IIT JAM Physics

Sample Paper – 10

Instructions

- This paper contains **60 questions** carrying a maximum of **100 marks**. Duration: **3 hours (180 minutes)**.
- **Section A** (**Q1–Q30**, MCQ): Q1–Q10 carry **+1 mark** each; wrong response: $-\frac{1}{3}$ mark. Q11–Q30 carry **+2 marks** each; wrong response: $-\frac{2}{3}$ mark. Choose the **single best** option.
- **Section B** (**Q31–Q40**, MSQ): Each carries **+2 marks**. **No negative marking. One or more** options may be correct; full marks only if *all* correct options are selected; no partial credit.
- **Section C** (**Q41–Q60**, NAT): Q41–Q50 carry **+1 mark**; Q51–Q60 carry **+2 marks**. **No negative marking.** Enter the numerical value using the virtual keypad (integer or decimal as appropriate).
- **Calculator policy:** Personal calculators, log tables, and electronic devices are **strictly prohibited**. A virtual scientific calculator is provided in the CBT interface. Rough sheets are supplied at the examination centre.
- Once you move to the next section, you cannot return to a previous section. Manage time accordingly.

Section A — Multiple Choice Questions (MCQ)

*Q1–Q10: 1 mark each ($-\frac{1}{3}$ for wrong). Choose the **single** correct option.*

Q1. The function $f(z) = z^{1/3}$ (the principal cube root) is a multi-valued function of the complex variable z. Under analytic continuation around $z = 0$ by 2π, the function acquires a phase $e^{2\pi i/3}$ and is not single-valued. Which of the following statements about its branch points is correct?

(A) $f(z)$ has exactly one branch point, located at $z = 0$ only

(B) $f(z)$ has branch points at both $z = 0$ and $z = \infty$

(C) $f(z)$ has a branch point only at $z = \infty$, not at $z = 0$

(D) $f(z)$ is single-valued everywhere in the complex plane and has no branch points

Q2. The Hamiltonian for a particle in one dimension is $H = \dfrac{p^2}{2m} + V(q)$. Using Hamilton's equation of motion $\dot{q} = \dfrac{\partial H}{\partial p}$, the generalised velocity $\dot{q}$ is:

(A) $\dot{q} = \frac{2p}{m}$
(B) $\dot{q} = \frac{p}{2m}$
(C) $\dot{q} = \frac{p}{m}$
(D) $\dot{q} = -\frac{\partial V}{\partial q}$

Q3. A diffraction grating has $N = 10{,}000$ lines ruled uniformly. In the second-order diffraction ($m = 2$) at wavelength $\lambda = 500\,\text{nm}$, the resolving power is $R = mN$. The minimum resolvable wavelength difference $\delta\lambda = \lambda/R$ is:

(A) $0.1\,\text{nm}$
(B) $0.05\,\text{nm}$
(C) $0.5\,\text{nm}$
(D) $0.025\,\text{nm}$

Q4. A non-magnetic dielectric medium ($\mu_r \approx 1$) has relative permittivity $\varepsilon_r = 2.25$. The refractive index $n = \sqrt{\varepsilon_r \mu_r}$ of this medium is:

(A) 1.5
(B) 2.25
(C) 4.5
(D) 1.13

Q5. The chemical potential of an ideal classical monatomic gas of number density n at temperature T is $\mu = k_B T \ln(n/n_Q)$, where $n_Q = (2\pi m k_B T/h^2)^{3/2}$ is the quantum concentration. In the classical (high-temperature) regime where $n \ll n_Q$:

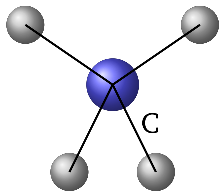
(A) $\mu > 0$
(B) $\mu < 0$
(C) $\mu = 0$
(D) $\mu = k_B T$

Q6. In quantum mechanics, when a measurement of the observable $\hat{A}$ on a system in state $|\psi\rangle$ yields the eigenvalue a_n, the state of the system *immediately after* the measurement is:

(A) Unchanged: $|\psi\rangle$
(B) An equal superposition of all eigenstates of $\hat{A}$
(C) The eigenstate $|a_n\rangle$ corresponding to eigenvalue a_n
(D) The vacuum (ground) state

Q7. In the diamond cubic crystal structure of carbon, each atom is bonded covalently to its nearest neighbours in a tetrahedral arrangement, as shown. The coordination number (number of nearest neighbours) of each carbon atom is:

Tetrahedral bonding in diamond cubic



(A) 6
(B) 8
(C) 12
(D) 4

Q8. An XOR (exclusive-OR) logic gate produces an output of 1 *only* when its two input bits differ. Which of the following entries of the XOR truth table is *correct*?

(A) Inputs $A = 1,\ B = 1$ produce output 0
(B) Inputs $A = 1,\ B = 0$ produce output 0
(C) Inputs $A = 0,\ B = 0$ produce output 1
(D) Inputs $A = 0,\ B = 1$ produce output 0

Q9. In thermal-neutron-induced fission of ${}^{235}_{92}\text{U}$, one possible reaction channel is:

$${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \longrightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + k \cdot {}^{1}_{0}\text{n}$$

Applying conservation of mass number A, the number of prompt neutrons k released in this fission channel is:

(A) 1

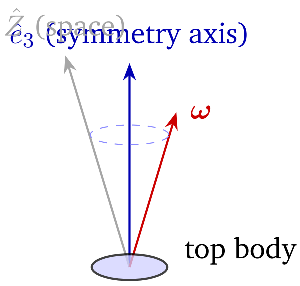
(B) 3
(C) 5
(D) 2

Q10. The Legendre polynomial of degree 2 is $P_2(x) = \dfrac{3x^2 - 1}{2}$. The numerical value of $P_2(\cos 60°)$ is:

(A) $\dfrac{1}{4}$
(B) $-\dfrac{1}{4}$
(C) $-\dfrac{1}{8}$
(D) $\dfrac{3}{4}$

***Q11–Q30**: 2 marks each ($-\frac{2}{3}$ for wrong). Choose the **single** correct option.*

Q11. For a torque-free symmetric top with principal moments of inertia $I_1 = I_2 \neq I_3$, the Euler equations are $I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3$, $I_1\dot{\omega}_2 = (I_3 - I_1)\omega_1\omega_3$, and $I_3\dot{\omega}_3 = 0$. The precessional motion is illustrated below. Which quantity is therefore an exact constant of the motion?

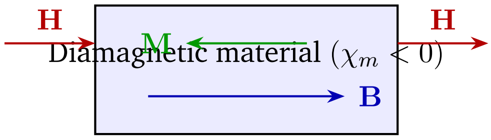


(A) ω_1 (component along body axis 1)
(B) ω_2 (component along body axis 2)
(C) The vector $\boldsymbol{\omega}$ is fixed in the body frame
(D) ω_3 (component along the symmetry axis $\hat{e}_3$)

Q12. In Compton scattering, the wavelength shift of a photon scattered from a free electron at rest is $\Delta\lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$. For *backscattering* at $\theta = 180°$, the shift $\Delta\lambda$ in picometres (pm) is approximately (use $h/(m_e c) = 2.426\,\text{pm}$):

(A) $4.85\,\text{pm}$
(B) $2.43\,\text{pm}$
(C) $1.21\,\text{pm}$
(D) $9.70\,\text{pm}$

Q13. In a linear isotropic diamagnetic material, the magnetisation satisfies $\mathbf{M} = \chi_m \mathbf{H}$ with susceptibility $\chi_m < 0$, and $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$ as illustrated. The relative permeability $\mu_r = 1 + \chi_m$ satisfies:



(A) $\mu_r > 1$: the material enhances the field ($B > \mu_0 H$)
(B) $\mu_r < 1$: the material weakly reduces the field ($B < \mu_0 H$)
(C) $B = 0$ always inside a diamagnetic material (perfect diamagnet)
(D) $\mathbf{M}$ is parallel to $\mathbf{H}$ in a diamagnetic material

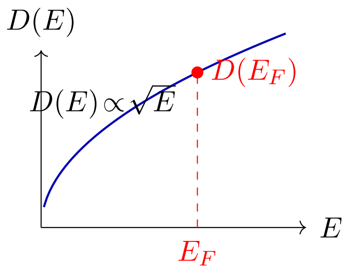
Q14. The Joule–Thomson (throttling) process is an isenthalpic expansion of a gas through a porous plug. The Joule–Thomson coefficient $\mu_{JT} = \left(\frac{\partial T}{\partial P}\right)_H$ measures the temperature change with pressure at constant enthalpy. For an *ideal* gas:

(A) μ_{JT} is large and positive (ideal gas always cools on throttling)
(B) μ_{JT} is large and negative (ideal gas always heats on throttling)
(C) $\mu_{JT} = 0$ (temperature does not change on throttling)
(D) $\mu_{JT} = 2a/(Rb)$, where a, b are van der Waals constants

Q15. In the ground state of the helium atom, both electrons occupy the $1s$ orbital ($1s^2$ configuration). The Pauli exclusion principle requires their spins to be antiparallel. The spectroscopic term symbol of the helium ground state is:

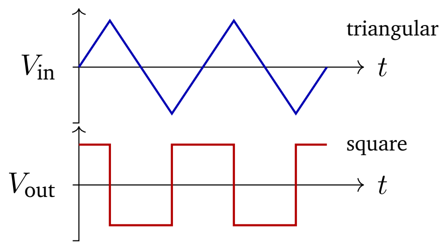
(A) ${}^3S_1 \quad (S = 1,\ \text{triplet})$
(B) ${}^1P_1 \quad (L = 1,\ \text{singlet})$
(C) ${}^3P_0 \quad (L = 1,\ \text{triplet})$
(D) ${}^1S_0 \quad (S = 0,\ \text{singlet})$

Q16. In the free-electron model, the electronic density of states is $D(E) \propto \sqrt{E}$, as sketched below. For copper ($n = 8.5 \times 10^{28}\,\mathrm{m^{-3}}$, $E_F = 7.04\,\mathrm{eV}$), using $D(E_F) = 3n/(2E_F)$, the DOS at the Fermi level in units of $\mathrm{eV^{-1}m^{-3}}$ is approximately:



(A) $1.81 \times 10^{28}\,\mathrm{eV^{-1}m^{-3}}$
(B) $3.62 \times 10^{28}\,\mathrm{eV^{-1}m^{-3}}$
(C) $9.05 \times 10^{27}\,\mathrm{eV^{-1}m^{-3}}$
(D) $7.24 \times 10^{28}\,\mathrm{eV^{-1}m^{-3}}$

Q17. An RC-differentiating op-amp circuit has input capacitor $C_1 = 0.01\,\mu\mathrm{F}$ and feedback resistor $R_f = 100\,\mathrm{k\Omega}$ (time constant $\tau = R_f C_1 = 10^{-3}\,\mathrm{s}$). When a *triangular* wave of frequency $1\,\mathrm{kHz}$ is applied at the input, the output waveform is approximately:



(A) Triangular wave (same shape as the input)
(B) Square wave
(C) Sinusoidal wave
(D) A train of narrow impulse spikes

Q18. In the nuclear shell model, large energy gaps at *magic numbers* arise from spin–orbit coupling ($\propto \mathbf{l} \cdot \mathbf{s}$) which lifts degeneracies and creates energy gaps. The first six magic numbers correctly predicted by the shell model are:

(A) $2,\ 6,\ 14,\ 28,\ 50,\ 82$
(B) $2,\ 8,\ 14,\ 28,\ 50,\ 82$
(C) $2,\ 8,\ 20,\ 28,\ 50,\ 82$
(D) $2,\ 8,\ 20,\ 40,\ 70,\ 112$

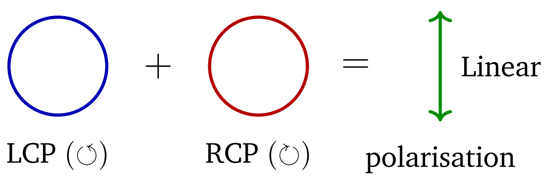
Q19. The function $f(x) = x$ on $[-\pi, \pi]$ (period 2π) is odd, so its Fourier series contains only sine terms: $f(x) = \sum_{n=1}^{\infty} b_n \sin(nx)$. The Fourier sine coefficient b_n is:

(A) $b_n = \dfrac{2}{n}$
(B) $b_n = \dfrac{(-1)^n}{n}$
(C) $b_n = \dfrac{2(-1)^n}{n}$
(D) $b_n = \dfrac{2(-1)^{n+1}}{n}$

Q20. A particle on Earth's surface at latitude $\lambda = 45^\circ$ moves due north at speed $v = 10\,\mathrm{m/s}$. The Coriolis acceleration has magnitude $2\Omega v \sin\lambda$, where $\Omega = 7.27 \times 10^{-5}$ rad/s is Earth's angular speed. This magnitude is approximately:

(A) $\approx 10^{-3}\,\mathrm{m/s^2}$
(B) $\approx 10^{-2}\,\mathrm{m/s^2}$
(C) $\approx 10^{-4}\,\mathrm{m/s^2}$
(D) $\approx 10^{-5}\,\mathrm{m/s^2}$

Q21. A linearly polarised electromagnetic wave can be decomposed into circularly polarised waves. The illustration shows that the superposition of a left-circularly polarised (LCP) and a right-circularly polarised (RCP) wave of *equal* amplitude produces:



(A) Two orthogonal linearly polarised components
(B) Linear polarisation

(C) Elliptical polarisation with unequal axes

(D) Circular polarisation of the same handedness

Q22. In the grand canonical ensemble, the grand partition function is $\mathcal{Z} = \sum_{N,s} e^{-\beta(E_s - \mu N)}$. The grand potential $\Omega(T, V, \mu)$ — from which all thermodynamic quantities follow — is related to $\mathcal{Z}$ by:

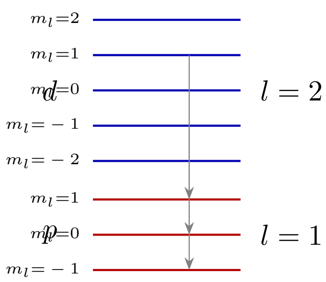
(A) $\Omega = k_B T \ln \mathcal{Z}$

(B) $\Omega = -k_B T / \mathcal{Z}$

(C) $\Omega = -k_B T \ln \mathcal{Z}$

(D) $\Omega = k_B T / \mathcal{Z}$

Q23. In the *normal* Zeeman effect, a magnetic field B splits an atomic level with orbital quantum number l into $2l + 1$ sub-levels separated by $\Delta E = \mu_B B$. For a $d \to p$ transition ($l = 2 \to l = 1$), the electric-dipole selection rule $\Delta m_l = 0, \pm 1$ restricts the number of allowed spectral lines. The diagram below shows the level splitting. The number of distinct spectral lines in the normal Zeeman pattern is:



(A) 5

(B) 7

(C) 9

(D) 3

Q24. For a p-n junction with equal doping densities $N_A = N_D = N$, relative permittivity ε_r, and equilibrium built-in potential V_{bi}, the total depletion width $W = x_p + x_n$ at equilibrium is:

(A) $W = 2\sqrt{\dfrac{\varepsilon_r \varepsilon_0 V_{bi}}{eN}}$

(B) $W = \sqrt{\dfrac{\varepsilon_r \varepsilon_0 V_{bi}}{eN}}$
(C) $W = 4\sqrt{\dfrac{\varepsilon_r \varepsilon_0 V_{bi}}{eN}}$
(D) $W = \dfrac{\varepsilon_r \varepsilon_0 V_{bi}}{eN}$

Q25. An inverting Schmitt trigger is built with an ideal op-amp having saturation voltages $\pm V_{\text{sat}} = \pm 15\,\text{V}$. Positive-feedback resistors satisfy $R_1 = R_2 = 10\,\text{k}\Omega$ (from output to inverting input, and from non-inverting input to ground respectively). The upper and lower switching thresholds (V_{TH}, V_{TL}) are:

(A) $(+15\,\text{V},\ -15\,\text{V})$
(B) $(+7.5\,\text{V},\ -7.5\,\text{V})$
(C) $(+5.0\,\text{V},\ -5.0\,\text{V})$
(D) $(+10\,\text{V},\ -10\,\text{V})$

Q26. The Wigner–Seitz cell of a lattice is constructed by drawing perpendicular bisector planes between a given lattice point and all its neighbours; the resulting region is the primitive cell. For the *BCC* (body-centred cubic) lattice, the Wigner–Seitz cell is:

(A) A cube
(B) A rhombic dodecahedron
(C) A truncated octahedron (8 hexagonal faces and 6 square faces)
(D) A regular tetrahedron

Q27. In the kinetic theory of ideal gases, the thermal conductivity is $\kappa = \frac{1}{3}\,\bar{v}\,\lambda\,C_v n$. Since the mean speed $\bar{v} \propto \sqrt{T}$ and the mean free path satisfies $\lambda \propto 1/n$ (so the product $\lambda n = \text{const}$ at fixed T), the dependence of κ on temperature T and pressure P at fixed composition is:

(A) $\kappa \propto T$, independent of P
(B) $\kappa \propto 1/\sqrt{T}$, decreases as T increases
(C) $\kappa \propto P$ at fixed T
(D) $\kappa \propto \sqrt{T}$, independent of P

Q28. The radiation resistance of a short electric dipole antenna of length $\ell \ll \lambda$ is $R_{\text{rad}} = 80\pi^2(\ell/\lambda)^2\,\Omega$. For an antenna of length $\ell = \lambda/20$, the radiation resistance is approximately:

(A) $2\,\Omega$
(B) $8\,\Omega$
(C) $20\,\Omega$
(D) $0.2\,\Omega$

Q29. The Rayleigh criterion for angular resolution of a circular aperture of diameter D at wavelength λ is $\theta_{\text{min}} = 1.22\lambda/D$. For an optical telescope with $D = 5$ m and $\lambda = 550$ nm, using 1 rad $= 2.06 \times 10^5$ arcsec, the angular resolution is approximately:

(A) 0.11 arcsec
(B) 0.028 arcsec
(C) 0.056 arcsec
(D) 0.014 arcsec

Q30. A monochromatic light wave of vacuum wavelength λ_{vac} enters a medium of refractive index $n > 1$. The frequency ν of the light remains unchanged, but the phase velocity becomes $v = c/n$. The wavelength of the light *inside* the medium is:

(A) $\lambda_{\text{medium}} = n\,\lambda_{\text{vac}}$
(B) $\lambda_{\text{medium}} = \lambda_{\text{vac}}$
(C) $\lambda_{\text{medium}} = \lambda_{\text{vac}}/n$
(D) $\lambda_{\text{medium}} = \lambda_{\text{vac}}/n^2$

Section B — Multiple Select Questions (MSQ)

Q31–Q40**: 2 marks each.* ***No negative marking. One or more *options may be correct. Full marks only if* all *correct options (and no incorrect options) are chosen.*

Q31. In special relativity, which of the following quantities are *conserved* in *all* particle collisions (whether elastic or inelastic)?

(A) The total four-momentum $p^\mu = (E/c,\, \mathbf{p})$ of the system

(B) The total energy $E_{\text{total}} = \sum_i E_i$ (including rest-mass energies $m_i c^2$)
(C) The rest mass of each individual particle
(D) The total kinetic energy of all particles

Q32. Which of the following statements about the electrostatic potential $V(\mathbf{r})$ are correct?

(A) The electric field is $\mathbf{E} = -\nabla V$
(B) V is a vector field
(C) V is constant on every equipotential surface
(D) For a point charge q at the origin: $V(r) = q/(4\pi\varepsilon_0 r)$

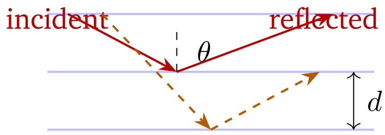
Q33. The microcanonical ensemble describes an isolated system in thermal equilibrium. Which of the following are properties of the microcanonical ensemble?

(A) Temperature T is a fixed (externally imposed) parameter
(B) The total energy E of the system is fixed
(C) The thermodynamic entropy is $S = k_B \ln \Omega$, where Ω is the number of accessible microstates
(D) The microcanonical ensemble is appropriate for isolated systems

Q34. Which of the following statements about junction field-effect transistors (JFETs) are correct?

(A) The gate–channel p-n junction is reverse-biased in normal (active) operation
(B) For an n-channel JFET, increasing $|V_{GS}|$ (more negative V_{GS}) narrows the conducting channel
(C) A JFET is a current-controlled device (like a BJT)
(D) Gate current is significant in normal JFET operation

Q35. X-ray diffraction from a crystal is described by Bragg's law $2d \sin\theta = n\lambda$, illustrated below for crystal planes of spacing d at glancing angle θ. Which statements about X-ray diffraction from crystals are correct?



(A) Bragg's law for constructive interference is $2d\sin\theta = n\lambda$, where θ is the glancing angle

(B) X-ray diffraction occurs only in crystals with cubic symmetry

(C) The scattered intensity at each reflection depends on the *structure factor* of the unit cell

(D) Systematic absences in the diffraction pattern reveal the Bravais lattice type of the crystal

Q36. Hamilton's equations of motion $\dot{q}_i = \partial H/\partial p_i$ and $\dot{p}_i = -\partial H/\partial q_i$ govern a system with N degrees of freedom. Which of the following statements are true?

(A) The total number of first-order differential equations is N

(B) The total number of first-order differential equations is $2N$

(C) For a conservative system (no explicit time dependence), $H = T + V$

(D) Hamilton's equations preserve the volume element in phase space (Liouville's theorem)

Q37. The $^{238}_{92}$U radioactive decay series ends at a stable lead isotope. Which of the following statements are correct?

(A) The series terminates at the stable isotope $^{206}_{82}$Pb

(B) The total number of α-decay steps from ^{238}U to ^{206}Pb is 8

(C) The series terminates at $^{208}_{82}$Pb

(D) The series involves only α decays, with no β decays

Q38. For a glass prism of apex angle A and refractive index n, at the angle of *minimum deviation* D_m, which of the following statements are correct?

(A) The angle of incidence equals the angle of emergence: $i_1 = i_2$

(B) The refracted ray inside the prism is perpendicular to the bisector of the apex angle

(C) The refractive index satisfies $n = \sin\left(\frac{A+D_m}{2}\right) \Big/ \sin\left(\frac{A}{2}\right)$

(D) The deviation angle has a unique minimum as the incidence angle is varied

Q39. The time-independent Schrödinger equation for a free particle ($V = 0$) in one dimension is $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$. Which of the following are correct?

(A) The energy eigenvalues form a discrete (quantised) spectrum

(B) The function $\psi_k(x) = e^{ikx}$ is an energy eigenstate for any real k

(C) The energy eigenvalue corresponding to ψ_k is $E_k = \hbar^2k^2/(2m)$

(D) The probability current for eigenstate $\psi_k = Ae^{ikx}$ is $j = (\hbar k/m)|A|^2$

Q40. Which of the following statements about *photons* are correct?

(A) Photons have zero rest mass ($m_0 = 0$)

(B) The spin quantum number of a photon is $s = 1$ (photon is a spin-1 boson)

(C) A single photon incident on an ideal 50:50 beamsplitter is split into two half-energy photons

(D) The momentum of a photon is the same in a medium of refractive index n as in vacuum ($p = E/c$)

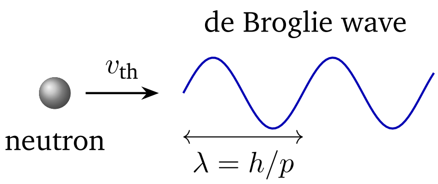
Section C — Numerical Answer Type (NAT)

***Q41–Q50**: 1 mark each. **No negative marking.** Enter the exact numerical value.*

Q41. A projectile is launched from the ground at $45°$ to the horizontal with initial speed $v_0 = 20\,\text{m/s}$ (take $g = 9.8\,\text{m/s}^2$, no air resistance). The horizontal range $R = v_0^2 \sin(2 \times 45°)/g$. Calculate R in metres, correct to two decimal places.

Q42. A capacitor of capacitance $C = 50\,\mu\text{F}$ is charged to voltage $V = 200\,\text{V}$. Calculate the energy $U = \frac{1}{2}CV^2$ stored in the capacitor in joules, correct to two decimal places.

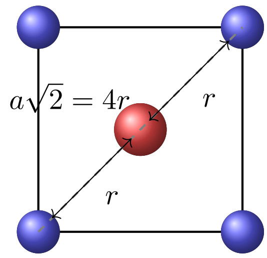
Q43. The de Broglie wavelength of a thermal neutron is estimated using momentum $p = \sqrt{2m_nk_BT}$ (most probable energy $= k_BT$). For $T = 293\,\text{K}$, calculate $\lambda = h/p$ in nm, correct to three decimal places. ($m_n = 1.675 \times 10^{-27}\,\text{kg}$; $k_B = 1.38 \times 10^{-23}\,\text{J/K}$; $h = 6.626 \times 10^{-34}\,\text{J s}$.)



Q44. For nitrogen gas ($m = 4.65 \times 10^{-26}$ kg/molecule) at $T = 300\,\text{K}$ and $P = 10^5\,\text{Pa}$, the number density is $n = P/(k_B T)$ and the mean speed is $\bar{v} = \sqrt{8k_B T/(\pi m)}$. Calculate the molecular flux $\Phi = n\bar{v}/4$ in units of $10^{27}\,\text{m}^{-2}\text{s}^{-1}$, correct to two decimal places. ($k_B = 1.38 \times 10^{-23}$ J/K.)

Q45. Calculate the specific activity of $^{60}_{27}$Co (half-life $T_{1/2} = 5.27\,\text{yr} = 1.663 \times 10^8\,\text{s}$; molar mass $\approx 60\,\text{g/mol}$; $N_A = 6.022 \times 10^{23}\,\text{mol}^{-1}$) in units of 10^{13} Bq/g, correct to two decimal places.

Q46. A metal crystallises in an FCC structure with lattice constant $a = 4.05\,\text{Å}$. In FCC, atoms touch along the face diagonal ($4r = a\sqrt{2}$). Calculate the atomic radius r in Å, correct to two decimal places.



FCC face (atoms touch along diagonal)

Q47. Given vectors $\mathbf{A} = \hat{x} + 2\hat{y} + 0\hat{z}$ and $\mathbf{B} = 0\hat{x} + \hat{y} + 3\hat{z}$, calculate the magnitude $|\mathbf{A} \times \mathbf{B}|$, correct to two decimal places.

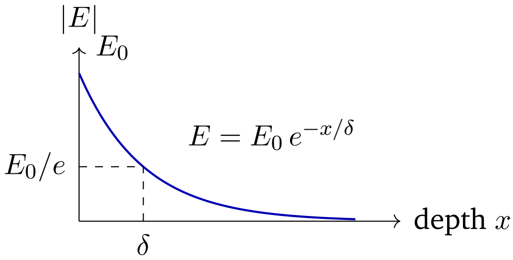
Q48. A series RLC circuit has inductance $L = 10\,\text{mH}$ and capacitance $C = 0.1\,\mu\text{F}$. Calculate the resonance frequency $f_0 = 1/(2\pi\sqrt{LC})$ in kHz, correct to two decimal places.

Q49. An ideal op-amp comparator has its non-inverting input at $V_+ = 2.5\,\text{V}$ and its inverting input at $V_- = 3.0\,\text{V}$. The op-amp is powered by $\pm 12\,\text{V}$ rails. Determine the output voltage V_{out} in volts.

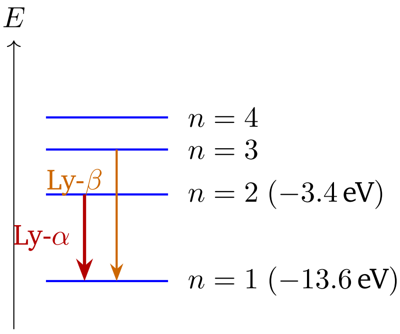
Q50. A series RLC circuit is driven at its resonance frequency $f_0 = 1/(2\pi\sqrt{LC})$. At resonance, the inductive and capacitive reactances cancel exactly ($X_L = X_C$), making the impedance purely resistive. The power factor $\cos\phi$ of the circuit at resonance is:

***Q51–Q60**: 2 marks each. **No negative marking.** Enter the exact numerical value.*

Q51. The skin depth of an electromagnetic wave in a good conductor is $\delta = \sqrt{2/(\omega\mu_0\sigma)}$. For copper ($\sigma = 6\times10^7$ S/m, $\mu_r = 1$) at frequency $f = 1$ MHz, the field decays as $e^{-x/\delta}$ below the surface (as shown). Calculate δ in μm, correct to one decimal place. ($\mu_0 = 4\pi \times 10^{-7}$ H/m.)



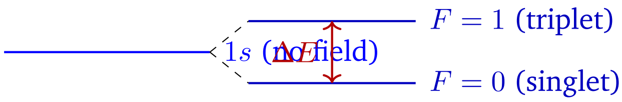
Q52. In the hydrogen atom, the first Lyman series line is the $n = 2 \to n = 1$ transition. Using $1/\lambda = R_H(1/1^2 - 1/2^2)$ with $R_H = 1.097 \times 10^7\,\text{m}^{-1}$, calculate λ in nm correct to two decimal places.



Q53. When 0.5 mol of ideal gas A and 0.5 mol of ideal gas B (both at the same T and P) are mixed irreversibly, the entropy of mixing is $\Delta S_{\text{mix}} = -nR(x_A \ln x_A + x_B \ln x_B)$ with $n = 1$ mol and $x_A = x_B = 0.5$. Calculate ΔS_{mix} in J/K, correct to two decimal places. ($R = 8.314\,\text{J mol}^{-1}\text{K}^{-1}$; $\ln 2 = 0.6931$.)

Q54. Using the free-electron formula $D(E_F) = 3n/(2E_F)$ for the density of states at the Fermi level in copper ($n = 8.5 \times 10^{28}\,\mathrm{m}^{-3}$, $E_F = 7.04\,\mathrm{eV}$), calculate $D(E_F)$ in units of $10^{28}\,\mathrm{eV}^{-1}\mathrm{m}^{-3}$, correct to two decimal places.

Q55. The $1s$ ground state of atomic hydrogen is split by hyperfine interaction into two levels ($F = 1$ triplet and $F = 0$ singlet) separated by frequency $f_{HFS} = 1.42041 \times 10^9\,\mathrm{Hz}$ (the 21 cm line). Calculate the hyperfine splitting energy $\Delta E = hf_{HFS}$ in μeV, correct to two decimal places. ($h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$; $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$.)



$\Delta E = 5.88\,\mu\mathrm{eV}$; $f = 1.42\,\mathrm{GHz}$ (21 cm)

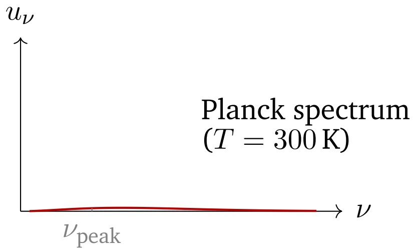
Q56. Calculate the speed of sound in argon gas at $T = 300\,\mathrm{K}$, using $v = \sqrt{\gamma RT/M}$ with $\gamma = 5/3$, $R = 8.314\,\mathrm{J\,mol^{-1}K^{-1}}$, and $M = 40\,\mathrm{g/mol} = 0.040\,\mathrm{kg/mol}$. Give your answer in m/s to the nearest integer.

Q57. A radioactive sample is observed after exactly 2.5 half-lives. The fraction of the original number of atoms remaining is $N/N_0 = (1/2)^{2.5}$. Calculate this fraction correct to three decimal places.

Q58. The Fresnel cosine integral $C(\infty) = \int_0^\infty \cos\left(\frac{\pi t^2}{2}\right) dt$ has a well-known exact value. Evaluate $C(\infty)$ correct to four decimal places.

Q59. A capacitor ($C = 100\,\mu\mathrm{F}$, initially charged to $V_0 = 10\,\mathrm{V}$) discharges through a resistor $R = 100\,\Omega$. The voltage decays as $V(t) = V_0 e^{-t/\tau}$ where $\tau = RC$. Calculate the time constant τ in milliseconds (ms), correct to one decimal place.

Q60. The photon number density in blackbody radiation at temperature T is $n_\gamma = \frac{2\zeta(3)}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3$ with $\zeta(3) = 1.202$. At $T = 300\,\mathrm{K}$, calculate n_γ in units of $10^{14}\,\mathrm{m}^{-3}$, correct to two decimal places. ($k_B = 1.38 \times 10^{-23}\,\mathrm{J/K}$; $\hbar = 1.055 \times 10^{-34}\,\mathrm{J\,s}$; $c = 3 \times 10^8\,\mathrm{m/s}$.)


u_ν
Planck spectrum
($T = 300\,\mathrm{K}$)
ν
ν_{peak}

Solutions

IIT JAM Physics – Sample Paper 10

Section A Solutions — MCQ

Solution

Concept — Branch Points of Multi-Valued Functions:
A branch point of $f(z) = z^{1/3}$ occurs at points where analytic continuation around a closed loop changes the value of f.

Step 1 — Branch point at $z = 0$:
Write $z = re^{i\theta}$. Then $z^{1/3} = r^{1/3}e^{i\theta/3}$. Under $\theta \to \theta + 2\pi$ (one loop around $z = 0$): $e^{i(\theta+2\pi)/3} = e^{2\pi i/3}e^{i\theta/3} \neq e^{i\theta/3}$. So f is not single-valued around $z = 0$ — it is a branch point.

Step 2 — Branch point at $z = \infty$:
Let $w = 1/z$, so $f = (1/w)^{1/3} = w^{-1/3}$. This has a branch point at $w = 0$, i.e. $z = \infty$.

Why other options are wrong:

Q1.
- (A) Only at $z = 0$: incomplete — $z = \infty$ is also a branch point.
- (C) Only at $z = \infty$: incorrect; $z = 0$ is clearly a branch point.
- (D) Single-valued: $z^{1/3}$ is triple-valued and requires a branch cut.

Final Answer: Branch points at $z = 0$ and $z = \infty \Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q1*

Solution

Concept — Hamilton's Equations of Motion:
For $H = p^2/(2m) + V(q)$, Hamilton's first equation gives:

$$\dot{q} = \frac{\partial H}{\partial p} = \frac{\partial}{\partial p}\left(\frac{p^2}{2m}\right) = \frac{2p}{2m} = \frac{p}{m}$$

Physical interpretation: $\dot{q} = p/m$ is simply the relation between momentum and velocity, $p = m\dot{q}$.

Why other options are wrong:

Q2.
- (A) $2p/m$: off by factor of 2; $\partial(p^2/2m)/\partial p = p/m$.
- (B) $p/(2m)$: off by factor of 2 in the denominator.
- (D) $-\partial V/\partial q$: This is $\dot{p}$ (from Hamilton's second equation), not $\dot{q}$.

Final Answer: $\dot{q} = p/m \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q2*

Solution

Concept — Resolving Power of a Diffraction Grating:
The resolving power is $R = mN$ where m is the diffraction order and N is the number of lines.

Step 1 — Resolving power:

$$R = mN = 2 \times 10{,}000 = 20{,}000$$

Step 2 — Minimum resolvable wavelength difference:

$$\delta\lambda = \frac{\lambda}{R} = \frac{500\,\text{nm}}{20{,}000} = 0.025\,\text{nm}$$

Why other options are wrong:

Q3.
- (A) $0.1\,\text{nm}$: corresponds to $R = 5{,}000$ (only first order with $N = 5{,}000$).
- (B) $0.05\,\text{nm}$: corresponds to $R = 10{,}000$ (first order, $m = 1$).
- (C) $0.5\,\text{nm}$: factor of 20 too large; incorrect resolving power.

Final Answer: $\delta\lambda = 0.025\,\text{nm} \Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q3*

Solution

Concept — Refractive Index from Permittivity:
For a non-magnetic medium ($\mu_r = 1$):

$$n = \sqrt{\varepsilon_r \mu_r} = \sqrt{\varepsilon_r} = \sqrt{2.25}$$

Computation:

$$\sqrt{2.25} = \sqrt{9/4} = 3/2 = 1.5$$

Why other options are wrong:

Q4.
- (B) 2.25: This is ε_r itself, not $\sqrt{\varepsilon_r}$.
- (C) 4.5: Double the correct value; no physical basis.
- (D) 1.13: Would correspond to $\varepsilon_r \approx 1.28$, not 2.25.

Final Answer: $n = 1.5 \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q4*

Solution

Concept — Chemical Potential of Classical Ideal Gas:
$\mu = k_B T \ln(n/n_Q)$. In the classical regime, $n \ll n_Q$, so $n/n_Q \ll 1$, giving $\ln(n/n_Q) < 0$.

Conclusion:

$$\mu = k_B T \underbrace{\ln(n/n_Q)}_{<0} < 0$$

The chemical potential of a classical ideal gas is always negative. Physically, this means adding a particle to the system (at constant T, V) *lowers* the free energy.

Why other options are wrong:

Q5.
- (A) $\mu > 0$: Occurs only for fermions below the Fermi energy (degenerate quantum gas).
- (C) $\mu = 0$: Applies to photons and phonons (non-conserved bosons), not ideal gas.
- (D) $\mu = k_B T$: Dimensionally consistent but numerically wrong for classical gas.

Final Answer: $\mu < 0 \Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q5*

Solution

Concept — Measurement Postulate (Projection / State Collapse):
The measurement postulate (fourth postulate of QM) states: if a measurement of observable $\hat{A}$ on state $|\psi\rangle$ yields eigenvalue a_n, the state collapses instantaneously to the normalised eigenstate $|a_n\rangle$.

Probability of outcome a_n:

$$P(a_n) = |\langle a_n|\psi\rangle|^2$$

State after measurement: $|\psi\rangle \to |a_n\rangle$ (wave-function collapse).

Why other options are wrong:

Q6.
- (A) Unchanged $|\psi\rangle$: Only if $|\psi\rangle$ is already an eigenstate of $\hat{A}$.
- (B) Equal superposition: The post-measurement state is the *specific* eigenstate $|a_n\rangle$, not all eigenstates.
- (D) Vacuum state: No physical basis; the system remains in the Hilbert space of the observable.

Final Answer: $|\psi\rangle \to |a_n\rangle \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q6*

Q7.

Solution

Concept — Diamond Cubic Structure:
In the diamond cubic lattice, each atom forms 4 covalent sp^3 bonds directed toward the corners of a regular tetrahedron. The lattice is FCC with a two-atom basis (one at a corner, one displaced by $a(1/4, 1/4, 1/4)$).

Coordination number: Each carbon atom is bonded to exactly **4** nearest neighbours at tetrahedral angles ($\approx 109.47°$).

Why other options are wrong:

- (A) 6: Coordination number of simple cubic; not applicable here.
- (B) 8: BCC coordination number; diamond has a more open structure.
- (C) 12: FCC or HCP coordination number; diamond packing fraction is only $\approx 34\%$.

Final Answer: Coordination number $= 4 \Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q7*

Q8.

Solution

Concept — XOR Gate Truth Table:
The XOR gate outputs 1 *only* when inputs differ; it outputs 0 when inputs are the same.

Full truth table:

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0 $\leftarrow$ option (A)

Why other options are wrong:

- (B) $A = 1, B = 0 \to 0$: Incorrect; inputs differ, so output = 1.
- (C) $A = 0, B = 0 \to 1$: Incorrect; inputs are the same, so output = 0.
- (D) $A = 0, B = 1 \to 0$: Incorrect; inputs differ, so output = 1.

Final Answer: $A = 1, B = 1 \to 0 \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q8*

Solution

Concept — Mass Number Conservation in Nuclear Fission:
In any nuclear reaction, the total mass number A and total charge Z are conserved.

Mass number balance (A):

$$235 + 1 = 141 + 92 + k \times 1$$

$$236 = 233 + k \quad \Rightarrow \quad k = 3$$

Charge balance (Z):

$$92 + 0 = 56 + 36 + 0 = 92 \checkmark$$

Why other options are wrong:

Q9.

- (A) $k = 1$: Mass sum $141 + 92 + 1 = 234 \neq 236$.
- (C) $k = 5$: Mass sum $141 + 92 + 5 = 238 \neq 236$.
- (D) $k = 2$: Mass sum $141 + 92 + 2 = 235 \neq 236$.

Final Answer: $k = 3$ prompt neutrons $\Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q9*

Solution

Concept — Legendre Polynomial P_2:
$P_2(x) = \dfrac{3x^2 - 1}{2}$.
Step 1 — Substitute $x = \cos 60° = 1/2$:

$$P_2\left(\frac{1}{2}\right) = \frac{3(1/2)^2 - 1}{2} = \frac{3/4 - 1}{2} = \frac{-1/4}{2} = -\frac{1}{8}$$

Why other options are wrong:

Q10.

- (A) $1/4$: Would need $3x^2 - 1 = 1/2$, i.e. $x^2 = 1/2$, $x = 1/\sqrt{2} = \cos 45°$, not $\cos 60°$.
- (B) $-1/4$: Missing the factor of $1/2$ in the denominator.
- (D) $3/4$: This is $3(1/2)^2 = 3/4$ before subtracting 1 and dividing by 2.

Final Answer: $P_2(\cos 60°) = -1/8 \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q10*

Solution

Concept — Euler's Equations for Torque-Free Symmetric Top:
The three Euler equations are:

$$I_1\dot{\omega}_1 = (I_2 - I_3)\omega_2\omega_3, \quad I_1\dot{\omega}_2 = (I_3 - I_1)\omega_1\omega_3, \quad I_3\dot{\omega}_3 = 0$$

Analysis of the third equation:

$$I_3\dot{\omega}_3 = 0 \;\Rightarrow\; \dot{\omega}_3 = 0 \;\Rightarrow\; \omega_3 = \text{constant}$$

The component of angular velocity along the symmetry axis $\hat{e}_3$ is a constant of the motion.

From $\omega_3 = \Omega_0$ (const), the first two equations become equations for a circular oscillation of (ω_1, ω_2) with precession frequency $\Omega_p = (I_3 - I_1)\Omega_0/I_1$ — the body-frame precession (Euler precession).

Why other options are wrong:

Q11.

- (A,B) ω_1 or ω_2: These precess in the body frame; neither is individually constant.
- (C) $\boldsymbol{\omega}$ fixed in body frame: The vector $\boldsymbol{\omega}$ traces a cone in the body frame (body-cone motion), not fixed.

Final Answer: $\omega_3 = \text{const} \Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q11*

Solution

Concept — Compton Scattering (Backscattering):
The Compton shift: $\Delta\lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$.

Step 1 — Substitute $\theta = 180°$:

$$\cos 180° = -1 \;\Rightarrow\; 1 - \cos 180° = 1 - (-1) = 2$$

Step 2 — Compute shift:

$$\Delta\lambda = 2 \times \frac{h}{m_e c} = 2 \times 2.426\,\text{pm} = 4.852\,\text{pm} \approx 4.85\,\text{pm}$$

This is the *maximum possible* Compton shift (backscattering).

Why other options are wrong:

Q12.

- (B) $2.43\,\text{pm}$: This is $\Delta\lambda$ for $\theta = 90^\circ$ (perpendicular scattering).
- (C) $1.21\,\text{pm}$: Corresponds to $1 - \cos\theta = 1/2$, i.e. $\theta = 60^\circ$.
- (D) $9.70\,\text{pm}$: Double the correct backscattering shift; incorrect.

Final Answer: $\Delta\lambda = 4.85\,\text{pm} \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q12*

Solution

Concept — Diamagnetic Materials:
For a diamagnetic material: $\mathbf{M} = \chi_m \mathbf{H}$ with $\chi_m < 0$. Relative permeability: $\mu_r = 1 + \chi_m$. Since $\chi_m < 0$: $\mu_r = 1 + \chi_m < 1$.

Consequence for B:

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H} = \mu_0(1 + \chi_m)\mathbf{H} < \mu_0 \mathbf{H}$$

The field inside the diamagnetic material is *slightly weaker* than the applied $\mu_0 H$.

Why other options are wrong:

Q13.

- (A) $\mu_r > 1$: This describes *paramagnetic* materials ($\chi_m > 0$).
- (C) $B = 0$: Only for a *perfect* superconductor (Meissner effect), where $\chi_m = -1$; ordinary diamagnets have $|\chi_m| \ll 1$.
- (D) $\mathbf{M}$ parallel to $\mathbf{H}$: $\mathbf{M} = \chi_m \mathbf{H}$ with $\chi_m < 0$ means $\mathbf{M}$ is *antiparallel* to $\mathbf{H}$.

Final Answer: $\mu_r < 1$, $B < \mu_0 H \Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q13*

Solution

Concept — Joule–Thomson Coefficient for Ideal Gas:
The Joule–Thomson coefficient is:

$$\mu_{JT} = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]$$

For an ideal gas: $PV = nRT \Rightarrow \left(\frac{\partial V}{\partial T}\right)_P = \frac{nR}{P} = \frac{V}{T}$.

$$\mu_{JT} = \frac{1}{C_P}\left[T \cdot \frac{V}{T} - V\right] = \frac{1}{C_P}[V - V] = 0$$

Physical interpretation: An ideal gas has no intermolecular interactions, so throttling neither cools nor heats it. The inversion temperature T_{inv} of an ideal gas is infinite.

Why other options are wrong:

Q14.

- (A,B) Large positive or negative: Applies to real gases (van der Waals), not ideal.
- (D) $2a/(Rb)$: This is the inversion temperature of a van der Waals gas, not μ_{JT} for an ideal gas.

Final Answer: $\mu_{JT} = 0 \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q14*

Solution

Concept — Ground State of Helium Atom:
Both electrons in $1s^2$: orbital angular momentum $L = 0$ (S-state). By the Pauli exclusion principle, the two electrons must have opposite spins, giving total spin $S = 0$ (singlet state, antisymmetric spin wavefunction). The total angular momentum $J = L + S = 0$.

Term symbol: ${}^{2S+1}L_J = {}^1S_0$ (singlet S-state, $J = 0$).

Why other options are wrong:

Q15.

- (A) ${}^3S_1$: $S = 1$ (triplet) requires parallel spins, which is *forbidden* in $1s^2$ by Pauli exclusion.
- (B) ${}^1P_1$: $L = 1$ requires one electron in a p-orbital; not the ground state ($1s^2$).
- (C) ${}^3P_0$: Both $L \neq 0$ and $S \neq 0$ would require excited configurations.

Final Answer: ${}^1S_0$ (singlet) $\Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q15*

Solution

Concept — Density of States at the Fermi Level:
In the free-electron model, $D(E_F) = \dfrac{3n}{2E_F}$.

Step 1 — Convert E_F to joules:

$$E_F = 7.04\,\text{eV} \times 1.6 \times 10^{-19}\,\text{J/eV} = 1.1264 \times 10^{-18}\,\text{J}$$

Step 2 — Compute $D(E_F)$ in J^{-1}m^{-3}:

$$D(E_F) = \frac{3 \times 8.5 \times 10^{28}}{2 \times 1.1264 \times 10^{-18}} = \frac{25.5 \times 10^{28}}{2.2528 \times 10^{-18}} = 1.132 \times 10^{47}\,\mathrm{J^{-1}m^{-3}}$$

Step 3 — Convert to eV^{-1}m^{-3} (multiply by J/eV = 1.6×10^{-19}):

$$D(E_F) = 1.132 \times 10^{47} \times 1.6 \times 10^{-19} = 1.811 \times 10^{28}\,\mathrm{eV^{-1}m^{-3}}$$

Why other options are wrong:

Q16.
- (B) 3.62×10^{28}: Double the correct value (factor of 2 error).
- (C) 9.05×10^{27}: Half the correct value.
- (D) 7.24×10^{28}: Four times the correct value.

Final Answer: $D(E_F) \approx 1.81 \times 10^{28}\,\mathrm{eV^{-1}m^{-3}} \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q16*

Solution

Concept — RC Differentiating Op-Amp:
The output of an ideal differentiator is $V_{\text{out}} = -\tau\, dV_{\text{in}}/dt$ where $\tau = R_f C_1$.

For a triangular wave input: A triangular wave has a piecewise-constant derivative (slope alternates between $+A/T_h$ and $-A/T_h$). Therefore its derivative is a square wave.

With $\tau = 10^{-3}$ s and $f = 1$ kHz ($T = 10^{-3}$ s): The time constant equals the period, giving clean differentiation. The output is a bipolar square wave of amplitude $\propto A/T_h$.

Why other options are wrong:

Q17.
- (A) Triangular: The derivative of a triangular wave is not triangular.
- (C) Sinusoidal: The derivative of a sinusoid is a cosine (shifted sinusoid); triangular is not sinusoidal.
- (D) Impulses: Ideal differentiator of a square wave gives impulses, not of a triangular wave.

Final Answer: Square wave $\Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q17*

Solution

Concept — Nuclear Shell Model and Magic Numbers:
Without spin–orbit coupling, harmonic-oscillator shells give closed shells at $2, 8, 20, 40, 70, 112\ldots$ — these do *not* match experimental magic numbers.

Adding the $\boldsymbol{l}\cdot\boldsymbol{s}$ spin–orbit term splits each nl-shell. The large gaps (magic numbers) shift to:

$$2,\ 8,\ 20,\ 28,\ 50,\ 82,\ 126$$

Physical origin: The spin–orbit interaction is large in nuclei (unlike atoms). It depresses the $j = l + \frac{1}{2}$ subshell into the lower shell, creating new large gaps at 28, 50, 82, and 126.

Why other options are wrong:

Q18.
- (A) $6, 14, \ldots$: Not supported by experiment or the shell model.
- (B) $14, \ldots$: 14 is not a magic number; experimentally, 20 comes after 8.
- (D) $40, 70, \ldots$: These are the pure harmonic-oscillator closed shells (before spin–orbit correction).

Final Answer: $2, 8, 20, 28, 50, 82 \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q18*

Solution

Concept — Fourier Sine Coefficient of $f(x) = x$:
$b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} x\sin(nx)\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin(nx)\,dx$
Since $x\sin(nx)$ is an even function:

$$b_n = \frac{2}{\pi}\int_0^{\pi} x\sin(nx)\,dx$$

Integration by parts: $u = x$, $dv = \sin(nx)\,dx$:

$$\int_0^{\pi} x\sin(nx)\,dx = \left[-\frac{x\cos(nx)}{n}\right]_0^{\pi} + \frac{1}{n}\int_0^{\pi}\cos(nx)\,dx = -\frac{\pi\cos(n\pi)}{n} + 0 = \frac{\pi(-1)^{n+1}}{n}$$

Therefore:

$$b_n = \frac{2}{\pi}\cdot\frac{\pi(-1)^{n+1}}{n} = \frac{2(-1)^{n+1}}{n}$$

Verification: $b_1 = 2$, $b_2 = -1$, $b_3 = 2/3, \ldots$ giving $f(x) = 2\sin x - \sin 2x + \frac{2}{3}\sin 3x - \ldots$

Why other options are wrong:

Q19.

- (A) $2/n$: Missing the alternating sign $(-1)^{n+1}$.
- (B) $(-1)^n/n$: Missing the factor of 2 from the symmetric integration.
- (C) $2(-1)^n/n$: Wrong sign pattern; b_1 would be -2 but the expansion starts with $+2\sin x$.

Final Answer: $b_n = 2(-1)^{n+1}/n \Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q19*

Solution

Concept — Coriolis Acceleration:
$|\mathbf{a}_{\text{Cor}}| = 2\Omega v \sin\lambda$

Calculation:

$$\begin{aligned}
|\mathbf{a}_{\text{Cor}}| &= 2 \times 7.27 \times 10^{-5}\,\text{rad/s} \times 10\,\text{m/s} \times \sin 45^\circ \\
&= 2 \times 7.27 \times 10^{-5} \times 10 \times \frac{1}{\sqrt{2}} \\
&= 2 \times 7.27 \times 10^{-5} \times 10 \times 0.7071 \\
&= 2 \times 5.14 \times 10^{-4} = 1.03 \times 10^{-3}\,\text{m/s}^2
\end{aligned}$$

This is $\approx 10^{-3}\,\text{m/s}^2$, far smaller than $g = 9.8\,\text{m/s}^2$, explaining why Coriolis effects are only noticeable at large scales.

Why other options are wrong:

Q20.

- (B) 10^{-2}: Ten times too large; requires $v \approx 100\,\text{m/s}$.
- (C) 10^{-4}: Ten times too small.
- (D) 10^{-5}: Hundred times too small.

Final Answer: $\approx 10^{-3}\,\text{m/s}^2 \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q20*

Solution

Concept — Decomposition of Linear Polarisation:
A linearly polarised wave $\hat{E} = E_0 \hat{x} e^{i(kz-\omega t)}$ can be written as:

$$\hat{E} = \frac{E_0}{2}\underbrace{(\hat{x} + i\hat{y})e^{i(kz-\omega t)}}_{\text{LCP}} + \frac{E_0}{2}\underbrace{(\hat{x} - i\hat{y})e^{i(kz-\omega t)}}_{\text{RCP}}$$

The superposition of LCP and RCP waves of *equal* amplitude gives linear polarisation (the circular motions cancel and the resultant oscillates in a fixed direction). This is the basis of optical activity: a medium with different refractive indices for LCP and RCP (circular birefringence) rotates the plane of polarisation.

Why other options are wrong:

Q21.
- (A) Two orthogonal linear components: That is the alternative decomposition of the *same* wave; the question asks about the sum of LCP and RCP.
- (C) Elliptical: Elliptical polarisation results from LCP + RCP of *unequal* amplitudes.
- (D) Same-handedness circular: Cannot produce linear polarisation.

Final Answer: Linear polarisation $\Rightarrow$ **(B)**

Answer: (B) ← Go back to Q21

Solution

Concept — Grand Potential and Grand Partition Function:
By analogy with the canonical ensemble ($F = -k_B T \ln Z$), the grand potential is:

$$\Omega(T, V, \mu) = -k_B T \ln \mathcal{Z}$$

Verification: From $\Omega = -k_B T \ln \mathcal{Z}$ one derives:

$$S = -\left(\frac{\partial\Omega}{\partial T}\right)_{V,\mu}, \quad P = -\left(\frac{\partial\Omega}{\partial V}\right)_{T,\mu}, \quad \langle N\rangle = -\left(\frac{\partial\Omega}{\partial \mu}\right)_{T,V}$$

and the relation $\Omega = F - \mu\langle N\rangle = U - TS - \mu\langle N\rangle = -PV$.

Why other options are wrong:

Q22.
- (A) $+k_B T \ln \mathcal{Z}$: Wrong sign; would give $S < 0$ in many cases.
- (B,D) $\pm k_B T/\mathcal{Z}$: Not the correct functional form; logarithm is required.

Final Answer: $\Omega = -k_B T \ln \mathcal{Z} \Rightarrow$ **(C)**

Answer: (C) ← Go back to Q22

Solution

Concept — Normal Zeeman Effect: Spectral Lines:
For a d-state ($l = 2$, $m_l = -2, -1, 0, 1, 2$) and a p-state ($l = 1$, $m_l = -1, 0, 1$) transition:

Selection rule: $\Delta m_l = 0, \pm 1$.

Allowed Δm_l values and their frequencies:

Q23.

- $\Delta m_l = +1$: $E = E_0 + \mu_B B$ (one shifted line)
- $\Delta m_l = 0$: $E = E_0$ (unshifted central line)
- $\Delta m_l = -1$: $E = E_0 - \mu_B B$ (one shifted line)

Even though there are multiple transitions for each Δm_l value (e.g. $m_l = 2 \to 1$, $1 \to 0$, $0 \to -1$, $-1 \to -1$ all have $\Delta m_l = 0$ but the same energy shift), they all contribute to the *same* three frequencies in the normal Zeeman pattern: the normal Zeeman effect always gives **3 spectral lines**.

Why other options are wrong:

- (A) 5: The number of m_l sub-levels of the d-state, not the number of spectral lines.
- (B) 7, (C) 9: Counts individual transitions, not distinct frequencies.

Final Answer: 3 spectral lines $\Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q23*

Solution

Concept — p-n Junction Depletion Width:
For equal doping $N_A = N_D = N$: the depletion extends equally on both sides, $x_p = x_n = W/2$.

Poisson's equation on each side gives:

$$W = x_n + x_p = 2x_n = 2\sqrt{\frac{\varepsilon_r \varepsilon_0 V_{bi}}{eN}}$$

Verification for Si ($\varepsilon_r = 11.7$, $V_{bi} = 0.7\,\text{V}$, $N = 10^{23}\,\text{m}^{-3}$):

$$W = 2\sqrt{\frac{11.7 \times 8.85 \times 10^{-12} \times 0.7}{1.6 \times 10^{-19} \times 10^{23}}} \approx 96\,\text{nm}$$

Why other options are wrong:

Q24.

- (B) Missing factor of 2: This would be x_n alone (or x_p alone), not the total width W.
- (C) $4\sqrt{\cdots}$: Overestimates by factor of 2.
- (D) Not a square root: Incorrect dimensional dependence on V_{bi} and N.

Final Answer: $W = 2\sqrt{\varepsilon_r \varepsilon_0 V_{bi}/(eN)} \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q24*

Solution

Concept — Schmitt Trigger Thresholds:
For an inverting Schmitt trigger with positive feedback: $V_{TH} = +V_{\text{sat}} \cdot \dfrac{R_1}{R_1 + R_2}$.

Calculation:

$$V_{TH} = +15\,\text{V} \times \frac{10\,\text{k}\Omega}{10\,\text{k}\Omega + 10\,\text{k}\Omega} = 15 \times \frac{1}{2} = +7.5\,\text{V}$$

$$V_{TL} = -7.5\,\text{V} \quad \text{(by symmetry)}$$

Why other options are wrong:

Q25.
- (A) $\pm 15\,\text{V}$: This would require $R_2 = 0$ (no voltage divider); the thresholds equal the supply only with no feedback division.
- (C) $\pm 5\,\text{V}$: Requires $R_1/R_2 = 1/2$ (e.g. $R_1 = 5\,\text{k}\Omega$, $R_2 = 10\,\text{k}\Omega$).
- (D) $\pm 10\,\text{V}$: Requires $R_1/R_2 = 2/1$; inconsistent with equal resistors.

Final Answer: $(V_{TH}, V_{TL}) = (+7.5\,\text{V}, -7.5\,\text{V}) \Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q25*

Solution

Concept — Wigner–Seitz Cell of BCC:
The BCC lattice has each lattice point surrounded by 8 nearest neighbours (body-centre corners) and 6 next-nearest neighbours (face-centre directions). Drawing bisector planes to all 14 neighbours produces a **truncated octahedron**: 8 regular hexagonal faces (bisecting NN bonds) and 6 square faces (bisecting NNN bonds). It has 14 faces in total.

This cell encloses exactly 1 lattice point (primitive cell volume $= a^3/2$ for a BCC crystal).

Why other options are wrong:

Q26.
- (A) Cube: This is the conventional BCC unit cell (2 atoms), not the Wigner–Seitz cell.
- (B) Rhombic dodecahedron: This is the Wigner–Seitz cell of the *FCC* lattice (12 rhombic faces).
- (D) Tetrahedron: Cannot fill space by itself; not a Wigner–Seitz cell of any Bravais lattice.

Final Answer: Truncated octahedron $\Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q26*

Q27.

Solution

Concept — Thermal Conductivity of Ideal Gas:
$\kappa = \frac{1}{3}\bar{v}\lambda C_v n$

Key substitutions:

- $\bar{v} = \sqrt{8k_B T/(\pi m)} \propto \sqrt{T}$
- Mean free path: $\lambda = 1/(\sqrt{2}\pi d^2 n) \propto 1/n$, *independent of* T (at fixed n)
- So λn = const and C_v = const for monatomic ideal gas.

Result:

$$\kappa = \frac{1}{3}\bar{v}\cdot(\lambda n)\cdot C_v \propto \sqrt{T}\times \text{const} \propto \sqrt{T}$$

Since λn is independent of pressure P, so is κ: **thermal conductivity is independent of pressure**.

Why other options are wrong:

- (A) $\kappa \propto T$: This would arise if $\bar{v} \propto T$ (incorrect; $\bar{v} \propto \sqrt{T}$).
- (B) $\kappa \propto 1/\sqrt{T}$: Wrong direction; κ increases with temperature.
- (C) $\kappa \propto P$: Incorrect; both $\lambda \propto 1/n$ and $n \propto P$ cancel exactly.

Final Answer: $\kappa \propto \sqrt{T}$, independent of $P \Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q27*

Q28.

Solution

Concept — Radiation Resistance of a Short Dipole:
$R_{\text{rad}} = 80\pi^2\left(\frac{\ell}{\lambda}\right)^2\ \Omega$

Substitute $\ell = \lambda/20$**:**

$$R_{\text{rad}} = 80\pi^2\left(\frac{1}{20}\right)^2 = \frac{80\pi^2}{400} = \frac{\pi^2}{5} \approx \frac{9.870}{5} \approx 1.97\,\Omega \approx 2\,\Omega$$

Physical significance: The small radiation resistance ($\ll 50\,\Omega$) explains why electrically short antennas are inefficient — most power is wasted in ohmic resistance.

Why other options are wrong:

- (B) $8\,\Omega$: Would correspond to $\ell = \lambda/10$, giving $R_{\text{rad}} = 80\pi^2/100 \approx 7.9\,\Omega$.
- (C) $20\,\Omega$: Much too large for $\ell = \lambda/20$.
- (D) $0.2\,\Omega$: Would require $\ell = \lambda/200$, ten times shorter.

Final Answer: $R_{\text{rad}} \approx 2\,\Omega \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q28*

Solution

Concept — Rayleigh Resolution:
$\theta_{\text{min}} = \frac{1.22\lambda}{D}$
Step 1 — Compute in radians:

$$\theta_{\text{min}} = \frac{1.22 \times 550 \times 10^{-9}\,\text{m}}{5\,\text{m}} = \frac{6.71 \times 10^{-7}}{5} = 1.342 \times 10^{-7}\,\text{rad}$$

Step 2 — Convert to arcseconds:

$$\theta_{\text{min}} = 1.342 \times 10^{-7} \times 2.06 \times 10^{5}\,\text{arcsec/rad} = 0.0276\,\text{arcsec} \approx 0.028\,\text{arcsec}$$

Why other options are wrong:

Q29.

- (A) 0.11 arcsec: Would correspond to $D \approx 1.2$ m (a much smaller telescope).
- (C) 0.056 arcsec: Double the correct value; roughly corresponds to $D \approx 2.5$ m.
- (D) 0.014 arcsec: Half the correct value; corresponds to $D \approx 10$ m.

Final Answer: $\theta_{\text{min}} \approx 0.028\,\text{arcsec} \Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q29*

Solution

Concept — Wavelength of Light in a Medium:
The frequency ν of a wave is set by the source and does not change when entering a medium. The phase velocity changes from c to $v = c/n$.

Using $v = \lambda_{\text{medium}}\nu = c/n$ and $c = \lambda_{\text{vac}}\nu$:

$$\lambda_{\text{medium}} = \frac{v}{\nu} = \frac{c/n}{\nu} = \frac{\lambda_{\text{vac}}}{n}$$

Physical interpretation: In a medium with $n > 1$, the wavelength is compressed by factor n (shorter wavelength, same frequency, slower speed).

Why other options are wrong:

Q30.

- (A) $n\lambda_{\text{vac}}$: This would imply the wave *speeds up* inside the medium ($v > c$).
- (B) λ_{vac}: The wavelength would be unchanged only if $n = 1$ (vacuum).
- (D) λ_{vac}/n^2: Over-compressed by extra factor of n; no physical basis.

Final Answer: $\lambda_{\text{medium}} = \lambda_{\text{vac}}/n \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q30*

Section B Solutions — MSQ

Q31.

Solution

Concept — Conservation Laws in Relativistic Collisions:

(A) Total four-momentum p^μ: CORRECT. Conservation of four-momentum $\sum p^\mu = \text{const}$ is the fundamental relativistic conservation law and holds in all collision types (elastic and inelastic). It encodes simultaneous conservation of energy and three-momentum.

(B) Total energy E_{total}: CORRECT. The time component of four-momentum conservation gives $\sum E_i = \text{const}$, where $E_i = \gamma_i m_i c^2$ includes rest-mass energy. This holds universally.

(C) Rest mass of each individual particle: NOT CORRECT. In an inelastic relativistic collision (e.g. $e^+ + e^- \to 2\gamma$), individual particles can be annihilated and their rest mass converted to photon energy.

(D) Total kinetic energy: NOT CORRECT. Kinetic energy is conserved only in *elastic* collisions. Inelastic collisions involve conversion of KE to internal energy or rest mass.

Correct options: (A) and (B).

Answer: (AB) ← *Go back to Q31*

Q32.

Solution

Concept — Properties of Electrostatic Potential:

(A) $\mathbf{E} = -\nabla V$: CORRECT. This is a fundamental relation in electrostatics, obtained from $\nabla \times \mathbf{E} = 0$, which allows $\mathbf{E}$ to be written as the gradient of a scalar.

(B) V is a vector field: WRONG. V is a *scalar* field; it assigns a single number (potential) to each point in space.

(C) V is constant on equipotential surfaces: CORRECT. An equipotential surface is defined as the locus where $V = \text{const}$. By construction, V is constant on it and $\mathbf{E}$ is perpendicular to it.

(D) $V = q/(4\pi\varepsilon_0 r)$ for a point charge: CORRECT. This is Coulomb's potential for a point charge at the origin.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q32*

Q33.

Solution

Concept — Microcanonical Ensemble:

(A) Temperature is a fixed external parameter: WRONG. In the microcanonical ensemble, the total energy E, volume V, and particle number N are fixed. Temperature is a *derived* quantity: $1/T = \partial S/\partial E|_{N,V}$.

(B) Total energy E is fixed: CORRECT. The defining feature of the microcanonical ensemble: $E = \text{const}$ (isolated system, no energy exchange with surroundings).

(C) $S = k_B \ln \Omega$: CORRECT. This is Boltzmann's entropy formula, fundamental to the microcanonical ensemble. Ω is the number of microstates consistent with the macroscopic constraints (E, V, N).

(D) Appropriate for isolated systems: CORRECT. The microcanonical ensemble is precisely the statistical description of an isolated system with fixed E, V, N (no exchange of energy or particles with the environment).

Correct options: (B), (C), and (D).

Answer: (BCD) ← *Go back to Q33*

Q34.

Solution

Concept — JFET Operation:

(A) Gate–channel junction is reverse-biased: CORRECT. In normal operation, the gate–channel junction (a p-n junction) is reverse-biased. This keeps the gate current negligibly small and prevents the junction from conducting.

(B) Channel narrows as $|V_{GS}|$ increases: CORRECT. For an n-channel JFET, making V_{GS} more negative increases the reverse bias on the gate–channel junction, widening the depletion region and *narrowing* the conducting channel. At $V_{GS} = V_p$ (pinch-off voltage), the channel is fully depleted.

(C) JFET is a current-controlled device: WRONG. A JFET is a *voltage-controlled* device: the drain current I_D is controlled by the gate-source voltage V_{GS}, not by gate current (which is essentially zero).

(D) Gate current is significant: WRONG. Because the gate junction is reverse-biased, the gate current I_G is extremely small (typically nA range), which is one of the JFET's key advantages over BJTs.

Correct options: (A) and (B).

Answer: (AB) ← *Go back to Q34*

Q35.

Solution

Concept — X-ray Diffraction from Crystals:

(A) Bragg's law $2d \sin\theta = n\lambda$: CORRECT. Bragg's law describes the condition for constructive interference of X-rays reflected from parallel crystal planes. θ is the glancing angle (complement of the angle of incidence from the normal).

(B) Only cubic crystals: WRONG. X-ray diffraction occurs in crystals of any symmetry — cubic, tetragonal, hexagonal, monoclinic, etc. The Bragg condition $2d \sin\theta = n\lambda$ applies universally, with d being the spacing of the relevant set of planes.

(C) Intensity depends on structure factor: CORRECT. The structure factor $F_{hkl} = \sum_j f_j e^{2\pi i(hx_j+ky_j+lz_j)}$ determines the amplitude (and hence intensity) of the diffracted beam from the (hkl) planes. It depends on the atomic scattering factors f_j and atomic positions.

(D) Systematic absences reveal Bravais lattice: CORRECT. When the structure factor $F_{hkl} = 0$ for certain (hkl) combinations, the corresponding reflections are *absent*. These systematic absences (e.g. $h + k + l$ odd for BCC, h, k, l not all same parity for FCC) identify the Bravais lattice.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q35*

Q36.

Solution

Concept — Properties of Hamilton's Equations:

(A) Total number of equations is N: WRONG. Hamilton's equations come in pairs: $\dot{q}_i = \partial H/\partial p_i$ and $\dot{p}_i = -\partial H/\partial q_i$ for each $i = 1, \ldots, N$. That gives $2N$ first-order equations total.

(B) Total number is $2N$: CORRECT. As above, Hamilton's formulation converts the N second-order Lagrange equations into $2N$ first-order equations in phase space (q_i, p_i).

(C) $H = T + V$ for conservative systems: CORRECT. When the constraints are time-independent and the forces are derivable from a potential $V(q)$, the Hamiltonian equals the total mechanical energy $T + V$.

(D) Phase-space volume preserved (Liouville's theorem): CORRECT. Hamilton's equations lead to $\nabla_\Gamma \cdot \dot{\Gamma} = 0$ (incompressibility of the phase-space flow), so the volume of any region in phase space is preserved under time evolution.

Correct options: (B), (C), and (D).

Answer: (BCD) ← *Go back to Q36*

Q37.

Solution

Concept — ^{238}U Decay Series:

(A) Terminates at $^{206}_{82}$Pb: CORRECT. The uranium-238 decay series (also called the $4n+2$ series) ends at ^{206}Pb, which is stable.

(B) Number of α decays = 8: CORRECT. Each α decay reduces A by 4. From $A = 238$ to $A = 206$: $\Delta A = 32 = 4 \times 8$. So there are exactly 8 α decays.

Charge check: 8 α decays reduce Z by 16: $92 - 16 = 76$. But $Z(^{206}\text{Pb}) = 82$. The deficit of $82 - 76 = 6$ protons is supplied by 6 β^- decays (each increases Z by 1). So the series also involves 6 β^- decays.

(C) Terminates at ^{208}Pb: WRONG. ^{208}Pb is the end product of the thorium-232 ($4n$ series), not the uranium-238 series.

(D) Only α decays: WRONG. As shown above, 6 β^- decays are required to reach $Z = 82$.

Correct options: (A) and (B).

Answer: (AB) ← *Go back to Q37*

Q38.

Solution

Concept — Prism at Minimum Deviation:

(A) $i_1 = i_2$ at minimum deviation: CORRECT. At the angle of minimum deviation, the refracted ray inside the prism is parallel to the base (for an isoceles prism), and the passage is symmetric: the angle of incidence i_1 equals the angle of emergence i_2.

(B) Refracted ray perpendicular to bisector: WRONG. At minimum deviation, the refracted ray inside is *parallel to the base* (for a symmetric prism), not perpendicular to the bisector. Perpendicular to the bisector would be perpendicular to the prism axis, which is not generally true.

(C) $n = \sin[(A + D_m)/2]/\sin(A/2)$: CORRECT. This is the standard prism formula. At minimum deviation with $i_1 = i_2 = i$, the geometry gives $r = A/2$ (refraction angle) and $i = (A + D_m)/2$, leading directly to this result via Snell's law.

(D) δ has a unique minimum: CORRECT. As i_1 varies from $0°$ to $90°$, the deviation δ decreases, reaches a minimum D_m, then increases. This minimum is the angle of minimum deviation and corresponds to option (A).

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q38*

Q39.

Solution

Concept — Free Particle in Quantum Mechanics:

(A) Discrete energy spectrum: WRONG. A free particle has a *continuous* energy spectrum $E = \hbar^2k^2/(2m)$ for all real k; no quantisation occurs without boundary conditions.

(B) e^{ikx} is an eigenstate for any real k: CORRECT. Substituting $\psi = e^{ikx}$ into $-\hbar^2\psi''/(2m) = E\psi$: $-\hbar^2(ik)^2e^{ikx}/(2m) = \hbar^2k^2e^{ikx}/(2m) = Ee^{ikx}$. So it is an eigenstate with $E = \hbar^2k^2/(2m)$ for any real k.

(C) $E_k = \hbar^2k^2/(2m)$: CORRECT. As verified above, this is the energy eigenvalue for momentum eigenstate e^{ikx}.

(D) Probability current $j = (\hbar k/m)|A|^2$: CORRECT. The quantum probability current is $j = \hbar\text{Im}(\psi^*\partial_x\psi)/m = \hbar\text{Im}(A^*e^{-ikx} \cdot ikAe^{ikx})/m = \hbar k|A|^2/m$.

Correct options: (B), (C), and (D).

Answer: (BCD) ← *Go back to Q39*

Q40.

Solution

Concept — Properties of Photons:

(A) Zero rest mass: CORRECT. Photons are massless particles ($m_0 = 0$). Their energy is $E = \hbar\omega = pc$ and they travel at speed c in vacuum.

(B) Spin quantum number $s = 1$: CORRECT. Photons are spin-1 bosons with two polarisation states (helicities $\pm\hbar$). The $s_z = 0$ state is absent because massless particles cannot be at rest.

(C) Single photon splits at beamsplitter: WRONG. A single photon is *not* split into two photons at a beamsplitter (that would violate energy conservation). Instead, the photon is in a quantum superposition of the two output paths and is detected with probability 1/2 in each arm. Photon number is conserved.

(D) Photon momentum same in medium as in vacuum: WRONG. In a medium of refractive index n, the photon momentum changes. The Minkowski momentum (commonly used) is $p = nE/c > E/c$, while the Abraham momentum is $p = E/(nc) < E/c$. Either way, the momentum differs from E/c (the vacuum value).

Correct options: (A) and (B).

Answer: (AB) ← *Go back to Q40*

Section C Solutions — NAT

Q41.

Solution

Concept — Projectile Range:
$R = v_0^2 \sin(2\theta)/g$

Calculation (with $\theta = 45°$, $\sin 90° = 1$):

$$R = \frac{(20)^2 \times 1}{9.8} = \frac{400}{9.8} = 40.816\,\mathrm{m}$$

$$\boxed{R \approx 40.82\,\mathrm{m}}$$

Answer: (40.82) ← *Go back to Q41*

Q42.

Solution

Concept — Energy Stored in a Capacitor:
$U = \frac{1}{2}CV^2$

Calculation:

$$U = \frac{1}{2} \times 50 \times 10^{-6}\,\mathrm{F} \times (200)^2\,\mathrm{V}^2 = \frac{1}{2} \times 50 \times 10^{-6} \times 40{,}000 = \frac{1}{2} \times 2\,\mathrm{J} = 1.0\,\mathrm{J}$$

$$\boxed{U = 1.00\,\mathrm{J}}$$

Answer: (1.00) ← *Go back to Q42*

Q43.

Solution

Concept — de Broglie Wavelength of Thermal Neutron:
Using momentum $p = \sqrt{2m_n k_B T}$:

Step 1 — Compute p:

$$\begin{aligned} p &= \sqrt{2 \times 1.675 \times 10^{-27} \times 1.38 \times 10^{-23} \times 293} \\ &= \sqrt{2 \times 1.675 \times 10^{-27} \times 4.041 \times 10^{-21}} \\ &= \sqrt{2 \times 6.769 \times 10^{-48}} = \sqrt{1.354 \times 10^{-47}} \\ &= 3.680 \times 10^{-24}\,\text{kg m s}^{-1} \end{aligned}$$

Step 2 — Wavelength:

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{3.680 \times 10^{-24}} = 1.800 \times 10^{-10}\,\text{m} = 0.180\,\text{nm}$$

$$\boxed{\lambda = 0.180\,\text{nm}}$$

Answer: (0.180) ← *Go back to Q43*

Q44.

Solution

Concept — Molecular Flux:
$\Phi = n\bar{v}/4$ where $n = P/(k_B T)$ and $\bar{v} = \sqrt{8k_B T/(\pi m)}$.

Step 1 — Number density:

$$n = \frac{10^5}{1.38 \times 10^{-23} \times 300} = \frac{10^5}{4.14 \times 10^{-21}} = 2.415 \times 10^{25}\,\text{m}^{-3}$$

Step 2 — Mean speed:

$$\bar{v} = \sqrt{\frac{8 \times 1.38 \times 10^{-23} \times 300}{\pi \times 4.65 \times 10^{-26}}} = \sqrt{\frac{3.312 \times 10^{-20}}{1.461 \times 10^{-25}}} = \sqrt{2.267 \times 10^5} = 476.1\,\text{m/s}$$

Step 3 — Flux:

$$\Phi = \frac{2.415 \times 10^{25} \times 476.1}{4} = \frac{1.150 \times 10^{28}}{4} = 2.874 \times 10^{27}\,\text{m}^{-2}\text{s}^{-1}$$

$$\boxed{\Phi \approx 2.87 \times 10^{27}\,\text{m}^{-2}\text{s}^{-1} \quad \Rightarrow \quad 2.87 \text{ (in units of } 10^{27})}$$

Answer: (2.87) ← *Go back to Q44*

Q45.

Solution

Concept — Specific Activity of ^{60}Co:
Specific activity $= \lambda N_A/M$ where $\lambda = \ln 2/T_{1/2}$.

Step 1:

$$\lambda = \frac{\ln 2}{1.663 \times 10^8\,\text{s}} = \frac{0.6931}{1.663 \times 10^8} = 4.168 \times 10^{-9}\,\text{s}^{-1}$$

Step 2 — Specific activity:

$$A/m = \lambda \cdot \frac{N_A}{M} = 4.168 \times 10^{-9} \times \frac{6.022 \times 10^{23}}{60\,\text{g/mol}}$$

$$= 4.168 \times 10^{-9} \times 1.004 \times 10^{22} = 4.183 \times 10^{13}\,\text{Bq/g}$$

$$\boxed{A/m \approx 4.18 \times 10^{13}\,\text{Bq/g} \quad \Rightarrow \quad 4.18 \text{ (in units of } 10^{13})}$$

Answer: (4.18) ← *Go back to Q45*

Q46.

Solution

Concept — Atomic Radius in FCC Structure:
In FCC, atoms touch along the face diagonal: $4r = a\sqrt{2}$, so:

$$r = \frac{a\sqrt{2}}{4} = \frac{a}{2\sqrt{2}}$$

Calculation:

$$r = \frac{4.05\,\text{Å} \times \sqrt{2}}{4} = \frac{4.05 \times 1.41421}{4} = \frac{5.7276}{4} = 1.4319\,\text{Å}$$

$$\boxed{r \approx 1.43\,\text{Å}}$$

Answer: (1.43) ← *Go back to Q46*

Q47.

Solution

Concept — Magnitude of Cross Product:
$\mathbf{A} = (1, 2, 0)$, $\mathbf{B} = (0, 1, 3)$.

Cross product:

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 1 & 2 & 0\\ 0 & 1 & 3\end{vmatrix}=\hat{x}(2\cdot 3-0\cdot 1)-\hat{y}(1\cdot 3-0\cdot 0)+\hat{z}(1\cdot 1-2\cdot 0)$$

$$=(6)\hat{x}+(-3)\hat{y}+(1)\hat{z}=(6,-3,1)$$

Magnitude:

$$|\mathbf{A}\times\mathbf{B}|=\sqrt{36+9+1}=\sqrt{46}\approx 6.782$$

$$\boxed{|\mathbf{A}\times\mathbf{B}|\approx 6.78}$$

Answer: (6.78) ← *Go back to Q47*

Q48.

Solution

Concept — LC Resonance Frequency:

$f_0=\dfrac{1}{2\pi\sqrt{LC}}$

Calculation:

$$LC=10\times 10^{-3}\,\mathrm{H}\times 0.1\times 10^{-6}\,\mathrm{F}=10^{-9}\,\mathrm{H\,F}$$

$$\sqrt{LC}=\sqrt{10^{-9}}=10^{-4.5}=3.162\times 10^{-5}\,\mathrm{s}$$

$$f_0=\frac{1}{2\pi\times 3.162\times 10^{-5}}=\frac{1}{1.987\times 10^{-4}}=5033\,\mathrm{Hz}=5.033\,\mathrm{kHz}$$

$$\boxed{f_0\approx 5.03\,\mathrm{kHz}}$$

Answer: (5.03) ← *Go back to Q48*

Q49.

Solution

Concept — Op-Amp Comparator:

For an ideal op-amp comparator with infinite open-loop gain:

$$V_{\text{out}}=\begin{cases}+V_{\text{sat}} & \text{if } V_+>V_-\\ -V_{\text{sat}} & \text{if } V_+<V_-\end{cases}$$

Given: $V_+=2.5\,\mathrm{V}<V_-=3.0\,\mathrm{V}$.

Since $V_+ < V_-$, the output saturates to the negative rail:

$$V_{\text{out}} = -12\,\text{V}$$

$$\boxed{V_{\text{out}} = -12\,\text{V}}$$

Answer: (-12) ← *Go back to Q49*

Q50.

Solution

Concept — Power Factor at Resonance:
At resonance, $X_L = \omega_0 L = 1/(\omega_0 C) = X_C$, so they cancel. The total impedance is:

$$Z = R + j(X_L - X_C) = R + j \cdot 0 = R \quad \text{(purely real)}$$

The phase angle ϕ between voltage and current: $\tan\phi = (X_L - X_C)/R = 0$, so $\phi = 0°$.

$$\text{Power factor} = \cos\phi = \cos 0° = 1$$

All the power supplied by the source is dissipated in the resistor (maximum power transfer at resonance).

$$\boxed{\cos\phi = 1.00}$$

Answer: (1.00) ← *Go back to Q50*

Q51.

Solution

Concept — Skin Depth in a Good Conductor:
$\delta = \sqrt{\dfrac{2}{\omega\mu_0\sigma}}$

Step 1 — Parameters: $f = 10^6\,\text{Hz}$, $\omega = 2\pi \times 10^6\,\text{rad/s}$, $\mu_0 = 4\pi \times 10^{-7}\,\text{H/m}$, $\sigma = 6 \times 10^7\,\text{S/m}$.

Step 2 — Denominator $\omega\mu_0\sigma$:

$$\begin{aligned}\omega\mu_0\sigma &= 2\pi \times 10^6 \times 4\pi \times 10^{-7} \times 6 \times 10^7 \\ &= 8\pi^2 \times 6 \times 10^6 = 48\pi^2 \times 10^6 \\ &= 48 \times 9.8696 \times 10^6 = 473.7 \times 10^6 = 4.737 \times 10^8\,\text{s}^{-1}\text{H}^{-1}\text{m}^{-1}\text{S}\end{aligned}$$

Step 3 — Skin depth:

$$\delta = \sqrt{\frac{2}{4.737 \times 10^8}} = \sqrt{4.223 \times 10^{-9}} = 6.498 \times 10^{-5}\,\text{m} = 64.98\,\mu\text{m}$$

$$\boxed{\delta \approx 65.0\,\mu\text{m}}$$

Answer: (65.0) ← *Go back to Q51*

Q52.

Solution

Concept — Hydrogen Lyman Alpha Line:
$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R_H \times \frac{3}{4}$

Step 1:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{3}{4} = 8.228 \times 10^6\,\text{m}^{-1}$$

Step 2:

$$\lambda = \frac{1}{8.228 \times 10^6} = 1.2153 \times 10^{-7}\,\text{m} = 121.53\,\text{nm}$$

Using $R_H = 1.097 \times 10^7$ exactly: $\lambda = 4/(3 \times 1.097 \times 10^7) = 4/3.291 \times 10^7 = 121.50\,\text{nm}$.

$$\boxed{\lambda \approx 121.50\,\text{nm}}$$

Answer: (121.50) ← *Go back to Q52*

Q53.

Solution

Concept — Entropy of Mixing:
$\Delta S_{\text{mix}} = -nR(x_A \ln x_A + x_B \ln x_B)$

With $n = 1$ mol and $x_A = x_B = 0.5$:

$$\Delta S_{\text{mix}} = -1 \times 8.314 \times \left[0.5\ln(0.5) + 0.5\ln(0.5)\right] = -8.314 \times \left[2 \times 0.5 \times (-0.6931)\right]$$

$$= -8.314 \times (-0.6931) = 8.314 \times 0.6931 = 5.763\,\text{J/K}$$

$$\boxed{\Delta S_{\text{mix}} \approx 5.76\,\text{J/K}}$$

Answer: (5.76) ← *Go back to Q53*

Q54.

Solution

Concept — Density of States at Fermi Level:
$D(E_F) = \frac{3n}{2E_F}$

Step 1 — In $\mathbf{J^{-1}m^{-3}}$ (convert E_F to joules):

$$E_F = 7.04 \times 1.6 \times 10^{-19} = 1.1264 \times 10^{-18}\,\text{J}$$

$$D(E_F) = \frac{3 \times 8.5 \times 10^{28}}{2 \times 1.1264 \times 10^{-18}} = \frac{25.5 \times 10^{28}}{2.2528 \times 10^{-18}} = 1.132 \times 10^{47}\,\text{J}^{-1}\text{m}^{-3}$$

Step 2 — Convert to $\mathbf{eV^{-1}m^{-3}}$ (multiply by J/eV $= 1.6 \times 10^{-19}$):

$$D(E_F) = 1.132 \times 10^{47} \times 1.6 \times 10^{-19} = 1.811 \times 10^{28}\,\text{eV}^{-1}\text{m}^{-3}$$

$$\boxed{D(E_F) \approx 1.81 \times 10^{28}\,\text{eV}^{-1}\text{m}^{-3} \quad \Rightarrow \quad 1.81 \text{ (in units of } 10^{28})}$$

Answer: (1.81) ← *Go back to Q54*

Q55.

Solution

Concept — Hydrogen 21 cm Hyperfine Splitting:
$\Delta E = h f_{HFS}$

Step 1 — Energy in joules:

$$\Delta E = 6.626 \times 10^{-34}\,\text{J s} \times 1.42041 \times 10^{9}\,\text{Hz} = 9.411 \times 10^{-25}\,\text{J}$$

Step 2 — Convert to eV:

$$\Delta E = \frac{9.411 \times 10^{-25}}{1.6 \times 10^{-19}} = 5.882 \times 10^{-6}\,\text{eV} = 5.88\,\mu\text{eV}$$

This corresponds to a photon wavelength $\lambda = c/f = 3 \times 10^8 / 1.42 \times 10^9 = 0.211\,\text{m} = 21\,\text{cm}$, the famous 21 cm hydrogen line used in radio astronomy.

$$\boxed{\Delta E \approx 5.88\,\mu\text{eV}}$$

Answer: (5.88) ← *Go back to Q55*

Q56.

Solution

Concept — Speed of Sound in an Ideal Gas:
$v = \sqrt{\gamma RT/M}$ for an ideal gas.

Calculation for argon ($\gamma = 5/3$, $M = 0.040\,$kg/mol, $T = 300\,$K):

$$\begin{aligned} v &= \sqrt{\frac{5}{3} \times 8.314 \times 300 \div 0.040} \\ &= \sqrt{1.6667 \times 8.314 \times 7500} \\ &= \sqrt{1.6667 \times 62355} \\ &= \sqrt{103925} \\ &= 322.4\,\text{m/s} \end{aligned}$$

$$\boxed{v \approx 322\,\text{m/s}}$$

Note: The measured speed of sound in Ar at $20°$C ($293\,$K) is about $319\,$m/s; at $300\,$K the calculation gives $322\,$m/s, consistent with $v \propto \sqrt{T}$.

Answer: (322) ← *Go back to Q56*

Q57.

Solution

Concept — Radioactive Decay over Non-Integer Half-Lives:
$N/N_0 = (1/2)^n$ where $n = 2.5$.

Calculation:

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{2.5} = \frac{1}{2^{5/2}} = \frac{1}{4\sqrt{2}} = \frac{1}{4 \times 1.41421} = \frac{1}{5.6569} = 0.17678$$

$$\boxed{N/N_0 \approx 0.177}$$

Answer: (0.177) ← *Go back to Q57*

Q58.

Solution

Concept — Fresnel Cosine Integral:
The Fresnel integrals are defined as:

$$C(x) = \int_0^x \cos\left(\frac{\pi t^2}{2}\right) dt, \quad S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right) dt$$

Known result:

$$C(\infty) = \int_0^\infty \cos\left(\frac{\pi t^2}{2}\right) dt = \frac{1}{2}$$

Derivation sketch: Setting $u = \pi t^2/2$, the Fresnel integral reduces (via the standard Gaussian integral and contour integration in the complex plane) to $(1/2)(1+i)/\sqrt{2} \cdot \mathrm{Re}(\ldots) = 1/2$. Both $C(\infty) = S(\infty) = 1/2$ exactly.

This result arises in diffraction theory (Cornu spiral) and in optics.

$$\boxed{C(\infty) = 0.5000}$$

Answer: (0.5000) ← *Go back to Q58*

Q59.

Solution

Concept — RC Discharge Time Constant:
$\tau = RC$

Calculation:

$$\tau = R \times C = 100\,\Omega \times 100 \times 10^{-6}\,\mathrm{F} = 100 \times 10^{-4}\,\mathrm{s} = 10^{-2}\,\mathrm{s} = 10.0\,\mathrm{ms}$$

At $t = \tau$, the voltage has decayed to $V_0/e \approx 0.368\,V_0 = 3.68\,\mathrm{V}$.

$$\boxed{\tau = 10.0\,\mathrm{ms}}$$

Answer: (10.0) ← *Go back to Q59*

Q60.

Solution

Concept — Blackbody Photon Number Density:
$n_\gamma = \dfrac{2\zeta(3)}{\pi^2}\left(\dfrac{k_B T}{\hbar c}\right)^3$

Step 1 — Compute $k_B T/(\hbar c)$:

$$\frac{k_B T}{\hbar c} = \frac{1.38 \times 10^{-23} \times 300}{1.055 \times 10^{-34} \times 3 \times 10^8} = \frac{4.14 \times 10^{-21}}{3.165 \times 10^{-26}} = 1.308 \times 10^5\,\mathrm{m}^{-1}$$

Step 2 — Cube:

$$\left(\frac{k_B T}{\hbar c}\right)^3 = (1.308 \times 10^5)^3 = 2.237 \times 10^{15}\,\mathrm{m}^{-3}$$

Step 3 — Photon density:

$$n_\gamma = \frac{2 \times 1.202}{\pi^2} \times 2.237 \times 10^{15} = \frac{2.404}{9.870} \times 2.237 \times 10^{15} = 0.2436 \times 2.237 \times 10^{15} = 5.45 \times 10^{14}\,\mathrm{m}^{-3}$$

$$\boxed{n_\gamma \approx 5.45 \times 10^{14}\,\mathrm{m}^{-3} \quad \Rightarrow \quad 5.45 \text{ (in units of } 10^{14})}$$

Note: At $T = 300\,\mathrm{K}$, thermal radiation peaks in the infrared ($\sim 10\,\mu\mathrm{m}$), and there are $\sim 5 \times 10^{14}$ photons per cubic metre.

Answer: (5.45) ← *Go back to Q60*

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	D	4	A	5	B
6	C	7	D	8	A	9	B	10	C
11	D	12	A	13	B	14	C	15	D
16	A	17	B	18	C	19	D	20	A
21	B	22	C	23	D	24	A	25	B
26	C	27	D	28	A	29	B	30	C
31	AB	32	ACD	33	BCD	34	AB	35	ACD
36	BCD	37	AB	38	ACD	39	BCD	40	AB
41	40.82	42	1.00	43	0.180	44	2.87	45	4.18
46	1.43	47	6.78	48	5.03	49	-12	50	1.00
51	65.0	52	121.50	53	5.76	54	1.81	55	5.88
56	322	57	0.177	58	0.5000	59	10.0	60	5.45