysis of $f'(x) = 6(x-1)(x-2)$:

- $x < 1$: both factors negative $\Rightarrow f'(x) > 0$ (increasing)
- $1 < x < 2$: $(x-1) > 0$, $(x-2) < 0 \Rightarrow f'(x) < 0$ (**decreasing**)
- $x > 2$: both factors positive $\Rightarrow f'(x) > 0$ (increasing)

Step 3: Conclude.

f is strictly decreasing on $(1, 2)$.

Why other options are wrong:

- (A) $(0, 1)$: $f'(x) > 0$ here (both factors negative, product positive).
- (B) $(2, \infty)$: $f'(x) > 0$ here (both factors positive).
- (C) $(0, 2)$: Includes $(0, 1)$ where f is increasing.

Answer: (D) Go Back to Q11

Q12.

Solution

Concept: Integration using substitution or the rule $\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C$.

Step 1: Apply the rule with $a = 2$.

$$\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C$$

Step 2: Verify by differentiation.

$$\frac{d}{dx}\left[\frac{1}{2}e^{2x} + C\right] = \frac{1}{2} \cdot 2e^{2x} = e^{2x}\checkmark$$

Alternative via substitution: Let $u = 2x$, so $du = 2\,dx$, i.e. $dx = du/2$.

$$\int e^{2x}\,dx = \int e^{u} \cdot \frac{du}{2} = \frac{1}{2}e^{u} + C = \frac{1}{2}e^{2x} + C$$

Why other options are wrong:

- (B) $2e^{2x} + C$: Differentiates to $4e^{2x}$, not e^{2x}.
- (C) $e^{2x} + C$: Differentiates to $2e^{2x}$, not e^{2x}.

- (D) $\frac{1}{2}e^x + C$: Arises from integrating e^x, not e^{2x}.

Answer: (A) Go Back to Q12

Q13.

Solution

Concept: Definite integral using substitution.

Step 1: Substitute $u = x^2 + 1$.

Then $du = 2x\,dx$, so $x\,dx = \dfrac{du}{2}$.

When $x = 1$: $u = 2$. When $x = 2$: $u = 5$.

Step 2: Transform the integral.

$$\int_1^2 \frac{x}{x^2+1}\,dx = \int_2^5 \frac{1}{u}\cdot\frac{du}{2} = \frac{1}{2}\int_2^5 \frac{du}{u} = \frac{1}{2}\Big[\ln u\Big]_2^5$$

Step 3: Evaluate.

$$= \frac{1}{2}(\ln 5 - \ln 2) = \frac{1}{2}\ln\frac{5}{2}$$

Why other options are wrong:

- (A) $\frac{1}{2}\ln 3$: Arises from limits $u = 2$ to $u = 5$ with denominator 3; incorrect.
- (C) $\ln 5 - \ln 2$: Forgets the factor of $\frac{1}{2}$ from the substitution.
- (D) $\frac{1}{4}\ln\frac{5}{2}$: Uses $\frac{1}{4}$ instead of $\frac{1}{2}$; arises from $du = 4x\,dx$ (incorrect differentiation).

Answer: (B) Go Back to Q13

Q14.

Solution

Concept: To form a differential equation, differentiate the given relation and eliminate the arbitrary constant.

Step 1: Start with $y = ce^x$.

Step 2: Differentiate both sides with respect to x.

$$\frac{dy}{dx} = ce^x$$

Step 3: Eliminate the constant c.

From $y = ce^x$, we get $c = y/e^x$. Substituting:

$$\frac{dy}{dx} = \frac{y}{e^x} \cdot e^x = y$$

Hence the differential equation is $\frac{dy}{dx} = y$.

Step 4: Verify.

The general solution of $\frac{dy}{dx} = y$ is $y = Ce^x$ (separable: $dy/y = dx \Rightarrow \ln y = x + k$). This matches the given relation. ✓

Why other options are wrong:

- (A) $dy/dx = c$: The constant c has not been eliminated.
- (B) $dy/dx = e^x$: This gives solution $y = e^x + C$, not $y = ce^x$.
- (D) $dy/dx = y/x$: This gives the solution $y = kx$ (linear), not $y = ce^x$.

Answer: (C) Go Back to Q14

Q15.

Solution

Concept: Area of a triangle formed by a line with coordinate axes.

Step 1: Find the intercepts of $3x + 4y = 12$.

- x-intercept: set $y = 0$: $3x = 12 \Rightarrow x = 4$. Point: $(4, 0)$.
- y-intercept: set $x = 0$: $4y = 12 \Rightarrow y = 3$. Point: $(0, 3)$.

Step 2: Compute the area of the triangle with vertices $O(0, 0)$, $A(4, 0)$, $B(0, 3)$.

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 3 = 6 \text{ sq. units}$$

Why other options are wrong:

- (A) 12: This is $4 \times 3 = 12$, the full rectangle area (forgetting the factor of $1/2$).
- (B) 3: This is only the y-intercept value.
- (C) 4: This is only the x-intercept value.

Answer: (D) Go Back to Q15

Q16.

Solution

Concept: General equation of a circle: $x^2 + y^2 + Dx + Ey + F = 0$.

Step 1: Use point $(0, 0)$**.**

$$0 + 0 + 0 + 0 + F = 0 \implies F = 0$$

Step 2: Use point $(1, 0)$**.**

$$1 + 0 + D(1) + E(0) + 0 = 0 \implies 1 + D = 0 \implies D = -1$$

Step 3: Use point $(0, 1)$**.**

$$0 + 1 + D(0) + E(1) + 0 = 0 \implies 1 + E = 0 \implies E = -1$$

Step 4: Write the equation.

$$x^2 + y^2 - x - y = 0$$

Verify centre and radius: completing the square gives $(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{1}{2}$. Centre $(\frac{1}{2}, \frac{1}{2})$, $r = \frac{1}{\sqrt{2}}$.

Why other options are wrong:

- (B) $x^2 + y^2 + x + y = 0$: Gives $D = 1, E = 1$; substituting $(1, 0)$ gives $1 + 1 + 0 = 2 \neq 0$.
- (C) $x^2 + y^2 = 1$: Does not pass through $(1, 0)$... wait, it does; but $(0, 0)$: $0 \neq 1$. Fails.
- (D) $x^2 + y^2 + x - y = 0$: Substituting $(0, 1)$: $0 + 1 + 0 - 1 = 0$ (passes), but $(1, 0)$: $1 + 0 + 1 - 0 = 2 \neq 0$. Fails.

Answer: (A) Go Back to Q16

Q17.

Solution

Concept: Standard form of upward-opening parabola: $x^2 = 4ay$. Latus rectum $= 4a$.

Step 1: Compare $x^2 = 12y$ with $x^2 = 4ay$.

$$4a = 12 \implies a = 3$$

Step 2: Compute the length of the latus rectum.

$$\text{Latus rectum} = 4a = 12$$

Additional properties: Focus $= (0, a) = (0, 3)$; Directrix: $y = -a = -3$.

Why other options are wrong:

- (A) 3: This is a, the focal length, not the latus rectum.
- (C) 6: This is $2a$, half the latus rectum.
- (D) 4: This comes from misreading $4a = 4$ (i.e. using $a = 1$ from $y^2 = 4x$).

Answer: (B) Go Back to Q17

Q18.

Solution

Concept: Standard hyperbola with foci on the x-axis: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $c^2 = a^2 + b^2$.

Step 1: Extract a from the transverse axis.

Transverse axis length $= 2a = 8 \implies a = 4 \implies a^2 = 16$.

Step 2: Extract c from the foci.

Foci at $(\pm 5, 0) \implies c = 5 \implies c^2 = 25$.

Step 3: Compute b^2.

$$b^2 = c^2 - a^2 = 25 - 16 = 9$$

Step 4: Write the equation.

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Why other options are wrong:

- (A) $x^2/9 - y^2/16 = 1$: Swaps a^2 and b^2; this has transverse axis $2a = 2\sqrt{9} = 6 \neq 8$.
- (B) $x^2/25 - y^2/9 = 1$: Uses $a^2 = 25$ (i.e. $a = 5$); but transverse axis would be $10 \neq 8$.
- (D) $x^2/16 - y^2/25 = 1$: Uses $b^2 = 25$; then $c^2 = 16 + 25 = 41$, so $c = \sqrt{41} \neq 5$.

Answer: (C) Go Back to Q18

Q19.

Solution

Concept: In triangle ABC: $A + B + C = \pi$, so $A + B = \pi - C$.

Step 1: Express $A + B$ using the triangle angle sum.

$$A + B + C = \pi \implies A + B = \pi - C$$

Step 2: Apply the sine function.

$$\sin(A + B) = \sin(\pi - C)$$

Step 3: Use the identity $\sin(\pi - \theta) = \sin\theta$.

$$\sin(\pi - C) = \sin C$$

Therefore:

$$\sin(A + B) = \sin C$$

Why other options are wrong:

- (A) $\sin A$: Would require $B + C = 0$, impossible in a triangle.
- (B) $\sin B$: Would require $A + C = 0$, impossible in a triangle.
- (C) $\sin(A - B)$: This equals $\sin(A + B)$ only if $B = 0$, degenerate case.

Answer: (D) Go Back to Q19

Q20.

Solution

Concept: Addition formula for inverse sine: $\sin^{-1} p + \sin^{-1} q = \sin^{-1}\left(p\sqrt{1-q^2} + q\sqrt{1-p^2}\right)$ when $p^2 + q^2 \leq 1$, or $= \pi/2$ when appropriate.

Direct approach — use the sine addition rule:

Let $\alpha = \sin^{-1}(3/5)$ and $\beta = \sin^{-1}(4/5)$. Both $\alpha, \beta \in [0, \pi/2]$.

Step 1: Find $\cos\alpha$ and $\cos\beta$.

$$\cos\alpha = \sqrt{1-(3/5)^2} = \sqrt{1-9/25} = \sqrt{16/25} = \frac{4}{5}$$

$$\cos\beta = \sqrt{1-(4/5)^2} = \sqrt{1-16/25} = \sqrt{9/25} = \frac{3}{5}$$

Step 2: Compute $\sin(\alpha+\beta)$.

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = \frac{3}{5}\cdot\frac{3}{5} + \frac{4}{5}\cdot\frac{4}{5} = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1$$

Step 3: Conclude.

$\sin(\alpha+\beta) = 1$ and $\alpha + \beta \in [0, \pi]$, so $\alpha + \beta = \frac{\pi}{2}$.

Why other options are wrong:

- (B) $\pi/3$: $\sin(\pi/3) = \sqrt{3}/2 \neq 1$.
- (C) $\pi/4$: $\sin(\pi/4) = 1/\sqrt{2} \neq 1$.
- (D) π: $\sin\pi = 0 \neq 1$.

Answer: (A) Go Back to Q20

Section B — Integer Answer Type Solutions

Q21.

Solution

Concept: Dot product: $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$.

Step 1: Substitute the given values.

$$\vec{a}\cdot\vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta = 2\times 3\times\cos 60^\circ = 6\times\frac{1}{2} = 3$$

Step 2: State the answer.

$\vec{a} \cdot \vec{b} = \mathbf{3}$.

Answer: (3) Go Back to Q21

Q22.

Solution

Concept: For any vector, the sum of the squares of its direction cosines equals 1: $l^2 + m^2 + n^2 = 1$.

Step 1: Verify using the given vector $\vec{v} = 2\hat{i} - \hat{j} + 2\hat{k}$.

$$|\vec{v}| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

$$l = \frac{2}{3}, \quad m = \frac{-1}{3}, \quad n = \frac{2}{3}$$

$$l^2 + m^2 + n^2 = \frac{4}{9} + \frac{1}{9} + \frac{4}{9} = \frac{9}{9} = 1$$

Step 2: Compute $9(l^2 + m^2 + n^2)$.

$$9(l^2 + m^2 + n^2) = 9 \times 1 = \mathbf{9}$$

Answer: (9) Go Back to Q22

Q23.

Solution

Concept: Probability using combinations.

Step 1: Total ways to choose 2 cards from 52.

$$\binom{52}{2} = \frac{52 \times 51}{2} = 1326$$

Step 2: Favourable ways to choose 2 aces from 4.

$$\binom{4}{2} = \frac{4 \times 3}{2} = 6$$

Step 3: Compute P and then $221 \times P$.

$$P = \frac{6}{1326} = \frac{6}{1326} = \frac{1}{221}$$

$$221 \times P = 221 \times \frac{1}{221} = \mathbf{6}$$

Note: $1326 = 6 \times 221$, confirming the calculation. ✓

Answer: (6) Go Back to Q23

Q24.

Solution

Concept: Coefficient of Variation (CV) $= \frac{\text{SD}}{\text{Mean}} \times 100$.

Step 1: Substitute.

$$\text{CV} = \frac{5}{25} \times 100 = \frac{1}{5} \times 100 = 20$$

Interpretation: A CV of 20 means the standard deviation is 20% of the mean.

Answer: (20) Go Back to Q24

Q25.

Solution

Concept: The nth term of a sequence is $a_n = S_n - S_{n-1}$ for $n \geq 2$.

Step 1: Compute S_7 and S_6.

$$S_7 = 7^2 + 7 = 49 + 7 = 56$$

$$S_6 = 6^2 + 6 = 36 + 6 = 42$$

Step 2: Find the 7th term.

$$a_7 = S_7 - S_6 = 56 - 42 = 14$$

Alternative: $a_n = S_n - S_{n-1} = (n^2 + n) - ((n-1)^2 + (n-1)) = n^2 + n - n^2 + 2n - 1 - n + 1 = 2n$.

So $a_7 = 2 \times 7 = \mathbf{14}$. ✓

Answer: (14) Go Back to Q25

Q26.

Solution

Concept: Related rates using differentiation.

Step 1: Write the area formula for a circle.

$$A = \pi r^2$$

Step 2: Differentiate with respect to time t.

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$$

Step 3: Substitute $dA/dt = 4\pi$ cm^2/s and $r = 2$ cm.

$$4\pi = 2\pi(2) \cdot \frac{dr}{dt} = 4\pi \cdot \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{4\pi}{4\pi} = 1 \text{ cm/s}$$

Answer: The rate of increase of the radius is **1** cm/s.

Answer: (1) Go Back to Q26

Q27.

Solution

Concept: Definite integral of a linear function.

Step 1: Find the antiderivative.

$$\int (2x+1)\,dx = x^2 + x + C$$

Step 2: Apply the limits.

$$\int_0^3 (2x+1)\,dx = \Big[x^2 + x\Big]_0^3 = (9+3) - (0+0) = 12$$

Geometric interpretation: The integral equals the area of a trapezoid with parallel sides $f(0) = 1$ and $f(3) = 7$, width 3: Area $= \frac{1}{2}(1+7)(3) = 12$. ✓

Answer: (12) Go Back to Q27

Q28.

Solution

Concept: Sum of all coefficients in a polynomial $p(x, y)$ is obtained by setting all variables equal to 1.

Step 1: Substitute $x = y = 1$ in $(2x + y)^5$.

$$(2(1) + 1)^5 = 3^5 = 243$$

Step 2: Verify by partial expansion.

$(2x + y)^5 = \sum_{k=0}^{5} \binom{5}{k} (2x)^k y^{5-k} = \sum_{k=0}^{5} \binom{5}{k} 2^k x^k y^{5-k}$.

Sum of coefficients $= \sum_{k=0}^{5} \binom{5}{k} 2^k = (2 + 1)^5 = 3^5 =$ **243.**

Answer: (243) Go Back to Q28

Q29.

Solution

Concept: Simplify z first, then raise to the 4th power.

Step 1: Rationalise $z = \dfrac{1-i}{1+i}$.

$$z = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{(1-i)^2}{1+1} = \frac{1-2i+i^2}{2} = \frac{1-2i-1}{2} = \frac{-2i}{2} = -i$$

Step 2: Compute z^4.

$$z^4 = (-i)^4 = (i)^4 = i^4 = (i^2)^2 = (-1)^2 = 1$$

Verification: $|z| = |-i| = 1$, so $|z^4| = 1$. Also $\arg(z) = \arg(-i) = -\pi/2$, so $\arg(z^4) = -2\pi \equiv 0$, confirming $z^4 = 1$. ✓

Answer: (1) Go Back to Q29

Q30.

Solution

Concept: Properties of idempotent matrices and determinants.

Step 1: Start from the idempotent condition $A^2 = A$.

$$A^2 - A = O \implies A(A - I) = O$$

Step 2: Take determinants on both sides.

$$\det(A) \cdot \det(A - I) = \det(O) = 0$$

Step 3: Show $\det(A - I) = 0$.

For a non-trivial idempotent ($A \neq O$, $A \neq I$), the eigenvalues of A are 0 and 1 (since $\lambda^2 = \lambda \Rightarrow \lambda \in \{0, 1\}$). Therefore the eigenvalues of $A - I$ are $0 - 1 = -1$ and $1 - 1 = 0$. Since 0 is an eigenvalue of $A - I$:

$$\det(A - I) = 0$$

Verification with example: Let $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$. Then $A^2 = A$. $A - I = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$. $\det(A - I) = 0 \times (-1) - 0 \times 0 = 0$. ✓

$\boxed{\text{Answer: (0)}}$ Go Back to Q30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	D	4	A	5	B
6	C	7	D	8	A	9	B	10	C
11	D	12	A	13	B	14	C	15	D
16	A	17	B	18	C	19	D	20	A
21	3	22	9	23	6	24	20	25	14
26	1	27	12	28	243	29	1	30	0