CAT Quantitative Aptitude

Sample Paper – 9

Duration: 40 Minutes **Maximum Marks: 66**

Instructions

- This paper contains **22 MCQ questions (single-correct)**, modelled on the Quantitative Aptitude section of **CAT**.
- Each correct answer carries **+3 marks**. There is a penalty of **-1 mark per wrong MCQ**; **0** marks for unattempted questions.
- Only **one** option is correct per question. Choose carefully.
- **No personal calculators** are permitted; a basic on-screen calculator is provided in the actual CAT exam.
- **Rough sheets** are provided at the test centre; use them freely for calculations.
- Syllabus level: **Arithmetic, Algebra, Geometry & Mensuration, and Modern Mathematics** at the level of the CAT Quantitative Aptitude section.

Questions (Q1 – Q22)

Q1. What is the units digit of 3^{151}?

(A) 3

(B) 9

(C) 1

(D) 7

Q2. The product of the LCM and HCF of two numbers is 2160. If one of the numbers is 36, find the other number.

(A) 30

(B) 60

(C) 45

(D) 72

Q3. Ten years ago, Raman's age was 3 times Shyam's age. After 5 years, Raman's age will be twice Shyam's age. What are their present ages (Raman, Shyam)?

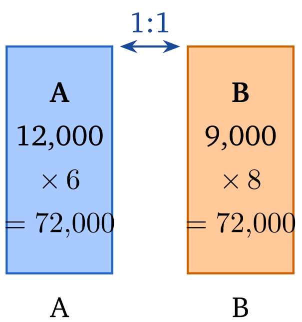
(A) (60, 30)

(B) (50, 25)

(C) (55, 25)

(D) (45, 20)

Q4. A invests Rs. 12,000 for 6 months and B invests Rs. 9,000 for 8 months in a business. The total profit at the end of the year is Rs. 13,800. Find A's share of the profit.



Profit share ratio $= 1 : 1$

(A) Rs. 6,900

(B) Rs. 7,200

(C) Rs. 6,600

(D) Rs. 7,500

Q5. A's salary is 20% more than B's salary. B's salary is 25% less than C's salary. By what percentage is A's salary less than C's salary?

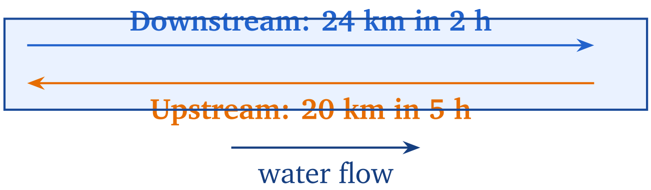
(A) 5%

(B) 8%

(C) 12%

(D) 10%

Q6. A boat covers 24 km downstream in 2 hours and 20 km upstream in 5 hours. Find the speed of the current (stream).



(A) 3 km/h

(B) 4 km/h

(C) 5 km/h

(D) 6 km/h

Q7. Pipes A and B can fill a tank in 12 hours and 18 hours respectively. Pipe C (outlet) can empty the full tank in 9 hours. If all three are opened simultaneously, how many hours will it take to fill 25% of the tank?

(A) 6 hours

(B) 8 hours

(C) 9 hours

(D) 12 hours

Q8. A mixture of 20 litres contains milk and water in the ratio 3:2. How many litres of water must be added to make the ratio 3:4?

(A) 8 litres

(B) 10 litres

(C) 6 litres

(D) 12 litres

Q9. Two trains start simultaneously from stations P and Q, which are 450 km apart, and move towards each other. Train 1 travels at 60 km/h and Train 2 at 90 km/h. After how many hours do they meet?

(A) 2 hours

(B) 2.5 hours

(C) 4 hours

(D) 3 hours

Q10. At what rate per annum simple interest will a principal amount triple itself in 20 years?

(A) 8%

(B) 10%

(C) 12%

(D) 15%

Q11. If x and y are positive real numbers such that $x + y = 10$, find the maximum value of xy.

(A) 25

(B) 20

(C) 24

(D) 30

Q12. The sum of the first 15 terms of an AP is 600 and the first term is 10. Find the 15^{th} term.

(A) 60

(B) 65

(C) 70

(D) 75

Q13. Find the number of integers satisfying $|3x - 7| \leq 5$.

(A) 2

(B) 3

(C) 5

(D) 4

Q14. Let $f(x) = 2x + 3$. If $g(f(x)) = 6x + 11$, what is $g(x)$?

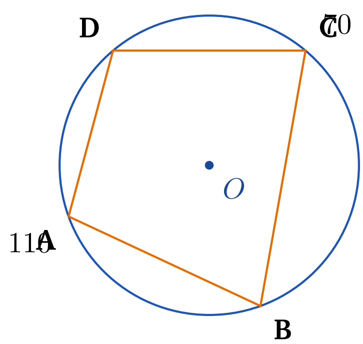
(A) $3x + 5$

(B) $3x + 2$

(C) $2x + 3$

(D) $6x + 2$

Q15. In a cyclic quadrilateral $ABCD$, $\angle A = 110$ and $\angle B = 80$. Find $\angle C$.



(A) 80

(B) 90

(C) 60

(D) 70

Q16. In triangle ABC, $AB = 8$ cm, $AC = 6$ cm, and $\angle A = 30$. Find the area of the triangle.

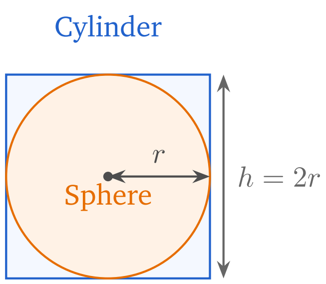
(A) 12 cm^2

(B) 16 cm^2

(C) 18 cm^2

(D) 24 cm^2

Q17. A sphere of radius r is exactly inscribed in a right circular cylinder (touching the bases and the curved surface). What is the ratio of the volume of the sphere to the volume of the cylinder?



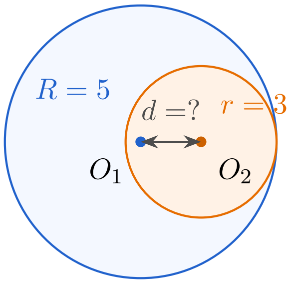
(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) $\frac{2}{3}$

(D) $\frac{1}{3}$

Q18. Two circles of radii 5 cm and 3 cm touch each other internally. Find the distance between their centres.



(A) 8 cm

(B) 2 cm

(C) 4 cm

(D) 3 cm

Q19. Find the equation of the perpendicular bisector of the line segment joining $A(2, 4)$ and $B(6, 8)$.

(A) $x + y = 10$

(B) $x - y = 10$

(C) $2x + y = 14$

(D) $x + 2y = 16$

Q20. A fair coin is tossed 5 times. Find the probability of getting exactly 3 heads.

(A) $\frac{1}{4}$

(B) $\frac{3}{8}$

(C) $\frac{5}{16}$

(D) $\frac{3}{16}$

Q21. In how many ways can 6 people be seated at a round table such that 2 specific people always sit next to each other?

(A) 120

(B) 24

(C) 60

(D) 48

Q22. In a group of 70 students, 40 like Mathematics, 35 like Science, and every student likes at least one subject. How many students like both Mathematics and Science?

(A) 3

(B) 5

(C) 7

(D) 10

Detailed Solutions

Q1.

Solution

Concept – Cyclicity of units digit: The units digits of powers of 3 repeat in a cycle of length 4: $3^1 \to 3$, $3^2 \to 9$, $3^3 \to 7$, $3^4 \to 1$, $3^5 \to 3$, $\dots$

Step 1 – Find the position in the cycle: Divide the exponent by the cycle length: $151 \div 4 = 37$ remainder 3.

Step 2 – Read off the units digit: A remainder of 3 corresponds to the 3rd term in the cycle. The 3rd term is 3^3, whose units digit is **7**.

Why other options are wrong:

- Option A (3): Units digit of $3^1, 3^5, \dots$ (remainder 1 from cycle), not remainder 3.
- Option B (9): Units digit of $3^2, 3^6, \dots$ (remainder 2), not remainder 3.
- Option C (1): Units digit of $3^4, 3^8, \dots$ (remainder 0), not remainder 3.

Final Answer: Units digit of 3^{151} is $7 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 1

Q2.

Solution

Concept – LCM $\times$ HCF = product of the two numbers: For any two positive integers a and b:

$$\text{LCM}(a, b) \times \text{HCF}(a, b) = a \times b.$$

Step 1 – Set up the equation: Let the two numbers be 36 and x. Then:

$$\text{LCM} \times \text{HCF} = 36 \times x = 2160.$$

Step 2 – Solve for x:

$$x = \frac{2160}{36} = 60.$$

Why other options are wrong:

- Option A (30): $36 \times 30 = 1080 \neq 2160$.
- Option C (45): $36 \times 45 = 1620 \neq 2160$.

- Option D (72): $36 \times 72 = 2592 \neq 2160$.

Final Answer: The other number is $60 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 2

Q3.

Solution

Concept – Age problems using linear equations: Let Raman's present age $= R$ and Shyam's present age $= S$. Translate the given conditions into equations.

Step 1 – Form the equations:

$$\begin{aligned} (R-10) = 3(S-10) \quad &\Rightarrow \quad R-10 = 3S-30 \quad \Rightarrow \quad R = 3S-20 \quad \cdots (1) \\ (R+5) = 2(S+5) \quad &\Rightarrow \quad R+5 = 2S+10 \quad \Rightarrow \quad R = 2S+5 \quad \cdots (2) \end{aligned}$$

Step 2 – Solve the system: Equating (1) and (2):

$$3S - 20 = 2S + 5 \quad \Rightarrow \quad S = 25.$$

Substituting back: $R = 2(25) + 5 = 55$.

Verification: 10 years ago: Raman $= 45 = 3 \times 15 = 3$ times Shyam's age (15). After 5 years: Raman $= 60 = 2 \times 30 =$ twice Shyam's age (30). Both conditions satisfied.

Why other options are wrong:

- Option A (60, 30): $60 - 10 = 50 \neq 3(30-10) = 60$; first condition fails.
- Option B (50, 25): $50 - 10 = 40 \neq 3(25-10) = 45$; first condition fails.
- Option D (45, 20): $45 - 10 = 35 \neq 3(20-10) = 30$; first condition fails.

Final Answer: Raman $= 55$ years, Shyam $= 25$ years $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 3

Q4.

Solution

Concept – Partnership profit sharing: Profit is shared in proportion to (Capital $\times$ Time). This is called the "capital-time product" or investment ratio.

Step 1 – Calculate each partner's investment ratio:

$$\text{A's share} = 12{,}000 \times 6 = 72{,}000.$$

$$\text{B's share} = 9{,}000 \times 8 = 72{,}000.$$

Step 2 – Find the profit ratio and A's share: Profit ratio $= 72{,}000 : 72{,}000 = 1 : 1$. The profits are split equally.

$$\text{A's profit} = \frac{1}{2} \times 13{,}800 = \text{Rs.}\, 6{,}900.$$

Why other options are wrong:

- Option B (Rs. 7,200): Incorrectly uses capital ratio $12{,}000 : 9{,}000 = 4 : 3$ without accounting for time, giving $A = \frac{4}{7} \times 13{,}800 \approx 7{,}886$.
- Option C (Rs. 6,600): Arithmetic error in computing the equal split.
- Option D (Rs. 7,500): Uses an incorrect ratio.

Final Answer: A's share = Rs. 6,900 $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 4

Q5.

Solution

Concept – Percentage chains: Work with a base value of 100 for C, then apply successive percentage changes to find A's salary relative to C.

Step 1 – Set up the chain: Let $C = 100$.

$$B = C \times (1 - 0.25) = 100 \times 0.75 = 75.$$

$$A = B \times (1 + 0.20) = 75 \times 1.20 = 90.$$

Step 2 – Compare A to C:

$$\text{A is less than C by} = \frac{C - A}{C} \times 100 = \frac{100 - 90}{100} \times 100 = 10\%.$$

Why other options are wrong:

- Option A (5%): Ignores the compounding of the two percentages.
- Option B (8%): Arithmetic error in computing 75×1.2.
- Option C (12%): Simply adds/subtracts the raw percentages: $25 - 20 = 5$ or $25 - 20 +$ error.

Final Answer: A's salary is 10% less than C's salary $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 5

Q6.

Solution

Concept – Boats and Streams: Let the speed of the boat in still water $= v$ km/h and speed of the stream $= u$ km/h. Then:

$$\text{Speed downstream} = v + u, \qquad \text{Speed upstream} = v - u.$$

Step 1 – Compute speeds from the given data:

$$v + u = \frac{24}{2} = 12 \text{ km/h} \quad \cdots (1)$$

$$v - u = \frac{20}{5} = 4 \text{ km/h} \quad \cdots (2)$$

Step 2 – Find stream speed: Subtract (2) from (1):

$$2u = 12 - 4 = 8 \quad \Rightarrow \quad u = 4 \text{ km/h}.$$

Why other options are wrong:

- Option A (3 km/h): Would require downstream speed of 11 km/h and upstream speed of 5 km/h, inconsistent with the given data.
- Option C (5 km/h): Would give $v + u = 17$ and $v - u = -6$, impossible (negative boat speed).
- Option D (6 km/h): Would give $2u = 12$, requiring upstream speed $= 0$, contradicting the 20 km in 5 h data.

Final Answer: Speed of current $= 4$ km/h $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 6

Q7.

Solution

Concept – Pipes and Cisterns (work-rate method): Express each pipe's rate as a fraction of the tank filled (or emptied) per hour. The net rate is the algebraic sum.

Step 1 – Find individual rates and net rate:

$$\text{Rate of A} = \frac{1}{12}, \quad \text{Rate of B} = \frac{1}{18}, \quad \text{Rate of C (outlet)} = -\frac{1}{9}.$$

$$\text{Net rate} = \frac{1}{12} + \frac{1}{18} - \frac{1}{9} = \frac{3}{36} + \frac{2}{36} - \frac{4}{36} = \frac{1}{36} \text{ tank/hour.}$$

Step 2 – Time to fill 25% of the tank:

$$\text{Time} = \frac{0.25}{1/36} = 0.25 \times 36 = 9 \text{ hours.}$$

Why other options are wrong:

- Option A (6 hours): Would require a net rate of $1/24$ tank/hour, which is not the case.
- Option B (8 hours): Would require a net rate of $1/32$ tank/hour; arithmetic error.
- Option D (12 hours): Corresponds to filling the entire tank at this net rate, not just 25%.

Final Answer: Time to fill 25% of tank $= 9$ hours $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 7

Q8.

Solution

Concept – Mixture and Alligation: When water is added to a mixture, only the water quantity changes; the milk quantity remains constant. Use this to find how much water is needed for the new ratio.

Step 1 – Find initial quantities: Total $= 20$ L, ratio $3 : 2$.

$$\text{Milk} = \frac{3}{5} \times 20 = 12 \text{ L}, \qquad \text{Water} = \frac{2}{5} \times 20 = 8 \text{ L}.$$

Step 2 – Find water needed for ratio 3:4: Milk stays at 12 L. For ratio $3 : 4$, the water needed is:

$$\text{Required water} = \frac{4}{3} \times 12 = 16 \text{ L.}$$

$$\text{Water to be added} = 16 - 8 = 8 \text{ litres.}$$

Verification: New mixture $= 12$ L milk $+16$ L water $= 28$ L. Ratio $= 12 : 16 = 3 : 4$. Correct.

Why other options are wrong:

- Option B (10 litres): Total water would be 18 L; ratio $= 12 : 18 = 2 : 3 \neq 3 : 4$.
- Option C (6 litres): Total water would be 14 L; ratio $= 12 : 14 = 6 : 7 \neq 3 : 4$.
- Option D (12 litres): Total water would be 20 L; ratio $= 12 : 20 = 3 : 5 \neq 3 : 4$.

Final Answer: Add 8 litres of water $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 8

Q9.

Solution

Concept – Relative speed (trains moving towards each other): When two objects move towards each other, their relative speed is the *sum* of their individual speeds.

Step 1 – Calculate relative speed:

$$\text{Relative speed} = 60 + 90 = 150 \text{ km/h.}$$

Step 2 – Find the time to meet:

$$\text{Time} = \frac{\text{Distance}}{\text{Relative speed}} = \frac{450}{150} = 3 \text{ hours.}$$

Verification: In 3 hours, Train 1 covers $60 \times 3 = 180$ km and Train 2 covers $90 \times 3 = 270$ km. Total $= 180 + 270 = 450$ km. Correct.

Why other options are wrong:

- Option A (2 hours): $60 \times 2 + 90 \times 2 = 300 \neq 450$ km.
- Option B (2.5 hours): $150 \times 2.5 = 375 \neq 450$ km.

- Option C (4 hours): $150 \times 4 = 600 \neq 450$ km.

Final Answer: The trains meet after 3 hours $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 9

Q10.

Solution

Concept – Simple Interest formula: SI $= \dfrac{P \times r \times t}{100}$, where P is principal, r is rate per annum, and t is time in years.

Step 1 – Set up the equation: For the amount to triple, the interest earned $= 2P$ (i.e., SI equals twice the principal). With $t = 20$ years:

$$2P = \frac{P \times r \times 20}{100}.$$

Step 2 – Solve for r: Cancel P from both sides:

$$2 = \frac{20r}{100} = \frac{r}{5} \quad \Rightarrow \quad r = 10\%.$$

Why other options are wrong:

- Option A (8%): At 8% for 20 years, SI $= 160\%$ of P; amount $= 2.6P$, not triple.
- Option C (12%): At 12% for 20 years, SI $= 240\%$ of P; amount $= 3.4P$, overshoots.
- Option D (15%): At 15% for 20 years, SI $= 300\%$ of P; amount $= 4P$, far too high.

Final Answer: Rate $= 10\%$ per annum $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 10

Q11.

Solution

Concept – AM-GM Inequality: For positive reals x and y:

$$\frac{x+y}{2} \geq \sqrt{xy} \quad \Rightarrow \quad xy \leq \left(\frac{x+y}{2}\right)^2.$$

Equality holds when $x = y$.

Step 1 – Apply the bound with $x + y = 10$:

$$xy \leq \left(\frac{10}{2}\right)^2 = 5^2 = 25.$$

Step 2 – Achieve the maximum: Equality holds when $x = y = 5$. Then $xy = 5 \times 5 = 25$.

Why other options are wrong:

- Option B (20): $xy = 20$ is achievable (e.g., $x = 4, y = 6$) but is *not* the maximum.
- Option C (24): $xy = 24$ requires $x + y > 10$ (since $\sqrt{24} \cdot 2 > 10$); impossible under the constraint.
- Option D (30): $xy = 30$ would require $x + y \geq 2\sqrt{30} \approx 10.95 > 10$; violates the constraint.

Final Answer: Maximum value of $xy = 25 \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q 11**

Q12.

Solution

Concept – AP sum formula: For an AP with n terms, first term a_1, and last term a_n:

$$S_n = \frac{n}{2}(a_1 + a_n).$$

Step 1 – Use the given information: $n = 15$, $S_{15} = 600$, $a_1 = 10$.

$$600 = \frac{15}{2}(10 + a_{15}) \quad \Rightarrow \quad 10 + a_{15} = \frac{600 \times 2}{15} = 80.$$

Step 2 – Solve for a_{15}:

$$a_{15} = 80 - 10 = 70.$$

Verification: Common difference $d = \frac{a_{15} - a_1}{14} = \frac{60}{14} = \frac{30}{7}$. Sum $= \frac{15}{2}(10 + 70) = \frac{15}{2} \times 80 = 600$. Correct.

Why other options are wrong:

- Option A (60): Would give $S_{15} = \frac{15}{2}(10 + 60) = 525 \neq 600$.
- Option B (65): Would give $S_{15} = \frac{15}{2}(75) = 562.5 \neq 600$.
- Option D (75): Would give $S_{15} = \frac{15}{2}(85) = 637.5 \neq 600$.

Final Answer: $a_{15} = 70 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 12

Q13.

Solution

Concept – Absolute value inequality: $|A| \leq k$ is equivalent to $-k \leq A \leq k$. Remove the absolute value signs and solve the compound inequality.

Step 1 – Remove the absolute value:

$$|3x - 7| \leq 5 \quad \Rightarrow \quad -5 \leq 3x - 7 \leq 5.$$

Step 2 – Solve for x:

$$-5 + 7 \leq 3x \leq 5 + 7 \quad \Rightarrow \quad 2 \leq 3x \leq 12 \quad \Rightarrow \quad \frac{2}{3} \leq x \leq 4.$$

Step 3 – Count integers in $\left[\frac{2}{3}, 4\right]$: The integers satisfying $\frac{2}{3} \leq x \leq 4$ are $x = 1, 2, 3, 4$. That gives 4 integers.

Why other options are wrong:

- Option A (2): Only counts boundary integers, missing the interior values.
- Option B (3): Misses $x = 1$ by taking the interval as $(1, 4)$ instead of $[2/3, 4]$.
- Option C (5): Incorrectly includes $x = 0$ (which gives $|-7| = 7 > 5$, failing the inequality).

Final Answer: 4 integers $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 13

Q14.

Solution

Concept – Function composition: Given $g(f(x)) = 6x + 11$ and $f(x) = 2x + 3$, substitute $y = f(x)$ and express x in terms of y to find $g(y)$.

Step 1 – Express x in terms of $y = f(x)$:

$$y = 2x + 3 \quad \Rightarrow \quad x = \frac{y-3}{2}.$$

Step 2 – Find $g(y)$: Since $g(f(x)) = 6x + 11$, substitute $x = \dfrac{y-3}{2}$:

$$g(y) = 6 \cdot \frac{y-3}{2} + 11 = 3(y-3) + 11 = 3y - 9 + 11 = 3y + 2.$$

Verification: $g(f(x)) = g(2x + 3) = 3(2x + 3) + 2 = 6x + 9 + 2 = 6x + 11$. Correct.

Why other options are wrong:

- Option A $(3x + 5)$: $g(f(x)) = 3(2x + 3) + 5 = 6x + 14 \neq 6x + 11$.
- Option C $(2x+3)$: Same as $f(x)$; $g(f(x)) = 2(2x+3)+3 = 4x+9 \neq 6x+11$.
- Option D $(6x + 2)$: $g(f(x)) = 6(2x + 3) + 2 = 12x + 20 \neq 6x + 11$.

Final Answer: $g(x) = 3x + 2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 14

Q15.

Solution

Concept – Cyclic quadrilateral property: In a cyclic quadrilateral, opposite angles are supplementary (they add up to 180). That is, $\angle A + \angle C = 180$ and $\angle B + \angle D = 180$.

Step 1 – Apply the property:

$$\angle A + \angle C = 180 \quad \Rightarrow \quad 110 + \angle C = 180 \quad \Rightarrow \quad \angle C = 70.$$

Step 2 – Verify using $\angle B$:

$$\angle B + \angle D = 180 \quad \Rightarrow \quad 80 + \angle D = 180 \quad \Rightarrow \quad \angle D = 100.$$

Sum of all angles: $110 + 80 + 70 + 100 = 360$. Correct.

Why other options are wrong:

- Option A (80): This equals $\angle B$; confuses opposite angles with adjacent angles.
- Option B (90): Only applies if the quadrilateral were a rectangle, which is not given.
- Option C (60): Would give $\angle A + \angle C = 170 \neq 180$; violates the cyclic property.

Final Answer: $\angle C = 70 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 15

Q16.

Solution

Concept – Area of a triangle using SAS formula: When two sides and the included angle are known, the area is:

$$\text{Area} = \frac{1}{2} \times \text{side}_1 \times \text{side}_2 \times \sin(\text{included angle}).$$

Step 1 – Identify the values: $AB = 8$ cm, $AC = 6$ cm, $\angle A = 30$. Note that $\sin 30 = \frac{1}{2}$.

Step 2 – Compute the area:

$$\text{Area} = \frac{1}{2} \times 8 \times 6 \times \sin 30 = \frac{1}{2} \times 8 \times 6 \times \frac{1}{2} = \frac{1}{2} \times 8 \times 3 = 12 \text{ cm}^2.$$

Why other options are wrong:

- Option B (16 cm^2): Uses $\sin 30 = 2/3$ instead of the correct $1/2$.
- Option C (18 cm^2): Uses $\frac{1}{2} \times 6 \times 6 = 18$, incorrectly substituting one side twice.
- Option D (24 cm^2): Omits the $\sin 30$ factor entirely: $\frac{1}{2} \times 8 \times 6 = 24$.

Final Answer: Area $= 12$ cm$^2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 16

Q17.

Solution

Concept – Volume ratio of inscribed sphere and cylinder: When a sphere of radius r is inscribed in a right circular cylinder, the cylinder has radius r and height $h = 2r$ (sphere touches both bases and the curved surface).

Step 1 – Write the volumes:

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3, \qquad V_{\text{cylinder}} = \pi r^2 \times 2r = 2\pi r^3.$$

Step 2 – Compute the ratio:

$$\frac{V_{\text{sphere}}}{V_{\text{cylinder}}} = \frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{4/3}{2} = \frac{4}{3} \times \frac{1}{2} = \frac{2}{3}.$$

Why other options are wrong:

- Option A $(1/2)$: Uses $V_{\text{sphere}} = \pi r^3$ (forgetting the $4/3$ factor).
- Option B $(3/4)$: Inverts the ratio partially; swaps numerator and denominator erroneously.
- Option D $(1/3)$: Uses height $h = r$ instead of the correct $h = 2r$ for the cylinder.

Final Answer: Ratio $= \frac{2}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 17

Q18.

Solution

Concept – Distance between centres of internally tangent circles: When two circles touch internally, the distance between their centres equals the *difference* of their radii: $d = |R - r|$.

Step 1 – Identify the radii: Large circle radius $R = 5$ cm, small circle radius $r = 3$ cm.

Step 2 – Compute the distance:

$$d = |R - r| = |5 - 3| = 2 \text{ cm}.$$

Why other options are wrong:

- Option A (8 cm): This equals $R + r = 5 + 3 = 8$, which is the distance for *externally* tangent circles, not internally tangent.
- Option C (4 cm): Off by 2; a common arithmetic slip.
- Option D (3 cm): Equals only the radius of the smaller circle; no geometric basis.

Final Answer: Distance between centres $= 2$ cm $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 18

Q19.

Solution

Concept – Perpendicular bisector: The perpendicular bisector of a segment passes through the midpoint of the segment and is perpendicular to it (i.e., its slope is the negative reciprocal of the segment's slope).

Step 1 – Find the midpoint of AB:

$$M = \left(\frac{2+6}{2}, \frac{4+8}{2}\right) = (4,\ 6).$$

Step 2 – Find the slope of AB and the perpendicular slope:

$$\text{Slope of } AB = \frac{8-4}{6-2} = \frac{4}{4} = 1.$$

$$\text{Slope of perpendicular bisector} = -\frac{1}{1} = -1.$$

Step 3 – Write the equation through $(4, 6)$ with slope -1:

$$y - 6 = -1 \cdot (x - 4) \quad \Rightarrow \quad y - 6 = -x + 4 \quad \Rightarrow \quad x + y = 10.$$

Verification: Point $A(2, 4)$: $2 + 4 = 6 \neq 10$; point $B(6, 8)$: $6 + 8 = 14 \neq 10$ (correct, endpoints are not on the bisector). Midpoint $(4, 6)$: $4 + 6 = 10$. Correct.

Why other options are wrong:

- Option B ($x - y = 10$): Has slope $+1$ (same as AB), not perpendicular.
- Option C ($2x + y = 14$): Passes through the midpoint but has slope -2, not -1.

- Option D ($x + 2y = 16$): Has slope $-1/2$, not -1.

Final Answer: Equation is $x + y = 10 \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q 19**

Q20.

Solution

Concept – Binomial Probability: For n independent Bernoulli trials each with success probability p, the probability of exactly k successes is:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}.$$

Step 1 – Identify parameters: $n = 5$, $k = 3$, $p = \frac{1}{2}$ (fair coin).

Step 2 – Compute the probability:

$$P(X = 3) = \binom{5}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = 10 \times \frac{1}{8} \times \frac{1}{4} = \frac{10}{32} = \frac{5}{16}.$$

Why other options are wrong:

- Option A ($1/4 = 4/16$): Uses $\binom{5}{3} = 8$ (arithmetic error); $8/32 = 1/4$.
- Option B ($3/8 = 6/16$): Uses $\binom{5}{3} = 6$ (confuses with $\binom{4}{2}$); $6/32 = 3/16$, not $3/8$.
- Option D ($3/16$): Uses $\binom{5}{3} = 6$ giving $6/32 = 3/16$; wrong binomial coefficient.

Final Answer: $P = \frac{5}{16} \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q 20**

Q21.

Solution

Concept – Circular permutations with restrictions: In a circular arrangement of n distinct objects, there are $(n-1)!$ ways. When two specific people must always sit together, treat them as a single combined unit.

Step 1 – Treat the pair as one unit: Combining the 2 specific people into 1 unit gives 5 units to arrange around a round table.

$$\text{Circular arrangements of 5 units} = (5-1)! = 4! = 24.$$

Step 2 – Account for internal arrangement of the pair: The 2 specific people can swap places within their unit in $2! = 2$ ways.

$$\text{Total arrangements} = 24 \times 2 = 48.$$

Why other options are wrong:

- Option A (120): This is $(6-1)! = 5! = 120$, the total circular arrangements of all 6 people *without* any restriction.
- Option B (24): Forgets to multiply by the $2!$ internal arrangements of the constrained pair.
- Option C (60): Incorrect; possibly computes $\frac{5!}{2} = 60$, misapplying a divisor.

Final Answer: Total arrangements $= 48 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 21

Q22.

Solution

Concept – Inclusion-Exclusion Principle (Venn diagram): For two sets M and S:

$$n(M \cup S) = n(M) + n(S) - n(M \cap S).$$

Step 1 – Identify the values: Total students $= 70$, and every student likes at least one subject, so $n(M \cup S) = 70$. $n(M) = 40$, $n(S) = 35$.

Step 2 – Solve for $n(M \cap S)$:

$$70 = 40 + 35 - n(M \cap S) \quad \Rightarrow \quad n(M \cap S) = 75 - 70 = 5.$$

Verification: Students who like only Mathematics $= 40 - 5 = 35$. Students who like only Science $= 35 - 5 = 30$. Both $= 35 + 30 + 5 = 70$. Correct.

Why other options are wrong:

- Option A (3): Would give total $= 40 + 35 - 3 = 72 \neq 70$.
- Option C (7): Would give total $= 40 + 35 - 7 = 68 \neq 70$.
- Option D (10): Would give total $= 40 + 35 - 10 = 65 \neq 70$.

Final Answer: Students liking both $= 5 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q 22

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	B	3	C	4	A	5	D
6	B	7	C	8	A	9	D	10	B
11	A	12	C	13	D	14	B	15	D
16	A	17	C	18	B	19	A	20	C
21	D	22	B						

Quick Reference: Correct Answers

Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	7	C	13	D	19	A
2	B	8	A	14	B	20	C
3	C	9	D	15	D	21	D
4	A	10	B	16	A	22	B
5	D	11	A	17	C		
6	B	12	C	18	B		