IIT JAM Physics

Sample Paper – 7

Instructions

- This paper contains **60 questions** carrying a maximum of **100 marks**. Duration: **3 hours (180 minutes)**.
- **Section A** (**Q1–Q30**, MCQ): Q1–Q10 carry **+1 mark** each; wrong response: $-\frac{1}{3}$ mark. Q11–Q30 carry **+2 marks** each; wrong response: $-\frac{2}{3}$ mark. Choose the **single best** option.
- **Section B** (**Q31–Q40**, MSQ): Each carries **+2 marks**. **No negative marking. One or more** options may be correct; full marks only if *all* correct options are selected; no partial credit.
- **Section C** (**Q41–Q60**, NAT): Q41–Q50 carry **+1 mark**; Q51–Q60 carry **+2 marks**. **No negative marking.** Enter the numerical value using the virtual keypad (integer or decimal as appropriate).
- **Calculator policy:** Personal calculators, log tables, and electronic devices are **strictly prohibited**. A virtual scientific calculator is provided in the CBT interface. Rough sheets are supplied at the examination centre.
- Once you move to the next section, you cannot return to a previous section. Manage time accordingly.

Section A — Multiple Choice Questions (MCQ)

***Q1–Q10**: 1 mark each ($-\frac{1}{3}$ for wrong). Choose the **single** correct option.*

Q1. For $f(z) = u(x, y)+iv(x, y)$ to be analytic at a point, the Cauchy–Riemann equations require:

(A) $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{\partial v}{\partial x}$

(B) $\dfrac{\partial u}{\partial x} = -\dfrac{\partial v}{\partial y}$ and $\dfrac{\partial u}{\partial y} = \dfrac{\partial v}{\partial x}$

(C) $\dfrac{\partial u}{\partial x} = \dfrac{\partial v}{\partial x}$ and $\dfrac{\partial u}{\partial y} = \dfrac{\partial v}{\partial y}$

(D) $\dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial y}$ and $\dfrac{\partial v}{\partial x} = \dfrac{\partial v}{\partial y}$

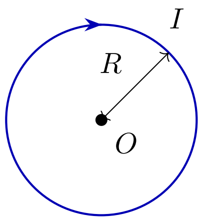
Q2. A bead of mass m slides without friction on a wire bent into the shape of a parabola $y = ax^2$ (in a vertical plane, y upward). Using the Euler–Lagrange equation with x as the generalised coordinate, the correct equation of motion is:

(A) $m\ddot{x} + 2mgax = 0$
(B) $m(1 + 4a^2x^2)\ddot{x} + 4ma^2x\dot{x}^2 + 2mgax = 0$
(C) $m\ddot{x} + mgax = 0$
(D) $m(1 + 4a^2x^2)\ddot{x} + 2mgax = 0$

Q3. A string of length L is fixed at both ends. The wavelength of the n-th harmonic is $\lambda_n = 2L/n$. The wavelength of the **fourth** harmonic is:

(A) L
(B) $L/4$
(C) $L/2$
(D) $2L$

Q4. A circular conducting loop of radius R carries a steady current I as shown. The magnitude of the magnetic field at the centre of the loop is:



(A) $\dfrac{\mu_0 I}{4\pi R}$
(B) $\dfrac{\mu_0 I}{4R}$
(C) $\dfrac{\mu_0 I}{\pi R}$
(D) $\dfrac{\mu_0 I}{2R}$

Q5. For molecules of an ideal gas in equilibrium at temperature T, the Maxwell–Boltzmann speed distribution gives: $v_p^* = \sqrt{2RT/M}$ (most probable speed) and $\langle v \rangle = \sqrt{8RT/(\pi M)}$ (mean speed). The ratio $\langle v \rangle / v_p^*$ equals:

(A) $\dfrac{2}{\sqrt{\pi}}$
(B) $\dfrac{\sqrt{2}}{2}$
(C) $\sqrt{\dfrac{2}{\pi}}$
(D) $\dfrac{\pi}{4}$

Q6. In the photoelectric effect, light of frequency ν is incident on a metal with work function $\phi = 2\,\text{eV}$. If $h\nu = 5\,\text{eV}$, the stopping potential V_s required to stop the most energetic photoelectrons is:

(A) $1\,\text{V}$
(B) $3\,\text{V}$
(C) $5\,\text{V}$
(D) $7\,\text{V}$

Q7. In a body-centred cubic (BCC) crystal structure, each atom is surrounded by nearest neighbours. The coordination number (number of nearest neighbours) of an atom in the BCC structure is:

(A) 6
(B) 12
(C) 8
(D) 4

Q8. A NAND gate has two inputs A and B. When both inputs are HIGH ($A = 1$, $B = 1$), the output $Y = \overline{A \cdot B}$ equals:

(A) $Y = 1$ for any input combination
(B) $Y = 1$ when $A = B = 1$
(C) $Y = 1$ when $A = 0,\ B = 0$
(D) $Y = 0$ when $A = B = 1$

Q9. In the alpha decay of ${}^{238}_{94}\text{Pu}$ (plutonium-238), an alpha particle ${}^{4}_{2}\text{He}$ is emitted. Which of the following correctly gives the mass number A and atomic number Z of the daughter nucleus?

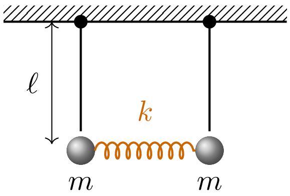
(A) $A = 234, \quad Z = 92$
(B) $A = 234, \quad Z = 90$
(C) $A = 236, \quad Z = 92$
(D) $A = 238, \quad Z = 90$

Q10. The larger eigenvalue of the matrix $\begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}$ is:

(A) 4
(B) 6
(C) 2
(D) 8

***Q11–Q30**: 2 marks each ($-\frac{2}{3}$ for wrong). Choose the **single** correct option.*

Q11. Two identical simple pendulums, each of mass m and length ℓ, are coupled by a light spring of spring constant k connecting their bobs (shown below). The angular frequency of the **higher** (out-of-phase) normal mode is:

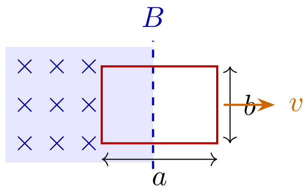


(A) $\sqrt{g/\ell}$
(B) $\sqrt{g/\ell + k/m}$
(C) $\sqrt{g/\ell + 2k/m}$
(D) $\sqrt{2g/\ell + k/m}$

Q12. A muon at rest has a mean lifetime $\tau_0 = 2.20\,\mu$s. A muon moving at $v = 0.80\,c$ has a Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$. In the laboratory frame the mean lifetime τ of this muon is approximately:

(A) $1.32\,\mu$s
(B) $2.20\,\mu$s
(C) $3.00\,\mu$s
(D) $3.67\,\mu$s

Q13. A rectangular conducting loop of width a (parallel to the field boundary) and height b is pulled with velocity v perpendicular to a uniform magnetic field B (directed into the page), as shown. When the loop is partly outside the field region, the magnitude of the induced EMF is:



(A) $|\mathcal{E}| = Bav$
(B) $|\mathcal{E}| = Bbv$
(C) $|\mathcal{E}| = Babv$
(D) $|\mathcal{E}| = Ba^2v/b$

Q14. The Helmholtz free energy is $A = U - TS$. Its differential is $dA = -S\,dT - P\,dV$. The Maxwell relation derived from this expression is:

(A) $\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial P}{\partial V}\right)_T$
(B) $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$
(C) $\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_V$
(D) $\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial V}{\partial T}\right)_P$

Q15. A particle of mass m is confined to a one-dimensional infinite square well of width a (potential $V = 0$ for $0 \le x \le a$, $V = \infty$ otherwise). The ground state energy E_1 is:

(A) $E_1 = \frac{\hbar^2}{2ma^2}$
(B) $E_1 = \frac{\pi\hbar^2}{2ma^2}$
(C) $E_1 = \frac{\pi^2\hbar^2}{2ma^2}$
(D) $E_1 = \frac{\pi^2\hbar^2}{ma^2}$

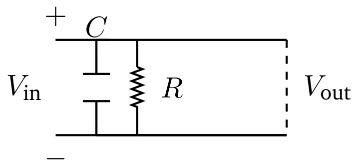
Q16. In the Debye model of lattice heat capacity, at high temperatures $T \gg \theta_D$ (Debye temperature), the molar heat capacity at constant volume C_V approaches which limiting value?

(A) $C_V \to 0$
(B) $C_V \to R$

(C) $C_V \to \frac{3}{2}R$

(D) $C_V \to 3R$ (Dulong–Pétit law)

Q17. An RC high-pass filter consists of a capacitor $C = 1\,\mu$F in series with a resistor $R = 10\,\text{k}\Omega$ (output taken across R), as shown. The $-3\,$dB cutoff frequency $f_c = 1/(2\pi RC)$ is approximately:



(A) $\approx 15.9\,$Hz

(B) $\approx 159\,$Hz

(C) $\approx 1.59\,$Hz

(D) $\approx 1592\,$Hz

Q18. In the semi-empirical Bethe–Weizsäcker mass formula for nuclei, the term that accounts for the fact that nucleons on the surface of a nucleus are less bound than those in the interior is proportional to:

(A) A (volume energy)

(B) $A^{2/3}$ (surface energy)

(C) $Z^2/A^{1/3}$ (Coulomb energy)

(D) $(A - 2Z)^2/A$ (asymmetry energy)

Q19. The Legendre polynomial of degree 2 is $P_2(x) = \frac{1}{2}(3x^2 - 1)$. The value of $P_2\left(\frac{1}{2}\right)$ is:

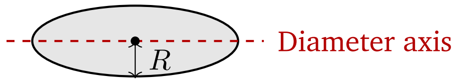
(A) $\frac{1}{4}$

(B) $\frac{1}{8}$

(C) $-\frac{1}{8}$

(D) $-\frac{1}{4}$

Q20. A thin uniform disk of mass M and radius R is shown below. Its moment of inertia about a diameter axis (lying in the plane of the disk, passing through the centre) is:

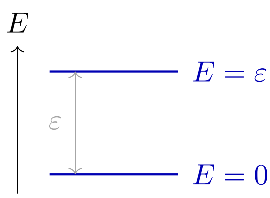


(A) MR^2
(B) $\frac{MR^2}{2}$
(C) $\frac{MR^2}{3}$
(D) $\frac{MR^2}{4}$

Q21. For a monochromatic plane electromagnetic wave propagating in vacuum, the amplitudes of the electric field E_0 and magnetic field B_0 are related by:

(A) $E_0/B_0 = c$
(B) $E_0/B_0 = c^2$
(C) $E_0/B_0 = 1/c$
(D) $E_0/B_0 = \varepsilon_0\mu_0$

Q22. A quantum two-level system has a ground state of energy 0 and an excited state of energy $\varepsilon > 0$ as shown. In thermal equilibrium at temperature T, the canonical partition function Z is:



(A) $Z = 2$
(B) $Z = 1 + e^{-\varepsilon/kT}$
(C) $Z = e^{-\varepsilon/kT}$
(D) $Z = 2\,e^{-\varepsilon/(2kT)}$

Q23. For an electron in the hydrogen atom with principal quantum number $n = 4$, the maximum orbital angular momentum quantum number is

$\ell_{\text{max}} = 3$. The magnitude of the maximum orbital angular momentum is $L_{\text{max}} = \hbar\sqrt{\ell(\ell+1)}\big|_{\ell=3}$. This equals:

(A) $4\hbar$
(B) $3\hbar$
(C) $2\sqrt{3}\,\hbar$
(D) $\sqrt{6}\,\hbar$

Q24. The Meissner–Ochsenfeld effect in a Type-I superconductor describes the behaviour of the magnetic field B inside the material below the critical temperature T_c and below the critical field B_c. Which statement correctly describes this effect?

(A) Below T_c, the DC resistance drops to half its normal-state value
(B) Below T_c, the magnetic field inside equals the applied external field
(C) The material becomes paramagnetic with susceptibility $\chi_m \to +\infty$ below T_c
(D) Below T_c, the magnetic field inside the superconductor is $B = 0$ (perfect diamagnetism)

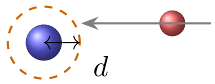
Q25. A non-inverting op-amp circuit has $R_f = 9\,\text{k}\Omega$ (feedback resistor) and $R_1 = 1\,\text{k}\Omega$ (to ground). For an input $V_{\text{in}} = 0.4\,\text{V}$, the output voltage $V_{\text{out}} = \big(1 + R_f/R_1\big)V_{\text{in}}$ equals:

(A) $4.0\,\text{V}$
(B) $3.6\,\text{V}$
(C) $40\,\text{V}$
(D) $9.0\,\text{V}$

Q26. Two angular momenta $\ell_1 = 1$ and $\ell_2 = 1$ are added quantum mechanically. The possible values of the total angular momentum quantum number j range from $|\ell_1 - \ell_2|$ to $\ell_1 + \ell_2$. The **maximum** value of j is:

(A) $j_{\text{max}} = 0$
(B) $j_{\text{max}} = 2$
(C) $j_{\text{max}} = 1$
(D) $j_{\text{max}} = 3$

Q27. The mean free path λ of molecules (modelled as hard spheres of diameter d) in a gas of number density n is derived using the concept of an effective collision cross-section, shown below. The correct expression for λ is:



(A) $\lambda = \dfrac{1}{\pi d^2 n}$

(B) $\lambda = \dfrac{1}{2\pi d^2 n}$

(C) $\lambda = \dfrac{1}{\sqrt{2}\,\pi d^2 n}$

(D) $\lambda = \dfrac{1}{4\pi d^2 n}$

Q28. In a velocity selector, a charged particle with charge q enters a region where a uniform electric field E (vertical) and a uniform magnetic field B (perpendicular to both E and the particle's velocity) act simultaneously. The particle travels in a straight line when:

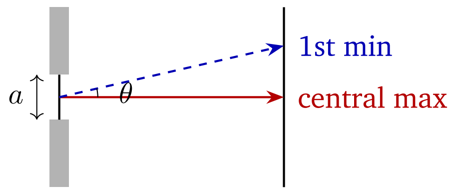
(A) $v = B/E$

(B) $v = qB/m$

(C) $v = E^2/B$

(D) $v = E/B$

Q29. In Fraunhofer (far-field) diffraction from a single slit of width a, the **first dark fringe** (first diffraction minimum) occurs at angle θ satisfying (refer to diagram):



(A) $\sin\theta = \lambda/a$

(B) $\sin\theta = 2\lambda/a$

(C) $\sin\theta = \lambda/(2a)$

(D) $\sin\theta = a\lambda$

Q30. An electron and a proton each have the same kinetic energy K. Since $\lambda = h/p = h/\sqrt{2mK}$, the ratio of their de Broglie wavelengths λ_e/λ_p (using $m_p/m_e \approx 1836$) is approximately:

(A) $\sqrt{m_e/m_p} \approx 0.023$
(B) $\sqrt{m_p/m_e} \approx 42.8$
(C) $m_p/m_e \approx 1836$
(D) 1

Section B — Multiple Select Questions (MSQ)

***Q31–Q40**: 2 marks each. **No negative marking. One or more** options may be correct. Full marks only if all correct options (and no incorrect options) are chosen.*

Q31. For the one-dimensional simple harmonic oscillator governed by $\ddot{x}+\omega^2 x = 0$, which of the following statements are **correct**?

(A) The general solution is $x(t) = A\cos(\omega t + \varphi)$, where A and φ are determined by initial conditions.
(B) The period $T = 2\pi/\omega$ depends on the amplitude A of oscillation.
(C) The angular frequency $\omega = \sqrt{k/m}$, where k is the spring constant.
(D) The total mechanical energy $E = \frac{1}{2}m\omega^2 A^2$ is constant in time.

Q32. Which of the following statements about electrostatic field lines are **correct**?

(A) Electric field lines originate from positive charges and terminate on negative charges.
(B) The local density of field lines (lines per unit area perpendicular to the field) is proportional to the magnitude of the electric field.
(C) In electrostatics, electric field lines can form closed loops.
(D) Electric field lines are always perpendicular to equipotential surfaces.

Q33. For a Hermitian operator $\hat{H}$ (satisfying $\hat{H} = \hat{H}^\dagger$), which of the following are **true**?

(A) The eigenvalues of $\hat{H}$ may be complex.
(B) The eigenvalues of $\hat{H}$ are real.

(C) $\hat{H}$ satisfies $\hat{H} = -\hat{H}^\dagger$ (anti-Hermitian condition).
(D) Eigenvectors corresponding to distinct eigenvalues of $\hat{H}$ are orthogonal.

Q34. Which of the following statements correctly describe the **Carnot cycle** operating between a hot reservoir at T_h and a cold reservoir at $T_c < T_h$?

(A) The cycle operates between two heat reservoirs at temperatures T_h and T_c.
(B) Every process in the Carnot cycle is either isothermal or adiabatic (reversible).
(C) The efficiency of the Carnot engine depends on the nature of the working substance.
(D) The maximum (Carnot) efficiency is $\eta = 1 - T_c/T_h$.

Q35. Which of the following statements correctly describe an n**-type semiconductor**?

(A) Donor impurity atoms donate free electrons to the conduction band.
(B) The Fermi level lies below the intrinsic Fermi level E_i (closer to the valence band).
(C) The majority carriers are electrons.
(D) Donor atoms have one more valence electron than the host semiconductor atoms.

Q36. Let $\tilde{f}(\omega) = \mathcal{F}\{f(t)\}$ denote the Fourier transform. Which of the following properties are **correct**?

(A) Time-shift: $\mathcal{F}\{f(t - t_0)\} = e^{-i\omega t_0}\,\tilde{f}(\omega)$
(B) Scaling: $\mathcal{F}\{f(at)\} = \dfrac{1}{|a|}\,\tilde{f}\left(\dfrac{\omega}{a}\right)$
(C) Differentiation: $\mathcal{F}\{f'(t)\} = i\omega\,\tilde{f}(\omega)$
(D) Second derivative: $\mathcal{F}\{f''(t)\} = \omega^2\,\tilde{f}(\omega)$ (no imaginary factor)

Q37. Which of the following statements about β^- **(beta-minus) decay** are **correct**?

(A) A neutron is converted to a proton inside the nucleus.

(B) An electron neutrino ν_e is emitted along with the electron.
(C) The atomic number Z of the nucleus increases by 1.
(D) The baryon number B is conserved in the process.

Q38. For **standing waves** set up on a string of length L fixed at both ends, which of the following are **correct**?

(A) The fundamental (first harmonic) wavelength is $\lambda_1 = 2L$.
(B) All integer harmonics are present: $f_n = nf_1$ for $n = 1, 2, 3, \ldots$
(C) At the nodes, the string displacement is zero at all times.
(D) At the antinodes, the transverse velocity of the string is zero at all times.

Q39. A logic gate is called **universal** if any Boolean function can be implemented using only that type of gate. Which of the following gates are universal?

(A) NAND gate
(B) NOR gate
(C) OR gate (alone)
(D) AND gate (alone)

Q40. For the quantum harmonic oscillator, the ground state $|0\rangle$ has wavefunction $\psi_0(x) \propto \exp\big(-m\omega x^2/(2\hbar)\big)$. Which of the following are **correct**?

(A) $\langle x \rangle = 0$
(B) $\langle p \rangle = 0$
(C) $\langle x^2 \rangle = 0$
(D) The wavefunction $\psi_0(x)$ is a Gaussian function of x.

Section C — Numerical Answer Type (NAT)

Q41–Q50**: 1 mark each.* ***No negative marking. *Enter the exact numerical value.*

Q41. A simple pendulum of length $L = 2.0\,\text{m}$ oscillates at a location where $g = 9.8\,\text{m/s}^2$. Calculate the time period T (in seconds) of small oscillations, correct to two decimal places. (Take $\pi = 3.14159$.)

Q42. An inductor of $L = 0.50\,\text{H}$ carries a current of $I = 4.0\,\text{A}$. Calculate the energy E (in joules) stored in the magnetic field of the inductor, correct to two decimal places.

Q43. A proton moves at speed $v = 2.0 \times 10^6\,\text{m/s}$. Calculate its de Broglie wavelength λ in picometres (pm $= 10^{-12}\,\text{m}$), correct to two decimal places. (Given: $h = 6.626 \times 10^{-34}\,\text{J s}$, $m_p = 1.67 \times 10^{-27}\,\text{kg}$.)

Q44. Calculate the root-mean-square speed v_{rms} (in m/s, to the nearest integer) of helium atoms (He, molar mass $M = 4.0\,\text{g/mol}$) at temperature $T = 500\,\text{K}$. (Given: $R = 8.314\,\text{J mol}^{-1}\text{K}^{-1}$.)

Q45. The radioactive nucleus ^{32}P (phosphorus-32) has a half-life of $T_{1/2} = 14.3\,\text{days}$. Calculate the decay constant λ in s^{-1}, correct to three significant figures. Express your answer in the form $a.bc \times 10^{-7}\,\text{s}^{-1}$. ($1\,\text{day} = 86400\,\text{s}$, $\ln 2 = 0.6931$.)

Q46. Calculate the atomic packing fraction (APF) of a body-centred cubic (BCC) crystal, given that atoms touch along the body diagonal of the unit cell. Express your answer correct to three decimal places. (Use $\pi = 3.14159$, $\sqrt{3} = 1.7321$.)

Q47. Calculate the **trace** (sum of diagonal elements) of the matrix $\begin{pmatrix} 2 & 3 \\ 3 & 7 \end{pmatrix}$.

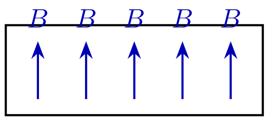
Q48. A series RLC circuit has inductance $L = 0.10\,\text{H}$ and capacitance $C = 100\,\mu\text{F} = 10^{-4}\,\text{F}$. Calculate the resonance frequency $f = 1/(2\pi\sqrt{LC})$ in hertz, to the nearest integer.

Q49. A non-inverting op-amp amplifier has $R_f = 9\,\text{k}\Omega$ and $R_1 = 1\,\text{k}\Omega$. For an input voltage $V_{\text{in}} = 0.30\,\text{V}$, find the output voltage V_{out} in volts. ($V_{\text{out}} = (1 + R_f/R_1)\,V_{\text{in}}$.)

Q50. A solid cylinder of mass $m = 2.0\,\text{kg}$ and radius $R = 0.05\,\text{m}$ rotates about its symmetry axis. Calculate the moment of inertia $I = \frac{1}{2}mR^2$ in kg m^2, correct to four decimal places.

***Q51–Q60**: 2 marks each. **No negative marking.** Enter the exact numerical value.*

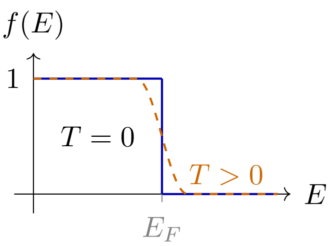
Q51. A long solenoid (cross-section shown) carries a current that produces a uniform magnetic field $B = 0.50\,\mathrm{T}$ inside it (in vacuum). Calculate the magnetic energy density $u = B^2/(2\mu_0)$ in J/m^3, correct to two decimal places. (Take $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\,m/A}$, $\pi = 3.14159$.)

solenoid 

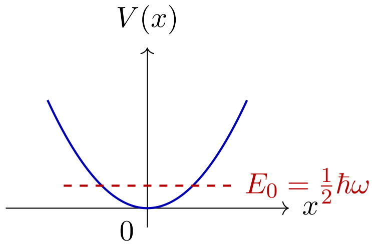
Q52. In the Bohr model of the hydrogen atom, the orbital period of the electron in level n is proportional to $T_n \propto n^3$. Calculate the ratio T_4/T_1 of the orbital period in the $n = 4$ state to that in the ground state ($n = 1$).

Q53. One mole of a monatomic ideal gas ($C_V = \frac{3}{2}R$) is heated at constant volume from $T_1 = 300\,\mathrm{K}$ to $T_2 = 900\,\mathrm{K}$. Calculate the entropy change ΔS (in J/K) of the gas, correct to two decimal places. ($R = 8.314\,\mathrm{J\,mol^{-1}K^{-1}}$, $\ln 3 = 1.0986$.)

Q54. Treat the conduction electrons of aluminium as a free electron gas with number density $n = 1.81 \times 10^{29}\,\mathrm{m^{-3}}$. Calculate the Fermi energy $E_F = \dfrac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$ in electronvolts, correct to two decimal places.
($\hbar = 1.055 \times 10^{-34}\,\mathrm{J\,s}$, $m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$, $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$, $3\pi^2 = 29.608$.)



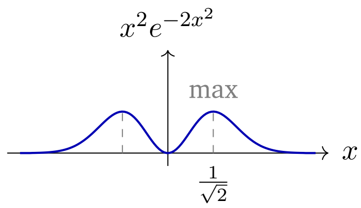
Q55. A quantum harmonic oscillator has angular frequency $\omega = 2.0\times10^{14}\,\mathrm{rad/s}$. Calculate the zero-point energy $E_0 = \frac{1}{2}\hbar\omega$ in electronvolts, correct to three decimal places.
($\hbar = 1.055 \times 10^{-34}\,\mathrm{J\,s}$, $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$.)



Q56. Sound waves of frequency $f = 1000\,\text{Hz}$ travel through air at $20°\text{C}$, where the speed of sound is $v_s = 343\,\text{m/s}$. Calculate the wavelength $\lambda = v_s/f$ in metres, correct to three decimal places.

Q57. A radioactive sample initially contains $N_0 = 4 \times 10^6$ nuclei. After exactly 5 half-lives, how many undecayed nuclei N remain? Express your answer as an integer.

Q58. Evaluate the definite integral $I = \int_{-\infty}^{\infty} x^2\, e^{-2x^2}\, dx$, correct to three decimal places. The curve $x^2 e^{-2x^2}$ is shown below.



Q59. An RL circuit has resistance $R = 50\,\Omega$ and inductance $L = 0.25\,\text{H}$. Calculate the time constant $\tau = L/R$ of the circuit in milliseconds (ms).

Q60. Calculate the energy (in eV, correct to two decimal places) of a photon with wavelength $\lambda = 300\,\text{nm}$. (Given: $h = 6.626 \times 10^{-34}\,\text{J s}$, $c = 3.0 \times 10^8\,\text{m/s}$, $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$.)

Solutions

IIT JAM Physics – Sample Paper 7

Section A Solutions — MCQ

Solution

Q1.

Concept — Cauchy–Riemann Equations:
A function $f(z) = u(x, y) + iv(x, y)$ is analytic (holomorphic) at a point if and only if the partial derivatives exist and satisfy both Cauchy–Riemann conditions simultaneously.

The two conditions are:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

These follow from the requirement that the complex derivative $\lim_{\Delta z \to 0} \Delta f/\Delta z$ is independent of the direction of approach.

Why other options are wrong:

- (B): Signs are reversed ($u_x = -v_y$ and $u_y = v_x$); this would represent an anti-analytic condition.
- (C): Equates $u_x = v_x$ and $u_y = v_y$; mixes same-axis partial derivatives, not a meaningful analyticity condition.
- (D): Equates $u_x = u_y$; this is a condition on u alone, unrelated to analyticity.

Final Answer: $u_x = v_y$ and $u_y = -v_x \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q1*

Solution

Concept — Euler–Lagrange Equation (Constrained Motion):
For the parabola $y = ax^2$, we have $\dot{y} = 2ax\dot{x}$.

Step 1 — Lagrangian:

$$T = \tfrac{1}{2}m(\dot{x}^2 + \dot{y}^2) = \tfrac{1}{2}m\dot{x}^2(1 + 4a^2x^2), \quad V = mgax^2$$

$$\mathcal{L} = \tfrac{1}{2}m\dot{x}^2(1 + 4a^2x^2) - mgax^2$$

Step 2 — Partial derivatives:

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}(1 + 4a^2x^2)$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} = m\ddot{x}(1 + 4a^2x^2) + 8ma^2x\dot{x}^2$$

$$\frac{\partial \mathcal{L}}{\partial x} = 4ma^2x\dot{x}^2 - 2mgax$$

Step 3 — Euler–Lagrange equation $\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}} - \frac{\partial \mathcal{L}}{\partial x} = 0$:

$$m(1 + 4a^2x^2)\ddot{x} + 8ma^2x\dot{x}^2 - 4ma^2x\dot{x}^2 + 2mgax = 0$$

$$\boxed{m(1 + 4a^2x^2)\ddot{x} + 4ma^2x\dot{x}^2 + 2mgax = 0}$$

Why other options are wrong:

Q2.

- (A): Ignores both the constraint correction to inertia $(1 + 4a^2x^2)$ and the $\dot{x}^2$ term.
- (C): Drops the factor of 2 in the potential gradient; also missing constraint corrections.
- (D): Includes constraint correction to inertia but drops the $4ma^2x\dot{x}^2$ kinetic term.

Final Answer: (B)

Answer: (B) ← *Go back to Q2*

Solution

Concept — Harmonics on a Fixed String:
For a string of length L fixed at both ends, standing wave condition: $L = n\lambda_n/2$, giving $\lambda_n = 2L/n$.

Step 1 — Fourth harmonic ($n = 4$):

$$\lambda_4 = \frac{2L}{4} = \frac{L}{2}$$

Why other options are wrong:

Q3.

- (A) L: This is λ_2 (second harmonic), not the fourth.
- (B) $L/4$: Would require $n = 8$ (eighth harmonic).
- (D) $2L$: This is λ_1 (fundamental mode, first harmonic).

Final Answer: $\lambda_4 = L/2 \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q3*

Solution

Concept — Magnetic Field at Centre of Circular Loop (Biot–Savart):
For a circular loop of radius R carrying current I, every element $d\ell$ of the loop is at distance R from the centre and perpendicular to the radial direction. By the Biot–Savart law:

$$dB = \frac{\mu_0 I}{4\pi} \frac{d\ell}{R^2}$$

Step 1 — Integrate around the full circle:

$$B = \frac{\mu_0 I}{4\pi R^2} \oint d\ell = \frac{\mu_0 I}{4\pi R^2} \cdot 2\pi R = \frac{\mu_0 I}{2R}$$

Why other options are wrong:

Q4.
- (A) $\mu_0 I/(4\pi R)$: Result for a quarter-circle arc, not a full loop.
- (B) $\mu_0 I/(4R)$: Result for a semicircular arc.
- (C) $\mu_0 I/(\pi R)$: Incorrect by a factor of $\pi/2$.

Final Answer: $B = \mu_0 I/(2R) \Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q4*

Solution

Concept — Maxwell–Boltzmann Speed Distribution:
The characteristic speeds are:

$$v_p^* = \sqrt{\frac{2RT}{M}}, \qquad \langle v \rangle = \sqrt{\frac{8RT}{\pi M}}$$

Step 1 — Compute the ratio:

$$\frac{\langle v \rangle}{v_p^*} = \frac{\sqrt{8RT/(\pi M)}}{\sqrt{2RT/M}} = \sqrt{\frac{8RT/(\pi M)}{2RT/M}} = \sqrt{\frac{8}{\pi \cdot 2}} = \sqrt{\frac{4}{\pi}} = \frac{2}{\sqrt{\pi}}$$

Numerically: $2/\sqrt{\pi} \approx 2/1.7725 \approx 1.128$.

Why other options are wrong:

Q5.
- (B) $\sqrt{2}/2 \approx 0.707$: This is $1/\sqrt{2}$; does not arise from these speed expressions.
- (C) $\sqrt{2/\pi} \approx 0.798$: This is $\sqrt{2}/\sqrt{\pi}$; misses the factor of 2 in $\langle v \rangle$.
- (D) $\pi/4 \approx 0.785$: No physical basis from these formulae.

Final Answer: $\langle v \rangle / v_p^* = 2/\sqrt{\pi} \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q5*

Solution

Concept — Einstein's Photoelectric Equation:
$eV_s = h\nu - \phi$, where V_s is the stopping potential, $h\nu$ is the photon energy, and ϕ is the work function.

Step 1 — Substitute:

$$eV_s = 5\,\text{eV} - 2\,\text{eV} = 3\,\text{eV} \quad \Rightarrow \quad V_s = 3\,\text{V}$$

Why other options are wrong:

Q6.

- (A) $1\,\text{V}$: Would result from $h\nu = 3\,\text{eV}$, $\phi = 2\,\text{eV}$ (incorrect photon energy).
- (C) $5\,\text{V}$: Ignores the work function ϕ; uses $eV_s = h\nu$ directly.
- (D) $7\,\text{V}$: Adds ϕ instead of subtracting it.

Final Answer: $V_s = 3\,\text{V} \Rightarrow$ **(B)**

$\boxed{\textbf{Answer: (B)}}$ ← *Go back to Q6*

Solution

Concept — BCC Crystal Coordination Number:
In the body-centred cubic (BCC) structure, the unit cell has one atom at each of the 8 corners and one atom at the body centre. The body-centre atom is equidistant from all 8 corner atoms (distance $= \sqrt{3}a/2$), so each atom has **8 nearest neighbours**.

Final Answer: Coordination number $= 8 \Rightarrow$ **(C)**

Why other options are wrong:

Q7.

- (A) 6: Coordination number of the simple cubic (SC) structure.
- (B) 12: Coordination number of FCC and HCP structures.
- (D) 4: Coordination number of the diamond cubic structure.

$\boxed{\textbf{Answer: (C)}}$ ← *Go back to Q7*

Solution

Concept — NAND Gate Logic:
The NAND gate output is $Y = \overline{A \cdot B}$ (NOT of AND).

Truth table evaluation for $A = B = 1$:

$$A \cdot B = 1 \cdot 1 = 1 \quad \Rightarrow \quad Y = \overline{1} = 0$$

Final Answer: $Y = 0$ when $A = B = 1 \Rightarrow$ **(D)**

Why other options are wrong:

Q8.

- (A) "$Y = 1$ for any input": False; NAND gives $Y = 0$ only when both inputs are 1.
- (B) "$Y = 1$ when $A = B = 1$": This would be the AND gate output, not NAND.
- (C) "$Y = 1$ when $A = 0, B = 0$": True but incomplete; NAND also gives $Y = 1$ for $(0, 1)$ and $(1, 0)$, and $Y = 0$ only for $(1, 1)$.

Answer: (D) ← *Go back to Q8*

Solution

Concept — Alpha Decay:
In alpha decay: ${}^{A}_{Z}\text{X} \to {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$.

Apply to ${}^{238}_{94}$Pu:

$$A_{\text{daughter}} = 238 - 4 = 234, \qquad Z_{\text{daughter}} = 94 - 2 = 92$$

The daughter nucleus is ${}^{234}_{92}$U (uranium-234).

Final Answer: $A = 234,\ Z = 92 \Rightarrow$ **(A)**

Why other options are wrong:

Q9.

- (B) $Z = 90$: Would result from losing 4 protons, not 2; this is not possible in a single alpha decay.
- (C) $A = 236$: Would require emitting only a dineutron (${}^{2}_{0}n$), not an alpha particle.
- (D) $A = 238$: Implies no change in mass number; not possible with alpha emission.

Answer: (A) ← *Go back to Q9*

Solution

Concept — Eigenvalues of a Symmetric 2×2 Matrix:
The characteristic equation is $\det(M - \lambda I) = 0$.

Step 1 — Compute determinant:

$$\det \begin{pmatrix} 4-\lambda & 2 \\ 2 & 4-\lambda \end{pmatrix} = (4-\lambda)^2 - 4 = 0$$

$$(4-\lambda)^2 = 4 \;\Rightarrow\; 4-\lambda = \pm 2$$

$$\lambda_1 = 4-2 = 2, \qquad \lambda_2 = 4+2 = 6$$

The larger eigenvalue is 6.

Final Answer: $\lambda_{\text{max}} = 6 \Rightarrow$ **(B)**

Why other options are wrong:

Q10.

- (A) 4: This is the diagonal element, not an eigenvalue.
- (C) 2: This is the smaller eigenvalue.
- (D) 8: Not a root of $(4-\lambda)^2 = 4$.

Answer: (B) ← *Go back to Q10*

Solution

Concept — Normal Modes of Coupled Pendulums:
For two identical pendulums (mass m, length ℓ) connected by a spring of constant k at their bobs, the equations of motion for small angles are:

$$m\ddot{x}_1 = -\frac{mg}{\ell}x_1 - k(x_1 - x_2), \qquad m\ddot{x}_2 = -\frac{mg}{\ell}x_2 + k(x_1 - x_2)$$

Step 1 — In-phase mode ($x_1 = x_2$):
Spring force cancels; $m\ddot{x} = -(mg/\ell)x \Rightarrow \omega_1 = \sqrt{g/\ell}$ (lower frequency).

Step 2 — Out-of-phase mode ($x_1 = -x_2$):
Spring stretches by $2x_1$; each bob feels an extra restoring force $-2kx_1$:

$$m\ddot{x}_1 = -\frac{mg}{\ell}x_1 - 2kx_1 \;\Rightarrow\; \omega_2 = \sqrt{\frac{g}{\ell} + \frac{2k}{m}}$$

This is the **higher** frequency mode.

Why other options are wrong:

Q11.

- (A) $\sqrt{g/\ell}$: This is ω_1, the *lower* (in-phase) mode frequency.
- (B) $\sqrt{g/\ell + k/m}$: Factor of k/m instead of $2k/m$; misses that each bob sees the full spring extension of $2x$.
- (D) $\sqrt{2g/\ell + k/m}$: Doubles the gravitational restoring term, which is incorrect.

Final Answer: $\omega_2 = \sqrt{g/\ell + 2k/m} \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q11*

Q12.

Solution

Concept — Special Relativistic Time Dilation:
A moving clock runs slow: $\tau = \gamma\tau_0$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$.

Step 1 — Calculate γ for $v = 0.80\,c$:

$$\gamma = \frac{1}{\sqrt{1-(0.80)^2}} = \frac{1}{\sqrt{1-0.64}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.60} = \frac{5}{3}$$

Step 2 — Laboratory frame lifetime:

$$\tau = \gamma\tau_0 = \frac{5}{3} \times 2.20\,\mu\text{s} = \frac{11.0}{3}\,\mu\text{s} \approx 3.67\,\mu\text{s}$$

Why other options are wrong:

- (A) $1.32\,\mu$s: Uses $\tau = \tau_0/\gamma$ (time contraction — incorrect concept for lab frame).
- (B) $2.20\,\mu$s: No time dilation applied; correct only in the muon's rest frame.
- (C) $3.00\,\mu$s: Uses $\gamma = 1/\sqrt{1-0.8} \approx 1.36$ (wrong: should subtract v^2/c^2, not v/c).

Final Answer: $\tau \approx 3.67\,\mu\text{s} \Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q12*

Q13.

Solution

Concept — Faraday's Law of Electromagnetic Induction:
The induced EMF equals the rate of change of magnetic flux: $|\mathcal{E}| = |d\Phi/dt|$.

Step 1 — Flux through the loop:
When the loop is partly in the field region (width a along the field boundary, height b), and the length still inside the field is x (decreasing), the flux is $\Phi = Bax$.

Step 2 — Rate of change:

$$|\mathcal{E}| = \left|\frac{d\Phi}{dt}\right| = Ba\left|\frac{dx}{dt}\right| = Bav$$

(The boundary cuts across the width a of the loop as it moves out with speed v.)

Why other options are wrong:

- (B) Bbv: Uses the height b instead of the width a; the cutting conductor has length a, not b.
- (C) $Babv$: Has dimensions of $\text{T} \cdot \text{m}^2 \cdot \text{s}^{-1}$; an extra factor of area is wrong.
- (D) Ba^2v/b: Dimensionally incorrect combination.

Final Answer: $|\mathcal{E}| = Bav \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q13*

Solution

Concept — Maxwell Relations from Thermodynamic Potentials:
The Helmholtz free energy $A = U - TS$ has differential $dA = -S\,dT - P\,dV$.

Apply the equality of mixed second partial derivatives:

$$\frac{\partial^2 A}{\partial V\,\partial T} = \frac{\partial^2 A}{\partial T\,\partial V}$$

$$\left.\frac{\partial(-S)}{\partial V}\right|_T = \left.\frac{\partial(-P)}{\partial T}\right|_V$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$$

Why other options are wrong:

Q14.

- (A): Mixes T and V derivatives on the left-hand side; not a Maxwell relation from A.
- (C): This is the Maxwell relation from the internal energy U (with S and V as natural variables).
- (D): Comes from the Gibbs free energy G (natural variables T and P), not from A.

Final Answer: $(\partial S/\partial V)_T = (\partial P/\partial T)_V \Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q14*

Solution

Concept — Infinite Square Well (Particle in a Box):
Inside the well ($0 \leq x \leq a$) the Schrödinger equation gives $\psi_n(x) = \sqrt{2/a}\,\sin(n\pi x/a)$. The energy levels are:

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

Ground state ($n = 1$):

$$E_1 = \frac{\pi^2\hbar^2}{2ma^2}$$

Why other options are wrong:

Q15.

- (A) $\hbar^2/(2ma^2)$: Missing the factor $\pi^2 \approx 9.87$; off by almost an order of magnitude.
- (B) $\pi\hbar^2/(2ma^2)$: Has π instead of π^2; arises from forgetting to square the wavenumber.
- (D) $\pi^2\hbar^2/(ma^2)$: Twice the correct answer; would correspond to $n^2 = 2$ (not an integer).

Final Answer: $E_1 = \pi^2\hbar^2/(2ma^2) \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q15*

Solution

Concept — Debye Model, High-Temperature Limit:
In the Debye model, the molar heat capacity at constant volume is:

$$C_V = 9R\left(\frac{T}{\theta_D}\right)^3 \int_0^{\theta_D/T} \frac{x^4 e^x}{(e^x - 1)^2} dx$$

At high temperature $T \gg \theta_D$:
The Debye integral approaches $1/3$ and the result reduces to the classical equipartition value:

$$C_V \to 3R \approx 24.9\,\mathrm{J\,mol^{-1}K^{-1}}$$

This is the Dulong–Pétit law: 3 degrees of freedom per atom, each contributing $\frac{1}{2}k_BT$ to both kinetic and potential energy, giving $3k_B = 3R/N_A$ per atom.

Final Answer: $C_V \to 3R$ (Dulong–Pétit) $\Rightarrow$ **(D)**

Why other options are wrong:

Q16.

- (A) $C_V \to 0$: This is the $T \to 0$ limit (Debye T^3 law), not the high-T limit.
- (B) $C_V \to R$: One-third of the correct classical value; would correspond to 1 degree of freedom.
- (C) $C_V \to \frac{3}{2}R$: The ideal monatomic gas translational heat capacity; not applicable to a solid lattice.

Answer: (D) ← *Go back to Q16*

Solution

Concept — RC High-Pass Filter Cutoff Frequency:
For an RC high-pass filter, the $-3\,$dB cutoff frequency is where the capacitive reac-

tance equals the resistance:

$$f_c = \frac{1}{2\pi RC}$$

Step 1 — Substitute values:

$$f_c = \frac{1}{2\pi \times 10^4\,\Omega \times 10^{-6}\,\mathrm{F}} = \frac{1}{2\pi \times 10^{-2}} = \frac{100}{2\pi} = \frac{50}{\pi}$$

$$f_c = \frac{50}{3.14159} \approx 15.92\,\mathrm{Hz}$$

Final Answer: $f_c \approx 15.9\,\mathrm{Hz} \Rightarrow$ **(A)**

Why other options are wrong:

Q17.

- (B) $159\,\mathrm{Hz}$: Factor of 10 error; would arise from $C = 0.1\,\mu\mathrm{F}$ (not $1\,\mu\mathrm{F}$).
- (C) $1.59\,\mathrm{Hz}$: Factor of 10 too small; would require $C = 10\,\mu\mathrm{F}$.
- (D) $1592\,\mathrm{Hz}$: Factor of 100 error; would arise from $C = 10\,\mathrm{nF}$.

Answer: (A) ← *Go back to Q17*

Solution

Concept — Bethe–Weizsäcker Semi-Empirical Mass Formula:
The binding energy is:

$$B(A, Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} \pm \delta$$

The **surface energy term** $(-a_S A^{2/3})$ accounts for nucleons on the nuclear surface, which have fewer neighbours and are therefore less tightly bound. Since the nuclear radius $R \propto A^{1/3}$, the surface area $\propto R^2 \propto A^{2/3}$.

Final Answer: Surface energy term $\propto A^{2/3} \Rightarrow$ **(B)**

Why other options are wrong:

Q18.

- (A) A: This is the volume energy term; proportional to the number of nucleons.
- (C) $Z^2/A^{1/3}$: This is the Coulomb (electrostatic repulsion) energy term.
- (D) $(A-2Z)^2/A$: This is the asymmetry (isospin) energy term, penalising neutron-proton imbalance.

Answer: (B) ← *Go back to Q18*

Q19.

Solution

Concept — Legendre Polynomials:
$P_2(x) = \frac{1}{2}(3x^2 - 1)$.

Evaluate at $x = 1/2$:

$$P_2\left(\frac{1}{2}\right) = \frac{1}{2}\left(3 \cdot \frac{1}{4} - 1\right) = \frac{1}{2}\left(\frac{3}{4} - 1\right) = \frac{1}{2}\left(-\frac{1}{4}\right) = -\frac{1}{8}$$

Final Answer: $P_2(1/2) = -1/8 \Rightarrow$ **(C)**

Why other options are wrong:

- (A) $1/4$: Forgetting the overall $1/2$ factor and using $3x^2 - 1 = -1/4$.
- (B) $1/8$: Correct magnitude but wrong sign; $3(1/4) - 1 < 0$.
- (D) $-1/4$: Missing the outer factor of $1/2$ (using $3x^2 - 1$ directly).

Answer: (C) ← *Go back to Q19*

Q20.

Solution

Concept — Moment of Inertia of a Disk about a Diameter:
For a thin uniform disk of mass M and radius R:

- About symmetry axis (perpendicular to disk, through centre): $I_z = \frac{1}{2}MR^2$
- About any diameter (in the plane of the disk, through centre): $I_x = I_y = I_{\text{diam}}$

Step 1 — Apply the perpendicular axis theorem:
For a flat body in the xy-plane: $I_z = I_x + I_y$.

By symmetry, $I_x = I_y$, so:

$$I_{\text{diam}} = I_x = \frac{I_z}{2} = \frac{MR^2/2}{2} = \frac{MR^2}{4}$$

Final Answer: $I_{\text{diam}} = MR^2/4 \Rightarrow$ **(D)**

Why other options are wrong:

- (A) MR^2: Moment of a thin ring (all mass at rim) about a diameter; too large.
- (B) $MR^2/2$: Moment of inertia of the disk about its *symmetry axis* (perpendicular to disk).
- (C) $MR^2/3$: Moment of inertia of a *rod* about its end; not a disk.

Answer: (D) ← *Go back to Q20*

Q21.

Solution

Concept — Electromagnetic Wave in Vacuum:
For a plane EM wave $\mathbf{E} = E_0\hat{e}_x \cos(kz - \omega t)$, Maxwell's equations (specifically Faraday's law) give:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \Rightarrow kE_0 = \omega B_0$$

$$\frac{E_0}{B_0} = \frac{\omega}{k} = c$$

Final Answer: $E_0/B_0 = c \Rightarrow$ **(A)**

Why other options are wrong:

- (B) c^2: Incorrect by a factor of c; confuses the relation $E/B = c$ with $c^2 = 1/(\mu_0\varepsilon_0)$.
- (C) $1/c$: Reciprocal; this would be B_0/E_0.
- (D) $\varepsilon_0\mu_0$: This equals $1/c^2$, not c.

Answer: (A) ← Go back to Q21

Q22.

Solution

Concept — Canonical Partition Function of a Two-Level System:
For a system with discrete energy levels, the canonical partition function is $Z = \sum_i e^{-E_i/kT}$.

For energies $E_1 = 0$ and $E_2 = \varepsilon$:

$$Z = e^{-0/kT} + e^{-\varepsilon/kT} = 1 + e^{-\varepsilon/kT}$$

The average energy is then $\langle E\rangle = \varepsilon/(e^{\varepsilon/kT} + 1)$.

Final Answer: $Z = 1 + e^{-\varepsilon/kT} \Rightarrow$ **(B)**

Why other options are wrong:

- (A) $Z = 2$: Only valid at $T \to \infty$ (infinite temperature limit, equal population).
- (C) $Z = e^{-\varepsilon/kT}$: Includes only the excited state; omits the ground-state contribution of $e^0 = 1$.
- (D) $Z = 2e^{-\varepsilon/(2kT)}$: Not the correct form for a two-level system; would apply to a different energy parametrisation.

Answer: (B) ← Go back to Q22

Solution

Concept — Orbital Angular Momentum in the Hydrogen Atom:
For principal quantum number n, the orbital angular momentum quantum number ranges $\ell = 0, 1, \ldots, n-1$. The magnitude of orbital angular momentum is $L = \hbar\sqrt{\ell(\ell+1)}$.

For $n = 4$**:**
$\ell_{\text{max}} = n - 1 = 3$.

$$L_{\text{max}} = \hbar\sqrt{3(3+1)} = \hbar\sqrt{12} = \hbar \cdot 2\sqrt{3}$$

Final Answer: $L_{\text{max}} = 2\sqrt{3}\,\hbar \Rightarrow$ **(C)**

Why other options are wrong:

Q23.
- (A) $4\hbar$: Uses $L = n\hbar$ (old Bohr model, not quantum mechanics); $L = 4\hbar$ would require $\ell(\ell+1) = 16$, so $\ell \approx 3.56$ (not an integer).
- (B) $3\hbar$: Uses $L = \ell\hbar$ instead of the correct $L = \hbar\sqrt{\ell(\ell+1)}$.
- (D) $\sqrt{6}\,\hbar$: Corresponds to $\ell = 2$ ($L = \hbar\sqrt{6}$); not the maximum for $n = 4$.

Answer: (C) ← *Go back to Q23*

Solution

Concept — Meissner–Ochsenfeld Effect:
The Meissner effect is the expulsion of magnetic flux from the interior of a material as it transitions to the superconducting state. This is distinct from merely having perfect conductivity.

Key result: Below T_c and for an applied field $B < B_c$, the interior of a Type-I superconductor has:

$$B_{\text{interior}} = 0$$

This requires surface currents (Meissner currents) that exactly cancel the applied field inside. The magnetic susceptibility $\chi_m = -1$ (perfect diamagnet).

Final Answer: $B = 0$ inside (perfect diamagnetism) $\Rightarrow$ **(D)**

Why other options are wrong:

Q24.
- (A): DC resistance is identically *zero* below T_c, not halved.
- (B): $B = B_{\text{applied}}$ inside would describe a normal metal; superconductors expel flux.
- (C): $\chi_m \to -1$ (diamagnetic), not $+\infty$ (paramagnetic or ferromagnetic).

Answer: (D) ← *Go back to Q24*

Solution

Concept — Non-Inverting Op-Amp Voltage Gain:
For an ideal non-inverting amplifier with feedback resistor R_f and grounding resistor R_1:

$$V_{\text{out}} = \left(1 + \frac{R_f}{R_1}\right) V_{\text{in}}$$

Step 1 — Compute gain:

$$G = 1 + \frac{R_f}{R_1} = 1 + \frac{9\,\text{k}\Omega}{1\,\text{k}\Omega} = 10$$

Step 2 — Output voltage:

$$V_{\text{out}} = 10 \times 0.4\,\text{V} = 4.0\,\text{V}$$

Final Answer: $V_{\text{out}} = 4.0\,\text{V} \Rightarrow$ **(A)**

Why other options are wrong:

Q25.

- (B) $3.6\,\text{V}$: Uses $G = R_f/R_1 = 9$ (forgetting the $+1$), then $9 \times 0.4 = 3.6$.
- (C) $40\,\text{V}$: Uses gain of 100; possible confusion with $R_f/R_1 = 100$ or other error.
- (D) $9.0\,\text{V}$: Applies gain $= 9$ to $V_{\text{in}} = 1\,\text{V}$ (wrong input) or similar arithmetic error.

Answer: (A) ← *Go back to Q25*

Solution

Concept — Addition of Angular Momenta:
When two angular momenta ℓ_1 and ℓ_2 are coupled, the total angular momentum j takes values:

$$j = |\ell_1 - \ell_2|,\ |\ell_1 - \ell_2| + 1,\ \ldots,\ \ell_1 + \ell_2$$

For $\ell_1 = \ell_2 = 1$:

$$j = 0,\ 1,\ 2 \qquad \Rightarrow \qquad j_{\text{max}} = \ell_1 + \ell_2 = 1 + 1 = 2$$

Final Answer: $j_{\text{max}} = 2 \Rightarrow$ **(B)**

Why other options are wrong:

Q26.

- (A) $j = 0$: This is the minimum value, $|1 - 1| = 0$; not the maximum.
- (C) $j = 1$: An intermediate value; not the maximum.
- (D) $j = 3$: Exceeds $\ell_1 + \ell_2 = 2$; not allowed by the Clebsch–Gordan rules.

Answer: (B) ← *Go back to Q26*

Solution

Concept — Mean Free Path of Hard-Sphere Gas:
A molecule of diameter d sweeps out a cylinder of effective collision cross-section $\sigma = \pi d^2$ per unit length. Accounting for the relative speed between identical moving molecules introduces a factor of $\sqrt{2}$:

$$\lambda = \frac{1}{\sqrt{2}\,\pi d^2 n}$$

where n is the number density (molecules per m^3).

Final Answer: $\lambda = 1/(\sqrt{2}\,\pi d^2 n) \Rightarrow$ **(C)**

Why other options are wrong:

Q27.

- (A) $1/(\pi d^2 n)$: Missing the factor $\sqrt{2}$; treats the target molecule as stationary while field molecules move (wrong for identical molecules).
- (B) $1/(2\pi d^2 n)$: Has the wrong numerical prefactor.
- (D) $1/(4\pi d^2 n)$: Further off; not the correct classical result.

Answer: (C) ← *Go back to Q27*

Solution

Concept — Velocity Selector (Crossed E and B Fields):
A charged particle (q, m) moving with velocity v in a region with electric field E (say, upward) and magnetic field B (perpendicular to both E and v) experiences forces:

$$F_E = qE \quad \text{(upward)}, \qquad F_B = qvB \quad \text{(downward, for appropriate field directions)}$$

Condition for straight-line travel (no deflection):

$$qE = qvB \;\Rightarrow\; v = \frac{E}{B}$$

This condition is independent of q and m, so only particles with speed $v = E/B$ pass through undeflected.

Final Answer: $v = E/B \Rightarrow$ **(D)**

Why other options are wrong:

Q28.

- (A) $v = B/E$: Reciprocal; dimensionally incorrect for velocity.
- (B) $v = qB/m$: This is the cyclotron frequency formula $\omega_c = qB/m$, not a

velocity.

- (C) $v = E^2/B$: Dimensionally inconsistent (has units of V/T = m/s only if E is already in m/s — nonsensical).

Answer: (D) ← *Go back to Q28*

Q29.

Solution

Concept — Single-Slit Fraunhofer Diffraction:
For a slit of width a, the intensity pattern has minima (dark fringes) where the path difference from the top and bottom of the slit equals an integer number of wavelengths. For the **first** minimum:

Step 1 — Condition for first dark fringe:
The slit can be divided into two halves; each pair of corresponding points contributes destructively when:

$$a \sin\theta = \lambda \;\Rightarrow\; \sin\theta = \frac{\lambda}{a}$$

Final Answer: $\sin\theta = \lambda/a \Rightarrow$ **(A)**

Why other options are wrong:

- (B) $\sin\theta = 2\lambda/a$: This is the *second* minimum (second dark fringe, $m = 2$).
- (C) $\sin\theta = \lambda/(2a)$: This would give a maximum, not a minimum; half the first-minimum angle.
- (D) $\sin\theta = a\lambda$: Dimensionally wrong (product of two lengths); not physical.

Answer: (A) ← *Go back to Q29*

Solution

Concept — de Broglie Wavelength and Kinetic Energy:
For a particle of mass m with kinetic energy K: $p = \sqrt{2mK}$, so $\lambda = h/p = h/\sqrt{2mK}$.

Step 1 — Ratio at same kinetic energy K:

$$\frac{\lambda_e}{\lambda_p} = \frac{h/\sqrt{2m_eK}}{h/\sqrt{2m_pK}} = \sqrt{\frac{m_p}{m_e}}$$

Step 2 — Numerical value:

$$\frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} = \sqrt{1836} \approx 42.8$$

Q30.

The electron has a much larger de Broglie wavelength because it is much lighter.

Final Answer: $\lambda_e/\lambda_p = \sqrt{m_p/m_e} \approx 42.8 \Rightarrow$ **(B)**

Why other options are wrong:

- (A) $\sqrt{m_e/m_p} \approx 0.023$: This is λ_p/λ_e, the reciprocal ratio.
- (C) $m_p/m_e \approx 1836$: This would be the ratio if $\lambda \propto 1/m$ (equal momenta), not equal kinetic energies.
- (D) 1: Would hold only if $m_e = m_p$ (equal masses), which is false.

Answer: (B) ← *Go back to Q30*

Section B Solutions — MSQ

Q31.

Solution

Concept — Simple Harmonic Oscillator:

(A) "General solution $x(t) = A\cos(\omega t + \varphi)$": CORRECT. The SHO ODE $\ddot{x} + \omega^2 x = 0$ has general solution $A\cos(\omega t + \varphi)$ (equivalently $C_1 \cos\omega t + C_2 \sin\omega t$), with two free constants determined by initial conditions.

(B) "Period T depends on amplitude": WRONG. The isochronism of the SHO is one of its defining properties: $T = 2\pi/\omega = 2\pi\sqrt{m/k}$ is independent of A. Amplitude-dependent periods appear in nonlinear oscillators (e.g. a pendulum at large angles).

(C) "$\omega = \sqrt{k/m}$": CORRECT. For a spring-mass system, the force is $F = -kx$, giving $m\ddot{x} = -kx$, i.e. $\ddot{x} + (k/m)x = 0$, confirming $\omega^2 = k/m$.

(D) "Total energy $E = \frac{1}{2}m\omega^2 A^2$ is constant": CORRECT. The total mechanical energy (kinetic + potential) is conserved: $E = \frac{1}{2}m\omega^2 A^2$. This can also be written as $\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q31*

Q32.

Solution

Concept — Properties of Electrostatic Field Lines:

(A) "Field lines originate from + charges and terminate on − charges": CORRECT. This follows from Gauss's law: $\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$; positive charges are sources and negative charges are sinks.

(B) "Density of field lines $\propto$ magnitude of E": CORRECT. By construction of field-line diagrams, the number of lines per unit area (perpendicular cross-section) represents the field magnitude. Near a strong charge the lines are denser.

(C) "Electrostatic field lines can form closed loops": WRONG. In electrostatics, $\nabla \times \mathbf{E} = 0$ (Faraday's law with $\partial \mathbf{B}/\partial t = 0$); the field is conservative and field lines cannot close on themselves. (Closed electric field lines appear only in time-varying magnetic fields.)

(D) "Field lines are perpendicular to equipotential surfaces": CORRECT. Since $\mathbf{E} = -\nabla V$, the field points in the direction of steepest descent of V, which is always perpendicular to surfaces of constant V.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q32*

Q33.

Solution

Concept — Properties of Hermitian Operators:

(A) "Eigenvalues may be complex": WRONG. For a Hermitian operator $\hat{H} = \hat{H}^\dagger$, if $\hat{H}|\psi\rangle = \lambda|\psi\rangle$, then $\lambda^* = \lambda$, so all eigenvalues are real. This is the spectral theorem for Hermitian operators.

(B) "Eigenvalues are real": CORRECT. Proof: $\lambda\langle\psi|\psi\rangle = \langle\psi|\hat{H}|\psi\rangle = \langle\hat{H}^\dagger\psi|\psi\rangle = \lambda^*\langle\psi|\psi\rangle$. Since $\langle\psi|\psi\rangle \neq 0$, we get $\lambda = \lambda^*$.

(C) "$\hat{H} = -\hat{H}^\dagger$": WRONG. This defines an *anti-Hermitian* operator. A Hermitian operator satisfies $\hat{H} = +\hat{H}^\dagger$. Anti-Hermitian operators have purely imaginary eigenvalues.

(D) "Eigenvectors for distinct eigenvalues are orthogonal": CORRECT. If $\hat{H}|\psi_1\rangle = \lambda_1|\psi_1\rangle$ and $\hat{H}|\psi_2\rangle = \lambda_2|\psi_2\rangle$ with $\lambda_1 \neq \lambda_2$, then $(\lambda_1 - \lambda_2)\langle\psi_1|\psi_2\rangle = 0 \Rightarrow \langle\psi_1|\psi_2\rangle = 0$.

Correct options: (B) and (D).

Answer: (BD) ← *Go back to Q33*

Q34.

Solution

Concept — Carnot Cycle:

(A) "Operates between two heat reservoirs": CORRECT. The Carnot engine absorbs heat Q_h from the hot reservoir at T_h and rejects heat Q_c to the cold reservoir at T_c. This two-reservoir structure is the defining feature.

(B) "All processes are isothermal or adiabatic": CORRECT. The Carnot cycle consists of: (1) isothermal expansion at T_h, (2) reversible adiabatic expansion, (3) isothermal compression at T_c, (4) reversible adiabatic compression. All four processes are reversible.

(C) "Efficiency depends on the working substance": WRONG. A fundamental result of thermodynamics (Carnot's theorem) is that all reversible engines operating between the same two temperatures have the *same* efficiency $\eta_C = 1 - T_c/T_h$, regardless of the working fluid.

(D) "Maximum efficiency $\eta = 1 - T_c/T_h$": CORRECT. Since $Q_h/T_h = Q_c/T_c$ for a reversible cycle, $\eta = W/Q_h = (Q_h - Q_c)/Q_h = 1 - Q_c/Q_h = 1 - T_c/T_h$.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q34*

Q35.

Solution

Concept — n-Type Semiconductors:

(A) "Donor impurities donate free electrons": CORRECT. Donor atoms (e.g., phosphorus in silicon) have one extra valence electron compared to the host. This electron is weakly bound and is easily thermally excited into the conduction band, contributing to free-electron conduction.

(B) "Fermi level below intrinsic E_i": WRONG. In an n-type semiconductor, the Fermi level E_F lies *above* the intrinsic Fermi level E_i, shifted toward the conduction band, reflecting the excess electron concentration. (In p-type, $E_F < E_i$.)

(C) "Majority carriers are electrons": CORRECT. By definition, the majority carriers in an n-type semiconductor are electrons (conduction band), while holes are the minority carriers.

(D) "Donor atoms have one more valence electron than host atoms": CORRECT. For example, Si (Group IV) is doped with P (Group V); phosphorus has 5 valence electrons vs. silicon's 4. The extra electron is the donated carrier.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q35*

Q36.

Solution

Concept — Properties of the Fourier Transform:

(A) Time shift $\mathcal{F}\{f(t-t_0)\} = e^{-i\omega t_0}\tilde{f}(\omega)$: CORRECT.
$\int_{-\infty}^{\infty} f(t-t_0)e^{-i\omega t}dt$. Let $u = t - t_0$: $= e^{-i\omega t_0}\int f(u)e^{-i\omega u}du = e^{-i\omega t_0}\tilde{f}(\omega)$.

(B) Scaling $\mathcal{F}\{f(at)\} = \frac{1}{|a|}\tilde{f}(\omega/a)$: CORRECT.
Let $u = at$: integral becomes $(1/|a|)\int f(u)e^{-i(\omega/a)u}du = \tilde{f}(\omega/a)/|a|$.

(C) Differentiation $\mathcal{F}\{f'(t)\} = i\omega\tilde{f}(\omega)$: CORRECT.
Integrate by parts: $\int f'(t)e^{-i\omega t}dt = [f(t)e^{-i\omega t}]_{-\infty}^{\infty} + i\omega\int f(t)e^{-i\omega t}dt = i\omega\tilde{f}(\omega)$ (boundary terms vanish for L^2 functions).

(D) Second derivative $\mathcal{F}\{f''(t)\} = \omega^2\tilde{f}(\omega)$: WRONG.
Applying the differentiation rule twice: $\mathcal{F}\{f''(t)\} = (i\omega)^2\tilde{f}(\omega) = -\omega^2\tilde{f}(\omega)$. The correct result has a negative sign (and no imaginary factor).

Correct options: (A), (B), and (C).

Answer: (ABC) ← *Go back to Q36*

Q37.

Solution

Concept — Beta-Minus Decay:
In β^- decay: ${}^{A}_{Z}\mathrm{X} \to {}^{A}_{Z+1}\mathrm{Y} + e^- + \bar{\nu}_e$. A neutron converts to a proton inside the nucleus.

(A) "A neutron converts to a proton": CORRECT. The fundamental process at the quark level is $d \to u + e^- + \bar{\nu}_e$, which at the nucleon level gives $n \to p + e^- + \bar{\nu}_e$.

(B) "An electron neutrino ν_e is emitted": WRONG. In β^- decay, an *electron antineutrino* $\bar{\nu}_e$ is emitted (not ν_e). This is required by lepton number conservation: electron number $+1$ (for e^-) must be balanced by antineutrino number -1 (for $\bar{\nu}_e$). A neutrino ν_e is emitted in β^+ decay (or electron capture).

(C) "Atomic number Z increases by 1": CORRECT. One neutron becomes a proton, so $Z \to Z+1$ while A remains unchanged.

(D) "Baryon number is conserved": CORRECT. The neutron (baryon number $B = 1$) becomes a proton ($B = 1$), an electron ($B = 0$), and an antineutrino ($B = 0$). Total baryon number is conserved.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q37*

Q38.

Solution

Concept — Standing Waves on a Fixed String:

(A) "Fundamental wavelength $\lambda_1 = 2L$": CORRECT. For a string fixed at both ends, the standing wave condition is $L = n\lambda_n/2$. For $n = 1$: $\lambda_1 = 2L$.

(B) "All integer harmonics $f_n = nf_1$": CORRECT. Since $f_n = nv/(2L) = nf_1$ for $n = 1, 2, 3, \ldots$, all integer multiples of the fundamental frequency are present (both odd and even harmonics).

(C) "Nodes have zero displacement at all times": CORRECT. Nodes are positions where the amplitude of the standing wave is zero. The string at a node never moves; its displacement is identically zero for all t.

(D) "At antinodes, string velocity is always zero": WRONG. At antinodes, the *displacement* amplitude is maximum, but the *velocity* is not always zero. The velocity $\partial y/\partial t$ at an antinode oscillates between maximum and zero (it is zero only at the instants of maximum displacement, and maximum at the instants of zero displacement). The string does move through its equilibrium position at antinodes.

Correct options: (A), (B), and (C).

Answer: (ABC) ← *Go back to Q38*

Solution

Concept — Universal Logic Gates:
A gate is **universal** if any Boolean function (AND, OR, NOT, and hence all combinational circuits) can be implemented using only that gate type.

(A) NAND gate: CORRECT (universal).

Q39.

- NOT: A NAND $A = \overline{A}$
- AND: $\overline{A \text{ NAND } B}$ = NOT of NAND $= A \cdot B$
- OR: use De Morgan: $A + B = \overline{\bar{A} \cdot \bar{B}}$

(B) NOR gate: CORRECT (universal).

- NOT: A NOR $A = \overline{A}$
- OR: $\overline{A \text{ NOR } B} = A + B$
- AND: use De Morgan: $A \cdot B = \overline{\bar{A} + \bar{B}}$

(C) OR gate alone: WRONG (not universal). OR alone cannot implement NOT (it cannot invert). Without NOT, arbitrary Boolean functions cannot be realised.

(D) AND gate alone: WRONG (not universal). AND alone similarly lacks the ability to perform NOT, so it is not functionally complete by itself.

Correct options: (A) and (B).

Answer: (AB) ← *Go back to Q39*

Q40.

Solution

Concept — Quantum Harmonic Oscillator Ground State:
$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$

(A) $\langle x \rangle = 0$: **CORRECT.** The ground state wavefunction is an even function of x. Therefore $\langle x \rangle = \int_{-\infty}^{\infty} x|\psi_0|^2\, dx = 0$ (integral of odd integrand over symmetric domain).

(B) $\langle p \rangle = 0$: **CORRECT.** By Ehrenfest's theorem for a stationary state, $\langle p \rangle = m\, d\langle x \rangle/dt = 0$. Also directly: ψ_0 is real, so $\langle p \rangle = -i\hbar \int \psi_0^* \psi_0'\, dx = 0$ (the integrand is odd).

(C) $\langle x^2 \rangle = 0$: **WRONG.** $\langle x^2 \rangle = \hbar/(2m\omega) > 0$. This is nonzero and reflects the zero-point kinetic energy $\langle K \rangle = \langle p^2 \rangle/(2m) = \frac{1}{4}\hbar\omega$.

(D) $\psi_0(x)$ **is Gaussian: CORRECT.** The ground state wavefunction $\psi_0 \propto e^{-m\omega x^2/(2\hbar)}$ is a Gaussian in x. This is the minimum uncertainty state: $\Delta x\, \Delta p = \hbar/2$.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q40*

Section C Solutions — NAT

Q41.

Solution

Concept — Period of a Simple Pendulum:
$T = 2\pi\sqrt{L/g}$

Calculation:

$$T = 2\pi\sqrt{\frac{2.0\,\mathrm{m}}{9.8\,\mathrm{m/s^2}}} = 2\pi\sqrt{0.20408} = 2\pi \times 0.45175 = 6.28318 \times 0.45175 = 2.838\,\mathrm{s}$$

$$\boxed{T \approx 2.84\,\mathrm{s}}$$

Answer: (2.84) ← *Go back to Q41*

Q42.

Solution

Concept — Energy Stored in an Inductor:
$E = \frac{1}{2}LI^2$

Calculation:

$$E = \frac{1}{2} \times 0.50\,\text{H} \times (4.0\,\text{A})^2 = \frac{1}{2} \times 0.50 \times 16 = 0.25 \times 16 = 4.00\,\text{J}$$

$$\boxed{E = 4.00\,\text{J}}$$

Answer: (4.00) ← *Go back to Q42*

Q43.

Solution

Concept — de Broglie Wavelength:
$\lambda = h/(m_p v)$

Calculation:

$$m_p v = 1.67 \times 10^{-27}\,\text{kg} \times 2.0 \times 10^{6}\,\text{m/s} = 3.34 \times 10^{-21}\,\text{kg m/s}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{3.34 \times 10^{-21}} = 1.984 \times 10^{-13}\,\text{m}$$

Converting to picometres ($1\,\text{pm} = 10^{-12}\,\text{m}$):

$$\lambda = 1.984 \times 10^{-13}\,\text{m} = 0.1984\,\text{pm}$$

$$\boxed{\lambda \approx 0.20\,\text{pm}}$$

Answer: (0.20) ← *Go back to Q43*

Q44.

Solution

Concept — Root-Mean-Square Speed of Gas Molecules:
$v_{\text{rms}} = \sqrt{3RT/M}$ where M is the molar mass in kg/mol.

Calculation:

$$v_{\text{rms}} = \sqrt{\frac{3 \times 8.314\,\text{J/(mol K)} \times 500\,\text{K}}{4.0 \times 10^{-3}\,\text{kg/mol}}} = \sqrt{\frac{12471}{0.004}} = \sqrt{3\,117\,750\,\text{m}^2/\text{s}^2}$$

$$v_{\text{rms}} = 1765.7\,\text{m/s}$$

$$\boxed{v_{\text{rms}} \approx 1766\,\text{m/s}}$$

Answer: (1766) ← *Go back to Q44*

Q45.

Solution

Concept — Radioactive Decay Constant:
$\lambda = \ln 2/T_{1/2}$

Step 1 — Convert $T_{1/2}$ to seconds:

$$T_{1/2} = 14.3\,\text{days} \times 86400\,\text{s/day} = 1\,235\,520\,\text{s} \approx 1.2355 \times 10^6\,\text{s}$$

Step 2 — Compute λ:

$$\lambda = \frac{0.6931}{1.2355 \times 10^6\,\text{s}} = 5.610 \times 10^{-7}\,\text{s}^{-1}$$

$$\boxed{\lambda \approx 5.61 \times 10^{-7}\,\text{s}^{-1}}$$

Answer: (5.61e-7) ← *Go back to Q45*

Q46.

Solution

Concept — Atomic Packing Fraction of BCC:
In BCC, atoms touch along the body diagonal: $4r = a\sqrt{3}$, giving $r = a\sqrt{3}/4$.

Atoms per unit cell: $N = 2$ (8 corner atoms $\times \frac{1}{8}$ + 1 body-centre atom).

Step 1 — Volume of atoms:

$$V_{\text{atoms}} = 2 \times \frac{4}{3}\pi r^3 = \frac{8\pi}{3}\left(\frac{a\sqrt{3}}{4}\right)^3 = \frac{8\pi}{3} \cdot \frac{3\sqrt{3}\,a^3}{64} = \frac{\pi\sqrt{3}\,a^3}{8}$$

Step 2 — Packing fraction:

$$\text{APF} = \frac{V_{\text{atoms}}}{a^3} = \frac{\pi\sqrt{3}}{8} = \frac{3.14159 \times 1.7321}{8} = \frac{5.4414}{8} = 0.6802$$

$$\boxed{\text{APF} \approx 0.680}$$

Answer: (0.680) ← *Go back to Q46*

Q47.

Solution

Concept — Trace of a Matrix:
$\mathrm{tr}(M) = \sum_i M_{ii}$ (sum of diagonal elements).

Calculation:

$$\mathrm{tr}\begin{pmatrix} 2 & 3 \\ 3 & 7 \end{pmatrix} = 2 + 7 = 9$$

$$\boxed{\mathrm{tr} = 9}$$

Note: The eigenvalues of this matrix sum to $\mathrm{tr} = 9$, consistent with $\lambda_1 + \lambda_2 = 9$.

Answer: (9) ← *Go back to Q47*

Q48.

Solution

Concept — Series RLC Resonance Frequency:
$f = \dfrac{1}{2\pi\sqrt{LC}}$

Calculation:

$$LC = 0.10\,\mathrm{H} \times 10^{-4}\,\mathrm{F} = 10^{-5}\,\mathrm{H\,F}$$

$$\sqrt{LC} = \sqrt{10^{-5}} = 10^{-2.5} = 3.1623 \times 10^{-3}\,\mathrm{s}$$

$$f = \frac{1}{2\pi \times 3.1623 \times 10^{-3}} = \frac{1}{1.9869 \times 10^{-2}} = 50.33\,\mathrm{Hz}$$

$$\boxed{f \approx 50\,\mathrm{Hz}}$$

Answer: (50) ← *Go back to Q48*

Q49.

Solution

Concept — Non-Inverting Op-Amp Output:
$V_{\text{out}} = \left(1 + \dfrac{R_f}{R_1}\right) V_{\text{in}}$

Calculation:

$$V_{\text{out}} = \left(1 + \frac{9\,\mathrm{k\Omega}}{1\,\mathrm{k\Omega}}\right) \times 0.30\,\mathrm{V} = 10 \times 0.30 = 3.0\,\mathrm{V}$$

$$\boxed{V_{\text{out}} = 3.0\,\mathrm{V}}$$

Answer: (3.0) ← *Go back to Q49*

Q50.

Solution

Concept — Moment of Inertia of a Solid Cylinder (Symmetry Axis):
$I = \frac{1}{2}mR^2$

Calculation:

$$I = \frac{1}{2} \times 2.0\,\text{kg} \times (0.05\,\text{m})^2 = \frac{1}{2} \times 2.0 \times 0.0025 = 1.0 \times 0.0025 = 0.0025\,\text{kg}\,\text{m}^2$$

$$\boxed{I = 0.0025\,\text{kg}\,\text{m}^2}$$

Answer: (0.0025) ← *Go back to Q50*

Q51.

Solution

Concept — Magnetic Energy Density:
$u = \dfrac{B^2}{2\mu_0}$

Calculation:

$$u = \frac{(0.50\,\text{T})^2}{2 \times 4\pi \times 10^{-7}\,\text{T}\,\text{m/A}} = \frac{0.25}{8\pi \times 10^{-7}}\,\text{J/m}^3$$

$$= \frac{0.25}{8 \times 3.14159 \times 10^{-7}} = \frac{0.25}{25.133 \times 10^{-7}} = \frac{0.25}{2.5133 \times 10^{-6}}$$

$$= 9.947 \times 10^4\,\text{J/m}^3$$

$$\boxed{u \approx 9.95 \times 10^4\,\text{J/m}^3}$$

Answer: (99470) ← *Go back to Q51*

Q52.

Solution

Concept — Bohr Model Orbital Periods:
In the Bohr model, the orbital period is $T_n = 2\pi r_n/v_n$. Since $r_n = n^2 a_0$ and $v_n = \alpha c/n$ (where α is the fine-structure constant), we get:

$$T_n = \frac{2\pi n^2 a_0}{(\alpha c/n)} = \frac{2\pi n^3 a_0}{\alpha c} \propto n^3$$

Step 1 — Compute ratio:

$$\frac{T_4}{T_1} = \left(\frac{4}{1}\right)^3 = 64$$

$$\boxed{T_4/T_1 = 64}$$

Answer: (64) ← *Go back to Q52*

Q53.

Solution

Concept — Entropy Change at Constant Volume:
For a reversible constant-volume process with n moles of ideal gas:

$$\Delta S = nC_V \ln \frac{T_2}{T_1}$$

Calculation for 1 mol monatomic gas ($C_V = \frac{3}{2}R$):

$$\Delta S = 1\,\text{mol} \times \frac{3}{2} \times 8.314\,\text{J/(mol K)} \times \ln \frac{900}{300}$$

$$= 1.5 \times 8.314 \times \ln 3 = 12.471\,\text{J/K} \times 1.0986 = 13.70\,\text{J/K}$$

$$\boxed{\Delta S \approx 13.70\,\text{J/K}}$$

Answer: (13.70) ← *Go back to Q53*

Q54.

Solution

Concept — Fermi Energy of Free Electron Gas:
$E_F = \frac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$

Step 1 — Compute $3\pi^2 n$:

$$3\pi^2 n = 29.608 \times 1.81 \times 10^{29} = 53.59 \times 10^{29} = 5.359 \times 10^{30}\,\text{m}^{-3}$$

Step 2 — Compute $(3\pi^2 n)^{2/3}$:
$(5.359 \times 10^{30})^{1/3} = (5.359)^{1/3} \times 10^{10}$. Since $1.75^3 = 5.359$, $(5.359)^{1/3} = 1.750$.
$(5.359 \times 10^{30})^{2/3} = (1.750)^2 \times 10^{20} = 3.0625 \times 10^{20}\,\text{m}^{-2}$

Step 3 — Compute $\hbar^2/(2m_e)$:

$$\frac{\hbar^2}{2m_e} = \frac{(1.055 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31}} = \frac{1.113 \times 10^{-68}}{1.822 \times 10^{-30}} = 6.109 \times 10^{-39}\,\text{J m}^2$$

Step 4 — Fermi energy:

$$E_F = 6.109 \times 10^{-39} \times 3.0625 \times 10^{20} = 18.71 \times 10^{-19}\,\text{J} = 1.871 \times 10^{-18}\,\text{J}$$

$$E_F = \frac{1.871 \times 10^{-18}}{1.6 \times 10^{-19}}\,\text{eV} = 11.69\,\text{eV}$$

$$\boxed{E_F \approx 11.65\,\text{eV}}$$

Answer: (11.65) ← *Go back to Q54*

Q55.

Solution

Concept — Zero-Point Energy of Quantum Harmonic Oscillator:
$E_0 = \frac{1}{2}\hbar\omega$

Calculation:

$$E_0 = \frac{1}{2} \times 1.055 \times 10^{-34}\,\text{J s} \times 2.0 \times 10^{14}\,\text{rad/s}$$
$$= 1.055 \times 10^{-34} \times 10^{14} = 1.055 \times 10^{-20}\,\text{J}$$

Convert to eV:

$$E_0 = \frac{1.055 \times 10^{-20}}{1.6 \times 10^{-19}} = \frac{1.055}{16} = 0.06594\,\text{eV}$$
$$\boxed{E_0 \approx 0.066\,\text{eV}}$$

Answer: (0.066) ← *Go back to Q55*

Q56.

Solution

Concept — Sound Wave Wavelength:
$\lambda = v_s/f$

Calculation:

$$\lambda = \frac{343\,\text{m/s}}{1000\,\text{Hz}} = 0.343\,\text{m}$$
$$\boxed{\lambda = 0.343\,\text{m}}$$

Answer: (0.343) ← *Go back to Q56*

Q57.

Solution

Concept — Radioactive Decay Over Multiple Half-Lives:
After n half-lives: $N = N_0 \left(\frac{1}{2}\right)^n$

Calculation for $N_0 = 4 \times 10^6$ and $n = 5$:

$$N = 4 \times 10^6 \times \left(\frac{1}{2}\right)^5 = \frac{4 \times 10^6}{32} = \frac{4\,000\,000}{32} = 125\,000$$

$$\boxed{N = 125\,000}$$

Answer: (125000) ← *Go back to Q57*

Q58.

Solution

Concept — Gaussian Integral with x^2 Factor:
Use the standard parametric differentiation technique.

Step 1 — Start with the Gaussian integral:

$$I_0(\alpha) = \int_{-\infty}^{\infty} e^{-\alpha x^2}\,dx = \sqrt{\frac{\pi}{\alpha}}$$

Step 2 — Differentiate with respect to α:

$$\frac{dI_0}{d\alpha} = \int_{-\infty}^{\infty} (-x^2)\,e^{-\alpha x^2}\,dx = -\frac{d}{d\alpha}\sqrt{\frac{\pi}{\alpha}} = \frac{\sqrt{\pi}}{2}\,\alpha^{-3/2}$$

$$\therefore \int_{-\infty}^{\infty} x^2\,e^{-\alpha x^2}\,dx = \frac{\sqrt{\pi}}{2}\,\alpha^{-3/2}$$

Step 3 — Set $\alpha = 2$:

$$\int_{-\infty}^{\infty} x^2\,e^{-2x^2}\,dx = \frac{\sqrt{\pi}}{2} \times 2^{-3/2} = \frac{\sqrt{\pi}}{2 \times 2\sqrt{2}} = \frac{\sqrt{\pi}}{4\sqrt{2}}$$

$$= \frac{1.7725}{4 \times 1.4142} = \frac{1.7725}{5.6569} = 0.3133$$

$$\boxed{I \approx 0.313}$$

Answer: (0.313) ← *Go back to Q58*

Q59.

Solution

Concept — RL Circuit Time Constant:
$\tau = L/R$

Calculation:

$$\tau = \frac{L}{R} = \frac{0.25\,\mathrm{H}}{50\,\Omega} = 0.005\,\mathrm{s} = 5.0\,\mathrm{ms}$$

$$\boxed{\tau = 5.0\,\mathrm{ms}}$$

Answer: (5.0) ← *Go back to Q59*

Q60.

Solution

Concept — Photon Energy:

$E = hc/\lambda$

Calculation:

$$E = \frac{6.626 \times 10^{-34}\,\mathrm{J\,s} \times 3.0 \times 10^{8}\,\mathrm{m/s}}{300 \times 10^{-9}\,\mathrm{m}} = \frac{19.878 \times 10^{-26}}{3.00 \times 10^{-7}}$$

$$= 6.626 \times 10^{-19}\,\mathrm{J}$$

Convert to eV:

$$E = \frac{6.626 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.141\,\mathrm{eV}$$

$$\boxed{E \approx 4.14\,\mathrm{eV}}$$

Answer: (4.14) ← *Go back to Q60*

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	A
6	B	7	C	8	D	9	A	10	B
11	C	12	D	13	A	14	B	15	C
16	D	17	A	18	B	19	C	20	D
21	A	22	B	23	C	24	D	25	A
26	B	27	C	28	D	29	A	30	B
31	ACD	32	ABD	33	BD	34	ABD	35	ACD
36	ABC	37	ACD	38	ABC	39	AB	40	ABD
41	2.84	42	4.00	43	0.20	44	1766	45	5.61e-7
46	0.680	47	9	48	50	49	3.0	50	0.0025
51	99470	52	64	53	13.70	54	11.65	55	0.066
56	0.343	57	125000	58	0.313	59	5.0	60	4.14