

IIT JAM Mathematics

Sample Paper – 2

Instructions

- Total Questions: **60** Total Marks: **100** Duration: **180 minutes**
- **Section A** (Q1–Q30, MCQ, single correct): Q1–Q10: 1 mark each; *wrong answer: $-1/3$ mark*. Q11–Q30: 2 marks each; *wrong answer: $-2/3$ mark*.
- **Section B** (Q31–Q40, MSQ, one or more correct): 2 marks each; *no negative marking*. Full marks only if ALL correct options selected; partial/wrong selection: 0 marks.
- **Section C** (Q41–Q60, NAT, numerical answer): Q41–Q50: 1 mark each; Q51–Q60: 2 marks each; *no negative marking*.
- Use of calculators is **NOT** allowed.
- All questions are compulsory.

SECTION A — Multiple Choice Questions (Single Correct)

Q1–Q10: 1 Mark each | Wrong answer: $-\frac{1}{3}$

Q1. The series $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n$ is tested using the *root test*. Which of the following statements is correct?

- (A) The series converges; the root test gives $\limsup_{n \rightarrow \infty} (a_n)^{1/n} = 0 < 1$.
- (B) The series diverges; the root test gives limit > 1 .
- (C) The root test is inconclusive for this series.
- (D) The series converges, but only by the ratio test, not the root test.

Q2. A continuous function f defined on a closed bounded interval $[a, b]$ is always:

- (A) Differentiable on (a, b)
- (B) Monotone on $[a, b]$
- (C) Lipschitz continuous on $[a, b]$

(D) Uniformly continuous on $[a, b]$

Q3. Let $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $J = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Both have characteristic polynomial $(\lambda - 1)^2$. Which statement is TRUE?

(A) I_2 and J are similar matrices.

(B) I_2 and J are row equivalent only.

(C) I_2 and J are **NOT** similar, since their minimal polynomials differ.

(D) I_2 and J are equal as matrices.

Q4. The alternating group A_4 (even permutations of 4 elements) has order:

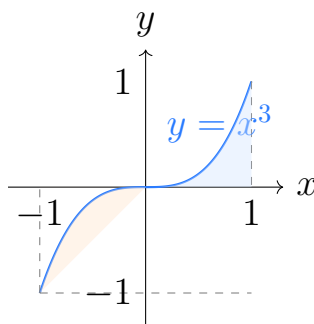
(A) 24

(B) 12

(C) 6

(D) 8

Q5. The graph below shows $y = x^3$ on $[-1, 1]$, which is an *odd* function.



Evaluate $\int_{-1}^1 x^3 dx$.

(A) 0

(B) $\frac{1}{2}$

(C) 2

(D) 1

Q6. Which is the **complete and correct** set of hypotheses required for Rolle's Theorem?



- (A) f is continuous on the open interval (a, b) only.
- (B) f is differentiable on the closed interval $[a, b]$.
- (C) f is continuous on $[a, b]$ and $f(a) = f(b)$.
- (D) f is continuous on $[a, b]$, differentiable on (a, b) , and $f(a) = f(b)$.

Q7. The general solution of the exact equation $y dx + x dy = 0$ is:

- (A) $y = Cx$
- (B) $xy = C$
- (C) $x + y = C$
- (D) $x^2 + y^2 = C$

Q8. Compute the curl $\nabla \times \mathbf{F}$ for $\mathbf{F} = x\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$.

- (A) $\hat{\mathbf{i}}$
- (B) $x\hat{\mathbf{j}}$
- (C) $\mathbf{0}$
- (D) $-\hat{\mathbf{k}}$

Q9. The gradient of $f(x, y) = x^2 + y^2$ at the point $(1, 1)$ is:

- (A) $(2, 2)$
- (B) $(1, 1)$
- (C) $(0, 0)$
- (D) $(4, 4)$

Q10. The geometric series $\sum_{n=0}^{\infty} x^n$ converges if and only if:

- (A) $|x| \leq 1$
- (B) $|x| < 2$
- (C) for all real x
- (D) $|x| < 1$



Q11–Q30: 2 Marks each | Wrong answer: $-\frac{2}{3}$

Q11. A monotone function f defined on a closed interval $[a, b]$ is:

- (A) Riemann integrable on $[a, b]$.
- (B) Necessarily differentiable on (a, b) .
- (C) Necessarily continuous on $[a, b]$.
- (D) Unbounded on $[a, b]$.

Q12. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges by:

- (A) The ratio test
- (B) The root test
- (C) The comparison test
- (D) The Leibniz alternating series test

Q13. The Cayley–Hamilton theorem states that:

- (A) Every square matrix has only real eigenvalues.
- (B) $\det(A)$ equals the product of the eigenvalues (counted with multiplicity).
- (C) Every square matrix satisfies its own characteristic polynomial.
- (D) Every square matrix is diagonalizable.

Q14. For $n \times n$ matrices A and B , the *rank sub-additivity* inequality states:

- (A) $\text{rank}(A + B) \leq |\text{rank}(A) - \text{rank}(B)|$
- (B) $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$
- (C) $\text{rank}(A + B) \leq \text{rank}(A) \cdot \text{rank}(B)$
- (D) $\text{rank}(A + B) \leq \min(\text{rank}(A), \text{rank}(B))$

Q15. Let $H = \{0, 2, 4\}$ be the subgroup of $\mathbb{Z}_6$. The quotient group $\mathbb{Z}_6/H$ is isomorphic to:

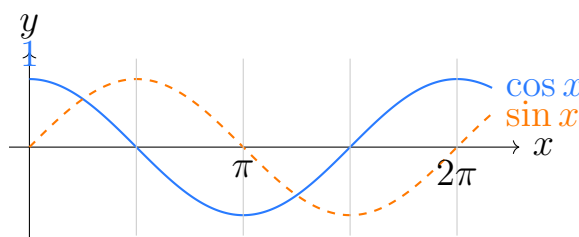


- (A) $\mathbb{Z}_6$
- (B) $\mathbb{Z}_3$
- (C) $\mathbb{Z}_4$
- (D) $\mathbb{Z}_2$

Q16. Every subgroup of index 2 in a group G is:

- (A) Normal in G
- (B) Necessarily cyclic
- (C) Necessarily abelian
- (D) Of prime order

Q17. The figure shows the two linearly independent solutions of the ODE $y'' + y = 0$.



The general solution of $y'' + y = 0$ is:

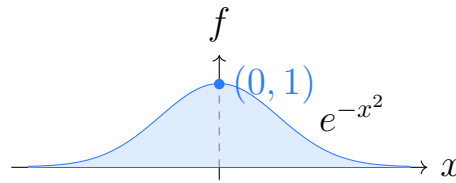
- (A) $C_1 e^x + C_2 e^{-x}$
- (B) $C_1 e^x + C_2$
- (C) $C_1 \cos x + C_2 \sin x$
- (D) $e^x (C_1 \cos x + C_2 \sin x)$

Q18. The Bernoulli ODE $y' + P(x)y = Q(x)y^2$ is linearised by the substitution:

- (A) $v = y^2$
- (B) $v = 1/y$
- (C) $v = y$
- (D) $v = e^y$



Q19. The cross-section of $f(x, y) = e^{-(x^2+y^2)}$ along $y = 0$ is shown below.



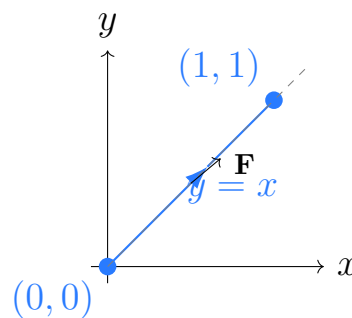
For $f(x, y) = e^{-(x^2+y^2)}$, the point $(0, 0)$ is a:

- (A) Global maximum
- (B) Saddle point
- (C) Local minimum
- (D) Inflection point

Q20. The second-order Taylor polynomial of f at $(0, 0)$, given that $f(0, 0) = 1$, $f_x(0, 0) = f_y(0, 0) = 0$, $f_{xx}(0, 0) = 2$, $f_{yy}(0, 0) = 4$, $f_{xy}(0, 0) = 1$, is:

- (A) $1 + 2x^2 + y^2$
- (B) $1 + x^2 + y^2$
- (C) $x^2 + y^2$
- (D) $1 + x^2 + xy + 2y^2$

Q21. The work done by $\mathbf{F} = (x^2, y^2)$ along the path $y = x$ from $(0, 0)$ to $(1, 1)$ is shown below.



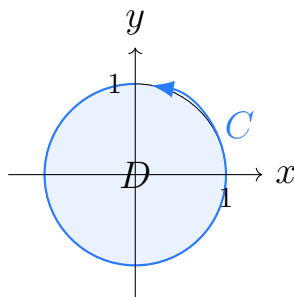
Compute $\int_C x^2 dx + y^2 dy$ along $y = x$, $0 \leq x \leq 1$.

- (A) 0
- (B) $\frac{1}{2}$



- (C) $\frac{2}{3}$
 (D) 1

Q22. Use Green's theorem to evaluate $\oint_C (y \, dx + x^2 \, dy)$ where C is the boundary of the unit disk D traversed counter-clockwise.



With $P = y$, $Q = x^2$: $Q_x - P_y = 2x - 1$. The answer is:

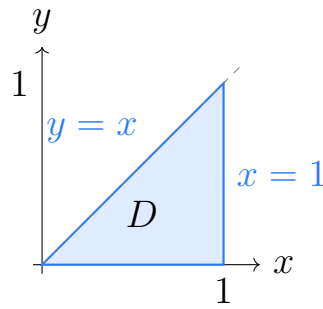
- (A) 0
 (B) $-\pi$
 (C) π
 (D) 2π

Q23. The radius of convergence of $\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$ (by Stirling's approximation or ratio test, $n!/n^n \rightarrow e^{-n}$) is:

- (A) 0
 (B) 1
 (C) $\frac{1}{e}$
 (D) e

Q24. The integral $\int_0^1 \int_y^1 e^{x^2} \, dx \, dy$ is evaluated by switching the order of integration. The triangular region of integration is shown below.





After switching: $\int_0^1 \int_0^x e^{x^2} dy dx = \int_0^1 x e^{x^2} dx = \frac{e^{x^2}}{2} \Big|_0^1$. The value is:

- (A) $\frac{e-1}{2}$
- (B) $e-1$
- (C) $\frac{e+1}{2}$
- (D) 1

Q25. If $f'(x) > 0$ for all $x \in (a, b)$, then f is:

- (A) Constant on (a, b)
- (B) Strictly decreasing on (a, b)
- (C) Strictly increasing on (a, b)
- (D) Necessarily convex on (a, b)

Q26. If W is a subspace of $\mathbb{R}^n$ and $W^\perp$ its orthogonal complement, then:

- (A) $\dim(W^\perp) = \dim(W)$
- (B) $\dim(W) + \dim(W^\perp) = n$
- (C) $\dim(W) + \dim(W^\perp) = n + \dim(W)$
- (D) $\dim(W^\perp) = n - \dim(W) + 1$

Q27. The characteristic polynomial of $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is:

- (A) $\lambda^2 + 1$
- (B) $\lambda^2 - 1$
- (C) λ^2



(D) $\lambda^2 + \lambda + 1$

Q28. Lagrange's theorem for finite groups states that:

- (A) $|H| = |G|/2$ for every subgroup H .
- (B) Every subgroup of G is normal.
- (C) G/H is a group for every subgroup H .
- (D) The order of every subgroup H divides the order of G .

Q29. The additive group $(\mathbb{Z}, +)$ is:

- (A) Finite
- (B) Non-abelian
- (C) Cyclic (generated by 1)
- (D) A non-abelian group of finite order

Q30. The infimum of the set $\{\frac{1}{n} : n \in \mathbb{N}\}$ is:

- (A) 1
- (B) 0
- (C) Does not exist
- (D) $-\infty$

**SECTION B — Multiple Select
Questions (One or More Correct)**

Q31–Q40: 2 Marks each | No negative marking

Full marks awarded only if ALL correct options are selected.

Q31. Consider the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^\alpha}$. Which of the following statements are TRUE?

- (A) The series converges *conditionally* for $\alpha = \frac{1}{2}$.
- (B) The series converges *absolutely* for $\alpha = \frac{1}{2}$.
- (C) The series converges *absolutely* for $\alpha = 2$.



(D) The series *diverges* for $\alpha = 0$.

Q32. Which of the following statements about the $n \times n$ identity matrix I_n are TRUE?

(A) I_n has n *distinct* eigenvalues.

(B) I_n is diagonalizable.

(C) $\det(I_n) = 1$.

(D) The only eigenvalue of I_n is 0.

Q33. Let $T : V \rightarrow W$ be a linear transformation between finite-dimensional vector spaces. Which are TRUE?

(A) $\ker(T)$ is a subspace of V .

(B) $\text{Im}(T)$ is a subspace of W .

(C) $\dim(\ker T) = \dim(V)$ always.

(D) $\dim(\ker T) + \dim(\text{Im } T) = \dim(V)$ (rank–nullity theorem).

Q34. Which of the following must be TRUE about every finite abelian group G ?

(A) G is isomorphic to a direct product of cyclic groups.

(B) G has prime order.

(C) Every subgroup of G is normal in G .

(D) G is necessarily cyclic.

Q35. Let $f(x) = |x|$. Which of the following are TRUE?

(A) f is differentiable at $x = 0$.

(B) f is continuous at $x = 0$.

(C) $f'(0) = 0$.

(D) f is Riemann integrable on $[-1, 1]$.

Q36. Which of the following are solutions of $y'' + 4y = 0$?



- (A) $y = \cos 2x$
- (B) $y = \sin 2x$
- (C) $y = 3 \cos 2x - \sin 2x$
- (D) $y = e^{2x}$

Q37. Green's theorem $\oint_C (P dx + Q dy) = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$ applies when which conditions hold?

- (A) P and Q have continuous first-order partial derivatives on D .
- (B) C is any smooth curve.
- (C) C is a piecewise-smooth, positively oriented, simple closed curve.
- (D) D is the region enclosed by C .

Q38. Consider the ODE $y'' - y = 0$. Which statements are TRUE?

- (A) The general solution is $C_1 \cos x + C_2 \sin x$.
- (B) The general solution is $C_1 e^x + C_2 e^{-x}$.
- (C) The characteristic roots are $\pm i$.
- (D) $y = \cosh x$ is a particular solution.

Q39. Let $f(x, y) = xy(x - y)$. Which of the following are TRUE about the point $(0, 0)$?

- (A) $(0, 0)$ is a critical point of f .
- (B) $f(0, 0) = 0$.
- (C) $(0, 0)$ is a local minimum of f .
- (D) $(0, 0)$ is a saddle point of f .

Q40. Let $\mathbf{F}(x, y) = (2x, 2y)$. Which statements are TRUE?

- (A) $\nabla \times \mathbf{F} \neq \mathbf{0}$
- (B) $\mathbf{F}$ is a conservative vector field.
- (C) $\mathbf{F} = \nabla(x^2 + y^2)$.



(D) $\nabla \cdot \mathbf{F} = 0$.

SECTION C — Numerical Answer Type

Q41–Q50: 1 Mark each | Q51–Q60: 2 Marks each | No negative marking*Enter your answer as a number in the box on screen. No options are provided.*

Q41. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. The matrix is schematically shown below (shaded columns indicate pivot columns).

1	2	3
4	5	6
7	8	9

2 pivot cols (rank 2), 1 free col

Find the nullity of A (i.e., $\dim(\ker A)$).

Q42. Find the order of the element 4 in the additive group $(\mathbb{Z}_{12}, +)$. (Recall: $\text{order} = 12 / \gcd(4, 12)$.)

Q43. Evaluate $\int_{-\pi}^{\pi} \sin(x) \cos(x) dx$. (Hint: $\sin x \cos x = \frac{1}{2} \sin 2x$, which is an odd function on $[-\pi, \pi]$.)

Q44. Find the number of distinct Sylow 2-subgroups of the symmetric group S_3 . (The Sylow 2-subgroups of S_3 are the three subgroups generated by the transpositions $(12), (13), (23)$.)

Q45. Evaluate $\left. \frac{d^2}{dx^2} e^{2x} \right|_{x=0}$.

Q46. Find the number of distinct real roots of the equation $x^3 - 3x = 0$.

Q47. Evaluate $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x}$.

Q48. Evaluate $\int_{-1}^1 x^5 dx$.



Q49. A 2×2 matrix A has eigenvalues 1 and -1 . Find $\det(A)$.

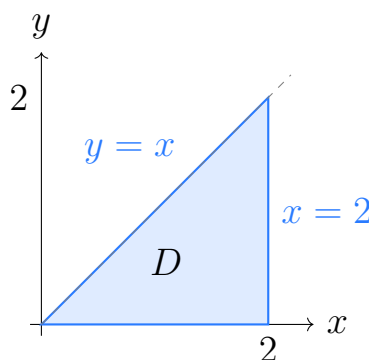
Q50. Find the order of the symmetric group S_5 .

Q51. Let A be a 3×5 matrix with $\text{rank}(A) = 2$. Find the nullity of A .

Q52. Find the number of elements of order 3 in the cyclic group $\mathbb{Z}_9 = (\mathbb{Z}/9\mathbb{Z}, +)$.

Q53. Find a particular constant solution y_p to the ODE $y'' + 4y = 8$.

Q54. Evaluate $\int_0^2 \int_0^x (x+y) dy dx$.



Q55. Use Green's theorem to evaluate $\oint_C (x^2 dx + xy dy)$ where C is the triangle with vertices $(0,0)$, $(1,0)$, $(1,1)$ traversed counter-clockwise. (Enter your answer correct to 2 decimal places.)

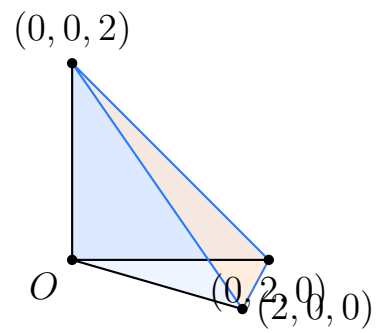
Q56. Find the sum of the series $\sum_{n=0}^{\infty} \frac{1}{n!}$. (Enter your answer to 3 decimal places.)

Q57. A 3×3 matrix A has eigenvalues 1, 1, 2. Find $\text{tr}(A)$.

Q58. Solve the IVP $y' = y$, $y(0) = 1$. Find $y(\ln 2)$.

Q59. Find the volume of the tetrahedron with vertices at $(0,0,0)$, $(2,0,0)$, $(0,2,0)$, and $(0,0,2)$.





(Enter your answer to 3 decimal places.)

Q60. Find the number of elements in the group $\mathbb{Z}_2 \times \mathbb{Z}_3$.

— End of Question Paper —

Solutions start on the next page.



IIT JAM Mathematics

Sample Paper 2 — Detailed Solutions

SECTION A — Solutions (MCQ)

Q1. Answer: (A)

Solution

Apply the **root test** to $a_n = \left(\frac{1}{n}\right)^n$:

$$L = \limsup_{n \rightarrow \infty} (a_n)^{1/n} = \lim_{n \rightarrow \infty} \frac{1}{n} = 0 < 1.$$

Since $L < 1$, the series $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n$ **converges** by the root test. Option (A) is correct.

Q2. Answer: (D)

Solution

By the **Heine–Cantor theorem**, every continuous function on a *compact* (closed and bounded) set is uniformly continuous. Since $[a, b]$ is compact, the function is uniformly continuous. The other options are false in general (e.g., $f(x) = |x|$ on $[-1, 1]$ is continuous but not differentiable at 0).

Q3. Answer: (C)

Solution

I_2 and $J = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ share the same characteristic polynomial $(\lambda - 1)^2$. However:

- Minimal polynomial of I_2 : $(\lambda - 1)$.
- Minimal polynomial of J : $(\lambda - 1)^2$.

Similar matrices have identical minimal polynomials. Since the minimal polynomials differ, I_2 and J are **NOT similar**. Option (C) is correct.

Q4. Answer: (B)



Solution

The symmetric group S_4 has order $4! = 24$. The alternating group A_4 consists of the even permutations of S_4 , which is exactly half:

$$|A_4| = \frac{|S_4|}{2} = \frac{24}{2} = \boxed{12}.$$

Option (B) is correct.

Q5. **Answer: (A)**

Solution

Since $f(x) = x^3$ is an **odd function** (i.e., $f(-x) = -f(x)$) and the interval $[-1, 1]$ is symmetric about the origin:

$$\int_{-1}^1 x^3 dx = 0.$$

The positive and negative areas (shaded in the figure) cancel exactly. Option (A) is correct.

Q6. **Answer: (D)**

Solution

Rolle's Theorem requires three hypotheses:

- (i) f is *continuous* on $[a, b]$,
- (ii) f is *differentiable* on (a, b) ,
- (iii) $f(a) = f(b)$.

Only option (D) lists all three correctly. (Note: continuity is required on the *closed* interval, differentiability on the *open* interval.)

Q7. **Answer: (B)**

Solution

The equation $y dx + x dy = 0$ is recognised as the total differential of xy :

$$d(xy) = y dx + x dy = 0.$$

Integrating: $xy = C$ (constant). Option (B) is correct.



Q8. Answer: (C)

Solution

For $\mathbf{F} = (x, 0, 0)$, the curl is:

$$\nabla \times \mathbf{F} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \partial_x & \partial_y & \partial_z \\ x & 0 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0 - 0) - \hat{\mathbf{j}}(0 - 0) + \hat{\mathbf{k}}(0 - 0) = \mathbf{0}.$$

Since $\mathbf{F}$ depends only on x and points only in the x -direction, the curl is zero. Option (C) is correct.

Q9. Answer: (A)

Solution

For $f(x, y) = x^2 + y^2$:

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = (2x, 2y).$$

At $(1, 1)$: $\nabla f(1, 1) = (2 \cdot 1, 2 \cdot 1) = (2, 2)$. Option (A) is correct.

Q10. Answer: (D)

Solution

The geometric series $\sum_{n=0}^{\infty} x^n$ has common ratio x . By the standard result, it converges (to $\frac{1}{1-x}$) if and only if $|x| < 1$. At $|x| = 1$ the series diverges. Option (D) is correct.

Q11. Answer: (A)

Solution

A **monotone function** on $[a, b]$ has at most countably many discontinuities (all of jump type). Since the set of discontinuities has measure zero, the function is **Riemann integrable**. (Lebesgue's criterion: f is Riemann integrable iff its discontinuities form a set of measure zero.) Option (A) is correct.

Q12. Answer: (D)



Solution

The **Leibniz alternating series test**: $\sum_{n=1}^{\infty} (-1)^n b_n$ converges if (i) $b_n \searrow 0$ monotonically. Here $b_n = \frac{1}{n} \rightarrow 0$ decreasingly. Hence $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges. (It converges conditionally: $\sum 1/n$ diverges.) Option (D) is correct.

Q13. **Answer: (C)****Solution**

The **Cayley–Hamilton theorem**: if $p(\lambda) = \det(\lambda I - A)$ is the characteristic polynomial of A , then $p(A) = \mathbf{0}$ (the zero matrix). That is, every square matrix satisfies its own characteristic equation. Option (C) is correct.

Q14. **Answer: (B)****Solution**

Rank sub-additivity: The column space of $A + B$ is contained in $\text{col}(A) + \text{col}(B)$, so:

$$\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B).$$

Option (B) is correct.

Q15. **Answer: (D)****Solution**

$H = \{0, 2, 4\}$ is a subgroup of $\mathbb{Z}_6$ of order 3. By Lagrange: $[\mathbb{Z}_6 : H] = 6/3 = 2$. The quotient $\mathbb{Z}_6/H$ has order 2, and a group of order 2 is isomorphic to $\mathbb{Z}_2$. Option (D) is correct.

Q16. **Answer: (A)****Solution**

If $[G : H] = 2$, then H has exactly two left cosets: H and $G \setminus H$. For any $g \in G$:

- If $g \in H$: $gHg^{-1} \subseteq H$.
- If $g \notin H$: $gH = G \setminus H = Hg$, so $gHg^{-1} = H$.

In both cases $gHg^{-1} = H$, so H is **normal** in G . Option (A) is correct.



Q17. Answer: (C)

Solution

The characteristic equation of $y'' + y = 0$ is $r^2 + 1 = 0$, giving $r = \pm i$ (purely imaginary roots). The general solution is:

$$y = C_1 \cos x + C_2 \sin x.$$

This corresponds to the two curves shown in the figure. Option (C) is correct.

Q18. Answer: (B)

Solution

For the Bernoulli equation $y' + P(x)y = Q(x)y^n$ (here $n = 2$), set $v = y^{1-n} = y^{-1} = \frac{1}{y}$.

Then $v' = -y^{-2}y'$, and after substitution the equation becomes the *linear* ODE $v' - P(x)v = -Q(x)$. Option (B) is correct.

Q19. Answer: (A)

Solution

For $f(x, y) = e^{-(x^2+y^2)}$, note $f \leq 1$ for all (x, y) and $f(0, 0) = e^0 = 1$. Since $-(x^2 + y^2) \leq 0$ with equality only at the origin, the **global maximum** value is 1, attained at $(0, 0)$.

The Hessian at $(0, 0)$ has $f_{xx} = f_{yy} = -2 < 0$ and $D = f_{xx}f_{yy} - f_{xy}^2 = 4 > 0$ (local max), and the function decays to 0 at $\pm\infty$, confirming it is a global maximum. Option (A) is correct.

Q20. Answer: (D)

Solution

The second-order Taylor polynomial at $(0, 0)$ is:

$$f(x, y) \approx f(0, 0) + f_x x + f_y y + \frac{1}{2}f_{xx}x^2 + f_{xy}xy + \frac{1}{2}f_{yy}y^2.$$

Substituting the given values:

$$= 1 + 0 \cdot x + 0 \cdot y + \frac{1}{2}(2)x^2 + (1)xy + \frac{1}{2}(4)y^2 = 1 + x^2 + xy + 2y^2.$$

Option (D) is correct.



Q21. Answer: (C)

Solution

Along $y = x$ from $(0, 0)$ to $(1, 1)$: parameterise by $x = t, y = t, t \in [0, 1]$:

$$\int_C x^2 dx + y^2 dy = \int_0^1 (t^2 \cdot 1 + t^2 \cdot 1) dt = \int_0^1 2t^2 dt = \frac{2t^3}{3} \Big|_0^1 = \frac{2}{3}.$$

Option (C) is correct.

Q22. Answer: (B)

Solution

By Green's theorem with $P = y, Q = x^2$:

$$\oint_C (y dx + x^2 dy) = \iint_D \left(\frac{\partial x^2}{\partial x} - \frac{\partial y}{\partial y} \right) dA = \iint_D (2x - 1) dA.$$

Over the unit disk D : $\iint_D 2x dA = 0$ (odd integrand on symmetric region) and $\iint_D 1 dA = \pi(1)^2 = \pi$. Therefore the integral $= 0 - \pi = -\pi$. Option (B) is correct.

Q23. Answer: (D)

Solution

Apply the ratio test to $a_n = \frac{n!}{n^n}$:

$$\frac{a_{n+1}}{a_n} = \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = \left(\frac{n}{n+1} \right)^n \rightarrow e^{-1}.$$

The radius of convergence is $R = 1/e^{-1} = e$. Option (D) is correct.

Q24. Answer: (A)

Solution

The original region is $0 \leq y \leq x \leq 1$. Switching order (fixing x, y from 0 to x):

$$\int_0^1 \int_y^1 e^{x^2} dx dy = \int_0^1 \int_0^x e^{x^2} dy dx = \int_0^1 x e^{x^2} dx.$$



Let $u = x^2$, $du = 2x dx$:

$$= \frac{1}{2} \int_0^1 e^u du = \frac{1}{2} [e^u]_0^1 = \frac{e-1}{2}.$$

Option (A) is correct.

Q25. **Answer: (C)**

Solution

If $f'(x) > 0$ on (a, b) , then by the **mean value theorem**, for any $x_1 < x_2$ in (a, b) :

$$f(x_2) - f(x_1) = f'(c)(x_2 - x_1) > 0,$$

so $f(x_2) > f(x_1)$. Hence f is **strictly increasing**. Option (C) is correct.

Q26. **Answer: (B)**

Solution

By the **fundamental theorem of linear algebra**, $\mathbb{R}^n = W \oplus W^\perp$ (orthogonal direct sum), so:

$$\dim(W) + \dim(W^\perp) = n.$$

Option (B) is correct.

Q27. **Answer: (A)**

Solution

$$\det(\lambda I - A) = \det \begin{pmatrix} \lambda & -1 \\ 1 & \lambda \end{pmatrix} = \lambda^2 - (-1) = \lambda^2 + 1.$$

The eigenvalues are $\pm i$ (purely imaginary). Option (A) is correct.

Q28. **Answer: (D)**

Solution

Lagrange's theorem: If G is a finite group and $H \leq G$, then $|H|$ divides $|G|$. This follows from the fact that G is partitioned into $[G : H] = |G|/|H|$ distinct left cosets, each of size $|H|$. Option (D) is correct.



Q29. Answer: (C)

Solution

$(\mathbb{Z}, +)$ is **cyclic**: every integer n can be written as $n \cdot 1$ (i.e., 1 added n times) or (-1) added $|n|$ times. So $\mathbb{Z} = \langle 1 \rangle = \langle -1 \rangle$. It is infinite and abelian. Option (C) is correct.

Q30. Answer: (B)

Solution

The set $S = \{1/n : n \in \mathbb{N}\} = \{1, 1/2, 1/3, \dots\}$. The infimum is the greatest lower bound. Since $1/n > 0$ for all n , we have 0 as a lower bound. For any $\varepsilon > 0$, choose $n > 1/\varepsilon$, then $1/n < \varepsilon$, so 0 is the *greatest* lower bound:

$$\inf S = 0.$$

(Note: $0 \notin S$, so it is not attained.) Option (B) is correct.

SECTION B — Solutions (MSQ)

Q31. Answer: (ACD)

Solution

For $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^\alpha}$:

- **(A) True** ($\alpha = 1/2$): $\sum 1/n^{1/2}$ diverges (p-series, $p \leq 1$), but $b_n = 1/\sqrt{n} \rightarrow 0$ decreasingly, so by Leibniz it converges *conditionally*.
- **(B) False**: $\sum 1/\sqrt{n}$ diverges, so not absolutely convergent for $\alpha = 1/2$.
- **(C) True** ($\alpha = 2$): $\sum 1/n^2$ converges (p-series, $p = 2 > 1$), so the series converges *absolutely*.
- **(D) True** ($\alpha = 0$): Terms are $(-1)^n$, which do not tend to 0, so the series *diverges*.

Correct options: A, C, D.

Q32. Answer: (BC)



Solution

- **(A) False:** The only eigenvalue of I_n is 1 (with multiplicity n). It does NOT have n distinct eigenvalues (unless $n = 1$).
- **(B) True:** I_n is already diagonal ($I_n = I_n I_n I_n^{-1}$), so it is diagonalizable.
- **(C) True:** $\det(I_n) = 1$ (product of diagonal entries).
- **(D) False:** The eigenvalue of I_n is 1, not 0.

Correct options: **B, C.**

Q33. **Answer: (ABD)**

Solution

- **(A) True:** The kernel $\ker(T) = \{v \in V : T(v) = 0\}$ is always a subspace of V .
- **(B) True:** The image $\text{Im}(T) = \{T(v) : v \in V\}$ is always a subspace of W .
- **(C) False:** $\dim(\ker T) = \dim(V)$ only if $T = 0$. In general $\dim(\ker T) \leq \dim(V)$.
- **(D) True:** The **rank–nullity theorem** states $\dim(\ker T) + \dim(\text{Im } T) = \dim(V)$.

Correct options: **A, B, D.**

Q34. **Answer: (AC)**

Solution

- **(A) True:** The **fundamental theorem of finite abelian groups**: every finite abelian group is isomorphic to a direct product of cyclic groups of prime-power order.
- **(B) False:** E.g., $\mathbb{Z}_4$ has order 4 (not prime).
- **(C) True:** In any abelian group, *every* subgroup is normal (since $gHg^{-1} = H$ trivially when $gh = hg$).
- **(D) False:** $\mathbb{Z}_2 \times \mathbb{Z}_2$ is abelian but not cyclic.

Correct options: **A, C.**

Q35. **Answer: (BD)**



Solution

- **(A) False:** The left derivative of $|x|$ at 0 is -1 and right derivative is $+1$; they differ, so f is NOT differentiable at 0.
- **(B) True:** $\lim_{x \rightarrow 0} |x| = 0 = f(0)$, so f is continuous at 0.
- **(C) False:** $f'(0)$ does not exist (see (A)).
- **(D) True:** $f = |x|$ is bounded and monotone on each half of $[-1, 1]$, hence Riemann integrable on $[-1, 1]$.

Correct options: **B, D.**

Q36. **Answer: (ABC)**

Solution

The general solution of $y'' + 4y = 0$ is $y = C_1 \cos 2x + C_2 \sin 2x$.

- **(A) True:** $y = \cos 2x$ (set $C_1 = 1, C_2 = 0$). Check: $y'' = -4 \cos 2x$; $y'' + 4y = 0$. ✓
- **(B) True:** $y = \sin 2x$ (set $C_1 = 0, C_2 = 1$). Check: $y'' = -4 \sin 2x$; $y'' + 4y = 0$. ✓
- **(C) True:** $y = 3 \cos 2x - \sin 2x$ (linear combination). ✓
- **(D) False:** $y = e^{2x}$; $y'' = 4e^{2x}$; $y'' + 4y = 8e^{2x} \neq 0$. ✗

Correct options: **A, B, C.**

Q37. **Answer: (ACD)**

Solution

Green's theorem requires:

- **(A) True:** P and Q must have continuous first partial derivatives on the closed region $\overline{D}$.
- **(B) False:** C cannot be any curve; it must be a simple closed curve (not self-intersecting).
- **(C) True:** C must be piecewise smooth and positively (counter-clockwise) oriented.



- (D) **True:** D is specifically the region *enclosed* by C .

Correct options: **A, C, D.**

Q38. **Answer: (BD)**

Solution

The characteristic equation of $y'' - y = 0$ is $r^2 - 1 = 0$, giving $r = \pm 1$ (real roots).

- (A) **False:** $\cos x, \sin x$ come from $r = \pm i$, not $r = \pm 1$.
- (B) **True:** General solution is $y = C_1 e^x + C_2 e^{-x}$.
- (C) **False:** Characteristic roots are ± 1 , not $\pm i$.
- (D) **True:** $\cosh x = \frac{e^x + e^{-x}}{2}$ is a particular solution (with $C_1 = C_2 = 1/2$).
Check: $(\cosh x)'' - \cosh x = \cosh x - \cosh x = 0$. ✓

Correct options: **B, D.**

Q39. **Answer: (ABD)**

Solution

Let $f(x, y) = xy(x - y) = x^2y - xy^2$.

- (A) **True:** $f_x = 2xy - y^2 = y(2x - y)$, $f_y = x^2 - 2xy = x(x - 2y)$. At $(0, 0)$: $f_x = f_y = 0$, so $(0, 0)$ is a critical point.
- (B) **True:** $f(0, 0) = 0 \cdot 0 \cdot (0 - 0) = 0$.
- (C) **False:** Along $y = x > 0$: $f = x \cdot x \cdot 0 = 0$. Along $y = 0$: $f = 0$. Along $y = 2x$: $f = x(2x)(x - 2x) = -2x^3 < 0$ for $x > 0$ and > 0 for $x < 0$. The function changes sign, so $(0, 0)$ is NOT a local minimum.
- (D) **True:** Since f takes both positive and negative values in every neighbourhood of $(0, 0)$, it is a **saddle point**.

Correct options: **A, B, D.**

Q40. **Answer: (BC)**



Solution

Let $\mathbf{F} = (2x, 2y)$.

- **(A) False:** In 2D, $\nabla \times \mathbf{F} = \frac{\partial(2y)}{\partial x} - \frac{\partial(2x)}{\partial y} = 0 - 0 = 0$. So $\nabla \times \mathbf{F} = \mathbf{0}$, not $\neq \mathbf{0}$.
- **(B) True:** Since $\nabla \times \mathbf{F} = \mathbf{0}$ (on a simply connected domain), $\mathbf{F}$ is conservative.
- **(C) True:** $\nabla(x^2 + y^2) = (2x, 2y) = \mathbf{F}$. ✓
- **(D) False:** $\nabla \cdot \mathbf{F} = \frac{\partial(2x)}{\partial x} + \frac{\partial(2y)}{\partial y} = 2 + 2 = 4 \neq 0$.

Correct options: **B, C**.

SECTION C — Solutions (NAT)

Q41. **Answer: (1)**

Solution

Row reduce $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$:

$$R_2 \leftarrow R_2 - 4R_1, R_3 \leftarrow R_3 - 7R_1 \Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{pmatrix}.$$

$R_3 \leftarrow R_3 - 2R_2$:

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & 0 & 0 \end{pmatrix}.$$

$\text{rank}(A) = 2$. By rank-nullity: $\text{nullity} = 3 - 2 = \boxed{1}$.

Q42. **Answer: (3)**

Solution

In $(\mathbb{Z}_{12}, +)$, the order of element a is $12/\gcd(a, 12)$:

$$\text{ord}(4) = \frac{12}{\gcd(4, 12)} = \frac{12}{4} = \boxed{3}.$$

Verify: $4 + 4 + 4 = 12 \equiv 0 \pmod{12}$. ✓



Q43. Answer: (0)

Solution

Use the double-angle identity: $\sin x \cos x = \frac{1}{2} \sin 2x$.

$$\int_{-\pi}^{\pi} \sin x \cos x \, dx = \frac{1}{2} \int_{-\pi}^{\pi} \sin 2x \, dx.$$

Since $\sin 2x$ is an **odd function** and $[-\pi, \pi]$ is symmetric:

$$\int_{-\pi}^{\pi} \sin 2x \, dx = 0 \implies \text{answer} = \boxed{0}.$$

Q44. Answer: (3)

Solution

$|S_3| = 6 = 2 \cdot 3$. By Sylow's theorem, the number n_2 of Sylow 2-subgroups satisfies $n_2 \mid 3$ and $n_2 \equiv 1 \pmod{2}$, so $n_2 \in \{1, 3\}$. Since S_3 is non-abelian, $n_2 = 3$. Explicitly, the three Sylow 2-subgroups are $\langle(12)\rangle$, $\langle(13)\rangle$, $\langle(23)\rangle$. Answer: $\boxed{3}$.

Q45. Answer: (4)

Solution

$$\frac{d}{dx} e^{2x} = 2e^{2x}; \quad \frac{d^2}{dx^2} e^{2x} = 4e^{2x}.$$

$$\text{At } x = 0: 4e^0 = \boxed{4}.$$

Q46. Answer: (3)

Solution

$$x^3 - 3x = 0 \implies x(x^2 - 3) = 0 \implies x = 0, x = \sqrt{3}, x = -\sqrt{3}.$$

All three roots are real and distinct. Answer: $\boxed{3}$.

Q47. Answer: (2)



Solution

Using the standard limit $\lim_{x \rightarrow 0} \frac{e^{ax} - 1}{x} = a$ (or L'Hôpital):

$$\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{x} = \lim_{x \rightarrow 0} \frac{2e^{2x}}{1} = 2e^0 = \boxed{2}.$$

Q48. **Answer: (0)**

Solution

$f(x) = x^5$ is an **odd function**: $f(-x) = (-x)^5 = -x^5 = -f(x)$.

On the symmetric interval $[-1, 1]$:

$$\int_{-1}^1 x^5 dx = 0.$$

Q49. **Answer: (-1)**

Solution

The determinant of a matrix equals the **product of its eigenvalues** (with multiplicity):

$$\det(A) = \lambda_1 \cdot \lambda_2 = 1 \times (-1) = \boxed{-1}.$$

Q50. **Answer: (120)**

Solution

The symmetric group S_n consists of all permutations of n elements:

$$|S_5| = 5! = 5 \times 4 \times 3 \times 2 \times 1 = \boxed{120}.$$

Q51. **Answer: (3)**

Solution

By the rank-nullity theorem: for an $m \times n$ matrix, $\text{rank} + \text{nullity} = n$ (number of columns).

Here $n = 5$, $\text{rank} = 2$:

$$\text{nullity} = 5 - 2 = \boxed{3}.$$

Q52. **Answer: (2)**



Solution

In $\mathbb{Z}_9$, the order of element a is $9/\gcd(a, 9)$. Elements of order 3:

$$\frac{9}{\gcd(a, 9)} = 3 \iff \gcd(a, 9) = 3 \iff a \in \{3, 6\}.$$

So there are exactly $\boxed{2}$ elements of order 3 in $\mathbb{Z}_9$.

Q53. **Answer: (2)**

Solution

Try a constant particular solution $y_p = k$. Then $y'_p = y''_p = 0$:

$$0 + 4k = 8 \implies k = 2.$$

So $y_p = \boxed{2}$.

Q54. **Answer: (4)**

Solution

$$\begin{aligned} \int_0^2 \int_0^x (x+y) dy dx &= \int_0^2 \left[xy + \frac{y^2}{2} \right]_0^x dx = \int_0^2 \left(x^2 + \frac{x^2}{2} \right) dx = \int_0^2 \frac{3x^2}{2} dx \\ &= \frac{3}{2} \cdot \frac{x^3}{3} \Big|_0^2 = \frac{1}{2} \cdot 8 = \boxed{4}. \end{aligned}$$

Q55. **Answer: (0.17)**

Solution

By Green's theorem with $P = x^2$, $Q = xy$ over triangle D with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$:

$$Q_x - P_y = y - 0 = y.$$

The region D (for fixed $x \in [0, 1]$, y from 0 to x):

$$\iint_D y dA = \int_0^1 \int_0^x y dy dx = \int_0^1 \frac{x^2}{2} dx = \frac{1}{2} \cdot \frac{x^3}{3} \Big|_0^1 = \frac{1}{6} \approx \boxed{0.17}.$$

Q56. **Answer: (2.718)**



Solution

The Taylor series of e^x about $x = 0$ is:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

At $x = 1$:

$$\sum_{n=0}^{\infty} \frac{1}{n!} = e^1 = e \approx \boxed{2.718}.$$

Q57. **Answer: (4)**

Solution

The trace of a matrix equals the **sum of its eigenvalues** (with multiplicity):

$$\text{tr}(A) = 1 + 1 + 2 = \boxed{4}.$$

Q58. **Answer: (2)**

Solution

The IVP $y' = y$, $y(0) = 1$ has unique solution $y(x) = e^x$.

At $x = \ln 2$: $y(\ln 2) = e^{\ln 2} = \boxed{2}$.

Q59. **Answer: (1.333)**

Solution

The tetrahedron with one vertex at the origin and three adjacent vertices at $(2, 0, 0)$, $(0, 2, 0)$, $(0, 0, 2)$:

$$V = \frac{1}{6} |\det[\mathbf{a}, \mathbf{b}, \mathbf{c}]|, \quad \text{where } \mathbf{a} = (2, 0, 0), \mathbf{b} = (0, 2, 0), \mathbf{c} = (0, 0, 2).$$

$$\det \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} = 8, \quad V = \frac{8}{6} = \frac{4}{3} \approx \boxed{1.333}.$$

Q60. **Answer: (6)**



Solution

The direct product $\mathbb{Z}_2 \times \mathbb{Z}_3$ has order $|\mathbb{Z}_2| \times |\mathbb{Z}_3| = 2 \times 3 = \boxed{6}$.

By the Chinese Remainder Theorem (since $\gcd(2, 3) = 1$), $\mathbb{Z}_2 \times \mathbb{Z}_3 \cong \mathbb{Z}_6$.

Answer Key — IIT JAM Mathematics Sample Paper 2

Section A (MCQ):

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	D	3	C	4	B	5	A
6	D	7	B	8	C	9	A	10	D
11	A	12	D	13	C	14	B	15	D
16	A	17	C	18	B	19	A	20	D
21	C	22	B	23	D	24	A	25	C
26	B	27	A	28	D	29	C	30	B
31	ACD	32	BC	33	ABD	34	AC	35	BD
36	ABC	37	ACD	38	BD	39	ABD	40	BC
41	1	42	3	43	0	44	3	45	4
46	3	47	2	48	0	49	-1	50	120
51	3	52	2	53	2	54	4	55	0.17
56	2.718	57	4	58	2	59	1.333	60	6

Section B (MSQ):

Q31	Q32	Q33	Q34	Q35	Q36	Q37	Q38	Q39	Q40
ACD	BC	ABD	AC	BD	ABC	ACD	BD	ABD	BC

Section C (NAT):

Q41	Q42	Q43	Q44	Q45	Q46	Q47	Q48	Q49	Q50
1	3	0	3	4	3	2	0	-1	120
Q51	Q52	Q53	Q54	Q55	Q56	Q57	Q58	Q59	Q60
3	2	2	4	0.17	2.718	4	2	1.333	6



Prepared by Collegedunia Editorial Team ·

<https://collegedunia.com/exams/iit-jam/sample-paper>

