TIFR GS Computer Science

Sample Paper – 5

Duration: 180 Minutes **Maximum Marks: 120**

Instructions

- This paper contains **30 Multiple Choice Questions** (single correct answer), divided into **Part A: Common (Q1–Q15)** and **Part B: Computer Science (Q16–Q30)**.
- **Part A** must be attempted by all candidates. **Part B** must be attempted for Computer Science (CS) programme admission.
- Each correct answer carries **+4 marks**. **1 mark is deducted** for each wrong answer. Unattempted questions carry **0 marks**.
- Only **one** option is correct per question.
- Use of calculators, mobile phones, laptops, tablets, or any electronic device is strictly **NOT permitted**. Rough sheets are provided at the examination centre.
- Syllabus level: **Graduate-level Computer Science** — Discrete Mathematics, Algorithms & Data Structures, Theory of Computation, Programming Languages, Operating Systems, Digital Circuits, Databases.

Part A: Common (Q1–Q15)

Q1. **[Boolean Algebra – Duality Principle]** The *dual* of a Boolean expression is obtained by swapping the operators $+$ and $\cdot$, and the constants 0 and 1. What is the dual of the Boolean expression $(A + B) \cdot (C + D)$?

(A) $(A + B) \cdot (C + D)$

(B) $(A \cdot B) + (C \cdot D) + 1$

(C) $(\overline{A} + \overline{B}) \cdot (\overline{C} + \overline{D})$

(D) $(A \cdot B) + (C \cdot D)$

Q2. [Combinatorics – Cayley's Formula] By Cayley's formula, the number of distinct *labeled* spanning trees of the complete graph K_n is n^{n-2}. How many labeled spanning trees does K_4 have?

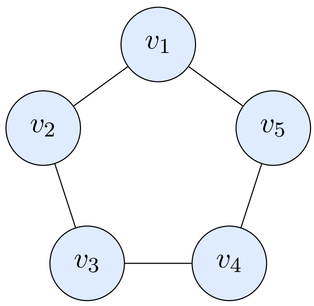
(A) 16

(B) 12

(C) 8

(D) 24

Q3. [Graph Theory – Bipartite Graphs] The graph C_5 is the cycle on 5 vertices, as shown below. Is C_5 a bipartite graph?



(A) Yes, because it has exactly 5 vertices

(B) No, because C_5 contains an odd cycle of length 5

(C) Yes, because C_5 is a planar graph

(D) No, because C_5 is not a connected graph

Q4. [Number Theory – Euclidean Algorithm] Using the Euclidean algorithm, compute $\gcd(48, 36)$.

(A) 4

(B) 6

(C) 12

(D) 18

Q5. [Set Theory – Cardinality] Let $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, and $\mathbb{R}$ denote the natural numbers, integers, rationals, and reals respectively. Which of the following statements is **TRUE**?

(A) $|\mathbb{N}| < |\mathbb{Z}|$

(B) $|\mathbb{N}| < |\mathbb{Q}|$

(C) $|\mathbb{R}| = |\mathbb{N}|$

(D) $|\mathbb{N}| = |\mathbb{Z}|$

Q6. [Probability – Negative Binomial Distribution] Let X denote the number of independent Bernoulli trials (each with success probability $p = \frac{1}{3}$) needed to obtain the **3rd** success. What is $\mathbb{E}[X]$?

(A) 9

(B) 3

(C) 6

(D) 12

Q7. [Linear Algebra – Matrix Rank] Consider the matrix

$$M = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}.$$

What is $\mathrm{rank}(M)$?

(A) 2

(B) 1

(C) 0

(D) 4

Q8. [Predicate Logic – Skolemization] The Skolem normal form of the formula $\exists x\, \forall y\, \exists z\;\; P(x, y, z)$ is obtained by replacing existentially quantified variables with Skolem constants or functions. Which of the following is the correct Skolem normal form?

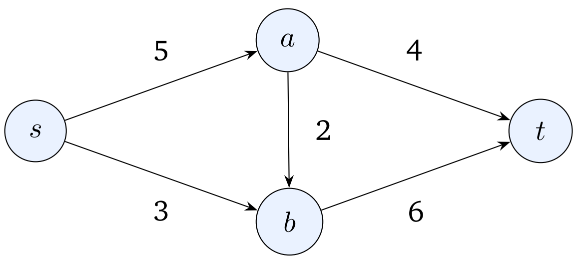
(A) $\exists x\, \forall y\, \forall z\; P(x, y, z)$

(B) $\forall x\, \forall y\, \exists z\; P(x, y, z)$

(C) $\forall y\; P(a,\; y,\; f(y))$

(D) $\forall y\, \exists z\; P(a,\; y,\; z)$

Q9. [Graph Theory – Max-Flow Min-Cut] Consider the flow network shown below (edge labels are capacities). Using the max-flow min-cut theorem, what is the value of the maximum flow from s to t?



(A) 5

(B) 6

(C) 7

(D) 8

Q10. [Probability – Joint Distribution] Random variables X and Y have the following joint probability mass function:

	$Y = 0$	$Y = 1$
$X = 0$	0.1	0.2
$X = 1$	0.3	0.4

What is $P(X = 1)$?

(A) 0.7

(B) 0.4

(C) 0.5

(D) 0.6

Q11. [Combinatorics – Occupancy Problem] Suppose $n = 10$ balls are thrown uniformly at random into $n = 10$ bins (each ball independently and uniformly). What is the **exact** expected number of *empty* bins?

(A) $10/e$ (exact)

(B) $10 \cdot \left(\frac{9}{10}\right)^{10}$

(C) $10 \cdot \left(\frac{1}{2}\right)^{10}$

(D) 5

Q12. [Number Theory – Primitive Roots] An integer g is a *primitive root* modulo a prime p if $\mathrm{ord}_p(g) = \varphi(p) = p - 1$. Which of the following is a primitive root modulo 7?

(A) 2

(B) 5

(C) 3

(D) 4

Q13. [Relations – Well-Ordering Principle] The *well-ordering principle* states that every non-empty subset of $\mathbb{N}$ has a least element. Over the natural numbers, this principle is logically equivalent to which of the following?

(A) The Axiom of Choice

(B) Zorn's Lemma

(C) Transfinite induction on ordinals

(D) The principle of mathematical induction

Q14. [Logic – Craig's Interpolation Theorem] Craig's interpolation theorem states: if $A \vdash B$ in first-order logic, then there exists a formula C (called an *interpolant*) such that $A \vdash C$ and $C \vdash B$. Which statement about the interpolant C is **TRUE**?

(A) C uses only non-logical symbols that appear in **both** A and B

(B) No interpolant generally exists in first-order logic

(C) C may use any non-logical symbols whatsoever

(D) The theorem holds only for propositional logic, not first-order logic

Q15. [Recurrence – Arithmetic Sequence] Solve the recurrence $T(n) = T(n-1) + 2$, with $T(0) = 1$.

(A) $2^n - 1$

(B) $2n + 1$

(C) $n^2 + 1$

(D) $3n - 1$

Part B: Computer Science (Q16–Q30)

Q16. [Algorithms – Strassen's Matrix Multiplication] Strassen's algorithm multiplies two 2×2 matrices using fewer multiplications than the naive algorithm. Exactly how many scalar multiplications does Strassen's algorithm use per recursive call (for 2×2 matrices)?

(A) 4

(B) 6

(C) 7

(D) 8

Q17. [Data Structures – Skip List] In a skip list, each element is promoted to higher levels independently with probability p (typically $p = \frac{1}{2}$). What is the **expected** number of levels in a skip list storing n elements?

(A) $O(n)$

(B) $O(\sqrt{n})$

(C) $O(n/\log n)$

(D) $O(\log n)$

Q18. [Theory of Computation – CFL Closure Properties] Let $L_1 = \{a^n b^n c^m \mid n, m \geq 1\}$ and $L_2 = \{a^m b^n c^n \mid n, m \geq 1\}$. Both L_1 and L_2 are context-free languages (CFLs). Consider $L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 1\}$. Which statement correctly describes closure properties of CFLs?

(A) Closed under union, but **not** under intersection

(B) Closed under both union and intersection

(C) Closed under intersection, but not under union

(D) Closed under neither union nor intersection

Q19. [Algorithms – Greedy Activity Selection] In the interval scheduling maximization problem, the greedy algorithm always selects the activity with the **earliest finish time** among the remaining compatible activities. Why is this greedy choice optimal?

(A) It maximises the total duration of the selected activities

(B) Choosing the earliest finish time leaves maximum room (time) for future activities, satisfying an optimal substructure property

(C) It is equivalent to solving the longest-common-subsequence problem

(D) It reduces the problem to the fractional knapsack problem

Q20. [Data Structures – Union-Find] A Union-Find (Disjoint Set Union) structure supports `Union` and `Find` operations. With

both path compression and union by rank applied simultaneously, what is the **amortized** time per operation for a sequence of m operations on n elements?

(A) $O(\log n)$

(B) $O(\log^* n)$

(C) $O(\alpha(n))$, where α is the inverse Ackermann function

(D) $O(1)$

Q21. [Theory of Computation – Chomsky Hierarchy] The Chomsky hierarchy classifies formal languages into four nested families. Which ordering correctly summarises the hierarchy from most restrictive (innermost) to least restrictive (outermost)?

(A) CFL $\subset$ Regular $\subset$ Decidable $\subset$ RE

(B) Decidable $\subset$ CFL $\subset$ Regular $\subset$ RE

(C) Regular $\subset$ Decidable $\subset$ CFL $\subset$ RE

(D) Regular $\subset$ CFL $\subset$ Decidable $\subset$ RE

Q22. [Operating Systems – SCAN Disk Scheduling] A disk has tracks numbered 0–199. The read/write head is currently at track 53 and is moving toward **lower-numbered** tracks. The pending service queue contains the following track requests (in arrival order): 98, 183, 37, 122, 14, 124, 65, 67. Using the **SCAN** (elevator) algorithm, which track is serviced **first**?

(A) 37

(B) 14

(C) 65

(D) 67

Q23. [Compilers – Three-Address Code] A compiler generates three-address code by breaking compound expressions into sequences of simple assignments, each with at most one operator. Which of the following is the correct three-address code for the assignment $x = a + b \times c$?

(A) `x = a + b * c` (single instruction)

(B) `t1 = b * c` then `x = a + t1`

(C) `t1 = a + b` then `x = t1 * c`

(D) `t1 = a * b` then `x = t1 + c`

Q24. [Theory of Computation – PDA for $a^n b^{2n}$] A pushdown automaton (PDA) accepts $L = \{a^n b^{2n} \mid n \geq 1\}$ by the following strategy: push one stack symbol for each a read; then, while reading b's, pop one symbol for every **pair** of b's consumed. In the transition for reading the **second** b in each pair, what does the PDA do to its stack?

(A) Read b, push one symbol onto the stack

(B) Read b, leave the stack unchanged

(C) Read b, pop one symbol from the stack

(D) Read ε, pop one symbol from the stack

Q25. [Algorithms – KMP Failure Function] The KMP failure function (prefix function) π for a pattern P satisfies: $\pi[i]$ is the length of the longest proper prefix of $P[0..i]$ that is also a suffix of $P[0..i]$. For the pattern $P = \texttt{abcabc}$, compute $\pi[5]$ (0-indexed, so $P[5] = \texttt{c}$).

(A) 0

(B) 1

(C) 2

(D) 3

Q26. [Operating Systems – Buddy System Memory Allocation] The buddy system allocates memory in powers of two. To satisfy a 10 KB request, a 128 KB free block is successively split: $128 \to 64 + 64$, $64 \to 32 + 32$, $32 \to 16 + 16$, yielding a 16 KB block for allocation. The partner block released alongside the allocated 16 KB block (its *buddy*) has what size?

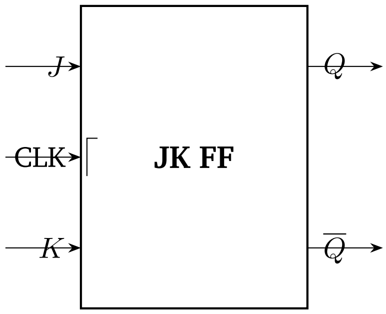
(A) 16 KB

(B) 32 KB

(C) 64 KB

(D) 10 KB

Q27. [Digital Circuits – JK Flip-Flop] A JK flip-flop is shown below. On a rising clock edge with inputs $J = 1$ and $K = 1$, the next-state equation is $Q^+ = J\overline{Q} + \overline{K}Q$. If the current state is $Q = 0$, what is Q^+?



(A) $Q^+ = 0$

(B) $Q^+ = \overline{Q}$ (the flip-flop *toggles*)

(C) $Q^+ = 1$ (set)

(D) $Q^+ = Q$ (no change)

Q28. [Databases – Normal Forms] Consider relation $R(A, B, C)$ with functional dependencies $A \to B$ and $A \to C$. The only candidate key is A (since $A^+ = \{A, B, C\}$). In which highest normal form does R reside?

(A) R is in 1NF only

(B) R is in 2NF but not 3NF

(C) R is in 3NF (and also in BCNF)

(D) R is not in any standard normal form

Q29. [Programming Languages – Lambda Calculus, β-Reduction] Apply one β-reduction step to the following λ-term:

$$(\lambda x.\ (\lambda y.\ x + y))\ 3$$

What is the result?

(A) 3

(B) $\lambda x.\ 3 + x$

(C) $\lambda y.\, x + y$

(D) $\lambda y.\, 3 + y$

Q30. **[Algorithms – Amortised Analysis, Table Doubling]** A dynamic array doubles its capacity whenever it becomes full. Using the **accounting method** of amortised analysis, what is the amortised cost per insertion, and the total cost for n insertions starting from an empty array of capacity 1?

(A) $O(1)$ amortised per insertion; $O(n)$ total

(B) $O(\log n)$ amortised per insertion

(C) $O(n)$ amortised per insertion

(D) $O(\sqrt{n})$ amortised per insertion

Detailed Solutions

Q1.

Solution

[Boolean Algebra – Duality Principle]

The dual of a Boolean expression is obtained by simultaneously replacing every $+$ with $\cdot$ and every $\cdot$ with $+$, and also swapping constants $0 \leftrightarrow 1$ (variables are left unchanged).

Apply to $(A + B) \cdot (C + D)$:

$$(A + B) \cdot (C + D) \xrightarrow{\text{dual}} (A \cdot B) + (C \cdot D).$$

Complementation ($\overline{\ }$) is *not* applied by the dual operation, so option (C) is incorrect.

Hence the dual is $(A \cdot B) + (C \cdot D)$, which is option **(D)**.

Answer: (D) ↑ Go Back to Q1

Q2.

Solution

[Combinatorics – Cayley's Formula]

Cayley's formula: the number of distinct labeled spanning trees of the complete graph K_n is

$$\tau(K_n) = n^{n-2}.$$

For $n = 4$:

$$\tau(K_4) = 4^{4-2} = 4^2 = \mathbf{16}.$$

This can be verified directly: K_4 has $\binom{4}{2} = 6$ edges and any spanning tree uses exactly $4 - 1 = 3$ of them; counting carefully via Prüfer sequences (each sequence of length $n - 2 = 2$ with entries in $\{1, 2, 3, 4\}$) yields $4^2 = 16$ trees.

Answer: **(A) 16**.

Answer: (A) ↑ Go Back to Q2

Q3.

Solution

[Graph Theory – Bipartite Graphs]

A graph is bipartite if and only if it contains *no* cycle of odd length.

C_5 is the 5-cycle $v_1 - v_2 - v_3 - v_4 - v_5 - v_1$, which is a single cycle of length **5** (odd). Therefore C_5 is **not** bipartite.

Verification by 2-colouring attempt: Colour v_1 red. Then v_2 must be blue, v_3 red, v_4 blue, v_5 red. But v_5 and v_1 are adjacent and both red — contradiction. No valid 2-colouring exists.

Answer: **(B) No, because C_5 contains an odd cycle of length 5.**

Answer: (B) ↑ Go Back to Q3

Q4.

Solution

[Number Theory – Euclidean Algorithm]

Apply the Euclidean algorithm to $\gcd(48, 36)$:

$$48 = 1 \cdot 36 + 12,$$
$$36 = 3 \cdot 12 + 0.$$

The last non-zero remainder is **12**, so $\gcd(48, 36) = 12$.

Check: $12 \mid 48$ and $12 \mid 36$; no integer larger than 12 divides both.

Answer: **(C) 12.**

Answer: (C) ↑ Go Back to Q4

Q5.

Solution

[Set Theory – Cardinality]

Evaluate each option:

- **(A)** $|\mathbb{N}| < |\mathbb{Z}|$: **False**. The bijection $n \mapsto 0, 1, -1, 2, -2, \ldots$ shows $|\mathbb{Z}| = |\mathbb{N}|$ (both countably infinite).
- **(B)** $|\mathbb{N}| < |\mathbb{Q}|$: **False**. $\mathbb{Q}$ is countable (Cantor's diagonal enumeration), so $|\mathbb{Q}| = |\mathbb{N}|$.
- **(C)** $|\mathbb{R}| = |\mathbb{N}|$: **False**. Cantor's theorem shows $|\mathbb{R}|$ is uncountable; $|\mathbb{R}| > |\mathbb{N}|$.
- **(D)** $|\mathbb{N}| = |\mathbb{Z}|$: **True** (see A above).

Answer: **(D)** $|\mathbb{N}| = |\mathbb{Z}|$.

Answer: (D) ↑ Go Back to Q5

Q6.

Solution

[Probability – Negative Binomial Distribution]

If X is the number of trials needed to achieve the r-th success in a sequence of independent Bernoulli(p) trials, then X follows a *Negative Binomial* distribution with:

$$\mathbb{E}[X] = \frac{r}{p}.$$

Here $r = 3$ and $p = \frac{1}{3}$:

$$\mathbb{E}[X] = \frac{3}{1/3} = 3 \times 3 = \mathbf{9}.$$

Answer: **(A) 9**.

Answer: (A) ↑ Go Back to Q6

Q7.

Solution

[Linear Algebra – Matrix Rank]

$$M = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}.$$

Observe that Row 2 $= \frac{1}{2}\cdot$ Row 1. The two rows are *linearly dependent*, so the row space has dimension 1.

Formally, $\det(M) = 2 \cdot 2 - 4 \cdot 1 = 0$, confirming that M is singular. The rank equals the number of linearly independent rows (or columns), which is **1**.

Answer: **(B) 1**.

Answer: (B) ↑ Go Back to Q7

Solution

[Predicate Logic – Skolemization]

Skolemization eliminates existential quantifiers by replacing them with *Skolem functions* (or constants) whose arguments are the universally quantified variables that appear above the existential in the prefix:

$$\exists x\, \forall y\, \exists z\ P(x, y, z)$$

Q8.

- x has no universal variable above it $\Rightarrow$ replace x by a Skolem *constant* a.
- z has y universally quantified above it $\Rightarrow$ replace z by a Skolem *function* $f(y)$.

Result (universal quantifier retained, all $\exists$ removed):

$$\forall y\ P(a,\ y,\ f(y)).$$

Answer: **(C)** $\forall y\ P(a,\ y,\ f(y))$.

Answer: (C) ↑ Go Back to Q8

Q9.

Solution

[Graph Theory – Max-Flow Min-Cut]

Find augmenting paths using Ford-Fulkerson:

(a) Path $s \to a \to t$: bottleneck $= \min(5,4) = 4$. Flow $= 4$. Residual: $s \to a$ has 1 left; $a \to t$ is saturated.

(b) Path $s \to b \to t$: bottleneck $= \min(3,6) = 3$. Flow $= 3$. Residual: $s \to b$ is saturated; $b \to t$ has 3 left.

(c) Path $s \to a \to b \to t$: bottleneck $= \min(1,2,3) = 1$. Flow $= 1$.

Total max flow $= 4 + 3 + 1 = \mathbf{8}$.

Min-cut verification: Cut $\{s\} \mid \{a,b,t\}$ has capacity $\text{cap}(s \to a) + \text{cap}(s \to b) = 5 + 3 = 8$. By the max-flow min-cut theorem, min cut $= 8 =$ max flow. ✓

Answer: **(D) 8**.

Answer: (D) ↑ Go Back to Q9

Q10.

Solution

[Probability – Joint Distribution]

The marginal probability $P(X=1)$ is obtained by summing over all values of Y:

$$P(X=1) = P(X=1, Y=0) + P(X=1, Y=1) = 0.3 + 0.4 = \mathbf{0.7}.$$

Sanity check: $P(X=0) = 0.1 + 0.2 = 0.3$; total $= 0.3 + 0.7 = 1.0$. ✓

Answer: **(A) 0.7**.

Answer: (A) ↑ Go Back to Q10

Q11.

Solution

[Combinatorics – Occupancy Problem]

For bin i, let $E_i = \mathbf{1}[\text{bin } i \text{ is empty}]$. Since each ball is placed independently and uniformly in one of $n = 10$ bins:

$$P(\text{bin } i \text{ is empty}) = \left(1 - \frac{1}{n}\right)^n = \left(\frac{9}{10}\right)^{10}.$$

By linearity of expectation:

$$\mathbb{E}[\text{empty bins}] = n \cdot \left(\frac{n-1}{n}\right)^n = 10 \cdot \left(\frac{9}{10}\right)^{10}.$$

(Note: $10/e$ is the *asymptotic* approximation as $n \to \infty$, not the exact value for $n = 10$.)

Answer: **(B)** $10 \cdot \left(\frac{9}{10}\right)^{10}$.

Answer: (B) ↑ Go Back to Q11

Q12.

Solution

[Number Theory – Primitive Roots]

g is a primitive root mod p iff $\text{ord}_p(g) = p - 1 = \varphi(p)$. For $p = 7$, need $\text{ord}_7(g) = 6$.

Check $g = 3$ (option C):

$$3^1 \equiv 3, \quad 3^2 \equiv 2, \quad 3^3 \equiv 6, \quad 3^4 \equiv 4, \quad 3^5 \equiv 5, \quad 3^6 \equiv 1 \pmod 7.$$

The powers of 3 generate all six non-zero residues $\{1, 2, 3, 4, 5, 6\}$, so $\text{ord}_7(3) = 6$. Hence 3 is a primitive root mod 7.

Check option A ($g = 2$): $2^1 = 2, 2^2 = 4, 2^3 = 1 \pmod 7$. Order = 3, not 6. Not a primitive root.

Answer: **(C) 3**.

Answer: (C) ↑ Go Back to Q12

Q13.

Solution

[Relations – Well-Ordering Principle]

Over the natural numbers $\mathbb{N}$, the following are logically equivalent:

- The *well-ordering principle*: every non-empty $S \subseteq \mathbb{N}$ has a least element.
- The *principle of (strong/weak) mathematical induction*.

Both can be proved from each other using only basic set-theoretic axioms for $\mathbb{N}$. They are *not* equivalent to the Axiom of Choice or Zorn's Lemma (those are set-theoretic axioms about arbitrary sets, not specifically $\mathbb{N}$); transfinite induction on ordinals is a broader generalisation.

Answer: **(D) The principle of mathematical induction**.

Answer: (D) ↑ Go Back to Q13

Q14.

Solution

[Logic – Craig's Interpolation Theorem]

Craig's interpolation theorem (1957): if $A \vdash B$ in first-order logic, there exists a formula C (the *interpolant*) satisfying:

(a) $A \vdash C$,

(b) $C \vdash B$, and

(c) every non-logical symbol (predicate, function, constant) occurring in C occurs in *both* A and B.

This is the key property: C is "in the common language" of A and B.

The theorem holds for full first-order logic (not only propositional), so option (D) is false. Options (B) and (C) misstate the theorem.

Answer: **(A)** C **uses only non-logical symbols that appear in both** A **and** B.

Answer: (A) ↑ Go Back to Q14

Q15.

Solution

[Recurrence – Arithmetic Sequence]

Unroll the recurrence $T(n) = T(n-1) + 2$, $T(0) = 1$:

$$\begin{aligned} T(n) &= T(n-1) + 2 \\ &= T(n-2) + 2 + 2 \\ &= \cdots \\ &= T(0) + 2n \\ &= 1 + 2n = \mathbf{2n+1}. \end{aligned}$$

Verification: $T(0) = 1\checkmark$; $T(1) = 3 = 2(1) + 1\checkmark$; $T(2) = 5 = 2(2) + 1\checkmark$.

Answer: **(B)** $2n + 1$.

Answer: (B) ↑ Go Back to Q15

Q16.

Solution

[Algorithms – Strassen's Matrix Multiplication]

Strassen (1969) showed that two 2×2 matrices can be multiplied using only **7 scalar multiplications** (compared to 8 in the naïve algorithm) by defining seven auxiliary products $M_1, \ldots, M_7$ with suitable additions and subtractions.

This yields the recurrence $T(n) = 7\,T(n/2) + O(n^2)$, which by the Master theorem gives $T(n) = O(n^{\log_2 7}) \approx O(n^{2.807})$.

Answer: **(C) 7**.

Answer: (C) ↑ Go Back to Q16

Q17.

Solution

[Data Structures – Skip List]

In a skip list with n elements and promotion probability p, the expected height (number of levels) is $O(\log_{1/p} n) = O(\log n)$ (for any fixed $0 < p < 1$).

Reasoning: The probability that any given element reaches level k is p^k. The expected maximum level is about $\log_{1/p} n$ because beyond that level, the expected number of elements surviving is less than 1.

This gives $O(\log n)$ expected search time and $O(\log n)$ expected space overhead per element.

Answer: **(D)** $O(\log n)$.

Answer: (D) ↑ Go Back to Q17

Q18.

Solution

[Theory of Computation – CFL Closure Properties]

Closed under union: Given CFGs G_1 for L_1 and G_2 for L_2, introduce a new start symbol S with rules $S \to S_1 \mid S_2$. The resulting grammar generates $L_1 \cup L_2$. So CFLs are *closed* under union.

Not closed under intersection: Take $L_1 = \{a^n b^n c^m \mid n, m \geq 1\}$ and $L_2 = \{a^m b^n c^n \mid n, m \geq 1\}$. Both are CFLs (each needs only one "counter"). But

$$L_1 \cap L_2 = \{a^n b^n c^n \mid n \geq 1\},$$

which is a classic non-CFL by the pumping lemma for CFLs.

Answer: **(A) Closed under union, but not under intersection.**

Answer: (A) ↑ Go Back to Q18

Q19.

Solution

[Algorithms – Greedy Activity Selection]

In the interval scheduling problem, the greedy rule is: *always select the compatible activity with the smallest finish time*.

The correctness argument is an **exchange argument**: given any optimal solution that does *not* begin with the earliest-finishing activity a^*, we can swap a^* in for the first chosen activity without reducing the total number of activities selected. This is possible precisely because a^* finishes no later than any other activity, leaving at least as much (in fact, maximum) remaining time for subsequent compatible activities.

Option (B) correctly captures this intuition: choosing the earliest finish time leaves the maximum possible time interval for future activities (optimal substructure via the exchange argument).

Answer: **(B) Leaving maximum remaining time after each selection is an optimal substructure**.

Answer: (B) ↑ Go Back to Q19

Q20.

Solution

[Data Structures – Union-Find]

With **path compression** alone, the amortised cost per operation is $O(\log n)$.

With **union by rank** alone, the amortised cost is $O(\log n)$.

With **both** path compression and union by rank simultaneously, Tarjan (1975) proved the amortised cost per operation is $O(\alpha(n))$, where α is the *inverse Ackermann function* – a function that grows so slowly it is effectively constant ($\alpha(n) \leq 4$ for any n arising in practice).

This is also known to be essentially optimal.

Answer: **(C) $O(\alpha(n))$, where α is the inverse Ackermann function**.

Answer: (C) ↑ Go Back to Q20

Q21.

Solution

[Theory of Computation – Chomsky Hierarchy]

The Chomsky hierarchy, from most restrictive (innermost class) to least restrictive:

$$\underbrace{\text{Regular}}_{\text{Type 3}} \subset \underbrace{\text{Context-Free}}_{\text{Type 2}} \subset \underbrace{\text{Context-Sensitive} \subseteq \text{Decidable}}_{\text{Type 1}} \subset \underbrace{\text{Recursively Enumerable (RE)}}_{\text{Type 0}}.$$

For the options given (which omit Context-Sensitive and group Decidable separately):

$$\text{Regular} \subset \text{CFL} \subset \text{Decidable} \subset \text{RE}.$$

Answer: **(D) Regular $\subset$ CFL $\subset$ Decidable $\subset$ RE.**

Answer: (D) ↑ Go Back to Q21

Q22.

Solution

[Operating Systems – SCAN Disk Scheduling]

Current head position: 53, moving toward **lower** track numbers.

Pending requests sorted: 14, 37, 65, 67, 98, 122, 124, 183.

SCAN (elevator) algorithm services all requests in the current direction of head movement, then reverses.

Moving toward lower tracks from 53: the pending requests below 53 are **37** and **14** (in decreasing order). The **closest** request below 53 (i.e., the first encountered while moving down) is **37**.

Service order: $53 \to 37 \to 14 \to$ (reverse, move up) $\to 65 \to 67 \to 98 \to 122 \to 124 \to 183$.

The **first** request serviced is **37**.

Answer: **(A) 37**.

Answer: (A) ↑ Go Back to Q22

Q23.

Solution

[Compilers – Three-Address Code]

Three-address code (TAC) allows *at most one operator* per instruction. The expression $x = a + b \times c$ must respect operator precedence ($\times$ before $+$):

(a) Compute $b \times c$ first: `t1 = b * c`

(b) Compute $a +$ `t1`: `x = a + t1`

Option (A) uses two operators in one instruction (not TAC). Options (C) and (D) violate precedence.

Answer: **(B)** `t1 = b * c` **then** `x = a + t1`.

Answer: (B) ↑ Go Back to Q23

Q24.

Solution

[Theory of Computation – PDA for $a^n b^{2n}$]

Design: push one symbol Z for each a read. Then, for each *pair* of b's, pop one Z:

- Reading the **first** b of a pair: transition to an intermediate state; do *not* pop (stack unchanged).
- Reading the **second** b of a pair: **pop** one Z from the stack; return to the "reading b-pairs" state.

Accept when the stack is empty (all Z's popped) and input is exhausted.

The question asks specifically about the transition on the **second** b: it reads b from input and **pops one symbol** from the stack.

Answer: **(C) Read b, pop one symbol from the stack**.

Answer: (C) ↑ Go Back to Q24

Q25.

Solution

[Algorithms – KMP Failure Function]

For pattern P = `abcabc` (0-indexed: $P[0..5]$), compute $\pi[i]$ = length of longest proper prefix of $P[0..i]$ that is also a suffix:

i	0	1	2	3	4	5
$P[i]$	a	b	c	a	b	c
$\pi[i]$	0	0	0	1	2	3

For $i = 5$: $P[0..5]$ = `abcabc`. The longest proper prefix that equals a suffix is `abc` (length 3), since $P[0..2] = P[3..5]$ = `abc`.

Hence $\pi[5] = $ **3**.

Answer: **(D) 3**.

Answer: (D) ↑ Go Back to Q25

Q26.

Solution

[Operating Systems – Buddy System]

In the buddy system, memory is allocated in blocks of size 2^k. When a block of size 2^k is split, it produces two *buddy* blocks each of size 2^{k-1}.

Splitting 128 KB $\rightarrow 64+64 \rightarrow 32+32 \rightarrow 16+16$: one 16 KB block is allocated; its buddy (the partner block produced in the same split) is also **16** KB.

When the allocated block is freed, it can be merged back with its 16 KB buddy to form a 32 KB block (and so on up the tree).

Answer: **(A) 16 KB**.

Answer: (A) ↑ Go Back to Q26

Solution

[Digital Circuits – JK Flip-Flop]

The characteristic equation of a JK flip-flop is:

$$Q^+ = J\overline{Q} + \overline{K}Q.$$

With $J = 1$, $K = 1$, and current state $Q = 0$:

$$Q^+ = 1 \cdot \overline{0} + \overline{1} \cdot 0 = 1 \cdot 1 + 0 \cdot 0 = 1.$$

In general, when $J = K = 1$, the flip-flop *toggles*: $Q^+ = \overline{Q}$.

Q27.

J	K	Q^+	Operation
0	0	Q	No change
0	1	0	Reset
1	0	1	Set
1	1	$\overline{Q}$	Toggle

Answer: **(B)** $Q^+ = \overline{Q}$ **(the flip-flop toggles)**.

Answer: (B) ↑ Go Back to Q27

Q28.

Solution

[Databases – Normal Forms]

Relation $R(A, B, C)$ with FDs $\{A \rightarrow B,\ A \rightarrow C\}$.

Candidate key: $A^+ = \{A, B, C\}$ = all attributes, so A is the (only) candidate key.

BCNF check: For every non-trivial FD $X \rightarrow Y$, is X a superkey? The only non-trivial FDs are $A \rightarrow B$ and $A \rightarrow C$, and A is indeed a superkey. So R is in **BCNF**

(Boyce-Codd Normal Form), which implies 3NF, 2NF, and 1NF.

Answer: **(C)** R **is in 3NF (and BCNF).**

Answer: (C) ↑ Go Back to Q28

Q29.

Solution

[Programming Languages – λ-Calculus, β-Reduction]

The term is $(\lambda x.\,(\lambda y.\,x+y))\;3$.

Applying one β-reduction step: the outer abstraction λx is applied to argument 3. Substitute $x := 3$ in the body $(\lambda y.\,x+y)$:

$$(\lambda x.\,(\lambda y.\,x+y))\;3 \;\to_\beta\; \lambda y.\,3+y.$$

The result is a function (an abstraction over y) that adds 3 to its argument. No further β-reduction is possible without supplying an argument for y.

Answer: **(D)** $\lambda y.\,3+y$.

Answer: (D) ↑ Go Back to Q29

Q30.

Solution

[Algorithms – Amortised Analysis, Table Doubling]

Accounting method: Charge each insertion an amortised cost of $c = 3$ units (a constant). Of these:

- 1 unit pays for the insertion itself.
- 2 units are saved as "credit" on the newly inserted element.

When the table doubles from capacity m to $2m$, we must copy m existing elements. Each of the $m/2$ elements inserted *since the last doubling* has saved 2 credit units, giving m credit units in total – exactly enough to pay for all m copies.

Since the amortised cost per operation is $O(1)$ (constant), the total actual cost for n insertions is $O(n)$.

Answer: **(A)** $O(1)$ **amortised per insertion;** $O(n)$ **total.**

Answer: (A) ↑ Go Back to Q30

Answer Key

Part A: Common (Q1–Q15)

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	A	3	B	4	C	5	D
6	A	7	B	8	C	9	D	10	A
11	B	12	C	13	D	14	A	15	B

Part B: Computer Science (Q16–Q30)

Q	Ans	Q	Ans
16	C	17	D
18	A	19	B
20	C	21	D
22	A	23	B
24	C	25	D
26	A	27	B
28	C	29	D
30	A		

Scan the QR code on each page footer to access more TIFR GS sample papers and solutions on Collegedunia.