JEE Main Mathematics

Sample Paper – 7

Duration: 60 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **30 Questions** divided into two sections, modelled on the Mathematics section of **JEE Main**.
- **Section A (Q1–Q20):** Single Correct MCQs. Each correct answer carries **+4 marks**; each wrong answer carries $-$**1 mark**. Unattempted questions carry 0 marks.
- **Section B (Q21–Q30):** Integer Answer Type. Each correct answer carries **+4 marks**. There is **no negative marking**. Attempt any **5** of 10 questions.
- Syllabus level: **Class 11–12 (NCERT) — JEE Main Mathematics syllabus.**
- Personal calculators, log tables, mobile phones and electronic gadgets are strictly prohibited. Rough sheets will be provided at the exam centre.

Section A — Single Correct Answer Type (Q1–Q20)

*Each correct answer: **+4 marks**. Wrong answer: $-$**1 mark**. Unattempted: 0.*

Q1. If $A = \{1, 2, 3\}$, the number of elements in the power set $P(A)$ is:

(A) 3

(B) 6

(C) 8

(D) 9

Q2. If α and β are the roots of $x^2 - 5x + 7 = 0$, the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is:

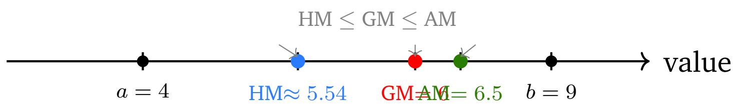
(A) $x^2 - 5x + 7 = 0$

(B) $7x^2 + 5x + 1 = 0$

(C) $x^2 + 5x + 7 = 0$

(D) $7x^2 - 5x + 1 = 0$

Q3. For the positive numbers $a = 4$ and $b = 9$, the geometric mean GM equals:



(A) 6

(B) 6.5

(C) 5

(D) 7

Q4. If $z = x + iy$ satisfies $|z - 2| = |z + 2|$, the locus of z is:

(A) the x-axis

(B) the y-axis

(C) a circle of radius 2

(D) an ellipse

Q5. The number of ways in which 6 persons can be divided into 2 groups of 3 each is:

(A) 20

(B) 15

(C) 10

(D) 60

Q6. Using Pascal's identity $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$, the value of $\binom{5}{2} + \binom{5}{3}$ is:

(A) $\binom{5}{5}$

(B) $\binom{10}{5}$

(C) $\binom{6}{2}$

(D) $\binom{6}{3}$

Q7. Matrix A has $\det(A) = 6$. If one row of A is multiplied by $k = 3$ to obtain matrix B, then $\det(B)$ equals:

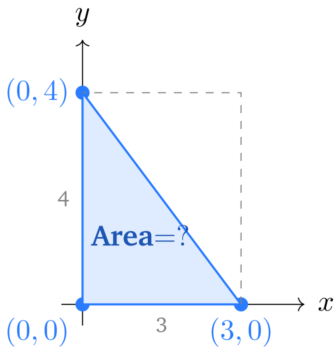
(A) 18

(B) 6

(C) 2

(D) 54

Q8. The area of the triangle with vertices $(0,0)$, $(3,0)$ and $(0,4)$, computed using the determinant formula, is:



(A) 12

(B) 6

(C) 3

(D) 24

Q9. The value of $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$ is:

(A) 0

(B) 2

(C) 4

(D) undefined

Q10. Let $f(x) = x^3 - x - 1$. Which of the following correctly explains why there is a root of f in the interval $(1,\ 2)$?

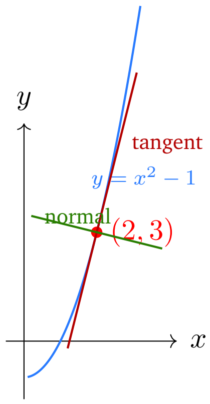
(A) $f(1) = 0$

(B) $f(2) = 0$

(C) $f(1) < 0$ and $f(2) < 0$, so a root exists by IVT

(D) $f(1) < 0$ and $f(2) > 0$, so by IVT a root exists in $(1, 2)$

Q11. The equation of the normal to the curve $y = x^2 - 1$ at the point $(2,\ 3)$ is:



(A) $x + 4y = 14$

(B) $4x + y = 11$

(C) $x - 4y = -10$

(D) $4x - y = 5$

Q12. $\displaystyle\int \frac{dx}{x^2 + 4}$ equals:

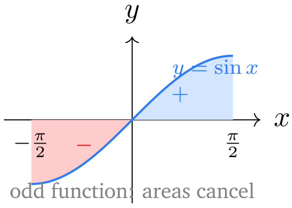
(A) $\tan^{-1}\left(\frac{x}{4}\right) + C$

(B) $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

(C) $\dfrac{1}{4}\tan^{-1}\left(\dfrac{x}{4}\right) + C$

(D) $2\tan^{-1}\left(\dfrac{x}{2}\right) + C$

Q13. The value of $\displaystyle\int_{-\pi/2}^{\pi/2} \sin x \, dx$ is:



(A) 2

(B) π

(C) 0

(D) 1

Q14. The general solution of the differential equation $\dfrac{dy}{dx} + y = e^x$ is:

(A) $y = e^x + C$

(B) $y = Ce^x$

(C) $y = \dfrac{e^x}{2}$

(D) $y = \dfrac{e^x}{2} + Ce^{-x}$

Q15. If the lines $2x + y = 5$, $x + 3y = 9$ and $kx - y = 1$ are concurrent, the value of k is:

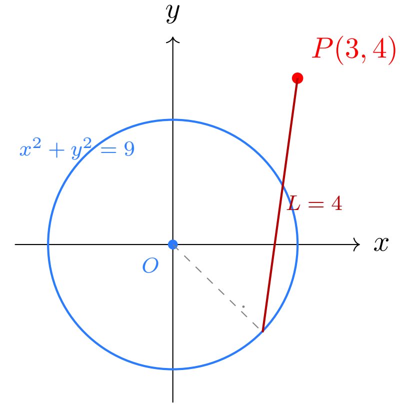
(A) 3

(B) 2

(C) -3

(D) 1

Q16. The length of the tangent drawn from the point $(3, 4)$ to the circle $x^2 + y^2 = 9$ is:



(A) 2

(B) 4

(C) 5

(D) 3

Q17. A point on the parabola $y^2 = 4ax$ is represented parametrically as:

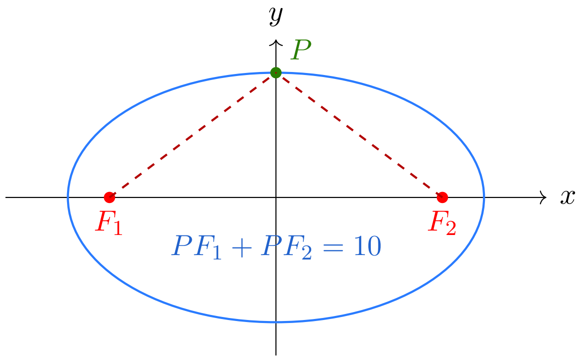
(A) $(2at,\ at^2)$

(B) $\left(\frac{a}{t^2}, \frac{a}{t}\right)$

(C) $(at^2,\ 2at)$

(D) $(2at^2,\ at)$

Q18. For the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, the sum of the distances from any point P on the ellipse to the two foci F_1 and F_2 equals:



(A) 9

(B) 5

(C) 6

(D) 10

Q19. The maximum value of $\sin\theta + \cos\theta$ is:

(A) $\sqrt{2}$

(B) 2

(C) 1

(D) $\sqrt{3}$

Q20. The value of $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$ is:

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{2}$

Section B — Integer Answer Type (Q21–Q30)

Each correct answer: ***+4 marks****. No negative marking. Attempt any* ***5*** *of 10 questions. Write the integer answer in the box provided.*

Q21. Let $\vec{a} = (1, 2, 3)$ and $\vec{b} = (2, 1, -1)$. If $\vec{a} \times \vec{b} = (p, q, r)$, find the value of $p^2 + q^2 + r^2$ (i.e. $|\vec{a} \times \vec{b}|^2$). ______.

Q22. The angle (in degrees) between the planes $x + y = 0$ and $y + z = 0$ is ______.

Q23. A fair coin is tossed 5 times. If P is the probability of getting exactly 3 heads, find the value of $32P$. ______.

Q24. The variance of the data set $\{3, 5, 7, 9, 11\}$ is ______.

Q25. In an A.P., the 3rd term is 7 and the 7th term is 15. The 10th term is ______.

Q26. For $f(x) = 2x^3 - 3x^2$, let x_1 and x_2 be the values of x at which f has local extrema. Find $x_1 + x_2$. ______.

Q27. The value of $\int_0^1 \left(3x^2 + 2x + 1\right) dx$ is ______.

Q28. In the expansion of $(x+y)^{10}$, the sum of the coefficients of all terms (set $x = y = 1$) is ______.

Q29. If $z = \cos\left(\frac{\pi}{3}\right) + i\sin\left(\frac{\pi}{3}\right)$, the value of $|z^{12}|$ is ______.

Q30. If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, the trace of A^2 (i.e. sum of diagonal elements of A^2) is ______.

Detailed Solutions

Q1.

Solution

Concept: The power set $P(A)$ of a set A with n elements contains 2^n elements.

Step 1: Count the elements of A.

$A = \{1, 2, 3\}$ has $n = 3$ elements.

Step 2: Apply the power-set formula.

$$|P(A)| = 2^n = 2^3 = 8$$

Step 3: List the elements of $P(A)$ to verify.

$$P(A) = \{\emptyset,\ \{1\},\ \{2\},\ \{3\},\ \{1,2\},\ \{1,3\},\ \{2,3\},\ \{1,2,3\}\} \quad \checkmark$$

Why other options are wrong:

- (A) 3: This is $|A|$ itself, not $|P(A)|$.
- (B) 6: This is $2 \times 3 = 6$; uses $2n$ instead of 2^n.
- (D) 9: This is $3^2 = 9$; incorrect base and exponent.

Answer: (C) Go Back to Q1

Q2.

Solution

Concept: If α, β are roots of $ax^2 + bx + c = 0$, the equation with roots $\frac{1}{\alpha}, \frac{1}{\beta}$ is obtained by replacing x with $\frac{1}{x}$ and clearing fractions.

Step 1: Use Vieta's formulas for $x^2 - 5x + 7 = 0$.

$$\alpha + \beta = 5, \qquad \alpha\beta = 7$$

Step 2: Find the sum and product of the new roots.

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{5}{7}, \qquad \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{7}$$

Step 3: Form the new quadratic equation.

$$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha\beta} = 0 \implies x^2 - \frac{5}{7}x + \frac{1}{7} = 0$$

Multiply throughout by 7:

$$7x^2 - 5x + 1 = 0$$

Why other options are wrong:

- (A) $x^2 - 5x + 7 = 0$: The original equation, not the transformed one.
- (B) $7x^2 + 5x + 1 = 0$: Sign of the middle term is wrong (uses $+5x$ instead of $-5x$).
- (C) $x^2 + 5x + 7 = 0$: Neither coefficient nor sign is correct for the reciprocal equation.

Answer: (D) Go Back to Q2

Q3.

Solution

Concept: The Geometric Mean of two positive numbers a and b is $\text{GM} = \sqrt{ab}$.

Step 1: Identify the numbers.

$$a = 4, \qquad b = 9$$

Step 2: Compute the GM.

$$\text{GM} = \sqrt{4 \times 9} = \sqrt{36} = 6$$

Step 3: Verify the HM $\leq$ GM $\leq$ AM inequality.

$$\text{AM} = \frac{4+9}{2} = 6.5, \quad \text{GM} = 6, \quad \text{HM} = \frac{2 \times 4 \times 9}{4+9} = \frac{72}{13} \approx 5.54$$

$$5.54 \leq 6 \leq 6.5\checkmark$$

Why other options are wrong:

- (B) 6.5: This is the Arithmetic Mean $\text{AM} = (4+9)/2$, not GM.
- (C) 5: Not the GM; $\sqrt{25} = 5$ would require $ab = 25$.

- (D) 7: Not the GM; $\sqrt{49} = 7$ would require $ab = 49$.

$\boxed{\text{Answer: (A)}}$ Go Back to Q3

Q4.

Solution

Concept: $|z - z_1| = |z - z_2|$ is the perpendicular bisector of the segment joining z_1 and z_2 in the Argand plane.

Step 1: Write the condition in coordinate form.

With $z = x + iy$:

$$|z-2| = |(x-2)+iy| = \sqrt{(x-2)^2 + y^2}$$

$$|z+2| = |(x+2)+iy| = \sqrt{(x+2)^2 + y^2}$$

Step 2: Set them equal and simplify.

$$(x-2)^2 + y^2 = (x+2)^2 + y^2$$

$$x^2 - 4x + 4 = x^2 + 4x + 4$$

$$-4x = 4x \implies 8x = 0 \implies x = 0$$

Step 3: Identify the locus.

$x = 0$ is the y-axis.

Why other options are wrong:

- (A) x-axis ($y = 0$): The perpendicular bisector of a segment on the x-axis is perpendicular to that axis, i.e., the y-axis, not the x-axis.
- (C) Circle of radius 2: The condition $|z-2| = 2$ (not $|z+2|$) gives a circle.
- (D) Ellipse: An ellipse arises when the sum (not the equality) of distances to two foci is constant.

$\boxed{\text{Answer: (B)}}$ Go Back to Q4

Q5.

Solution

Concept: Dividing $2n$ distinct objects into 2 equal groups of n each (groups are unlabelled) is $\frac{\binom{2n}{n}}{2!}$.

Step 1: Choose 3 persons from 6 for the first group.

$$\binom{6}{3} = \frac{6!}{3!\,3!} = 20$$

Step 2: The remaining 3 form the second group automatically. Since the two groups are unlabelled (indistinguishable), divide by $2!$ to avoid double-counting.

$$\text{Number of ways} = \frac{20}{2!} = \frac{20}{2} = 10$$

Why other options are wrong:

- (A) 20: This is $\binom{6}{3}$ without dividing by 2, which counts each division twice.
- (B) 15: This is $\binom{6}{2}$; incorrect group size.
- (D) 60: This is $\binom{6}{3} \times 3 = 60$; an over-count.

$\boxed{\textbf{Answer: (C)}}$ Go Back to Q5

Q6.

Solution

Concept: Pascal's Identity: $\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$.

Step 1: Match the identity with $n = 5$, $r = 2$.

$$\binom{5}{2} + \binom{5}{3} = \binom{6}{3}$$

Step 2: Verify numerically.

$$\binom{5}{2} = 10, \quad \binom{5}{3} = 10, \quad \binom{5}{2} + \binom{5}{3} = 20$$

$$\binom{6}{3} = \frac{6!}{3!\,3!} = \frac{6 \times 5 \times 4}{6} = 20\checkmark$$

Why other options are wrong:

- (A) $\binom{5}{5} = 1$: Far too small.
- (B) $\binom{10}{5} = 252$: Incorrect application of the identity.
- (C) $\binom{6}{2} = 15 \neq 20$: Wrong second index in Pascal's identity.

Answer: (D) Go Back to Q6

Q7.

Solution

Concept: If one row of a matrix is multiplied by scalar k, the determinant is multiplied by k.

Step 1: State the row-operation property of determinants.

Multiplying one row by k scales $\det$ by k:

$$\det(B) = k \cdot \det(A)$$

Step 2: Substitute the given values.

$$\det(B) = 3 \times 6 = 18$$

Why other options are wrong:

- (B) 6: This would be $\det(A)$ unchanged; ignores the row operation.
- (C) 2: This is $\det(A)/k = 6/3$; divides instead of multiplies.
- (D) 54: This would result if all three rows were multiplied by $k = 3$ (for a 3×3 matrix: $k^3 \det(A) = 27 \times 6$), but only one row was scaled here.

Answer: (A) Go Back to Q7

Q8.

Solution

Concept: Area of triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$:

$$\text{Area} = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$$

Step 1: Write the determinant with the given vertices.

$$\begin{vmatrix} 0 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & 4 & 1 \end{vmatrix}$$

Step 2: Expand along the first row.

$$= 0 \cdot (0 \cdot 1 - 1 \cdot 4) - 0 \cdot (3 \cdot 1 - 1 \cdot 0) + 1 \cdot (3 \cdot 4 - 0 \cdot 0) = 0 - 0 + 12 = 12$$

Step 3: Halve to get the area.

$$\text{Area} = \frac{1}{2}|12| = 6$$

Alternate check: Base $= 3$, height $= 4$, Area $= \frac{1}{2} \times 3 \times 4 = 6$. ✓

Why other options are wrong:

- (A) 12: Forgets to multiply by $\frac{1}{2}$ in the formula.
- (C) 3: Takes only half the base without the height.
- (D) 24: Doubles instead of halving the determinant value.

Answer: (B) Go Back to Q8

Q9.

Solution

Concept: Factor the numerator to cancel the indeterminate form.

Step 1: Factorise $x^2 - 4$.

$$x^2 - 4 = (x - 2)(x + 2)$$

Step 2: Cancel the common factor $(x - 2)$ (valid since $x \to 2$, not $x = 2$).

$$\lim_{x\to 2} \frac{x^2 - 4}{x - 2} = \lim_{x\to 2} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x\to 2}(x + 2) = 2 + 2 = 4$$

Why other options are wrong:

- (A) 0: Direct substitution gives $0/0$ (indeterminate); zero is not the answer.
- (B) 2: This equals x evaluated at $x = 2$, not $x + 2$.
- (D) undefined: After factoring, the limit is well-defined as 4.

Answer: (C) Go Back to Q9

Q10.

Solution

Concept: Intermediate Value Theorem (IVT): If f is continuous on $[a, b]$ and $f(a)$ and $f(b)$ have opposite signs, then $\exists\, c \in (a, b)$ such that $f(c) = 0$.

Step 1: Evaluate f at the endpoints.

$$f(1) = 1^3 - 1 - 1 = 1 - 1 - 1 = -1 < 0$$

$$f(2) = 2^3 - 2 - 1 = 8 - 2 - 1 = 5 > 0$$

Step 2: Apply IVT.

Since f is a polynomial (hence continuous on $[1, 2]$) and $f(1) < 0$ while $f(2) > 0$, by IVT there exists $c \in (1, 2)$ such that $f(c) = 0$.

Why other options are wrong:

- (A) $f(1) = 0$: Computed above, $f(1) = -1 \neq 0$.
- (B) $f(2) = 0$: Computed above, $f(2) = 5 \neq 0$.
- (C) $f(1) < 0$ and $f(2) < 0$: Incorrect; $f(2) = 5 > 0$.

Answer: (D) Go Back to Q10

Q11.

Solution

Concept: The normal to a curve at a point is perpendicular to the tangent; if the tangent slope is m, the normal slope is $-1/m$.

Step 1: Differentiate the curve and find the tangent slope at $(2, 3)$.

$$y = x^2 - 1 \implies \frac{dy}{dx} = 2x \implies \left.\frac{dy}{dx}\right|_{x=2} = 2(2) = 4$$

Step 2: Verify that $(2, 3)$ lies on the curve.

$$y = (2)^2 - 1 = 4 - 1 = 3\checkmark$$

Step 3: Find the normal slope.

$$m_{\text{normal}} = -\frac{1}{4}$$

Step 4: Write the equation of the normal.

$$y - 3 = -\frac{1}{4}(x - 2) \implies 4(y - 3) = -(x - 2) \implies 4y - 12 = -x + 2$$

$$x + 4y = 14$$

Why other options are wrong:

- (B) $4x + y = 11$: This uses slope 4 (the tangent slope) for the normal instead of $-1/4$.
- (C) $x - 4y = -10$: Uses slope $+1/4$ (positive reciprocal) instead of $-1/4$.
- (D) $4x - y = 5$: Slope would be 4, which is the tangent, not the normal.

Answer: (A) Go Back to Q11

Q12.

Solution

Concept: Standard integral: $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$.

Step 1: Identify $a^2 = 4$, so $a = 2$.

Step 2: Apply the formula.

$$\int \frac{dx}{x^2 + 4} = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

Step 3: Verify by differentiation.

$$\frac{d}{dx}\left[\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)\right] = \frac{1}{2}\cdot\frac{1}{1+(x/2)^2}\cdot\frac{1}{2} = \frac{1}{4}\cdot\frac{1}{1+x^2/4} = \frac{1}{4}\cdot\frac{4}{4+x^2} = \frac{1}{x^2+4}\checkmark$$

Why other options are wrong:

- (A) $\tan^{-1}(x/4) + C$: Missing the $1/a$ factor and uses wrong argument $x/4$ instead of $x/2$.
- (C) $\frac{1}{4}\tan^{-1}(x/4) + C$: Uses $a = 4$ instead of $a = 2$.
- (D) $2\tan^{-1}(x/2) + C$: Has coefficient 2 instead of $1/2$.

$\boxed{\textbf{Answer: (B)}}$ Go Back to Q12

Q13.

Solution

Concept: Odd function property of definite integrals: if $f(-x) = -f(x)$, then $\int_{-a}^{a} f(x)\,dx = 0$.

Step 1: Check whether $\sin x$ is odd.

$$\sin(-x) = -\sin(x) \implies \sin x \text{ is an odd function.}$$

Step 2: Apply the odd-function property.

$$\int_{-\pi/2}^{\pi/2} \sin x\,dx = 0$$

Step 3: Verify directly.

$$\int_{-\pi/2}^{\pi/2} \sin x\, dx = \Big[-\cos x\Big]_{-\pi/2}^{\pi/2} = -\cos\tfrac{\pi}{2} + \cos\left(-\tfrac{\pi}{2}\right) = -0 + 0 = 0\checkmark$$

Why other options are wrong:

- (A) 2: This equals $\displaystyle\int_0^{\pi} \sin x\, dx = 2$; different limits.
- (B) π: Not the value of any standard integral of $\sin x$ over these limits.
- (D) 1: This equals $\displaystyle\int_0^{\pi/2} \sin x\, dx = 1$; only the right half.

Answer: (C) Go Back to Q13

Q14.

Solution

Concept: Linear first-order ODE $\dfrac{dy}{dx} + P(x)\, y = Q(x)$; integrating factor $\mu = e^{\int P\,dx}$.

Step 1: Identify $P(x) = 1$. Compute the integrating factor.

$$\mu = e^{\int 1\,dx} = e^x$$

Step 2: Multiply both sides of the ODE by e^x.

$$e^x \frac{dy}{dx} + e^x y = e^x \cdot e^x = e^{2x}$$

$$\frac{d}{dx}\left(ye^x\right) = e^{2x}$$

Step 3: Integrate both sides.

$$ye^x = \int e^{2x}\, dx = \frac{e^{2x}}{2} + C$$

Step 4: Solve for y.

$$y = \frac{e^{2x}}{2e^x} + \frac{C}{e^x} = \frac{e^x}{2} + Ce^{-x}$$

Why other options are wrong:

- (A) $y = e^x + C$: Does not satisfy the ODE; substituting gives $e^x + 0 + e^x + C \neq e^x$.

- (B) $y = Ce^x$: The homogeneous solution only; ignores the particular solution from e^x.
- (C) $y = e^x/2$: A particular solution only; the general solution must include the arbitrary constant C.

$\boxed{\text{Answer: (D)}}$ Go Back to Q14

Q15.

Solution

Concept: Three lines are concurrent if the system formed by any two of them satisfies the third.

Step 1: Solve the first two lines simultaneously.

$$2x + y = 5 \quad \text{(i)}$$
$$x + 3y = 9 \quad \text{(ii)}$$

From (ii): $x = 9 - 3y$. Substitute into (i):

$$2(9-3y) + y = 5 \implies 18 - 6y + y = 5 \implies -5y = -13 \implies y = \frac{13}{5}$$

$$x = 9 - 3 \cdot \frac{13}{5} = 9 - \frac{39}{5} = \frac{45-39}{5} = \frac{6}{5}$$

Step 2: Substitute the intersection point $\left(\frac{6}{5}, \frac{13}{5}\right)$ into the third line $kx - y = 1$.

$$k \cdot \frac{6}{5} - \frac{13}{5} = 1 \implies \frac{6k-13}{5} = 1 \implies 6k - 13 = 5 \implies 6k = 18 \implies k = 3$$

Why other options are wrong:

- (B) $k = 2$: Gives $\frac{12-13}{5} = -\frac{1}{5} \neq 1$.
- (C) $k = -3$: Gives $\frac{-18-13}{5} = -\frac{31}{5} \neq 1$.
- (D) $k = 1$: Gives $\frac{6-13}{5} = -\frac{7}{5} \neq 1$.

$\boxed{\text{Answer: (A)}}$ Go Back to Q15

Q16.

Solution

Concept: Length of tangent from external point (x_1, y_1) to circle $x^2 + y^2 = r^2$:

$$L = \sqrt{x_1^2 + y_1^2 - r^2}$$

Step 1: Identify the given values.

$$(x_1, y_1) = (3, 4), \qquad r^2 = 9$$

Step 2: Apply the formula.

$$L = \sqrt{3^2 + 4^2 - 9} = \sqrt{9 + 16 - 9} = \sqrt{16} = 4$$

Why other options are wrong:

- (A) 2: $\sqrt{4} = 2$ would require $x_1^2 + y_1^2 - r^2 = 4$, but it equals 16.
- (C) 5: This is $\sqrt{x_1^2 + y_1^2} = \sqrt{25} = 5$, the distance from the origin to $(3, 4)$, not the tangent length.
- (D) 3: The radius of the circle, not the tangent length.

Answer: (B) Go Back to Q16

Q17.

Solution

Concept: For the parabola $y^2 = 4ax$, the standard parametric form of any point on it is $(at^2, 2at)$.

Step 1: Substitute the parametric point into the equation to verify.

$$y^2 = (2at)^2 = 4a^2t^2, \qquad 4ax = 4a(at^2) = 4a^2t^2$$

$$y^2 = 4ax\checkmark$$

Step 2: Check each option by substitution.

- (C) $(at^2, 2at)$: satisfies the equation as shown above. ✓
- (A) $(2at, at^2)$: gives $(at^2)^2 = a^2t^4$ and $4a(2at) = 8a^2t$; not equal in general.
- (B) $(a/t^2, a/t)$: gives $(a/t)^2 = a^2/t^2$ and $4a(a/t^2) = 4a^2/t^2$; not equal.

- (D) $(2at^2, at)$: gives $(at)^2 = a^2t^2$ and $4a(2at^2) = 8a^2t^2$; not equal.

Answer: (C) Go Back to Q17

Q18.

Solution

Concept: For an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$, the sum of focal radii from any point on the ellipse to the two foci equals $2a$ (the major axis length).

Step 1: Identify $a^2 = 25$, so $a = 5$.

Step 2: Apply the focal-radii property.

$$PF_1 + PF_2 = 2a = 2 \times 5 = 10$$

Step 3: Find the eccentricity and foci for context.

$$c = \sqrt{a^2 - b^2} = \sqrt{25 - 9} = \sqrt{16} = 4 \implies \text{foci at } (\pm 4, 0)$$

Why other options are wrong:

- (A) 9: This is b^2, not $2a$.
- (B) 5: This is a (semi-major axis), not $2a$.
- (C) 6: This is twice the value of $b = 3$ (i.e., $2b$), not $2a$.

Answer: (D) Go Back to Q18

Q19.

Solution

Concept: Write $\sin\theta + \cos\theta$ in the form $R\sin(\theta + \phi)$ to find its maximum.

Step 1: Express as a single sinusoidal.

$$\sin\theta + \cos\theta = \sqrt{2}\,\sin\left(\theta + \frac{\pi}{4}\right)$$

This follows from the identity $a\sin\theta + b\cos\theta = \sqrt{a^2 + b^2}\,\sin(\theta + \phi)$ with $a = b = 1$:

$$\sqrt{a^2 + b^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Step 2: State the maximum.

The maximum value of $\sqrt{2}\sin(\cdot)$ is $\sqrt{2}$ (attained at $\theta + \pi/4 = \pi/2$, i.e. $\theta = \pi/4$).

Verification: At $\theta = 45^\circ$: $\sin 45^\circ + \cos 45^\circ = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$. ✓

Why other options are wrong:

- (B) 2: Requires $\sin\theta = \cos\theta = 1$ simultaneously, which is impossible.
- (C) 1: The maximum of each term separately is 1, but together they can exceed 1.
- (D) $\sqrt{3}$: No standard argument yields this maximum for $\sin\theta + \cos\theta$.

$\boxed{\text{Answer: (A)}}$ Go Back to Q19

Q20.

Solution

Concept: Addition formula for inverse tangent:

$$\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right), \quad \text{when } xy < 1.$$

Step 1: Apply with $x = \frac{1}{2}$ and $y = \frac{1}{3}$.

Check: $xy = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} < 1$, so the formula applies directly.

Step 2: Compute the argument.

$$\frac{x+y}{1-xy} = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

Step 3: Evaluate.

$$\tan^{-1}(1) + \tan^{-1}\left(\frac{1}{3}\right) \longrightarrow \tan^{-1}(1) = \frac{\pi}{4}$$

$\boxed{\text{Answer: (B)}}$ Go Back to Q20

Section B — Integer Answer Type Solutions

Q21.

Solution

Concept: Cross product $\vec{a} \times \vec{b}$ for $\vec{a} = (a_1, a_2, a_3)$ and $\vec{b} = (b_1, b_2, b_3)$:

$$\vec{a} \times \vec{b} = (a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1)$$

Step 1: Compute each component.

$$p = a_2b_3 - a_3b_2 = (2)(-1) - (3)(1) = -2 - 3 = -5$$

$$q = a_3b_1 - a_1b_3 = (3)(2) - (1)(-1) = 6 + 1 = 7$$

$$r = a_1b_2 - a_2b_1 = (1)(1) - (2)(2) = 1 - 4 = -3$$

So $\vec{a} \times \vec{b} = (-5, 7, -3)$.

Step 2: Compute $p^2 + q^2 + r^2 = |\vec{a} \times \vec{b}|^2$.

$$(-5)^2 + 7^2 + (-3)^2 = 25 + 49 + 9 = 83$$

Answer: (83) Go Back to Q21

Q22.

Solution

Concept: Angle θ between planes with normal vectors $\vec{n}_1$ and $\vec{n}_2$:

$$\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$$

Step 1: Identify normal vectors.

$$\text{Plane 1: } x + y = 0 \implies \vec{n}_1 = (1, 1, 0)$$

$$\text{Plane 2: } y + z = 0 \implies \vec{n}_2 = (0, 1, 1)$$

Step 2: Compute the dot product and magnitudes.

$$\vec{n}_1 \cdot \vec{n}_2 = (1)(0) + (1)(1) + (0)(1) = 1$$

$$|\vec{n}_1| = \sqrt{1 + 1 + 0} = \sqrt{2}, \qquad |\vec{n}_2| = \sqrt{0 + 1 + 1} = \sqrt{2}$$

Step 3: Find θ.

$$\cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \implies \theta = 60^\circ$$

Answer: (60) Go Back to Q22

Q23.

Solution

Concept: Binomial probability: $P(\text{exactly } k \text{ heads in } n \text{ tosses}) = \binom{n}{k}\left(\frac{1}{2}\right)^n$.

Step 1: Set $n = 5$, $k = 3$.

$$P = \binom{5}{3}\left(\frac{1}{2}\right)^5 = \frac{10}{32}$$

Step 2: Compute $32P$.

$$32P = 32 \times \frac{10}{32} = 10$$

Answer: (10) Go Back to Q23

Q24.

Solution

Concept: Variance $= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$.

Step 1: Compute the mean.

$$\bar{x} = \frac{3+5+7+9+11}{5} = \frac{35}{5} = 7$$

Step 2: Compute squared deviations from the mean.

$$(3-7)^2 = 16, \quad (5-7)^2 = 4, \quad (7-7)^2 = 0, \quad (9-7)^2 = 4, \quad (11-7)^2 = 16$$

Step 3: Compute the variance.

$$\text{Variance} = \frac{16+4+0+4+16}{5} = \frac{40}{5} = 8$$

Answer: (8) Go Back to Q24

Q25.

Solution

Concept: In an AP with first term a and common difference d, the n-th term is $a_n = a + (n-1)d$.

Step 1: Set up the equations from the given terms.

$$a_3 = a + 2d = 7 \qquad \text{(i)}$$

$$a_7 = a + 6d = 15 \qquad \text{(ii)}$$

Step 2: Subtract (i) from (ii) to find d.

$$4d = 8 \implies d = 2$$

Step 3: Find a using (i).

$$a + 2(2) = 7 \implies a = 3$$

Step 4: Compute the 10th term.

$$a_{10} = 3 + (10-1)(2) = 3 + 18 = 21$$

Answer: (21) Go Back to Q25

Q26.

Solution

Concept: Local extrema occur at critical points where $f'(x) = 0$.

Step 1: Differentiate.

$$f(x) = 2x^3 - 3x^2 \implies f'(x) = 6x^2 - 6x = 6x(x-1)$$

Step 2: Solve $f'(x) = 0$.

$$6x(x-1) = 0 \implies x = 0 \text{ or } x = 1$$

Step 3: Confirm these are local extrema (sign change in f').

f' changes from $+$ to $-$ at $x = 0$ (local max) and from $-$ to $+$ at $x = 1$ (local min). Both are local extrema.

Step 4: Sum the critical values.

$$x_1 + x_2 = 0 + 1 = 1$$

Answer: (1) Go Back to Q26

Q27.

Solution

Concept: Power rule for integration: $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$.

Step 1: Integrate term by term.

$$\int_0^1 (3x^2 + 2x + 1)\, dx = \left[x^3 + x^2 + x\right]_0^1$$

Step 2: Evaluate at the limits.

$$\left[x^3 + x^2 + x\right]_0^1 = (1 + 1 + 1) - (0 + 0 + 0) = 3$$

Answer: (3) Go Back to Q27

Q28.

Solution

Concept: Sum of all coefficients in $(x + y)^n$ is obtained by setting $x = y = 1$: the result is 2^n.

Step 1: Set $x = y = 1$ in $(x + y)^{10}$.

$$(1 + 1)^{10} = 2^{10} = 1024$$

Verification of a few terms: The coefficients are $\binom{10}{0}, \binom{10}{1}, \ldots, \binom{10}{10}$; their sum is $2^{10} = 1024$.

Answer: (1024) Go Back to Q28

Q29.

Solution

Concept: $z = \cos\theta + i\sin\theta$ has modulus $|z| = 1$ (it lies on the unit circle). Therefore $|z^n| = |z|^n = 1^n = 1$ for all n.

Step 1: Compute $|z|$.

$$|z| = \sqrt{\cos^2\left(\frac{\pi}{3}\right) + \sin^2\left(\frac{\pi}{3}\right)} = \sqrt{1} = 1$$

Step 2: Compute $|z^{12}|$.

$$|z^{12}| = |z|^{12} = 1^{12} = 1$$

Note: By De Moivre's theorem, $z^{12} = \cos(12\pi/3) + i\sin(12\pi/3) = \cos(4\pi) + i\sin(4\pi) = 1 + 0i$, confirming $|z^{12}| = 1$.

Answer: (1) Go Back to Q29

Q30.

Solution

Concept: Matrix multiplication and trace (sum of diagonal elements).

Step 1: Compute $A^2 = A \cdot A$.

$$A^2 = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 3\cdot 3 + 1\cdot(-1) & 3\cdot 1 + 1\cdot 2 \\ (-1)\cdot 3 + 2\cdot(-1) & (-1)\cdot 1 + 2\cdot 2 \end{pmatrix} = \begin{pmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{pmatrix} = \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix}$$

Step 2: Compute the trace.

$$\mathrm{tr}(A^2) = 8 + 3 = 11$$

Alternate check using trace identity: $\mathrm{tr}(A^2) = [\mathrm{tr}(A)]^2 - 2\det(A)$. $\mathrm{tr}(A) = 3+2 = 5$, $\det(A) = 6-(-1) = 7$. So $\mathrm{tr}(A^2) = 25 - 14 = 11$. ✓

Answer: (11) Go Back to Q30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	D	3	A	4	B	5	C
6	D	7	A	8	B	9	C	10	D
11	A	12	B	13	C	14	D	15	A
16	B	17	C	18	D	19	A	20	B
21	83	22	60	23	10	24	8	25	21
26	1	27	3	28	1024	29	1	30	11