IIT JAM Physics

Sample Paper – 8

Instructions

- This paper contains **60 questions** carrying a maximum of **100 marks**. Duration: **3 hours (180 minutes)**.
- **Section A** (**Q1–Q30**, MCQ): Q1–Q10 carry **+1 mark** each; wrong response: $-\frac{1}{3}$ mark. Q11–Q30 carry **+2 marks** each; wrong response: $-\frac{2}{3}$ mark. Choose the **single best** option.
- **Section B** (**Q31–Q40**, MSQ): Each carries **+2 marks**. **No negative marking. One or more** options may be correct; full marks only if *all* correct options are selected; no partial credit.
- **Section C** (**Q41–Q60**, NAT): Q41–Q50 carry **+1 mark**; Q51–Q60 carry **+2 marks**. **No negative marking.** Enter the numerical value using the virtual keypad (integer or decimal as appropriate).
- **Calculator policy:** Personal calculators, log tables, and electronic devices are **strictly prohibited**. A virtual scientific calculator is provided in the CBT interface. Rough sheets are supplied at the examination centre.
- Once you move to the next section, you cannot return to a previous section. Manage time accordingly.

Section A — Multiple Choice Questions (MCQ)

***Q1–Q10**: 1 mark each ($-\frac{1}{3}$ for wrong). Choose the **single** correct option.*

Q1. Using the Cauchy integral formula $\oint_C \dfrac{f(z)}{z-z_0}\,dz = 2\pi i\, f(z_0)$, evaluate the contour integral

$$\oint_{|z|=2} \frac{\sin z}{z-1}\,dz$$

taken counter-clockwise. The result is:

(A) $2\pi i$

(B) $\pi i \sin(1)$

(C) $2\pi i \sin(1)$

(D) 0

Q2. In the central force problem with effective potential $V_{\text{eff}}(r) = V(r) + \dfrac{L^2}{2\mu r^2}$, for an attractive Coulomb potential $V(r) = -k/r$ (with $k > 0$), the radius

r_c of the circular orbit (obtained by setting $dV_{\text{eff}}/dr = 0$) is:

(A) $r_c = \dfrac{L^2}{2\mu k}$

(B) $r_c = \dfrac{2L^2}{\mu k}$

(C) $r_c = \dfrac{L}{\mu k}$

(D) $r_c = \dfrac{L^2}{\mu k}$

Q3. A diffraction grating has $N = 500$ lines/mm and is illuminated at normal incidence by light of wavelength $\lambda = 600\,\text{nm}$ (so the grating spacing is $d = 2000\,\text{nm}$). The angle θ of the first-order principal maximum (satisfying $\sin\theta = \lambda/d$) is approximately:

(A) 17.5°

(B) 30.0°

(C) 45.0°

(D) 8.6°

Q4. By Gauss's law in electrostatics, the total electric flux Φ_E through a closed spherical surface of radius $2R$ that encloses a point charge Q at its centre is:

(A) $\Phi_E = \dfrac{Q}{4\varepsilon_0}$

(B) $\Phi_E = \dfrac{Q}{\varepsilon_0}$

(C) $\Phi_E = \dfrac{Q}{2\varepsilon_0}$

(D) $\Phi_E = \dfrac{4Q}{\varepsilon_0}$

Q5. When equal amounts ($N/2$ molecules each) of two different ideal gases are mixed irreversibly at the same temperature and pressure, the entropy increase *per molecule* of the mixture ($\Delta S_{\text{mix}}/N$) equals:

(A) 0

(B) $k_B/2$

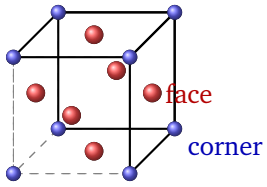
(C) $k_B \ln 2$

(D) $2\,k_B \ln 2$

Q6. An electron (mass $m_e = 9.11 \times 10^{-31}\,\text{kg}$, charge $e = 1.6 \times 10^{-19}\,\text{C}$) is accelerated from rest through a potential difference of $V = 100\,\text{V}$. Its de Broglie wavelength $\lambda = h/\sqrt{2m_e eV}$ (with $h = 6.626 \times 10^{-34}\,\text{J s}$) is approximately:

(A) $0.61\,\text{Å}$
(B) $2.46\,\text{Å}$
(C) $0.39\,\text{Å}$
(D) $1.23\,\text{Å}$

Q7. A face-centred cubic (FCC) unit cell is shown below. Counting the corner atoms (each shared by 8 unit cells) and face-centre atoms (each shared by 2 unit cells), the total number of atoms *per unit cell* of an FCC crystal is:



(A) 4
(B) 2
(C) 8
(D) 6

Q8. A NOR gate receives inputs $A = 1$ and $B = 0$. Its output $Y = \overline{A + B}$ is:

(A) 1
(B) 0
(C) Same as input A
(D) Indeterminate without a clock signal

Q9. In beta-plus (β^+) decay, a proton in the nucleus converts to a neutron with emission of a positron and a neutrino. Using atomic masses M_P (parent) and M_D (daughter), the Q-value of β^+ decay is:

(A) $Q = (M_P - M_D)\,c^2$

(B) $Q = (M_D - M_P)\,c^2$
(C) $Q = (M_P - M_D - 2m_e)\,c^2$
(D) $Q = (M_P - M_D + 2m_e)\,c^2$

Q10. The Wronskian $W[y_1, y_2](x) = y_1 y_2' - y_1' y_2$ for the pair $y_1 = \sin x$ and $y_2 = \cos x$ equals:

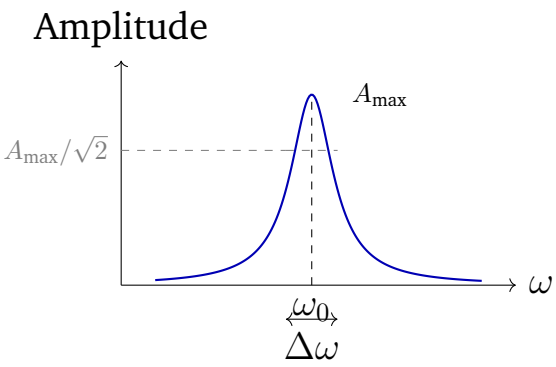
(A) 0
(B) 1
(C) $\sin 2x$
(D) -1

***Q11–Q30**: 2 marks each ($-\frac{2}{3}$ for wrong). Choose the **single** correct option.*

Q11. By Kepler's second law (conservation of angular momentum), if a planet's aphelion distance is $r_{\max}$ and perihelion distance is $r_{\min}$, then $r_{\min} v_{\max} = r_{\max} v_{\min}$. For $r_{\max}/r_{\min} = 3$, the ratio $v_{\max}/v_{\min}$ is:

(A) 3
(B) 9
(C) $\sqrt{3}$
(D) $1/3$

Q12. A lightly damped oscillator ($Q \gg 1$) driven near resonance ω_0 has the amplitude response shown below. The full width at half-power (bandwidth) $\Delta\omega$ is related to the quality factor Q by:

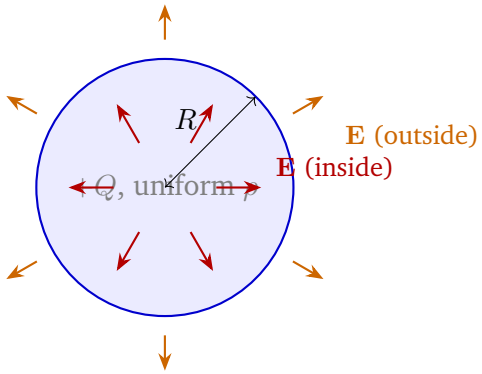


(A) $\Delta\omega = \omega_0\, Q$
(B) $\Delta\omega = \omega_0/Q$

(C) $\Delta\omega = \omega_0/Q^2$

(D) $\Delta\omega = Q/\omega_0$

Q13. A uniformly charged solid sphere of radius R carries total charge Q (with volume charge density $\rho = 3Q/(4\pi R^3)$). Applying Gauss's law, the electric field at a point $r < R$ inside the sphere is shown in the cross-section below. Its magnitude is:



(A) $E(r) = \dfrac{Q}{4\pi\varepsilon_0 r^2}$

(B) $E(r) = 0$

(C) $E(r) = \dfrac{Qr}{4\pi\varepsilon_0 R^3}$

(D) $E(r) = \dfrac{Qr^2}{4\pi\varepsilon_0 R^4}$

Q14. A Carnot engine operates between a hot reservoir at $T_H = 600\,\text{K}$ and a cold reservoir at $T_C = 300\,\text{K}$. Its thermodynamic efficiency $\eta = 1 - T_C/T_H$ is:

(A) 25%

(B) 33%

(C) 75%

(D) 50%

Q15. For the 3D isotropic quantum harmonic oscillator with Hamiltonian $H = \mathbf{p}^2/(2m) + \frac{1}{2}m\omega^2 r^2$, which statement correctly gives the ground state energy E_0 and its degeneracy, and the first excited state energy E_1 and its degeneracy?

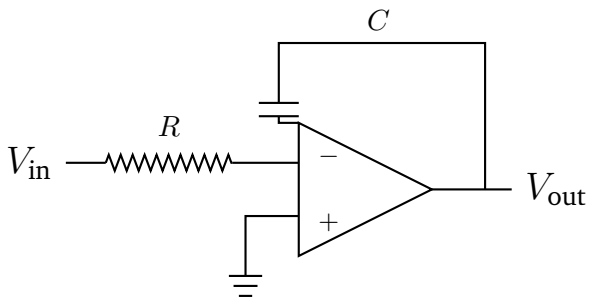
(A) $E_0 = \frac{3}{2}\hbar\omega$, degeneracy $= 1$; $\quad E_1 = \frac{5}{2}\hbar\omega$, degeneracy $= 3$

(B) $E_0 = \frac{1}{2}\hbar\omega$, degeneracy $= 1$; $E_1 = \frac{3}{2}\hbar\omega$, degeneracy $= 3$
(C) $E_0 = \frac{3}{2}\hbar\omega$, degeneracy $= 3$; $E_1 = \frac{5}{2}\hbar\omega$, degeneracy $= 5$
(D) $E_0 = \hbar\omega$, degeneracy $= 1$; $E_1 = 2\hbar\omega$, degeneracy $= 3$

Q16. Which statement correctly describes a *direct-band-gap* semiconductor and gives an example?

(A) GaAs is an indirect-gap semiconductor; optical transitions require phonon assistance to conserve momentum
(B) GaAs is a direct-gap semiconductor with its conduction band minimum and valence band maximum both at $\mathbf{k} = 0$; optical transitions are phonon-free with $E_g \approx 1.42$ eV at room temperature
(C) Silicon (Si) is a direct-gap semiconductor with $E_g \approx 1.12$ eV at room temperature
(D) In a direct-gap semiconductor, optical (band-to-band) transitions are forbidden by selection rules

Q17. The inverting op-amp integrator circuit shown below (with resistor R at the input and capacitor C in the feedback path) produces an output:



(A) $V_{\text{out}} = -\dfrac{R}{C}\dfrac{dV_{\text{in}}}{dt}$
(B) $V_{\text{out}} = \dfrac{1}{RC}\displaystyle\int V_{\text{in}}\,dt$
(C) $V_{\text{out}} = -\dfrac{1}{RC}\displaystyle\int V_{\text{in}}\,dt$
(D) $V_{\text{out}} = -RC\dfrac{dV_{\text{in}}}{dt}$

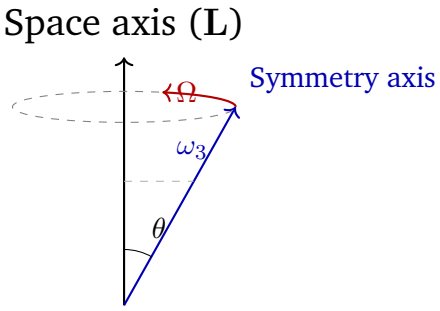
Q18. In the nuclear shell model, filled shells produce unusually stable nuclei at the *magic numbers*. Which of the following correctly lists the first four magic numbers?

(A) $2,\ 6,\ 14,\ 28$
(B) $2,\ 8,\ 18,\ 32$
(C) $2,\ 8,\ 14,\ 20$
(D) $2,\ 8,\ 20,\ 28$

Q19. The physicist's Hermite polynomials are defined recursively. The first few are $H_0(x) = 1$, $H_1(x) = 2x$, $H_2(x) = 4x^2 - 2$. The value of $H_2(1)$ is:

(A) 2
(B) 4
(C) -2
(D) 6

Q20. A torque-free symmetric top (with $I_1 = I_2 \neq I_3$) undergoes steady precession about its symmetry axis. The angular velocity of this (body-frame) precession Ω in terms of the spin component ω_3 along the symmetry axis is shown in the diagram. The expression for Ω is:



(A) $\Omega = \dfrac{I_3}{I_1}\,\omega_3$
(B) $\Omega = \dfrac{I_3 - I_1}{I_1}\,\omega_3$
(C) $\Omega = \dfrac{I_1 - I_3}{I_1}\,\omega_3$
(D) $\Omega = \dfrac{I_3 - I_1}{I_3}\,\omega_3$

Q21. The Poynting vector $\mathbf{S} = (\mathbf{E}\times\mathbf{B})/\mu_0$ gives the electromagnetic energy flux density. For a monochromatic plane wave in vacuum with peak electric field $E_0 = 300\,\text{V/m}$, the time-averaged intensity $\langle S\rangle = E_0^2/(2\mu_0 c)$ (with $\mu_0 = 4\pi \times 10^{-7}\,\text{T m/A}$, $c = 3 \times 10^8\,\text{m/s}$) is approximately:

(A) $47.8\,\text{W/m}^2$

(B) $60.0\,\mathrm{W/m^2}$
(C) $119.4\,\mathrm{W/m^2}$
(D) $238.8\,\mathrm{W/m^2}$

Q22. At high temperature ($k_BT \gg \varepsilon-\mu$), the Bose–Einstein distribution $\langle n_{\mathrm{BE}}\rangle = 1/(e^{(\varepsilon-\mu)/k_BT}-1)$ and the Fermi–Dirac distribution $\langle n_{\mathrm{FD}}\rangle = 1/(e^{(\varepsilon-\mu)/k_BT}+1)$ behave as:

(A) They remain distinct at all temperatures; $\langle n_{\mathrm{BE}}\rangle \neq \langle n_{\mathrm{FD}}\rangle$ even as $T \to \infty$
(B) Both approach $\langle n\rangle \to 1/2$ at high temperature
(C) $\langle n_{\mathrm{BE}}\rangle \to \infty$ while $\langle n_{\mathrm{FD}}\rangle \to 1$ as $T \to \infty$
(D) Both approach the Maxwell–Boltzmann distribution $\langle n\rangle \approx e^{-(\varepsilon-\mu)/k_BT}$ at high temperature

Q23. For the quantum harmonic oscillator with energy eigenstates $|n\rangle$ (energy $E_n = (n+\frac{1}{2})\hbar\omega$), which of the following is correct regarding the expectation values of position?

(A) $\langle x\rangle = 0$ for *all* eigenstates $|n\rangle$; and $\langle x^2\rangle = \left(n+\frac{1}{2}\right)\hbar/(m\omega)$
(B) $\langle x\rangle = \sqrt{(n+\frac{1}{2})\hbar/(m\omega)}$ for the n-th eigenstate
(C) $\langle x\rangle = 0$ only for the ground state $|0\rangle$; for $n \geq 1$, $\langle x\rangle \neq 0$
(D) $\langle x\rangle = 0$ and $\langle x^2\rangle = 0$ for all energy eigenstates

Q24. The built-in (contact) potential V_{bi} of a p-n junction in thermal equilibrium, which compensates the concentration gradient and prevents net current flow, is given by:

(A) $V_{\mathrm{bi}} = \dfrac{k_BT}{e}\ln\left(\dfrac{N_A}{N_D}\right)$
(B) $V_{\mathrm{bi}} = \dfrac{k_BT}{e}\ln\left(\dfrac{N_AN_D}{n_i^2}\right)$
(C) $V_{\mathrm{bi}} = k_BT\ln\left(\dfrac{N_AN_D}{n_i^2}\right)$
(D) $V_{\mathrm{bi}} = \dfrac{e}{k_BT}\ln\left(\dfrac{N_AN_D}{n_i^2}\right)$

Q25. An inverting summing amplifier has two input voltages V_1 and V_2, each

applied through a resistor R to the inverting input of an ideal op-amp. The feedback resistor is also $R_f = R$. The output voltage is:

(A) $V_{\text{out}} = V_1 + V_2$
(B) $V_{\text{out}} = -\dfrac{V_1 + V_2}{2}$
(C) $V_{\text{out}} = -(V_1 + V_2)$
(D) $V_{\text{out}} = \dfrac{V_1 - V_2}{2}$

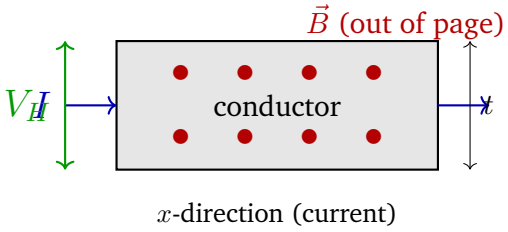
Q26. When orbital angular momentum $\ell = 1$ and spin $s = 1/2$ are added via Clebsch–Gordan rules, which of the following correctly states the possible total j values, their degeneracies, and the ground state of hydrogen?

(A) Only $j = 1$ and $j = 0$ are possible (4 states total); the H ground state has $j = 0$
(B) Only $j = 3/2$ is possible with degeneracy 6; the H ground state has $j = 3/2$
(C) $j = 3/2$ (4 states) or $j = 1/2$ (2 states); the H ground state has $j = 3/2$ since it has $\ell = 1$
(D) $j = 3/2$ (4 states) or $j = 1/2$ (2 states); the H ground state ($\ell = 0$) has $j = s = 1/2$

Q27. By the equipartition theorem applied to a diatomic ideal gas at moderate temperature (where vibrational modes are not excited), the molar heat capacity at constant volume C_V is:

(A) 5 degrees of freedom active at moderate temperature; $C_V = \frac{5}{2}R$
(B) 3 degrees of freedom (translational only); $C_V = \frac{3}{2}R$
(C) 7 degrees of freedom including vibration; $C_V = \frac{7}{2}R$ at all temperatures
(D) A diatomic gas has the same C_V as a monatomic ideal gas

Q28. In the Hall effect, a current I flows in the x-direction through a conducting slab of thickness t in the y-direction. A magnetic field B is applied in the z-direction. The diagram below shows the geometry. The Hall voltage V_H (across the y-direction) and the Hall coefficient R_H for conduction by electrons (carrier density n, charge $-e$) are:



(A) $V_H = R_H\, I\, B\, t$ where $R_H = ne$
(B) $V_H = \dfrac{IB}{net}$ where $R_H = \dfrac{1}{ne}$ for electrons
(C) $V_H = \dfrac{ne\, IB}{t}$
(D) $V_H = R_H\, I\, B\, t$ where $R_H = ne^2$

Q29. In Young's double-slit experiment with slit separation $d = 0.5\,\text{mm}$, screen distance $D = 1\,\text{m}$, and wavelength $\lambda = 500\,\text{nm}$, the fringe width $\Delta y = \lambda D/d$ is:

(A) $0.5\,\text{mm}$
(B) $2.0\,\text{mm}$
(C) $1.0\,\text{mm}$
(D) $0.25\,\text{mm}$

Q30. Using the Bohr model of hydrogen ($E_n = -13.6/n^2\,\text{eV}$), the energy of the photon emitted in a transition from $n = 4$ to $n = 2$ is $\Delta E = E_4 - E_2$ (with $E_4 = -0.85\,\text{eV}$ and $E_2 = -3.40\,\text{eV}$). The photon frequency $\nu = \Delta E/h$ corresponds to ΔE equal to:

(A) $3.40\,\text{eV}$
(B) $1.51\,\text{eV}$
(C) $0.66\,\text{eV}$
(D) $2.55\,\text{eV}$

Section B — Multiple Select Questions (MSQ)

__Q31–Q40__: 2 marks each. __No negative marking. One or more__ options may be correct. Full marks only if all *correct options (and no incorrect options) are chosen.*

Q31. For a rigid body rotating about a fixed axis with angular velocity ω, which of the following statements are true?

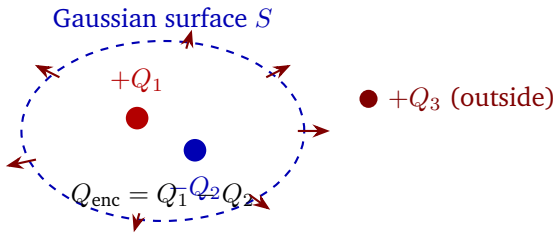
(A) All particles of the rigid body have the same linear velocity

(B) All particles of the rigid body have the same angular velocity ω about the rotation axis

(C) The angular momentum of the body about the rotation axis is $L = I\omega$, where I is the moment of inertia

(D) The kinetic energy of the body equals $\frac{1}{2}Mv_{\text{cm}}^2$, where v_{cm} is the speed of the centre of mass

Q32. Which of the following statements about Gauss's law for electrostatics $\oint_S \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\varepsilon_0$ are correct? The diagram below shows a closed Gaussian surface S of arbitrary shape enclosing charge Q_{enc}.



(A) The total electric flux through any closed surface equals $Q_{\text{enc}}/\varepsilon_0$, where Q_{enc} is the total charge enclosed

(B) Gauss's law can only be applied when the Gaussian surface has spherical symmetry

(C) Gauss's law holds for any closed (Gaussian) surface, regardless of its shape, as long as Q_{enc} is well defined

(D) For a conductor in electrostatic equilibrium, Gauss's law implies that the electric field inside the conductor is zero

Q33. For a system in contact with a heat bath at temperature $T = 1/(k_B\beta)$ (canonical ensemble), which of the following relations are correct?

(A) The canonical partition function is $Z = \sum_n e^{-\beta E_n}$, where the sum runs over all microstates n

(B) The Helmholtz free energy is $F = -k_B T \ln Z$

(C) The entropy is $S = -\partial Z/\partial T$

(D) The mean energy is $\langle E \rangle = -\partial(\ln Z)/\partial\beta$

Q34. For a bipolar junction transistor (BJT) operating in the common-emitter (CE) configuration in the *active* mode, which of the following are true?

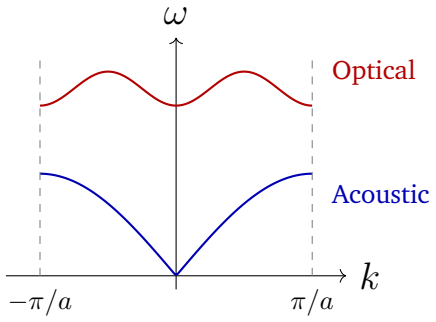
(A) The base-emitter junction is reverse biased

(B) The collector current is $I_C = \beta\, I_B$, where β is the current gain

(C) The emitter current satisfies $I_E = I_B + I_C$

(D) The collector-emitter junction is forward biased

Q35. Phonons are quantised lattice vibrations. The dispersion curve for a 1D diatomic chain is sketched below (showing acoustic and optical branches). Which statements about phonons are correct?



(A) Phonons are the quanta of lattice vibrational energy; a mode with frequency ω has energy $\hbar\omega$ per phonon

(B) Phonons obey Fermi–Dirac statistics because they are associated with lattice vibrations of ions

(C) The average occupation number of a phonon mode of frequency ω is given by the Bose–Einstein distribution $\bar{n} = 1/(e^{\hbar\omega/k_B T} - 1)$

(D) Acoustic phonons satisfy $\omega \to 0$ as $k \to 0$ (i.e. they are gapless in the long-wavelength limit)

Q36. Which of the following statements about the Gauss (Divergence) theorem $\oint_S \mathbf{F} \cdot d\mathbf{A} = \int_V (\nabla \cdot \mathbf{F})\, dV$ are true?

(A) The theorem equates the outward flux of $\mathbf{F}$ through a closed surface S to the volume integral of the divergence of $\mathbf{F}$ over the enclosed volume V

(B) The theorem requires that $\mathbf{F}$ be continuously differentiable (with bounded partial derivatives) inside V and on S

(C) The theorem applies only to divergence-free fields ($\nabla \cdot \mathbf{F} = 0$)

(D) Applying the divergence theorem to the electric field $\mathbf{E}$ of a point charge immediately yields Gauss's law

Q37. Which of the following statements about gamma (γ) decay are correct?

(A) The atomic number Z of the nucleus changes in γ decay

(B) A high-energy photon (gamma ray) is emitted in γ decay

(C) The mass number A of the nucleus remains unchanged in γ decay

(D) The neutron number N of the nucleus changes in γ decay

Q38. Which of the following conditions are *necessary* to observe stable interference fringes in a two-beam interference experiment?

(A) The two interfering beams must be coherent, i.e. they must maintain a fixed (or slowly varying) phase difference over the observation time

(B) The two beams must be linearly polarised in exactly the same plane

(C) The optical path difference between the two beams must be less than the coherence length of the source

(D) The two beams must have the same frequency (or wavelength)

Q39. For a particle in a 3D cubic box of side L, the energy eigenfunctions are ψ_{n_x,n_y,n_z} with energy $E \propto n_x^2 + n_y^2 + n_z^2$ ($n_x, n_y, n_z = 1, 2, 3, \ldots$). Which of the following statements are correct?

(A) Each ψ_{n_x,n_y,n_z} is an eigenstate of the Hamiltonian

(B) States with different quantum number triples (n_x, n_y, n_z) but the same value of $n_x^2 + n_y^2 + n_z^2$ are degenerate

(C) The states $\psi_{1,1,1}$ and $\psi_{1,1,2}$ are degenerate (have the same energy)

(D) The ground state $\psi_{1,1,1}$ is non-degenerate

Q40. Which of the following statements about a p-n junction diode are correct?

(A) In forward bias, the depletion layer widens as applied voltage increases

(B) In reverse bias, the depletion layer widens as the magnitude of the applied reverse voltage increases

(C) The built-in potential (contact potential) V_{bi} at the junction opposes the diffusion of majority carriers across the junction in equilibrium

(D) Minority carriers carry the majority of the current in the forward-biased diode

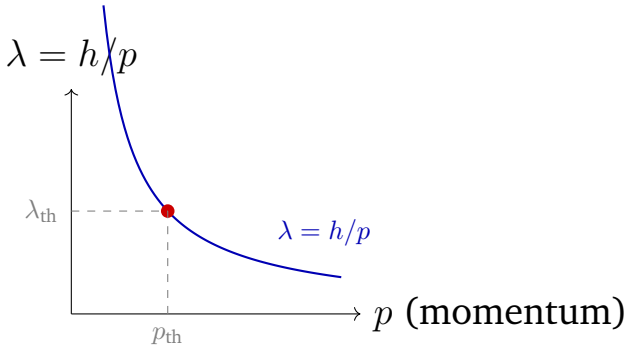
Section C — Numerical Answer Type (NAT)

***Q41–Q50**: 1 mark each. **No negative marking.** Enter the exact numerical value.*

Q41. A particle undergoes simple harmonic motion with angular frequency $\omega = 4\pi$ rad/s. Calculate the time period T of oscillation in seconds, correct to one decimal place.

Q42. A spherical capacitor has inner radius $a = 5\,\text{cm}$ and outer radius $b = 10\,\text{cm}$. Its capacitance is $C = 4\pi\varepsilon_0 ab/(b-a)$ (take $\varepsilon_0 = 8.854 \times 10^{-12}\,\text{F/m}$). Calculate C in picofarads (pF), correct to two decimal places.

Q43. The thermal de Broglie wavelength of a nitrogen molecule (mass $m = 2.33 \times 10^{-26}\,\text{kg}$) at temperature $T = 300\,\text{K}$ is $\lambda_{\text{th}} = h/\sqrt{2\pi m k_B T}$ (with $h = 6.626 \times 10^{-34}\,\text{J s}$, $k_B = 1.38 \times 10^{-23}\,\text{J/K}$). The graph of $\lambda = h/p$ vs momentum p is shown below. Calculate λ_{th} in nanometres, correct to two decimal places.



Q44. The mean free path of gas molecules in air (molecular diameter $d = 3\,\text{Å} = 3 \times 10^{-10}\,\text{m}$) at pressure $P = 1 \times 10^5\,\text{Pa}$ and temperature $T = 300\,\text{K}$ is $\lambda = 1/(\sqrt{2}\,\pi d^2 n)$, where $n = P/(k_B T)$ is the number density ($k_B = 1.38 \times 10^{-23}\,\text{J/K}$). Calculate λ in nanometres, to the nearest integer.

Q45. A radioactive isotope has a half-life $T_{1/2} = 3.0\,\mu$s. Its mean lifetime $\tau = T_{1/2}/\ln 2$ is how many microseconds? Give your answer correct to two decimal places.

Q46. Calculate the atomic packing fraction (packing efficiency) of the simple cubic (SC) crystal structure, where the unit cell contains one atom of radius $r = a/2$ (atoms touch along the cube edge of length a). Give your answer correct to three decimal places.

Q47. Find the determinant of the matrix $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$.

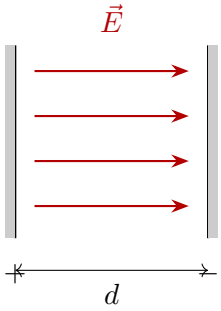
Q48. An RLC series circuit has resistance $R = 20\,\Omega$, inductance $L = 0.1\,\text{H}$, and capacitance $C = 10\,\mu\text{F} = 10^{-5}\,\text{F}$. The resonance angular frequency is $\omega_0 = 1/\sqrt{LC}$. Calculate the quality factor $Q = \omega_0 L/R$ of the circuit, correct to one decimal place.

Q49. An inverting op-amp amplifier has feedback resistor $R_f = 47\,\text{k}\Omega$ and input resistor $R_{\text{in}} = 10\,\text{k}\Omega$. The magnitude of the voltage gain is $|G| = R_f/R_{\text{in}}$. Calculate $|G|$, correct to one decimal place.

Q50. A resistor of resistance $R = 50\,\Omega$ carries a steady current $I = 2\,\text{A}$. The power dissipated in the resistor is $P = I^2 R$. Calculate P in watts.

***Q51–Q60**: 2 marks each. **No negative marking.** Enter the exact numerical value.*

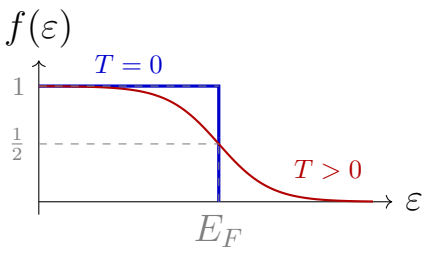
Q51. The energy density of an electric field in vacuum is $u = \frac{1}{2}\varepsilon_0 E^2$. For a uniform field $E = 3 \times 10^5\,\text{V/m}$ (such as between the plates of a capacitor as shown below), calculate u in J/m^3, correct to two decimal places. (Take $\varepsilon_0 = 8.854 \times 10^{-12}\,\text{F/m}$.)



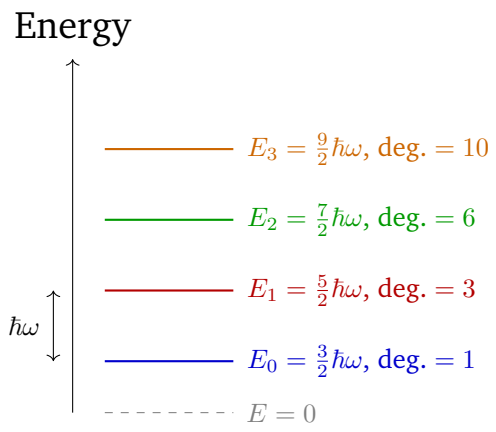
Q52. In the Bohr model of hydrogen, the de Broglie wavelength of the electron in the n-th circular orbit satisfies $\lambda_n = 2\pi r_n/n$ (since the circumference equals n wavelengths). Since $r_n = n^2 a_0$, we have $\lambda_n \propto n$. Calculate the ratio λ_3/λ_1 (the wavelength in the $n = 3$ orbit to that in the $n = 1$ orbit).

Q53. One mole of an ideal gas at temperature $T = 300\,\text{K}$ is allowed to expand freely (Joule expansion) from volume V to volume $3V$. Although no work is done and $\Delta T = 0$, the entropy change is $\Delta S = nR\ln(3)$ (as for a reversible isothermal expansion to the same final state). Calculate ΔS in J/K, correct to two decimal places. ($R = 8.314\,\text{J mol}^{-1}\text{K}^{-1}$, $\ln 3 = 1.0986$).

Q54. Estimate the Fermi energy E_F (in eV, correct to two decimal places) for sodium (Na), treating the conduction electrons as a free Fermi gas with number density $n = 2.65 \times 10^{28}\,\text{m}^{-3}$, using $E_F = \dfrac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$. The Fermi–Dirac distribution at $T = 0$ and $T > 0$ is shown schematically below. (Use $\hbar = 1.055 \times 10^{-34}\,\text{J s}$, $m_e = 9.11 \times 10^{-31}\,\text{kg}$, $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$.)



Q55. A 3D isotropic harmonic oscillator has angular frequency $\omega = 2\times10^{14}\,\text{rad/s}$. The first excited state has energy $E_1 = \frac{5}{2}\hbar\omega$. The energy level diagram is sketched below. Calculate E_1 in eV, correct to three decimal places. ($\hbar = 1.055 \times 10^{-34}\,\text{J s}$; $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$.)



Q56. A transverse wave travels along a string under tension $T = 100\,\text{N}$ with linear mass density $\mu = 0.04\,\text{kg/m}$. The wave speed is $v = \sqrt{T/\mu}$. Calculate v in m/s.

Q57. A radioactive sample initially contains $N_0 = 2 \times 10^6$ nuclei. After a time equal to 4 half-lives ($t = 4\,T_{1/2}$), how many undecayed nuclei N remain? Express your answer as an integer.

Q58. Evaluate the definite integral $I = \displaystyle\int_0^{\infty} x^3\, e^{-x}\, dx$ using the gamma function $\Gamma(n) = \int_0^{\infty} x^{n-1} e^{-x}\, dx = (n-1)!$ for integer n.

Q59. In an RC charging circuit, a capacitor $C = 1\,\mu\text{F} = 10^{-6}\,\text{F}$ is connected in series with a resistor $R = 10\,\text{k}\Omega = 10^4\,\Omega$. The capacitor voltage reaches 63.2% of the supply voltage after one time constant $\tau = RC$. Calculate τ in milliseconds (ms), correct to one decimal place.

Q60. An alpha particle ($m_\alpha = 4 \times 1.67 \times 10^{-27}\,\text{kg} = 6.68 \times 10^{-27}\,\text{kg}$) has kinetic energy $KE = 0.5\,\text{eV} = 0.5 \times 1.6 \times 10^{-19}\,\text{J}$. Its de Broglie wavelength is $\lambda = h/\sqrt{2\,m_\alpha\,KE}$. Calculate λ in nanometres, correct to three decimal places. ($h = 6.626 \times 10^{-34}\,\text{J s}$).

Solutions

IIT JAM Physics – Sample Paper 8

Section A Solutions — MCQ

Solution

Concept — Cauchy Integral Formula:
The Cauchy integral formula states: if $f(z)$ is analytic inside and on a simple closed contour C, and z_0 is inside C, then

$$\oint_C \frac{f(z)}{z - z_0}\, dz = 2\pi i\, f(z_0).$$

Step 1 — Identify $f(z)$ and z_0:
The integrand is $\dfrac{\sin z}{z-1}$. So $f(z) = \sin z$ and $z_0 = 1$.

Step 2 — Check that z_0 lies inside the contour:
$|z_0| = |1| = 1 < 2$, so $z_0 = 1$ lies inside $|z| = 2$. Also $f(z) = \sin z$ is entire (analytic everywhere).

Step 3 — Apply the formula:

$$\oint_{|z|=2} \frac{\sin z}{z-1}\, dz = 2\pi i\, f(1) = 2\pi i \sin(1).$$

Why other options are wrong:

Q1.

- (A) $2\pi i$: Would require $f(z_0) = 1$, but $\sin(1) \approx 0.841 \neq 1$.
- (B) $\pi i \sin(1)$: Missing the factor of 2; the formula gives $2\pi i$, not πi.
- (D) 0: Would be the result if there were no poles inside C, but $z_0 = 1$ is inside $|z| = 2$.

Final Answer: $2\pi i \sin(1) \Rightarrow$ **(C)**

Answer: (C) *← Go back to Q1*

Solution

Concept — Circular Orbit in Central Force:
The effective potential is $V_{\text{eff}}(r) = -k/r + L^2/(2\mu r^2)$. A circular orbit occurs at the minimum of V_{eff}, i.e. where $dV_{\text{eff}}/dr = 0$.

Step 1 — Differentiate and set to zero:

$$\frac{dV_{\text{eff}}}{dr} = \frac{k}{r^2} - \frac{L^2}{\mu r^3} = 0$$

Step 2 — Solve for r_c:

$$\frac{k}{r_c^2} = \frac{L^2}{\mu r_c^3} \;\Rightarrow\; k\, r_c = \frac{L^2}{\mu} \;\Rightarrow\; r_c = \frac{L^2}{\mu k}$$

Why other options are wrong:

Q2.

- (A) $L^2/(2\mu k)$: Off by factor of 2; the 2 in the effective potential denominator cancels with the factor from differentiation.
- (B) $2L^2/(\mu k)$: Off by a factor of 2 in the opposite direction.
- (C) $L/(\mu k)$: Incorrect dimensions; $L^2/(\mu k)$ has dimensions of length.

Final Answer: $r_c = L^2/(\mu k) \Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q2*

Solution

Concept — Diffraction Grating Equation:
For a grating with spacing d, the condition for the m-th order principal maximum at normal incidence is $d \sin\theta = m\lambda$.

Step 1 — Grating spacing:
$N = 500$ lines/mm $\Rightarrow d = 1/500\,\text{mm} = 2000\,\text{nm}$.

Step 2 — First-order maximum ($m = 1$):

$$\sin\theta = \frac{m\lambda}{d} = \frac{600\,\text{nm}}{2000\,\text{nm}} = 0.300$$

$$\theta = \arcsin(0.300) \approx 17.46^\circ \approx 17.5^\circ$$

Why other options are wrong:

Q3.

- (B) 30.0°: Corresponds to $\sin\theta = 0.5$, i.e. $\lambda/d = 0.5$ or $\lambda = 1000\,\text{nm}$ (not 600 nm).
- (C) 45.0°: Corresponds to $\sin\theta = 1/\sqrt{2} \approx 0.707$, which would require $\lambda = 1414\,\text{nm}$.
- (D) 8.6°: Corresponds to $\sin\theta \approx 0.15$, roughly half the correct value.

Final Answer: $\theta \approx 17.5^\circ \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q3*

Solution

Concept — Gauss's Law for Electric Flux:

Gauss's law states $\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\varepsilon_0$ for any closed surface.

Key point: The flux depends only on the *total enclosed charge*, not on the size or shape of the Gaussian surface.

Application:

A sphere of radius $2R$ encloses the point charge Q at its centre. Therefore $Q_{\text{enc}} = Q$, and:

$$\Phi_E = \frac{Q}{\varepsilon_0}$$

The fact that the radius is $2R$ (rather than R or $3R$) is irrelevant — the flux is the same for any closed surface enclosing the same Q.

Why other options are wrong:

Q4.

- (A) $Q/(4\varepsilon_0)$: Incorrect; the 4 has no physical basis here.
- (C) $Q/(2\varepsilon_0)$: Incorrect; halving the flux would require only half the charge to be enclosed.
- (D) $4Q/\varepsilon_0$: Incorrect; the area of the sphere $\propto (2R)^2 = 4R^2$ does not multiply the flux.

Final Answer: $\Phi_E = Q/\varepsilon_0 \Rightarrow$ **(B)**

Answer: (B) *← Go back to Q4*

Solution

Concept — Entropy of Mixing of Ideal Gases:

The entropy of mixing for N_1 molecules of gas 1 and N_2 molecules of gas 2 (both at the same T and P) is:

$$\Delta S_{\text{mix}} = -k_B(N_1 \ln x_1 + N_2 \ln x_2)$$

where $x_i = N_i/(N_1 + N_2)$ are the mole fractions.

Step 1 — Equal amounts: $N_1 = N_2 = N/2$**, so** $x_1 = x_2 = 1/2$**:**

$$\Delta S_{\text{mix}} = -k_B\left(\frac{N}{2}\ln\frac{1}{2} + \frac{N}{2}\ln\frac{1}{2}\right) = -k_B \cdot N \ln\frac{1}{2} = Nk_B \ln 2$$

Step 2 — Per molecule $(\Delta S_{\text{mix}}/N)$**:**

$$\frac{\Delta S_{\text{mix}}}{N} = k_B \ln 2$$

Why other options are wrong:

Q5.

- (A) 0: Zero entropy of mixing applies only to identical gases (not physically distinct gases).
- (B) $k_B/2$: Incorrect; $\ln(1/2) = -\ln 2 \approx -0.693$, so the per-molecule entropy is $k_B \ln 2 \neq k_B/2$.
- (D) $2k_B \ln 2$: Twice the correct answer; the total is $Nk_B \ln 2$, not $2Nk_B \ln 2$ per molecule.

Final Answer: $\Delta S_{\text{mix}}/N = k_B \ln 2 \Rightarrow$ **(C)**

Answer: (C) *← Go back to Q5*

Solution

Concept — de Broglie Wavelength of an Accelerated Electron:

An electron accelerated through potential V acquires kinetic energy eV. Its de Broglie wavelength is $\lambda = h/p = h/\sqrt{2m_e eV}$.

Calculation:

$$\begin{aligned} 2m_e eV &= 2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19} \times 100 \\ &= 2 \times 9.11 \times 1.6 \times 10^{-48} \times 10^2 = 2 \times 14.576 \times 10^{-48} \\ &= 2.9152 \times 10^{-47}\,\text{kg}^2\text{m}^2\text{s}^{-2} \end{aligned}$$

$$\sqrt{2m_e eV} = \sqrt{2.9152 \times 10^{-47}} = 5.399 \times 10^{-24}\,\text{kg m s}^{-1}$$

$$\lambda = \frac{6.626 \times 10^{-34}}{5.399 \times 10^{-24}} = 1.227 \times 10^{-10}\,\text{m} = 1.23\,\text{Å}$$

Why other options are wrong:

Q6.

- (A) 0.61 Å: Half the correct value; would result from using $4m_e eV$ instead of $2m_e eV$ under the square root.
- (B) 2.46 Å: Double the correct value; corresponds to $V = 25$ V, not 100 V.
- (C) 0.39 Å: About $1/3$ of correct value; no simple physical basis.

Final Answer: $\lambda \approx 1.23$ Å $\Rightarrow$ **(D)**

Answer: (D) *← Go back to Q6*

Solution

Concept — Atoms per Unit Cell of FCC:

In the face-centred cubic structure:

Q7.

- **Corner atoms:** 8 corners, each shared by 8 unit cells $\Rightarrow 8 \times \frac{1}{8} = 1$ atom
- **Face-centre atoms:** 6 faces, each shared by 2 unit cells $\Rightarrow 6 \times \frac{1}{2} = 3$ atoms

Total atoms per unit cell: $1 + 3 = 4$

Physical consequence: This high coordination (12 nearest neighbours) and packing fraction ($\eta = \pi\sqrt{2}/6 \approx 0.740$) makes FCC very dense. Examples: Cu, Al, Ag, Au, Ni.

Why other options are wrong:

- (B) 2: This is the count for BCC (body-centred cubic: $8 \times 1/8 + 1 = 2$).
- (C) 8: This would be the case if all 8 corners were entirely within one cell (none shared), which is not the case.
- (D) 6: Counts only the 6 face-centre atoms without proper sharing fractions.

Final Answer: 4 atoms per unit cell $\Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q7*

Solution

Concept — NOR Gate Logic:

The NOR gate computes $Y = \overline{A + B}$ (NOT OR). Its truth table is:

Q8.

A	B	$A + B$	$Y = \overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

For $A = 1$, $B = 0$:

$A + B = 1 + 0 = 1$, so $Y = \overline{1} = 0$.

Why other options are wrong:

- (A) 1: The OR of $A = 1$ and $B = 0$ is 1; the NOR output is the complement, i.e. 0, not 1.
- (C) Same as input A: $A = 1$, but the output is $0 \neq 1$.
- (D) Indeterminate: NOR is a combinational logic gate with a deterministic output for given inputs; no clock is needed.

Final Answer: $Y = 0 \Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q8*

Solution

Concept — Q-value of Beta-Plus Decay:

In β^+ decay, atomic masses include the electron cloud. When writing M_P and M_D as *atomic* masses, the positron mass must be accounted for by subtracting $2m_e$ (one m_e for the emitted positron, another because the daughter atom has one fewer atomic electron than its parent).

Derivation:

The nuclear masses are $M_P^{\text{nuc}} = M_P - Z_P m_e$ and $M_D^{\text{nuc}} = M_D - Z_D m_e = M_D - (Z_P - 1)m_e$. In β^+ decay the positron has mass m_e, so:

$$Q = \left[M_P^{\text{nuc}} - M_D^{\text{nuc}} - m_e\right]c^2 = \left[M_P - Z_P m_e - M_D + (Z_P - 1)m_e - m_e\right]c^2$$

$$= \left[M_P - M_D - 2m_e\right]c^2$$

Why other options are wrong:

Q9.

- (A) $(M_P - M_D)c^2$: Ignores the two extra electron mass terms arising from the change in atomic number.
- (B) $(M_D - M_P)c^2$: Sign is wrong; parent must be heavier for β^+ to be energetically allowed.
- (D) $(M_P - M_D + 2m_e)c^2$: Wrong sign on the $2m_e$ correction.

Final Answer: $Q = (M_P - M_D - 2m_e)c^2 \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q9*

Solution

Concept — Wronskian of Two Functions:

$W[y_1, y_2] = y_1 y_2' - y_1' y_2$

Step 1 — Compute derivatives:

$y_1 = \sin x \;\Rightarrow\; y_1' = \cos x$

$y_2 = \cos x \;\Rightarrow\; y_2' = -\sin x$

Step 2 — Evaluate:

$$W = \sin x \cdot (-\sin x) - \cos x \cdot \cos x = -\sin^2 x - \cos^2 x = -(sin^2 x + \cos^2 x) = -1$$

The Wronskian is a non-zero constant ($-1 \neq 0$), confirming that $\sin x$ and $\cos x$ are linearly independent solutions of $y'' + y = 0$.

Why other options are wrong:

Q10.

- (A) 0: A zero Wronskian would imply linear dependence, but $\sin x$ and $\cos x$

are clearly independent.

- (B) 1: Differs in sign from the correct result.
- (C) $\sin 2x$: $\sin 2x = 2\sin x \cos x$, which is x-dependent; the Wronskian is a constant.

Final Answer: $W = -1 \Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q10*

Solution

Concept — Kepler's Second Law and Conservation of Angular Momentum:
At perihelion and aphelion, the velocity is perpendicular to the radius. Conservation of angular momentum $L = \mu r v =$ const gives:

$$r_{\text{min}}\, v_{\text{max}} = r_{\text{max}}\, v_{\text{min}}$$

Step 1 — Apply the relation:

$$\frac{v_{\text{max}}}{v_{\text{min}}} = \frac{r_{\text{max}}}{r_{\text{min}}} = 3$$

Physical note: The planet moves fastest at perihelion and slowest at aphelion, sweeping equal areas in equal times.

Why other options are wrong:

Q11.

- (B) 9: Would require $v \propto 1/r^2$, but angular momentum conservation gives $v \propto 1/r$.
- (C) $\sqrt{3}$: Incorrect; the ratio of speeds equals the ratio of distances, not its square root.
- (D) $1/3$: This is $v_{\text{min}}/v_{\text{max}}$ (the ratio is inverted).

Final Answer: $v_{\text{max}}/v_{\text{min}} = 3 \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q11*

Solution

Concept — Quality Factor and Resonance Bandwidth:
For a damped oscillator with equation $\ddot{x} + 2\gamma\dot{x} + \omega_0^2 x = F_0 \cos(\omega t)/m$, the amplitude response has a peak at $\omega \approx \omega_0$ with maximum value $A_{\text{max}} \propto 1/(2\gamma)$.

Half-power bandwidth: The frequencies at which amplitude falls to $A_{\text{max}}/\sqrt{2}$ are

at $\omega = \omega_0 \pm \gamma$, giving:

$$\Delta\omega = 2\gamma = \frac{\omega_0}{Q}$$

Quality factor: $Q = \omega_0/(2\gamma) = \omega_0 m/b$ where b is the damping coefficient, so:

$$\Delta\omega = \frac{\omega_0}{Q}$$

Why other options are wrong:

Q12.

- (A) $\Delta\omega = \omega_0 Q$: This is the inverse relation; high-Q oscillators have *narrow* bandwidth, not wide.
- (C) $\Delta\omega = \omega_0/Q^2$: Over-counts the Q factor by one power.
- (D) $\Delta\omega = Q/\omega_0$: Incorrect dimensions ($[Q/\omega_0]$ is time, not angular frequency).

Final Answer: $\Delta\omega = \omega_0/Q \Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q12*

Solution

Concept — Electric Field Inside a Uniformly Charged Sphere:
Apply Gauss's law with a spherical Gaussian surface of radius $r < R$ concentric with the charged sphere.

Step 1 — Charge enclosed:

$$Q_{\text{enc}} = \rho \cdot \frac{4}{3}\pi r^3 = \frac{3Q}{4\pi R^3} \cdot \frac{4\pi r^3}{3} = Q\left(\frac{r}{R}\right)^3$$

Step 2 — Gauss's law (spherical symmetry $\Rightarrow$ E is radial and uniform on Gaussian surface):

$$E(r) \cdot 4\pi r^2 = \frac{Q_{\text{enc}}}{\varepsilon_0} = \frac{Q\, r^3}{\varepsilon_0 R^3}$$

$$E(r) = \frac{Q\, r}{4\pi\varepsilon_0 R^3}$$

The field grows linearly with r inside the sphere.

Why other options are wrong:

Q13.

- (A) $Q/(4\pi\varepsilon_0 r^2)$: This is the *outside* field ($r > R$), not the inside field.
- (B) 0: Zero field inside is the result for a *conducting* shell, not a solid insulating sphere.
- (D) $Qr^2/(4\pi\varepsilon_0 R^4)$: Wrong power of r; Gauss's law gives $E \propto r$, not r^2, inside.

Final Answer: $E(r) = Qr/(4\pi\varepsilon_0 R^3) \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q13*

Solution

Concept — Carnot Efficiency:

The Carnot cycle gives the maximum efficiency for a heat engine operating between temperatures T_H and T_C:

$$\eta_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$$

Calculation:

$$\eta = 1 - \frac{300\,\text{K}}{600\,\text{K}} = 1 - \frac{1}{2} = \frac{1}{2} = 50\%$$

Why other options are wrong:

Q14.

- (A) 25%: Corresponds to $T_C/T_H = 3/4$, e.g. $T_C = 450\,\text{K}$ for $T_H = 600\,\text{K}$.
- (B) 33%: Corresponds to $T_C/T_H = 2/3$, e.g. $T_C = 400\,\text{K}$ for $T_H = 600\,\text{K}$.
- (C) 75%: Over-estimated; Carnot efficiency is limited by the temperature ratio.

Final Answer: $\eta = 50\% \Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q14*

Solution

Concept — 3D Isotropic Harmonic Oscillator Energy Levels:

The Hamiltonian separates into three independent 1D oscillators. The energy eigenvalues are:

$$E_N = \left(N + \frac{3}{2}\right)\hbar\omega, \quad N = n_x + n_y + n_z = 0, 1, 2, \ldots$$

Degeneracy of level N: number of ways to write $N = n_x + n_y + n_z$ with non-negative integers $= \frac{(N+1)(N+2)}{2}$.

Step 1 — Ground state ($N = 0$):

$E_0 = \frac{3}{2}\hbar\omega$; degeneracy $= \frac{1\times 2}{2} = 1$ (only $n_x = n_y = n_z = 0$).

Step 2 — First excited state ($N = 1$):

$E_1 = \frac{5}{2}\hbar\omega$; degeneracy $= \frac{2\times 3}{2} = 3$ (states: $(1,0,0), (0,1,0), (0,0,1)$).

Why other options are wrong:

Q15.

- (B) $E_0 = \frac{1}{2}\hbar\omega$: This is the ground state of a 1D oscillator; the 3D ground state

has three zero-point energies.

- (C) Ground state degeneracy $= 3$: False; the ground state $(0, 0, 0)$ is unique.
- (D) $E_0 = \hbar\omega$: Incorrect; the zero-point contribution is $\frac{3}{2}\hbar\omega$ not $\hbar\omega$.

Final Answer: $E_0 = \frac{3}{2}\hbar\omega$ (deg. 1), $E_1 = \frac{5}{2}\hbar\omega$ (deg. 3) $\Rightarrow$ **(A)**

Answer: (A) *← Go back to Q15*

Solution

Concept — Direct vs Indirect Band Gap:

In a *direct-gap* semiconductor the conduction band minimum (CBM) and valence band maximum (VBM) occur at the **same** k-point (typically $\mathbf{k} = 0$ in the Brillouin zone, the Γ point). An electron can make a vertical transition in k-space by absorbing or emitting a photon, without needing a phonon to conserve momentum.

GaAs: CBM and VBM are both at Γ ($\mathbf{k} = 0$). Band gap $E_g \approx 1.42\,\text{eV}$ at room temperature. This makes GaAs ideal for LEDs and laser diodes.

Contrast with Si: Silicon has an *indirect* gap ($E_g \approx 1.12\,\text{eV}$); its CBM is near the X-point, so optical transitions require phonon assistance.

Why other options are wrong:

Q16.

- (A) "GaAs is indirect-gap": Factually wrong; GaAs is the archetypal direct-gap semiconductor.
- (C) "Si is direct-gap": Factually wrong; Si is indirect-gap.
- (D) "optical transitions forbidden": In a direct-gap semiconductor, optical transitions are *allowed* (no momentum mismatch to bridge).

Final Answer: GaAs is direct-gap, $E_g \approx 1.42\,\text{eV} \Rightarrow$ **(B)**

Answer: (B) *← Go back to Q16*

Solution

Concept — Inverting Op-Amp Integrator:

For an ideal op-amp with virtual earth at the inverting input (since $V^+ = 0$ and $V^- = V^+ = 0$):

Q17.

- Current through R: $i_R = V_{\text{in}}/R$ (flowing into the summing node)
- Current through C: $i_C = C\,dV_{\text{out}}/dt$ (flowing from output back to summing node)

KCL at summing node: $i_R + i_C = 0$ (no current into ideal op-amp):

$$\frac{V_{\text{in}}}{R} + C\,\frac{dV_{\text{out}}}{dt} = 0 \;\Rightarrow\; \frac{dV_{\text{out}}}{dt} = -\frac{V_{\text{in}}}{RC}$$

$$V_{\text{out}}(t) = -\frac{1}{RC}\int V_{\text{in}}\,dt + \text{const}$$

Why other options are wrong:

- (A) $-(R/C)dV_{\text{in}}/dt$: This would be the output of a *differentiator* (R in feedback, C at input).
- (B) $(1/RC)\int V_{\text{in}}\,dt$: Missing the minus sign; the inverting configuration always introduces a sign inversion.
- (D) $-RC\,dV_{\text{in}}/dt$: This is the differentiator formula with wrong coefficient (RC instead of $1/(RC)$).

Final Answer: $V_{\text{out}} = -\frac{1}{RC}\int V_{\text{in}}\,dt \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q17*

Solution

Concept — Nuclear Magic Numbers:
Magic numbers arise from large gaps in the single-particle energy levels of the nuclear shell model. At magic numbers, both proton and neutron shells are completely filled, giving extra binding energy and stability.

Magic numbers (in order): $2, 8, 20, 28, 50, 82, 126$

Physical evidence: Nuclei with magic Z or N (e.g. ^{4}He, ^{16}O, ^{40}Ca, ^{48}Ca, ^{208}Pb) have unusual stability, larger binding energy per nucleon, and reduced neutron capture cross-sections.

Why other options are wrong:

Q18.

- (A) $2, 6, 14, 28$: The value 6 is not a magic number; 8 (filling the $1p$ shell) is.
- (B) $2, 8, 18, 32$: These are the electron magic numbers of atomic shells (noble gases), not nuclear magic numbers.
- (C) $2, 8, 14, 20$: The value 14 is not a magic number; the correct sequence jumps from 8 to 20.

Final Answer: $2, 8, 20, 28 \Rightarrow$ **(D)**

Answer: (D) ← *Go back to Q18*

Solution

Concept — Physicist's Hermite Polynomials:
The physicist's Hermite polynomials are: $H_0(x) = 1$, $H_1(x) = 2x$, $H_2(x) = 4x^2 - 2$, $H_3(x) = 8x^3 - 12x$, ...

They satisfy the Hermite ODE $H_n'' - 2xH_n' + 2nH_n = 0$ and the recurrence $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$.

Evaluation at $x = 1$:

$$H_2(1) = 4(1)^2 - 2 = 4 - 2 = 2$$

These polynomials appear as the position-space wave functions of the quantum harmonic oscillator: $\psi_n(x) \propto H_n(\alpha x)\, e^{-\alpha^2 x^2/2}$.

Why other options are wrong:

Q19.

- (B) 4: This is $H_2(1)$ without the constant term: $4 \cdot 1^2 = 4$, but $H_2(x)$ includes -2.
- (C) -2: This confuses the constant term with the entire polynomial value.
- (D) 6: This might arise from using $H_3(1) = 8 - 12 = -4$ or some other wrong polynomial.

Final Answer: $H_2(1) = 2 \Rightarrow$ **(A)**

Answer: (A) *← Go back to Q19*

Solution

Concept — Torque-Free Precession of a Symmetric Top (Euler's Equations):
For a torque-free symmetric top with $I_1 = I_2 \neq I_3$, Euler's equations give:

$$\dot{\omega}_1 = \frac{I_2 - I_3}{I_1}\omega_2\omega_3, \qquad \dot{\omega}_2 = \frac{I_3 - I_1}{I_2}\omega_1\omega_3, \qquad \dot{\omega}_3 = 0$$

Step 1 — ω_3 is constant.

Step 2 — Precession of ω_1 and ω_2:
Let $\omega_\perp = \omega_1 + i\omega_2$. Euler's equations give $\dot{\omega}_\perp = -i\Omega\,\omega_\perp$ where:

$$\Omega = \frac{I_3 - I_1}{I_1}\,\omega_3$$

This is the angular frequency of precession of the angular velocity vector about the symmetry axis (body frame).

Why other options are wrong:

Q20.

- (A) $(I_3/I_1)\omega_3$: Ignores the term -1 from the difference; dimensionally plausi-

ble but incorrect.

- (C) $[(I_1 - I_3)/I_1]\omega_3$: Sign error; if $I_3 > I_1$ (oblate top), the precession is prograde; (C) would give the opposite sign.
- (D) $[(I_3 - I_1)/I_3]\omega_3$: Wrong denominator; Euler's equation gives I_1 in the denominator, not I_3.

Final Answer: $\Omega = [(I_3 - I_1)/I_1]\,\omega_3 \Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q20*

Solution

Concept — Poynting Vector and EM Wave Intensity:
For a linearly polarised plane wave in vacuum with peak electric field amplitude E_0, the time-averaged intensity (Poynting magnitude) is:

$$\langle S \rangle = \frac{E_0^2}{2\mu_0 c}$$

where $\mu_0 c = 4\pi \times 10^{-7} \times 3 \times 10^8 = 4\pi \times 30 = 120\pi \approx 376.99\,\Omega$ is the impedance of free space.

Calculation:

$$\langle S \rangle = \frac{(300)^2}{2 \times 376.99} = \frac{90000}{753.98} \approx 119.4\,\text{W/m}^2$$

Why other options are wrong:

Q21.

- (A) $47.8\,\text{W/m}^2$: Off by a factor of ≈ 2.5; possibly computed with the wrong formula.
- (B) $60.0\,\text{W/m}^2$: Approximately half the correct value.
- (D) $238.8\,\text{W/m}^2$: Exactly double the correct answer; omits the factor of 2 in the denominator (uses $E_0^2/(\mu_0 c)$ instead of $E_0^2/(2\mu_0 c)$).

Final Answer: $\langle S \rangle \approx 119.4\,\text{W/m}^2 \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q21*

Solution

Concept — High-Temperature Limit of Quantum Statistics:
At high temperature where $k_B T \gg \varepsilon - \mu$, let $x = (\varepsilon - \mu)/(k_B T) \ll 1$.

Bose–Einstein: $\langle n_{\text{BE}} \rangle = \dfrac{1}{e^x - 1} \approx \dfrac{1}{x} - \dfrac{1}{2} + \cdots$ — diverges as $x \to 0^+$ (chemical potential must be negative for bosons).

More precisely (when $e^x \gg 1$, i.e. x is large enough that $e^x \gg 1$):

$$\langle n_{\text{BE}} \rangle \approx e^{-x} + e^{-2x} + \cdots \approx e^{-x}, \quad \langle n_{\text{FD}} \rangle \approx e^{-x} - e^{-2x} + \cdots \approx e^{-x}$$

Both approach the Maxwell–Boltzmann distribution $e^{-(\varepsilon-\mu)/k_BT}$ in the limit $e^{(\varepsilon-\mu)/k_BT} \gg 1$ (which is the classical, high-temperature, low-density limit).

Why other options are wrong:

Q22.

- (A) Remain distinct: They converge to the same classical limit at high T.
- (B) Both $\to 1/2$: Incorrect; $\langle n_{\text{FD}} \rangle \to 1/2$ only at $\varepsilon = \mu$, not universally at high T.
- (C) $\langle n_{\text{BE}} \rangle \to \infty$: Only at $\varepsilon \to \mu^+$ from above; for fixed $\varepsilon > \mu$ at high T both vanish as $e^{-(\varepsilon-\mu)/k_BT}$.

Final Answer: Both $\to e^{-(\varepsilon-\mu)/k_BT}$ (Maxwell–Boltzmann) $\Rightarrow$ **(D)**

Answer: (D) *← Go back to Q22*

Solution

Concept — Expectation Values for QHO Energy Eigenstates:
The energy eigenstates $|n\rangle$ of the quantum harmonic oscillator have definite parity: $\psi_n(-x) = (-1)^n \psi_n(x)$.

Expectation value of x:
Since $|\psi_n(x)|^2$ is an even function of x (for all n), and x is an odd function:

$$\langle x \rangle = \int_{-\infty}^{\infty} x\, |\psi_n(x)|^2\, dx = 0 \quad \forall\, n$$

(Odd integrand integrated over symmetric domain.)

Expectation value of x^2:
Using $\hat{x} = \sqrt{\hbar/(2m\omega)}(\hat{a} + \hat{a}^\dagger)$ and $\hat{x}^2 = [\hbar/(2m\omega)](\hat{a}^2 + \hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger + \hat{a}^{\dagger 2})$:

$$\langle n|\hat{x}^2|n\rangle = \frac{\hbar}{2m\omega}\langle n|(2\hat{a}^\dagger\hat{a} + 1)|n\rangle = \frac{\hbar}{2m\omega}(2n+1) = \left(n + \frac{1}{2}\right)\frac{\hbar}{m\omega}$$

Why other options are wrong:

Q23.

- (B) $\langle x \rangle = \sqrt{(n+1/2)\hbar/(m\omega)}$: The expectation value of x (not $\sqrt{\langle x^2 \rangle}$) is zero.
- (C) $\langle x \rangle = 0$ only for $n = 0$: Parity argument makes $\langle x \rangle = 0$ for *all* n, not just $n = 0$.
- (D) $\langle x^2 \rangle = 0$: $\langle x^2 \rangle \neq 0$ for any bound state (Heisenberg uncertainty: $\Delta x \neq 0$).

Final Answer: $\langle x \rangle = 0$; $\langle x^2 \rangle = (n + \frac{1}{2})\hbar/(m\omega) \Rightarrow$ **(A)**

Answer: (A) ← *Go back to Q23*

Solution

Concept — Built-in Potential of a p-n Junction:
In thermal equilibrium, the Fermi level is constant throughout the device. The built-in potential is the energy difference between the Fermi levels before junction formation, divided by e:

Derivation: For the n-side, $n_n = N_D$ and $n_p = n_i^2/N_A$ (minority carriers on p-side). The potential difference satisfies:

$$eV_{\text{bi}} = k_B T \ln\left(\frac{n_n}{n_p}\right) = k_B T \ln\left(\frac{N_D}{n_i^2/N_A}\right) = k_B T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$V_{\text{bi}} = \frac{k_B T}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Why other options are wrong:

Q24.

- (A) $\ln(N_A/N_D)$: Misses the n_i^2 factor and ignores one type of carrier.
- (C) $k_B T \ln(\cdots)$: Missing the division by e; the built-in potential has units of volts, so the factor is $k_B T/e$.
- (D) $(e/k_B T)\ln(\cdots)$: The ratio $e/k_B T$ is inverted; should be $k_B T/e$.

Final Answer: $V_{\text{bi}} = (k_B T/e)\ln(N_A N_D/n_i^2) \Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q24*

Solution

Concept — Inverting Summing Amplifier:
For an ideal op-amp in inverting configuration with n inputs each through a resistor R and feedback resistor R_f, the output is:

$$V_{\text{out}} = -\frac{R_f}{R}(V_1 + V_2 + \cdots + V_n)$$

For $R_f = R$ and two inputs V_1, V_2:

$$V_{\text{out}} = -\frac{R}{R}(V_1 + V_2) = -(V_1 + V_2)$$

Physical reason: The virtual earth at the inverting input forces the sum of currents $(V_1/R + V_2/R)$ to flow entirely through R_f, and the inverting configuration gives a sign reversal.

Why other options are wrong:

Q25.

- (A) $+(V_1 + V_2)$: Correct magnitude but wrong sign; the inverting op-amp always produces a sign inversion.
- (B) $-(V_1 + V_2)/2$: Would require $R_f = R/2$, not $R_f = R$.
- (D) $(V_1 - V_2)/2$: This is the output of a difference amplifier, not a summing amplifier.

Final Answer: $V_{\text{out}} = -(V_1 + V_2) \Rightarrow$ **(C)**

Answer: (C) *← Go back to Q25*

Solution

Concept — Clebsch–Gordan Decomposition ($\ell = 1$, $s = 1/2$):
When adding angular momenta $j_1 = 1$ and $j_2 = 1/2$, the possible total j values range from $|j_1 - j_2|$ to $j_1 + j_2$ in integer steps:

$$j = \frac{3}{2} \quad \text{or} \quad j = \frac{1}{2}$$

Degeneracies ($2j + 1$ states each):

$$j = \frac{3}{2} : \; 2 \times \frac{3}{2} + 1 = 4 \text{ states}; \quad j = \frac{1}{2} : \; 2 \times \frac{1}{2} + 1 = 2 \text{ states}$$

Check: Total $= 4 + 2 = 6 = (2 \times 1 + 1)(2 \times \frac{1}{2} + 1) = 3 \times 2$ ✓

Hydrogen ground state: The $n = 1$ state has $\ell = 0$ (the $1s$ orbital), so $j = s = 1/2$. The Clebsch–Gordan decomposition for $\ell = 1$ is *not* relevant to the ground state.

Why other options are wrong:

Q26.

- (A) $j = 1$ or $j = 0$: These would be the result for adding two spin-1/2 particles, not $\ell = 1$ and $s = 1/2$.
- (B) Only $j = 3/2$: This is only one of the two allowed values.
- (C) H ground state has $j = 3/2$: Incorrect; the ground state has $\ell = 0$, hence $j = 1/2$.

Final Answer: $j = 3/2$ (4 states) or $j = 1/2$ (2 states); H ground state $j = 1/2 \Rightarrow$ **(D)**

Answer: (D) *← Go back to Q26*

Solution

Concept — Equipartition Theorem for Diatomic Gas:
The equipartition theorem assigns $\frac{1}{2}k_BT$ of energy to each *quadratic* degree of freedom in the Hamiltonian.

For a diatomic molecule at moderate temperature (where vibrational modes are not yet excited):

Q27.

- **3 translational** degrees of freedom: $\frac{3}{2}k_BT$ per molecule
- **2 rotational** degrees of freedom (rotation about two axes perpendicular to the molecular axis): $\frac{2}{2}k_BT = k_BT$ per molecule

Total: $\frac{5}{2}k_BT$ per molecule; molar heat capacity:

$$C_V = \frac{5}{2}N_Ak_B = \frac{5}{2}R \approx 20.8\,\text{J mol}^{-1}\text{K}^{-1}$$

Why other options are wrong:

- (B) 3 DOF, $C_V = \frac{3}{2}R$: This applies to a *monatomic* ideal gas (only translational modes).
- (C) 7 DOF, $C_V = \frac{7}{2}R$: This includes vibrational modes (2 extra: 1 kinetic + 1 potential), which are not excited at moderate temperatures.
- (D) Same as monatomic: Incorrect; the additional rotational modes increase C_V.

Final Answer: 5 DOF; $C_V = \frac{5}{2}R \Rightarrow$ **(A)**

Answer: (A) *← Go back to Q27*

Solution

Concept — Hall Effect:
When current I flows in the x-direction through a slab of thickness t in a magnetic field $\mathbf{B} = B\hat{z}$, the Lorentz force deflects charge carriers in the y-direction until equilibrium.

For electrons (negative carriers):

Q28.

- Electron drift is in $-x$ direction (opposite to conventional current I)
- Lorentz force $\mathbf{F} = q\mathbf{v} \times \mathbf{B} = (-e)(-v_d\hat{x}) \times (B\hat{z}) = (-e)(-v_dB)(\hat{x} \times \hat{z}) = -ev_dB(-\hat{y}) = ev_dB\hat{y}$
- Electrons accumulate at the $+y$ face; Hall field E_H builds up in $-y$ direction.

Equilibrium: $eE_H = ev_dB \Rightarrow E_H = v_dB$.

Hall voltage: $V_H = E_H \cdot t = v_d\,B\,t$.

Current: $I = nev_d A_{\text{cross}} = ne\, v_d\, (t \cdot w)$, so $v_d = I/(net \cdot w)$.

Therefore: $V_H = \dfrac{I}{new} \cdot B \cdot t \cdot \dfrac{w}{t}$... Using $V_H = E_H t = v_d B t = \dfrac{I}{ne\, t\, w} \cdot B\, t = \dfrac{IB}{ne\, w}$.
But with the geometry of the problem where t is the dimension along B:

$$V_H = \frac{IB}{net}$$

The Hall coefficient is $R_H = 1/(ne)$ for electrons.

Why other options are wrong:

- (A) $R_H = ne$: Sign and magnitude wrong; the Hall coefficient has units of m^3/C, not C/m^3.
- (C) $V_H = neIB/t$: Inverted formula; would give enormous values for typical carrier densities.
- (D) $R_H = ne^2$: Wrong units; the Hall coefficient should have units of m^3/C.

Final Answer: $V_H = IB/(net)$, $R_H = 1/(ne) \Rightarrow$ **(B)**

Answer: (B) ← *Go back to Q28*

Solution

Concept — Young's Double-Slit Fringe Width:
The fringe spacing (distance between consecutive bright or dark fringes) in Young's double-slit experiment is:

$$\Delta y = \frac{\lambda D}{d}$$

where λ is the wavelength, D is the slit-to-screen distance, and d is the slit separation.

Calculation:

$$\Delta y = \frac{500 \times 10^{-9}\,\text{m} \times 1\,\text{m}}{0.5 \times 10^{-3}\,\text{m}} = \frac{5 \times 10^{-7}}{5 \times 10^{-4}} = 1 \times 10^{-3}\,\text{m} = 1.0\,\text{mm}$$

Why other options are wrong:

Q29.

- (A) $0.5\,\text{mm}$: Half the correct value; would require $d = 1\,\text{mm}$ or $D = 0.5\,\text{m}$.
- (B) $2.0\,\text{mm}$: Double the correct value; would require $d = 0.25\,\text{mm}$.
- (D) $0.25\,\text{mm}$: Quarter of the correct value; would require $d = 2\,\text{mm}$.

Final Answer: $\Delta y = 1.0\,\text{mm} \Rightarrow$ **(C)**

Answer: (C) ← *Go back to Q29*

Solution

Concept — Bohr Model Photon Energy ($n = 4 \to n = 2$):
The Bohr energy formula gives $E_n = -13.6/n^2$ eV.

Step 1 — Energies:

$$E_4 = -\frac{13.6}{16} = -0.850\,\text{eV}, \qquad E_2 = -\frac{13.6}{4} = -3.400\,\text{eV}$$

Step 2 — Photon energy:

$$\Delta E = E_4 - E_2 = -0.850 - (-3.400) = 2.550\,\text{eV}$$

This photon (of energy 2.55 eV) falls in the visible green region (the H_β line of the Balmer series, $\lambda \approx 486$ nm).

Frequency: $\nu = \Delta E/h = 2.55 \times 1.6 \times 10^{-19}/(6.626 \times 10^{-34}) \approx 6.17 \times 10^{14}$ Hz.

Why other options are wrong:

Q30.

- (A) 3.40 eV: This is $|E_2|$, not the transition energy ΔE.
- (B) 1.51 eV: This is $|E_3| = 13.6/9$ (energy of $n = 3$ state), not the $4 \to 2$ transition energy.
- (C) 0.66 eV: This is approximately $E_4 - E_3 = -0.85 + 1.51 = 0.66$ eV (wrong transition).

Final Answer: $\Delta E = 2.55$ eV $\Rightarrow$ **(D)**

Answer: (D) *← Go back to Q30*

Section B Solutions — MSQ

Q31.

Solution

Concept — Kinematics of Rigid Body Rotation About a Fixed Axis:

(A) "All particles have the same linear velocity": WRONG. For rotation about a fixed axis, the linear speed of a particle at distance r from the axis is $v = r\omega$. Since different particles are at different distances r, they have different linear speeds (even though they sweep through the same angle per unit time).

(B) "All particles have the same angular velocity ω": CORRECT. By definition of rigid body rotation, every particle rotates with the same angular velocity ω about the fixed axis. This is what makes the body "rigid".

(C) "$L = I\omega$": CORRECT. The angular momentum about the rotation axis is $L = I\omega$, where $I = \sum m_i r_i^2$ is the moment of inertia about that axis.

(D) "$KE = \frac{1}{2}Mv_{\text{cm}}^2$": WRONG. The rotational kinetic energy is $KE = \frac{1}{2}I\omega^2$ (not $\frac{1}{2}Mv_{\text{cm}}^2$). The formula $\frac{1}{2}Mv_{\text{cm}}^2$ gives only the translational kinetic energy of the centre of mass; for pure rotation about a fixed axis, there is no translational motion of the centre of mass.

Correct options: (B) and (C).

Answer: (BC) ← *Go back to Q31*

Q32.

Solution

Concept — Gauss's Law in Electrostatics:

(A) "Total flux equals $Q_{\text{enc}}/\varepsilon_0$": CORRECT. This is precisely Gauss's law: $\oint_S \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\varepsilon_0$ for any closed surface S.

(B) "Only for spherical surfaces": WRONG. Gauss's law holds for *any* closed surface, regardless of shape. Spherical Gaussian surfaces are merely computationally convenient when there is spherical symmetry; the law itself is general.

(C) "Holds for any closed Gaussian surface": CORRECT. As a direct consequence of Coulomb's law and the superposition principle, Gauss's law is valid for any closed surface in any geometry.

(D) "$E = 0$ inside a conductor follows from Gauss's law": CORRECT. In a conductor in electrostatic equilibrium, any net charge resides on the surface. Applying a Gaussian surface just inside the conductor surface: $Q_{\text{enc}} = 0 \Rightarrow \oint \mathbf{E} \cdot d\mathbf{A} = 0$. By the symmetry of the conductor, this implies $\mathbf{E} = 0$ everywhere inside.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q32*

Q33.

Solution

Concept — Canonical Ensemble Thermodynamics:

(A) $Z = \sum_n e^{-\beta E_n}$: CORRECT. The canonical partition function sums $e^{-\beta E_n}$ (the Boltzmann factor) over all accessible microstates n.

(B) $F = -k_B T \ln Z$: CORRECT. The Helmholtz free energy $F = U - TS$ is related to the partition function by $F = -k_B T \ln Z$. This is derived from the relation between entropy and Z: $S = k_B(\ln Z + \beta\langle E\rangle)$.

(C) $S = -\partial Z/\partial T$: WRONG. The entropy is $S = -\partial F/\partial T = k_B(\ln Z + T\,\partial \ln Z/\partial T)$, *not* $-\partial Z/\partial T$. Note that differentiating Z (not F) directly gives the wrong formula.

(D) $\langle E\rangle = -\partial(\ln Z)/\partial\beta$**: CORRECT.** The mean energy is $\langle E\rangle = \sum_n E_n e^{-\beta E_n}/Z = -\partial(\ln Z)/\partial\beta$. This is a fundamental result of the canonical ensemble.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q33*

Q34.

Solution

Concept — BJT in Common-Emitter Active Mode:

(A) "Base-emitter junction is reverse biased": WRONG. In active mode, the base-emitter (BE) junction is *forward* biased (typically $V_{BE} \approx 0.6$–$0.7\,\text{V}$ for silicon). This forward bias is what injects minority carriers into the base.

(B) $I_C = \beta I_B$**: CORRECT.** In active mode, the collector current is β times the base current, where β (the DC current gain, or h_{FE}) is typically 50–300 for silicon BJTs.

(C) $I_E = I_B + I_C$**: CORRECT.** By KCL at the transistor node: the emitter current equals the sum of base and collector currents. Since $I_C = \beta I_B$: $I_E = (1+\beta)I_B$.

(D) "Collector-emitter junction is forward biased": WRONG. In active mode, the collector-base (CB) junction is *reverse* biased. This is what sweeps the injected minority carriers across the thin base into the collector.

Correct options: (B) and (C).

Answer: (BC) ← *Go back to Q34*

Q35.

Solution

Concept — Phonon Properties:

(A) "Phonons are quanta of lattice vibrational energy": CORRECT. A phonon in a mode of frequency ω carries energy $\hbar\omega$. This quantisation arises from treating the lattice Hamiltonian (in the harmonic approximation) as a sum of independent quantum harmonic oscillators.

(B) "Phonons obey Fermi–Dirac statistics": WRONG. Phonons are bosons (integer spin particles). They obey *Bose–Einstein* statistics. Their occupation number is not limited to 0 or 1.

(C) "Average occupation given by Bose–Einstein distribution": CORRECT. The

average number of phonons in a mode of angular frequency ω at temperature T is:

$$\bar{n}_{\text{phonon}} = \frac{1}{e^{\hbar\omega/k_BT} - 1}$$

(Here chemical potential $\mu = 0$ because phonon number is not conserved.)

(D) "Acoustic phonons: $\omega \to 0$ as $k \to 0$": CORRECT. Acoustic branches are gapless: $\omega = v_s k$ in the long-wavelength limit (where v_s is the sound velocity). This contrasts with optical phonons, which have a non-zero frequency at $k = 0$.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q35*

Q36.

Solution

Concept — Gauss's Divergence Theorem:

(A) $\oint_S \mathbf{F} \cdot d\mathbf{A} = \int_V (\nabla \cdot \mathbf{F})\, dV$: CORRECT. This is exactly the Divergence (Gauss's) theorem, one of the fundamental theorems of vector calculus.

(B) "Requires $\mathbf{F}$ to be continuously differentiable": CORRECT. The theorem requires that all first partial derivatives of $\mathbf{F}$ be continuous on V and its boundary $S = \partial V$. Without this smoothness, $\nabla \cdot \mathbf{F}$ may not be integrable.

(C) "Applies only when $\nabla \cdot \mathbf{F} = 0$": WRONG. The theorem holds for any sufficiently smooth $\mathbf{F}$, divergence-free or not. If $\nabla \cdot \mathbf{F} = 0$, the theorem simply says the net outward flux is zero.

(D) "Applied to $\mathbf{E}$ of a point charge gives Gauss's law": CORRECT. Applying the divergence theorem to $\mathbf{E}$ with $\nabla \cdot \mathbf{E} = \rho/\varepsilon_0$ (Maxwell's equation) yields $\oint \mathbf{E} \cdot d\mathbf{A} = Q_{\text{enc}}/\varepsilon_0$.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q36*

Q37.

Solution

Concept — Gamma Decay:

(A) "Z changes in γ decay": WRONG. Gamma decay is a nuclear de-excitation in which the nucleus emits a photon (γ ray) to transition from an excited state to a lower energy state. Neither Z (proton number) nor N (neutron number) changes; only the nuclear energy level changes.

(B) "A photon is emitted": CORRECT. The emitted γ radiation is a high-energy photon (typically 0.01 to 10 MeV). The photon carries away the energy difference between the initial and final nuclear states.

(C) "Mass number A is unchanged": CORRECT. Since $A = Z + N$ and neither Z nor N changes in γ decay, A remains constant. The nucleus is the same nuclide, just in a lower energy (less excited) state.

(D) "Neutron number changes": WRONG. The neutron number $N = A - Z$ is unchanged in γ decay (neither A nor Z changes).

Correct options: (B) and (C).

Answer: (BC) *← Go back to Q37*

Q38.

Solution

Concept — Conditions for Two-Beam Interference:

(A) "Beams must be coherent": CORRECT. Coherence (fixed or slowly varying phase difference) is essential for stable fringes. With random phase differences, the fringe pattern averages out to uniform illumination.

(B) "Both beams must be polarised in the same plane": WRONG. While identical polarisation maximises fringe contrast (since the dot product $\mathbf{E}_1 \cdot \mathbf{E}_2$ is maximised), fringes can be observed even with partially overlapping polarisations as long as the interfering components are parallel. Completely orthogonal polarisations give *no* fringes, but this is not a requirement of "the same plane".

(C) "Path difference $<$ coherence length": CORRECT. The coherence length $L_c = c/\Delta\nu$ is the path difference over which fringes remain visible. If the path difference exceeds L_c, the phase difference becomes random and fringes wash out.

(D) "Same frequency": CORRECT. If the two beams have different frequencies $\omega_1 \neq \omega_2$, the interference pattern oscillates in time at the beat frequency $|\omega_1 - \omega_2|$ and cannot be observed as a static pattern. Stable fringes require the same frequency (monochromatic or narrow-bandwidth light).

Correct options: (A), (C), and (D).

Answer: (ACD) *← Go back to Q38*

Q39.

Solution

Concept — Particle in a 3D Cubic Box:

The energy eigenvalues are $E_{n_x,n_y,n_z} = \dfrac{\pi^2\hbar^2}{2mL^2}(n_x^2 + n_y^2 + n_z^2)$, with $n_x, n_y, n_z = 1, 2, 3, \ldots$

(A) "Each ψ_{n_x,n_y,n_z} is an eigenstate of $\hat{H}$": CORRECT. The wave functions $\psi_{n_x,n_y,n_z} = A\sin(n_x\pi x/L)\sin(n_y\pi y/L)\sin(n_z\pi z/L)$ are solutions of the time-independent Schrödinger equation and hence energy eigenstates.

(B) "States with same $n_x^2 + n_y^2 + n_z^2$ are degenerate": CORRECT. Since $E \propto n_x^2+n_y^2+n_z^2$, any permutation of (n_x, n_y, n_z) that gives the same sum of squares yields the same energy — these states are degenerate. For example, $(1, 2, 3)$, $(2, 1, 3)$, etc., all give $E \propto 14$.

(C) "$\psi_{1,1,1}$ and $\psi_{1,1,2}$ are degenerate": WRONG. $E_{1,1,1} \propto 1^2 + 1^2 + 1^2 = 3$, while $E_{1,1,2} \propto 1^2 + 1^2 + 2^2 = 6$. They have different energies, so they are *not* degenerate.

(D) "Ground state $\psi_{1,1,1}$ is non-degenerate": CORRECT. The only way to write $1^2 + 1^2 + 1^2 = 3$ with positive integers is $(1, 1, 1)$; there are no other combinations giving the same sum of squares.

Correct options: (A), (B), and (D).

Answer: (ABD) *← Go back to Q39*

Q40.

Solution

Concept — p-n Junction Diode Behaviour:

(A) "Forward bias widens depletion layer": WRONG. Forward bias *narrows* the depletion layer. The applied voltage opposes the built-in potential, reducing the width of the depletion region and allowing majority carrier injection.

(B) "Reverse bias widens depletion layer": CORRECT. Reverse bias adds to the built-in potential, increasing the width of the depletion region (and hence the barrier height). The depletion width $W \propto \sqrt{V_{\text{bi}} + V_R}$ increases with reverse voltage V_R.

(C) "Built-in potential opposes majority carrier diffusion": CORRECT. At equilibrium, the drift current due to V_{bi} (sweeping minority carriers) exactly balances the diffusion current (of majority carriers). The built-in field opposes the net flow of majority carriers across the junction.

(D) "Minority carriers carry majority of forward current": WRONG. In forward bias, the current is dominated by majority carrier injection across the junction (electrons from n-side into p-side and holes from p-side into n-side). Once injected, these

become minority carriers in the other region, but the dominant *source* of current is majority carrier injection.

Correct options: (B) and (C).

Answer: (BC) ← *Go back to Q40*

Section C Solutions — NAT

Q41.

Solution

Concept — Period of SHM from Angular Frequency:

$T = 2\pi/\omega$

Calculation:

$$T = \frac{2\pi}{4\pi\,\text{rad/s}} = \frac{2\pi}{4\pi} = \frac{1}{2} = 0.5\,\text{s}$$

$$\boxed{T = 0.5\,\text{s}}$$

Answer: (0.5) ← *Go back to Q41*

Q42.

Solution

Concept — Spherical Capacitor:

$C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$

Step 1 — Convert: $a = 0.05\,\text{m}$, $b = 0.10\,\text{m}$, $b - a = 0.05\,\text{m}$.

Step 2 — Calculate:

$$C = 4\pi \times 8.854 \times 10^{-12} \times \frac{0.05 \times 0.10}{0.05}$$

$$= 4\pi \times 8.854 \times 10^{-12} \times 0.10$$

$$= 4 \times 3.14159 \times 8.854 \times 10^{-13}$$

$$= 12.566 \times 8.854 \times 10^{-13} = 111.3 \times 10^{-13}\,\text{F} = 11.13\,\text{pF}$$

$$\boxed{C \approx 11.13\,\text{pF}}$$

Answer: (11.13) ← *Go back to Q42*

Q43.

Solution

Concept — Thermal de Broglie Wavelength:

$\lambda_{\text{th}} = \dfrac{h}{\sqrt{2\pi m k_B T}}$

Calculation:

$$\begin{aligned} 2\pi m k_B T &= 2\pi \times 2.33 \times 10^{-26} \times 1.38 \times 10^{-23} \times 300 \\ &= 2\pi \times 2.33 \times 1.38 \times 300 \times 10^{-49} \\ &= 2\pi \times 964.98 \times 10^{-49} = 2\pi \times 9.6498 \times 10^{-47} \\ &= 6.0636 \times 10^{-46} \end{aligned}$$

$$\sqrt{6.0636 \times 10^{-46}} = 2.4625 \times 10^{-23}\,\text{kg m s}^{-1}$$

$$\lambda_{\text{th}} = \frac{6.626 \times 10^{-34}}{2.4625 \times 10^{-23}} = 2.691 \times 10^{-11}\,\text{m} = 0.027\,\text{nm}$$

Rounded to two decimal places in nm: $\lambda_{\text{th}} \approx 0.03\,\text{nm}$.

$$\boxed{\lambda_{\text{th}} \approx 0.03\,\text{nm}}$$

Answer: (0.03) *← Go back to Q43*

Q44.

Solution

Concept — Mean Free Path of Gas Molecules:

$\lambda = \dfrac{1}{\sqrt{2}\,\pi\, d^2\, n}$ where $n = P/(k_B T)$

Step 1 — Number density:

$$n = \frac{10^5}{1.38 \times 10^{-23} \times 300} = \frac{10^5}{4.14 \times 10^{-21}} = 2.415 \times 10^{25}\,\text{m}^{-3}$$

Step 2 — Mean free path:

$$\begin{aligned} \lambda &= \frac{1}{\sqrt{2}\,\pi\,(3 \times 10^{-10})^2 \times 2.415 \times 10^{25}} \\ &= \frac{1}{1.4142 \times 3.14159 \times 9 \times 10^{-20} \times 2.415 \times 10^{25}} \\ &= \frac{1}{1.4142 \times 3.14159 \times 21.74 \times 10^{5}} \\ &= \frac{1}{9.654 \times 10^{6}} = 1.036 \times 10^{-7}\,\text{m} \end{aligned}$$

$$\lambda = 1.036 \times 10^{-7}\,\text{m} = 103.6\,\text{nm} \approx 104\,\text{nm}$$

$$\boxed{\lambda \approx 104\,\text{nm}}$$

Answer: (104) ← Go back to Q44

Q45.

Solution

Concept — Mean Lifetime from Half-Life:

$\tau = T_{1/2}/\ln 2$

Calculation:

$$\tau = \frac{3.0\,\mu s}{0.6931} = 4.330\,\mu s$$

$$\boxed{\tau \approx 4.33\,\mu s}$$

Answer: (4.33) ← Go back to Q45

Q46.

Solution

Concept — Packing Fraction of Simple Cubic Crystal:

In the SC structure: 1 atom per unit cell (corner atoms: $8 \times \frac{1}{8} = 1$). Atoms touch along the cube edge: $2r = a$, so $r = a/2$.

Volume of atom sphere vs unit cell:

$$\text{APF} = \frac{N_{\text{atoms}} \times \frac{4}{3}\pi r^3}{a^3} = \frac{1 \times \frac{4}{3}\pi (a/2)^3}{a^3} = \frac{\frac{4\pi}{3} \cdot \frac{1}{8}}{1} = \frac{\pi}{6}$$

$$\text{APF} = \frac{\pi}{6} = \frac{3.14159}{6} = 0.5236$$

$$\boxed{\text{APF} \approx 0.524}$$

Answer: (0.524) ← Go back to Q46

Q47.

Solution

Concept — Determinant of a 2×2 Matrix:

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

Calculation:

$$\det\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix} = (2)(4) - (3)(1) = 8 - 3 = 5$$

$$\boxed{\det = 5}$$

Answer: (5) ← *Go back to Q47*

Q48.

Solution

Concept — Quality Factor of RLC Series Circuit:

$Q = \omega_0 L/R$ where $\omega_0 = 1/\sqrt{LC}$

Step 1 — Resonance frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.1 \times 10^{-5}}} = \frac{1}{\sqrt{10^{-6}}} = \frac{1}{10^{-3}} = 10^3 \text{ rad/s}$$

Step 2 — Quality factor:

$$Q = \frac{\omega_0 L}{R} = \frac{10^3 \times 0.1}{20} = \frac{100}{20} = 5.0$$

$$\boxed{Q = 5.0}$$

Answer: (5.0) ← *Go back to Q48*

Q49.

Solution

Concept — Inverting Op-Amp Voltage Gain:

$|G| = R_f/R_{\text{in}}$

Calculation:

$$|G| = \frac{47\,\text{k}\Omega}{10\,\text{k}\Omega} = \frac{47}{10} = 4.7$$

$$\boxed{|G| = 4.7}$$

Answer: (4.7) ← *Go back to Q49*

Q50.

Solution

Concept — Power Dissipation in a Resistor:

$P = I^2 R$

Calculation:

$$P = (2\,\text{A})^2 \times 50\,\Omega = 4 \times 50 = 200\,\text{W}$$

$$\boxed{P = 200\,\text{W}}$$

Answer: (200) ← Go back to Q50

Q51.

Solution

Concept — Electric Field Energy Density:

$u = \frac{1}{2}\varepsilon_0 E^2$

Calculation:

$$\begin{aligned} u &= \frac{1}{2} \times 8.854 \times 10^{-12} \times (3 \times 10^5)^2 \\ &= \frac{1}{2} \times 8.854 \times 10^{-12} \times 9 \times 10^{10} \\ &= \frac{1}{2} \times 8.854 \times 9 \times 10^{-2} \\ &= \frac{1}{2} \times 79.686 \times 10^{-2} \\ &= 39.843 \times 10^{-2} = 0.3984\,\text{J/m}^3 \end{aligned}$$

$$\boxed{u \approx 0.40\,\text{J/m}^3}$$

Answer: (0.40) ← Go back to Q51

Q52.

Solution

Concept — de Broglie Wavelength in Bohr Orbits:

In the n-th Bohr orbit, n wavelengths fit along the circumference: $n\lambda_n = 2\pi r_n$.

So $\lambda_n = 2\pi r_n / n$.

Since $r_n = n^2 a_0$:

$$\lambda_n = \frac{2\pi n^2 a_0}{n} = 2\pi n\, a_0 \;\Rightarrow\; \lambda_n \propto n$$

Ratio:

$$\frac{\lambda_3}{\lambda_1} = \frac{3}{1} = 3$$

$$\boxed{\lambda_3/\lambda_1 = 3}$$

Answer: (3) ← Go back to Q52

Q53.

Solution

Concept — Entropy Change in Free Expansion:

For an ideal gas, internal energy depends only on T. In free expansion ($\Delta T = 0$, $W = 0$, $Q = 0$), the entropy change equals that of a reversible isothermal expansion between the same initial and final states:

$$\Delta S = nR \ln\left(\frac{V_f}{V_i}\right) = nR \ln 3$$

Calculation:

$$\Delta S = 1\,\text{mol} \times 8.314\,\text{J mol}^{-1}\text{K}^{-1} \times 1.0986 = 9.134\,\text{J/K}$$

$$\boxed{\Delta S \approx 9.13\,\text{J/K}}$$

Answer: (9.13) ← *Go back to Q53*

Q54.

Solution

Concept — Fermi Energy (Free Electron Gas, Sodium):

$E_F = \dfrac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3}$

Step 1 — Compute $3\pi^2 n$:

$$3\pi^2 n = 3 \times 9.8696 \times 2.65 \times 10^{28} = 29.608 \times 2.65 \times 10^{28} = 78.46 \times 10^{28} = 7.846 \times 10^{29}\,\text{m}^{-3}$$

Step 2 — Compute $(7.846 \times 10^{29})^{2/3}$:

$(7.846 \times 10^{29})^{1/3}$: note $10^{29} = 10^{27} \times 100 \Rightarrow (10^{29})^{1/3} = 10^{9.667}$.

$(7.846)^{1/3} \approx 1.988$ (since $2^3 = 8$, slightly less). So: $(7.846 \times 10^{29})^{2/3} = (1.988)^2 \times 10^{19.333} = 3.952 \times 10^{19.333}$.

$10^{19.333} = 10^{19} \times 10^{0.333} = 10^{19} \times 2.154 = 2.154 \times 10^{19}$.

$(3\pi^2 n)^{2/3} = 3.952 \times 2.154 \times 10^{19} = 8.513 \times 10^{19}\,\text{m}^{-2}$

Step 3 — Fermi energy:

$$\begin{aligned} E_F &= \frac{(1.055 \times 10^{-34})^2}{2 \times 9.11 \times 10^{-31}} \times 8.513 \times 10^{19} \\ &= \frac{1.113 \times 10^{-68}}{1.822 \times 10^{-30}} \times 8.513 \times 10^{19} \\ &= 6.11 \times 10^{-39} \times 8.513 \times 10^{19} = 5.20 \times 10^{-19}\,\text{J} \end{aligned}$$

$$E_F = \frac{5.20 \times 10^{-19}}{1.6 \times 10^{-19}}\,\text{eV} = 3.25\,\text{eV}$$

$$\boxed{E_F \approx 3.24\,\text{eV}}$$

(The accepted value for Na is $E_F \approx 3.24\,\text{eV}$.)

Answer: (3.24) ← *Go back to Q54*

Q55.

Solution

Concept — First Excited State of 3D Isotropic HO:

$E_1 = \frac{5}{2}\hbar\omega$

Calculation:

$$\begin{aligned} E_1 &= \frac{5}{2} \times 1.055 \times 10^{-34}\,\text{J s} \times 2 \times 10^{14}\,\text{rad/s} \\ &= \frac{5}{2} \times 2.110 \times 10^{-20}\,\text{J} \\ &= 5.275 \times 10^{-20}\,\text{J} \end{aligned}$$

$$E_1 = \frac{5.275 \times 10^{-20}}{1.6 \times 10^{-19}}\,\text{eV} = 0.3297\,\text{eV}$$

$$\boxed{E_1 \approx 0.330\,\text{eV}}$$

Answer: (0.330) ← *Go back to Q55*

Q56.

Solution

Concept — Wave Speed on a Stretched String:

$v = \sqrt{T/\mu}$ where T is the tension and μ is the linear mass density.

Calculation:

$$v = \sqrt{\frac{100\,\text{N}}{0.04\,\text{kg/m}}} = \sqrt{2500\,\text{m}^2\text{s}^{-2}} = 50\,\text{m/s}$$

$$\boxed{v = 50\,\text{m/s}}$$

Answer: (50) ← *Go back to Q56*

Q57.

Solution

Concept — Radioactive Decay After n Half-Lives:

$$N = N_0 \left(\frac{1}{2}\right)^n$$

Calculation for $n = 4$:

$$N = 2 \times 10^6 \times \left(\frac{1}{2}\right)^4 = 2 \times 10^6 \times \frac{1}{16} = \frac{2 \times 10^6}{16} = 125000$$

$$\boxed{N = 125000}$$

Answer: (125000) ← Go back to Q57

Q58.

Solution

Concept — Gamma Function and Definite Integral:

The gamma function is defined as $\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx$, which satisfies $\Gamma(n) = (n-1)!$ for positive integers n.

Identification:

$$I = \int_0^\infty x^3\, e^{-x}\, dx = \Gamma(4)$$

Calculation:

$$\Gamma(4) = (4-1)! = 3! = 3 \times 2 \times 1 = 6$$

$$\boxed{I = 6}$$

Answer: (6) ← Go back to Q58

Q59.

Solution

Concept — RC Time Constant:

$\tau = RC$

Calculation:

$$\tau = 10^4\,\Omega \times 10^{-6}\,\text{F} = 10^{-2}\,\text{s} = 10.0\,\text{ms}$$

$$\boxed{\tau = 10.0\,\text{ms}}$$

Answer: (10.0) ← Go back to Q59

Q60.

Solution

Concept — de Broglie Wavelength of Alpha Particle:

$\lambda = h/p = h/\sqrt{2\,m_\alpha\,KE}$

Step 1 — Kinetic energy in joules:

$$KE = 0.5 \times 1.6 \times 10^{-19} = 8.0 \times 10^{-20}\,\text{J}$$

Step 2 — Momentum:

$$\begin{aligned} 2\,m_\alpha\,KE &= 2 \times 6.68 \times 10^{-27} \times 8.0 \times 10^{-20} \\ &= 2 \times 53.44 \times 10^{-47} = 106.88 \times 10^{-47} = 1.069 \times 10^{-45}\,\text{kg}^2\text{m}^2\text{s}^{-2} \end{aligned}$$

$$p = \sqrt{1.069 \times 10^{-45}} = \sqrt{10.69 \times 10^{-46}} = \sqrt{10.69} \times 10^{-23} = 3.270 \times 10^{-23}\,\text{kg m s}^{-1}$$

Step 3 — Wavelength:

$$\lambda = \frac{6.626 \times 10^{-34}}{3.270 \times 10^{-23}} = 2.027 \times 10^{-11}\,\text{m} = 0.020\,\text{nm}$$

$$\boxed{\lambda \approx 0.020\,\text{nm}}$$

Answer: (0.020) ← *Go back to Q60*

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	D	3	A	4	B	5	C
6	D	7	A	8	B	9	C	10	D
11	A	12	B	13	C	14	D	15	A
16	B	17	C	18	D	19	A	20	B
21	C	22	D	23	A	24	B	25	C
26	D	27	A	28	B	29	C	30	D
31	BC	32	ACD	33	ABD	34	BC	35	ACD
36	ABD	37	BC	38	ACD	39	ABD	40	BC
41	0.5	42	11.13	43	0.03	44	104	45	4.33
46	0.524	47	5	48	5.0	49	4.7	50	200
51	0.40	52	3	53	9.13	54	3.24	55	0.330
56	50	57	125000	58	6	59	10.0	60	0.020