IIT JAM Physics

Sample Paper – 9

Instructions

- This paper contains **60 questions** carrying a maximum of **100 marks**. Duration: **3 hours (180 minutes)**.
- **Section A** (**Q1–Q30**, MCQ): Q1–Q10 carry **+1 mark** each; wrong response: $-\frac{1}{3}$ mark. Q11–Q30 carry **+2 marks** each; wrong response: $-\frac{2}{3}$ mark. Choose the **single best** option.
- **Section B** (**Q31–Q40**, MSQ): Each carries **+2 marks**. **No negative marking. One or more** options may be correct; full marks only if *all* correct options are selected; no partial credit.
- **Section C** (**Q41–Q60**, NAT): Q41–Q50 carry **+1 mark**; Q51–Q60 carry **+2 marks**. **No negative marking.** Enter the numerical value using the virtual keypad (integer or decimal as appropriate).
- **Calculator policy:** Personal calculators, log tables, and electronic devices are **strictly prohibited**. A virtual scientific calculator is provided in the CBT interface. Rough sheets are supplied at the examination centre.
- Once you move to the next section, you cannot return to a previous section. Manage time accordingly.

Section A — Multiple Choice Questions (MCQ)

***Q1–Q10**: 1 mark each ($-\frac{1}{3}$ for wrong). Choose the **single** correct option.*

Q1. The Laurent expansion of $f(z) = \dfrac{e^z}{z^2}$ about $z = 0$ is written as

$$\frac{e^z}{z^2} = \sum_{n=-2}^{\infty} c_n\, z^n.$$

The coefficient c_{-1} (the residue of $f(z)$ at $z = 0$) equals:

(A) 0
(B) $\dfrac{1}{2}$
(C) 2
(D) 1

Q2. Using the relativistic energy–momentum relation $E^2 = (pc)^2 + (m_0c^2)^2$, find the magnitude of the relativistic momentum p of a particle of rest mass m_0 when its total energy is $E = 2m_0c^2$:

(A) $p = \sqrt{3}\,m_0c$
(B) $p = \sqrt{2}\,m_0c$
(C) $p = \sqrt{5}\,m_0c$
(D) $p = m_0c$

Q3. A converging lens of focal length $f = 10\,\text{cm}$ forms an image of an object placed $30\,\text{cm}$ in front of it (on the same side as the incoming light). Using the thin-lens formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ with $u = -30\,\text{cm}$, the image distance v is:

(A) $v = 30\,\text{cm}$
(B) $v = 15\,\text{cm}$
(C) $v = 20\,\text{cm}$
(D) $v = 7.5\,\text{cm}$

Q4. According to Maxwell's equations, the displacement current density between the plates of a parallel-plate capacitor that is being charged by a time-varying current I is related to the conduction current in the external leads. Which statement is correct?

(A) The displacement current is always much smaller than the conduction current.
(B) The displacement current flows only along the edges of the capacitor plates.
(C) The total displacement current between the plates equals the conduction current in the leads.
(D) The displacement current is zero because there is no charge between the plates.

Q5. An ideal gas is carried through a reversible **isochoric** (constant-volume) process. The work done *by* the gas during this process is:

(A) $W = nC_V\,\Delta T$
(B) $W = nRT\ln(V_2/V_1)$
(C) $W = P\,\Delta V = nR\,\Delta T$

(D) $W = 0$

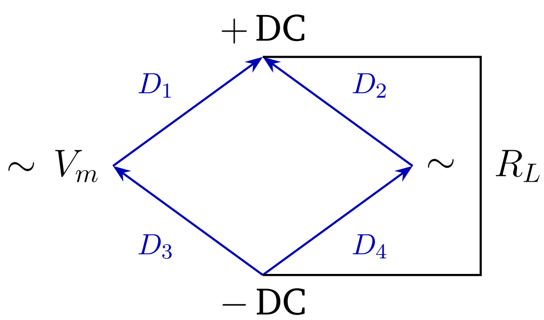
Q6. The Heisenberg uncertainty principle states $\Delta x\, \Delta p_x \geq \hbar/2$. A particle is confined to a region of width $\Delta x = 0.1\,\text{nm}$. The minimum uncertainty in momentum Δp_{min} is approximately (take $\hbar = 1.055 \times 10^{-34}\,\text{J s}$):

(A) $5.3 \times 10^{-25}\,\text{kg m/s}$
(B) $5.3 \times 10^{-30}\,\text{kg m/s}$
(C) $5.3 \times 10^{-20}\,\text{kg m/s}$
(D) $5.3 \times 10^{-15}\,\text{kg m/s}$

Q7. In the free-electron model, the density of states at the Fermi energy $D(E_F)$ is proportional to $n^{1/3}$, where n is the electron number density. If the free-electron density is doubled (e.g. by applying pressure), the density of states at the new Fermi level changes by a factor of:

(A) 2
(B) $2^{1/3}$
(C) $2^{2/3}$
(D) $1/2$

Q8. A full-wave bridge rectifier uses four silicon diodes (each with forward voltage drop $V_D \approx 0.7\,\text{V}$). The transformer secondary delivers a sinusoidal voltage with peak amplitude $V_m = 12\,\text{V}$. The peak output voltage V_{peak} across the load resistor is (the circuit is shown below):



(A) $12.0\,\text{V}$
(B) $11.3\,\text{V}$
(C) $10.6\,\text{V}$
(D) $9.2\,\text{V}$

Q9. The binding energy of the ${}^4_2$He nucleus is approximately $28.3\,$MeV. The binding energy *per nucleon* (BE$/A$) of ^{4}He is:

(A) $1.79\,$MeV/nucleon
(B) $3.54\,$MeV/nucleon
(C) $5.48\,$MeV/nucleon
(D) $7.07\,$MeV/nucleon

Q10. The gradient operator in spherical coordinates (r, θ, ϕ) is

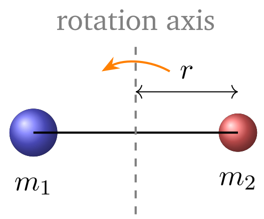
$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}.$$

For the scalar field $f(r, \theta, \phi) = r^2 \sin\theta$, the radial component $(\nabla f)_r = \partial f/\partial r$ is:

(A) $2r\sin\theta$
(B) $r^2\cos\theta$
(C) $2r\cos\theta$
(D) $r\sin\theta$

*Q11–Q30: 2 marks each ($-\frac{2}{3}$ for wrong). Choose the **single** correct option.*

Q11. A diatomic molecule (modelled as a rigid rotor) has moment of inertia $I = \mu r^2$ about its centre of mass (shown below). The quantised rotational energy levels are $E_J = \dfrac{\hbar^2 J(J+1)}{2I}$. The energy of the photon emitted in the $J = 1 \to J = 0$ transition is:



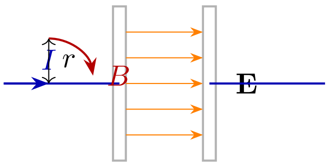
(A) $\dfrac{\hbar^2}{2I}$
(B) $\dfrac{\hbar^2}{I}$
(C) $\dfrac{2\hbar^2}{I}$

(D) $\dfrac{\hbar^2}{4I}$

Q12. A particle of rest mass m moves with velocity $v = 0.8c$. The Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2} = 5/3$. The relativistic momentum $p = \gamma m v$ of the particle is:

(A) $\dfrac{1}{3}mc$
(B) mc
(C) $\dfrac{4}{3}mc$
(D) $\dfrac{5}{3}mc$

Q13. The Ampere–Maxwell law states $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0(I_{\text{enc}} + \varepsilon_0\, d\Phi_E/dt)$. A parallel-plate capacitor with circular plates of area A is being charged by a steady current I (shown below). At a distance r from the axis of the connecting wire (outside the capacitor), the magnitude of the magnetic field B is:

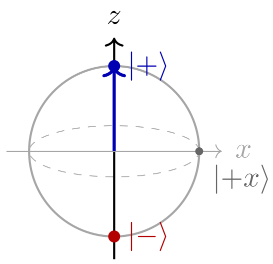


(A) $\dfrac{\mu_0 I}{4\pi r}$
(B) $\dfrac{\mu_0 I}{4r}$
(C) $\dfrac{\mu_0 I}{\pi r}$
(D) $\dfrac{\mu_0 I}{2\pi r}$

Q14. The van der Waals equation for one mole of gas is $\left(P + \dfrac{a}{V_m^2}\right)(V_m - b) = RT$, where a and b are constants. The critical molar volume V_c at the critical point is:

(A) $V_c = 3b$
(B) $V_c = b$
(C) $V_c = 2b$
(D) $V_c = 6b$

Q15. A spin-$\frac{1}{2}$ particle is in the eigenstate $|+\rangle$ of $\hat{S}_z$ with eigenvalue $+\hbar/2$. The Bloch sphere representation of spin eigenstates is shown below. The expectation value $\langle \hat{S}_x \rangle$ in the state $|+\rangle$ is:

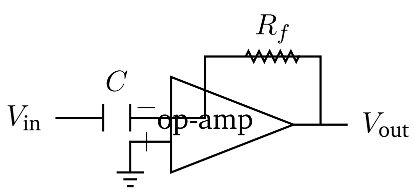


(A) $\langle \hat{S}_x \rangle = \hbar/2$
(B) $\langle \hat{S}_x \rangle = 0$
(C) $\langle \hat{S}_x \rangle = -\hbar/2$
(D) $\langle \hat{S}_x \rangle = \hbar/4$

Q16. In the Drude model, the mean free time between electron collisions in a metal is $\tau = m_e\sigma/(ne^2)$. For copper at $300\,\text{K}$ with conductivity $\sigma = 6 \times 10^7\,(\Omega{\cdot}\text{m})^{-1}$, electron density $n = 8.5\times10^{28}\,\text{m}^{-3}$, and $m_e = 9.11\times10^{-31}\,\text{kg}$, $e = 1.6 \times 10^{-19}\,\text{C}$, the mean collision time τ is approximately:

(A) $\tau \approx 2.5 \times 10^{-10}\,\text{s}$
(B) $\tau \approx 2.5 \times 10^{-12}\,\text{s}$
(C) $\tau \approx 2.5 \times 10^{-14}\,\text{s}$
(D) $\tau \approx 2.5 \times 10^{-16}\,\text{s}$

Q17. An op-amp differentiator circuit (capacitor C at the input, resistor R_f in the feedback, as shown below) has $R_fC = 0.01\,\text{s}$. For an input $V_{\text{in}}(t) = V_0\sin(\omega t)$ with $V_0 = 1\,\text{V}$ and $\omega = 1000\,\text{rad/s}$, the magnitude of the maximum output voltage is:



(A) $|V_{\text{out}}|_{\text{max}} = 100\,\text{V}$
(B) $|V_{\text{out}}|_{\text{max}} = 1\,\text{V}$

(C) $|V_{\text{out}}|_{\max} = 0.1\,\text{V}$
(D) $|V_{\text{out}}|_{\max} = 10\,\text{V}$

Q18. The binding energy per nucleon (BE/A) is plotted as a function of mass number A for all stable nuclei. The nucleus with the *highest* binding energy per nucleon (the peak of the BE/A curve) is:

(A) ${}^{56}_{26}\text{Fe}$
(B) ${}^{4}_{2}\text{He}$
(C) ${}^{238}_{92}\text{U}$
(D) ${}^{16}_{8}\text{O}$

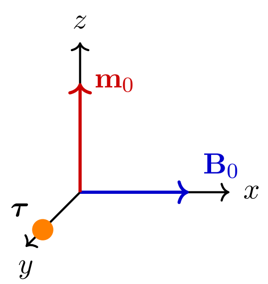
Q19. For the regular Sturm–Liouville eigenvalue problem

$$\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] - q(x)y + \lambda w(x)y = 0, \quad x \in [a, b],$$

with appropriate homogeneous boundary conditions and $p, w > 0$, which statement is correct?

(A) All eigenvalues are complex (non-real).
(B) The eigenvalues are real and the corresponding eigenfunctions are orthogonal with weight $w(x)$.
(C) Only finitely many eigenvalues exist.
(D) All eigenvalues are necessarily negative.

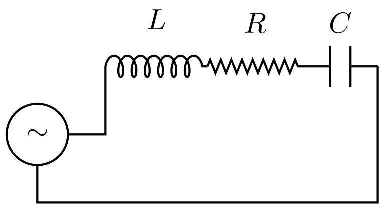
Q20. A magnetic dipole with moment $\mathbf{m} = m_0\hat{z}$ is placed in a uniform external magnetic field $\mathbf{B} = B_0\hat{x}$ (as shown below). The torque $\boldsymbol{\tau} = \mathbf{m}\times\mathbf{B}$ exerted on the dipole has magnitude:



(A) $|\boldsymbol{\tau}| = m_0 B_0/2$
(B) $|\boldsymbol{\tau}| = m_0 B_0/\sqrt{2}$

(C) $|\boldsymbol{\tau}| = m_0 B_0$
(D) $|\boldsymbol{\tau}| = 2m_0 B_0$

Q21. A series RLC circuit is driven by an AC voltage source at angular frequency ω, as shown below. The complex impedance of the circuit is $Z = R + i\left(\omega L - \dfrac{1}{\omega C}\right)$. At the resonance frequency $\omega_0 = 1/\sqrt{LC}$, the impedance reduces to:



(A) $Z = \omega L$
(B) $Z = 1/(\omega C)$
(C) $Z = \sqrt{R^2 + (\omega L - 1/(\omega C))^2}$
(D) $Z = R$

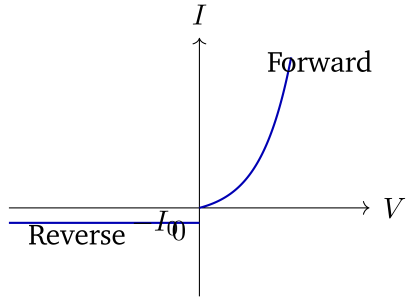
Q22. The ratio of populations of two non-degenerate energy levels $E_2 > E_1$ at temperature T is given by the Boltzmann factor $N_2/N_1 = e^{-(E_2-E_1)/kT}$. When $(E_2 - E_1) = kT$, this ratio equals:

(A) $e^{-1} \approx 0.368$
(B) $e^{-2} \approx 0.135$
(C) $1/2 = 0.500$
(D) $e^{-1/2} \approx 0.607$

Q23. The parity operator $\hat{P}$ acts as $\hat{P}\psi(\mathbf{r}) = \psi(-\mathbf{r})$. For the hydrogen atom in a state with orbital quantum number l, the parity eigenvalue is:

(A) $+1$ for all values of l
(B) $(-1)^l$
(C) $(-1)^{l+1}$
(D) 0 always

Q24. The current–voltage (I–V) characteristic of an ideal p–n junction diode follows the Shockley equation $I = I_0(e^{eV/kT} - 1)$, where I_0 is the reverse saturation current. The I–V curve is sketched below. In the limit of strong reverse bias ($V \to -\infty$):



(A) $I \to +\infty$ (forward current grows unboundedly)
(B) $I \to 0$ (no current)
(C) $I \to -I_0$ (saturation current)
(D) $I \to -\infty$ (breakdown)

Q25. An inverting Schmitt trigger op-amp circuit has equal resistors $R_1 = R_2 = 10\,\text{k}\Omega$ and saturation voltages $V_{\text{sat}} = \pm 10\,\text{V}$. The upper and lower threshold voltages (hysteresis limits) are:

(A) $V_{\text{UTP}} = +1\,\text{V}, \quad V_{\text{LTP}} = -1\,\text{V}$
(B) $V_{\text{UTP}} = +2.5\,\text{V}, \quad V_{\text{LTP}} = -2.5\,\text{V}$
(C) $V_{\text{UTP}} = +10\,\text{V}, \quad V_{\text{LTP}} = -10\,\text{V}$
(D) $V_{\text{UTP}} = +5\,\text{V}, \quad V_{\text{LTP}} = -5\,\text{V}$

Q26. For a two-state quantum system with orthonormal basis $\{|+\rangle, |-\rangle\}$, the completeness (closure) relation asserts that the sum of the projection operators equals:

(A) $|+\rangle\langle+| + |-\rangle\langle-| = \hat{1}$ (the identity operator)
(B) $|+\rangle\langle+| + |-\rangle\langle-| = \hat{0}$ (the zero operator)
(C) $|+\rangle\langle+| + |-\rangle\langle-| = \hat{S}_z/\hbar$
(D) $|+\rangle\langle+| + |-\rangle\langle-| = |+\rangle\langle-| + |-\rangle\langle+|$

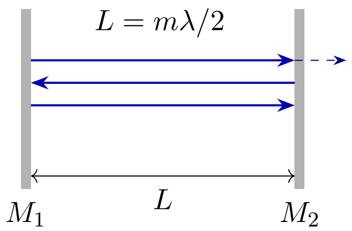
Q27. The dynamic viscosity η of an ideal gas in kinetic theory is given by $\eta = \frac{1}{3}\rho\langle v\rangle\lambda$, where ρ is the mass density, $\langle v\rangle \propto \sqrt{T}$ is the mean molecular speed, and $\lambda \propto 1/n$ (mean free path) is independent of T at fixed density. The dependence of η on temperature T (at constant volume, so ρ fixed) is:

(A) $\eta \propto T$
(B) $\eta \propto T^{1/2}$
(C) $\eta \propto T^{-1}$
(D) η is independent of temperature

Q28. An electromagnetic plane wave of intensity $I = 1000\,\text{W/m}^2$ is normally incident on a perfectly absorbing (black) surface. The radiation pressure P_{rad} on the surface is (take $c = 3 \times 10^8\,\text{m/s}$):

(A) $P_{\text{rad}} = 2.00 \times 10^{-6}\,\text{Pa}$
(B) $P_{\text{rad}} = 1.00 \times 10^{-3}\,\text{Pa}$
(C) $P_{\text{rad}} = 3.33 \times 10^{-6}\,\text{Pa}$
(D) $P_{\text{rad}} = 6.67 \times 10^{-6}\,\text{Pa}$

Q29. A Fabry–Pérot etalon consists of two partially reflecting parallel mirrors separated by distance L, as shown. Resonance occurs when $L = m\lambda/2$ (m integer), giving a discrete set of resonance frequencies $f_m = mc/(2L)$. The *free spectral range* (FSR), i.e. the frequency spacing between adjacent resonances, is:



(A) $\Delta f = c/L$
(B) $\Delta f = 2c/L$
(C) $\Delta f = c/(4L)$
(D) $\Delta f = c/(2L)$

Q30. In a photoelectric experiment, the threshold frequency (below which no electrons are emitted) is measured to be $\nu_0 = 5.5 \times 10^{14}\,\text{Hz}$. The work function ϕ of the metal is (take $h = 6.626 \times 10^{-34}\,\text{J s}$, $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$):

(A) $\phi = 2.28\,\text{eV}$
(B) $\phi = 3.64\,\text{eV}$

(C) $\phi = 1.52\,\text{eV}$
(D) $\phi = 4.56\,\text{eV}$

Section B — Multiple Select Questions (MSQ)

__Q31–Q40__: 2 marks each. __No negative marking. One or more__ options may be correct. Full marks only if all correct options (and no incorrect options) are chosen.

Q31. Which of the following quantities are Lorentz invariants in special relativity (i.e. take the same numerical value in all inertial frames)?

(A) The spacetime interval $s^2 = c^2t^2 - x^2 - y^2 - z^2$
(B) The energy E of a particle
(C) The rest mass energy m_0c^2 (invariant mass)
(D) The four-momentum magnitude $p_\mu p^\mu = (E/c)^2 - |\mathbf{p}|^2 = m_0^2c^2$

Q32. In Maxwell's equations for electromagnetic fields in **free space** (vacuum, no free charges or currents), which of the following statements are correct?

(A) $\nabla \cdot \mathbf{E} = 0$ (Gauss's law in free space)
(B) $\nabla \cdot \mathbf{B} = 0$ (no magnetic monopoles)
(C) $\nabla \times \mathbf{E} = 0$ even when $\mathbf{B}$ is time-varying
(D) Electromagnetic waves propagate in vacuum at speed $c = 1/\sqrt{\mu_0\varepsilon_0}$

Q33. Which of the following statements about thermodynamic entropy S are correct?

(A) Entropy is a path function (depends on how a process is carried out, not just on the initial and final states).
(B) For a reversible process: $dS = \delta Q_{\text{rev}}/T$.
(C) The total entropy of an isolated system (universe) never decreases: $\Delta S_{\text{univ}} \geq 0$ (second law of thermodynamics).
(D) Entropy is always positive because it measures disorder.

Q34. Which of the following statements about a Metal-Oxide-Semiconductor Field-Effect Transistor (MOSFET) are correct?

(A) The gate current is essentially zero in normal operation (very high input impedance).
(B) It is a bipolar device that uses both electrons and holes as majority carriers.
(C) The conductivity of the channel is controlled by the gate-to-source voltage V_{GS}.
(D) It can operate in both depletion mode and enhancement mode.

Q35. Which of the following are true properties of **diamagnetic** materials?

(A) The magnetic susceptibility $\chi_m < 0$ (negative).
(B) A diamagnetic sample is repelled by a non-uniform external magnetic field.
(C) Each atom in a diamagnetic material carries a permanent magnetic dipole moment.
(D) Superconductors below T_c are perfect diamagnets (Meissner–Ochsenfeld effect: $\mathbf{B} = 0$ inside).

Q36. For the Legendre differential equation $(1-x^2)y'' - 2xy' + l(l+1)y = 0$ on $[-1, 1]$, which of the following are true?

(A) Solutions exist only for non-integer values of l.
(B) For non-negative integer l, the Legendre polynomial $P_l(x)$ is the solution that is regular (finite) at both $x = \pm 1$.
(C) The Legendre polynomials satisfy the orthogonality relation $\int_{-1}^{1} P_l(x)P_m(x)\,dx =$ 0 for $l \neq m$.
(D) $P_l(x)$ has exactly $l+1$ zeros in the interval $(-1, 1)$.

Q37. Which of the following statements about nuclear binding energy are correct?

(A) The binding energy per nucleon (BE/A) increases with mass number A up to a maximum near $A \approx 56$ (iron group).
(B) Nuclear fission releases energy for nuclei with $A < 56$ (lighter than iron).

(C) For very heavy nuclei ($A \gtrsim 200$), the binding energy per nucleon decreases with increasing A.

(D) The binding energy per nucleon is a measure of nuclear stability: higher BE$/A$ implies a more tightly bound (stable) nucleus.

Q38. Total internal reflection (TIR) occurs when light travels from medium 1 (refractive index n_1) to medium 2 (refractive index n_2). Which of the following conditions are necessary for TIR?

(A) Light must travel from a denser medium to a rarer medium (i.e. the light goes from a medium of higher refractive index to one of lower refractive index).

(B) The angle of incidence must exceed the critical angle $\theta_c = \arcsin(n_2/n_1)$.

(C) The interface between the two media must be perfectly smooth and flat.

(D) The incident medium must have $n_1 > n_2$ (the incident medium is optically denser).

Q39. Bloch's theorem for an electron in a perfectly periodic crystal potential $V(\mathbf{r}) = V(\mathbf{r} + \mathbf{R})$ states that the energy eigenfunction takes the form $\psi_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r})\, e^{i\mathbf{k}\cdot\mathbf{r}}$, where $u_{\mathbf{k}}(\mathbf{r})$ has the periodicity of the lattice. Which of the following statements about Bloch's theorem are correct?

(A) The Bloch wave function $\psi_{\mathbf{k}}(\mathbf{r})$ itself is periodic with the lattice period (i.e. $\psi_{\mathbf{k}}(\mathbf{r} + \mathbf{R}) = \psi_{\mathbf{k}}(\mathbf{r})$).

(B) The crystal momentum $\hbar\mathbf{k}$ is a conserved quantum number (good quantum number) for the periodic system.

(C) Energy bands arise as a consequence of the periodic crystal potential, even for arbitrarily weak potential.

(D) The energy eigenvalues $E_n(\mathbf{k})$ are the same for all $\mathbf{k}$ in the first Brillouin zone (flat bands always).

Q40. At absolute zero ($T = 0$) for a system of **free** (non-interacting) electrons in three dimensions, which of the following are true?

(A) All energy states up to the Fermi energy E_F are occupied and all states above E_F are empty.

(B) The average energy per electron equals E_F.

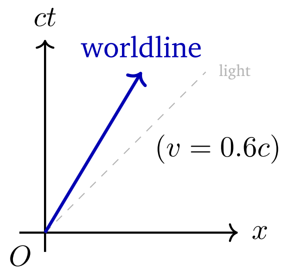
(C) The Fermi–Dirac distribution function becomes a unit step function: $f(E) = 1$ for $E < E_F$ and $f(E) = 0$ for $E > E_F$.

(D) No electron can have energy greater than E_F at $T = 0$.

Section C — Numerical Answer Type (NAT)

***Q41–Q50**: 1 mark each. **No negative marking.** Enter the exact numerical value.*

Q41. A proton (rest mass energy $m_pc^2 = 938.3\,\text{MeV}$) moves at velocity $v = 0.6c$. The spacetime worldline of the proton is shown schematically below. Calculate the relativistic kinetic energy $K = (\gamma - 1)m_pc^2$ of the proton in MeV, correct to one decimal place.



Q42. Two concentric conducting spherical shells have inner radius $a = 10\,\text{cm}$ and outer radius $b = 20\,\text{cm}$. The capacitance of this spherical capacitor is $C = 4\pi\varepsilon_0 ab/(b-a)$. Calculate C in picofarads (pF), correct to one decimal place. (Take $\varepsilon_0 = 8.854 \times 10^{-12}\,\text{F/m}$, $1/(4\pi\varepsilon_0) = 9 \times 10^9\,\text{N}\,\text{m}^2\text{C}^{-2}$.)

Q43. The thermal de Broglie wavelength of a particle of mass m at temperature T is defined as $\lambda_{\text{th}} = h/\sqrt{2\pi mkT}$. For a hydrogen atom (mass $m_H = 1.67 \times 10^{-27}\,\text{kg}$) at $T = 300\,\text{K}$, calculate λ_{th} in picometres ($\text{pm} = 10^{-12}\,\text{m}$), to the nearest integer. (Use $h = 6.626 \times 10^{-34}\,\text{J}\,\text{s}$, $k = 1.38 \times 10^{-23}\,\text{J/K}$.)

Q44. Calculate the heat capacity at constant volume C_V (in J/K) of 2 moles of a monatomic ideal gas. For a monatomic ideal gas, $C_V = \frac{3}{2}nR$. ($R = 8.314\,\text{J}\,\text{mol}^{-1}\text{K}^{-1}$.) Give the answer correct to one decimal place.

Q45. The nuclear radius is approximated by $R = R_0A^{1/3}$ with $R_0 = 1.2\,\text{fm}$ ($1\,\text{fm} = 10^{-15}\,\text{m}$). Calculate the nuclear radius of $^{208}_{82}\text{Pb}$ (lead, $A = 208$) in femtometres, correct to two decimal places.

Q46. In the sodium chloride (NaCl) crystal structure (rock-salt structure), each Na^+ ion sits at the centre of an octahedron of Cl^- ions and vice versa. What is the coordination number of Na^+ in the NaCl structure (i.e. the number of nearest-neighbour Cl^- ions around each Na^+)?

Q47. The matrix $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ has inverse A^{-1}. Determine the $(1, 2)$ element of A^{-1} (i.e. the element in row 1, column 2 of A^{-1}).

Q48. A parallel LC tank circuit has inductance $L = 10\,\text{mH} = 0.01\,\text{H}$ and capacitance $C = 1\,\mu\text{F} = 10^{-6}\,\text{F}$. The resonance frequency is $f_0 = 1/(2\pi\sqrt{LC})$. Calculate f_0 in Hz, to the nearest integer.

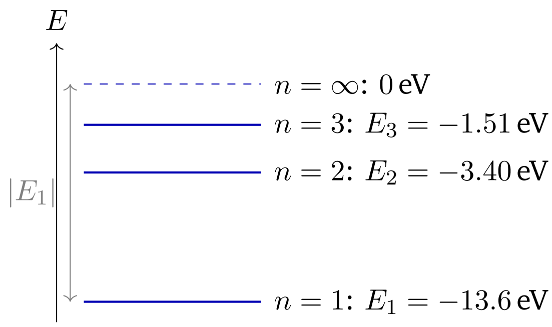
Q49. An op-amp difference amplifier has four equal resistors $R_1 = R_2 = R_3 = R_4 = 10\,\text{k}\Omega$. Voltage $V_1 = 4\,\text{V}$ is applied (through R_3) to the non-inverting terminal, and $V_2 = 2\,\text{V}$ is applied (through R_1) to the inverting terminal. The output voltage is $V_{\text{out}} = (V_1 - V_2)\,R_2/R_1$. Calculate V_{out} in volts.

Q50. A solid disk has moment of inertia $I = 0.5\,\text{kg}\,\text{m}^2$ and rotates with angular velocity $\omega = 20\,\text{rad/s}$ about its symmetry axis. Calculate its angular momentum $L = I\omega$ in J s.

Q51–Q60**: 2 marks each.* ***No negative marking. *Enter the exact numerical value.*

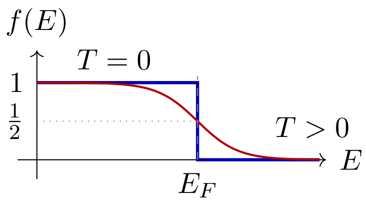
Q51. A particle moves at $v = 0.95c$. Calculate its Lorentz factor $\gamma = 1/\sqrt{1 - v^2/c^2}$, correct to two decimal places.

Q52. The ground-state energy of the hydrogen atom is $E_1 = -13.6\,\text{eV}$ (as shown on the energy-level diagram below). Calculate the magnitude $|E_1|$ in electron-volts, correct to two decimal places.

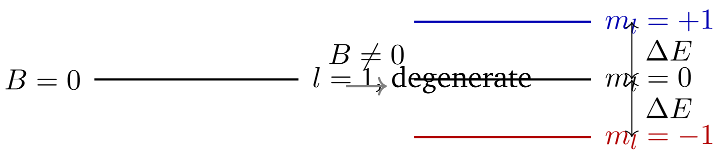


Q53. One mole of an ideal gas is taken through a complete **Carnot cycle** operating between a hot reservoir at $T_H = 600\,\text{K}$ and a cold reservoir at $T_C = 300\,\text{K}$. Calculate the net entropy change ΔS of the gas (in J/K) for the complete cycle, correct to two decimal places.

Q54. The Fermi temperature for copper is $T_F = E_F/k_B$, where $E_F = 7.04\,\text{eV}$ is the Fermi energy. The Fermi–Dirac distribution (shown below) becomes a step function at $T = 0$ with the step at E_F. Calculate T_F in kelvin, to the nearest 100 K. ($k_B = 1.38 \times 10^{-23}\,\text{J/K}$, $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$.)



Q55. An atom in a state with orbital quantum number $l = 1$ is placed in an external magnetic field $B = 1\,\text{T}$. The three magnetic sublevels ($m_l = -1, 0, +1$) are shown below. The energy splitting between adjacent sublevels due to the normal Zeeman effect is $\Delta E = \mu_B B$. Calculate ΔE in units of 10^{-5} eV, correct to two decimal places. ($\mu_B = 9.274 \times 10^{-24}\,\text{J/T}$, $1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$.)



Q56. In the Drude model, the mean time between electron collisions in copper is $\tau = m_e\sigma/(ne^2)$. Using $\sigma = 6 \times 10^7\,(\Omega \cdot \text{m})^{-1}$, $n = 8.5 \times 10^{28}\,\text{m}^{-3}$, $m_e = 9.11 \times 10^{-31}\,\text{kg}$, and $e = 1.6 \times 10^{-19}\,\text{C}$, calculate τ in picoseconds (ps $= 10^{-12}$ s), correct to one decimal place.

Q57. A radioactive sample initially contains N_0 atoms. After exactly $t = 5.0$ half-lives, the fraction of atoms remaining is $N/N_0 = (1/2)^{5.0}$. Calculate this fraction, correct to three decimal places.

Q58. The error function is defined as $\text{erf}(x) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^x e^{-t^2}\,dt$. The tabulated value $\text{erf}(1)$ is approximately:

Q59. A series RL circuit (no capacitor) has resistance $R = 200\,\Omega$ and inductance $L = 0.1\,\mathrm{H}$. The inductive time constant of the circuit is $\tau = L/R$. Calculate τ in milliseconds (ms), correct to two decimal places.

Q60. The de Broglie wavelength of a thermal neutron at room temperature ($T = 293\,\mathrm{K}$) is $\lambda = h/\sqrt{2m_n kT}$, where $m_n = 1.675 \times 10^{-27}$ kg. Calculate λ in picometres (pm), to the nearest integer. ($h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$, $k = 1.38 \times 10^{-23}$ J/K.)

Solutions

IIT JAM Physics – Sample Paper 9

Section A Solutions — MCQ

Solution

Concept — Laurent Series of e^z/z^2:
Expand e^z in its Taylor series about $z = 0$ and divide term by term by z^2.

Step 1 — Taylor series of e^z:

$$e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \cdots$$

Step 2 — Divide by z^2:

$$\frac{e^z}{z^2} = \frac{1}{z^2} + \frac{1}{z} + \frac{1}{2} + \frac{z}{6} + \frac{z^2}{24} + \cdots$$

The coefficient of z^{-1} is the residue: $c_{-1} = 1$.

Final Answer: $c_{-1} = 1 \Rightarrow$ **(D)**

Why other options are wrong:

Q1.

- (A) 0: The pole at $z = 0$ is of order 2; the coefficient of z^{-1} in the Laurent series is not zero.
- (B) $1/2$: This is the constant term c_0 in the Laurent series (the z^0 coefficient), not c_{-1}.
- (C) 2: No power of the Taylor expansion gives a factor of 2 for the z^{-1} term.

Answer: (D) ← *Go back to Q1*

Solution

Concept — Relativistic Energy–Momentum Relation:
$E^2 = (pc)^2 + (m_0c^2)^2$.

Step 1 — Substitute $E = 2m_0c^2$:

$$(2m_0c^2)^2 = (pc)^2 + (m_0c^2)^2$$

$$4m_0^2c^4 = p^2c^2 + m_0^2c^4$$

$$p^2c^2 = 3m_0^2c^4$$

Step 2 — Solve for p:

$$p = \sqrt{3}\, m_0 c$$

Final Answer: $p = \sqrt{3}\, m_0 c \Rightarrow$ **(A)**

Why other options are wrong:

Q2.

- (B) $\sqrt{2}\, m_0 c$: Would require $E^2 - m_0^2 c^4 = 3m_0^2 c^4$; $\sqrt{2}$ arises from $E = \sqrt{2}\, m_0 c^2$, not $2m_0 c^2$.
- (C) $\sqrt{5}\, m_0 c$: Comes from $E = \sqrt{6}\, m_0 c^2$; numerical error.
- (D) $m_0 c$: Corresponds to $E = \sqrt{2}\, m_0 c^2$ (ultra-relativistic approximation, incorrect here).

Answer: (A) ← *Go back to Q2*

Solution

Concept — Thin-Lens Formula:
$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$, where $u < 0$ for a real object (object on the incoming-light side).

Step 1 — Substitute $f = 10$ cm, $u = -30$ cm:

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-30} = \frac{1}{v} + \frac{1}{30}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{30} = \frac{3-1}{30} = \frac{2}{30} = \frac{1}{15}$$

Step 2 — Image distance:

$$v = 15\,\text{cm}$$

The positive sign indicates a real, inverted image on the other side of the lens.

Final Answer: $v = 15\,\text{cm} \Rightarrow$ **(B)**

Why other options are wrong:

Q3.

- (A) $30\,\text{cm}$: Arises if the object is at $2f = 20\,\text{cm}$; here object is at $3f$.
- (C) $20\,\text{cm}$: Corresponds to an object at $u = -20\,\text{cm}$ (at $2f$), not at $3f$.
- (D) $7.5\,\text{cm}$: Would require $1/v = 1/10 + 1/30$ (wrong sign on the u term).

Answer: (B) ← *Go back to Q3*

Solution

Concept — Maxwell's Displacement Current:

Maxwell modified Ampère's law to $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(I_c + I_D)$, where $I_D = \varepsilon_0 \frac{d\Phi_E}{dt}$ is the displacement current.

Step 1 — Apply to a charging capacitor:
Choose a surface that passes between the capacitor plates (no conduction current through it): the enclosed $I_c = 0$, but $I_D = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 A \frac{dE}{dt}$.

Since $E = Q/(\varepsilon_0 A)$: $I_D = \varepsilon_0 A \frac{d}{dt}\left(\frac{Q}{\varepsilon_0 A}\right) = \frac{dQ}{dt} = I_c$ (the conduction current in the leads).

Final Answer: Displacement current between plates equals conduction current in leads $\Rightarrow$ **(C)**

Why other options are wrong:

Q4.

- (A): False; for slowly charging capacitors, $I_D = I_c$ exactly.
- (B): Displacement current flows across the entire plate area, not only at edges.
- (D): Displacement current is precisely nonzero when E between plates changes; it is the mechanism for magnetic field continuity.

Answer: (C) ← *Go back to Q4*

Solution

Concept — Work in Thermodynamic Processes:
Work done *by* a gas is $W = \int_{V_i}^{V_f} P\,dV$.

Isochoric (constant volume) process: $V_i = V_f$, so $dV = 0$ throughout.

$$W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_i} P\,dV = 0$$

Final Answer: $W = 0 \Rightarrow$ **(D)**

Why other options are wrong:

Q5.

- (A) $nC_V\Delta T$: This is the heat added in an isochoric process ($Q = nC_V\Delta T$), not the work done.
- (B) $nRT\ln(V_2/V_1)$: This is the work in an *isothermal* expansion, not isochoric.
- (C) $nR\Delta T$: This would equal work only in an isobaric process ($W = P\Delta V = nR\Delta T$).

Answer: (D) ← *Go back to Q5*

Q6.

Solution

Concept — Heisenberg Uncertainty Principle:
$\Delta x\, \Delta p_x \geq \hbar/2$. The minimum momentum uncertainty is $\Delta p_{\text{min}} = \hbar/(2\Delta x)$.

Step 1 — Substitute $\Delta x = 0.1\,$nm $= 10^{-10}$ m:

$$\Delta p_{\text{min}} = \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}}{2 \times 10^{-10}} = \frac{1.055 \times 10^{-34}}{2 \times 10^{-10}} = 5.275 \times 10^{-25}\,\text{kg m/s}$$

$\approx 5.3 \times 10^{-25}\,$kg m/s.

Final Answer: $\Delta p_{\text{min}} \approx 5.3 \times 10^{-25}\,$kg m/s $\Rightarrow$ **(A)**

Why other options are wrong:

- (B) 10^{-30}: Would arise for $\Delta x \approx 10^{-5}\,$m (macroscopic scale), not atomic.
- (C) 10^{-20}: Would require $\Delta x \approx 10^{-15}\,$m (nuclear scale, far too small).
- (D) 10^{-15}: Corresponds to $\Delta x \sim 10^{-20}\,$m, far below any physical particle.

Answer: (A) ← *Go back to Q6*

Q7.

Solution

Concept — Free Electron Density of States at E_F:
$E_F = \dfrac{\hbar^2}{2m_e}(3\pi^2 n)^{2/3} \propto n^{2/3}$, and $D(E_F) = \dfrac{3n}{2E_F} \propto \dfrac{n}{n^{2/3}} = n^{1/3}$.

Step 1 — Scaling when $n \to 2n$:

$$\frac{D(E_F)_{\text{new}}}{D(E_F)_{\text{old}}} = \left(\frac{2n}{n}\right)^{1/3} = 2^{1/3} \approx 1.26$$

Final Answer: $D(E_F)$ increases by factor $2^{1/3} \Rightarrow$ **(B)**

Why other options are wrong:

- (A) Factor 2: $D(E_F) \propto n^{1/3}$, not n; linear scaling is wrong.
- (C) Factor $2^{2/3}$: This would be the scaling of E_F itself ($E_F \propto n^{2/3}$), not of $D(E_F)$.
- (D) Factor $1/2$: $D(E_F)$ increases with n (more electrons $\Rightarrow$ higher DOS).

Answer: (B) ← *Go back to Q7*

Solution

Concept — Bridge Rectifier Output Voltage:
In a full-wave bridge rectifier, the current flows through *two* diodes in series during each half cycle. Each diode drops $\approx V_D = 0.7\,$V in the forward direction.

Step 1 — Peak output voltage:

$$V_{\text{peak}} = V_m - 2V_D = 12\,\text{V} - 2 \times 0.7\,\text{V} = 12 - 1.4 = 10.6\,\text{V}$$

Final Answer: $V_{\text{peak}} = 10.6\,\text{V} \Rightarrow$ **(C)**

Why other options are wrong:

Q8.
- (A) $12.0\,\text{V}$: Neglects both diode voltage drops (ideal diodes); real diodes have $\approx 0.7\,\text{V}$ drop each.
- (B) $11.3\,\text{V}$: Subtracts only one diode drop $(12 - 0.7 = 11.3)$; the bridge uses two diodes per path.
- (D) $9.2\,\text{V}$: No physical basis for this value with the given parameters.

Answer: (C) ← *Go back to Q8*

Solution

Concept — Binding Energy per Nucleon:
For a nucleus of mass number A, $\text{BE}/A = \text{Total Binding Energy}/A$.

Step 1 — For ^{4}He ($A = 4$):

$$\frac{\text{BE}}{A} = \frac{28.3\,\text{MeV}}{4} = 7.075\,\text{MeV/nucleon} \approx 7.07\,\text{MeV/nucleon}$$

Final Answer: $\text{BE}/A = 7.07\,\text{MeV/nucleon} \Rightarrow$ **(D)**

Why other options are wrong:

Q9.
- (A) $1.79\,\text{MeV/nucleon}$: Approximately the binding energy per nucleon of *deuterium* (^{2}H).
- (B) $3.54\,\text{MeV/nucleon}$: No standard nucleus has this value; arises from dividing 28.3 by 8 instead of 4.
- (C) $5.48\,\text{MeV/nucleon}$: Typical of lighter nuclei (Li, Be region), not ^{4}He.

Answer: (D) ← *Go back to Q9*

Solution

Concept — Gradient in Spherical Coordinates:
$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}.$

Step 1 — Partial derivative with respect to r for $f = r^2 \sin\theta$:

$$(\nabla f)_r = \frac{\partial}{\partial r}(r^2 \sin\theta) = 2r \sin\theta$$

(Here $\sin\theta$ is treated as a constant when differentiating with respect to r.)

Final Answer: $(\nabla f)_r = 2r\sin\theta \Rightarrow$ **(A)**

Why other options are wrong:

Q10.

- (B) $r^2\cos\theta$: This is $\partial f/\partial\theta = r^2\cos\theta$, the θ-derivative, not the radial component.
- (C) $2r\cos\theta$: Confuses $\partial/\partial r$ with $\partial/\partial\theta$.
- (D) $r\sin\theta$: Missing the factor of 2 from differentiating r^2.

Answer: (A) ← *Go back to Q10*

Solution

Concept — Rigid Rotor (Rotational Spectrum):

Energy levels of a rigid rotor: $E_J = \dfrac{\hbar^2 J(J+1)}{2I}$.

Step 1 — Energies of $J = 1$ and $J = 0$:

$$E_1 = \frac{\hbar^2 \cdot 1 \cdot 2}{2I} = \frac{\hbar^2}{I}, \qquad E_0 = 0$$

Step 2 — Photon energy for $J = 1 \to 0$ transition:

$$\Delta E = E_1 - E_0 = \frac{\hbar^2}{I}$$

Final Answer: $\Delta E = \hbar^2/I \Rightarrow$ **(B)**

Why other options are wrong:

Q11.

- (A) $\hbar^2/(2I)$: This is $E_1/2$; the formula gives $E_J = \hbar^2 J(J+1)/(2I)$, so $E_1 = \hbar^2/I$ (not $\hbar^2/(2I)$).
- (C) $2\hbar^2/I$: Would correspond to the $J = 2 \to 1$ transition ($\Delta E = E_2 - E_1 = 6\hbar^2/(2I) - 2\hbar^2/(2I) = 2\hbar^2/I$), not $J = 1 \to 0$.
- (D) $\hbar^2/(4I)$: No physical basis from the rigid-rotor formula.

Answer: (B) ← *Go back to Q11*

Solution

Concept — Relativistic Momentum:
$p = \gamma mv$, where $\gamma = 1/\sqrt{1 - v^2/c^2}$.

Step 1 — Lorentz factor at $v = 0.8c$:

$$\gamma = \frac{1}{\sqrt{1 - 0.64}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.6} = \frac{5}{3}$$

Step 2 — Relativistic momentum:

$$p = \gamma mv = \frac{5}{3} \times m \times 0.8c = \frac{5 \times 0.8}{3} mc = \frac{4}{3} mc$$

Final Answer: $p = \dfrac{4}{3}mc \Rightarrow$ **(C)**

Why other options are wrong:

Q12.

- (A) $mc/3$: Off by factor 4; non-relativistic approximation gives $p \approx mv = 0.8mc$, far from $mc/3$.
- (B) mc: Would require $\gamma v/c = 1$; with $v = 0.8c$, $\gamma v/c = 5/3 \times 0.8 = 4/3 \neq 1$.
- (D) $5mc/3$: This equals γmc (the relativistic energy divided by c), not the momentum γmv.

Answer: (C) ← *Go back to Q12*

Solution

Concept — Ampere–Maxwell Law:
$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(I_{\text{enc}} + I_D)$, where $I_D = \varepsilon_0\, d\Phi_E/dt$ is the displacement current.

Step 1 — Outside the capacitor, in the lead wire region:
Choosing a circular Ampèrian loop of radius r centered on the wire axis: the conduction current enclosed is I. Since this loop is in the wire region (not between the plates), $I_D = 0$ on this surface and $I_{\text{enc}} = I$.

Step 2 — Apply Ampère's law (by symmetry, B is constant on the loop):

$$B \times 2\pi r = \mu_0 I \quad \Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

Key insight: The displacement current between the plates equals I, ensuring continuity; outside the plates in the wire, the result is the same as for an infinite straight wire.

Final Answer: $B = \mu_0 I/(2\pi r) \Rightarrow$ **(D)**

Why other options are wrong:

Q13.

- (A) $\mu_0 I/(4\pi r)$: This is the Biot–Savart result for a semi-infinite wire, not the standard Ampèrian result for a wire.
- (B) $\mu_0 I/(4r)$: Has incorrect denominator; missing the factor of π.
- (C) $\mu_0 I/(\pi r)$: Missing the factor of 2 in the denominator (off by factor of 2).

Answer: (D) ← *Go back to Q13*

Solution

Concept — Van der Waals Critical Point:
The critical point satisfies $\left(\frac{\partial P}{\partial V_m}\right)_T = 0$ and $\left(\frac{\partial^2 P}{\partial V_m^2}\right)_T = 0$ simultaneously.

Step 1 — Critical conditions for the van der Waals equation:
Solving the two simultaneous equations gives the three critical constants:

$$V_c = 3b, \qquad T_c = \frac{8a}{27Rb}, \qquad P_c = \frac{a}{27b^2}$$

The critical molar volume is $V_c = 3b$ (three times the excluded volume parameter).

Final Answer: $V_c = 3b \Rightarrow$ **(A)**

Why other options are wrong:

Q14.

- (B) $V_c = b$: V_c is always larger than b (gas volume must exceed excluded volume).
- (C) $V_c = 2b$: Incorrect algebraic factor; solving the critical equations unambiguously gives $V_c = 3b$.
- (D) $V_c = 6b$: Off by factor of 2; no physical basis.

Answer: (A) ← *Go back to Q14*

Solution

Concept — Expectation Values of Spin Operators:
For a spin-$1/2$ particle, the operators in matrix representation (in the $\hat{S}_z$ eigenbasis) are:

$$\hat{S}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad |+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Step 1 — Compute $\langle \hat{S}_x \rangle$:

$$\langle \hat{S}_x \rangle = \langle + | \hat{S}_x | + \rangle = \begin{pmatrix} 1 & 0 \end{pmatrix} \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

Physical interpretation: $|+\rangle$ is a superposition of equal weights of $|\hat{S}_x = +\hbar/2\rangle$ and $|\hat{S}_x = -\hbar/2\rangle$, giving zero average.

Final Answer: $\langle \hat{S}_x \rangle = 0 \Rightarrow$ **(B)**

Why other options are wrong:

Q15.

- (A) $+\hbar/2$: Only the eigenvalue of $\hat{S}_z$; $\hat{S}_x$ in state $|+\rangle$ has zero expectation.
- (C) $-\hbar/2$: Only the eigenvalue of $\hat{S}_z$ for $|-\rangle$; $\langle S_x \rangle = 0$ for any $\hat{S}_z$ eigenstate.
- (D) $\hbar/4$: No physical basis; spin-$1/2$ expectation values are multiples of $\hbar/2$ or zero.

Answer: (B) ← *Go back to Q15*

Solution

Concept — Drude Model Mean Collision Time:
$\tau = \dfrac{m_e \sigma}{ne^2}$

Step 1 — Numerator:

$$m_e\sigma = 9.11 \times 10^{-31} \times 6 \times 10^{7} = 54.66 \times 10^{-24}\,\text{kg}\,(\Omega \cdot \text{m})^{-1}$$

Step 2 — Denominator:

$$ne^2 = 8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 = 8.5 \times 10^{28} \times 2.56 \times 10^{-38} = 21.76 \times 10^{-10}\,\text{C}^2\text{m}^{-3}$$

Step 3 — Mean collision time:

$$\tau = \frac{54.66 \times 10^{-24}}{21.76 \times 10^{-10}} = \frac{54.66}{21.76} \times 10^{-14} = 2.51 \times 10^{-14}\,\text{s} \approx 2.5 \times 10^{-14}\,\text{s}$$

Final Answer: $\tau \approx 2.5 \times 10^{-14}\,\text{s} \Rightarrow$ **(C)**

Why other options are wrong:

Q16.

- (A) 10^{-10} s: Too large by four orders of magnitude; corresponds to a very poor conductor.
- (B) 10^{-12} s: Off by two orders of magnitude; the Drude time is in the femtosecond range.
- (D) 10^{-16} s: Too small by two orders of magnitude; would correspond to an unrealistically large scattering rate.

Answer: (C) ← *Go back to Q16*

Q17.

Solution

Concept — Op-Amp Differentiator:
For an ideal inverting differentiator with capacitor C at the input and resistor R_f in the feedback:

$$V_{\text{out}}(t) = -R_f C \frac{dV_{\text{in}}}{dt}$$

Step 1 — Differentiate $V_{\text{in}} = V_0 \sin(\omega t)$:

$$V_{\text{out}}(t) = -R_f C \cdot V_0 \omega \cos(\omega t)$$

Step 2 — Maximum magnitude:

$$|V_{\text{out}}|_{\text{max}} = R_f C \cdot V_0 \cdot \omega = 0.01 \times 1\,\text{V} \times 1000\,\text{rad/s} = 10\,\text{V}$$

Final Answer: $|V_{\text{out}}|_{\text{max}} = 10\,\text{V} \Rightarrow$ **(D)**

Why other options are wrong:

- (A) $100\,\text{V}$: Off by factor of 10; would arise if $\omega = 10^4$ rad/s.
- (B) $1\,\text{V}$: Neglects the frequency factor; correct only if $\omega = 100\,\text{rad/s}$.
- (C) $0.1\,\text{V}$: Would require $\omega = 10\,\text{rad/s}$ or $R_f C = 10^{-4}$ s.

Answer: (D) ← *Go back to Q17*

Q18.

Solution

Concept — Nuclear Binding Energy Curve:
The binding energy per nucleon (BE/A) rises steeply for light nuclei, reaches a broad maximum near $A \approx 56$, and slowly decreases for heavier nuclei.

Peak nucleus: ^{56}Fe (iron-56) has BE/$A \approx 8.8$ MeV/nucleon, the highest among all stable nuclei.

Final Answer: $^{56}_{26}$Fe $\Rightarrow$ **(A)**

Why other options are wrong:

- (B) ^{4}He: BE/$A = 7.07$ MeV/nucleon; lower than the iron-group peak due to the doubly-magic but small nucleus.
- (C) ^{238}U: BE/$A \approx 7.6$ MeV/nucleon; for heavy nuclei, Coulomb repulsion reduces BE/A.
- (D) ^{16}O: BE/$A \approx 8.0$ MeV/nucleon; close but less than ^{56}Fe.

Answer: (A) ← *Go back to Q18*

Q19.

Solution

Concept — Sturm–Liouville Theory:
For a regular Sturm–Liouville (S-L) problem on a finite interval $[a, b]$ with $p(x) > 0$, $w(x) > 0$, and appropriate (separated) homogeneous boundary conditions:

- Eigenvalues λ_n are **real** and form a countably infinite discrete set bounded below.
- Eigenfunctions $y_n(x)$ corresponding to *distinct* eigenvalues are **orthogonal** with weight $w(x)$:

$$\int_a^b y_n(x)\, y_m(x)\, w(x)\, dx = 0 \quad \text{for } n \neq m.$$

- The eigenfunctions form a **complete** set (basis for $L^2([a, b], w)$).

Final Answer: Eigenvalues real and eigenfunctions orthogonal (with weight w) $\Rightarrow$ **(B)**

Why other options are wrong:

- (A) Complex eigenvalues: The Hermitian nature of the S-L operator guarantees *real* eigenvalues.
- (C) Finitely many eigenvalues: A regular S-L problem has infinitely many eigenvalues ($\lambda_n \to +\infty$).
- (D) Negative eigenvalues only: Eigenvalues are bounded below, but whether they are positive depends on $q(x)$ and boundary conditions.

Answer: (B) ← Go back to Q19

Solution

Concept — Torque on a Magnetic Dipole:
$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B}$

Step 1 — Compute the cross product:
$\mathbf{m} = m_0\hat{z}$, $\mathbf{B} = B_0\hat{x}$:

$$\boldsymbol{\tau} = m_0\hat{z} \times B_0\hat{x} = m_0B_0(\hat{z} \times \hat{x}) = m_0B_0\hat{y}$$

Step 2 — Magnitude:

$$|\boldsymbol{\tau}| = m_0B_0\,|\hat{y}| = m_0B_0$$

The torque points in the $+y$ direction (out of the page in the xz-plane diagram), tending to rotate $\mathbf{m}$ toward alignment with $\mathbf{B}$.

Final Answer: $|\boldsymbol{\tau}| = m_0B_0 \Rightarrow$ **(C)**

Why other options are wrong:

Q20.

- (A) $m_0B_0/2$: The factor of $\sin 90 = 1$ between $\hat{z}$ and $\hat{x}$ gives no reduction.
- (B) $m_0B_0/\sqrt{2}$: Would arise if the angle between $\mathbf{m}$ and $\mathbf{B}$ were 45; here the angle is 90.
- (D) $2m_0B_0$: Magnitude of a cross product cannot exceed $|\mathbf{m}||\mathbf{B}| = m_0B_0$.

Answer: (C) ← *Go back to Q20*

Solution

Concept — Series RLC Resonance:

The impedance is $Z = R + i\left(\omega L - \dfrac{1}{\omega C}\right)$.

Step 1 — At resonance $\omega_0 = 1/\sqrt{LC}$:

$$\omega_0 L = \frac{1}{\sqrt{LC}} \cdot L = \sqrt{\frac{L}{C}}, \qquad \frac{1}{\omega_0 C} = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}}$$

Step 2 — Reactive part cancels:

$$\omega_0 L - \frac{1}{\omega_0 C} = \sqrt{\frac{L}{C}} - \sqrt{\frac{L}{C}} = 0$$

$$Z = R + i \times 0 = R$$

At resonance the circuit is purely resistive; impedance is minimised and current is maximised.

Final Answer: $Z = R \Rightarrow$ **(D)**

Why other options are wrong:

Q21.

- (A) $Z = \omega L$: The inductive reactance alone; at resonance the capacitive reactance cancels it.
- (B) $Z = 1/(\omega C)$: The capacitive reactance alone; same argument.
- (C) $Z = \sqrt{R^2 + (\cdots)^2}$: This is the general modulus; at resonance the bracket is zero.

Answer: (D) ← *Go back to Q21*

Solution

Concept — Boltzmann Population Ratio:

$\dfrac{N_2}{N_1} = e^{-(E_2-E_1)/kT}$

Step 1 — Substitute $(E_2 - E_1) = kT$:

$$\frac{N_2}{N_1} = e^{-kT/kT} = e^{-1} \approx 0.368$$

Final Answer: $N_2/N_1 = e^{-1} \approx 0.368 \Rightarrow$ **(A)**

Why other options are wrong:

Q22.
- (B) $e^{-2} \approx 0.135$: Corresponds to $(E_2 - E_1) = 2kT$, not kT.
- (C) $1/2 = 0.500$: Would require $e^{-1} = 1/2$, i.e. $\ln 2 = 1$, which is false.
- (D) $e^{-1/2} \approx 0.607$: Corresponds to $(E_2 - E_1) = kT/2$, half the given energy difference.

Answer: (A) ← Go back to Q22

Solution

Concept — Parity of Hydrogen Atom Eigenstates:
Under the parity transformation $\hat{P}\psi(\mathbf{r}) = \psi(-\mathbf{r})$, spherical harmonics transform as:

$$\hat{P}\, Y_l^m(\theta, \phi) = Y_l^m(\pi - \theta, \phi + \pi) = (-1)^l\, Y_l^m(\theta, \phi)$$

Since the radial wave function $R_{nl}(r)$ is unchanged by $r \to r$ (parity maps $r \to r$ in spherical coordinates), the full hydrogen eigenfunction has parity:

$$\hat{P}\, \psi_{nlm}(\mathbf{r}) = (-1)^l\, \psi_{nlm}(\mathbf{r})$$

Thus the parity eigenvalue is $(-1)^l$: even for $l = 0, 2, 4, \ldots$ (s, d, g, ...) and odd for $l = 1, 3, \ldots$ (p, f, ...).

Final Answer: $(-1)^l \Rightarrow$ **(B)**

Why other options are wrong:

Q23.
- (A) $+1$ always: p-orbitals ($l = 1$) have odd parity $(-1)^1 = -1$, not $+1$.
- (C) $(-1)^{l+1}$: Off by a factor of -1; s-orbitals ($l = 0$) would be odd, but they are even.
- (D) 0: $\hat{P}$ has eigenvalues ± 1, not zero.

Answer: (B) ← Go back to Q23

Solution

Concept — Shockley Diode Equation:
$I = I_0\left(e^{eV/kT} - 1\right)$

Step 1 — Reverse bias limit ($V \to -\infty$):
As $V \to -\infty$, $e^{eV/kT} \to 0$:

$$I \to I_0(0-1) = -I_0$$

The current saturates at $-I_0$ (the reverse saturation current); this is the small leakage current across the depletion region due to thermally generated minority carriers.

Final Answer: $I \to -I_0 \Rightarrow$ **(C)**

Why other options are wrong:

Q24.
- (A) $I \to +\infty$: Forward current grows exponentially for *positive* V; for reverse bias it saturates.
- (B) $I \to 0$: If $I_0 = 0$ exactly; physically, thermal generation always gives a small I_0.
- (D) $I \to -\infty$: Would imply diode breakdown (avalanche/Zener), not included in the simple Shockley model.

Answer: (C) *← Go back to Q24*

Solution

Concept — Schmitt Trigger Threshold Voltages:
For an inverting Schmitt trigger with feedback resistors R_1 (from output to non-inverting input) and R_2 (from non-inverting input to ground), the threshold voltages are:

$$V_{\text{UTP}} = V^{+}_{\text{sat}} \cdot \frac{R_2}{R_1 + R_2}, \qquad V_{\text{LTP}} = V^{-}_{\text{sat}} \cdot \frac{R_2}{R_1 + R_2}$$

Step 1 — Substitute $R_1 = R_2 = 10\,\text{k}\Omega$, $V^{\pm}_{\text{sat}} = \pm 10\,\text{V}$:

$$V_{\text{UTP}} = +10 \times \frac{10}{10+10} = +10 \times \frac{1}{2} = +5\,\text{V}$$

$$V_{\text{LTP}} = -10 \times \frac{1}{2} = -5\,\text{V}$$

The hysteresis band is $2 \times 5 = 10\,\text{V}$ wide, centred at 0.

Final Answer: $V_{\text{UTP}} = +5\,\text{V}$, $V_{\text{LTP}} = -5\,\text{V} \Rightarrow$ **(D)**

Why other options are wrong:

Q25.
- (A) $\pm 1\,\text{V}$: Would require $R_2/(R_1 + R_2) = 0.1$, i.e. $R_1 = 9R_2$.

- (B) $\pm 2.5\,\text{V}$: Would require $R_2/(R_1+R_2) = 0.25$, i.e. $R_1 = 3R_2$.
- (C) $\pm 10\,\text{V}$: Only possible if $R_2 \gg R_1$; with equal R the threshold is half the saturation.

Answer: (D) ← *Go back to Q25*

Q26.

Solution

Concept — Completeness (Closure) Relation in Quantum Mechanics:
For a complete orthonormal set $\{|n\rangle\}$, the identity (resolution of the identity) is:

$$\sum_n |n\rangle\langle n| = \hat{1}$$

For a two-state system $\{|+\rangle, |-\rangle\}$:

$$|+\rangle\langle +| + |-\rangle\langle -| = \hat{1}$$

Verification in matrix form:

$$\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} + \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \hat{1}$$

Final Answer: The identity operator $\hat{1} \Rightarrow$ **(A)**

Why other options are wrong:

- (B) Zero operator: Sum of two non-zero projectors is non-zero; $|+\rangle\langle +|$ alone is a rank-1 projector.
- (C) $\hat{S}_z/\hbar$: The Pauli $\sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ is not the identity.
- (D) $|+\rangle\langle -| + |-\rangle\langle +|$: This is $\hat{S}_x \times (2/\hbar)$ (off-diagonal matrix), not the identity.

Answer: (A) ← *Go back to Q26*

Solution

Concept — Viscosity of an Ideal Gas (Kinetic Theory):
$\eta = \dfrac{1}{3}\rho\langle v\rangle\lambda$, where $\langle v\rangle = \sqrt{8kT/(\pi m)} \propto T^{1/2}$ and $\lambda \propto 1/n$.

Step 1 — Identify T-dependence at fixed density ρ (fixed n):
At constant density: ρ is fixed, $\lambda \propto 1/n =$ const (since n is fixed at constant volume). Only $\langle v\rangle \propto \sqrt{T}$ varies with temperature.

Step 2 — Temperature dependence of η:

$$\eta = \frac{1}{3}\rho\langle v\rangle\lambda \propto \sqrt{T}$$

Key result: The viscosity of an ideal gas is independent of *pressure* (at fixed T) but increases as $\sqrt{T}$.

Final Answer: $\eta \propto T^{1/2} \Rightarrow$ **(B)**

Why other options are wrong:

Q27.

- (A) $\eta \propto T$: Overestimates; applies to dense fluids, not dilute gases.
- (C) $\eta \propto T^{-1}$: The viscosity of gases *increases* with temperature (unlike liquids).
- (D) Independent of T: Only if $\langle v\rangle$ were independent of T, which it is not.

Answer: (B) ← *Go back to Q27*

Solution

Concept — Radiation Pressure:
For a perfectly absorbing surface, all electromagnetic momentum is transferred to the surface:

$$P_{\text{rad}} = \frac{I}{c}$$

For a perfect reflector: $P_{\text{rad}} = 2I/c$ (momentum transferred is doubled).

Step 1 — Absorbing surface with $I = 10^3$ W/m^2:

$$P_{\text{rad}} = \frac{I}{c} = \frac{10^3\,\text{W/m}^2}{3\times10^8\,\text{m/s}} = 3.33\times10^{-6}\,\text{Pa}$$

Final Answer: $P_{\text{rad}} = 3.33\times10^{-6}\,\text{Pa} \Rightarrow$ **(C)**

Why other options are wrong:

Q28.

- (A) 2.00×10^{-6} Pa: No physical basis for this value; not I/c or $2I/c$ for $I = 10^3$.
- (B) 10^{-3} Pa: Larger by factor ~ 300; wrong order of magnitude.
- (D) $6.67\times10^{-6}\,\text{Pa} = 2I/c$: Correct formula for a *perfect reflector*, not an absorber.

Answer: (C) ← *Go back to Q28*

Solution

Concept — Fabry–Pérot Free Spectral Range:
Resonance condition: $L = m\lambda/2 \Rightarrow \lambda_m = 2L/m$. Resonance frequencies:

$$f_m = \frac{c}{\lambda_m} = \frac{mc}{2L}$$

Step 1 — Free spectral range (spacing between adjacent modes):

$$\Delta f = f_{m+1} - f_m = \frac{(m+1)c}{2L} - \frac{mc}{2L} = \frac{c}{2L}$$

Physical significance: $\Delta f = c/(2L)$ is the frequency interval at which the etalon is transparent; it determines the resolving power of the interferometer.

Final Answer: $\Delta f = c/(2L) \Rightarrow$ **(D)**

Why other options are wrong:

Q29.
- (A) c/L: Off by factor of 2; would be FSR if resonance condition were $L = m\lambda$ (full wavelength, not half).
- (B) $2c/L$: Factor of 4 too large; no physical basis for this formula.
- (C) $c/(4L)$: Factor of 2 too small; arises from confusing with quarter-wave resonators.

Answer: (D) ← *Go back to Q29*

Solution

Concept — Photoelectric Work Function:
The work function equals the energy of threshold photons: $\phi = h\nu_0$.

Step 1 — Compute:

$$\phi = h\nu_0 = 6.626 \times 10^{-34} \times 5.5 \times 10^{14}\,\text{J} = 36.44 \times 10^{-20}\,\text{J} = 3.644 \times 10^{-19}\,\text{J}$$

Step 2 — Convert to eV:

$$\phi = \frac{3.644 \times 10^{-19}}{1.6 \times 10^{-19}}\,\text{eV} = 2.278\,\text{eV} \approx 2.28\,\text{eV}$$

Final Answer: $\phi = 2.28\,\text{eV} \Rightarrow$ **(A)**

Why other options are wrong:

Q30.
- (B) $3.64\,\text{eV}$: This is the energy in units of 10^{-19} J, not in eV (forgot to divide by 1.6).
- (C) $1.52\,\text{eV}$: Corresponds to $\nu_0 \approx 3.7 \times 10^{14}$ Hz (red light), not 5.5×10^{14}.

- (D) $4.56\,\text{eV}$: Double the correct answer; arises from using $h\nu$ with twice the frequency.

Answer: (A) ← *Go back to Q30*

Section B Solutions — MSQ

Q31.

Solution

Concept — Lorentz Invariants in Special Relativity:

(A) Spacetime interval $s^2 = c^2t^2 - x^2 - y^2 - z^2$: CORRECT. By construction, Lorentz transformations are defined to leave the spacetime interval invariant; s^2 has the same value in all inertial frames.

(B) Energy E of a particle: WRONG. Energy is the time-component of the 4-momentum and is frame-dependent: it changes under boosts ($E' = \gamma(E - vp_x)$).

(C) Rest mass energy m_0c^2: CORRECT. The rest mass (invariant mass) m_0 is a Lorentz scalar; m_0c^2 is the same in all frames, defining the particle's identity.

(D) $p_\mu p^\mu = (E/c)^2 - |\mathbf{p}|^2 = m_0^2c^2$: CORRECT. The magnitude-squared of the 4-momentum is a Lorentz invariant equal to $(m_0c)^2$.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q31*

Q32.

Solution

Concept — Maxwell's Equations in Free Space:

(A) $\nabla \cdot \mathbf{E} = 0$ in free space: CORRECT. Gauss's law in free space (no free charges) gives $\nabla \cdot \mathbf{E} = \rho_{\text{free}}/\varepsilon_0 = 0$.

(B) $\nabla \cdot \mathbf{B} = 0$ always: CORRECT. The absence of magnetic monopoles is a fundamental law; $\nabla \cdot \mathbf{B} = 0$ holds universally.

(C) $\nabla \times \mathbf{E} = 0$ for time-varying $\mathbf{B}$: WRONG. Faraday's law: $\nabla \times \mathbf{E} = -\partial\mathbf{B}/\partial t \neq 0$ when $\mathbf{B}$ is time-varying. Only for magnetostatics ($\partial\mathbf{B}/\partial t = 0$) does $\nabla \times \mathbf{E} = 0$.

(D) EM waves travel at $c = 1/\sqrt{\mu_0\varepsilon_0}$: CORRECT. Maxwell derived this from his equations; $c \approx 3 \times 10^8$ m/s in vacuum.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q32*

Q33.

Solution

Concept — Properties of Entropy:

(A) "Entropy is a path function": WRONG. Entropy S is a *state function*: it depends only on the thermodynamic state of the system, not on the history of how that state was reached. $\Delta S = S_f - S_i$ regardless of path.

(B) "For a reversible process, $dS = \delta Q_{\text{rev}}/T$": CORRECT. This is the *definition* of entropy change for a reversible process (Clausius).

(C) "$\Delta S_{\text{univ}} \geq 0$": CORRECT. The second law of thermodynamics states that the entropy of the universe (system + surroundings) never decreases; equality holds for reversible processes.

(D) "Entropy is always positive": WRONG. Entropy is defined up to an additive constant (the third law fixes it at $S = 0$ for a perfect crystal at $T = 0$, but in statistical mechanics the Boltzmann entropy $k_B \ln \Omega \geq 0$). However, in classical thermodynamics, entropy changes can be positive or negative; the statement "always positive" is incorrect as an absolute claim.

Correct options: (B) and (C).

Answer: (BC) ← *Go back to Q33*

Q34.

Solution

Concept — MOSFET Properties:

(A) "Gate current ≈ 0": CORRECT. The gate is insulated from the channel by a thin oxide layer (SiO_2); the gate current is essentially zero (typically $< 10^{-12}$ A), giving very high input impedance.

(B) "Bipolar device using both carriers": WRONG. A MOSFET is a *unipolar* device: only one type of carrier (electrons in n-channel, holes in p-channel) constitutes the channel current. BJTs are bipolar.

(C) "Channel controlled by gate voltage V_{GS}": CORRECT. The gate-to-source voltage modulates the density of carriers in the inversion layer (channel), thereby controlling the drain current: $I_D \propto (V_{GS} - V_{th})^2$ in saturation.

(D) "Can operate in depletion and enhancement mode": CORRECT. Enhancement-mode MOSFETs require $|V_{GS}| > V_{th}$ to form a channel (off by default); depletion-mode devices have a pre-existing channel at $V_{GS} = 0$ (on by default).

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q34*

Q35.

Solution

Concept — Diamagnetic Materials:

(A) $\chi_m < 0$: **CORRECT.** Diamagnets have negative magnetic susceptibility; an applied field induces opposing magnetisation ($\mathbf{M} = \chi_m \mathbf{H}$, $\chi_m < 0$), so $\mathbf{M}$ opposes $\mathbf{H}$.

(B) "Repelled by external field": CORRECT. Since M opposes H, a diamagnetic sample is pushed toward weaker field regions (repulsion from a magnet).

(C) "Each atom has a permanent dipole moment": WRONG. Diamagnetism is due to *induced* orbital currents that oppose the applied field (Lenz's law at the atomic level). Atoms in perfect diamagnets have no permanent magnetic moment.

(D) "Superconductors are perfect diamagnets (Meissner effect)": CORRECT. Below T_c, a superconductor expels magnetic flux entirely ($\mathbf{B} = 0$ inside, $\chi_m = -1$); this is the Meissner–Ochsenfeld effect.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q35*

Q36.

Solution

Concept — Legendre Differential Equation:

(A) "Solutions exist only for non-integer l": WRONG. For *non-negative integer* l, the Legendre polynomial $P_l(x)$ is the polynomial solution that is finite at $x = \pm 1$. Non-integer l can arise in other contexts but is not required here.

(B) "$P_l(x)$ is the polynomial solution regular at $x = \pm 1$ for non-negative integer l": CORRECT. For $l = 0, 1, 2, \ldots$, the series for one solution terminates, giving a polynomial $P_l(x)$ of degree l. The other linearly independent solution $Q_l(x)$ diverges at $x = \pm 1$.

(C) "$\int_{-1}^{1} P_l P_m \, dx = 0$ for $l \neq m$": CORRECT. Legendre polynomials are orthogonal: $\int_{-1}^{1} P_l(x) P_m(x) \, dx = \dfrac{2}{2l+1} \delta_{lm}$.

(D) "$P_l(x)$ has $l + 1$ zeros in $(-1, 1)$": WRONG. The Legendre polynomial $P_l(x)$ has exactly l zeros in the open interval $(-1, 1)$ (by Sturm–Liouville theory).

Correct options: (B) and (C).

Answer: (BC) ← *Go back to Q36*

Q37.

Solution

Concept — Nuclear Binding Energy:

(A) "BE/A increases up to $A \approx 56$": CORRECT. The binding energy per nucleon rises from ≈ 1.1 MeV for ^{2}H to ≈ 8.8 MeV near $A \approx 56$ (iron group).

(B) "Fission releases energy for $A < 56$": WRONG. Fission (splitting a heavy nucleus) releases energy only when the products have *higher* BE/A than the parent — this occurs for *heavy* nuclei ($A \gtrsim 100$). For light nuclei ($A < 56$), *fusion* releases energy.

(C) "For $A \gtrsim 200$, BE/A decreases with A": CORRECT. For heavy nuclei, the Coulomb repulsion among protons (growing as Z^2) reduces the binding energy per nucleon; BE/$A \approx 7.6$ MeV for U, lower than the Fe peak.

(D) "BE/A measures nuclear stability": CORRECT. Higher binding energy per nucleon means more energy is needed to remove a nucleon; a larger BE/A indicates a more tightly bound (energetically stable) nucleus.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q37*

Q38.

Solution

Concept — Total Internal Reflection:

(A) "Light must travel from denser to rarer medium": CORRECT. TIR requires $n_1 > n_2$: the light must originate in the optically denser medium and attempt to enter a rarer medium.

(B) "Angle of incidence $> \theta_c = \arcsin(n_2/n_1)$": CORRECT. At the critical angle, the refracted ray grazes the interface ($\theta_r = 90$). For $\theta_i > \theta_c$, no refracted ray exists and all energy is reflected.

(C) "Interface must be smooth and flat": WRONG. TIR is a wave optics phenomenon that occurs at any interface (smooth or rough) between the appropriate media, as long as $n_1 > n_2$ and $\theta_i > \theta_c$. Surface roughness affects the quality of the reflection but is not a necessary condition.

(D) "$n_1 > n_2$": CORRECT. This is the same as (A), stated in terms of the refractive index directly. θ_c is only defined when $n_1 > n_2$.

Correct options: (A), (B), and (D).

Answer: (ABD) ← *Go back to Q38*

Q39.

Solution

Concept — Bloch's Theorem for Periodic Potentials:

(A) "$\psi_{\mathbf{k}}(\mathbf{r})$ itself is periodic with the lattice": WRONG. Bloch's theorem states $\psi_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}$, where $u_{\mathbf{k}}$ has the periodicity of the lattice. But $\psi_{\mathbf{k}}$ itself acquires a phase $e^{i\mathbf{k}\cdot\mathbf{R}}$ under a lattice translation ($\psi_{\mathbf{k}}(\mathbf{r}+\mathbf{R}) = e^{i\mathbf{k}\cdot\mathbf{R}}\psi_{\mathbf{k}}(\mathbf{r}) \neq \psi_{\mathbf{k}}(\mathbf{r})$ in general).

(B) "Crystal momentum $\hbar$k is a good quantum number": CORRECT. Due to the discrete translational symmetry of the lattice, k (the crystal momentum) is a conserved quantum number labeling energy eigenstates.

(C) "Energy bands arise from periodic potential": CORRECT. The Bloch form of the wave function, combined with boundary conditions (Born–von Karman), leads to allowed energy bands and forbidden gaps.

(D) "$E_n(\mathrm{k})$ is the same for all k": WRONG. Band structure describes how $E_n(\mathbf{k})$ varies with k in the Brillouin zone; flat bands are a special case (e.g. in certain topological systems), not the general rule.

Correct options: (B) and (C).

Answer: (BC) ← *Go back to Q39*

Q40.

Solution

Concept — Free Electron Gas at $T = 0$:

(A) "All states up to E_F filled, all above empty": CORRECT. At $T = 0$, the Fermi–Dirac distribution is a step function: $f(E) = 1$ for $E < E_F$ and $f(E) = 0$ for $E > E_F$. Electrons fill all states in the Fermi sphere.

(B) "Average energy per electron $= E_F$": WRONG. The average energy of electrons at $T = 0$ is:

$$\langle E\rangle = \frac{\int_0^{E_F} E\, D(E)\, dE}{\int_0^{E_F} D(E)\, dE} = \frac{3}{5}E_F$$

not E_F. (Using $D(E) \propto E^{1/2}$.)

(C) "Fermi–Dirac distribution becomes a step function": CORRECT. $f(E, T \to 0) = \theta(E_F - E)$ (unit step function), as the thermal smearing $\sim k_BT$ vanishes.

(D) "No electron has energy $> E_F$": CORRECT. At $T = 0$, $f(E) = 0$ for $E > E_F$, so no quantum state above the Fermi energy is occupied.

Correct options: (A), (C), and (D).

Answer: (ACD) ← *Go back to Q40*

Section C Solutions — NAT

Q41.

Solution

Concept — Relativistic Kinetic Energy:
$K = (\gamma - 1)m_p c^2$

Step 1 — Lorentz factor at $v = 0.6c$:

$$\gamma = \frac{1}{\sqrt{1-(0.6)^2}} = \frac{1}{\sqrt{1-0.36}} = \frac{1}{\sqrt{0.64}} = \frac{1}{0.8} = 1.25$$

Step 2 — Kinetic energy:

$$K = (1.25 - 1) \times 938.3\,\text{MeV} = 0.25 \times 938.3 = 234.575\,\text{MeV}$$

$$\boxed{K \approx 234.6\,\text{MeV}}$$

Answer: (234.6) ← *Go back to Q41*

Q42.

Solution

Concept — Spherical Capacitor:
$C = \dfrac{4\pi\varepsilon_0\, ab}{b-a} = \dfrac{ab}{k_e(b-a)}$, where $k_e = 9 \times 10^9\,\text{N}\,\text{m}^2/\text{C}^2$.

Step 1 — Substitute $a = 0.10\,\text{m}$, $b = 0.20\,\text{m}$:

$$C = \frac{0.10 \times 0.20}{9 \times 10^9 \times (0.20 - 0.10)} = \frac{0.02}{9 \times 10^9 \times 0.10} = \frac{0.02}{9 \times 10^8}$$

Step 2 — Evaluate:

$$C = \frac{2 \times 10^{-2}}{9 \times 10^8} = 2.222 \times 10^{-11}\,\text{F} = 22.22\,\text{pF}$$

$$\boxed{C \approx 22.2\,\text{pF}}$$

Answer: (22.2) ← *Go back to Q42*

Q43.

Solution

Concept — Thermal de Broglie Wavelength:

$$\lambda_{\text{th}} = \frac{h}{\sqrt{2\pi mkT}}$$

Step 1 — Compute the denominator:

$$2\pi m_H kT = 2\pi \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300$$

$$= 6.2832 \times 1.67 \times 1.38 \times 300 \times 10^{-50} = 6.2832 \times 691.4 \times 10^{-50}$$

$$= 4344 \times 10^{-50} = 4.344 \times 10^{-47}\,\text{kg}^2\text{m}^2\text{s}^{-2}$$

Step 2 — Square root:

$$\sqrt{4.344 \times 10^{-47}} = \sqrt{4.344} \times 10^{-23.5} = 2.084 \times 3.162 \times 10^{-24} = 6.591 \times 10^{-24}\,\text{kg m/s}$$

Step 3 — Wavelength:

$$\lambda_{\text{th}} = \frac{6.626 \times 10^{-34}}{6.591 \times 10^{-24}} = 1.005 \times 10^{-10}\,\text{m} = 100.5\,\text{pm}$$

$$\boxed{\lambda_{\text{th}} \approx 100\,\text{pm}}$$

Note: This is approximately equal to the Bohr radius ($a_0 \approx 53\,\text{pm}$), confirming that quantum effects are important for hydrogen atoms at room temperature.

Answer: (100) ← *Go back to Q43*

Q44.

Solution

Concept — Heat Capacity of Ideal Gas:
For a monatomic ideal gas, each atom has 3 translational degrees of freedom, each contributing $\frac{1}{2}R$ per mole to C_V:

$$C_V = n \cdot \frac{3}{2}R$$

Calculation:

$$C_V = 2\,\text{mol} \times \frac{3}{2} \times 8.314\,\text{J/(mol K)} = 2 \times 12.471 = 24.942\,\text{J/K}$$

$$\boxed{C_V \approx 24.9\,\text{J/K}}$$

Answer: (24.9) ← *Go back to Q44*

Q45.

Solution

Concept — Nuclear Radius Formula:
$R = R_0 A^{1/3}$, with $R_0 = 1.2\,\text{fm}$.

Step 1 — Compute $208^{1/3}$:
$5.9^3 = 5.9 \times 5.9 \times 5.9 = 34.81 \times 5.9 = 205.4$
$5.93^3 = 5.93 \times 5.93 \times 5.93 = 35.165 \times 5.93 = 208.5$
$208^{1/3} \approx 5.927$

Step 2 — Nuclear radius of ^{208}Pb:

$$R = 1.2 \times 5.927 = 7.112\,\text{fm}$$

$$\boxed{R \approx 7.11\,\text{fm}}$$

Answer: (7.11) ← *Go back to Q45*

Q46.

Solution

Concept — NaCl (Rock-Salt) Crystal Structure:
In the rock-salt structure, each ion sits at a face-centred cubic sublattice. Each Na^+ is surrounded by 6 nearest-neighbour Cl^- ions at the face centres of a cube: one above, one below, one in front, one behind, one to the left, and one to the right.

Coordination number of Na^+ in NaCl:

$$\boxed{6}$$

(Octahedral coordination; by symmetry, each Cl^- is likewise surrounded by 6 Na^+.)

Answer: (6) ← *Go back to Q46*

Q47.

Solution

Concept — Inverse of a 2×2 Matrix:
For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$: $A^{-1} = \dfrac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

Step 1 — Determinant:

$$\det(A) = 1 \times 1 - 2 \times 0 = 1$$

Step 2 — Inverse:

$$A^{-1} = \frac{1}{1}\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

Element at row 1, column 2:

$$\boxed{(A^{-1})_{12} = -2}$$

Verification: $A \cdot A^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$

Answer: (-2) ← *Go back to Q47*

Q48.

Solution

Concept — LC Tank Resonance Frequency:

$f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

Calculation:

$$LC = 0.01\,\text{H} \times 10^{-6}\,\text{F} = 10^{-8}\,\text{H F}$$

$$\sqrt{LC} = \sqrt{10^{-8}} = 10^{-4}\,\text{s}$$

$$f_0 = \frac{1}{2\pi \times 10^{-4}} = \frac{10^4}{2 \times 3.14159} = \frac{10^4}{6.2832} = 1591.5\,\text{Hz}$$

$$\boxed{f_0 \approx 1592\,\text{Hz}}$$

Answer: (1592) ← *Go back to Q48*

Q49.

Solution

Concept — Op-Amp Difference Amplifier (Equal Resistors):

For a standard difference amplifier with all four resistors equal ($R_1 = R_2 = R_3 = R_4 = R$):

$$V_{\text{out}} = \frac{R_2}{R_1}(V_1 - V_2) = V_1 - V_2$$

where V_1 is the non-inverting input and V_2 is the inverting input.

Calculation:

$$V_{\text{out}} = 4\,\text{V} - 2\,\text{V} = 2\,\text{V}$$

$$\boxed{V_{\text{out}} = 2\,\text{V}}$$

Answer: (2) ← Go back to Q49

Q50.

Solution

Concept — Angular Momentum of Rotating Body:
$L = I\omega$

Calculation:

$$L = 0.5\,\text{kg}\,\text{m}^2 \times 20\,\text{rad/s} = 10\,\text{J}\,\text{s}$$

$$\boxed{L = 10\,\text{J}\,\text{s}}$$

Answer: (10) ← Go back to Q50

Q51.

Solution

Concept — Lorentz Factor:
$\gamma = 1/\sqrt{1 - v^2/c^2}$

Calculation for $v = 0.95c$:

$$v^2/c^2 = 0.9025$$

$$1 - v^2/c^2 = 0.0975$$

$$\sqrt{0.0975} = 0.3122$$

$$\gamma = \frac{1}{0.3122} = 3.202$$

$$\boxed{\gamma \approx 3.20}$$

Answer: (3.20) ← Go back to Q51

Q52.

Solution

Concept — Hydrogen Atom Ground State Energy:
The ground state of hydrogen has principal quantum number $n = 1$ and energy:

$$E_1 = -\frac{m_e e^4}{8\varepsilon_0^2 h^2} = -13.6\,\text{eV}$$

Step 1 — Magnitude:

$$|E_1| = 13.60\,\text{eV}$$

This is the ionisation energy of hydrogen from the ground state (energy needed to remove the electron completely from $n = 1$ to $n = \infty$).

$$\boxed{|E_1| = 13.60\,\text{eV}}$$

Answer: (13.60) ← *Go back to Q52*

Q53.

Solution

Concept — Entropy Change in a Complete Cycle:
Entropy S is a *state function*. In any complete thermodynamic cycle, the system returns to its initial state, so:

$$\Delta S_{\text{cycle}} = S_{\text{final}} - S_{\text{initial}} = 0$$

For the Carnot cycle specifically:
Each step is reversible. The gas absorbs heat Q_H isothermally at T_H (entropy change $+Q_H/T_H$) and rejects heat Q_C isothermally at T_C (entropy change $-Q_C/T_C$). Since the Carnot cycle satisfies $Q_H/T_H = Q_C/T_C$, the two changes exactly cancel:

$$\Delta S_{\text{cycle}} = \frac{Q_H}{T_H} - \frac{Q_C}{T_C} = 0$$

$$\boxed{\Delta S = 0.00\,\text{J/K}}$$

Answer: (0.00) ← *Go back to Q53*

Q54.

Solution

Concept — Fermi Temperature:
$T_F = E_F/k_B$

Step 1 — Convert E_F to joules:

$$E_F = 7.04\,\text{eV} \times 1.6 \times 10^{-19}\,\text{J/eV} = 11.264 \times 10^{-19}\,\text{J}$$

Step 2 — Fermi temperature:

$$T_F = \frac{11.264 \times 10^{-19}}{1.38 \times 10^{-23}} = \frac{11.264}{1.38} \times 10^4 = 8.162 \times 10^4\,\text{K}$$

To the nearest 100 K: $T_F \approx 81600\,\text{K}$.

$$\boxed{T_F \approx 81600\,\mathrm{K}}$$

Physical note: At room temperature ($T \approx 300\,\mathrm{K}$), $T \ll T_F$, confirming that conduction electrons in copper are a highly degenerate quantum system at room temperature.

Answer: (81600) ← *Go back to Q54*

Q55.

Solution

Concept — Normal Zeeman Effect:
In an external magnetic field B, an orbital level with quantum number l splits into $2l+1$ sublevels at energies $E = m_l\,\mu_B B$ ($m_l = -l, \ldots, +l$). The energy spacing between adjacent sublevels is:

$$\Delta E = \mu_B B$$

Step 1 — Compute ΔE in joules:

$$\Delta E = \mu_B B = 9.274 \times 10^{-24}\,\text{J/T} \times 1\,\text{T} = 9.274 \times 10^{-24}\,\text{J}$$

Step 2 — Convert to eV:

$$\Delta E = \frac{9.274 \times 10^{-24}}{1.602 \times 10^{-19}}\,\text{eV} = 5.789 \times 10^{-5}\,\text{eV}$$

In units of 10^{-5} eV:

$$\Delta E \approx 5.79 \times 10^{-5}\,\text{eV}$$

$$\boxed{\Delta E = 5.79 \text{ (in units of } 10^{-5} \text{ eV)}}$$

Answer: (5.79) ← *Go back to Q55*

Q56.

Solution

Concept — Drude Model: Mean Collision Time in Copper:
$\tau = \dfrac{m_e \sigma}{n e^2}$

Step 1 — Numerator:

$$m_e \sigma = 9.11 \times 10^{-31} \times 6 \times 10^{7} = 54.66 \times 10^{-24}$$

Step 2 — Denominator:

$$ne^2 = 8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 = 8.5 \times 10^{28} \times 2.56 \times 10^{-38} = 21.76 \times 10^{-10}$$

Step 3 — Mean collision time:

$$\tau = \frac{54.66 \times 10^{-24}}{21.76 \times 10^{-10}} = 2.512 \times 10^{-14}\,\text{s} = 25.12\,\text{ps}$$

$$\boxed{\tau \approx 25.1\,\text{ps}}$$

Answer: (25.1) ← *Go back to Q56*

Q57.

Solution

Concept — Radioactive Decay Over Multiple Half-Lives:
After n half-lives: $N/N_0 = (1/2)^n$.

Calculation for $n = 5.0$:

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^5 = \frac{1}{32} = 0.03125$$

To three decimal places:

$$\boxed{\frac{N}{N_0} \approx 0.031}$$

Answer: (0.031) ← *Go back to Q57*

Q58.

Solution

Concept — Error Function $\mathrm{erf}(x)$:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt$$

The standard tabulated value at $x = 1$:

$$\mathrm{erf}(1) = \frac{2}{\sqrt{\pi}} \int_0^1 e^{-t^2}\,dt \approx 0.8427$$

This can also be obtained via numerical integration of the Gaussian, or using the series expansion:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{3} + \frac{x^5}{10} - \frac{x^7}{42} + \cdots\right)$$

At $x = 1$: $\frac{2}{\sqrt{\pi}}(1 - 0.333 + 0.100 - 0.024 + \cdots) \approx \frac{2}{\sqrt{\pi}} \times 0.747 \approx 0.843$.

$$\boxed{\mathrm{erf}(1) \approx 0.843}$$

Answer: (0.843) ← *Go back to Q58*

Q59.

Solution

Concept — Inductive Time Constant:
$\tau = L/R$

Calculation:

$$\tau = \frac{L}{R} = \frac{0.1\,\mathrm{H}}{200\,\Omega} = 5 \times 10^{-4}\,\mathrm{s} = 0.50\,\mathrm{ms}$$

$$\boxed{\tau = 0.50\,\mathrm{ms}}$$

Physical interpretation: The current in the RL circuit rises from 0 to $(1-1/e) \approx 63.2\%$ of its final value in one time constant $\tau = 0.50\,$ms.

Answer: (0.50) ← *Go back to Q59*

Q60.

Solution

Concept — de Broglie Wavelength of Thermal Neutron:
$\lambda = h/\sqrt{2m_n kT}$

Step 1 — Momentum:

$$2m_n kT = 2 \times 1.675 \times 10^{-27} \times 1.38 \times 10^{-23} \times 293$$

$$= 2 \times 1.675 \times 1.38 \times 293 \times 10^{-50}$$

$$= 2 \times 676.6 \times 10^{-50} = 1353.2 \times 10^{-50} = 1.3532 \times 10^{-47}$$

$$p = \sqrt{2m_n kT} = \sqrt{1.3532 \times 10^{-47}} = 3.679 \times 10^{-24}\,\mathrm{kg\,m/s}$$

Step 2 — Wavelength:

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{3.679 \times 10^{-24}} = 1.801 \times 10^{-10}\,\text{m} = 180.1\,\text{pm}$$

$$\boxed{\lambda \approx 180\,\text{pm}}$$

Note: 180 pm is comparable to typical interatomic spacings in crystals, which is why thermal neutrons are excellent probes of crystal structure via neutron diffraction.

Answer: (180) ← *Go back to Q60*

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	A	3	B	4	C	5	D
6	A	7	B	8	C	9	D	10	A
11	B	12	C	13	D	14	A	15	B
16	C	17	D	18	A	19	B	20	C
21	D	22	A	23	B	24	C	25	D
26	A	27	B	28	C	29	D	30	A
31	ACD	32	ABD	33	BC	34	ACD	35	ABD
36	BC	37	ACD	38	ABD	39	BC	40	ACD
41	234.6	42	22.2	43	100	44	24.9	45	7.11
46	6	47	-2	48	1592	49	2	50	10
51	3.20	52	13.60	53	0.00	54	81600	55	5.79
56	25.1	57	0.031	58	0.843	59	0.50	60	180