IIT JAM Physics

Sample Paper – 3

Instructions

- This paper contains **60 questions** carrying a maximum of **100 marks**. Duration: **3 hours (180 minutes)**.
- **Section A (Q1–Q30**, MCQ): Q1–Q10 carry **+1 mark** each; wrong response: $-\frac{1}{3}$ mark. Q11–Q30 carry **+2 marks** each; wrong response: $-\frac{2}{3}$ mark. Choose the **single best** option.
- **Section B (Q31–Q40**, MSQ): Each carries **+2 marks**. **No negative marking. One or more** options may be correct; full marks only if *all* correct options are selected; no partial credit.
- **Section C (Q41–Q60**, NAT): Q41–Q50 carry **+1 mark**; Q51–Q60 carry **+2 marks**. **No negative marking.** Enter the numerical value using the virtual keypad (integer or decimal as appropriate).
- **Calculator policy:** Personal calculators, log tables, and electronic devices are **strictly prohibited**. A virtual scientific calculator is provided in the CBT interface. Rough sheets are supplied at the examination centre.
- Once you move to the next section, you cannot return to a previous section. Manage time accordingly.

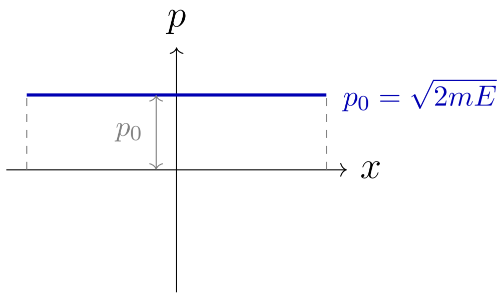
Section A — Multiple Choice Questions (MCQ)

***Q1–Q10**: 1 mark each ($-\frac{1}{3}$ for wrong). Choose the **single** correct option.*

Q1. Consider the power series $\sum_{n=0}^{\infty} n!\, x^n$. The radius of convergence R of this series, obtained using the ratio test $R = \lim_{n\to\infty} \dfrac{a_n}{a_{n+1}}$ where $a_n = n!$, is:

(A) $R = \infty$
(B) $R = 1$
(C) $R = e$
(D) $R = 0$

Q2. A free particle of mass m moves in one dimension with no potential ($H = p^2/2m$). Hamilton's equations give $\dot{p} = -\partial H/\partial x = 0$, so momentum is conserved. The trajectory of the particle in phase space (x, p) at constant energy E is shown below. Which description is correct?

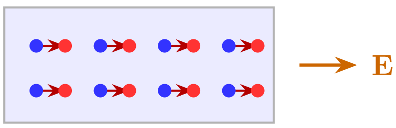


(A) A horizontal straight line $p = p_0 = \sqrt{2mE}$ (constant momentum)
(B) An ellipse centred at the origin
(C) A circle of radius $\sqrt{2mE}$
(D) A parabola opening along the x-axis

Q3. Brewster's angle θ_B is the angle of incidence at which the reflected light is completely plane-polarized. For a glass surface with refractive index $n = 1.5$, using the relation $\tan\theta_B = n$, the Brewster's angle is approximately:

(A) $\theta_B \approx 33.7°$
(B) $\theta_B \approx 56.3°$
(C) $\theta_B \approx 45.0°$
(D) $\theta_B \approx 63.4°$

Q4. A dielectric material of relative permittivity $\varepsilon_r = 4$ is placed in a uniform electric field $\mathbf{E}$. The induced dipoles are shown schematically below. The electric susceptibility χ_e defined by $\varepsilon_r = 1 + \chi_e$ and the polarization $\mathbf{P} = \varepsilon_0\chi_e\mathbf{E}$ give:



dielectric $(\varepsilon_r = 4)$

(A) $\chi_e = 4$
(B) $\chi_e = 2$
(C) $\chi_e = 3$
(D) $\chi_e = 1/4$

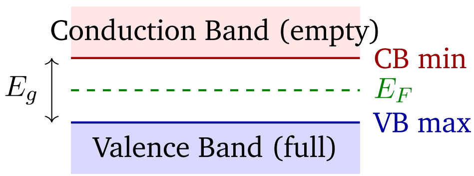
Q5. A Carnot engine operates between a hot reservoir at $T_H = 800\,\text{K}$ and a cold reservoir at $T_C = 400\,\text{K}$, giving efficiency $\eta = 50\%$. If the hot reservoir temperature is increased to $T'_H = 1000\,\text{K}$ while keeping $T_C = 400\,\text{K}$, the new efficiency η' is:

(A) 50%
(B) 55%
(C) 45%
(D) 60%

Q6. A particle of mass m and energy E is incident on a rectangular potential barrier of height $V_0 > E$ and width a. The transmission probability is $T \approx e^{-2\kappa a}$ where $\kappa = \sqrt{2m(V_0 - E)}/\hbar$. Which statement about quantum tunneling is correct?

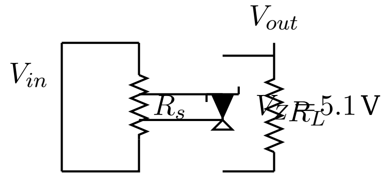
(A) A particle with $E < V_0$ can tunnel through the barrier with non-zero probability
(B) Tunneling probability increases exponentially as barrier width a increases
(C) Classically, a particle with $E < V_0$ can also penetrate the barrier
(D) The transmission probability is exactly zero for any finite-width barrier when $E < V_0$

Q7. At $T = 0\,\text{K}$, an intrinsic semiconductor has a completely filled valence band and completely empty conduction band separated by an energy gap E_g. The energy band diagram is shown below. The Fermi level E_F at $T = 0\,\text{K}$ lies:



(A) At the conduction band minimum
(B) Exactly in the middle of the band gap ($E_g/2$ above valence band maximum)
(C) At the valence band maximum
(D) Above the conduction band

Q8. A Zener diode with breakdown voltage $V_Z = 5.1\,\text{V}$ is used in the voltage regulator circuit shown below. In this circuit the Zener diode maintains a constant output voltage of $5.1\,\text{V}$ by operating in:



(A) Forward bias amplification mode
(B) Forward bias blocking mode
(C) Reverse breakdown (Zener breakdown) mode
(D) Avalanche breakdown in forward bias

Q9. The Q-value of a nuclear reaction is defined as the energy released (or absorbed) and equals the difference in rest-mass energies of reactants and products. For a reaction with initial total atomic mass M_i and final total atomic mass M_f (in atomic mass units), the Q-value is:

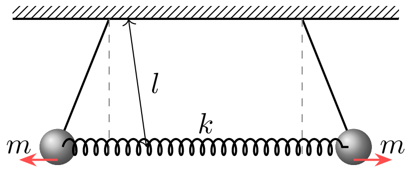
(A) $Q = (M_f - M_i)\,c^2$
(B) $Q = (M_i + M_f)\,c^2$
(C) $Q > 0$ always (all nuclear reactions are exothermic)
(D) $Q = (M_i - M_f)\,c^2$

Q10. Using Green's theorem in the plane, $\oint_C (P\,dx + Q\,dy) = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$, with $P = -y$ and $Q = x$ taken counter-clockwise around the unit circle C ($|z| = 1$), the value of the line integral is:

(A) 2π
(B) π
(C) 4π
(D) 0

***Q11–Q30**: 2 marks each ($-\frac{2}{3}$ for wrong). Choose the **single** correct option.*

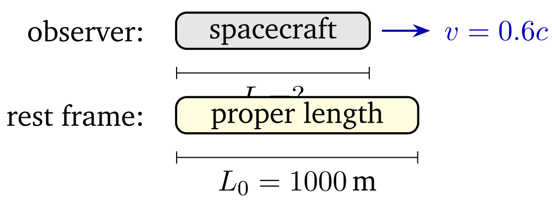
Q11. Two identical simple pendulums (each of mass m, length l) are connected at their bobs by a spring of spring constant k, as shown below. The system has two normal modes: a symmetric (in-phase) mode and an antisymmetric (out-of-phase) mode. Which pair of angular frequencies is correct?



antisymmetric mode

(A) $\omega_1 = \omega_2 = \sqrt{g/l}$ (degenerate)
(B) $\omega_1 = \sqrt{g/l}, \quad \omega_2 = \sqrt{g/l + 2k/m}$
(C) $\omega_1 = \sqrt{g/l - k/m}, \quad \omega_2 = \sqrt{g/l + k/m}$
(D) $\omega_1 = \sqrt{2k/m}, \quad \omega_2 = \sqrt{g/l}$

Q12. A spaceship has proper length $L_0 = 1000\,\text{m}$ and moves past an Earth observer at speed $v = 0.6c$. Due to length contraction, the observer measures the spaceship length $L = L_0\sqrt{1 - v^2/c^2}$, as shown below. The measured length is:



(A) $750\,\text{m}$
(B) $1000\,\text{m}$ (unchanged)
(C) $800\,\text{m}$
(D) $600\,\text{m}$

Q13. An electromagnetic plane wave propagates in the z-direction with electric field amplitude $E_0 = 100\,\text{V/m}$. The time-averaged intensity (Poynting vector magnitude) is $\langle S \rangle = \varepsilon_0 c E_0^2/2$. Using $\varepsilon_0 = 8.854 \times 10^{-12}\,\text{F/m}$ and $c = 3 \times 10^8\,\text{m/s}$, the value of $\langle S \rangle$ is approximately:

(A) $5.0\,\text{W/m}^2$
(B) $26.5\,\text{W/m}^2$
(C) $3.33\,\text{W/m}^2$

(D) $13.3\,\mathrm{W/m^2}$

Q14. For a thermodynamic system, the Gibbs free energy is $G = H - TS$ (relevant at constant T and P). The Maxwell relation derived from $dG = -S\,dT + V\,dP$ is:

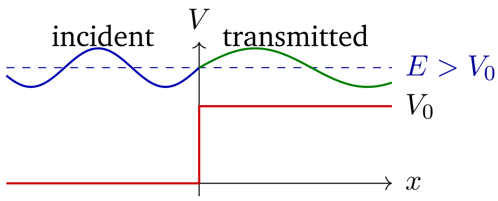
(A) $\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$

(B) $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$

(C) $\left(\frac{\partial T}{\partial P}\right)_S = -\left(\frac{\partial V}{\partial S}\right)_P$

(D) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$

Q15. A particle of mass m moves in a step potential $V(x) = 0$ for $x < 0$ and $V(x) = V_0$ for $x \geq 0$. For $E > V_0$, the wavefunctions in each region and the step are shown below. The wavenumber of the transmitted wave in the region $x > 0$ is:



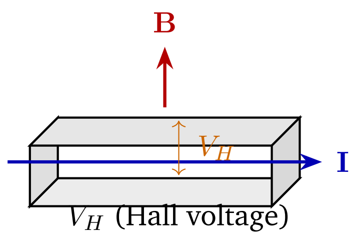
(A) $k_2 = \sqrt{2mE}/\hbar$ (same as incident)

(B) $k_2 = \sqrt{2m(E - V_0)}/\hbar$

(C) $k_2 = \sqrt{2m(E + V_0)}/\hbar$

(D) The wave is exponentially decaying for $E > V_0$

Q16. A semiconductor bar is placed in a magnetic field **B** perpendicular to the current **I**. The resulting Hall effect is illustrated below. For an n-type semiconductor with electron density n, the Hall coefficient R_H is:



(A) $R_H = +1/(ne)$
(B) $R_H = 0$
(C) $R_H = -1/(ne)$
(D) $R_H = ne$

Q17. In a phase-locked loop (PLL), a voltage-controlled oscillator (VCO) is driven by a phase detector that compares the VCO output with a reference signal of frequency f_{ref}. At steady-state lock, the relationship between the VCO output frequency f_{out} and the reference is:

(A) $f_{\text{out}} = 2f_{\text{ref}}$
(B) The VCO oscillates independently at its free-running frequency
(C) The phase detector output is maximum at lock
(D) $f_{\text{out}} = f_{\text{ref}}$

Q18. A nuclear power plant with electrical output $P_e = 1000\,\text{MW}_e$ operates at thermal efficiency $\eta = 33\%$, so the thermal power is $P_{\text{th}} = 3000\,\text{MW}$. Each ^{235}U fission releases $\approx 200\,\text{MeV}$. The number of fissions per second required to sustain this power is approximately:

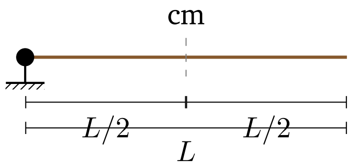
(A) $9.4 \times 10^{19}\,\text{s}^{-1}$
(B) $9.4 \times 10^{22}\,\text{s}^{-1}$
(C) $9.4 \times 10^{16}\,\text{s}^{-1}$
(D) $9.4 \times 10^{25}\,\text{s}^{-1}$

Q19. The function $f(x) = x$ on $[-\pi, \pi]$ is an odd function and has the Fourier series expansion $f(x) = \sum_{n=1}^{\infty} b_n \sin(nx)$, where $b_n = \frac{2}{\pi}\int_0^{\pi} x \sin(nx)\,dx = \frac{2(-1)^{n+1}}{n}$. The coefficient b_3 of $\sin(3x)$ is:

(A) $1/3$
(B) $2/3$
(C) $-2/3$
(D) $-1/3$

Q20. A thin uniform rod of mass m and length L is pivoted about one end. Using the parallel axis theorem $I_{\text{end}} = I_{\text{cm}} + md^2$ with $I_{\text{cm}} = mL^2/12$ and

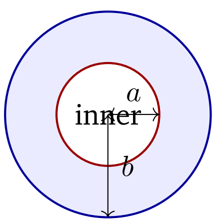
$d = L/2$, as illustrated below:



The moment of inertia about the end is:

(A) $mL^2/6$
(B) $mL^2/4$
(C) $mL^2/3$
(D) $mL^2/12$

Q21. A cylindrical capacitor consists of two coaxial cylinders with inner radius $a = 1\,\text{cm}$, outer radius $b = 2\,\text{cm}$, and length $L = 1\,\text{m}$. The cross-section is shown below. Using $C = 2\pi\varepsilon_0 L/\ln(b/a)$ and $\varepsilon_0 = 8.854 \times 10^{-12}\,\text{F/m}$, the capacitance is:



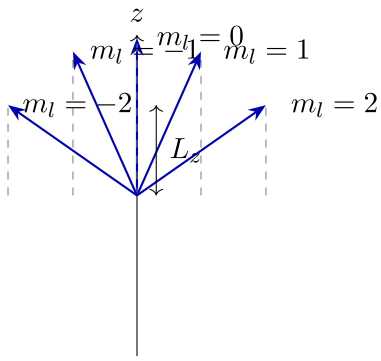
(A) $55.6\,\text{pF}$
(B) $25.6\,\text{pF}$
(C) $40.0\,\text{pF}$
(D) $80.3\,\text{pF}$

Q22. For blackbody radiation, the Planck distribution gives the average energy per mode as $\langle E\rangle = \hbar\omega/(e^{\beta\hbar\omega} - 1)$ where $\beta = 1/(k_B T)$. In the long-wavelength (Rayleigh–Jeans) limit $\hbar\omega \ll k_B T$, the average energy per mode approaches:

(A) $k_B T$ (equipartition result)
(B) $\hbar\omega/2$ (zero-point energy only)
(C) $\hbar\omega$ (single quantum)

(D) Infinity

Q23. For an electron in a d-state ($l = 2$), the orbital angular momentum magnitude is $L = \hbar\sqrt{l(l+1)} = \hbar\sqrt{6}$. The allowed values of $L_z = m_l\hbar$ and the vector model are shown below. The number of allowed m_l values and the possible values of L_z are:

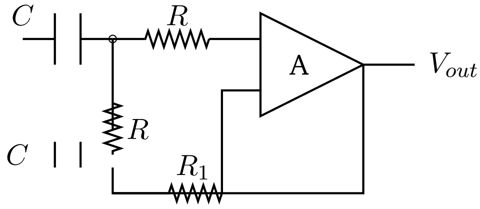


(A) 3 values: $m_l = -1,\ 0,\ +1$
(B) 5 values: $m_l = -2,\ -1,\ 0,\ +1,\ +2$
(C) 7 values: $m_l = -3,\ \ldots,\ +3$
(D) 2 values: $m_l = \pm 2$ only

Q24. According to BCS (Bardeen–Cooper–Schrieffer) theory of superconductivity, Cooper pairs responsible for the supercurrent are formed by:

(A) Two electrons with the same momentum $\mathbf{k}$ and same spin $\uparrow\uparrow$
(B) Two electrons with opposite momenta $(\mathbf{k}, -\mathbf{k})$ and same spin $\uparrow\uparrow$
(C) Two electrons with opposite momenta $(\mathbf{k}, -\mathbf{k})$ and opposite spins $(\uparrow, \downarrow)$
(D) Two electrons with the same momentum $\mathbf{k}$ and opposite spins $(\uparrow, \downarrow)$

Q25. A Wien bridge oscillator uses an RC network and an op-amp to produce a stable sinusoidal output. The circuit is shown schematically below. The oscillation frequency f is:



(A) $f = RC/(2\pi)$
(B) $f = R/(2\pi C)$
(C) $f = 2\pi/(RC)$
(D) $f = 1/(2\pi RC)$

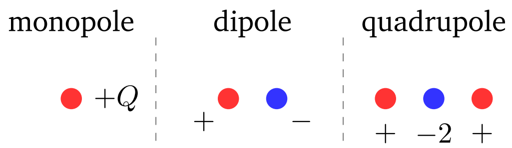
Q26. The spin–orbit interaction in the hydrogen atom gives an energy correction $\Delta E_{nlj} \propto \big[j(j+1) - l(l+1) - s(s+1)\big]$. For the $2p$ level ($l = 1$, $s = \frac{1}{2}$, so $j = \frac{3}{2}$ or $j = \frac{1}{2}$), which statement is correct?

(A) $\Delta E_{j=3/2} > 0$ and $\Delta E_{j=1/2} < 0$ (the $j = 3/2$ level lies higher in energy)
(B) $\Delta E_{j=3/2} < 0$ and $\Delta E_{j=1/2} > 0$ (the $j = 1/2$ level lies higher)
(C) Both corrections are positive; $j = 3/2$ has a larger positive shift
(D) Both corrections are zero for the $2p$ level

Q27. For an ideal gas, the difference between the molar heat capacity at constant pressure C_P and at constant volume C_V is given by $C_P - C_V = R$ (Mayer's relation). For a diatomic ideal gas at moderate temperatures ($C_V = \frac{5}{2}R$), the ratio of specific heats $\gamma = C_P/C_V$ is:

(A) $\gamma = 5/3$
(B) $\gamma = 7/5$
(C) $\gamma = 3/2$
(D) $\gamma = 4/3$

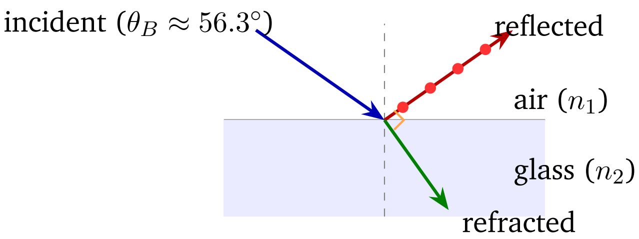
Q28. In the multipole expansion of the electric potential, for a charge distribution that is electrically neutral (total charge $= 0$) and has zero electric dipole moment ($\mathbf{p} = 0$), the three configurations shown below illustrate monopole, dipole, and quadrupole. The leading term in the potential at large r is:



(A) Monopole potential $\propto 1/r$ (since the total charge is Q)
(B) Dipole potential $\propto 1/r^2$
(C) Quadrupole potential $\propto 1/r^3$

(D) No electric potential (neutral distributions have $V = 0$)

Q29. When light is incident at Brewster's angle $\theta_B = \arctan(n_2/n_1)$ on an interface between two dielectrics, the situation is illustrated below (with $n_1 = 1$ for air and $n_2 = 1.5$ for glass). The reflected beam is:



(A) Polarized with $\mathbf{E}$ in the plane of incidence (p-polarized)
(B) Circularly polarized
(C) Unpolarized
(D) Polarized perpendicular to the plane of incidence (s-polarized)

Q30. In the Compton effect, a photon scatters off a free electron at rest. The Compton wavelength shift is $\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)$, where $h/(m_e c) = 2.43\,\text{pm}$ is the Compton wavelength. For a scattering angle $\theta = 60°$, the wavelength shift $\Delta\lambda$ is:

(A) $1.21\,\text{pm}$
(B) $2.43\,\text{pm}$
(C) $0\,\text{pm}$
(D) $4.86\,\text{pm}$

Section B — Multiple Select Questions (MSQ)

***Q31–Q40**: 2 marks each. **No negative marking. One or more** options may be correct. Full marks only if* all *correct options (and no incorrect options) are chosen.*

Q31. Which of the following statements about the Hamiltonian formulation of classical mechanics are correct?

(A) Hamilton's equations are $2N$ coupled first-order ODEs for a system with N degrees of freedom

(B) The Hamiltonian H is always equal to $T + V$ (kinetic plus potential energy), even for velocity-dependent potentials

(C) The Poisson bracket $\{H, H\} = 0$, and H is a constant of the motion whenever $\partial H/\partial t = 0$

(D) Phase-space volume (the volume of a region in (q, p) space) is conserved under Hamiltonian time evolution (Liouville's theorem)

Q32. Which statements about electromagnetic waves propagating in free space (vacuum) are correct?

(A) The fields $\mathbf{E}$ and $\mathbf{B}$ are mutually perpendicular and both perpendicular to the direction of propagation (transverse wave)

(B) The amplitudes are related by $E_0 = cB_0$ (where c is the speed of light)

(C) The wave carries electric charge along its direction of propagation

(D) The time-averaged energy density is $\langle u \rangle = \varepsilon_0 E_0^2/2 = B_0^2/(2\mu_0)$

Q33. Which of the following statements are true for an ideal gas undergoing a *reversible adiabatic* (isentropic) expansion?

(A) The temperature of the gas increases during the expansion

(B) The relation $PV^\gamma =$ constant holds throughout the process (where $\gamma = C_P/C_V$)

(C) The entropy change $\Delta S = 0$ (the process is isentropic)

(D) The work done by the gas equals the heat absorbed from the surroundings

Q34. Which of the following are properties of Hermitian operators $\hat{A}$ (with $\hat{A}^\dagger = \hat{A}$) in quantum mechanics?

(A) All eigenvalues of $\hat{A}$ are real

(B) Eigenvectors belonging to different eigenvalues are orthogonal

(C) A Hermitian operator cannot have zero as an eigenvalue

(D) The expectation value $\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle$ is real for any normalized state $|\psi\rangle$

Q35. Which statements about phonon dispersion in a one-dimensional diatomic chain (with masses $m_1 \neq m_2$) are correct?

(A) The dispersion curve has two branches: an acoustic branch and an optical branch

(B) The acoustic branch has a non-zero frequency at $k = 0$ (the long-wavelength limit)

(C) The optical branch has a non-zero frequency even at $k = 0$ (where the two sub-lattices vibrate against each other)

(D) At the first Brillouin zone boundary there is a frequency gap between the acoustic and optical branches

Q36. Which of the following properties of Bessel functions $J_n(x)$ of the first kind are correct?

(A) $J_n(x)$ satisfies the Bessel equation $x^2 J_n'' + xJ_n' + (x^2 - n^2)J_n = 0$

(B) The general solution of the Bessel equation is a linear combination of $J_n(x)$ (first kind) and $Y_n(x)$ (second kind, Neumann function)

(C) $J_n(x)$ has no positive zeros (i.e. $J_n(x) > 0$ for all $x > 0$)

(D) $J_0(0) = 1$ and $J_n(0) = 0$ for all integers $n \geq 1$

Q37. Which statements about the proton-to-neutron ratio and nuclear stability are correct?

(A) All stable nuclei have equal numbers of protons and neutrons ($N = Z$ exactly)

(B) For very heavy stable nuclei, the neutron number N exceeds the proton number Z ($N > Z$), due to the need to offset Coulomb repulsion

(C) A nucleus with N/Z too large (too many neutrons) tends to undergo β^- decay

(D) Nuclei with too few neutrons can *only* undergo alpha decay; no other decay mode is possible

Q38. Which of the following conditions hold at Brewster's angle θ_B (where $\tan\theta_B = n_2/n_1$)?

(A) The reflected ray and the refracted (transmitted) ray are perpendicular to each other

(B) Both the s-polarized and p-polarized components of the incident light are reflected equally

(C) The reflected beam is completely plane-polarized (only the s-component is reflected)

(D) Brewster's law states $\tan\theta_B = n_2/n_1$

Q39. Which of the following statements about an astable multivibrator are correct?

(A) An astable multivibrator has no stable state; it continuously switches between two quasi-stable states

(B) It generates a continuous square-wave (rectangular pulse) output without any external trigger

(C) An astable multivibrator requires an external trigger pulse to switch from one state to the other

(D) An astable multivibrator has one stable state and one quasi-stable state (it switches once per trigger)

Q40. For a quantum system with canonical partition function $Z = \sum_n e^{-\beta E_n}$ (where $\beta = 1/(k_B T)$), which of the following relations are correct?

(A) The Helmholtz free energy is $F = -k_B T \ln Z$

(B) The entropy is $S = -k_B \ln Z$ (independent of temperature)

(C) The mean energy is $\langle E \rangle = -\dfrac{\partial \ln Z}{\partial \beta}$

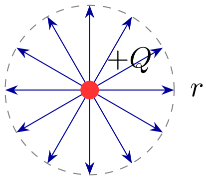
(D) For a non-degenerate multi-level system at finite $T > 0$, the partition function satisfies $Z > 1$

Section C — Numerical Answer Type (NAT)

***Q41–Q50**: 1 mark each. **No negative marking.** Enter the exact numerical value.*

Q41. A transverse wave travels along a string under tension $T = 200\,\mathrm{N}$ with linear mass density $\mu = 0.02\,\mathrm{kg/m}$. Using the wave speed formula $v = \sqrt{T/\mu}$, calculate the wave speed in m/s (to the nearest integer).

Q42. A point charge $Q = 10\,\text{nC}$ is located at the origin in air ($\varepsilon_0 = 8.85 \times 10^{-12}\,\text{F/m}$). The electric field lines radiate outward as shown below. Find the magnitude of the electric field at a distance $r = 1\,\text{m}$ from the charge in V/m (to the nearest integer).



Q43. An electron (mass $m_e = 9.11 \times 10^{-31}\,\text{kg}$, charge $e = 1.6 \times 10^{-19}\,\text{C}$) is accelerated through a potential difference of $V = 1000\,\text{V}$ (kinetic energy $= 1\,\text{keV}$). Using $p = \sqrt{2m_e eV}$ and $\lambda = h/p$ with $h = 6.626 \times 10^{-34}\,\text{J s}$, calculate the de Broglie wavelength in angströms (Å), correct to two decimal places.

Q44. Calculate the heat energy Q (in joules, to the nearest integer) required to raise the temperature of $m = 1\,\text{kg}$ of water by $\Delta T = 50\,°\text{C}$. The specific heat of water is $c = 4186\,\text{J}\,\text{kg}^{-1}\text{K}^{-1}$.

Q45. For a radioactive sample, 90% of the nuclei decay after time t. Using $N/N_0 = (1/2)^{t/T_{1/2}} = 0.10$, the number of half-lives $n = t/T_{1/2}$ required is $n = \log_2(10) = \ln(10)/\ln(2)$. Calculate n correct to two decimal places.

Q46. The atomic packing fraction (APF) of a face-centred cubic (FCC) crystal is computed as follows: there are 4 atoms per unit cell; atoms touch along the face diagonal ($4r = a\sqrt{2}$, so $r = a/(2\sqrt{2})$); each atom occupies volume $\frac{4}{3}\pi r^3$. The APF $= \pi/(3\sqrt{2}) = \pi\sqrt{2}/6$. Express your answer correct to four decimal places.

Q47. Find the sum of the eigenvalues of the matrix $M = \begin{pmatrix} 3 & -1 \\ 2 & 4 \end{pmatrix}$. (Recall: the sum of eigenvalues equals the trace of the matrix.)

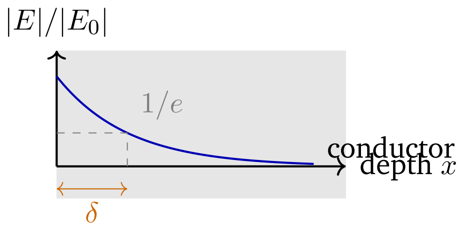
Q48. A series RLC circuit has resistance $R = 10\,\Omega$, inductance $L = 0.1\,\text{H}$, and capacitance $C = 10\,\mu\text{F}$. The resonance angular frequency is $\omega_0 = 1/\sqrt{LC}$ and the quality factor is $Q_f = \omega_0 L/R$. Calculate Q_f correct to two decimal places.

Q49. In an inverting op-amp amplifier, the inverting terminal is at virtual ground potential. The effective input resistance seen by the signal source equals the input resistor R_{in} (since the virtual ground is at the inverting node). If $R_{\text{in}} = 10\,\text{k}\Omega$, what is the input resistance in kΩ (to the nearest integer)?

Q50. An inverting op-amp has feedback resistor $R_f = 100\,\text{k}\Omega$ and input resistor $R_{\text{in}} = 4.7\,\text{k}\Omega$. The voltage gain magnitude is $|G| = R_f/R_{\text{in}}$. Calculate $|G|$ correct to two decimal places.

***Q51–Q60**: 2 marks each. **No negative marking.** Enter the exact numerical value.*

Q51. The skin depth (penetration depth) for electromagnetic fields in a conductor is $\delta = \sqrt{2/(\omega\mu_0\sigma)}$. For copper ($\sigma = 6 \times 10^7\,\text{S/m}$, $\mu_0 = 4\pi \times 10^{-7}\,\text{H/m}$) at frequency $f = 1\,\text{kHz}$, the exponential decay of the field inside the conductor is illustrated below. Calculate δ in mm, correct to two decimal places.

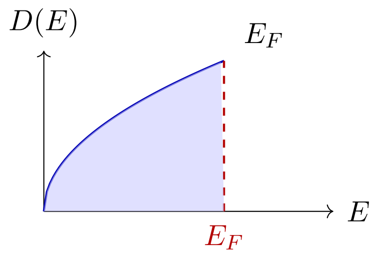


Q52. X-rays undergo Bragg diffraction from crystal planes of spacing $d = 2.82\,\text{Å}$ (sodium chloride). For first-order diffraction ($n = 1$) at a glancing angle $\theta = 15^\circ$, using $n\lambda = 2d\sin\theta$ and $\sin 15^\circ = 0.2588$, calculate the X-ray wavelength λ in angströms (Å), correct to two decimal places.

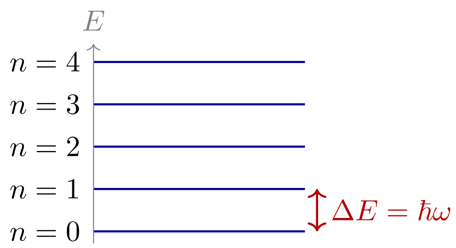
Q53. One mole of an ideal gas expands irreversibly into a vacuum (free expansion) at constant temperature $T = 300\,\text{K}$, doubling its volume from V to $2V$. Although no work is done ($W = 0$) and $\Delta U = 0$, the entropy change is $\Delta S = nR\ln(V_f/V_i)$. Calculate ΔS in J/K, correct to two decimal places. ($R = 8.314\,\text{J}\,\text{mol}^{-1}\text{K}^{-1}$, $\ln 2 = 0.6931$.)

Q54. The Fermi energy for free electrons in gold (Au) is given by $E_F = \dfrac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$, with number density $n = 5.90 \times 10^{28}\,\text{m}^{-3}$. Using $\hbar = 1.055 \times 10^{-34}\,\text{J s}$,

$m_e = 9.11 \times 10^{-31}$ kg, and the density of states shown below, calculate E_F in eV correct to two decimal places. ($1\,\text{eV} = 1.6 \times 10^{-19}$ J.)



Q55. A quantum harmonic oscillator in 3 dimensions has equally spaced energy levels with spacing $\Delta E = \hbar\omega$. For $\omega = 5 \times 10^{13}$ rad/s, the energy level ladder is shown below. Calculate the energy of the first allowed transition $\Delta E = \hbar\omega$ in eV, correct to three decimal places. ($\hbar = 1.055 \times 10^{-34}$ J s, $1\,\text{eV} = 1.6 \times 10^{-19}$ J.)



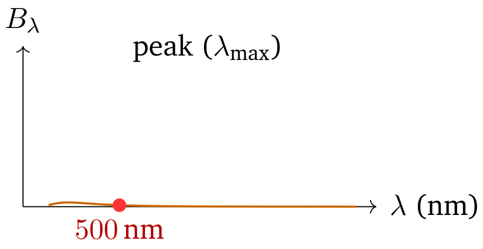
Q56. A laser operating at wavelength $\lambda = 633$ nm has output power $P = 10$ W. The energy per photon is $E_{\text{ph}} = hc/\lambda$. Calculate the number of photons emitted per second in units of $10^{18}\,\text{s}^{-1}$, correct to two decimal places. ($h = 6.626 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m/s.)

Q57. A radioactive sample starts with N_0 atoms. After a time $t = 4.5\,T_{1/2}$ (where $T_{1/2}$ is the half-life), the fraction of atoms remaining is $N/N_0 = (1/2)^{4.5} = 1/(16\sqrt{2})$. Calculate this fraction correct to four decimal places.

Q58. Evaluate the definite integral $I = \displaystyle\int_0^\infty t^3\, e^{-2t}\, dt$ using the formula $\displaystyle\int_0^\infty t^n e^{-\alpha t}\, dt = \Gamma(n+1)/\alpha^{n+1} = n!/\alpha^{n+1}$ for non-negative integers n. Give the answer correct to three decimal places.

Q59. An RC low-pass filter has $R = 10\,\text{k}\Omega$ and $C = 10$ nF. The -3 dB (half-power) cutoff frequency is $f_c = 1/(2\pi RC)$. Calculate f_c in kHz, correct to two decimal places.

Q60. Wien's displacement law states that the wavelength at which the Planck blackbody spectrum peaks satisfies $\lambda_{\text{max}} T = b = 2.898 \times 10^{-3}\,\text{m K}$. The Planck distribution and its peak at $\lambda_{\text{max}} = 500\,\text{nm}$ are shown below. Calculate the corresponding blackbody temperature T in K (to the nearest integer).



Solutions

IIT JAM Physics – Sample Paper 3

Section A Solutions — MCQ

Solution

Concept — Radius of Convergence (Ratio Test):
For the power series $\sum a_n x^n$ with $a_n = n!$, the radius of convergence is:

$$R = \lim_{n\to\infty} \frac{|a_n|}{|a_{n+1}|} = \lim_{n\to\infty} \frac{n!}{(n+1)!} = \lim_{n\to\infty} \frac{1}{n+1} = 0$$

Interpretation: $R = 0$ means the series $\sum n!\,x^n$ converges *only* at $x = 0$ and diverges everywhere else. The factorial growth of the coefficients far outpaces any polynomial power of x.

Final Answer: $R = 0 \Rightarrow$ **(D)**

Why other options are wrong:

Q1.
- (A) $R = \infty$: This would be a series like $\sum x^n/n!$ (exponential), not $\sum n!\,x^n$.
- (B) $R = 1$: Applies to geometric series $\sum x^n$ where $a_n = 1$.
- (C) $R = e$: No standard power series with $n!$ coefficients has $R = e$.

Answer: (D) ← *Go back to Q1*

Solution

Concept — Hamiltonian Mechanics, Phase Space Trajectory:
For a free particle, $H = p^2/(2m)$ with no x-dependence.

Hamilton's equations:

$$\dot{x} = \frac{\partial H}{\partial p} = \frac{p}{m}, \qquad \dot{p} = -\frac{\partial H}{\partial x} = 0$$

Since $\dot{p} = 0$, momentum p is a constant of the motion: $p(t) = p_0 = \sqrt{2mE}$.
Position increases linearly: $x(t) = x_0 + (p_0/m)\,t$.

Phase-space trajectory: At constant energy E, the trajectory is the horizontal line $p = p_0 =$ const traversed from left to right (or right to left depending on sign of p_0). This is a straight horizontal line.

Final Answer: A horizontal straight line $p = \sqrt{2mE} \Rightarrow$ **(A)**

Why other options are wrong:

Q2.
- (B) Ellipse: The SHO phase-space trajectory is an ellipse; free particle has no restoring force.
- (C) Circle: Only for SHO with specially normalised coordinates.
- (D) Parabola: No potential means no curved trajectory in phase space.

Answer: (A) ← *Go back to Q2*

Solution

Concept — Brewster's Angle:
At Brewster's angle θ_B, the reflected and refracted rays are perpendicular, so the reflected light is completely s-polarized. The condition is:

$$\tan\theta_B = \frac{n_2}{n_1}$$

For glass ($n_2 = 1.5$) in air ($n_1 = 1$):

$$\theta_B = \arctan(1.5) = \arctan(1.5)$$

Since $\tan 56.3^\circ \approx 1.5$ (noting $\tan 45^\circ = 1$ and $\tan 63.4^\circ = 2$, interpolating gives $\theta_B \approx 56.3^\circ$):

$$\theta_B \approx 56.3^\circ$$

Final Answer: $\theta_B \approx 56.3^\circ \Rightarrow$ **(B)**

Why other options are wrong:

Q3.
- (A) 33.7°: This is $90^\circ - \theta_B$ (the refracted ray angle), not θ_B.
- (C) 45°: Brewster angle is 45° only if $n = \tan 45^\circ = 1$ (trivial; no interface).
- (D) 63.4°: This corresponds to $\tan\theta = 2.0$ (glass with $n = 2$), not $n = 1.5$.

Answer: (B) ← *Go back to Q3*

Solution

Concept — Dielectric Susceptibility:
The relative permittivity and susceptibility are related by:

$$\varepsilon_r = 1 + \chi_e$$

Step 1 — Solve for χ_e:

$$\chi_e = \varepsilon_r - 1 = 4 - 1 = 3$$

Step 2 — Polarization:
$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E} = 3\varepsilon_0 \mathbf{E}$. The induced dipoles align with the external field as shown.

Final Answer: $\chi_e = 3 \Rightarrow$ **(C)**

Why other options are wrong:

Q4.
- (A) $\chi_e = 4$: This conflates ε_r with χ_e; the relation is $\chi_e = \varepsilon_r - 1$.
- (B) $\chi_e = 2$: Would give $\varepsilon_r = 3$, not 4.
- (D) $\chi_e = 1/4$: There is no physical basis; susceptibility is not the reciprocal of ε_r.

Answer: (C) ← *Go back to Q4*

Solution

Concept — Carnot Efficiency:
$\eta = 1 - T_C/T_H$

Step 1 — Original efficiency (check):

$$\eta = 1 - \frac{400}{800} = 1 - 0.5 = 50\% \quad \checkmark$$

Step 2 — New efficiency with $T'_H = 1000\,\mathrm{K}$:

$$\eta' = 1 - \frac{T_C}{T'_H} = 1 - \frac{400}{1000} = 1 - 0.4 = 0.60 = 60\%$$

Final Answer: $\eta' = 60\% \Rightarrow$ **(D)**

Why other options are wrong:

Q5.
- (A) 50%: Original efficiency; the new T_H is higher so efficiency must increase.
- (B) 55%: Arithmetic error; $400/1000 = 0.40$, giving $\eta' = 60\%$, not 55%.
- (C) 45%: Efficiency decreasing would require T_H to decrease, not increase.

Answer: (D) ← *Go back to Q5*

Solution

Concept — Quantum Tunneling:
The transmission probability through a rectangular barrier of height $V_0 > E$ and width a is:

$$T \approx e^{-2\kappa a}, \qquad \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar} > 0$$

Since $\kappa > 0$ and $a > 0$, we have $e^{-2\kappa a} > 0$. The transmission probability is *non-zero* even when $E < V_0$. This is a purely quantum phenomenon with no classical analogue.

Final Answer: Statement (A) is correct $\Rightarrow$ **(A)**

Why other options are wrong:

Q6.

- (B) T increases with a: False. $T \propto e^{-2\kappa a}$ *decreases* exponentially as a grows.
- (C) Classical particle can penetrate: False. Classically, $E < V_0$ means total reflection; no penetration.
- (D) $T = 0$ exactly: False. Quantum mechanics gives $T > 0$ for any finite barrier width.

Answer: (A) ← *Go back to Q6*

Solution

Concept — Fermi Level in Intrinsic Semiconductor at $T = 0\,$K:
At $T = 0\,$K, the intrinsic semiconductor has:

Q7.

- Valence band: completely filled with electrons
- Conduction band: completely empty

The Fermi level is defined as the energy at which the Fermi–Dirac distribution function equals $1/2$. At $T = 0$, the step function nature of the Fermi–Dirac distribution places E_F precisely in the **middle of the band gap**: $E_F = E_V + E_g/2$ (where E_V is the valence band maximum). This follows from the electron-hole symmetry of intrinsic semiconductors.

Final Answer: E_F lies exactly in the middle of the band gap $\Rightarrow$ **(B)**

Why other options are wrong:

- (A) At CB minimum: This would violate the symmetry; E_F would be at the CB edge only if the effective masses were infinite.
- (C) At VB maximum: This would place E_F inside the valence band, ignoring the gap.
- (D) Above the CB: At $T = 0$, the CB is empty, so E_F cannot be above it.

Answer: (B) ← *Go back to Q7*

Solution

Concept — Zener Diode as Voltage Regulator:
A Zener diode is specifically designed to operate in the **reverse breakdown** (Zener/avalanche) region. In this mode:

Q8.

- The voltage across the Zener remains clamped at V_Z (here, $5.1\,\text{V}$).
- Changes in input voltage or load current are absorbed by changes in Zener current.
- The output voltage $V_{\text{out}} = V_Z = 5.1\,\text{V}$ is maintained constant.

The series resistor R_s limits the Zener current to a safe value. The load R_L receives the regulated voltage.

Final Answer: Reverse breakdown (Zener breakdown) mode $\Rightarrow$ **(C)**

Why other options are wrong:

- (A) Forward bias amplification: Zener diodes do not amplify; regulators do not use forward bias.
- (B) Forward bias blocking: Diodes do not block in forward bias; they conduct at $\sim 0.7\,\text{V}$.
- (D) Avalanche in forward bias: Avalanche breakdown is a reverse-bias phenomenon.

Answer: (C) ← *Go back to Q8*

Solution

Concept — Q-Value of Nuclear Reaction:
The Q-value measures the energy released ($Q > 0$, exothermic) or absorbed ($Q < 0$, endothermic):

$$Q = \left(M_{\text{initial}} - M_{\text{final}}\right)c^2 = \left(\text{total mass of reactants} - \text{total mass of products}\right)c^2$$

In atomic mass units: $Q = (M_i - M_f) \times 931.5\,\text{MeV/u}$.

Physical interpretation:

Q9.

- $Q > 0$: Rest mass decreases $\rightarrow$ kinetic energy increases (exothermic)
- $Q < 0$: Rest mass increases $\rightarrow$ kinetic energy input required (endothermic)

Final Answer: $Q = (M_i - M_f)c^2 \Rightarrow$ **(D)**

Why other options are wrong:

- (A) $(M_f - M_i)c^2$: Sign is reversed; this gives negative Q for an exothermic reaction.

- (B) $(M_i + M_f)c^2$: Sum, not difference; dimensionally incorrect for energy *released*.
- (C) $Q > 0$ always: Many reactions are endothermic (e.g. photonuclear reactions) with $Q < 0$.

Answer: (D) *← Go back to Q9*

Solution

Concept — Green's Theorem:

Green's theorem: $\oint_C (P\,dx + Q\,dy) = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA.$

Step 1 — Compute the integrand with $P = -y$, $Q = x$:

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{\partial x}{\partial x} - \frac{\partial(-y)}{\partial y} = 1 - (-1) = 2$$

Step 2 — Apply Green's theorem over the unit disk D ($|z| \leq 1$, area $= \pi$):

$$\oint_C (-y\,dx + x\,dy) = \iint_D 2\,dA = 2 \times \text{Area}(D) = 2 \times \pi = 2\pi$$

Final Answer: $2\pi \Rightarrow$ **(A)**

Why other options are wrong:

Q10.

- (B) π: Missing factor of 2 from the integrand $(\partial Q/\partial x - \partial P/\partial y) = 2$.
- (C) 4π: Off by factor of 2; area of unit circle is π, not 2π.
- (D) 0: Would require the integrand to vanish, which it does not (it equals 2).

Answer: (A) *← Go back to Q10*

Solution

Concept — Coupled Pendulums (Normal Modes):

For two pendulums (each mass m, length l) connected by a spring k at their bobs:

Equations of motion (displacements x_1, x_2 from equilibrium):

$$m\ddot{x}_1 = -\frac{mg}{l}x_1 - k(x_1 - x_2), \qquad m\ddot{x}_2 = -\frac{mg}{l}x_2 + k(x_1 - x_2)$$

Normal modes (assume $x_{1,2} \propto e^{i\omega t}$):

Symmetric mode ($x_1 = x_2$): Spring unstretched, no spring force:

$$\omega_1^2 = \frac{g}{l}$$

Antisymmetric mode ($x_1 = -x_2$): Spring fully active:

$$\omega_2^2 = \frac{g}{l} + \frac{2k}{m}$$

Final Answer: $\omega_1 = \sqrt{g/l},\ \omega_2 = \sqrt{g/l + 2k/m} \Rightarrow$ **(B)**

Why other options are wrong:

Q11.

- (A) Degenerate modes: Only true if $k = 0$ (no coupling spring).
- (C) $\omega_1 = \sqrt{g/l - k/m}$: Spring would reduce the in-phase frequency — impossible (restoring force cannot become negative from a spring).
- (D) $\omega_1 = \sqrt{2k/m}$: Omits the gravitational restoring term g/l.

Answer: (B) *← Go back to Q11*

Solution

Concept — Lorentz Length Contraction:
An object at rest has proper length L_0. An observer moving at speed v relative to the object measures:

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

Step 1 — Compute the Lorentz factor:

$$\beta = v/c = 0.6, \qquad \sqrt{1-\beta^2} = \sqrt{1-0.36} = \sqrt{0.64} = 0.8$$

Step 2 — Contracted length:

$$L = 1000\,\mathrm{m} \times 0.8 = 800\,\mathrm{m}$$

Final Answer: $L = 800\,\mathrm{m} \Rightarrow$ **(C)**

Why other options are wrong:

Q12.

- (A) $750\,\mathrm{m}$: Would require $\sqrt{1 - v^2/c^2} = 0.75$, i.e. $v/c \approx 0.661$ (not 0.6).
- (B) $1000\,\mathrm{m}$: No contraction; this is the proper length (would hold if $v = 0$).
- (D) $600\,\mathrm{m}$: Would require $\sqrt{1 - v^2/c^2} = 0.6$, i.e. $v/c \approx 0.8$ (not $0.6c$).

Answer: (C) *← Go back to Q12*

Solution

Concept — Time-Averaged Intensity of EM Wave:
$\langle S \rangle = \dfrac{\varepsilon_0 c E_0^2}{2}$

Calculation:

$$\begin{aligned}\langle S \rangle &= \frac{1}{2} \times 8.854 \times 10^{-12} \times 3 \times 10^{8} \times (100)^2 \\ &= \frac{1}{2} \times 8.854 \times 10^{-12} \times 3 \times 10^{8} \times 10^{4} \\ &= \frac{1}{2} \times 8.854 \times 3 \times 10^{0} \\ &= \frac{1}{2} \times 26.562 \\ &= 13.28\,\mathrm{W/m^2} \approx 13.3\,\mathrm{W/m^2}\end{aligned}$$

Final Answer: $\langle S \rangle \approx 13.3\,\mathrm{W/m^2} \Rightarrow$ **(D)**

Why other options are wrong:

Q13.

- (A) $5.0\,\mathrm{W/m^2}$: Underestimate; incorrect intermediate computation.
- (B) $26.5\,\mathrm{W/m^2}$: Missing the factor of $1/2$ (gives peak intensity, not time-average).
- (C) $3.33\,\mathrm{W/m^2}$: Off by factor of 4; likely used $E_0^2/(2\mu_0 c)$ with a factor error.

Answer: (D) ← *Go back to Q13*

Solution

Concept — Maxwell Relations from Gibbs Free Energy:
The Gibbs free energy differential is $dG = -S\,dT + V\,dP$.

Step 1 — Apply the cross-derivative (Schwarz) condition:
Since dG is an exact differential, the mixed partial derivatives are equal:

$$G = G(T,P) \Rightarrow \frac{\partial}{\partial P}\left(\frac{\partial G}{\partial T}\right)_P = \frac{\partial}{\partial T}\left(\frac{\partial G}{\partial P}\right)_T$$

Step 2 — Identify the derivatives:

$$\left(\frac{\partial G}{\partial T}\right)_P = -S, \qquad \left(\frac{\partial G}{\partial P}\right)_T = V$$

Step 3 — Maxwell relation:

$$\left(\frac{\partial S}{\partial P}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_P$$

Final Answer: Option (A) $\Rightarrow$ **(A)**

Why other options are wrong:

Q14.

- (B): This Maxwell relation comes from the Helmholtz free energy $F(T,V)$: $(\partial S/\partial V)_T = (\partial P/\partial T)_V$.
- (C) and (D): These derive from the enthalpy $H(S,P)$ and internal energy $U(S,V)$ respectively.

Answer: (A) ← *Go back to Q14*

Solution

Concept — Step Potential ($E > V_0$):

For $x > 0$ with $V = V_0$ and $E > V_0$, the time-independent Schrödinger equation gives:

$$-\frac{\hbar^2}{2m}\psi'' + V_0\psi = E\psi \quad \Rightarrow \quad \psi'' = -k_2^2\psi$$

where

$$k_2 = \frac{\sqrt{2m(E - V_0)}}{\hbar}$$

This is a real, positive wavenumber (since $E > V_0$), so the transmitted wave is *oscillatory*: $\psi \propto e^{ik_2x}$.

Note: $k_2 < k_1 = \sqrt{2mE}/\hbar$ because $E - V_0 < E$, so the transmitted wave has a *longer* wavelength.

Final Answer: $k_2 = \sqrt{2m(E - V_0)}/\hbar \Rightarrow$ **(B)**

Why other options are wrong:

Q15.

- (A) $k_2 = \sqrt{2mE}/\hbar$: This is the incident wavenumber k_1, not k_2 (potential changes the kinetic energy).
- (C) $\sqrt{2m(E + V_0)}/\hbar$: Energy *gained* by the step would require a potential well, not a barrier.
- (D) Exponential decay: This applies for $E < V_0$ (evanescent wave), not $E > V_0$.

Answer: (B) ← *Go back to Q15*

Solution

Concept — Hall Coefficient:

For an n-type semiconductor with electron density n (electrons as majority carriers):

Physics: Current **I** flows in the $+x$ direction; electrons drift in the $-x$ direction. In field $\mathbf{B} = B\hat{z}$, the Lorentz force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$ on electrons is:

$$F_y = (-e)(v_x)(B) = -ev_xB \quad \Rightarrow \quad \text{electrons accumulate on } -y \text{ face}$$

This creates a Hall electric field E_H in the $-y$ direction (negative Hall voltage for n-type). The Hall coefficient is:

$$R_H = \frac{E_H}{J_xB} = -\frac{1}{ne}$$

The negative sign distinguishes n-type (negative R_H) from p-type (positive $R_H = +1/(pe)$).

Final Answer: $R_H = -1/(ne) \Rightarrow$ **(C)**

Why other options are wrong:

Q16.
- (A) $R_H = +1/(ne)$: Positive sign is for p-type (holes), not n-type.
- (B) $R_H = 0$: Hall coefficient is zero only if electron and hole densities are equal (intrinsic) and have equal mobilities.
- (D) $R_H = ne$: Dimensionally incorrect.

Answer: (C) ← *Go back to Q16*

Solution

Concept — Phase-Locked Loop (PLL):
A PLL consists of three main blocks:

Q17.
(a) **Phase Detector (PD):** Compares phases of VCO output and reference, produces error voltage proportional to phase difference.
(b) **Low-Pass Filter (LPF):** Smooths the error signal.
(c) **Voltage-Controlled Oscillator (VCO):** Its frequency is controlled by the filtered error voltage.

At lock: The feedback loop drives the phase error to zero (or to a constant), meaning the VCO output frequency exactly equals the reference frequency:

$$f_{\text{out}} = f_{\text{ref}}$$

The phase detector output at lock is a constant (not maximum); the VCO tracks any changes in f_{ref}.

Final Answer: $f_{\text{out}} = f_{\text{ref}} \Rightarrow$ **(D)**

Why other options are wrong:

- (A) $f_{out} = 2f_{ref}$: Frequency doubling requires a frequency divider in the feedback loop.
- (B) VCO at free-running frequency: When locked, VCO is driven away from its free-running frequency.
- (C) Phase detector output maximum at lock: At lock, the phase detector output is constant (not necessarily maximum), maintaining a DC correction.

Answer: (D) ← *Go back to Q17*

Q18.

Solution

Concept — Nuclear Fission Power:

Step 1 — Thermal power:

$P_{\text{th}} = P_e/\eta = 1000\,\text{MW}/0.33 = 3030\,\text{MW} \approx 3 \times 10^9\,\text{W}$

Step 2 — Energy per fission in joules:

$$E_{\text{fis}} = 200\,\text{MeV} = 200 \times 10^6 \times 1.6 \times 10^{-19}\,\text{J} = 3.2 \times 10^{-11}\,\text{J}$$

Step 3 — Fissions per second:

$$\dot{N} = \frac{P_{\text{th}}}{E_{\text{fis}}} = \frac{3 \times 10^9\,\text{W}}{3.2 \times 10^{-11}\,\text{J}} = \frac{3}{3.2} \times 10^{20} = 9.375 \times 10^{19}\,\text{s}^{-1} \approx 9.4 \times 10^{19}\,\text{s}^{-1}$$

Final Answer: $\approx 9.4 \times 10^{19}\,\text{s}^{-1} \Rightarrow$ **(A)**

Why other options are wrong:

- (B) 9.4×10^{22}: Off by factor 10^3; confusion of MW with kW or similar error.
- (C) 9.4×10^{16}: Off by factor 10^3 in the other direction.
- (D) 9.4×10^{25}: Off by factor 10^6.

Answer: (A) ← *Go back to Q18*

Solution

Concept — Fourier Series of $f(x) = x$:

Since $f(x) = x$ is odd on $[-\pi, \pi]$, only sine terms appear: $b_n = \dfrac{2}{\pi}\displaystyle\int_0^{\pi} x \sin(nx)\, dx$.

Step 1 — Evaluate by integration by parts:

$$b_n = \frac{2}{\pi}\left[-\frac{x\cos(nx)}{n} + \frac{\sin(nx)}{n^2}\right]_0^{\pi} = \frac{2}{\pi}\left(-\frac{\pi\cos(n\pi)}{n}\right) = \frac{2(-1)^{n+1}}{n}$$

Step 2 — Coefficient of $\sin(3x)$ ($n = 3$):

$$b_3 = \frac{2(-1)^{3+1}}{3} = \frac{2(-1)^4}{3} = \frac{2\times 1}{3} = \frac{2}{3}$$

The series is $f(x) = 2\left(\sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \cdots\right)$, so $b_3 = 2/3$.

Final Answer: $b_3 = 2/3 \Rightarrow$ **(B)**

Why other options are wrong:

Q19.
- (A) $1/3$: Missing the factor of 2 in the formula for b_n.
- (C) $-2/3$: Wrong sign; $(-1)^{3+1} = (-1)^4 = +1$, giving $+2/3$.
- (D) $-1/3$: Both magnitude and sign errors combined.

Answer: (B) ← *Go back to Q19*

Solution

Concept — Parallel Axis Theorem:
$I_{\text{end}} = I_{\text{cm}} + Md^2$, where $d = L/2$ (distance from cm to end).

Step 1 — Moment of inertia about cm:

$$I_{\text{cm}} = \frac{mL^2}{12}$$

Step 2 — Apply parallel axis theorem:

$$I_{\text{end}} = \frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{12} + \frac{mL^2}{4} = \frac{mL^2}{12} + \frac{3mL^2}{12} = \frac{4mL^2}{12} = \frac{mL^2}{3}$$

Final Answer: $I_{\text{end}} = mL^2/3 \Rightarrow$ **(C)**

Why other options are wrong:

Q20.
- (A) $mL^2/6$: No standard axis gives this; possibly $\frac{1}{2} \times I_{\text{end}}$ (factor error).
- (B) $mL^2/4$: Only the $m(L/2)^2$ term; forgot to add $I_{\text{cm}} = mL^2/12$.
- (D) $mL^2/12$: This is I_{cm} (about centre), not about the end.

Answer: (C) ← *Go back to Q20*

Solution

Concept — Cylindrical (Coaxial) Capacitor:
$C = \dfrac{2\pi\varepsilon_0 L}{\ln(b/a)}$

Step 1 — Compute $\ln(b/a)$**:**

$$\ln(b/a) = \ln(2\,\text{cm}/1\,\text{cm}) = \ln 2 = 0.6931$$

Step 2 — Compute capacitance:

$$\begin{aligned} C &= \frac{2\pi \times 8.854 \times 10^{-12} \times 1.0}{0.6931} \\ &= \frac{2 \times 3.14159 \times 8.854 \times 10^{-12}}{0.6931} \\ &= \frac{55.63 \times 10^{-12}}{0.6931} \\ &= 80.26 \times 10^{-12}\,\text{F} = 80.3\,\text{pF} \end{aligned}$$

Final Answer: $C \approx 80.3\,\text{pF} \Rightarrow$ **(D)**

Why other options are wrong:

Q21.

- (A) $55.6\,\text{pF}$: Forgot to divide by $\ln 2$; numerator is $2\pi\varepsilon_0 L = 55.6\,\text{pF}$ before the $\ln(b/a)$ division.
- (B) $25.6\,\text{pF}$: Incorrect; possibly used $\ln(b/a) = 2$ (error) instead of $\ln 2 = 0.693$.
- (C) $40.0\,\text{pF}$: Off by factor of 2; no physical basis.

Answer: (D) ← *Go back to Q21*

Solution

Concept — Planck Distribution, Rayleigh–Jeans Limit:

The average energy per mode: $\langle E\rangle = \dfrac{\hbar\omega}{e^{\hbar\omega/k_BT} - 1}$.

Long-wavelength (Rayleigh–Jeans) limit $\hbar\omega \ll k_BT$ ($x = \hbar\omega/k_BT \ll 1$):

Expand $e^x - 1 \approx x$ for small x:

$$\langle E\rangle \approx \frac{\hbar\omega}{\hbar\omega/k_BT} = k_BT$$

This is the classical equipartition result: each mode has average energy k_BT (or $\frac{1}{2}k_BT$ for each of two quadratic terms).

Final Answer: $\langle E\rangle \to k_BT \Rightarrow$ **(A)**

Why other options are wrong:

Q22.

- (B) $\hbar\omega/2$: This is the quantum zero-point energy, dominant in the short-wavelength limit (not the Rayleigh–Jeans limit).
- (C) $\hbar\omega$: One quantum of energy; not the classical average.
- (D) Infinity: The ultraviolet catastrophe predicted $\langle E \rangle = k_B T \propto \omega^2$ divergence for the total energy, not per mode.

Answer: (A) ← *Go back to Q22*

Solution

Concept — Orbital Angular Momentum ($l = 2$, d-state):
For quantum number $l = 2$, the magnetic quantum number m_l takes integer values from $-l$ to $+l$:

$$m_l = -2,\ -1,\ 0,\ +1,\ +2 \quad \Rightarrow \quad 2l + 1 = 5 \text{ values}$$

The z-component of angular momentum $L_z = m_l\hbar$ thus has 5 possible values: $-2\hbar,\ -\hbar,\ 0,\ +\hbar,\ +2\hbar$.

The total angular momentum magnitude is $L = \hbar\sqrt{l(l+1)} = \hbar\sqrt{6} \approx 2.449\hbar$.

Final Answer: 5 values: $m_l = -2,\ -1,\ 0,\ +1,\ +2 \Rightarrow$ **(B)**

Why other options are wrong:

Q23.

- (A) 3 values: This is for $l = 1$ (p-state), where $m_l = -1, 0, +1$.
- (C) 7 values: This is for $l = 3$ (f-state), where $m_l = -3, \ldots, +3$.
- (D) 2 values $m_l = \pm 2$ only: Not all m_l values in $[-l, +l]$ are allowed.

Answer: (B) ← *Go back to Q23*

Solution

Concept — BCS Theory of Superconductivity (Cooper Pairs):
In BCS theory (1957), the pairing mechanism is:

Q24.

(a) An electron moves through the lattice, creating a slight positive-charge distortion (via phonon emission).
(b) This distortion attracts a *second* electron, creating an effective electron-electron attractive interaction mediated by phonons.
(c) The paired electrons have *opposite momenta* $(\mathbf{k}, -\mathbf{k})$ and *opposite spins* $(\uparrow, \downarrow)$ — a spin-singlet state.

Cooper pairs have charge $-2e$, boson-like statistics, and condense into a macroscopic quantum state below T_c. The binding energy of a Cooper pair is 2Δ (the superconducting gap).

Final Answer: Opposite momenta and opposite spins $\Rightarrow$ **(C)**

Why other options are wrong:

- (A) Same momentum, same spin: Pauli exclusion forbids identical fermions in the same state.
- (B) Same spin: Triplet pairing ($S = 1$) occurs in exotic (p-wave) superconductors; conventional BCS is singlet ($S = 0$).
- (D) Same momentum: Pairs with zero total momentum ($\mathbf{k} + (-\mathbf{k}) = 0$) condense into one quantum state — this is the BCS ground state.

Answer: (C) ← *Go back to Q24*

Solution

Concept — Wien Bridge Oscillator Frequency:
The Wien bridge oscillator uses an RC network (series RC in one arm, parallel RC in another). The phase of the RC bridge network equals zero (required for oscillation) at frequency:

$$f_0 = \frac{1}{2\pi RC}$$

At this frequency, the voltage divider formed by the RC bridge passes the signal with zero phase shift and attenuation of $1/3$. The op-amp gain must be ≥ 3 to sustain oscillation.

Final Answer: $f = 1/(2\pi RC) \Rightarrow$ **(D)**

Why other options are wrong:

Q25.

- (A) $RC/(2\pi)$: Inverted expression; has units of $\Omega \cdot \mathrm{F} \cdot \mathrm{s} = \mathrm{s}^2$, not Hz.
- (B) $R/(2\pi C)$: Has units of $\Omega/\mathrm{F} = \mathrm{V}^2/\mathrm{J}$, not Hz.
- (C) $2\pi/(RC)$: This is angular frequency $\omega_0 = 2\pi f$, not frequency f.

Answer: (D) ← *Go back to Q25*

Solution

Concept — Spin–Orbit Coupling in Hydrogen:
The spin-orbit energy correction for the $2p$ state ($l = 1$, $s = \frac{1}{2}$) is:

$$\Delta E \propto j(j+1) - l(l+1) - s(s+1)$$

For $j = 3/2$:

$$\Delta E_{3/2} \propto \frac{3}{2}\cdot\frac{5}{2} - 1\cdot 2 - \frac{1}{2}\cdot\frac{3}{2} = \frac{15}{4} - 2 - \frac{3}{4} = \frac{15-8-3}{4} = \frac{4}{4} = 1 > 0$$

For $j = 1/2$:

$$\Delta E_{1/2} \propto \frac{1}{2}\cdot\frac{3}{2} - 1\cdot 2 - \frac{1}{2}\cdot\frac{3}{2} = \frac{3}{4} - 2 - \frac{3}{4} = -2 < 0$$

Thus the $j = 3/2$ level lies *above* the unsplit level and $j = 1/2$ lies *below*. The fine-structure splitting is $\Delta E_{3/2} - \Delta E_{1/2} > 0$.

Final Answer: $\Delta E_{j=3/2} > 0$, $\Delta E_{j=1/2} < 0 \Rightarrow$ **(A)**

Why other options are wrong:

Q26.

- (B) Inverted ordering: Incorrectly reverses the sign of the spin-orbit coupling.
- (C) Both positive: $j = 1/2$ gives a negative correction (calculation above).
- (D) Both zero: Non-zero l and s guarantee non-zero corrections (unless $\xi_{nl} = 0$, which is not physical).

Answer: (A) ← *Go back to Q26*

Solution

Concept — Heat Capacities and γ for Diatomic Gas:
Mayer's relation: $C_P - C_V = R$ (per mole, for any ideal gas).

For diatomic gas at moderate temperatures (3 translational + 2 rotational degrees of freedom):

$$C_V = \frac{5}{2}R, \qquad C_P = C_V + R = \frac{7}{2}R$$

$$\gamma = \frac{C_P}{C_V} = \frac{7/2}{5/2} = \frac{7}{5} = 1.4$$

Final Answer: $\gamma = 7/5 \Rightarrow$ **(B)**

Why other options are wrong:

Q27.

- (A) $\gamma = 5/3$: Correct for *monatomic* ideal gas ($C_V = \frac{3}{2}R$).
- (C) $\gamma = 3/2$: Does not correspond to any standard ideal gas.
- (D) $\gamma = 4/3$: Corresponds to an ultra-relativistic gas or a gas with many degrees of freedom ($C_V = 3R$).

Answer: (B) ← *Go back to Q27*

Q28.

Solution

Concept — Multipole Expansion:
The electric potential of a localized charge distribution at large distance r can be expanded:

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r} + \frac{\mathbf{p}\cdot\hat{r}}{r^2} + \frac{\text{quadrupole}}{r^3} + \cdots\right]$$

Given conditions:

- Total charge $Q = 0$ (electrically neutral) $\Rightarrow$ monopole term vanishes.
- Dipole moment $\mathbf{p} = 0 \Rightarrow$ dipole term also vanishes.

The leading non-vanishing term is therefore the **quadrupole** contribution, which falls off as $V \propto 1/r^3$.

Final Answer: Quadrupole term $\propto 1/r^3 \Rightarrow$ **(C)**

Why other options are wrong:

- (A) Monopole $\propto 1/r$: Requires $Q \neq 0$; but the distribution is neutral.
- (B) Dipole $\propto 1/r^2$: Requires $\mathbf{p} \neq 0$; given as zero.
- (D) $V = 0$ everywhere: False; a charge distribution with zero monopole and dipole moments still has a non-zero potential due to the quadrupole (and higher) terms.

Answer: (C) ← *Go back to Q28*

Q29.

Solution

Concept — Polarization at Brewster's Angle:
At Brewster's angle $\theta_B = \arctan(n_2/n_1)$:

- The reflected and refracted rays are *perpendicular* (at $90°$).
- The p-component (E in plane of incidence) of the reflected wave is *zero* (Fresnel reflection coefficient $r_p = 0$).
- Only the s-component (E perpendicular to plane of incidence) survives in reflection.

Therefore, the reflected beam is completely s**-polarized** (plane polarized with **E** perpendicular to the plane of incidence).

Final Answer: s-polarized (perpendicular to plane of incidence) $\Rightarrow$ **(D)**

Why other options are wrong:

- (A) p-polarized: The p-component has zero reflectance at θ_B; none is reflected.
- (B) Circularly polarized: No phase difference introduced at a single dielectric

interface at Brewster's angle.

- (C) Unpolarized: Completely polarized (not unpolarized) is the key property of Brewster's angle.

Answer: (D) ← *Go back to Q29*

Q30.

Solution

Concept — Compton Wavelength Shift:

$\Delta\lambda = \dfrac{h}{m_e c}(1-\cos\theta)$, with $h/(m_e c) = 2.43\,\text{pm}$.

Step 1 — Substitute $\theta = 60°$:

$$\Delta\lambda = 2.43\,\text{pm}\times(1-\cos 60°) = 2.43\,\text{pm}\times\left(1-\frac{1}{2}\right) = 2.43\,\text{pm}\times\frac{1}{2} = 1.215\,\text{pm} \approx 1.21\,\text{pm}$$

Final Answer: $\Delta\lambda \approx 1.21\,\text{pm} \Rightarrow$ **(A)**

Why other options are wrong:

- (B) $2.43\,\text{pm}$: Full Compton wavelength; corresponds to $\theta = 90°$ (where $1-\cos 90° = 1$).
- (C) $0\,\text{pm}$: Zero shift only at $\theta = 0°$ (forward scattering).
- (D) $4.86\,\text{pm}$: Corresponds to $\theta = 180°$ ($1-\cos 180° = 2$), the maximum shift.

Answer: (A) ← *Go back to Q30*

Section B Solutions — MSQ

Q31.

Solution

Concept — Hamiltonian Mechanics:

(A) "$2N$ first-order ODEs": CORRECT. For N degrees of freedom, there are N generalised coordinates q_i and N conjugate momenta p_i. Hamilton's equations $\dot{q}_i = \partial H/\partial p_i$ and $\dot{p}_i = -\partial H/\partial q_i$ give $2N$ coupled first-order ODEs.

(B) "H is always $T+V$": WRONG. The Hamiltonian equals $T+V$ only when the Lagrangian has no velocity-dependent potential (no magnetic forces, no non-holonomic constraints). For a particle in a magnetic field, $H = (\mathbf{p}-e\mathbf{A})^2/(2m)+e\phi$, which differs from $T+V$ when $\mathbf{A}\neq 0$.

(C) "$\{H,H\}=0$; H conserved if $\partial H/\partial t = 0$": CORRECT. The Poisson bracket is antisymmetric: $\{H,H\}=0$. The total time derivative of H is $dH/dt = \partial H/\partial t +$

$\{H, H\} = \partial H/\partial t$. So $H =$ const iff $\partial H/\partial t = 0$.

(D) "Liouville's theorem — phase-space volume conserved": CORRECT. The phase-space flow generated by any Hamiltonian is incompressible: $\partial\dot{q}/\partial q + \partial\dot{p}/\partial p = 0$, so the phase-space volume element $d^{2N}\Gamma$ is preserved.

Correct options: (A), (C), (D).

Answer: (ACD) ← *Go back to Q31*

Q32.

Solution

Concept — Electromagnetic Waves in Vacuum:

(A) "$\mathbf{E} \perp \mathbf{B} \perp \hat{k}$": CORRECT. From $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{B} = 0$ in a plane wave, $\mathbf{E}$ and $\mathbf{B}$ are both perpendicular to $\hat{k}$. From the curl equations, $\mathbf{E} \perp \mathbf{B}$.

(B) "$E_0 = cB_0$": CORRECT. From Faraday's law for a plane wave: $E_0 = \omega B_0/k = (ck/k)B_0 = cB_0$.

(C) "Wave carries electric charge": WRONG. EM waves are charge-free oscillations of $\mathbf{E}$ and $\mathbf{B}$ fields. There is no charge transport; the wave carries energy and momentum.

(D) "$\langle u \rangle = \varepsilon_0 E_0^2/2$": CORRECT. Time-averaged energy density: $\langle u_E \rangle = \varepsilon_0 E_0^2/4$ and $\langle u_B \rangle = B_0^2/(4\mu_0) = \varepsilon_0 E_0^2/4$. Total: $\langle u \rangle = \varepsilon_0 E_0^2/2 = B_0^2/(2\mu_0)$.

Correct options: (A), (B), (D).

Answer: (ABD) ← *Go back to Q32*

Q33.

Solution

Concept — Reversible Adiabatic Expansion of Ideal Gas:

(A) "Temperature increases": WRONG. In an adiabatic expansion, the gas does work ($W > 0$) with no heat input ($Q = 0$), so $\Delta U = -W < 0$. Since $U \propto T$ for an ideal gas, the temperature *decreases*.

(B) "$PV^\gamma =$ const": CORRECT. This is the defining relation for a reversible adiabatic (isentropic) process of an ideal gas. It follows from combining $PV = nRT$ with $dU = -P\,dV$ (adiabatic condition) and the definition of γ.

(C) "$\Delta S = 0$": CORRECT. A reversible adiabatic process is isentropic by definition: $\delta Q = 0$ (adiabatic) and $dS = \delta Q_{\text{rev}}/T = 0$.

(D) "Work = heat absorbed": WRONG. For an adiabatic process, $Q = 0$. By the first law: $W = -\Delta U = nC_V(T_i - T_f) > 0$. Work comes from internal energy, not from heat.

Correct options: (B) and (C).

Answer: (BC) ← *Go back to Q33*

Q34.

Solution

Concept — Properties of Hermitian Operators:

(A) "Eigenvalues are real": CORRECT. If $\hat{A}|\phi\rangle = a|\phi\rangle$, then $a = \langle\phi|\hat{A}|\phi\rangle/\langle\phi|\phi\rangle$. Since $\langle\phi|\hat{A}|\phi\rangle = \langle\phi|\hat{A}^\dagger|\phi\rangle^* = \langle\phi|\hat{A}|\phi\rangle^*$ for $\hat{A} = \hat{A}^\dagger$, we get $a = a^*$, so $a \in \mathbb{R}$.

(B) "Eigenvectors for different eigenvalues are orthogonal": CORRECT. If $\hat{A}|\phi_1\rangle = a_1|\phi_1\rangle$ and $\hat{A}|\phi_2\rangle = a_2|\phi_2\rangle$ with $a_1 \neq a_2$: $(a_1 - a_2)\langle\phi_2|\phi_1\rangle = 0 \Rightarrow \langle\phi_2|\phi_1\rangle = 0$.

(C) "Cannot have zero eigenvalue": WRONG. Zero is a perfectly valid eigenvalue. Example: the projection operator $\hat{P} = |\psi\rangle\langle\psi|$ is Hermitian and has eigenvalues 0 and 1.

(D) "$\langle\hat{A}\rangle$ is real for any $|\psi\rangle$": CORRECT. $\langle\hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle = \langle\psi|\hat{A}^\dagger|\psi\rangle^* = \langle\hat{A}\rangle^*$, so the expectation value is real.

Correct options: (A), (B), (D).

Answer: (ABD) ← *Go back to Q34*

Q35.

Solution

Concept — Phonon Dispersion in 1D Diatomic Chain:

(A) "Acoustic and optical branches": CORRECT. A diatomic chain (2 atoms/unit cell) yields two dispersion branches per polarisation mode: one acoustic (long-wavelength limit $\omega \to 0$) and one optical.

(B) "Acoustic branch non-zero at $k = 0$": WRONG. In the acoustic branch, all atoms in a unit cell move in *phase* (collective translation), and in the long-wavelength limit $k \to 0$, $\omega_{\text{ac}} \to 0$. This is the Goldstone mode of broken translational symmetry.

(C) "Optical branch non-zero at $k = 0$": CORRECT. At $k = 0$, the two sub-lattices vibrate against each other. The frequency is $\omega_{\text{op}}(0) = \sqrt{2\kappa(1/m_1 + 1/m_2)} > 0$ (where κ is the spring constant).

(D) "Gap at zone boundary": CORRECT. At the first Brillouin zone boundary

$k = \pi/a$ (where a is the unit cell length), the acoustic and optical branches are separated by a frequency gap $\Delta\omega = \omega_{\text{op}} - \omega_{\text{ac}} > 0$.

Correct options: (A), (C), (D).

Answer: (ACD) ← *Go back to Q35*

Q36.

Solution

Concept — Bessel Functions:

(A) "Bessel equation": CORRECT. By definition, $J_n(x)$ is the solution of $x^2y'' + xy' + (x^2 - n^2)y = 0$ regular at $x = 0$.

(B) "J_n and Y_n form complete solution": CORRECT. The Bessel equation is second-order linear, so its general solution is $y = AJ_n(x) + BY_n(x)$, where Y_n (Neumann function) is the second linearly independent solution (irregular at $x = 0$).

(C) "$J_n(x) > 0$ for all $x > 0$": WRONG. $J_n(x)$ oscillates and has infinitely many positive zeros. For example, $J_0(x) = 0$ at $x \approx 2.405, 5.520, 8.654, \ldots$

(D) "$J_0(0) = 1$, $J_n(0) = 0$ for $n \geq 1$": CORRECT. From the series definition $J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,\Gamma(k+n+1)} \left(\frac{x}{2}\right)^{2k+n}$: at $x = 0$, only the $k = 0$ term survives, giving $J_n(0) = 0^n/\Gamma(n+1)$. For $n = 0$: $J_0(0) = 1$. For $n \geq 1$: $J_n(0) = 0$.

Correct options: (A), (B), (D).

Answer: (ABD) ← *Go back to Q36*

Q37.

Solution

Concept — N/Z Ratio and Nuclear Stability:

(A) "All stable nuclei have $N = Z$": WRONG. This is approximately true only for light nuclei ($Z \lesssim 20$). For heavier stable nuclei, $N > Z$ due to Coulomb repulsion.

(B) "Heavy stable nuclei have $N > Z$": CORRECT. The strong nuclear force is short-ranged while Coulomb repulsion is long-ranged. Heavy nuclei need extra neutrons (which contribute attractive nuclear force without adding Coulomb repulsion) to maintain stability. For $^{208}_{82}$Pb: $N = 126 > Z = 82$.

(C) "Large $N/Z \to \beta^-$ decay": CORRECT. An excess of neutrons means a nucleus lies above the stability valley. A neutron converts to a proton via $n \to p + e^- + \bar{\nu}_e$ (β^- decay), moving the nucleus toward the stability line.

(D) "Too few neutrons $\to$ only alpha decay": WRONG. Nuclei with too few neutrons (proton-rich) can undergo β^+ decay (positron emission), electron capture, or, for heavy nuclei, alpha decay. Multiple decay modes are possible.

Correct options: (B) and (C).

Answer: (BC) ← *Go back to Q37*

Q38.

Solution

Concept — Brewster's Angle Conditions:

(A) "Reflected $\perp$ refracted": CORRECT. At Brewster's angle, $\theta_i + \theta_r = 90°$ (reflected and refracted rays are perpendicular). This follows from $\tan\theta_B = n_2/n_1$ combined with Snell's law.

(B) "s and p components equally reflected": WRONG. At Brewster's angle, the p-component (E in plane of incidence) has zero reflectance ($r_p = 0$), while the s-component ($r_s \neq 0$) is partially reflected. They are *not* equally reflected.

(C) "Reflected beam completely polarized": CORRECT. With $r_p = 0$ and $r_s \neq 0$, the reflected beam contains only the s-component, making it completely plane-polarized.

(D) "$\tan\theta_B = n_2/n_1$": CORRECT. This is the definition of Brewster's angle (Brewster's law).

Correct options: (A), (C), (D).

Answer: (ACD) ← *Go back to Q38*

Q39.

Solution

Concept — Astable Multivibrator:

(A) "No stable state": CORRECT. An astable multivibrator has no stable equilibrium state. It continuously alternates between two quasi-stable (metastable) states.

(B) "Generates continuous square wave": CORRECT. The astable circuit self-oscillates, producing a continuous periodic rectangular waveform. The frequency is determined by the RC time constants.

(C) "Requires external trigger": WRONG. An astable circuit is self-triggering (free-running). No external trigger is needed. (A *mono*stable or bistable requires a trigger.)

(D) "One stable and one quasi-stable state": WRONG. This describes a *mono*stable multivibrator: it has one stable state and one quasi-stable state, triggered by an external pulse.

Correct options: (A) and (B).

Answer: (AB) ← *Go back to Q39*

Q40.

Solution

Concept — Canonical Partition Function Relations:

(A) "$F = -k_B T \ln Z$": CORRECT. The Helmholtz free energy is the fundamental thermodynamic potential of the canonical ensemble: $F = -k_B T \ln Z$.

(B) "$S = -k_B \ln Z$": WRONG. The correct entropy is $S = -\partial F/\partial T = k_B \ln Z + k_B T\, \partial \ln Z/\partial T = k_B(\ln Z + \beta\langle E\rangle)$. In general $S \neq -k_B \ln Z$ (only if $\langle E\rangle = 0$, a special case).

(C) "$\langle E\rangle = -\partial \ln Z/\partial\beta$": CORRECT. $\langle E\rangle = \sum_n E_n e^{-\beta E_n}/Z = -\partial \ln Z/\partial\beta$.

(D) "$Z > 1$ for multi-level system at $T > 0$": CORRECT. For a system with ground state energy E_0 (defining reference $E_0 = 0$): $Z = 1 + e^{-\beta E_1} + \cdots$. With $E_1 > 0$ and $T > 0$: each excited-state term $e^{-\beta E_n} > 0$, so $Z > 1$.

Correct options: (A), (C), (D).

Answer: (ACD) ← *Go back to Q40*

Section C Solutions — NAT

Q41.

Solution

Concept — Speed of Transverse Wave on String:
$v = \sqrt{T/\mu}$

Calculation:

$$v = \sqrt{\frac{200\,\text{N}}{0.02\,\text{kg/m}}} = \sqrt{10000\,\text{m}^2\text{s}^{-2}} = 100\,\text{m/s}$$

$$\boxed{v = 100\,\text{m/s}}$$

Answer: (100) ← *Go back to Q41*

Q42.

Solution

Concept — Electric Field of a Point Charge:
$E = Q/(4\pi\varepsilon_0 r^2) = kQ/r^2$ where $k = 9 \times 10^9\,\mathrm{N\,m^2/C^2}$.

Calculation:

$$E = \frac{9 \times 10^9\,\mathrm{N\,m^2/C^2} \times 10 \times 10^{-9}\,\mathrm{C}}{(1\,\mathrm{m})^2} = \frac{90\,\mathrm{N/C}}{1} = 90\,\mathrm{V/m}$$

$$\boxed{E = 90\,\mathrm{V/m}}$$

Answer: (90) ← *Go back to Q42*

Q43.

Solution

Concept — de Broglie Wavelength (Electron, $KE = 1\,$keV):
$p = \sqrt{2m_e\,KE}$, $\lambda = h/p$.

Step 1 — Kinetic energy in Joules:

$$KE = 1000\,\mathrm{eV} = 1000 \times 1.6 \times 10^{-19}\,\mathrm{J} = 1.6 \times 10^{-16}\,\mathrm{J}$$

Step 2 — Momentum:

$$\begin{aligned} p &= \sqrt{2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-16}} \\ &= \sqrt{2.915 \times 10^{-46}} = 1.707 \times 10^{-23}\,\mathrm{kg\,m/s} \end{aligned}$$

Step 3 — Wavelength:

$$\lambda = \frac{6.626 \times 10^{-34}}{1.707 \times 10^{-23}} = 3.881 \times 10^{-11}\,\mathrm{m} = 0.3881\,\text{Å} \approx \boxed{0.39\,\text{Å}}$$

Answer: (0.39) ← *Go back to Q43*

Q44.

Solution

Concept — Calorimetry:
$Q = mc\Delta T$

Calculation:

$$Q = 1\,\mathrm{kg} \times 4186\,\mathrm{J/(kg\,K)} \times 50\,\mathrm{K} = 4186 \times 50\,\mathrm{J} = 209300\,\mathrm{J}$$

$$\boxed{Q = 209300\,\text{J}}$$

Answer: (209300) ← *Go back to Q44*

Q45.

Solution

Concept — Radioactive Decay, Number of Half-Lives to 90% Decay:
$N/N_0 = (1/2)^n = 0.10$ where $n = t/T_{1/2}$.

Calculation:

$$(1/2)^n = 0.10 \;\Rightarrow\; n\ln(1/2) = \ln(0.10)$$

$$n = \frac{\ln(0.10)}{\ln(1/2)} = \frac{-\ln 10}{-\ln 2} = \frac{\ln 10}{\ln 2} = \frac{2.3026}{0.6931} = 3.322$$

$$\boxed{n = 3.32 \text{ half-lives}}$$

Answer: (3.32) ← *Go back to Q45*

Q46.

Solution

Concept — FCC Atomic Packing Fraction:

Setup: In FCC, atoms touch along the face diagonal. If the cube side is a and atomic radius r:

$$4r = a\sqrt{2} \;\Rightarrow\; r = \frac{a}{2\sqrt{2}}$$

Packing fraction: 4 atoms per unit cell:

$$\text{APF} = \frac{4 \times \frac{4}{3}\pi r^3}{a^3} = \frac{\frac{16\pi}{3}\left(\frac{a}{2\sqrt{2}}\right)^3}{a^3} = \frac{16\pi}{3} \times \frac{1}{16\sqrt{2}} = \frac{\pi}{3\sqrt{2}}$$

Numerical value:

$$\frac{\pi}{3\sqrt{2}} = \frac{3.14159}{3 \times 1.41421} = \frac{3.14159}{4.24264} = 0.74048 \approx \boxed{0.7405}$$

Answer: (0.7405) ← *Go back to Q46*

Q47.

Solution

Concept — Sum of Eigenvalues = Trace:
For any matrix M, $\sum_i \lambda_i = \text{tr}(M)$.

Calculation:

$$\text{tr}\begin{pmatrix} 3 & -1 \\ 2 & 4 \end{pmatrix} = 3 + 4 = 7$$

Verification: Characteristic equation: $(3-\lambda)(4-\lambda) - (-1)(2) = \lambda^2 - 7\lambda + 14 = 0$. $\lambda_1 + \lambda_2 = 7$ (coefficient of $-\lambda$). ✓

$$\boxed{\text{Sum of eigenvalues} = 7}$$

Answer: (7) ← *Go back to Q47*

Q48.

Solution

Concept — Quality Factor of Series RLC Circuit:
$\omega_0 = 1/\sqrt{LC}, \quad Q_f = \omega_0 L/R$

Step 1 — Resonance frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1 \times 10 \times 10^{-6}}} = \frac{1}{\sqrt{10^{-6}}} = \frac{1}{10^{-3}} = 1000\,\text{rad/s}$$

Step 2 — Q factor:

$$Q_f = \frac{\omega_0 L}{R} = \frac{1000 \times 0.1}{10} = \frac{100}{10} = 10.00$$

$$\boxed{Q_f = 10.00}$$

Answer: (10.00) ← *Go back to Q48*

Q49.

Solution

Concept — Input Resistance of Inverting Op-Amp:
In an ideal inverting amplifier, the inverting $(-)$ terminal is held at virtual ground by the op-amp action. The signal source sees the inverting terminal as a ground potential. Therefore, the effective input resistance is simply:

$$R_{\text{input}} = R_{\text{in}}$$

since the voltage at the other end of R_{in} (the inverting terminal) is at 0 V (virtual ground).

For $R_{\text{in}} = 10\,\text{k}\Omega$:

$$\boxed{R_{\text{input}} = 10\,\text{k}\Omega}$$

Answer: (10) ← *Go back to Q49*

Q50.

Solution

Concept — Inverting Op-Amp Voltage Gain:
$|G| = R_f/R_{\text{in}}$

Calculation:

$$|G| = \frac{R_f}{R_{\text{in}}} = \frac{100\,\text{k}\Omega}{4.7\,\text{k}\Omega} = \frac{100}{4.7} = 21.2766\ldots \approx 21.28$$

$$\boxed{|G| = 21.28}$$

Answer: (21.28) ← *Go back to Q50*

Q51.

Solution

Concept — Skin Depth in a Conductor:
$\delta = \sqrt{\dfrac{2}{\omega\mu_0\sigma}}$

Step 1 — Angular frequency:

$$\omega = 2\pi f = 2\pi \times 10^3 = 6283.2\,\text{rad/s}$$

Step 2 — Compute $\omega\mu_0\sigma$:

$$\begin{aligned}\omega\mu_0\sigma &= 6283.2 \times 4\pi \times 10^{-7} \times 6 \times 10^7 \\ &= 6283.2 \times 4\pi \times 6 \\ &= 6283.2 \times 75.398 = 4.737 \times 10^5\,\text{s}^{-1}\text{m}^{-2}\end{aligned}$$

Step 3 — Skin depth:

$$\delta = \sqrt{\frac{2}{4.737 \times 10^5}} = \sqrt{4.222 \times 10^{-6}} = 2.055 \times 10^{-3}\,\text{m} = 2.055\,\text{mm} \approx \boxed{2.06\,\text{mm}}$$

Answer: (2.06) ← *Go back to Q51*

Q52.

Solution

Concept — Bragg's Law for X-ray Diffraction:
$n\lambda = 2d\sin\theta$ (first order: $n = 1$)

Step 1 — $\sin 15°$: Using $\sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$:

$$\sin 15° = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4} \approx 0.2588$$

Step 2 — Wavelength:

$$\lambda = 2d\sin\theta = 2 \times 2.82\,\text{Å} \times 0.2588 = 5.64 \times 0.2588 = 1.4596\,\text{Å} \approx \boxed{1.46\,\text{Å}}$$

Answer: (1.46) ← *Go back to Q52*

Q53.

Solution

Concept — Entropy Change for Free Expansion:
For irreversible free expansion into vacuum, $W = 0$ and $\Delta U = 0$ (ideal gas, isothermal). The entropy change is calculated via the reversible path:

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right)$$

Calculation:

$$\Delta S = 1\,\text{mol} \times 8.314\,\text{J/(mol K)} \times \ln(2) = 8.314 \times 0.6931 = 5.763\,\text{J/K} \approx \boxed{5.76\,\text{J/K}}$$

Note: Although the actual process is irreversible ($W = 0$, $\Delta S > 0$ violates reversibility), ΔS is a state function calculated on a reversible equivalent path.

Answer: (5.76) ← *Go back to Q53*

Q54.

Solution

Concept — Fermi Energy (Free Electron Model):
$E_F = \dfrac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$

Step 1 — Compute $3\pi^2 n$:

$$3\pi^2 = 29.608; \quad 3\pi^2 n = 29.608 \times 5.90 \times 10^{28} = 1.747 \times 10^{30}\,\text{m}^{-3}$$

Step 2 — Raise to power $2/3$:

$$(1.747)^{1/3}: \text{ since } 1.204^3 \approx 1.745 \Rightarrow (1.747)^{1/3} \approx 1.205$$

$$(1.747 \times 10^{30})^{2/3} = (1.205)^2 \times 10^{20} = 1.452 \times 10^{20}\,\text{m}^{-2}$$

Step 3 — Fermi energy:

$$\begin{aligned} E_F &= \frac{(1.055 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31}} \times 1.452 \times 10^{20} \\ &= \frac{1.113 \times 10^{-68}}{1.822 \times 10^{-30}} \times 1.452 \times 10^{20} \\ &= 6.11 \times 10^{-39} \times 1.452 \times 10^{20} = 8.872 \times 10^{-19}\,\text{J} \end{aligned}$$

$$E_F = \frac{8.872 \times 10^{-19}}{1.6 \times 10^{-19}} = 5.545\,\text{eV} \approx \boxed{5.54\,\text{eV}}$$

Answer: (5.54) *← Go back to Q54*

Q55.

Solution

Concept — 3D Harmonic Oscillator Energy Spacing:
The 3D isotropic harmonic oscillator has energy levels $E_N = (N + 3/2)\hbar\omega$ where $N = 0, 1, 2, \ldots$. Adjacent levels are separated by $\Delta E = \hbar\omega$.

Step 1 — Energy spacing:

$$\Delta E = \hbar\omega = 1.055 \times 10^{-34}\,\text{J s} \times 5 \times 10^{13}\,\text{rad/s} = 5.275 \times 10^{-21}\,\text{J}$$

Step 2 — Convert to eV:

$$\Delta E = \frac{5.275 \times 10^{-21}}{1.6 \times 10^{-19}}\,\text{eV} = \frac{5.275}{160}\,\text{eV} = 0.03297\,\text{eV} \approx \boxed{0.033\,\text{eV}}$$

Answer: (0.033) *← Go back to Q55*

Q56.

Solution

Concept — Photon Flux from Laser:
$N/t = P/E_{\text{ph}}$ where $E_{\text{ph}} = hc/\lambda$.

Step 1 — Energy per photon:

$$E_{\text{ph}} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{633 \times 10^{-9}} = \frac{1.988 \times 10^{-25}}{6.33 \times 10^{-7}} = 3.140 \times 10^{-19}\,\text{J}$$

Step 2 — Photon rate:

$$\frac{N}{t} = \frac{P}{E_{\text{ph}}} = \frac{10\,\text{W}}{3.140 \times 10^{-19}\,\text{J}} = 3.185 \times 10^{19}\,\text{s}^{-1} = 31.85 \times 10^{18}\,\text{s}^{-1}$$

$$\boxed{\frac{N}{t} \approx 31.85 \times 10^{18}\,\text{s}^{-1}}$$

Answer: (31.85) ← *Go back to Q56*

Q57.

Solution

Concept — Radioactive Decay after Non-Integer Half-Lives:
$N/N_0 = (1/2)^{4.5}$

Calculation:

$$(1/2)^{4.5} = \frac{1}{2^4 \times 2^{0.5}} = \frac{1}{16 \times \sqrt{2}} = \frac{1}{16 \times 1.41421} = \frac{1}{22.627}$$

$$N/N_0 = 0.04419 \approx \boxed{0.0442}$$

Verification: $\ln(1/2^{4.5}) = -4.5 \ln 2 = -4.5 \times 0.6931 = -3.119 \Rightarrow e^{-3.119} = 0.0442$ ✓

Answer: (0.0442) ← *Go back to Q57*

Q58.

Solution

Concept — Gamma Function Integral:
$\int_0^\infty t^n e^{-\alpha t}\,dt = \frac{n!}{\alpha^{n+1}}$ for non-negative integer n.
Apply with $n = 3$, $\alpha = 2$:

$$I = \int_0^\infty t^3 e^{-2t}\,dt = \frac{3!}{2^4} = \frac{6}{16} = \frac{3}{8} = 0.375$$

Verification by substitution $u = 2t$: $I = \frac{1}{2^4}\int_0^\infty u^3 e^{-u}\,du = \frac{\Gamma(4)}{16} = \frac{3!}{16} = \frac{6}{16} = 0.375$ ✓

$$\boxed{I = 0.375}$$

Answer: (0.375) ← *Go back to Q58*

Q59.

Solution

Concept — RC Low-Pass Filter Cutoff Frequency:

$f_c = \frac{1}{2\pi RC}$

Calculation:

$$RC = 10 \times 10^3\,\Omega \times 10 \times 10^{-9}\,\text{F} = 10^4 \times 10^{-8} = 10^{-4}\,\text{s}$$

$$f_c = \frac{1}{2\pi \times 10^{-4}} = \frac{10^4}{2\pi} = \frac{10000}{6.2832} = 1591.5\,\text{Hz} = 1.5915\,\text{kHz} \approx \boxed{1.59\,\text{kHz}}$$

Answer: (1.59) ← *Go back to Q59*

Q60.

Solution

Concept — Wien's Displacement Law:

$\lambda_{\text{max}} T = b = 2.898 \times 10^{-3}\,\text{m K}$

Calculation:

$$T = \frac{b}{\lambda_{\text{max}}} = \frac{2.898 \times 10^{-3}\,\text{m K}}{500 \times 10^{-9}\,\text{m}} = \frac{2.898 \times 10^{-3}}{5 \times 10^{-7}} = \frac{2.898}{5} \times 10^4 = 0.5796 \times 10^4 = 5796\,\text{K}$$

$$\boxed{T = 5796\,\text{K}}$$

This temperature is close to the surface temperature of the Sun ($\approx 5778\,\text{K}$), consistent with the solar spectrum peaking in the visible range around 500 nm.

Answer: (5796) ← *Go back to Q60*

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	A	3	B	4	C	5	D
6	A	7	B	8	C	9	D	10	A
11	B	12	C	13	D	14	A	15	B
16	C	17	D	18	A	19	B	20	C
21	D	22	A	23	B	24	C	25	D
26	A	27	B	28	C	29	D	30	A
31	ACD	32	ABD	33	BC	34	ABD	35	ACD
36	ABD	37	BC	38	ACD	39	AB	40	ACD
41	100	42	90	43	0.39	44	209300	45	3.32
46	0.7405	47	7	48	10.00	49	10	50	21.28
51	2.06	52	1.46	53	5.76	54	5.54	55	0.033
56	31.85	57	0.0442	58	0.375	59	1.59	60	5796