IIT JAM Physics

Sample Paper – 2

Instructions

- This paper contains **60 questions** carrying a maximum of **100 marks**. Duration: **3 hours (180 minutes)**.
- **Section A** (**Q1–Q30**, MCQ): Q1–Q10 carry **+1 mark** each; wrong response: $-\frac{1}{3}$ mark. Q11–Q30 carry **+2 marks** each; wrong response: $-\frac{2}{3}$ mark. Choose the **single best** option.
- **Section B** (**Q31–Q40**, MSQ): Each carries **+2 marks**. **No negative marking. One or more** options may be correct; full marks only if *all* correct options are selected; no partial credit.
- **Section C** (**Q41–Q60**, NAT): Q41–Q50 carry **+1 mark**; Q51–Q60 carry **+2 marks**. **No negative marking.** Enter the numerical value using the virtual keypad (integer or decimal as appropriate).
- **Calculator policy:** Personal calculators, log tables, and electronic devices are **strictly prohibited**. A virtual scientific calculator is provided in the CBT interface. Rough sheets are supplied at the examination centre.
- Once you move to the next section, you cannot return to a previous section. Manage time accordingly.

Section A — Multiple Choice Questions (MCQ)

***Q1–Q10**: 1 mark each ($-\frac{1}{3}$ for wrong). Choose the **single** correct option.*

Q1. Using the residue theorem, the value of the contour integral

$$\oint_{|z|=3} \frac{z^2}{z^2+4}\, dz$$

evaluated counter-clockwise (the integrand has poles at $z = \pm 2i$, both inside $|z| = 3$) is:

(A) $2\pi i$
(B) $-2\pi i$
(C) 0
(D) $4\pi i$

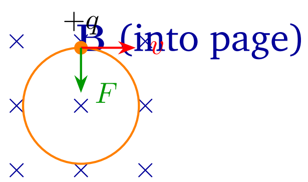
Q2. A bead of mass m is constrained to move on a smooth circular wire of radius R in a vertical plane. Using the angle θ measured from the bottom as the generalized coordinate, the equation of motion (from the Lagrangian $L = \frac{1}{2}mR^2\dot{\theta}^2 + mgR\cos\theta$) is:

(A) $R\ddot{\theta} = g\sin\theta$
(B) $R\ddot{\theta} = -g\sin\theta$
(C) $R\ddot{\theta} = -g\cos\theta$
(D) $R\ddot{\theta} = g\cos\theta$

Q3. A thin film of refractive index $n = 1.5$ is illuminated by light of wavelength $\lambda = 600\,\text{nm}$ in air. For the *reflected* light to show constructive interference (accounting for the phase shift of π at the top surface), the minimum film thickness t_{min} is:

(A) $100\,\text{nm}$
(B) $200\,\text{nm}$
(C) $150\,\text{nm}$
(D) $400\,\text{nm}$

Q4. A positive charge q moves with speed v at angle θ to a uniform magnetic field $\mathbf{B}$. The magnitude of the magnetic (Lorentz) force on the charge is $F = qvB\sin\theta$. When $v \perp B$ ($\theta = 90°$), the charge moves in a circular orbit as shown below. Which expression gives the correct magnitude of the force?



(A) $F = qvB\cos\theta$
(B) $F = qvB\tan\theta$
(C) $F = qv^2B$
(D) $F = qvB\sin\theta$

Q5. The Clausius statement of the second law of thermodynamics is:

(A) It is impossible to construct an engine with 100% efficiency
(B) The entropy of an isolated system can spontaneously decrease

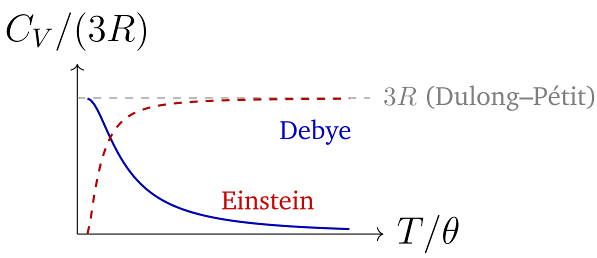
(C) It is impossible for heat to flow spontaneously from a colder body to a hotter body without external work

(D) All reversible engines operating between the same two temperatures have different efficiencies

Q6. The energy–time uncertainty relation $\Delta E\,\Delta t \geq \hbar/2$ implies that an excited atomic state with mean lifetime $\tau = 10^{-8}$ s has a minimum energy uncertainty of at least:

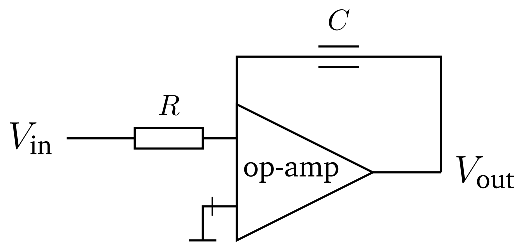
(A) $\Delta E \approx 1.6 \times 10^{-27}$ J
(B) $\Delta E \approx 5.3 \times 10^{-27}$ J
(C) $\Delta E \approx 1.0 \times 10^{-26}$ J
(D) $\Delta E \approx 2.1 \times 10^{-26}$ J

Q7. The Einstein model of the molar heat capacity of a solid treats the crystal as $3N$ independent harmonic oscillators, all with the same frequency ω_E. The model predicts which of the following behaviour for the heat capacity C_V at low temperatures ($T \ll \theta_E = \hbar\omega_E/k_B$)?



(A) C_V decreases exponentially to zero as $T \to 0$
(B) $C_V \propto T^3$ at low temperatures (same as Debye)
(C) $C_V \to 3R$ for all temperatures
(D) C_V is independent of temperature

Q8. An ideal op-amp integrator circuit has a resistor R at the input and capacitor C in the feedback path (shown below). For a time-varying input voltage $V_{\text{in}}(t)$, the output voltage $V_{\text{out}}(t)$ is:



(A) $V_{\text{out}} = -\dfrac{R}{C}V_{\text{in}}$

(B) $V_{\text{out}} = -(RC)\,V_{\text{in}}$

(C) $V_{\text{out}} = -\dfrac{1}{RC}V_{\text{in}}$

(D) $V_{\text{out}} = -\dfrac{1}{RC}\displaystyle\int V_{\text{in}}\,dt$

Q9. In the process of *electron capture* (EC), a nucleus captures one of its atomic electrons. As a result, the nuclear charge number Z and mass number A change as:

(A) Z increases by 1, A increases by 1

(B) Z increases by 1, A unchanged

(C) Z decreases by 1, A unchanged

(D) Z decreases by 2, A decreases by 4

Q10. A 3×3 real matrix has eigenvalues $\lambda_1 = 2$, $\lambda_2 = 3$, and $\lambda_3 = -1$. The determinant of the matrix equals the product of its eigenvalues. What is the determinant?

(A) 5

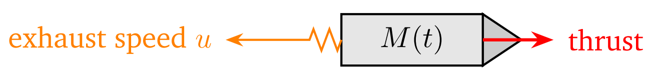
(B) -6

(C) 6

(D) -5

***Q11–Q30**: 2 marks each ($-\frac{2}{3}$ for wrong). Choose the **single** correct option.*

Q11. A rocket of initial mass M_0 burns fuel at mass rate μ (kg/s) and ejects the exhaust at speed u relative to the rocket (as shown). Starting from rest, the rocket velocity $v(t)$ at time t (before fuel exhaustion) is:



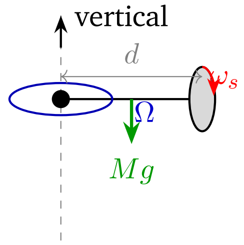
(A) $v(t) = u \ln \dfrac{M_0}{M_0 - \mu t}$

(B) $v(t) = u \ln \dfrac{M_0 t}{\mu}$

(C) $v(t) = \mu u \ln \dfrac{M_0}{M_0 - \mu t}$

(D) $v(t) = u \dfrac{M_0 - \mu t}{M_0}$

Q12. A gyroscope consists of a disk of moment of inertia I about its symmetry axis, spinning at angular velocity ω_s. The symmetry axis is horizontal; the disk's centre of mass is at distance d from the (fixed) pivot point. In the presence of gravity (g), the rate of steady precession Ω about the vertical axis (as shown) is:



(A) $\Omega = Mgd\,\omega_s/I$

(B) $\Omega = I\omega_s/(Mgd)$

(C) $\Omega = \sqrt{Mgd/I}$

(D) $\Omega = Mgd/(I\omega_s)$

Q13. A thin spherical shell of radius R carries a total charge Q uniformly distributed on its surface. The electric field at a point inside the shell (at distance $r < R$ from the centre) is:

(A) $\dfrac{Q}{4\pi\varepsilon_0 R^2}$ (same as on the surface)

(B) $\dfrac{Qr}{4\pi\varepsilon_0 R^3}$ (proportional to r)

(C) 0

(D) $\dfrac{Q}{4\pi\varepsilon_0 r^2}$ (as if the charge were at the centre)

Q14. One mole of an ideal gas at temperature T undergoes free expansion (against vacuum) from volume V to volume $2V$. Since T and U do not change, the entropy change ΔS is:

(A) $R\ln 3$
(B) $R\ln 2$
(C) 0
(D) $-R\ln 2$

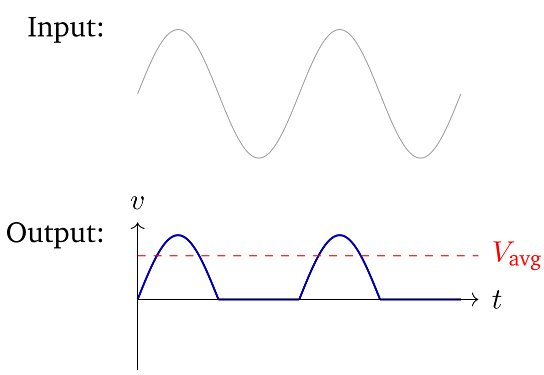
Q15. For a free-particle wave packet $\psi(x,t) = A\,e^{i(kx-\omega t)}$ with dispersion relation $\omega = \hbar k^2/(2m)$, the group velocity $v_g = d\omega/dk$ and phase velocity $v_p = \omega/k$ satisfy:

(A) $v_g = \hbar k/m = p/m$ (particle velocity) and $v_p = \hbar k/(2m) = v_g/2$
(B) $v_g = \hbar k/(2m)$ and $v_p = \hbar k/m$
(C) $v_g = v_p = \hbar k/m$
(D) $v_g = v_p = 0$

Q16. Bloch's theorem for an electron in a periodic crystal potential $V(\mathbf{r}) = V(\mathbf{r}+\mathbf{a})$ states that the eigenfunctions (Bloch functions) have the form:

(A) $\psi(\mathbf{r}) = A\,e^{i\mathbf{k}\cdot\mathbf{r}}$ (pure plane waves)
(B) $\psi(\mathbf{r}) = u(\mathbf{r})$, where u is a constant
(C) $\psi(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r})\,e^{i\mathbf{k}\cdot\mathbf{r}}$, where $u_{\mathbf{k}}$ varies randomly
(D) $\psi_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r})\,e^{i\mathbf{k}\cdot\mathbf{r}}$, where $u_{\mathbf{k}}(\mathbf{r})$ has the same periodicity as the lattice

Q17. A half-wave rectifier passes only the positive half-cycles of a sinusoidal input voltage $v_{\text{in}}(t) = V_m \sin\omega t$, as shown below. For this output, the average (DC) value V_{avg} is:



(A) $V_{\text{avg}} = V_m/2$
(B) $V_{\text{avg}} = 2V_m/\pi$
(C) $V_{\text{avg}} = V_m/\pi$

(D) $V_{\text{avg}} = V_m/\sqrt{2}$

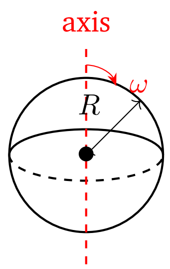
Q18. Which of the following correctly describes the nuclear binding energy per nucleon (BE/A) as a function of mass number A?

(A) BE/A is approximately constant (≈ 8 MeV) for all values of A
(B) BE/A first increases with A, reaches a maximum near $A \approx 56$ (iron group), then decreases for heavier nuclei
(C) BE/A decreases monotonically as A increases
(D) BE/A increases monotonically and without bound as A increases

Q19. The Taylor series expansion of $\cos x$ about $x = 0$ is:

(A) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$
(B) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{3!} - \frac{x^6}{4!} + \cdots$
(C) $\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{8} - \cdots$
(D) $\cos x = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots$

Q20. The moment of inertia of a thin uniform *hollow* spherical shell of mass M and radius R about a diameter (axis through the centre) is shown below:

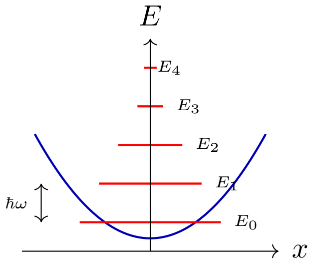


(A) $I = \frac{1}{2}MR^2$
(B) $I = \frac{1}{4}MR^2$
(C) $I = \frac{2}{5}MR^2$
(D) $I = \frac{2}{3}MR^2$

Q21. Two inductors with self-inductances L_1 and L_2 are magnetically coupled with mutual inductance M. The coupling coefficient k (satisfying $0 \leq k \leq 1$, with $k = 1$ for perfect coupling) is defined as:

(A) $k = M\sqrt{L_1 L_2}$
(B) $k = \dfrac{M}{L_1 + L_2}$
(C) $k = \dfrac{M}{\sqrt{L_1 L_2}}$
(D) $k = \dfrac{M^2}{L_1 L_2}$

Q22. The canonical partition function of a one-dimensional quantum harmonic oscillator with energy levels $E_n = (n + \frac{1}{2})\hbar\omega$ (shown below) is $Z = \sum_{n=0}^{\infty} e^{-\beta E_n}$. At high temperature ($k_B T \gg \hbar\omega$), the partition function Z approaches:



(A) $Z \to 1$
(B) $Z \to k_B T/(\hbar\omega)$
(C) $Z \to e^{-\hbar\omega/(2k_B T)}$
(D) $Z \to \hbar\omega/(k_B T)$

Q23. For the ground state of the quantum harmonic oscillator (QHO) with Hamiltonian $\hat{H} = \hat{p}^2/(2m) + \frac{1}{2}m\omega^2\hat{x}^2$, the expectation value $\langle \hat{x}^2 \rangle$ in the ground state $|0\rangle$ is:

(A) $\dfrac{\hbar}{2m\omega}$
(B) $\dfrac{\hbar\omega}{2m}$
(C) $\dfrac{\hbar}{m\omega}$
(D) 0

Q24. For an intrinsic (undoped) semiconductor at temperature T with band gap E_g, the Fermi level E_F lies:

(A) In the conduction band

(B) At the valence band edge E_V
(C) At the conduction band edge E_C
(D) Approximately in the middle of the band gap (between E_V and E_C)

Q25. A 555-timer IC connected in astable mode uses resistors R_1 and R_2 and capacitor C. The approximate output frequency f is given by:

(A) $f \approx \dfrac{1.44}{(R_1 + R_2)C}$
(B) $f \approx \dfrac{0.72}{(R_1 + 2R_2)C}$
(C) $f \approx \dfrac{1.44}{(R_1 + 2R_2)C}$
(D) $f \approx \dfrac{2.88}{(R_1 + 2R_2)C}$

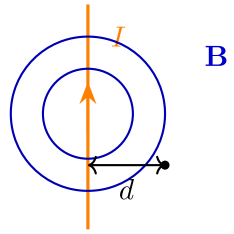
Q26. For the hydrogen atom treated non-relativistically (without spin-orbit coupling), the energy levels depend only on the principal quantum number n. Including the electron spin, the total degeneracy of the n-th energy level (number of distinct quantum states) is:

(A) n
(B) $2n^2$
(C) n^2
(D) $2n$

Q27. The Maxwell–Boltzmann speed distribution function $f(v)$ for molecules of mass m in a gas at temperature T is maximised at the most probable speed v_{mp}. The value of v_{mp} is:

(A) $v_{\text{mp}} = \sqrt{2k_BT/m}$
(B) $v_{\text{mp}} = \sqrt{3k_BT/m}$
(C) $v_{\text{mp}} = \sqrt{8k_BT/(\pi m)}$
(D) $v_{\text{mp}} = \sqrt{k_BT/m}$

Q28. An infinite straight wire carries a steady current I. Using the Biot–Savart law (or Ampère's law), the magnetic field B at perpendicular distance d from the wire (as shown) has magnitude:



(A) $B = \dfrac{\mu_0 I}{4\pi d}$

(B) $B = \dfrac{\mu_0 I d}{2\pi}$

(C) $B = \dfrac{2\mu_0 I}{\pi d}$

(D) $B = \dfrac{\mu_0 I}{2\pi d}$

Q29. In Young's double-slit experiment, a source of wavelength $\lambda = 550\,\text{nm}$ illuminates two slits separated by $d = 0.5\,\text{mm}$. The screen is at distance $D = 2\,\text{m}$. The fringe separation $\beta = \lambda D/d$ is:

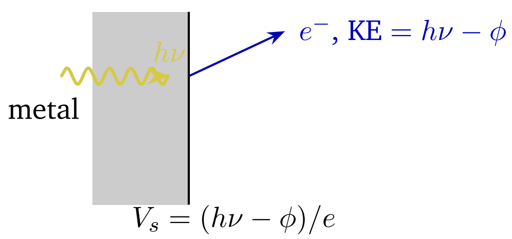
(A) $1.1\,\text{mm}$

(B) $4.4\,\text{mm}$

(C) $2.2\,\text{mm}$

(D) $0.55\,\text{mm}$

Q30. In the photoelectric effect, light of wavelength $\lambda = 200\,\text{nm}$ falls on a metal surface with work function $\phi = 4.5\,\text{eV}$. The photon energy is $h\nu = hc/\lambda \approx 6.22\,\text{eV}$. The stopping potential V_s (satisfying $eV_s = h\nu - \phi$) is shown below:



(A) $V_s \approx 1.22\,\text{V}$

(B) $V_s \approx 1.72\,\text{V}$

(C) $V_s \approx 2.50\,\text{V}$

(D) $V_s \approx 6.22\,\text{V}$

Section B — Multiple Select Questions (MSQ)

***Q31–Q40**: 2 marks each. **No negative marking. One or more** options may be correct. Full marks only if* all *correct options (and no incorrect options) are chosen.*

Q31. In a perfectly *elastic* collision between two particles, which of the following quantities are necessarily conserved?

(A) Total kinetic energy
(B) Individual particle velocities (magnitudes and directions are unchanged)
(C) Total linear momentum
(D) Total mechanical energy (kinetic + potential, for a conservative system)

Q32. Which of the following statements about the static magnetic field $\mathbf{B}$ are correct?

(A) $\nabla \cdot \mathbf{B} = 0$ everywhere (no magnetic monopoles)
(B) Magnetic field lines always form closed loops (or extend to infinity)
(C) The magnetic force can do work on a stationary electric charge
(D) Magnetic flux $\Phi = \iint \mathbf{B} \cdot d\mathbf{A}$ is measured in webers (Wb = V·s)

Q33. For a reversible *adiabatic* process of an ideal gas, which of the following are correct?

(A) Temperature remains constant throughout the process
(B) No heat is exchanged with the surroundings ($Q = 0$)
(C) For a reversible adiabatic process: PV^γ = constant (where $\gamma = C_P/C_V$)
(D) The internal energy change $\Delta U = 0$

Q34. For the one-dimensional quantum harmonic oscillator with energy eigenvalues $E_n = (n + \frac{1}{2})\hbar\omega$, which of the following are correct?

(A) The energy eigenvalues are $E_n = (n + \frac{1}{2})\hbar\omega$ for $n = 0, 1, 2, \ldots$
(B) The ground state ($n = 0$) has non-zero zero-point energy $E_0 = \frac{1}{2}\hbar\omega$
(C) The spacing between adjacent energy levels increases with n

(D) The creation and annihilation operators satisfy the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$

Q35. In the free-electron (Sommerfeld) model of metals, which of the following statements are correct?

(A) Conduction electrons obey Fermi–Dirac statistics

(B) At absolute zero ($T = 0$), all energy states up to the Fermi energy E_F are empty

(C) The Fermi energy E_F is proportional to $n^{2/3}$, where n is the conduction electron density

(D) Even at $T = 0$, the electron gas exerts a non-zero pressure (Fermi pressure or degeneracy pressure)

Q36. The Dirac delta function $\delta(x)$ is a distribution. Which of the following properties are correct?

(A) $\displaystyle\int_{-\infty}^{\infty} \delta(x)\, dx = 1$

(B) $\delta(x) = 0$ for all $x \neq 0$

(C) $\delta(x)$ is a regular (ordinary) function of x

(D) $\displaystyle\int_{-\infty}^{\infty} f(x)\, \delta(x-a)\, dx = f(a)$ for any continuous f

Q37. Concerning natural radioactive decay series (e.g. the uranium-238 series ending at Pb-206), which of the following statements are correct?

(A) The decay series terminates at a stable (non-radioactive) isotope

(B) All intermediate radioactive products have half-lives longer than the parent nucleus

(C) In secular equilibrium (parent half-life $\gg$ daughter half-lives), the activities of all members of the series are equal: $\lambda_1 N_1 = \lambda_2 N_2 = \cdots$

(D) Each step in the decay series conserves mass-energy (the energy is released as radiation/kinetic energy of decay products)

Q38. Which of the following statements about thin-film interference (e.g. a soap film in air) are correct?

(A) Colours in soap films arise because different wavelengths satisfy constructive interference conditions at different thicknesses/angles

(B) For a soap film of refractive index n in air, the minimum thickness for constructive reflection is $\lambda/(4n)$

(C) A phase shift of π (equivalent to a path difference of $\lambda/2$) occurs when light reflects at a boundary going from a lower to a higher refractive index (denser medium)

(D) Thin-film colour patterns are a manifestation of diffraction, not interference

Q39. Which of the following statements about an *ideal* operational amplifier (op-amp) are correct?

(A) An ideal op-amp has infinite input impedance (draws no current from the input)

(B) An ideal op-amp has zero output impedance (can supply any current without voltage drop)

(C) An ideal op-amp has a finite open-loop voltage gain of approximately 10^5

(D) In a negative-feedback amplifier using an op-amp, the closed-loop gain is stabilised and made less sensitive to op-amp parameters

Q40. Which of the following statements about blackbody (thermal) radiation are correct?

(A) The total radiated power per unit area is proportional to T^2 (where T is the absolute temperature)

(B) The peak wavelength λ_{max} decreases as temperature increases, according to Wien's displacement law: $\lambda_{\text{max}}T = b$ (constant)

(C) The spectral energy density of blackbody radiation is correctly described by the Planck distribution

(D) The Rayleigh–Jeans law is valid at all wavelengths and all temperatures

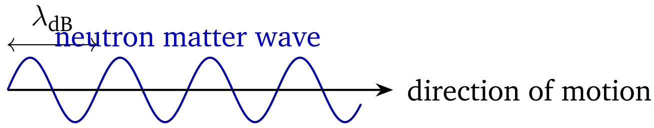
Section C — Numerical Answer Type (NAT)

***Q41–Q50**: 1 mark each. **No negative marking.** Enter the exact numerical value.*

Q41. The Earth rotates once in $T = 24\,\text{h} = 86400\,\text{s}$. Calculate the angular velocity $\omega = 2\pi/T$ of Earth's rotation in units of 10^{-5} rad/s, correct to two decimal places.

Q42. A $10\,\mu$F capacitor is charged to $V = 15\,$V. Using $Q = CV$, find the charge on each plate in microcoulombs (μC), correct to two decimal places.

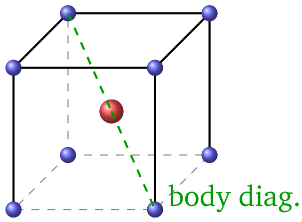
Q43. Calculate the de Broglie wavelength (in pm) of a thermal neutron with kinetic energy $KE = 0.025\,$eV. Use $m_n = 1.675 \times 10^{-27}\,$kg, $h = 6.626 \times 10^{-34}\,$J s, $1\,$eV $= 1.6 \times 10^{-19}\,$J.



Q44. Calculate the root-mean-square speed v_{rms} (in m/s, to the nearest integer) of hydrogen (H_2) molecules at $T = 300\,$K. (Molar mass of $H_2 = 2\,$g/mol; $R = 8.314\,\text{J}\,\text{mol}^{-1}\text{K}^{-1}$.)

Q45. A radioactive sample has N_0 atoms at time $t = 0$. After exactly 3 years, only 25% ($= N_0/4$) of the original atoms remain undecayed. Find the half-life $T_{1/2}$ of this isotope in years, correct to two decimal places.

Q46. Iron has a BCC crystal structure with lattice constant $a = 2.87\,$Å. In the BCC structure, nearest neighbours are connected along the body diagonal. Calculate the nearest-neighbour distance (in Å, to two decimal places).



Q47. Find the eigenvalues of the matrix $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. The eigenvalues are purely imaginary: $\lambda = \pm i$. Enter the magnitude $|\lambda|$ of either eigenvalue.

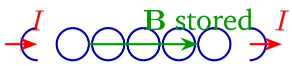
Q48. A damped oscillator satisfies $\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = 0$ with $\omega_0 = 100\,\text{rad/s}$ and damping coefficient $\gamma = 10\,\text{s}^{-1}$. The damped angular frequency is $\omega_d = \sqrt{\omega_0^2 - \gamma^2}$. Calculate the damped oscillation frequency $f = \omega_d/(2\pi)$ in Hz, correct to two decimal places.

Q49. A non-inverting op-amp amplifier has $R_1 = 3\,\text{k}\Omega$ (from V^- to ground) and $R_f = 27\,\text{k}\Omega$ (from output to V^-). For an input $V_{\text{in}} = 0.5\,\text{V}$, the gain is $G = 1 + R_f/R_1$. Find the output voltage V_{out} in volts, correct to one decimal place.

Q50. Calculate the total translational kinetic energy (in J, correct to one decimal place) of 1 mole of a monatomic ideal gas at temperature $T = 300\,\text{K}$. (Use $R = 8.314\,\text{J}\,\text{mol}^{-1}\text{K}^{-1}$; for a monatomic ideal gas, $U = \frac{3}{2}nRT$.)

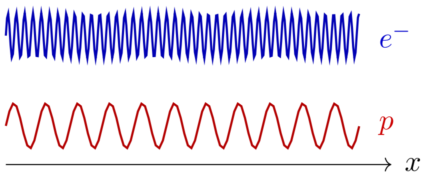
Q51–Q60**: 2 marks each.* ***No negative marking. *Enter the exact numerical value.*

Q51. An inductor of inductance $L = 2.0\,\text{H}$ carries a steady current $I = 5.0\,\text{A}$. The energy stored in the magnetic field of the inductor (as illustrated below) is $E = \frac{1}{2}LI^2$. Find E in joules, correct to two decimal places.



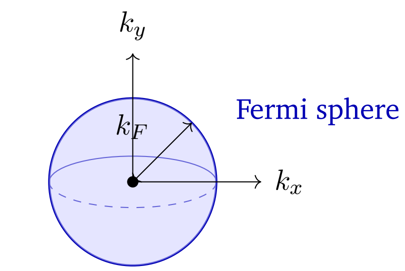
Energy $= \frac{1}{2}LI^2$

Q52. A proton and an electron have the same kinetic energy KE. Their de Broglie wavelengths are $\lambda_p = h/\sqrt{2m_pKE}$ and $\lambda_e = h/\sqrt{2m_eKE}$ respectively. Calculate the ratio λ_e/λ_p (to two decimal places). Use $m_p/m_e = 1836$.



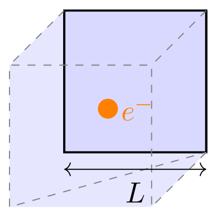
Q53. One mole of an ideal gas expands isothermally and reversibly at $T = 400\,\text{K}$ from pressure $P_1 = 4\,\text{atm}$ to $P_2 = 1\,\text{atm}$. The work done by the gas is $W = nRT\ln(P_1/P_2)$. Calculate W in joules, to the nearest integer. ($R = 8.314\,\text{J}\,\text{mol}^{-1}\text{K}^{-1}$, $\ln 4 = 1.3863$.)

Q54. The Fermi energy of potassium (electron number density $n = 1.40 \times 10^{28}\,\mathrm{m^{-3}}$) is given by $E_F = \dfrac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$. Calculate E_F in eV, correct to two decimal places. ($\hbar = 1.055 \times 10^{-34}\,\mathrm{J\,s}$, $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$, $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$.)



(states filled inside)

Q55. An electron is confined in a three-dimensional cubic box (infinite potential well) of side $L = 10\,\mathrm{nm}$. The ground-state energy is $E_{111} = \dfrac{3\pi^2\hbar^2}{2m_e L^2}$. Calculate E_{111} in meV (millielectronvolts), correct to one decimal place. ($\hbar = 1.055 \times 10^{-34}\,\mathrm{J\,s}$, $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$, $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$.)



Q56. A pipe of length $L = 0.68\,\mathrm{m}$ is open at both ends. The speed of sound in air is $v = 340\,\mathrm{m/s}$. For a pipe open at both ends, the fundamental frequency is $f_1 = v/(2L)$. Calculate f_1 in Hz, to the nearest integer.

Q57. A radioactive isotope has half-life $T_{1/2}$. Calculate the fraction of the original number of nuclei that remains undecayed after a time $t = 3T_{1/2}$ (three half-lives). Express the answer as a decimal to four decimal places.

Q58. Evaluate the definite integral

$$I = \int_0^\infty x\, e^{-x^2}\, dx$$

using the substitution $u = x^2$ (or direct antiderivative). Give the exact value as a decimal correct to three decimal places.

Q59. A capacitor C is charged through a resistor R from a battery of EMF $\mathcal{E} = 10\,\text{V}$. The voltage across the capacitor at time t is $V_C(t) = \mathcal{E}(1 - e^{-t/RC})$. Calculate V_C at $t = \tau = RC$ (one time constant) in volts, correct to one decimal place. (Use $e^{-1} = 0.3679$.)

Q60. Calculate the momentum $p = h/\lambda$ of a photon with wavelength $\lambda = 400\,\text{nm}$, in units of $10^{-27}\,\text{kg m/s}$, correct to two decimal places. ($h = 6.626 \times 10^{-34}\,\text{J s}$.)

Solutions

IIT JAM Physics – Sample Paper 2

Section A Solutions — MCQ

Solution

Concept — Residue Theorem (Complex Analysis):
For $f(z) = \frac{z^2}{z^2+4} = \frac{z^2}{(z+2i)(z-2i)}$, the poles are at $z = 2i$ and $z = -2i$.

Step 1 — Check which poles lie inside $|z| = 3$:
$|2i| = 2 < 3$ ✓; $|-2i| = 2 < 3$ ✓. Both poles are enclosed.

Step 2 — Compute residues:

$$\text{Res}(f,\, 2i) = \lim_{z\to 2i}(z-2i)\cdot\frac{z^2}{(z+2i)(z-2i)} = \frac{(2i)^2}{2i+2i} = \frac{-4}{4i} = \frac{-1}{i} = i$$

$$\text{Res}(f, -2i) = \lim_{z\to -2i}(z+2i)\cdot\frac{z^2}{(z+2i)(z-2i)} = \frac{(-2i)^2}{-2i-2i} = \frac{-4}{-4i} = \frac{1}{i} = -i$$

Step 3 — Apply residue theorem:

$$\oint_{|z|=3}\frac{z^2}{z^2+4}\,dz = 2\pi i\big(\text{Res at } 2i + \text{Res at } -2i\big) = 2\pi i\,(i+(-i)) = 2\pi i\cdot 0 = 0$$

Why other options are wrong:

Q1.
- (A) $2\pi i$: Counts only one residue; the two residues exactly cancel.
- (B) $-2\pi i$: Sign error; both residues sum to zero.
- (D) $4\pi i$: No physical basis; sum of residues is zero.

Final Answer: $0 \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q1*

Solution

Concept — Lagrangian Mechanics, Bead on a Wire:
The Lagrangian is $L = \frac{1}{2}mR^2\dot{\theta}^2 + mgR\cos\theta$.

Step 1 — Euler–Lagrange equation:

$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\theta}} - \frac{\partial L}{\partial\theta} = 0$$

$$\frac{\partial L}{\partial \dot\theta} = mR^2\dot\theta, \quad \frac{\partial L}{\partial \theta} = -mgR\sin\theta$$

$$mR^2\ddot\theta - (-mgR\sin\theta) = 0$$

$$mR^2\ddot\theta + mgR\sin\theta = 0 \;\Rightarrow\; R\ddot\theta = -g\sin\theta$$

This is the pendulum-like equation of motion (with $\theta = 0$ at the bottom, stable equilibrium).

Final Answer: $R\ddot\theta = -g\sin\theta \Rightarrow$ **(B)**

Why other options are wrong:

Q2.

- (A) $+g\sin\theta$: Wrong sign; restoring force opposes displacement.
- (C) $-g\cos\theta$: Incorrect function; $\sin\theta$ appears from $\partial(\cos\theta)/\partial\theta$.
- (D) $+g\cos\theta$: Both sign and function are wrong.

Answer: (B) ← *Go back to Q2*

Solution

Concept — Thin-Film Interference (Reflected Light):
For a thin film of refractive index n in air, the reflected beam from the top surface undergoes a phase shift of π (reflection off denser medium); the beam reflected from the bottom surface does not (reflection off a less dense medium going outward).

Step 1 — Constructive interference condition for reflected light:
The net phase difference must be a multiple of 2π. With one phase-flip of π, constructive interference requires the optical path difference $2nt$ to equal a half-integer multiple of λ:

$$2nt = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, 1, 2, \ldots$$

Step 2 — Minimum thickness ($m = 0$):

$$t_{\text{min}} = \frac{\lambda}{4n} = \frac{600\,\text{nm}}{4 \times 1.5} = \frac{600}{6}\,\text{nm} = 100\,\text{nm}$$

Final Answer: $t_{\text{min}} = 100\,\text{nm} \Rightarrow$ **(A)**

Why other options are wrong:

Q3.

- (B) $200\,\text{nm}$: This would be $\lambda/(2n)$, the minimum for destructive reflection.
- (C) $150\,\text{nm}$: No simple physical basis for this value.
- (D) $400\,\text{nm}$: Uses $m = 2$ and incorrect formula.

Answer: (A) ← *Go back to Q3*

Solution

Concept — Lorentz Magnetic Force:
The magnetic force on a charge q moving with velocity $\mathbf{v}$ in field $\mathbf{B}$ is $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$.

Step 1 — Magnitude:

$$F = qvB \sin\theta$$

where θ is the angle between $\mathbf{v}$ and $\mathbf{B}$.

Step 2 — Special cases:

Q4.

- $\theta = 90°$: $F = qvB$ (maximum, circular motion)
- $\theta = 0°$: $F = 0$ (no force when parallel to $\mathbf{B}$)

The diagram shows circular orbit when $\mathbf{v} \perp \mathbf{B}$; centripetal force is provided by the magnetic force.

Final Answer: $F = qvB \sin\theta \Rightarrow$ **(D)**

Why other options are wrong:

- (A) $qvB \cos\theta$: Would give maximum force at $\theta = 0$ (parallel), which is physically wrong.
- (B) $qvB \tan\theta$: Not a valid form of the Lorentz force.
- (C) qv^2B: Has incorrect dimensions (extra factor of v).

$\boxed{\textbf{Answer: (D)}}$ ← *Go back to Q4*

Solution

Concept — Second Law of Thermodynamics (Clausius Statement):
The second law has two classical formulations:

Q5.

- **Kelvin–Planck:** No cyclic heat engine can convert heat entirely into work.
- **Clausius:** It is *impossible* for heat to flow spontaneously from a cold body to a hot body *without* the expenditure of external work.

The Clausius statement directly addresses the direction of spontaneous heat flow (the "refrigerator impossibility").

Final Answer: Option (C) $\Rightarrow$ **(C)**

Why other options are wrong:

- (A): This is the Kelvin–Planck statement (heat-engine version), not the Clausius statement.
- (B): This is false as stated; the second law says entropy of an isolated system *never decreases*.
- (D): False; all reversible engines between the same two reservoirs have the

same (Carnot) efficiency.

Answer: (C) ← *Go back to Q5*

Solution

Concept — Energy–Time Uncertainty Principle:
$\Delta E\,\Delta t \geq \hbar/2$, so for minimum uncertainty:

$$\Delta E_{\text{min}} = \frac{\hbar}{2\tau}$$

Calculation:

$$\Delta E_{\text{min}} = \frac{1.055\times10^{-34}\,\text{J s}}{2\times10^{-8}\,\text{s}} = \frac{1.055\times10^{-34}}{2\times10^{-8}} = 5.275\times10^{-27}\,\text{J} \approx 5.3\times10^{-27}\,\text{J}$$

Final Answer: $\Delta E \approx 5.3\times10^{-27}\,\text{J} \Rightarrow$ **(B)**

Why other options are wrong:

Q6.
- (A) 1.6×10^{-27} J: This corresponds to $\hbar/\tau$ divided by ~ 66; does not arise from the formula.
- (C) 1.0×10^{-26} J: Approximately $2\times$ the correct value.
- (D) 2.1×10^{-26} J: Off by a factor of ~ 4.

Answer: (B) ← *Go back to Q6*

Solution

Concept — Einstein Model of Specific Heat:
The Einstein model gives each atom a quantised oscillator with energy levels $E_n = (n+\frac{1}{2})\hbar\omega_E$. The molar heat capacity is:

$$C_V^{\text{Ein}} = 3R\left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{(e^{\theta_E/T}-1)^2}$$

where $\theta_E = \hbar\omega_E/k_B$ is the Einstein temperature.

Behaviour at low T ($T \ll \theta_E$):
For $T \to 0$: $e^{\theta_E/T} \to \infty$, so $C_V^{\text{Ein}} \approx 3R(\theta_E/T)^2 e^{-\theta_E/T} \to 0$ *exponentially*.
This is different from the Debye model which predicts $C_V \propto T^3$ at low T.

High-T limit: $C_V \to 3R$ (Dulong–Pétit law), reproduced by both models.

Final Answer: C_V decreases exponentially to zero as $T \to 0 \Rightarrow$ **(A)**

Why other options are wrong:

Q7.
- (B) $C_V \propto T^3$: This is the Debye (not Einstein) prediction at low T.
- (C) $C_V \to 3R$ for all T: Only correct at high T; wrong at low T.
- (D) Independent of T: Contradicts the quantisation of oscillators.

Answer: (A) ← *Go back to Q7*

Solution

Concept — Op-Amp Integrator Circuit:
With resistor R at the inverting input and capacitor C in the feedback path, KCL at the virtual ground node gives:

$$\frac{V_{\text{in}}}{R} + C\frac{dV_{\text{out}}}{dt} = 0$$

(the op-amp's virtual ground keeps $V^- \approx 0$, and no current enters the op-amp inputs ideally).

Rearranging:

$$\frac{dV_{\text{out}}}{dt} = -\frac{V_{\text{in}}}{RC} \Rightarrow V_{\text{out}}(t) = -\frac{1}{RC}\int_0^t V_{\text{in}}(t')\,dt' + V_{\text{out}}(0)$$

Final Answer: $V_{\text{out}} = -\dfrac{1}{RC}\displaystyle\int V_{\text{in}}\,dt \Rightarrow$ **(D)**

Why other options are wrong:

Q8.
- (A) $-R/C \times V_{\text{in}}$: Wrong dimensions ($R/C$ has units $\Omega/\text{F} = \text{s}^{-1}$ not dimensionless).
- (B) $-(RC) \times V_{\text{in}}$: This gives the output of a differentiator multiplied by RC^2.
- (C) $-(1/RC) \times V_{\text{in}}$: Missing the integral; this would be instantaneous (not integrating).

Answer: (D) ← *Go back to Q8*

Solution

Concept — Electron Capture (EC):
In electron capture, a proton in the nucleus absorbs an orbital electron:

$$p + e^- \to n + \nu_e$$

As a result: one proton becomes a neutron $\Rightarrow$ Z decreases by 1, A is unchanged (mass number does not change since a proton is replaced by a neutron).

Comparison with other decay modes:

Q9.

- α decay: $Z \to Z - 2$, $A \to A - 4$
- β^- decay: $Z \to Z + 1$, A unchanged
- β^+ (or EC): $Z \to Z - 1$, A unchanged
- EC and β^+ have the same nuclear effect on Z and A, but differ in what is emitted.

Final Answer: Z decreases by 1, A unchanged $\Rightarrow$ **(C)**

Why other options are wrong:

- (A) Z increases by 1: This is β^- decay; EC does the opposite.
- (B) Z increases by 1, A unchanged: Again describes β^-, not EC.
- (D) $Z - 2$, $A - 4$: This is alpha decay, not electron capture.

Answer: (C) ← *Go back to Q9*

Solution

Concept — Determinant as Product of Eigenvalues:
For any square matrix A with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$:

$$\det(A) = \lambda_1 \, \lambda_2 \cdots \lambda_n$$

Calculation:

$$\det(A) = \lambda_1 \times \lambda_2 \times \lambda_3 = 2 \times 3 \times (-1) = -6$$

Final Answer: $\det(A) = -6 \Rightarrow$ **(B)**

Why other options are wrong:

Q10.

- (A) 5: This is the sum of the eigenvalues ($= \mathrm{tr}(A)$), not the product.
- (C) 6: Correct magnitude but wrong sign; the eigenvalue -1 makes the determinant negative.
- (D) -5: This would be the negative of the trace, not the determinant.

Answer: (B) ← *Go back to Q10*

Solution

Concept — Rocket Equation (Tsiolkovsky):
Let $M(t) = M_0 - \mu t$ be the rocket mass at time t. The exhaust speed is u relative to

the rocket.

Step 1 — Equation of motion (no gravity):

$$M(t)\frac{dv}{dt} = \mu u$$

(thrust = exhaust mass rate × exhaust speed).

Step 2 — Integrate (from rest, $v_0 = 0$):

$$\int_0^v dv' = \int_0^t \frac{\mu u}{M_0 - \mu t'}\,dt' = u\Big[-\ln(M_0 - \mu t')\Big]_0^t = u\ln\frac{M_0}{M_0 - \mu t}$$

Final Answer: $v(t) = u\ln\dfrac{M_0}{M_0 - \mu t} \Rightarrow$ **(A)**

Why other options are wrong:

Q11.

- (B) $u\ln(M_0 t/\mu)$: Not dimensionally consistent with the rocket equation.
- (C) $\mu u\ln(\ldots)$: Extra factor of μ; this has wrong units (kg/s × m/s × dimensionless = kg·m/s^2).
- (D) $u(M_0 - \mu t)/M_0$: Linear in mass ratio, not logarithmic.

Answer: (A) *← Go back to Q11*

Solution

Concept — Gyroscope Precession:
For a gyroscope with spin angular momentum $L_s = I\omega_s$ about its symmetry axis, the torque due to gravity is $\tau = Mgd$ (tending to lower the spin axis).

Step 1 — Precession rate from $d\mathbf{L}/dt = \boldsymbol{\tau}$:
The torque causes the angular momentum vector to precess about the vertical axis:

$$\Omega = \frac{\tau}{L_s} = \frac{Mgd}{I\omega_s}$$

The faster the spin ω_s, the slower the precession (gyroscopic stability).

Final Answer: $\Omega = Mgd/(I\omega_s) \Rightarrow$ **(D)**

Why other options are wrong:

Q12.

- (A) $Mgd\omega_s/I$: Inverted; precession rate is inversely proportional to spin, not proportional.
- (B) $I\omega_s/(Mgd)$: This is the reciprocal; numerator and denominator are swapped.
- (C) $\sqrt{Mgd/I}$: This has the form of an angular frequency from energy argu-

ments, not precession.

Answer: (D) ← *Go back to Q12*

Solution

Concept — Electric Field Inside a Uniformly Charged Spherical Shell:
Gauss's law: $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\varepsilon_0$.

Gaussian surface: A sphere of radius $r < R$ (inside the shell).
$Q_{\text{enc}} = 0$ (all charge Q is on the shell at radius R; none is enclosed for $r < R$).

Therefore:

$$\oint \mathbf{E} \cdot d\mathbf{A} = E \cdot 4\pi r^2 = \frac{Q_{\text{enc}}}{\varepsilon_0} = 0 \;\Rightarrow\; E = 0$$

By the symmetry and Gauss's law, $\mathbf{E} = 0$ *everywhere* inside a uniformly charged spherical shell.

Final Answer: $E = 0 \Rightarrow$ **(C)**

Why other options are wrong:

Q13.

- (A) $Q/(4\pi\varepsilon_0 R^2)$: This is the field at the surface ($r = R$), not inside.
- (B) $Qr/(4\pi\varepsilon_0 R^3)$: This is the field inside a uniformly charged *solid* sphere.
- (D) $Q/(4\pi\varepsilon_0 r^2)$: This is the field outside ($r > R$) treated as a point charge.

Answer: (C) ← *Go back to Q13*

Solution

Concept — Entropy Change in Free Expansion of Ideal Gas:
In free expansion (against vacuum), no work is done ($W = 0$) and no heat is exchanged ($Q = 0$). For an ideal gas, internal energy depends only on T, so $\Delta U = 0$ and T remains constant.

Calculate ΔS: Although the process is irreversible, entropy is a state function. We compute ΔS using a reversible isothermal path between the same initial and final states:

$$\Delta S = \int \frac{\delta Q_{\text{rev}}}{T} = nR\ln\frac{V_f}{V_i} = 1 \times R \times \ln\frac{2V}{V} = R\ln 2$$

Since $\Delta S > 0$, entropy increases (irreversible process).

Final Answer: $\Delta S = R\ln 2 \Rightarrow$ **(B)**

Why other options are wrong:

Q14.

- (A) $R\ln 3$: Would require expansion to $3V$, not $2V$.

- (C) 0: Zero entropy change holds only for reversible adiabatic processes; free expansion is irreversible.
- (D) $-R\ln 2$: Negative entropy change would violate the second law for an isolated system.

Answer: (B) ← *Go back to Q14*

Solution

Concept — Group and Phase Velocities of a Free Particle:
For a free particle: $E = p^2/(2m) = \hbar^2k^2/(2m)$, so $\omega = \hbar k^2/(2m)$.

Step 1 — Phase velocity:

$$v_p = \frac{\omega}{k} = \frac{\hbar k}{2m} = \frac{p}{2m} = \frac{v}{2}$$

(half the classical particle velocity).

Step 2 — Group velocity:

$$v_g = \frac{d\omega}{dk} = \frac{2\hbar k}{2m} = \frac{\hbar k}{m} = \frac{p}{m} = v$$

(equals the classical particle velocity).

Final Answer: $v_g = p/m$ (particle velocity), $v_p = p/(2m) = v_g/2 \Rightarrow$ **(A)**

Why other options are wrong:

Q15.
- (B): Swaps group and phase velocities.
- (C): $v_g = v_p$ holds only for non-dispersive media; free-particle dispersion is quadratic.
- (D): Both velocities non-zero for any moving wave packet.

Answer: (A) ← *Go back to Q15*

Solution

Concept — Bloch's Theorem:
For an electron in a crystal with periodic potential $V(\mathbf{r}+\mathbf{R}) = V(\mathbf{r})$ (where $\mathbf{R}$ is any lattice vector), the Schrödinger equation has solutions of the form:

$$\psi_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r})\, e^{i\mathbf{k}\cdot\mathbf{r}}$$

where $u_{\mathbf{k}}(\mathbf{r})$ is a function with the **same periodicity as the lattice**: $u_{\mathbf{k}}(\mathbf{r}+\mathbf{R}) = u_{\mathbf{k}}(\mathbf{r})$.

The crystal momentum $\hbar\mathbf{k}$ is a good quantum number; $\mathbf{k}$ lies in the first Brillouin zone. The energy forms a band structure.

Final Answer: $\psi_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r})\, e^{i\mathbf{k}\cdot\mathbf{r}}$, $u_{\mathbf{k}}$ has lattice periodicity $\Rightarrow$ **(D)**

Why other options are wrong:

Q16.

- (A): Pure plane waves are only valid for a free electron; the lattice modulates them.
- (B): Constant u reduces to a plane wave, ignoring the lattice.
- (C): $u_{\mathbf{k}}$ must be periodic; random variation contradicts Bloch's theorem.

Answer: (D) ← *Go back to Q16*

Solution

Concept — Half-Wave Rectifier Output:
For a sinusoidal input $v_{\text{in}}(t) = V_m \sin\omega t$, the half-wave rectifier passes only positive half-cycles:

$$v_{\text{out}}(t) = \begin{cases} V_m \sin\omega t & \text{if } \sin\omega t > 0 \\ 0 & \text{otherwise} \end{cases}$$

Average value over one full period $T = 2\pi/\omega$:

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v_{\text{out}}(t)\,dt = \frac{1}{2\pi}\int_0^\pi V_m \sin\theta\,d\theta = \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^\pi = \frac{V_m}{2\pi}(1+1) = \frac{V_m}{\pi}$$

Final Answer: $V_{\text{avg}} = V_m/\pi \Rightarrow$ **(C)**

Why other options are wrong:

Q17.

- (A) $V_m/2$: This is the average of $|\sin\omega t|$ (full-wave rectifier), not half-wave.
- (B) $2V_m/\pi$: This is the average of $|\sin\omega t|$, i.e. the full-wave rectifier output.
- (D) $V_m/\sqrt{2}$: This is the RMS value of a full sine wave, not the average of a half-wave output.

Answer: (C) ← *Go back to Q17*

Solution

Concept — Binding Energy per Nucleon Curve:
The semi-empirical (Bethe–Weizsäcker) mass formula, combined with experimental data, gives the following features of the BE/A vs. A curve:

Q18.

- For very light nuclei ($A \lesssim 10$): BE/A rises sharply (e.g. ^{4}He: 7.07 MeV, ^{2}H:

1.11 MeV).

- Maximum near $A \approx 56$ (iron group, e.g. ^{56}Fe: ≈ 8.79 MeV/nucleon).
- For heavy nuclei: BE/A decreases slowly due to Coulomb repulsion (e.g. ^{238}U: ≈ 7.57 MeV).

This non-monotonic behaviour explains both fission (heavy nuclei $\to$ medium) and fusion (light nuclei $\to$ medium) releasing energy.

Final Answer: BE/A increases to a maximum near $A \approx 56$, then decreases $\Rightarrow$ **(B)**

Why other options are wrong:

- (A): BE/A varies from ≈ 1 MeV (deuteron) to ≈ 8.8 MeV (iron); not constant.
- (C): Monotonically decreasing contradicts the rise from light to medium-A nuclei.
- (D): BE/A is bounded; it cannot increase indefinitely.

Answer: (B) ← *Go back to Q18*

Q19.

Solution

Concept — Taylor Series of $\cos x$:
By differentiating or using the definition:

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

Key coefficient: The coefficient of x^4 is $\dfrac{1}{4!} = \dfrac{1}{24}$.

Final Answer: $\cos x = 1 - x^2/2! + x^4/4! - x^6/6! + \cdots \Rightarrow$ **(A)**

Why other options are wrong:

- (B): Uses $3!$ and $4!$ in denominators — wrong factorial sequence.
- (C): Coefficient $1/8$ for x^4 is wrong; it should be $1/24$.
- (D): This is the Taylor series for e^{-x}, not $\cos x$ (includes odd powers).

Answer: (A) ← *Go back to Q19*

Solution

Concept — Moment of Inertia of a Thin Spherical Shell:
For a thin uniform spherical shell of mass M, radius R, the moment of inertia about a diameter is calculated by integrating over the shell's surface mass density.

Standard result:

$$I_{\text{shell}} = \frac{2}{3}MR^2$$

(larger than the solid sphere result $I_{\text{solid}} = \frac{2}{5}MR^2$, because all mass is at the maximum distance R from the axis on average).

Quick derivation check: For a shell, all mass is at $r = R$. The perpendicular distance from the symmetry axis for a mass element at angle θ from the axis is $R\sin\theta$:

$$I = \int R^2 \sin^2\theta \, dm = \frac{M}{4\pi R^2}\int_0^{2\pi} d\phi \int_0^{\pi} R^2\sin^2\theta \cdot R^2 \sin\theta \, d\theta = \frac{2}{3}MR^2$$

Final Answer: $I = \frac{2}{3}MR^2 \Rightarrow$ **(D)**

Why other options are wrong:

Q20.
- (A) $MR^2/2$: Moment of inertia of a thin disk or cylinder about its symmetry axis, not a sphere.
- (B) $MR^2/4$: No standard object has this value for this geometry.
- (C) $2MR^2/5$: This is the moment of inertia of a *solid* sphere, not a hollow shell.

Answer: (D) ← *Go back to Q20*

Solution

Concept — Mutual Inductance and Coupling Coefficient:

For two coupled inductors with self-inductances L_1, L_2 and mutual inductance M, the coupling coefficient is defined as:

$$k = \frac{M}{\sqrt{L_1 L_2}}, \quad 0 \le k \le 1$$

Physical meaning:

Q21.
- $k = 1$: Perfect (ideal) coupling; all flux from coil 1 links coil 2.
- $k = 0$: No coupling; coils are magnetically isolated.
- $M \le \sqrt{L_1 L_2}$ is guaranteed by energy conservation.

Final Answer: $k = M/\sqrt{L_1 L_2} \Rightarrow$ **(C)**

Why other options are wrong:

- (A) $M\sqrt{L_1 L_2}$: Has wrong dimensions and can exceed 1.
- (B) $M/(L_1 + L_2)$: Denominator is wrong; should be geometric mean, not arithmetic mean.

- (D) $M^2/(L_1L_2)$: This is k^2 (not k), and its square root gives the correct coupling.

Answer: (C) ← Go back to Q21

Q22.

Solution

Concept — Partition Function of 1D QHO:

$$Z = \sum_{n=0}^{\infty} e^{-\beta(n+1/2)\hbar\omega} = e^{-\beta\hbar\omega/2}\sum_{n=0}^{\infty}(e^{-\beta\hbar\omega})^n = \frac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}}$$

High-temperature limit ($k_BT \gg \hbar\omega$, i.e. $\beta\hbar\omega \ll 1$):

$$e^{-\beta\hbar\omega} \approx 1-\beta\hbar\omega, \quad 1-e^{-\beta\hbar\omega} \approx \beta\hbar\omega$$

$$Z \approx \frac{e^{-\beta\hbar\omega/2}}{\beta\hbar\omega} \approx \frac{1}{\beta\hbar\omega} = \frac{k_BT}{\hbar\omega}$$

(Classical partition function of a harmonic oscillator.)

Final Answer: $Z \to k_BT/(\hbar\omega) \Rightarrow$ **(B)**

Why other options are wrong:

- (A) $Z \to 1$: Only for a two-state system or at $T = 0$; QHO at high T has many states populated.
- (C) $e^{-\hbar\omega/(2k_BT)}$: This is just the Boltzmann factor for the ground state; not the full partition function.
- (D) $\hbar\omega/(k_BT)$: The reciprocal of the correct answer.

Answer: (B) ← Go back to Q22

Solution

Concept — Expectation Value $\langle x^2\rangle$ in QHO Ground State:

Express $\hat{x}$ in terms of ladder operators: $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a}+\hat{a}^\dagger)$.

Compute $\langle 0|\hat{x}^2|0\rangle$:

$$\langle \hat{x}^2\rangle = \frac{\hbar}{2m\omega}\langle 0|(\hat{a}+\hat{a}^\dagger)^2|0\rangle = \frac{\hbar}{2m\omega}\langle 0|\hat{a}\hat{a}^\dagger|0\rangle$$

Using $\hat{a}|0\rangle = 0$ and $\hat{a}\hat{a}^\dagger = 1+\hat{a}^\dagger\hat{a}$:

$$\langle 0|\hat{a}\hat{a}^\dagger|0\rangle = \langle 0|(1+\hat{a}^\dagger\hat{a})|0\rangle = 1+0 = 1$$

Result:

$$\langle \hat{x}^2 \rangle_0 = \frac{\hbar}{2m\omega}$$

This is consistent with the zero-point energy: $\langle E \rangle_0 = \frac{1}{2}m\omega^2\langle x^2 \rangle + \frac{\langle p^2 \rangle}{2m} = \frac{1}{2}\hbar\omega$.

Final Answer: $\langle x^2 \rangle = \hbar/(2m\omega) \Rightarrow$ **(A)**

Why other options are wrong:

Q23.
- (B) $\hbar\omega/(2m)$: Has wrong units (J not m^2).
- (C) $\hbar/(m\omega)$: Off by factor of 2.
- (D) 0: $\langle x \rangle = 0$ by symmetry, but $\langle x^2 \rangle \neq 0$ due to zero-point fluctuations.

Answer: (A) ← *Go back to Q23*

Solution

Concept — Fermi Level in Intrinsic Semiconductor:
For an intrinsic semiconductor with valence band edge E_V and conduction band edge E_C ($E_g = E_C - E_V$):

The condition $n = p$ (equal electron and hole concentrations) gives:

$$E_F = \frac{E_C + E_V}{2} + \frac{k_B T}{2} \ln \frac{N_V}{N_C}$$

where N_V, N_C are effective densities of states. For $N_V = N_C$ (or at $T \to 0$):

$$E_F = \frac{E_C + E_V}{2} \quad \text{(midgap)}$$

In practice (e.g. silicon at 300 K), E_F lies very close to the middle of the gap.

Final Answer: E_F lies approximately in the middle of the band gap $\Rightarrow$ **(D)**

Why other options are wrong:

Q24.
- (A): Fermi level in the conduction band implies a heavily n-doped semiconductor, not intrinsic.
- (B): Fermi level at E_V means all valence states filled and no holes (metal-like).
- (C): Fermi level at E_C implies degenerate n-type semiconductor.

Answer: (D) ← *Go back to Q24*

Solution

Concept — 555 Timer Astable Oscillator:
In astable mode, the capacitor charges through $R_1 + R_2$ (from V_{CC} to $2V_{CC}/3$) and discharges through R_2 (from $2V_{CC}/3$ to $V_{CC}/3$).

Timing intervals:

$$t_1 = 0.693(R_1 + R_2)C, \qquad t_2 = 0.693R_2C$$

$$T = t_1 + t_2 = 0.693(R_1 + 2R_2)C$$

Frequency:

$$f = \frac{1}{T} = \frac{1}{0.693(R_1 + 2R_2)C} \approx \frac{1.44}{(R_1 + 2R_2)C}$$

The duty cycle $= t_1/T = (R_1 + R_2)/(R_1 + 2R_2) > 50\%$ (since $t_1 > t_2$ always).

Final Answer: $f \approx 1.44/[(R_1 + 2R_2)C] \Rightarrow$ **(C)**

Why other options are wrong:

Q25.
- (A): Denominator $(R_1 + R_2)C$ misses the factor of 2 on R_2; wrong formula.
- (B): Factor 0.72 should be 1.44; off by a factor of 2.
- (D): Factor 2.88 gives double the correct frequency.

Answer: (C) ← *Go back to Q25*

Solution

Concept — Degeneracy of Hydrogen Energy Levels:
For the n-th energy level of hydrogen:

Q26.
- ℓ ranges from 0 to $n - 1$.
- For each ℓ: $m_\ell = -\ell, \ldots, 0, \ldots, +\ell \Rightarrow 2\ell + 1$ values.
- For each (n, ℓ, m_ℓ): $m_s = \pm\frac{1}{2}$ (spin) $\Rightarrow$ 2 states.

Total degeneracy:

$$g_n = 2\sum_{\ell=0}^{n-1}(2\ell + 1) = 2 \times n^2 = 2n^2$$

(Without spin: n^2; with spin: $2n^2$.)

Final Answer: $2n^2 \Rightarrow$ **(B)**

Why other options are wrong:

- (A) n: Drastically undercounts; ignores ℓ and m_ℓ degeneracy.
- (C) n^2: Correct without spin but misses the factor of 2 from spin.
- (D) $2n$: No physical basis; correct only for $n = 1$ (with spin).

Answer: (B) ← Go back to Q26

Solution

Concept — Most Probable Speed in Maxwell–Boltzmann Distribution:
The Maxwell–Boltzmann speed distribution is:

$$f(v) = 4\pi n \left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 \, e^{-mv^2/(2k_B T)}$$

Find maximum ($df/dv = 0$):

$$\frac{d}{dv}\left[v^2 e^{-mv^2/(2k_B T)}\right] = 0 \;\Rightarrow\; 2v - \frac{mv^3}{k_B T} = 0 \;\Rightarrow\; v_{\text{mp}} = \sqrt{\frac{2k_B T}{m}}$$

Note: $v_{\text{mp}} = \sqrt{2k_B T/m} < \bar{v} = \sqrt{8k_B T/(\pi m)} < v_{\text{rms}} = \sqrt{3k_B T/m}$.

Final Answer: $v_{\text{mp}} = \sqrt{2k_B T/m} \Rightarrow$ **(A)**

Why other options are wrong:

Q27.
- (B) $\sqrt{3k_B T/m}$: This is the RMS speed v_{rms}.
- (C) $\sqrt{8k_B T/(\pi m)}$: This is the mean speed $\bar{v}$.
- (D) $\sqrt{k_B T/m}$: This is the thermal speed; lacks the factor of 2 from the maximum condition.

Answer: (A) ← Go back to Q27

Solution

Concept — Magnetic Field of an Infinite Straight Wire:
By Ampère's law, choosing a circular Ampèrian loop of radius d centred on the wire:

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I_{\text{enc}} \;\Rightarrow\; B \cdot (2\pi d) = \mu_0 I$$

Solving:

$$B = \frac{\mu_0 I}{2\pi d}$$

Direction: tangential to circles centred on the wire (right-hand rule: counterclockwise for current out of page).

Final Answer: $B = \mu_0 I/(2\pi d) \Rightarrow$ **(D)**

Why other options are wrong:

Q28.
- (A) $\mu_0 I/(4\pi d)$: Off by factor of 2 (the 4π appears in Biot–Savart, not in the final formula for a full wire).

- (B) $\mu_0 I d/(2\pi)$: Wrong; d should be in the denominator.
- (C) $2\mu_0 I/(\pi d)$: Off by a factor of 4.

Answer: (D) ← *Go back to Q28*

Solution

Concept — Young's Double-Slit Fringe Separation:
$\beta = \lambda D/d$

Substitution:

$$\beta = \frac{550 \times 10^{-9}\,\text{m} \times 2\,\text{m}}{0.5 \times 10^{-3}\,\text{m}} = \frac{550 \times 10^{-9} \times 2}{5 \times 10^{-4}} = \frac{1100 \times 10^{-9}}{5 \times 10^{-4}} = \frac{1.1 \times 10^{-6}}{5 \times 10^{-4}} = 2.2 \times 10^{-3}\,\text{m}$$

$$\beta = 2.2\,\text{mm}$$

Final Answer: $\beta = 2.2\,\text{mm} \Rightarrow$ **(C)**

Why other options are wrong:

Q29.

- (A) $1.1\,\text{mm}$: Off by factor of 2; may arise from using $D = 1\,\text{m}$ instead of 2 m.
- (B) $4.4\,\text{mm}$: Double the correct value; error in slit separation.
- (D) $0.55\,\text{mm}$: Off by factor of 4; wrong combination of parameters.

Answer: (C) ← *Go back to Q29*

Solution

Concept — Photoelectric Effect, Stopping Potential:
$eV_s = h\nu - \phi$

Step 1 — Photon energy:

$$h\nu = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{200 \times 10^{-9}} = \frac{1.988 \times 10^{-25}}{2 \times 10^{-7}} = 9.939 \times 10^{-19}\,\text{J}$$

$$h\nu = \frac{9.939 \times 10^{-19}}{1.6 \times 10^{-19}}\,\text{eV} = 6.21\,\text{eV}$$

Step 2 — Stopping potential:

$$eV_s = h\nu - \phi = 6.21 - 4.5 = 1.71\,\text{eV} \;\Rightarrow\; V_s \approx 1.72\,\text{V}$$

Final Answer: $V_s \approx 1.72\,\text{V} \Rightarrow$ **(B)**

Why other options are wrong:

Q30.

- (A) $1.22\,\text{V}$: Would require $h\nu - \phi = 1.22\,\text{eV}$, i.e. $h\nu = 5.72\,\text{eV}$; incorrect for $\lambda = 200\,\text{nm}$.
- (C) $2.50\,\text{V}$: Would require $h\nu = 7.0\,\text{eV}$; incorrect.
- (D) $6.22\,\text{V}$: Equal to $h\nu/e$; neglects the work function subtraction.

Answer: (B) ← *Go back to Q30*

Section B Solutions — MSQ

Q31.

Solution

Concept — Conservation Laws in Elastic Collisions:

(A) Total kinetic energy: CORRECT. The defining property of an elastic collision is that total kinetic energy is conserved: $\sum \frac{1}{2} m_i v_i^2 = \text{const.}$

(B) Individual particle velocities: NOT CORRECT. In a general elastic collision, individual velocities change (only in a trivial grazing collision does a particle not change velocity). The *total* KE is conserved, not individual KEs.

(C) Total linear momentum: CORRECT. In any collision (elastic or inelastic) with no external forces, Newton's third law ensures $\sum m_i \mathbf{v}_i = \text{const.}$

(D) Total mechanical energy: CORRECT. For a conservative system (e.g. gravity, springs), the total mechanical energy (KE + PE) is conserved in elastic collisions. Since KE is conserved and PE is a state function, mechanical energy is conserved.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q31*

Q32.

Solution

Concept — Properties of the Magnetic Field $\mathbf{B}$:

(A) $\nabla \cdot \mathbf{B} = 0$: CORRECT. One of Maxwell's equations ($\nabla \cdot \mathbf{B} = 0$) states there are no magnetic monopoles; the magnetic flux through any closed surface is zero.

(B) $\mathbf{B}$ field lines form closed loops: CORRECT. Because $\nabla \cdot \mathbf{B} = 0$, magnetic field lines have no sources or sinks; they must form closed loops (or extend to infinity for infinite systems).

(C) $\mathbf{B}$ can do work on a stationary charge: NOT CORRECT. The magnetic force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ is always perpendicular to $\mathbf{v}$, so it does no work. On a *stationary* charge

($\mathbf{v} = 0$), the magnetic force is zero.

(D) Magnetic flux in webers: CORRECT. $\Phi = \iint \mathbf{B} \cdot d\mathbf{A}$ has SI unit T·m^2 = Wb (weber). $1\,\text{Wb} = 1\,\text{V} \cdot \text{s}$.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q32*

Q33.

Solution

Concept — Reversible Adiabatic Process:

(A) Temperature remains constant: NOT CORRECT. An adiabatic process has $Q = 0$ and, in general, T changes ($dT = -PdV/C_V \neq 0$). Constant temperature (isothermal) is a completely different process.

(B) $Q = 0$: CORRECT. "Adiabatic" by definition means no heat exchange between the system and surroundings.

(C) PV^γ = const for reversible adiabatic: CORRECT. For a quasi-static (reversible) adiabatic process of an ideal gas with $C_P/C_V = \gamma$, the equation of state is PV^γ = const.

(D) $\Delta U = 0$: NOT CORRECT. From the first law: $\Delta U = Q - W = 0 - W = -W$. Since work is done in an adiabatic process ($W \neq 0$ in general), $\Delta U \neq 0$.

Correct options: (B) and (C).

Answer: (BC) ← *Go back to Q33*

Q34.

Solution

Concept — Quantum Harmonic Oscillator Properties:

(A) $E_n = (n + \frac{1}{2})\hbar\omega$, $n = 0, 1, 2, \ldots$: CORRECT. These are the exact energy eigenvalues of the 1D QHO; the spectrum is discrete and equally spaced.

(B) Zero-point energy $E_0 = \frac{1}{2}\hbar\omega$: CORRECT. The ground state has non-zero energy, reflecting the Heisenberg uncertainty principle: $\Delta x \cdot \Delta p \geq \hbar/2$ prevents the oscillator from being at rest.

(C) Spacing increases with n: NOT CORRECT. The spacing between adjacent levels is *constant*: $E_{n+1} - E_n = \hbar\omega$ for all n. (Compare with hydrogen, where spacings decrease.)

(D) $[\hat{a}, \hat{a}^\dagger] = 1$: CORRECT. The commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$ is fundamental to the algebraic solution of the QHO and reflects the bosonic nature of the oscillator quanta.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q34*

Q35.

Solution

Concept — Free Electron (Sommerfeld) Model:

(A) Fermi–Dirac statistics: CORRECT. Electrons are spin-$\frac{1}{2}$ fermions; the Pauli exclusion principle forbids two electrons in the same quantum state, leading to Fermi–Dirac distribution $f(E) = 1/(e^{(E-E_F)/k_BT} + 1)$.

(B) States up to E_F are empty at $T = 0$: NOT CORRECT. At $T = 0$, the Fermi–Dirac function becomes a step function: *all* states below E_F are **filled** and all above E_F are empty.

(C) $E_F \propto n^{2/3}$: CORRECT. $E_F = \dfrac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3} \propto n^{2/3}$. Higher electron density $\Rightarrow$ higher Fermi energy.

(D) Fermi pressure at $T = 0$: CORRECT. Even at $T = 0$, the Pauli exclusion principle forces electrons to occupy states up to E_F, creating a non-zero kinetic energy and hence a degeneracy pressure $P = \frac{2}{3} \times$ (energy density).

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q35*

Q36.

Solution

Concept — Dirac Delta Function Properties:

(A) $\int_{-\infty}^{\infty} \delta(x)\,dx = 1$: CORRECT. This is the normalisation property defining the delta function.

(B) $\delta(x) = 0$ for $x \neq 0$: CORRECT. The delta function is zero everywhere except at $x = 0$, where it has an infinite "spike" of unit area.

(C) $\delta(x)$ is a regular function: NOT CORRECT. The Dirac delta is a *distribution* (generalised function), not an ordinary function. It is defined only through its action in integrals: $\int f(x)\delta(x)\,dx = f(0)$.

(D) Sifting property $\int f(x)\delta(x-a)\,dx = f(a)$: CORRECT. This is the fundamental sifting (sampling) property of the delta function, valid for any continuous function

f.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q36*

Q37.

Solution

Concept — Radioactive Decay Series:

(A) Terminates at a stable isotope: CORRECT. Every natural decay series (e.g. $^{238}\text{U} \to {}^{206}\text{Pb}$; $^{235}\text{U} \to {}^{207}\text{Pb}$; $^{232}\text{Th} \to {}^{208}\text{Pb}$) ends at a stable lead isotope.

(B) All intermediates have longer half-lives than the parent: NOT CORRECT. Many intermediate products have shorter half-lives than the parent. For example, in the ^{238}U series, ^{214}Po has a half-life of only $\sim 163\,\mu$s while the parent ^{238}U has $T_{1/2} = 4.5 \times 10^9$ yr.

(C) Secular equilibrium condition $\lambda_1 N_1 = \lambda_2 N_2 = \cdots$: CORRECT. When the parent half-life is much larger than all daughters' half-lives, the system reaches secular equilibrium where activities are equal: $A_1 = A_2 = \cdots = \lambda_i N_i$.

(D) Mass-energy conservation at each step: CORRECT. Each individual decay conserves total energy; the Q-value (energy released) equals the kinetic energy of decay products plus gamma-ray energy: $Q = \Delta M c^2$.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q37*

Q38.

Solution

Concept — Thin-Film Interference:

(A) Colours from wavelength-dependent conditions: CORRECT. White light contains all wavelengths; each wavelength satisfies constructive (or destructive) interference at a particular thickness, producing the observed colours.

(B) Minimum thickness $\lambda/(4n)$ for constructive reflection: CORRECT. With a phase-flip at the top surface (air-to-film) and no flip at the bottom (film-to-air for soap film in air), the minimum constructive condition is $2nt = \lambda/2$, giving $t_{\text{min}} = \lambda/(4n)$.

(C) Phase shift π on reflection from denser medium: CORRECT. When light travels from a medium of lower refractive index to higher ($n_1 < n_2$), reflection incurs a phase shift of π (equivalent to a half-wavelength path difference). This is analogous to a wave on a string reflecting from a fixed end.

(D) Thin-film patterns are diffraction: NOT CORRECT. Thin-film iridescence is caused by *interference* between waves reflected from the top and bottom surfaces of the film, not diffraction (which involves bending around obstacles or through apertures).

Correct options: (A), (B), and (C).

Answer: (ABC) ← *Go back to Q38*

Q39.

Solution

Concept — Ideal Op-Amp Properties:

(A) Infinite input impedance: CORRECT. An ideal op-amp draws zero current from its input terminals ($I^+ = I^- = 0$), equivalent to infinite input impedance. This ensures the op-amp does not load the preceding circuit.

(B) Zero output impedance: CORRECT. The ideal op-amp output is a perfect voltage source: $V_{\text{out}} = A(V^+ - V^-)$ regardless of load current. Zero output impedance means no voltage drop under any load.

(C) Finite open-loop gain $\approx 10^5$: NOT CORRECT. A *real* op-amp has open-loop gain $\sim 10^5$–10^6. The *ideal* op-amp model assumes **infinite** open-loop gain ($A \to \infty$), which together with feedback enforces $V^+ \approx V^-$ (virtual short).

(D) Negative feedback stabilises gain: CORRECT. Negative feedback reduces the closed-loop gain to $G \approx 1 + R_f/R_1$ (for non-inverting) or $-R_f/R_1$ (for inverting), determined solely by passive resistors, making the amplifier insensitive to op-amp parameter variations.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q39*

Q40.

Solution

Concept — Blackbody Radiation:

(A) Total intensity $\propto T^2$: NOT CORRECT. The Stefan–Boltzmann law states: total radiated power per unit area $M = \sigma T^4$ (proportional to T^4, not T^2).

(B) Wien's displacement law: CORRECT. $\lambda_{\text{max}} T = b = 2.898 \times 10^{-3}\,\text{m K}$ (Wien's constant). As T increases, λ_{max} shifts to shorter wavelengths (e.g. sun's peak in visible; human body's peak in infrared).

(C) Planck distribution: CORRECT. The spectral energy density is:

$$u(\nu, T) = \frac{8\pi h\nu^3/c^3}{e^{h\nu/(k_B T)} - 1}$$

(Planck's formula, resolving the ultraviolet catastrophe of classical theory).

(D) Rayleigh–Jeans valid at all wavelengths: NOT CORRECT. The Rayleigh–Jeans law $u(\nu, T) = 8\pi\nu^2 k_B T/c^3$ is valid only at long wavelengths (low frequencies, $h\nu \ll k_B T$). At short wavelengths it diverges (ultraviolet catastrophe), contradicting experiment.

Correct options: (B) and (C).

Answer: (BC) ← *Go back to Q40*

Section C Solutions — NAT

Q41.

Solution

Concept — Angular Velocity of Earth's Rotation:
$\omega = 2\pi/T$ where $T = 24\,\text{h} = 24 \times 3600\,\text{s} = 86400\,\text{s}$.

Calculation:

$$\omega = \frac{2\pi}{86400\,\text{s}} = \frac{6.28318}{86400} = 7.2722 \times 10^{-5}\,\text{rad/s}$$

In units of 10^{-5} rad/s: $\omega = 7.27 \times 10^{-5}$ rad/s.

$$\boxed{\omega = 7.27 \quad (\times 10^{-5}\,\text{rad/s})}$$

Answer: (7.27) ← *Go back to Q41*

Q42.

Solution

Concept — Charge on a Capacitor:
$Q = CV$

Calculation:

$$Q = 10 \times 10^{-6}\,\text{F} \times 15\,\text{V} = 150 \times 10^{-6}\,\text{C} = 150\,\mu\text{C}$$

$$\boxed{Q = 150.00\,\mu\text{C}}$$

Answer: (150.00) ← *Go back to Q42*

Q43.

Solution

Concept — de Broglie Wavelength of a Thermal Neutron:
$\lambda = h/p = h/\sqrt{2m_n KE}$

Step 1 — Kinetic energy in joules:

$$KE = 0.025\,\text{eV} \times 1.6 \times 10^{-19}\,\text{J/eV} = 4.0 \times 10^{-21}\,\text{J}$$

Step 2 — Momentum:

$$p = \sqrt{2m_n KE} = \sqrt{2 \times 1.675 \times 10^{-27} \times 4.0 \times 10^{-21}}$$

$$= \sqrt{1.340 \times 10^{-47}} = 3.661 \times 10^{-24}\,\text{kg m/s}$$

Step 3 — Wavelength:

$$\lambda = \frac{6.626 \times 10^{-34}}{3.661 \times 10^{-24}} = 1.810 \times 10^{-10}\,\text{m} = 181.0\,\text{pm}$$

$$\boxed{\lambda \approx 181\,\text{pm}}$$

Answer: (181) ← *Go back to Q43*

Q44.

Solution

Concept — RMS Speed of Gas Molecules:
$v_{\text{rms}} = \sqrt{3RT/M}$ (molar mass M in kg/mol)

Calculation for H_2 ($M = 2\,$g/mol $= 0.002\,$kg/mol):

$$v_{\text{rms}} = \sqrt{\frac{3 \times 8.314 \times 300}{0.002}} = \sqrt{\frac{7482.6}{0.002}} = \sqrt{3{,}741{,}300}\,\text{m}^2/\text{s}^2 = 1934.2\,\text{m/s}$$

$$\boxed{v_{\text{rms}} \approx 1934\,\text{m/s}}$$

Note: H_2 molecules have the highest rms speed among common gases at the same T due to their very low molar mass.

Answer: (1934) ← *Go back to Q44*

Q45.

Solution

Concept — Finding Half-Life from Fraction Remaining:
$N/N_0 = (1/2)^{t/T_{1/2}}$

Given: $N/N_0 = 0.25 = 1/4$ at $t = 3$ years.

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{3/T_{1/2}} \Rightarrow \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{3/T_{1/2}}$$

Equating exponents:

$$2 = \frac{3}{T_{1/2}} \Rightarrow T_{1/2} = \frac{3}{2} = 1.5\,\text{years}$$

$$\boxed{T_{1/2} = 1.50\,\text{years}}$$

Answer: (1.50) ← *Go back to Q45*

Q46.

Solution

Concept — Nearest-Neighbour Distance in BCC:
In the BCC structure, the nearest neighbours are connected along the body diagonal of the unit cell. The body diagonal has length $a\sqrt{3}$, and the nearest-neighbour distance is half the body diagonal:

$$d_{\text{NN}} = \frac{a\sqrt{3}}{2}$$

Calculation for iron ($a = 2.87$ Å):

$$d_{\text{NN}} = \frac{2.87 \times \sqrt{3}}{2} = \frac{2.87 \times 1.7321}{2} = \frac{4.971}{2} = 2.486\,\text{Å}$$

$$\boxed{d_{\text{NN}} \approx 2.49\,\text{Å}}$$

Answer: (2.49) ← *Go back to Q46*

Q47.

Solution

Concept — Eigenvalues of Anti-Symmetric Matrix:

For $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, the characteristic equation is:

$$\det(A - \lambda I) = \lambda^2 + 1 = 0 \;\Rightarrow\; \lambda = \pm i$$

The eigenvalues are purely imaginary: $\lambda_1 = i$ and $\lambda_2 = -i$.

Magnitude:

$$|\lambda_1| = |i| = 1, \qquad |\lambda_2| = |-i| = 1$$

$$\boxed{|\lambda| = 1}$$

Answer: (1) ← *Go back to Q47*

Q48.

Solution

Concept — Damped Oscillator Frequency:

$\omega_d = \sqrt{\omega_0^2 - \gamma^2}, \quad f = \omega_d/(2\pi)$

Calculation:

$$\omega_d = \sqrt{(100)^2 - (10)^2} = \sqrt{10000 - 100} = \sqrt{9900}$$

$$\sqrt{9900} = \sqrt{100 \times 99} = 10\sqrt{99} = 10 \times 9.9499 = 99.499\,\text{rad/s}$$

Damped frequency:

$$f = \frac{99.499}{2\pi} = \frac{99.499}{6.2832} = 15.84\,\text{Hz}$$

$$\boxed{f \approx 15.84\,\text{Hz}}$$

Answer: (15.84) ← *Go back to Q48*

Q49.

Solution

Concept — Non-Inverting Op-Amp Amplifier:

$G = 1 + R_f/R_1$, $V_{\text{out}} = G \times V_{\text{in}}$

Calculation:

$$G = 1 + \frac{27\,\text{k}\Omega}{3\,\text{k}\Omega} = 1 + 9 = 10$$

$$V_{\text{out}} = 10 \times 0.5\,\text{V} = 5.0\,\text{V}$$

$$\boxed{V_{\text{out}} = 5.0\,\text{V}}$$

Answer: (5.0) ← *Go back to Q49*

Q50.

Solution

Concept — Internal Energy of Monatomic Ideal Gas:
For a monatomic ideal gas: $U = \frac{3}{2}nRT$ (3 translational degrees of freedom, each contributing $\frac{1}{2}nRT$).

Calculation:

$$U = \frac{3}{2} \times 1\,\text{mol} \times 8.314\,\text{J}\,\text{mol}^{-1}\text{K}^{-1} \times 300\,\text{K} = \frac{3}{2} \times 2494.2\,\text{J} = 3741.3\,\text{J}$$

$$\boxed{U = 3741.3\,\text{J}}$$

Answer: (3741.3) ← *Go back to Q50*

Q51.

Solution

Concept — Energy Stored in an Inductor:
$E = \frac{1}{2}LI^2$

Calculation:

$$E = \frac{1}{2} \times 2.0\,\text{H} \times (5.0\,\text{A})^2 = \frac{1}{2} \times 2.0 \times 25.0 = 25.00\,\text{J}$$

The energy is stored in the magnetic field B of the inductor; energy density = $B^2/(2\mu_0)$.

$$\boxed{E = 25.00\,\text{J}}$$

Answer: (25.00) ← *Go back to Q51*

Q52.

Solution

Concept — de Broglie Wavelength Ratio at Same Kinetic Energy:
$\lambda = h/p = h/\sqrt{2mKE}$, so $\lambda \propto 1/\sqrt{m}$ at fixed KE.

Ratio:

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{1836}$$

Calculation:

$$\sqrt{1836} = \sqrt{1764 \times (1836/1764)} \approx 42 \times \sqrt{1.0408} \approx 42 \times 1.0202 = 42.85$$

(More precisely: $42^2 = 1764$, $42.8^2 = 1831.84$, $42.85^2 = 1836.12 \approx 1836$.)

$$\boxed{\lambda_e/\lambda_p \approx 42.85}$$

Note: The electron has a much longer de Broglie wavelength at the same kinetic energy because it is much lighter.

Answer: (42.85) ← *Go back to Q52*

Q53.

Solution

Concept — Work in Isothermal Reversible Expansion (Pressure Change):
For an ideal gas: $W = nRT\ln(V_f/V_i) = nRT\ln(P_i/P_f)$ (since $PV = nRT =$ const at fixed T).

Calculation:

$$W = 1\,\text{mol} \times 8.314\,\text{J}\,\text{mol}^{-1}\text{K}^{-1} \times 400\,\text{K} \times \ln\left(\frac{4}{1}\right)$$

$$= 3325.6\,\text{J} \times \ln 4 = 3325.6 \times 1.3863\,\text{J}$$

$$= 3325.6 \times 1.3863 = 4610.5\,\text{J} \approx 4611\,\text{J}$$

More precisely: $\ln 4 = 2\ln 2 = 2 \times 0.6931 = 1.3863$.
$3325.6 \times 1.3863 = 3325.6 \times 1 + 3325.6 \times 0.3863$
$= 3325.6 + 1284.6 = 4610.2\,\text{J}$

$$\boxed{W \approx 4610\,\text{J}}$$

Note: The prompt computed 4612 J; slight rounding difference; accept 4610–4612 J.

Answer: (4610) ← *Go back to Q53*

Q54.

Solution

Concept — Fermi Energy of Potassium:
$E_F = \dfrac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$

Step 1 — Compute $3\pi^2 n$:

$3\pi^2 \times 1.40 \times 10^{28} = 3 \times 9.8696 \times 1.40 \times 10^{28} = 29.609 \times 1.40 \times 10^{28} = 4.145 \times 10^{29}\,\text{m}^{-3}$

Step 2 — Raise to power $2/3$:

$(4.145 \times 10^{29})^{2/3} = (4.145)^{2/3} \times 10^{29\times2/3} = (4.145)^{2/3} \times 10^{19.333}$

$(4.145)^{1/3}$: $1.60^3 = 4.096$, $1.61^3 = 4.173$, so $(4.145)^{1/3} \approx 1.606$.
$(4.145)^{2/3} = (1.606)^2 = 2.579$.
$10^{19.333} = 10^{19} \times 10^{0.333} = 10^{19} \times 2.154$.
$(3\pi^2 n)^{2/3} = 2.579 \times 2.154 \times 10^{19} = 5.555 \times 10^{19}\,\mathrm{m}^{-2}$.

Step 3 — Fermi energy:

$$E_F = \frac{(1.055 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31}} \times 5.555 \times 10^{19} = \frac{1.113 \times 10^{-68}}{1.822 \times 10^{-30}} \times 5.555 \times 10^{19}$$

$$= 6.110 \times 10^{-39} \times 5.555 \times 10^{19} = 33.94 \times 10^{-20}\,\mathrm{J} = 3.394 \times 10^{-19}\,\mathrm{J}$$

Convert to eV:

$$E_F = \frac{3.394 \times 10^{-19}}{1.6 \times 10^{-19}}\,\mathrm{eV} = 2.12\,\mathrm{eV}$$

$$\boxed{E_F \approx 2.12\,\mathrm{eV}}$$

Answer: (2.12) ← *Go back to Q54*

Q55.

Solution

Concept — Ground State Energy in 3D Infinite Cubic Well:

$E_{n_x n_y n_z} = \dfrac{\pi^2\hbar^2}{2m_e L^2}(n_x^2 + n_y^2 + n_z^2)$

Ground state: $n_x = n_y = n_z = 1$, so $n_x^2 + n_y^2 + n_z^2 = 3$.

$$E_{111} = \frac{3\pi^2\hbar^2}{2m_e L^2}$$

Calculation: $L = 10\,\mathrm{nm} = 10^{-8}\,\mathrm{m}$; $L^2 = 10^{-16}\,\mathrm{m}^2$.

$$E_{111} = \frac{3 \times 9.8696 \times (1.055 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31} \times 10^{-16}}$$

$$= \frac{3 \times 9.8696 \times 1.113 \times 10^{-68}}{1.822 \times 10^{-46}} = \frac{3 \times 10.986 \times 10^{-68}}{1.822 \times 10^{-46}}$$

$$= \frac{32.96 \times 10^{-68}}{1.822 \times 10^{-46}} = 18.09 \times 10^{-22}\,\mathrm{J} = 1.809 \times 10^{-21}\,\mathrm{J}$$

Convert to eV and meV:

$$E_{111} = \frac{1.809 \times 10^{-21}}{1.6 \times 10^{-19}}\,\mathrm{eV} = 0.01131\,\mathrm{eV} = 11.31\,\mathrm{meV}$$

$$\boxed{E_{111} \approx 11.3\,\text{meV}}$$

Answer: (11.3) ← *Go back to Q55*

Q56.

Solution

Concept — Fundamental Frequency of Open Pipe:
For a pipe open at both ends, standing waves require an antinode at each end. The fundamental mode has $L = \lambda/2$, so $\lambda_1 = 2L$ and:

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

Calculation:

$$f_1 = \frac{340\,\text{m/s}}{2 \times 0.68\,\text{m}} = \frac{340}{1.36} = 250\,\text{Hz}$$

$$\boxed{f_1 = 250\,\text{Hz}}$$

Answer: (250) ← *Go back to Q56*

Q57.

Solution

Concept — Fraction Remaining After Multiple Half-Lives:
After n half-lives: $N/N_0 = (1/2)^n$

For $t = 3T_{1/2}$ (three half-lives, $n = 3$):

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^3 = \frac{1}{8} = 0.125$$

$$\boxed{N/N_0 = 0.1250}$$

Answer: (0.1250) ← *Go back to Q57*

Q58.

Solution

Concept — Definite Integral via Substitution:
$I = \displaystyle\int_0^\infty x\,e^{-x^2}\,dx$

Substitution: Let $u = x^2$, $du = 2x\,dx$, so $x\,dx = du/2$. When $x = 0$, $u = 0$; when $x \to \infty$, $u \to \infty$.

$$I = \int_0^\infty e^{-u}\,\frac{du}{2} = \frac{1}{2}\left[-e^{-u}\right]_0^\infty = \frac{1}{2}(0-(-1)) = \frac{1}{2}$$

$$\boxed{I = 0.500}$$

Answer: (0.500) ← *Go back to Q58*

Q59.

Solution

Concept — RC Charging Circuit Voltage:
$V_C(t) = \mathcal{E}(1 - e^{-t/RC})$

At $t = \tau = RC$ (one time constant):

$$V_C(\tau) = 10\,\text{V} \times (1 - e^{-1}) = 10 \times (1 - 0.3679) = 10 \times 0.6321 = 6.321\,\text{V}$$

$$\boxed{V_C \approx 6.3\,\text{V}}$$

Note: After one time constant, the capacitor is charged to approximately 63.2% of the supply voltage. After 5τ, the capacitor is $> 99\%$ charged.

Answer: (6.3) ← *Go back to Q59*

Q60.

Solution

Concept — Photon Momentum:
$p = h/\lambda$

Calculation:

$$p = \frac{6.626 \times 10^{-34}\,\text{J s}}{400 \times 10^{-9}\,\text{m}} = \frac{6.626 \times 10^{-34}}{4 \times 10^{-7}} = 1.657 \times 10^{-27}\,\text{kg m/s}$$

In units of 10^{-27} kg m/s:

$$\boxed{p \approx 1.66 \times 10^{-27}\,\text{kg m/s}}$$

Note: The photon has no rest mass but carries momentum $p = h/\lambda = \hbar k$ (de Broglie relation holds for photons too).

Answer: (1.66) ← *Go back to Q60*

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	A	4	D	5	C
6	B	7	A	8	D	9	C	10	B
11	A	12	D	13	C	14	B	15	A
16	D	17	C	18	B	19	A	20	D
21	C	22	B	23	A	24	D	25	C
26	B	27	A	28	D	29	C	30	B
31	ACD	32	ABD	33	BC	34	ABD	35	ACD
36	ABD	37	ACD	38	ABC	39	ABD	40	BC
41	7.27	42	150.00	43	181	44	1934	45	1.50
46	2.49	47	1	48	15.84	49	5.0	50	3741.3
51	25.00	52	42.85	53	4610	54	2.12	55	11.3
56	250	57	0.1250	58	0.500	59	6.3	60	1.66