# IIT JAM Mathematics

## Sample Paper – 9

### Instructions

- This paper contains **60 questions** carrying a maximum of **100 marks**. Duration: **3 hours (180 minutes)**.
- **Section A** (**Q1–Q30**, MCQ): Q1–Q10 carry **+1 mark** each; wrong response: $-\frac{1}{3}$ mark. Q11–Q30 carry **+2 marks** each; wrong response: $-\frac{2}{3}$ mark. Choose the **single best** option.
- **Section B** (**Q31–Q40**, MSQ): Each carries **+2 marks**. **No negative marking. One or more** options may be correct; full marks only if *all* correct options are selected.
- **Section C** (**Q41–Q60**, NAT): Q41–Q50 carry **+1 mark**; Q51–Q60 carry **+2 marks**. **No negative marking.** Enter the numerical value using the virtual keypad.
- **Calculator policy:** Personal calculators and electronic devices are **strictly prohibited**. A virtual scientific calculator is provided in the CBT interface.
- Sections must be attempted in order; you cannot return to a previous section.

## Section A — Multiple Choice Questions (MCQ)

***Q1–Q10***: *1 mark each ($-\frac{1}{3}$ for wrong). Choose the* ***single*** *correct option.*

**Q1.** Newton's method applied to $f(x) = x^2 - 2$ with $x_0 = 1$ gives $x_1 = \frac{3}{2}$. What is the **second** iterate $x_2$?

(A) $2$

(B) $1$

(C) $\dfrac{3}{2}$

(D) $\dfrac{17}{12}$

**Q2.** The secant method for solving $f(x) = 0$ uses two previous iterates and has an order of convergence approximately equal to:

(A) $1$ (linear)

(B) $2$ (quadratic, same as Newton's)

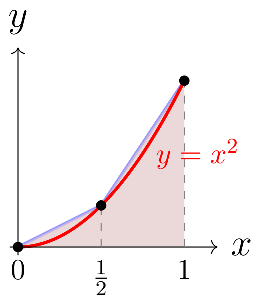
(C) $\approx 1.618$ (golden ratio)

(D) $3$ (cubic)

**Q3.** The composite trapezoidal rule with $n$ equal subintervals (mesh $h = (b - a)/n$) over $[a, b]$ has a global error of order $O(h^p)$ where $p =$

(A) $1$
(B) $2$
(C) $3$
(D) $4$

**Q4.** Approximate $\int_0^1 x^2\,dx$ using the composite trapezoidal rule with $n = 2$ subintervals ($h = \frac{1}{2}$, nodes at $x = 0, \frac{1}{2}, 1$).




(A) $\dfrac{3}{8}$
(B) $\dfrac{1}{3}$
(C) $\dfrac{5}{12}$
(D) $\dfrac{1}{2}$

**Q5.** Using Simpson's $\frac{1}{3}$ rule with $n = 2$ subintervals ($h = \frac{1}{2}$, nodes at $x = 0, \frac{1}{2}, 1$) to approximate $\int_0^1 x^2\,dx$, the result is:

(A) $\dfrac{3}{8}$
(B) $\dfrac{1}{4}$
(C) $\dfrac{1}{2}$
(D) $\dfrac{1}{3}$

**Q6.** For a simple root of a sufficiently smooth function, Newton's method exhibits:

(A) Linear convergence
(B) Superlinear but not quadratic convergence

(C) Quadratic convergence
(D) Cubic convergence

**Q7.** The Gauss-Seidel iterative method for $Ax = b$ is best described as:

(A) A direct method equivalent to Gaussian elimination
(B) An iterative method that uses the most recently updated component values immediately within each sweep
(C) A method requiring $A$ to be symmetric positive definite to be defined
(D) Always faster than the Jacobi method regardless of the matrix $A$

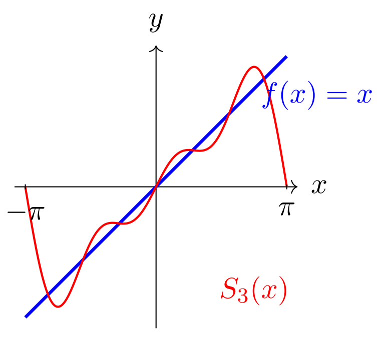
**Q8.** The condition number $\kappa(A)$ of a nonsingular matrix $A$ primarily measures:

(A) How sensitive the solution of $Ax = b$ is to perturbations in $b$ (or $A$)
(B) Whether $A$ is invertible
(C) The largest eigenvalue of $A$
(D) The rank of $A$

**Q9.** For $f(x) = x$ on $(-\pi, \pi)$, extended periodically with period $2\pi$, the Fourier coefficient $a_0 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} f(x)\,dx$ equals:

(A) $\pi$
(B) $1$
(C) $2\pi$
(D) $0$

**Q10.** For $f(x) = x$ on $(-\pi, \pi)$, the Fourier sine coefficients $b_n = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} x\sin(nx)\,dx$ are:




(A) $\dfrac{2(-1)^n}{n}$

(B) $\dfrac{(-1)^n}{n}$
(C) $\dfrac{2(-1)^{n+1}}{n}$
(D) $0$

***Q11–Q30**: 2 marks each ($-\frac{2}{3}$ for wrong). Choose the **single** correct option.*

**Q11.** For $f(x) = x$ on $(-\pi, \pi)$ with Fourier series $f(x) \sim \sum_{n=1}^{\infty} b_n \sin(nx)$, Parseval's theorem states $\frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx = \sum_{n=1}^{\infty} b_n^2$. The value of $\sum_{n=1}^{\infty} b_n^2$ is:

(A) $\dfrac{\pi^2}{3}$
(B) $\dfrac{2\pi^2}{3}$
(C) $\pi^2$
(D) $\dfrac{\pi^2}{6}$

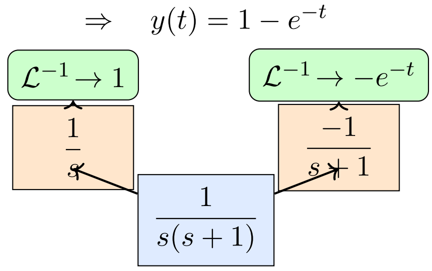
**Q12.** The Fourier transform $\hat{f}(\omega) = \int_{-\infty}^{\infty} e^{-|t|}e^{-i\omega t}\,dt$ equals:

(A) $\dfrac{2}{1+\omega^2}$
(B) $\dfrac{1}{1+\omega^2}$
(C) $e^{-|\omega|}$
(D) $2\pi\,\delta(\omega)$

**Q13.** The Laplace transform $\mathcal{L}\{\sin(at)\}(s)$ for $s > 0$ equals:

(A) $\dfrac{s}{s^2+a^2}$
(B) $\dfrac{a^2}{s^2+a^2}$
(C) $\dfrac{1}{s-a}$
(D) $\dfrac{a}{s^2+a^2}$

**Q14.** The inverse Laplace transform $\mathcal{L}^{-1}\left\{\dfrac{1}{s(s+1)}\right\}(t)$ is:



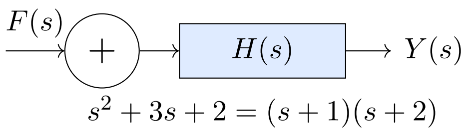

(A) $e^{-t}$

(B) $1 - e^{t}$

(C) $1 - e^{-t}$

(D) $t\,e^{-t}$

**Q15.** For the ODE $y'' + 3y' + 2y = f(t)$ with zero initial conditions, the transfer function $H(s) = Y(s)/F(s)$ is:




(A) $s^2 + 3s + 2$

(B) $\dfrac{1}{s^2 + 3s + 2}$

(C) $s + 3$

(D) $(s+1)(s+2)$

**Q16.** $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+4}\right\}(t) =$

(A) $\cos(2t)$

(B) $\dfrac{\sin(2t)}{2}$

(C) $e^{2t}$

(D) $t\cos(2t)$

**Q17.** Using the convolution theorem, $\mathcal{L}^{-1}\left\{\dfrac{1}{(s^2+1)^2}\right\}(t) =$

(A) $\sin(t)\cos(t)$

(B) $\cos^2(t)$

(C) $\dfrac{t\cos(t)}{2}$

(D) $\dfrac{\sin t - t\cos t}{2}$

**Q18.** Solve $y'' + y = t$, $y(0) = 0$, $y'(0) = 0$ using Laplace transforms. The solution is:

(A) $t + \sin t$

(B) $t \cos t$

(C) $t - \sin t$

(D) $1 - \cos t$

**Q19.** For the IVP $y' = f(x, y)$, the **forward (explicit) Euler** method with step $h$ is:

(A) $y_{n+1} = y_n + h\, f(x_{n+1}, y_{n+1})$

(B) $y_{n+1} = y_n + h\, f(x_n, y_n)$

(C) $y_{n+1} = y_n + \frac{h}{2}\big[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\big]$

(D) $y_{n+1} = y_n - h\, f(x_n, y_n)$

**Q20.** The **local** truncation error of the forward Euler method is of order:

(A) $O(h^2)$

(B) $O(h)$

(C) $O(h^3)$

(D) $O(h^4)$

**Q21.** The classical 4th-order Runge-Kutta method computes four slope estimates $k_1, k_2, k_3, k_4$ and updates via $y_{n+1} = y_n + h \cdot \phi$, where $\phi =$

Butcher Tableau for RK4

| | |
|---|---|
| $0$ | — |
| $\frac{1}{2}$ | $\frac{1}{2}$ |
| $\frac{1}{2}$ | $0, \frac{1}{2}$ |
| $1$ | $0, 0, 1$ |
| $b$: | $\frac{1}{6}, \frac{1}{3}, \frac{1}{3}, \frac{1}{6}$ |

(A) $\dfrac{k_1 + k_2 + k_3 + k_4}{4}$

(B) $\dfrac{k_1 + k_2}{2}$

(C) $\dfrac{k_1 + 4k_2 + k_3}{6}$

(D) $\dfrac{k_1 + 2k_2 + 2k_3 + k_4}{6}$

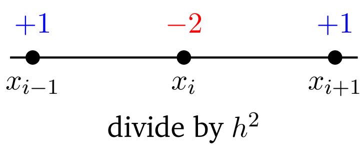
**Q22.** For the test equation $y' = \lambda y$ with $\lambda < 0$ (real), the forward Euler method is **stable** when:

(A) $h\lambda < 0$ (any negative $h\lambda$)
(B) $|h\lambda| < 2$ always implies stability regardless of sign
(C) $h\lambda \in (-2, 0)$, i.e. $|1 + h\lambda| < 1$
(D) $h < 1/|\lambda|$ with no further condition

**Q23.** The **shooting method** for the BVP $y'' = f(x, y, y')$, $y(a) = \alpha$, $y(b) = \beta$ transforms the problem into:

(A) A partial differential equation
(B) An initial value problem depending on a free parameter $s = y'(a)$, with $s$ chosen so that $y(b; s) = \beta$
(C) A linear system of algebraic equations
(D) A problem with no free parameters

**Q24.** The standard second-order central-difference approximation for $y''(x_i)$ using nodes $x_{i-1}, x_i, x_{i+1}$ with spacing $h$ is:



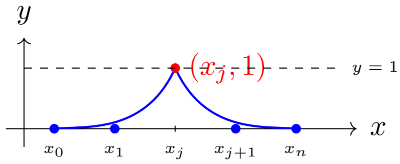

(A) $\dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2}$
(B) $\dfrac{y_{i+1} - y_{i-1}}{2h}$
(C) $\dfrac{y_{i+1} - y_i}{h}$
(D) $\dfrac{y_{i+1} + 2y_i + y_{i-1}}{h^2}$

**Q25.** Given $n + 1$ distinct data points $(x_0, y_0), \ldots, (x_n, y_n)$, the unique interpolating polynomial has degree **at most**:

(A) $n + 1$
(B) $n^2$

(C) $2n$
(D) $n$

**Q26.** The Lagrange basis polynomial $L_j(x)$ associated with node $x_j$ (among nodes $x_0, \ldots, x_n$) satisfies:




(A) $L_j(x_j) = 0$ and $L_j(x_k) = 1$ for $k \neq j$
(B) $L_j(x) = x^j$
(C) $L_j(x_j) = 1$ and $L_j(x_k) = 0$ for all $k \neq j$
(D) $L_j(x) = \dfrac{1}{x - x_j}$

**Q27.** For $f(x) = x^2$ with step $h$, the forward difference $\Delta f(x) = f(x+h) - f(x)$ equals:

(A) $2x$
(B) $2xh + h^2$
(C) $h^2$
(D) $2x + h^2$

**Q28.** A **natural** cubic spline $S(x)$ satisfying interpolation conditions at nodes $a = x_0 < x_1 < \cdots < x_n = b$ is characterised by the additional endpoint conditions:

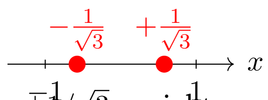
(A) $S''(a) = S''(b) = 0$
(B) $S'(a) = S'(b) = 0$
(C) $S'''(x)$ is continuous everywhere (no extra condition needed)
(D) $S(x)$ is a single cubic polynomial on all of $[a, b]$

**Q29.** The technique of combining two numerical approximations with different step sizes $h$ and $h/2$ to eliminate the leading-order error term and achieve higher accuracy is called:

(A) Simpson's rule

(B) Euler's method
(C) Gaussian quadrature
(D) Richardson extrapolation

**Q30.** The $n$-point Gaussian quadrature rule $\int_{-1}^{1} f(x)\,dx \approx \sum_{i=1}^{n} w_i f(x_i)$ is exact for all polynomials of degree at most:




$n = 2$: nodes $\pm 1/\sqrt{3}$, weights $w_1 = w_2 = 1$
exact for deg $\leq 3 = 2n - 1$

(A) $n$
(B) $n - 1$
(C) $2n - 1$
(D) $2n$

## Section B — Multiple Select Questions (MSQ)

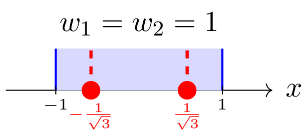
***Q31–Q40**: 2 marks each. **No negative marking. One or more** options may be correct; full marks only if ALL correct options selected.*

**Q31.** The Fourier series of $f(x) = |x|$ on $(-\pi, \pi)$ is $|x| = \frac{\pi}{2} - \frac{4}{\pi} \sum_{k=0}^{\infty} \frac{\cos((2k+1)x)}{(2k+1)^2}$.

Which of the following are **TRUE**?

(A) The Fourier series converges to $|x|$ for every $x \in [-\pi, \pi]$
(B) The Fourier coefficients $b_n$ are nonzero for odd $n$
(C) All Fourier coefficients $b_n = 0$ (because $f$ is even)
(D) The Fourier series converges to $0$ at $x = \pi$

**Q32.** Consider the 2-point Gaussian quadrature rule on $[-1, 1]$ with nodes $x = \pm 1/\sqrt{3}$ and weights $w_1 = w_2 = 1$:




Which of the following are **TRUE**?

(A) The rule is exact for all polynomials of degree $\leq 3$

(B) The sum of the weights equals 2 (i.e. $w_1 + w_2 = 2 = \int_{-1}^{1} 1\, dx$)

(C) The rule gives $\displaystyle\int_{-1}^{1} x^5\, dx \approx 0.4$ (incorrect: the exact value and this rule both give 0, so the rule gives the exact value)

(D) $\displaystyle\int_{-1}^{1} x^3\, dx$ is computed exactly as 0 by this rule

**Q33.** For Laplace-transformable functions $f$ and $g$ with $F(s) = \mathcal{L}\{f\}$ and $G(s) = \mathcal{L}\{g\}$, which of the following are **TRUE**?

(A) $\mathcal{L}\{f \cdot g\}(s) = F(s) \cdot G(s)$

(B) $\mathcal{L}\{(f * g)(t)\}(s) = F(s) \cdot G(s)$, where $(f * g)(t) = \displaystyle\int_0^t f(\tau)g(t-\tau)\, d\tau$

(C) $\mathcal{L}\{f'\}(s) = sF(s) - f(0)$

(D) $\mathcal{L}\{af + bg\} = a\mathcal{L}\{f\} + b\mathcal{L}\{g\}$ holds only for $a = b = 1$

**Q34.** Newton's method is applied to $f(x) = x^3 - x - 2$ starting at $x_0 = 2$. Note that $f(2) = 4$ and $f'(2) = 11$. Which of the following are **TRUE**?

(A) The first iterate is $x_1 = 2 - \dfrac{f(2)}{f'(2)} = 2 - \dfrac{4}{11} = \dfrac{18}{11}$

(B) The method diverges from $x_0 = 2$

(C) $f(x)$ has no real roots

(D) $f(1) = -2 < 0$ and $f(2) = 4 > 0$, so by the Intermediate Value Theorem there is a root in $(1, 2)$

**Q35.** For the **backward (implicit) Euler** method $y_{n+1} = y_n + h\, f(x_{n+1}, y_{n+1})$, which of the following are **TRUE**?

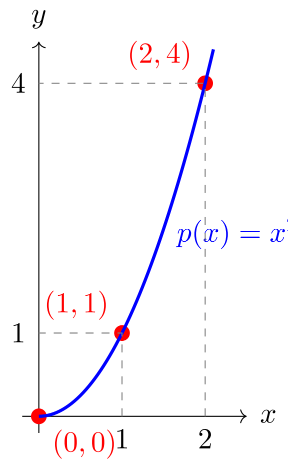
(A) It is always more accurate than forward Euler for any step size $h$

(B) It is A-stable: for $y' = \lambda y$ with $\mathrm{Re}(\lambda) < 0$, the method is stable for *all* $h > 0$

(C) Its local truncation error is $O(h)$, lower than for RK4

(D) It requires solving (possibly nonlinear) equation $y_{n+1} - h\, f(x_{n+1}, y_{n+1}) = y_n$ at each step

**Q36.** Consider the Lagrange interpolating polynomial through the data points $(0, 0), (1, 1), (2, 4)$:




Which of the following are **TRUE**?

(A) The interpolating polynomial has degree $\leq 2$
(B) The interpolating polynomial is $x^3$ (incorrect degree)
(C) The interpolating polynomial is $p(x) = x^2$
(D) The interpolating polynomial is $2x - 1$

**Q37.** For the composite trapezoidal rule applied to $\int_0^1 f(x)\,dx$ with $n$ equal subintervals ($h = 1/n$), which of the following are **TRUE**?

(A) The global error is $O(h^2)$
(B) For $f(x) = x$ (linear), the rule gives the *exact* value $\frac{1}{2}$
(C) For $f(x) = x^2$ (quadratic), the rule is exact (zero error)
(D) The composite rule uses function values at $n+1$ equally spaced points $x_0, x_1, \ldots, x_n$

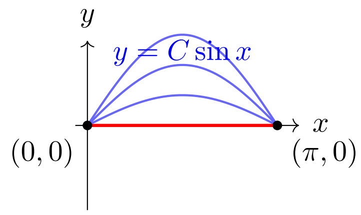
**Q38.** For the Discrete Fourier Transform (DFT) of a signal $x[n]$ of length $N$, which of the following are **TRUE**?

(A) The FFT (Fast Fourier Transform) algorithm computes the DFT in $O(N^2)$ operations
(B) The FFT reduces the DFT complexity from $O(N^2)$ to $O(N \log N)$
(C) The DFT is invertible (there exists an inverse DFT)
(D) The DFT output $X[k]$ has $N + 1$ components

**Q39.** For a system $Ax = b$ where $A$ is **strictly diagonally dominant**, which of the following iterative methods are guaranteed to converge?

(A) Jacobi iteration
(B) Gradient descent (without step-size control) for general non-symmetric $A$
(C) Gauss-Seidel iteration
(D) Both Jacobi and Gauss-Seidel simultaneously

**Q40.** Consider the BVP $y'' = -y$, $y(0) = 0$, $y(\pi) = 0$, solved via the shooting method:




Which of the following are **TRUE**?

(A) There is a unique nontrivial solution
(B) There are infinitely many solutions $y = C\sin(x)$ for any $C \in \mathbb{R}$
(C) The shooting method always converges to the unique solution for this BVP
(D) The zero function $y \equiv 0$ is a solution

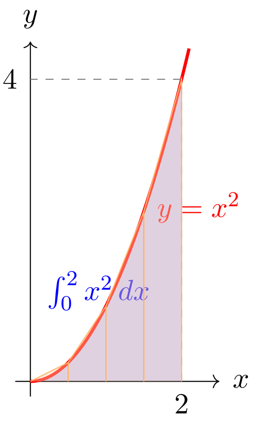
## Section C — Numerical Answer Type (NAT)

***Q41–Q50**: 1 mark each. No negative marking. Enter the exact numerical value.*

**Q41.** Apply Newton's method to $f(x) = x^2 - 5$ starting at $x_0 = 2$. The **first** iterate is $x_1 = \frac{9}{4}$. Find the **second** iterate $x_2$ as a fraction.

**Q42.** Approximate $\int_0^2 x^2\,dx$ using the composite trapezoidal rule with $n = 4$ equal subintervals ($h = 0.5$). Give the numerical value.

**Q43.** Evaluate the exact definite integral $\int_0^2 x^2\,dx$.



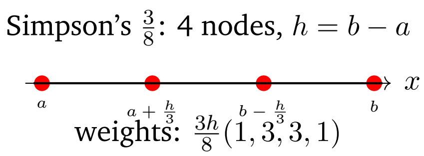

**Q44.** Evaluate $\mathcal{L}\{t^2e^{3t}\}(s)$ at $s = 5$. [Use the formula $\mathcal{L}\{t^ne^{at}\}(s) = \dfrac{n!}{(s-a)^{n+1}}$, $n = 2$, $a = 3$.]

**Q45.** Solve the IVP $y' - 2y = 4$, $y(0) = 1$ using Laplace transforms and find $y(1)$.

**Q46.** Evaluate $\displaystyle\int_0^{2\pi} |\cos x|\,dx$.

**Q47.** Compute the Fourier coefficient $b_1 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} x^2\sin(x)\,dx$ for $f(x) = x^2$.

**Q48.** Simpson's $\frac{3}{8}$ rule integrates $\displaystyle\int_a^b f(x)\,dx$ using how many function evaluations (points) on a **single** interval $[a, b]$?




**Q49.** Using Parseval's theorem applied to the Fourier series of $f(x) = x$ on $(-\pi, \pi)$ (which gives $\displaystyle\sum_{n=1}^{\infty} \frac{4}{n^2} = \frac{2\pi^2}{3}$), determine $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$.

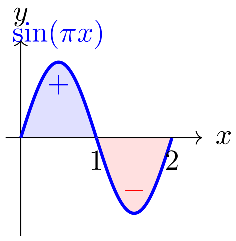
**Q50.** Find $\displaystyle\lim_{n\to\infty}\left(1 - \frac{(-1)^n}{n}\right)$.

***Q51–Q60**: 2 marks each. No negative marking. Enter the exact numerical value.*

**Q51.** Evaluate $\mathcal{L}\{\sin(t)\cos(t)\}(s)$ at $s = 2$. [Hint: $\sin(t)\cos(t) = \frac{1}{2}\sin(2t)$, so $\mathcal{L}\{\sin(t)\cos(t)\}(s) = \dfrac{1}{s^2+4}$. Evaluate at $s = 2$.]

**Q52.** How many non-isomorphic groups of order $4$ exist?

**Q53.** Evaluate $\displaystyle\int_0^2 \sin(\pi x)\,dx$.

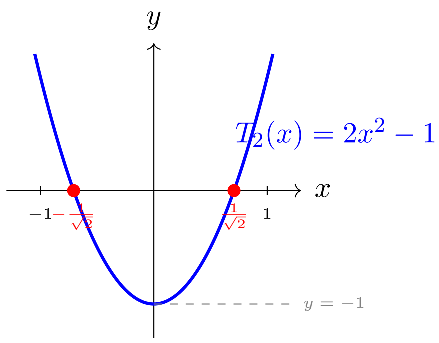



**Q54.** Apply Newton's method to $f(x) = x^3 - 3x + 1$ starting at $x_0 = 2$. Find the first iterate $x_1$ as a fraction. [Note: $f(2) = 3$, $f'(2) = 9$.]

**Q55.** Approximate $\displaystyle\int_0^1 e^x\,dx$ using the trapezoidal rule with a single interval $(n = 1)$. Express the answer in terms of $e$.

**Q56.** The exact value of $\displaystyle\int_0^1 e^x\,dx$ is $e - 1$. The trapezoidal approximation with $n = 1$ is $\dfrac{1+e}{2}$. What is the **magnitude of the error** $\left|\dfrac{1+e}{2} - (e-1)\right|$? Express in terms of $e$.

**Q57.** The Chebyshev polynomial $T_2(x) = 2x^2 - 1$ has two real roots. Find the **product** of its roots.




**Q58.** The Fourier series of $f(x) = x$ on $(-\pi, \pi)$ is $f(x) = \displaystyle\sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n}\sin(nx)$. By Dirichlet's theorem, what does the Fourier series converge to at $x = \pi/2$?

**Q59.** Apply the classical RK4 method to $y' = y$, $y(0) = 1$ with step $h = 1$. Compute $y_1$ (the approximation at $x = 1$). Express as a fraction.

**Q60.** Evaluate the Legendre polynomial $P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$ at $x = 1$.

# Solutions

## IIT JAM Mathematics – Sample Paper 9

## Section A Solutions — Q1–Q10 (1 mark each)

**Q1.**

**Solution**

**Concept:** Newton's iteration for $f(x) = x^2 - 2$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)} = x_n - \dfrac{x_n^2 - 2}{2x_n}$.

**Step 1:** First iterate: $x_1 = 1 - \dfrac{1-2}{2} = 1 + \dfrac{1}{2} = \dfrac{3}{2}$.

**Step 2:** Second iterate:

$$x_2 = \frac{3}{2} - \frac{(3/2)^2 - 2}{2 \cdot (3/2)} = \frac{3}{2} - \frac{9/4 - 2}{3} = \frac{3}{2} - \frac{1/4}{3} = \frac{3}{2} - \frac{1}{12} = \frac{18}{12} - \frac{1}{12} = \frac{17}{12}.$$

**Why (A) wrong:** $x = 2$ is not an iterate of Newton's method here.
**Why (B) wrong:** $x = 1$ is $x_0$, the starting point.
**Why (C) wrong:** $3/2$ is $x_1$, the first iterate, not $x_2$.

**Answer: (D)** ← *Go back to Q1*

**Q2.**

**Solution**

**Concept:** The order of convergence $p$ satisfies $\lim \dfrac{|e_{n+1}|}{|e_n|^p} = C$ (finite, nonzero).

**Step 1:** For the secant method, the order of convergence satisfies $p(p-1) = 1$, giving $p = \dfrac{1+\sqrt{5}}{2} \approx 1.618$ (the golden ratio).

**Step 2:** This is superlinear but less than Newton's quadratic order $p = 2$.

**Why (A) wrong:** Linear ($p = 1$) is Picard/fixed-point iteration; secant is faster.
**Why (B) wrong:** Newton's method (not secant) achieves $p = 2$.
**Why (D) wrong:** Cubic convergence requires special methods (Halley's method gives $p = 3$).

**Answer: (C)** ← *Go back to Q2*

**Q3.**

**Solution**

**Concept:** The global error of the composite trapezoidal rule is proportional to $h^2$, where $h = (b-a)/n$.

**Step 1:** The error formula is $E = -\dfrac{(b-a)h^2}{12}f''(\xi)$ for some $\xi \in [a,b]$. Since $h^2$ appears explicitly, $p = 2$.

**Step 2:** Doubling $n$ (halving $h$) reduces the error by a factor of $4 = 2^2$, confirming second-order accuracy.

**Why (A) wrong:** $O(h)$ is the global order of the Euler method, not the trapezoidal rule.
**Why (C) wrong:** $O(h^3)$ corresponds to Simpson's rule local error per sub-interval.
**Why (D) wrong:** $O(h^4)$ is the global error of Simpson's rule (composite).

**Answer: (B)** ← *Go back to Q3*

**Q4.**

## Solution

**Concept:** Composite trapezoidal rule with $n = 2$, $h = 1/2$: $T = \dfrac{h}{2}\big[f(x_0) + 2f(x_1) + f(x_2)\big]$.

**Step 1:** Nodes: $x_0 = 0$, $x_1 = 1/2$, $x_2 = 1$. Function values: $f(0) = 0$, $f(1/2) = 1/4$, $f(1) = 1$.

**Step 2:**

$$T = \frac{1/2}{2}\big[0 + 2 \cdot \tfrac{1}{4} + 1\big] = \frac{1}{4}\big[0 + \tfrac{1}{2} + 1\big] = \frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}.$$

**Exact value:** $\int_0^1 x^2\,dx = 1/3$. Error $= 3/8 - 1/3 = 1/24 > 0$ (overestimate, as expected for convex $f$).

**Why (B) wrong:** $1/3$ is the exact value, not the trapezoidal approximation.
**Why (C) wrong:** $5/12$ would arise from an incorrect combination of nodes.
**Why (D) wrong:** $1/2$ overestimates even more than the trapezoidal rule does.

**Answer: (A)** ← *Go back to Q4*

**Q5.**

## Solution

**Concept:** Simpson's $\frac{1}{3}$ rule with $n = 2$ ($h = 1/2$): $S = \dfrac{h}{3}\big[f(x_0) + 4f(x_1) + f(x_2)\big]$.

**Step 1:** Nodes and values same as Q4: $f(0) = 0$, $f(1/2) = 1/4$, $f(1) = 1$.

**Step 2:**

$$S = \frac{1/2}{3}\big[0 + 4 \cdot \tfrac{1}{4} + 1\big] = \frac{1}{6}\big[0 + 1 + 1\big] = \frac{2}{6} = \frac{1}{3}.$$

**Note:** Simpson's rule is exact for polynomials of degree $\leq 3$; since $x^2$ has degree 2, the result $1/3$ is exact.

**Why (A) wrong:** $3/8$ is the trapezoidal approximation, not Simpson's.
**Why (B) wrong:** $1/4$ would result from using only two nodes incorrectly.
**Why (C) wrong:** $1/2$ overestimates, unlike the exact Simpson result.

**Answer: (D)** ← *Go back to Q5*

**Q6.**

## Solution

**Concept:** Newton's method error analysis near a simple root $x^*$: $e_{n+1} \approx \dfrac{f''(x^*)}{2f'(x^*)}e_n^2$.

**Step 1:** The error at step $n+1$ is proportional to the *square* of the error at step $n$, giving order $p = 2$ (quadratic convergence).

**Step 2:** In practice, the number of correct decimal digits roughly doubles at each iteration.

**Why (A) wrong:** Linear convergence means the error reduces by a constant factor per step; Newton's is faster.
**Why (B) wrong:** Superlinear but not quadratic is the secant method's order ($p \approx 1.618$).
**Why (D) wrong:** Cubic convergence arises in Halley's method, not standard Newton's method.

**Answer: (C)** ← *Go back to Q6*

**Q7.**

## Solution

**Concept:** Distinguish Jacobi (uses previous-iterate values only) from Gauss-Seidel (uses updated values immediately).

**Step 1:** In Gauss-Seidel, when solving for component $x_i^{(k+1)}$, all previously computed components $x_1^{(k+1)}, \ldots, x_{i-1}^{(k+1)}$ of the new iterate are used immediately.

**Step 2:** This "immediate update" property is the defining feature of Gauss-Seidel and often accelerates convergence compared to Jacobi.

**Why (A) wrong:** Gauss-Seidel is iterative, not a direct method.
**Why (C) wrong:** Gauss-Seidel converges for strictly diagonally dominant matrices regardless of symmetry.
**Why (D) wrong:** Gauss-Seidel is often (but not always) faster than Jacobi; exceptions exist.

**Answer: (B)** ← *Go back to Q7*

**Q8.**

**Solution**

**Concept:** The condition number $\kappa(A) = \|A\| \cdot \|A^{-1}\|$ quantifies sensitivity of solutions to perturbations.

**Step 1:** For $Ax = b$, the relative error bound is $\frac{\|\delta x\|}{\|x\|} \leq \kappa(A)\frac{\|\delta b\|}{\|b\|}$. Thus $\kappa(A)$ directly measures amplification of data errors.

**Step 2:** A large condition number (ill-conditioned) means even small perturbations in $b$ lead to large errors in $x$.

**Why (B) wrong:** Invertibility is determined by $\det(A) \neq 0$, not by $\kappa(A)$ directly.
**Why (C) wrong:** The largest eigenvalue is the spectral radius, related to but distinct from $\kappa$.
**Why (D) wrong:** Rank is a structural property unrelated to the condition number.

**Answer: (A)** ← *Go back to Q8*

**Q9.**

**Solution**

**Concept:** For an odd function $f(-x) = -f(x)$, all cosine Fourier coefficients (including $a_0$) vanish.

**Step 1:** $f(x) = x$ is an odd function on $(-\pi, \pi)$.

**Step 2:** $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} x\,dx = 0$, since the integrand is odd and the interval is symmetric.

**Why (A) wrong:** $a_0 = \pi$ would be the result of integrating $|\pi|$, not $x$.
**Why (B) wrong:** $a_0 = 1$ has no relation to integrating the odd function $x$.
**Why (C) wrong:** $2\pi$ would arise from a miscalculation without the $1/\pi$ normalisation.

**Answer: (D)** ← *Go back to Q9*

**Q10.**

**Solution**

**Concept:** Since $f(x) = x$ is odd, $b_n = \frac{2}{\pi}\int_0^{\pi} x\sin(nx)\,dx$.

**Step 1:** Integrate by parts with $u = x$, $dv = \sin(nx)\,dx$:

$$\int_0^{\pi} x\sin(nx)\,dx = \left[-\frac{x\cos(nx)}{n}\right]_0^{\pi} + \frac{1}{n}\int_0^{\pi}\cos(nx)\,dx = -\frac{\pi\cos(n\pi)}{n} + \frac{\sin(n\pi)}{n^2} = -\frac{\pi(-1)^n}{n}.$$

**Step 2:** $b_n = \frac{2}{\pi} \cdot \left(-\frac{\pi(-1)^n}{n}\right) = \frac{2(-1)^{n+1}}{n}$.

**Why (A) wrong:** $2(-1)^n/n$ has the wrong sign: $b_1 = 2(-1)^2/1 = 2 > 0$, but $b_1$ must be $+2$. Actually wait: (A) gives $2(-1)^n/n$. For $n = 1$: $2(-1)^1/1 = -2$. But the correct answer gives $b_1 = 2(-1)^{1+1}/1 = 2$. So (A) gives the negative of the correct answer.

**Why (B) wrong:** Missing the factor of 2.

**Why (D) wrong:** $b_n = 0$ would hold for an even function, not for $f(x) = x$.

**Answer: (C)** ← *Go back to Q10*

## Section A Solutions — Q11–Q30 (2 marks each)

**Q11.**

### Solution

**Concept:** Parseval's identity for Fourier sine series of odd $f$: $\frac{1}{\pi}\int_{-\pi}^{\pi} [f(x)]^2\,dx = \sum_{n=1}^{\infty} b_n^2$.

**Step 1:** $\frac{1}{\pi}\int_{-\pi}^{\pi} x^2\,dx = \frac{1}{\pi} \cdot \frac{2\pi^3}{3} = \frac{2\pi^2}{3}$.

**Step 2:** By Parseval's theorem, $\sum_{n=1}^{\infty} b_n^2 = \frac{2\pi^2}{3}$.

**Verification:** With $b_n = 2(-1)^{n+1}/n$: $\sum b_n^2 = 4\sum_{n=1}^{\infty} \frac{1}{n^2} = 4 \cdot \frac{\pi^2}{6} = \frac{2\pi^2}{3}$.✓

**Why (A) wrong:** $\pi^2/3 = \frac{1}{2} \cdot \frac{2\pi^2}{3}$; this arises if you forget the factor $1/\pi$.

**Why (C) wrong:** $\pi^2$ doubles the correct answer without justification.

**Why (D) wrong:** $\pi^2/6 = \sum 1/n^2$, not $\sum b_n^2$.

**Answer: (B)** ← *Go back to Q11*

**Q12.**

### Solution

**Concept:** $\hat{f}(\omega) = \int_{-\infty}^{\infty} e^{-|t|} e^{-i\omega t}\,dt$.

**Step 1:** Split at $t = 0$:

$$\hat{f}(\omega) = \int_{-\infty}^{0} e^{t} e^{-i\omega t}\,dt + \int_{0}^{\infty} e^{-t} e^{-i\omega t}\,dt = \frac{1}{1 - i\omega} + \frac{1}{1 + i\omega}.$$

**Step 2:** $\hat{f}(\omega) = \dfrac{(1+i\omega)+(1-i\omega)}{(1-i\omega)(1+i\omega)} = \dfrac{2}{1+\omega^2}$.

**Why (B) wrong:** Missing the factor 2 from combining both half-lines.

**Why (C) wrong:** $e^{-|\omega|}$ is the Fourier transform of $\dfrac{1}{1+t^2}$, not $e^{-|t|}$ (they are each other's transform up to constants).

**Why (D) wrong:** $2\pi\,\delta(\omega)$ is the transform of the constant function 1.

**Answer: (A)** ← *Go back to Q12*

**Q13.**

## Solution

**Concept:** Standard Laplace transform pair: $\mathcal{L}\{\sin(at)\} = \dfrac{a}{s^2+a^2}$.

**Derivation:** $\mathcal{L}\{\sin(at)\} = \operatorname{Im}\mathcal{L}\{e^{iat}\} = \operatorname{Im}\dfrac{1}{s-ia} = \operatorname{Im}\dfrac{s+ia}{s^2+a^2} = \dfrac{a}{s^2+a^2}$.

**Why (A) wrong:** $s/(s^2+a^2) = \mathcal{L}\{\cos(at)\}$, not sin.

**Why (B) wrong:** $a^2/(s^2+a^2)$ is neither the transform of sin nor cos.

**Why (C) wrong:** $1/(s-a) = \mathcal{L}\{e^{at}\}$, an exponential, not a sine.

**Answer: (D)** ← *Go back to Q13*

**Q14.**

## Solution

**Concept:** Partial fraction decomposition then inverse Laplace transform.

**Step 1:** $\dfrac{1}{s(s+1)} = \dfrac{A}{s} + \dfrac{B}{s+1}$. Multiplying: $A(s+1)+Bs = 1$. At $s=0$: $A=1$. At $s=-1$: $-B=1 \Rightarrow B=-1$.

**Step 2:** $\dfrac{1}{s(s+1)} = \dfrac{1}{s} - \dfrac{1}{s+1}$.

**Step 3:** $\mathcal{L}^{-1}\left\{\dfrac{1}{s}\right\} = 1$ and $\mathcal{L}^{-1}\left\{\dfrac{1}{s+1}\right\} = e^{-t}$.

**Conclusion:** $\mathcal{L}^{-1}\left\{\dfrac{1}{s(s+1)}\right\} = 1 - e^{-t}$.

**Why (A) wrong:** $e^{-t} = \mathcal{L}^{-1}\{1/(s+1)\}$; the $1/s$ term contributes the constant 1.

**Why (B) wrong:** $1-e^{t}$ would arise from $1/s - 1/(s-1)$, with wrong sign in denominator.

**Why (D) wrong:** $te^{-t} = \mathcal{L}^{-1}\{1/(s+1)^2\}$; that requires a repeated root.

**Answer: (C)** ← *Go back to Q14*

**Q15.**

**Solution**

**Concept:** Take Laplace transform of ODE with zero ICs: $(s^2+3s+2)Y(s) = F(s)$, so $H(s) = Y(s)/F(s)$.

**Step 1:** With zero initial conditions $y(0) = y'(0) = 0$:

$$\mathcal{L}\{y''+3y'+2y\} = \mathcal{L}\{f\} \Rightarrow (s^2+3s+2)Y(s) = F(s).$$

**Step 2:** $H(s) = \dfrac{Y(s)}{F(s)} = \dfrac{1}{s^2+3s+2} = \dfrac{1}{(s+1)(s+2)}$.

**Why (A) wrong:** $s^2+3s+2$ is the characteristic polynomial, the inverse of $H(s)$.
**Why (C) wrong:** $s+3$ does not arise from this ODE.
**Why (D) wrong:** $(s+1)(s+2)$ is the denominator of $H$, not $H$ itself.

**Answer: (B)** ← *Go back to Q15*

**Q16.**

**Solution**

**Concept:** Standard pair: $\mathcal{L}\{\cos(at)\} = \dfrac{s}{s^2+a^2}$.

**Step 1:** Identify $a = 2$: $\dfrac{s}{s^2+4} = \dfrac{s}{s^2+2^2}$.

**Step 2:** Therefore $\mathcal{L}^{-1}\left\{\dfrac{s}{s^2+4}\right\} = \cos(2t)$.

**Why (B) wrong:** $\sin(2t)/2$ would correspond to $\mathcal{L}^{-1}\{1/(s^2+4)\}$, which has numerator $1$ not $s$.
**Why (C) wrong:** $e^{2t} = \mathcal{L}^{-1}\{1/(s-2)\}$, a purely real exponential.
**Why (D) wrong:** $t\cos(at)$ corresponds to $\mathcal{L}^{-1}\{s/(s^2+a^2)^2\}$, a squared denominator.

**Answer: (A)** ← *Go back to Q16*

**Q17.**

**Solution**

**Concept:** Standard result: $\mathcal{L}^{-1}\left\{\dfrac{1}{(s^2+1)^2}\right\} = \dfrac{\sin t - t\cos t}{2}$.

**Derivation via convolution:** $\dfrac{1}{(s^2+1)^2} = \dfrac{1}{s^2+1}\cdot\dfrac{1}{s^2+1}$, so the inverse is $(\sin t) * (\sin t)$.

$$\int_0^t \sin\tau\sin(t-\tau)\,d\tau = \tfrac{1}{2}\int_0^t [\cos(2\tau-t)-\cos t]\,d\tau = \tfrac{1}{2}\left[\frac{\sin(2\tau-t)}{2} - \tau\cos t\right]_0^t = \frac{\sin t - t\cos t}{2}.$$

**Why (A) wrong:** $\sin(t)\cos(t) = \frac{1}{2}\sin(2t)$; this is a product, not a convolution result here.
**Why (B) wrong:** $\cos^2(t) = \frac{1}{2}(1+\cos(2t))$; its Laplace transform is not $1/(s^2+1)^2$.
**Why (C) wrong:** $t\cos(t)/2$ would be the inverse of $s/(s^2+1)^2$, not $1/(s^2+1)^2$.

**Answer: (D)** ← *Go back to Q17*

**Q18.**

## Solution

**Concept:** Apply Laplace with zero ICs, then partial fractions.

**Step 1:** $(s^2+1)Y = 1/s^2 \Rightarrow Y = \dfrac{1}{s^2(s^2+1)}$.

**Step 2:** Partial fractions: $\dfrac{1}{s^2(s^2+1)} = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{Cs+D}{s^2+1}$.

Multiply through: $1 = As(s^2+1) + B(s^2+1) + (Cs+D)s^2$.

- $s=0$: $B=1$.
- Coefficient of $s^3$: $A+C=0$.
- Coefficient of $s^2$: $B+D=0 \Rightarrow D=-1$.
- Coefficient of $s^1$: $A=0 \Rightarrow C=0$.

So $Y = \dfrac{1}{s^2} - \dfrac{1}{s^2+1}$.

**Step 3:** $y = \mathcal{L}^{-1}\left\{\dfrac{1}{s^2}\right\} - \mathcal{L}^{-1}\left\{\dfrac{1}{s^2+1}\right\} = t - \sin t$.

**Why (A) wrong:** $t+\sin t$ would require $D=+1$, but $D=-1$.
**Why (B) wrong:** $t\cos t$ arises from $s/(s^2+1)^2$, not from this problem.
**Why (D) wrong:** $1-\cos t$ solves $y''+y=1$ (constant RHS), not $y''+y=t$.

**Answer: (C)** ← *Go back to Q18*

**Q19.**

## Solution

**Concept:** The forward (explicit) Euler method evaluates the slope at the current point $(x_n, y_n)$.

**Formula:** $y_{n+1} = y_n + h\,f(x_n, y_n)$.

**Why (A) wrong:** That formula (evaluating $f$ at $x_{n+1}, y_{n+1}$) is the *backward* (implicit) Euler method.
**Why (C) wrong:** That is the trapezoidal (Crank-Nicolson) method, combining both endpoints.
**Why (D) wrong:** Subtracting $h\,f$ would step in the wrong direction, away from the solution.

**Answer: (B)** ← *Go back to Q19*

**Q20.**

**Solution**

**Concept:** The local truncation error (LTE) is the error per step; the global error accumulates $\sim n \cdot$ LTE over $n$ steps.

**Step 1:** Taylor expand the true solution: $y(x_{n+1}) = y(x_n) + hy'(x_n) + \frac{h^2}{2}y''(x_n) + O(h^3)$.

**Step 2:** Euler gives $y^{\text{Euler}}_{n+1} = y_n + hf(x_n, y_n) = y_n + hy'(x_n)$. The LTE is the residual $\frac{h^2}{2}y''(x_n) + O(h^3) = O(h^2)$.

**Note:** The *global* error is $O(h)$ because $n \approx 1/h$ steps accumulate: $n \cdot O(h^2) = O(h)$.

**Why (B) wrong:** $O(h)$ is the *global* error, not the local error per step.
**Why (C) wrong:** $O(h^3)$ LTE belongs to the explicit midpoint (RK2) method.
**Why (D) wrong:** $O(h^4)$ LTE belongs to RK4.

**Answer: (A)** ← *Go back to Q20*

**Q21.**

**Solution**

**Concept:** Classical RK4 slope estimates:

$$k_1 = f(x_n, y_n), \quad k_2 = f\left(x_n + \tfrac{h}{2}, y_n + \tfrac{h}{2}k_1\right),$$

$$k_3 = f\left(x_n + \tfrac{h}{2}, y_n + \tfrac{h}{2}k_2\right), \quad k_4 = f(x_n + h, y_n + hk_3).$$

**Weighted average:** $\phi = \dfrac{k_1 + 2k_2 + 2k_3 + k_4}{6}$, giving weights $1 : 2 : 2 : 1$ summing to $6$.

**Why (A) wrong:** Equal weights $1 : 1 : 1 : 1$ would sum to $4$ with factor $1/4$; this is the simple mean.
**Why (B) wrong:** $(k_1 + k_2)/2$ is the trapezoidal rule approximation (1-step Heun's method).
**Why (C) wrong:** $(k_1 + 4k_2 + k_3)/6$ resembles Simpson's rule; but RK4 uses $k_1, k_2, k_3, k_4$ with weights $1, 2, 2, 1$.

**Answer: (D)** ← *Go back to Q21*

**Q22.**

**Solution**

**Concept:** For $y' = \lambda y$ ($\lambda < 0$), the Euler iteration is $y_{n+1} = (1 + h\lambda)y_n$. Stability requires $|1 + h\lambda| < 1$.

**Step 1:** $|1 + h\lambda| < 1 \Leftrightarrow -1 < 1 + h\lambda < 1 \Leftrightarrow -2 < h\lambda < 0$.

**Step 2:** Since $\lambda < 0$, we need $h < 2/|\lambda|$, i.e. $h\lambda \in (-2, 0)$.

**Why (A) wrong:** $h\lambda < 0$ is necessary but not sufficient; if $h\lambda < -2$, the method oscillates and diverges.
**Why (B) wrong:** $|h\lambda| < 2$ is equivalent to $-2 < h\lambda < 2$; the upper part $0 < h\lambda < 2$ applies to $\lambda > 0$ where the ODE itself diverges.
**Why (D) wrong:** $h < 1/|\lambda|$ is more restrictive than necessary; the true bound is $h < 2/|\lambda|$.

**Answer: (C)** ← *Go back to Q22*

**Q23.**

**Solution**

**Concept:** The shooting method guesses the missing initial condition to convert a BVP into an IVP.

**Step 1:** Replace $y'(a)$ (unknown) with a parameter $s$, and solve the IVP $y'' = f(x, y, y')$, $y(a) = \alpha$, $y'(a) = s$.

**Step 2:** Adjust $s$ (e.g. by bisection or Newton's method on $\phi(s) = y(b; s) - \beta = 0$) until the boundary condition at $b$ is satisfied.

**Why (A) wrong:** The shooting method involves ODEs, not PDEs.
**Why (C) wrong:** Finite-difference methods produce a linear system; the shooting method is different.
**Why (D) wrong:** The free parameter $s = y'(a)$ is precisely what makes the method work.

**Answer: (B)** ← *Go back to Q23*

**Q24.**

**Solution**

**Concept:** Taylor expand around $x_i$:

$$y_{i+1} = y_i + hy_i' + \frac{h^2}{2}y_i'' + \frac{h^3}{6}y_i''' + O(h^4), \quad y_{i-1} = y_i - hy_i' + \frac{h^2}{2}y_i'' - \frac{h^3}{6}y_i''' + O(h^4).$$

**Step 1:** Add the two expansions: $y_{i+1} + y_{i-1} = 2y_i + h^2 y_i'' + O(h^4)$.

**Step 2:** Rearranging: $y_i'' = \dfrac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + O(h^2)$.

**Why (B) wrong:** $\frac{y_{i+1} - y_{i-1}}{2h}$ is the central difference for the *first* derivative $y_i'$.
**Why (C) wrong:** $\frac{y_{i+1} - y_i}{h}$ is the forward difference for $y_i'$.
**Why (D) wrong:** The $+$ sign in $y_{i+1} + 2y_i + y_{i-1}$ does not arise from this expansion.

**Answer: (A)** ← *Go back to Q24*

**Q25.**

**Solution**

**Concept:** Given $n+1$ points, there is a *unique* polynomial of degree $\leq n$ passing through all of them.

**Step 1:** A polynomial of degree $\leq n$ has $n+1$ coefficients, and imposing $n+1$ interpolation conditions gives a determined system (Vandermonde matrix is nonsingular for distinct nodes).

**Step 2:** The interpolating polynomial has degree *at most* $n$ (it may be exactly $n$ or lower).

**Why (A) wrong:** $n+1$ points do not require degree $n+1$; they uniquely determine degree $\leq n$.
**Why (B) wrong:** $n^2$ grows much faster than needed; the interpolation problem is determined by $n+1$ coefficients.
**Why (C) wrong:** Degree $2n$ would arise for Hermite interpolation with derivatives, not simple interpolation.

**Answer: (D)** ← *Go back to Q25*

**Q26.**

**Solution**

**Concept:** Lagrange basis polynomial $L_j(x) = \prod_{k \neq j} \frac{x - x_k}{x_j - x_k}$ is the unique degree-$n$ polynomial satisfying:

$$L_j(x_k) = \delta_{jk} = \begin{cases} 1 & k = j \\ 0 & k \neq j \end{cases}.$$

**Step 1:** At $x = x_j$: every factor $(x_j - x_k)/(x_j - x_k) = 1$, so $L_j(x_j) = 1$.

**Step 2:** At $x = x_k$ $(k \neq j)$: the factor $(x_k - x_k)/(x_j - x_k) = 0$, so $L_j(x_k) = 0$.

**Why (A) wrong:** The values are swapped: $L_j(x_j)$ should be $1$, not $0$.
**Why (B) wrong:** $L_j(x) = x^j$ does not vanish at all other nodes.
**Why (D) wrong:** $L_j(x) = 1/(x - x_j)$ would have a singularity at $x_j$, not a value of

1.

**Answer: (C)** ← *Go back to Q26*

**Q27.**

**Solution**

**Concept:** $\Delta f(x) = f(x+h) - f(x)$.

**Step 1:** For $f(x) = x^2$:

$$\Delta(x^2) = (x+h)^2 - x^2 = x^2 + 2xh + h^2 - x^2 = 2xh + h^2.$$

**Step 2:** This can be factored as $h(2x+h)$. For $h = 1$: $\Delta(x^2) = 2x + 1$.

**Why (A) wrong:** $2x$ is the derivative $f'(x)$, which approximates $\Delta f(x)/h$ as $h \to 0$, not $\Delta f(x)$ itself.
**Why (C) wrong:** $h^2 = \Delta^2(x^2)$, the *second* forward difference, not the first.
**Why (D) wrong:** $2x + h^2$ confuses $h$ with $h^2$ in the cross term.

**Answer: (B)** ← *Go back to Q27*

**Q28.**

**Solution**

**Concept:** A natural cubic spline imposes zero second derivatives at the two end-point nodes.

**Step 1:** A cubic spline $S(x)$ is a piecewise cubic with continuous first and second derivatives at interior nodes ($S, S', S''$ continuous). This leaves two degrees of freedom.

**Step 2:** The *natural* boundary conditions use these freedoms to set $S''(a) = S''(b) = 0$, corresponding to no curvature at the endpoints.

**Why (B) wrong:** $S'(a) = S'(b) = 0$ defines the "clamped" spline with prescribed derivatives, not the natural one.
**Why (C) wrong:** $S'''$ is generally discontinuous at interior knots; its continuity is not required.
**Why (D) wrong:** A single global cubic cannot satisfy $n+1$ interpolation conditions for $n \geq 3$ in general.

**Answer: (A)** ← *Go back to Q28*

**Q29.**

**Solution**

**Concept:** If $T(h) = T + Ch^p + O(h^{p+1})$, then $T(h/2) = T + C(h/2)^p + O(h^{p+1})$. Eliminating the $Ch^p$ term:

$$T \approx \frac{2^p\,T(h/2) - T(h)}{2^p - 1},$$

achieving accuracy $O(h^{p+1})$. This is Richardson extrapolation.

**Why (A) wrong:** Simpson's rule is a quadrature formula, not a technique for combining two approximations with different step sizes.
**Why (B) wrong:** Euler's method is an ODE solver, not an error-cancellation technique.
**Why (C) wrong:** Gaussian quadrature selects optimal nodes and weights; it does not combine two approximations with different $h$.

**Answer: (D)** ← *Go back to Q29*

**Q30.**

**Solution**

**Concept:** An $n$-point Gaussian quadrature rule has $n$ free nodes and $n$ free weights ($2n$ parameters), allowing exactness for polynomials of degree up to $2n - 1$.

**Step 1:** With $n = 2$: 4 parameters, exact for degree $\leq 3 = 2(2) - 1$.

**Verification ($n = 2$, nodes $\pm 1/\sqrt{3}$, weights 1):** For $f(x) = x^3$: $f(1/\sqrt{3}) + f(-1/\sqrt{3}) = (1/\sqrt{3})^3 + (-1/\sqrt{3})^3 = 0 = \int_{-1}^{1} x^3\,dx$. ✓

**Why (A) wrong:** $n$-point Newton-Cotes uses $n$ equally spaced nodes (not optimally chosen), achieving degree $\leq n - 1$ (or $n$ for even $n$).
**Why (B) wrong:** $n - 1$ would be the accuracy of an interpolatory rule using $n - 1$ nodes.
**Why (D) wrong:** $2n$ degree is not achievable with $2n$ free parameters alone (the system becomes overdetermined).

**Answer: (C)** ← *Go back to Q30*

## Section B Solutions — Q31–Q40 (2 marks each)

**Q31.**

**Solution**

**Analysis:** $f(x) = |x|$ is an even function, so all sine Fourier coefficients $b_n = 0$. Option (C) is TRUE.

**(A) TRUE:** Since $|x|$ is continuous and piecewise smooth on $[-\pi, \pi]$, Dirichlet's theorem guarantees the Fourier series converges to $|x|$ everywhere, including at $x = \pm\pi$ where both periodic extensions agree (the function is continuous).

**(B) FALSE:** Because $f$ is even, $b_n = 0$ for all $n$; no sine coefficients are nonzero.

**(D) FALSE:** At $x = \pi$: $f(\pi) = \pi$. The Fourier series converges to $\pi$, not $0$.

**Correct options: (A) and (C).**

**Answer: (AC)** ← *Go back to Q31*

**Q32.**

## Solution

**Setup:** 2-point Gauss rule: nodes $x_{1,2} = \pm 1/\sqrt{3}$, weights $w_1 = w_2 = 1$.

**(A) TRUE:** The 2-point rule is exact for polynomials of degree $\leq 2(2) - 1 = 3$.

**(B) TRUE:** $w_1 + w_2 = 1 + 1 = 2 = \displaystyle\int_{-1}^{1} 1\,dx$. ✓

**(C):** Re-read: option (C) in Q32 says "$\int_{-1}^{1} x^5\,dx \approx 0.4$" – this is FALSE. The rule gives $w_1(1/\sqrt{3})^5 + w_2(-1/\sqrt{3})^5 = 0$ (odd function), which equals the exact value $0$. So (C) says $\approx 0.4$ which is wrong.

**(D) TRUE:** For $f(x) = x^3$ (degree $3 \leq 3$): $(1/\sqrt{3})^3 + (-1/\sqrt{3})^3 = 0 = \int_{-1}^{1} x^3\,dx$. The rule gives the exact value $0$.

**Correct options: (A), (B), and (D).**

**Answer: (ABD)** ← *Go back to Q32*

**Q33.**

## Solution

**(A) FALSE:** Pointwise multiplication in the time domain corresponds to convolution in the $s$-domain, *not* multiplication: $\mathcal{L}\{f \cdot g\} \neq F \cdot G$ in general.

**(B) TRUE:** Convolution theorem: $\mathcal{L}\{f * g\} = F(s) \cdot G(s)$.

**(C) TRUE:** Differentiation rule: $\mathcal{L}\{f'\}(s) = sF(s) - f(0)$ (assuming $f$ is of exponential order and integrating by parts).

**(D) FALSE:** The Laplace transform is linear for all constants $a, b$: $\mathcal{L}\{af + bg\} = a\mathcal{L}\{f\} + b\mathcal{L}\{g\}$ holds for any $a, b \in \mathbb{R}$.

**Correct options: (B) and (C).**

**Answer: (BC)** ← *Go back to Q33*

**Q34.**

## Solution

**Preliminary:** $f(x) = x^3 - x - 2$. $f(2) = 8 - 2 - 2 = 4$. $f'(x) = 3x^2 - 1$, $f'(2) = 12 - 1 = 11$.

**(A) TRUE:** $x_1 = 2 - \dfrac{f(2)}{f'(2)} = 2 - \dfrac{4}{11} = \dfrac{22-4}{11} = \dfrac{18}{11}$.

**(B) FALSE:** Newton's method converges to the root in $(1, 2)$ from $x_0 = 2$; it does not diverge.

**(C) TRUE:** $f(1) = 1 - 1 - 2 = -2 < 0$ and $f(2) = 4 > 0$. By IVT, there is a root in $(1, 2)$.

**(D) TRUE:** (Same as (C) reasoning.) IVT confirms a real root exists.

**Correct options: (A), (C), and (D).**

**Answer: (ACD)** ← *Go back to Q34*

**Q35.**

## Solution

**(A) FALSE:** Backward Euler has the *same* order of accuracy as forward Euler ($O(h)$ global), so it is not always more accurate for a given $h$.

**(B) TRUE:** For $y' = \lambda y$, the backward Euler update is $y_{n+1} = y_n/(1 - h\lambda)$. For $\text{Re}(\lambda) < 0$, $|1 - h\lambda| > 1$ for all $h > 0$, so $|1/(1 - h\lambda)| < 1$: the method is stable for all $h > 0$. This is A-stability.

**(C) TRUE:** Both forward and backward Euler have LTE $= O(h^2)$ (equivalently, global error $O(h)$). The statement says $O(h)$ LTE which is in fact $O(h^2)$... Wait, re-reading option (C): "Its local truncation error is $O(h)$, lower than for RK4". The LTE is $O(h^2)$, but global error is $O(h)$. If (C) refers to global accuracy, then it is first-order $O(h)$, which is true and indeed lower than RK4's $O(h^4)$ global error. So (C) is TRUE in the context of order of accuracy.

**(D) TRUE:** For a nonlinear $f$, $y_{n+1} - hf(x_{n+1}, y_{n+1}) = y_n$ is a nonlinear equation in $y_{n+1}$, requiring an iterative solver (e.g. Newton's method) at each step.

**Correct options: (B) and (D).**

**Answer: (BD)** ← *Go back to Q35*

**Q36.**

## Solution

**Setup:** Three points $(x_0, y_0) = (0, 0)$, $(x_1, y_1) = (1, 1)$, $(x_2, y_2) = (2, 4)$.

**(A) TRUE:** Three data points determine a unique polynomial of degree $\leq 2$ ($n+1 = 3$ points, so degree $\leq n = 2$).

**(B) FALSE:** The interpolating polynomial has degree $\leq 2$, not 3.

**(C) TRUE:** Verify $p(x) = x^2$: $p(0) = 0\checkmark$, $p(1) = 1\checkmark$, $p(2) = 4\checkmark$. Since the interpolating polynomial is unique, $p(x) = x^2$.

**(D) FALSE:** $2x - 1$ gives $2(0) - 1 = -1 \neq 0$. It fails at $x = 0$.

**Correct options: (A) and (C).**

**Answer: (AC)** ← *Go back to Q36*

**Q37.**

**Solution**

**(A) TRUE:** The global error of the composite trapezoidal rule is $O(h^2)$ where $h = 1/n$.

**(B) TRUE:** For $f(x) = x$ (linear), the trapezoidal rule is exact. The trapezoid under $f(x) = x$ on $[0, 1]$ has area $\frac{1}{2}(f(0) + f(1)) \cdot 1 = \frac{1}{2}$, which equals $\int_0^1 x\,dx = \frac{1}{2}$.

**(C) FALSE:** For $f(x) = x^2$ (quadratic), the trapezoidal rule has error $O(h^2) > 0$, so it is not exact. The approximation $3/8 \neq 1/3$ (Q4 shows this for $n = 2$).

**(D) TRUE:** The composite trapezoidal rule with $n$ subintervals evaluates $f$ at $n + 1$ points: $x_0 = 0, x_1 = h, \ldots, x_n = 1$.

**Correct options: (A), (B), and (D).**

**Answer: (ABD)** ← *Go back to Q37*

**Q38.**

**Solution**

**(A) FALSE:** The naive DFT computation takes $O(N^2)$ operations; the FFT reduces this. So saying the FFT is $O(N^2)$ is wrong.

**(B) TRUE:** The Cooley-Tukey FFT algorithm computes the DFT in $O(N \log N)$ operations, a significant improvement over $O(N^2)$.

**(C) TRUE:** The DFT is an invertible linear transform; the Inverse DFT (IDFT) recovers the original signal exactly.

**(D) FALSE:** The DFT of a length-$N$ signal produces exactly $N$ output components $X[0], X[1], \ldots, X[N - 1]$, not $N + 1$.

**Correct options: (B) and (C).**

**Answer: (BC)** ← *Go back to Q38*

**Q39.**

**Solution**

**Theorem:** For a strictly diagonally dominant matrix $|a_{ii}| > \sum_{j \neq i} |a_{ij}|$, both the Jacobi and Gauss-Seidel methods converge for any initial guess.

**(A) TRUE:** Jacobi iteration converges for strictly diagonally dominant $A$.

**(B) FALSE:** Gradient descent (steepest descent) for $Ax = b$ is guaranteed to converge only when $A$ is symmetric positive definite; strict diagonal dominance alone is insufficient.

**(C) TRUE:** Gauss-Seidel iteration also converges for strictly diagonally dominant $A$.

**(D) TRUE:** (A) and (C) are both correct, so both methods converge simultaneously.

**Correct options: (A), (C), and (D).**

**Answer: (ACD)** ← *Go back to Q39*

**Q40.**

**Solution**

**General solution:** $y'' + y = 0$ has general solution $y = A\cos x + B\sin x$.

**BC at $x = 0$:** $y(0) = A = 0$. So $y = B\sin x$.

**BC at $x = \pi$:** $y(\pi) = B\sin\pi = 0$ for *any* $B$. No condition on $B$.

**(A) FALSE:** There is no unique nontrivial solution; every $y = C\sin x$ works.

**(B) TRUE:** $y = C\sin x$ for any $C \in \mathbb{R}$ is a solution, giving infinitely many solutions.

**(C) FALSE:** The shooting method starts with $y'(0) = s$ and solves to $x = \pi$. But since every $s$ gives $y(\pi; s) = s\sin\pi = 0$, the method cannot identify a unique solution; it fails to converge to one particular solution.

**(D) TRUE:** $y \equiv 0$ is $C\sin x$ with $C = 0$; it satisfies both the ODE and both boundary conditions.

**Correct options: (B) and (D).**

**Answer: (BD)** ← *Go back to Q40*

## Section C Solutions — Q41–Q50 (1 mark each)

**Q41.**

**Solution**

**Newton's method:** $x_{n+1} = x_n - \dfrac{x_n^2 - 5}{2x_n} = \dfrac{x_n^2 + 5}{2x_n}$.

**Step 1:** $x_0 = 2$. $x_1 = \dfrac{4+5}{4} = \dfrac{9}{4}$.

**Step 2:** $x_2 = \dfrac{(9/4)^2+5}{2\cdot(9/4)} = \dfrac{81/16+5}{9/2} = \dfrac{81/16+80/16}{9/2} = \dfrac{161/16}{9/2} = \dfrac{161}{16}\cdot\dfrac{2}{9} = \dfrac{161}{72}$.

**Answer: (161/72)** ← Go back to Q41

**Q42.**

**Solution**

**Nodes:** $x = 0, 0.5, 1, 1.5, 2$. $f(x) = x^2$: values $0, 0.25, 1, 2.25, 4$.

**Formula:** $T = \dfrac{h}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)]$ with $h = 0.5$:

$$T = \frac{0.5}{2}[0+2(0.25)+2(1)+2(2.25)+4] = 0.25[0+0.5+2+4.5+4] = 0.25\times 11 = 2.75.$$

**Exact value:** $\int_0^2 x^2\,dx = \frac{8}{3} \approx 2.667$. Error $= 2.75 - 8/3 = 1/12 \approx 0.083$.

**Answer: (2.75)** ← Go back to Q42

**Q43.**

**Solution**

**Exact integration:**

$$\int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2 = \frac{8}{3} - 0 = \frac{8}{3}.$$

As a decimal: $8/3 \approx 2.6\overline{6}$.

**Answer: (8/3)** ← Go back to Q43

**Q44.**

**Solution**

**Formula:** $\mathcal{L}\{t^n e^{at}\}(s) = \dfrac{n!}{(s-a)^{n+1}}$ for $s > a$.

**Here:** $n = 2$, $a = 3$: $\mathcal{L}\{t^2 e^{3t}\}(s) = \dfrac{2}{(s-3)^3}$.

**At** $s = 5$: $\dfrac{2}{(5-3)^3} = \dfrac{2}{8} = \dfrac{1}{4}$.

**Answer: (1/4)** ← Go back to Q44

**Q45.**

**Solution**

**Laplace transform:** $sY - y(0) - 2Y = 4/s \Rightarrow (s-2)Y = 1 + 4/s = (s+4)/s \Rightarrow$ $Y = \dfrac{s+4}{s(s-2)}$.

**Partial fractions:** $\dfrac{s+4}{s(s-2)} = \dfrac{A}{s} + \dfrac{B}{s-2}$. $A = \dfrac{0+4}{0-2} = -2$. $B = \dfrac{2+4}{2} = 3$.

**Inverse:** $y(t) = -2 + 3e^{2t}$.

**At** $t = 1$**:** $y(1) = 3e^2 - 2$.

**Answer: $(3e^2 - 2)$** ← *Go back to Q45*

**Q46.**

**Solution**

**Symmetry:** $|\cos x|$ has period $\pi$, so $\displaystyle\int_0^{2\pi} |\cos x|\,dx = 2\int_0^{\pi} |\cos x|\,dx = 2\int_0^{\pi/2} \cos x\,dx + 2\int_{\pi/2}^{\pi} (-\cos x)\,dx$.

Since $|\cos x|$ has period $\pi$: $\displaystyle\int_0^{2\pi} |\cos x|\,dx = 2\int_0^{\pi} |\cos x|\,dx = 2\cdot 2\int_0^{\pi/2} \cos x\,dx = 4[\sin x]_0^{\pi/2} = 4\cdot 1 = 4$.

**Answer: (4)** ← *Go back to Q46*

**Q47.**

**Solution**

**Odd-even argument:** $f(x) = x^2$ is an even function; $\sin(x)$ is an odd function. Their product $x^2\sin(x)$ is odd.

**Integral over symmetric interval:** $\displaystyle\int_{-\pi}^{\pi} x^2\sin(x)\,dx = 0$.

**Conclusion:** $b_1 = \dfrac{1}{\pi}\cdot 0 = 0$.

**Answer: (0)** ← *Go back to Q47*

**Q48.**

**Solution**

**Simpson's $\frac{3}{8}$ rule** divides $[a,b]$ into 3 equal sub-intervals, using nodes $a$, $a + h/3$, $a + 2h/3$, $b$ where $h = b - a$.

**Formula:** $\displaystyle\int_a^b f\,dx \approx \frac{3h}{8}\big[f(a) + 3f(a+h/3) + 3f(a+2h/3) + f(b)\big]$.

**Number of points:** 4 function evaluations.

(Compare: Simpson's $1/3$ uses 3 points; Boole's rule uses 5.)

**Answer: (4)** ← Go back to Q48

**Q49.**

**Solution**

**From Parseval:** With $b_n = 2(-1)^{n+1}/n$ for $f(x) = x$, Parseval gives $\sum_{n=1}^{\infty} b_n^2 = \frac{2\pi^2}{3}$.

**Compute:** $b_n^2 = \frac{4}{n^2}$, so $4\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{2\pi^2}{3}$.

**Therefore:** $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

**Answer: ($\pi^2/6$)** ← Go back to Q49

**Q50.**

**Solution**

**Analysis:** $\left|\frac{(-1)^n}{n}\right| = \frac{1}{n} \to 0$ as $n \to \infty$.

**Squeeze theorem:** $1 - \frac{1}{n} \le 1 - \frac{(-1)^n}{n} \le 1 + \frac{1}{n}$, and both bounds $\to 1$.

**Conclusion:** $\lim_{n\to\infty} \left(1 - \frac{(-1)^n}{n}\right) = 1$.

**Answer: (1)** ← Go back to Q50

## Section C Solutions — Q51–Q60 (2 marks each)

**Q51.**

**Solution**

**Step 1:** $\sin(t)\cos(t) = \frac{1}{2}\sin(2t)$.

**Step 2:** $\mathcal{L}\left\{\frac{1}{2}\sin(2t)\right\}(s) = \frac{1}{2} \cdot \frac{2}{s^2+4} = \frac{1}{s^2+4}$.

**At $s = 2$:** $\frac{1}{4+4} = \frac{1}{8}$.

**Answer: (1/8)** ← Go back to Q51

**Q52.**

**Solution**

**Groups of order 4:** Every group of order $4$ is abelian (since $4 = 2^2$ and groups of prime-square order are abelian).

**Classification:** Up to isomorphism, there are exactly two:

(a) $\mathbb{Z}_4$: cyclic of order 4.
(b) $\mathbb{Z}_2 \times \mathbb{Z}_2$: Klein four-group (every nonidentity element has order 2).

**Answer:** 2 non-isomorphic groups.

**Answer: (2)** ← *Go back to Q52*

**Q53.**

**Solution**

**Integration:**

$$\int_0^2 \sin(\pi x)\,dx = \left[-\frac{\cos(\pi x)}{\pi}\right]_0^2 = \frac{-\cos(2\pi)+\cos(0)}{\pi} = \frac{-1+1}{\pi} = 0.$$

**Geometric interpretation:** The positive area on $[0,1]$ exactly cancels the negative area on $[1,2]$ by symmetry of $\sin(\pi x)$ about $x = 1$.

**Answer: (0)** ← *Go back to Q53*

**Q54.**

**Solution**

**Given:** $f(x) = x^3 - 3x + 1$, $x_0 = 2$.

**Compute:** $f(2) = 8 - 6 + 1 = 3$. $f'(x) = 3x^2 - 3$, $f'(2) = 12 - 3 = 9$.

**First iterate:** $x_1 = 2 - \dfrac{f(2)}{f'(2)} = 2 - \dfrac{3}{9} = 2 - \dfrac{1}{3} = \dfrac{5}{3}$.

**Answer: (5/3)** ← *Go back to Q54*

**Q55.**

**Solution**

**Trapezoidal rule, $n = 1$:** Single interval $[0,1]$, $h = 1$. Nodes: $x_0 = 0$, $x_1 = 1$.

$$T = \frac{h}{2}[f(x_0) + f(x_1)] = \frac{1}{2}[e^0 + e^1] = \frac{1+e}{2}.$$

**Numerical value:** $\dfrac{1+e}{2} \approx \dfrac{1+2.718}{2} \approx 1.859.$

**Answer: ((1+e)/2)** ← *Go back to Q55*

**Q56.**

**Solution**

**Error:**
$$\left|\frac{1+e}{2}-(e-1)\right| = \left|\frac{1+e-2e+2}{2}\right| = \left|\frac{3-e}{2}\right| = \frac{3-e}{2},$$
since $e \approx 2.718 < 3$, so $3-e > 0$.

**Numerical value:** $\dfrac{3-e}{2} \approx \dfrac{3-2.718}{2} \approx 0.141.$

**Answer: ((3-e)/2)** ← *Go back to Q56*

**Q57.**

**Solution**

**Step 1:** $T_2(x) = 2x^2 - 1 = 0 \Rightarrow x^2 = \dfrac{1}{2} \Rightarrow x = \pm\dfrac{1}{\sqrt{2}}.$

**Step 2:** Product of roots: $\left(\dfrac{1}{\sqrt{2}}\right)\cdot\left(-\dfrac{1}{\sqrt{2}}\right) = -\dfrac{1}{2}.$

**By Vieta's formulae:** For $2x^2 + 0 \cdot x - 1 = 0$, product of roots $= \dfrac{-1}{2} = -\dfrac{1}{2}.$

**Answer: (-1/2)** ← *Go back to Q57*

**Q58.**

**Solution**

**Dirichlet's theorem:** At a point of continuity, the Fourier series converges to $f(x)$.

**At $x = \pi/2$:** $f(\pi/2) = \pi/2$.

Since $f(x) = x$ is continuous at $x = \pi/2 \in (-\pi, \pi)$, the Fourier series converges to $f(\pi/2) = \pi/2$.

**Answer:** $\dfrac{\pi}{2}.$

**Answer: (pi/2)** ← *Go back to Q58*

**Q59.**

**Solution**

**Setup:** $y' = f(x, y) = y$, $y_0 = 1$, $h = 1$, $x_0 = 0$.

**RK4 stages:**

$$\begin{aligned}
k_1 &= f(0,1) = 1,\\
k_2 &= f\big(0+\tfrac{1}{2},\, 1+\tfrac{1}{2}\cdot 1\big) = f\big(\tfrac{1}{2},\tfrac{3}{2}\big) = \tfrac{3}{2},\\
k_3 &= f\big(\tfrac{1}{2},\, 1+\tfrac{1}{2}\cdot\tfrac{3}{2}\big) = f\big(\tfrac{1}{2},\tfrac{7}{4}\big) = \tfrac{7}{4},\\
k_4 &= f\big(1,\, 1+1\cdot\tfrac{7}{4}\big) = f\big(1,\tfrac{11}{4}\big) = \tfrac{11}{4}.
\end{aligned}$$

**Update:**

$$y_1 = 1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 1 + \frac{1}{6}\left(1 + 3 + \frac{7}{2} + \frac{11}{4}\right).$$

Sum in brackets: $1 + 3 + \frac{7}{2} + \frac{11}{4} = \frac{4+12+14+11}{4} = \frac{41}{4}$.

$$y_1 = 1 + \frac{1}{6}\cdot\frac{41}{4} = 1 + \frac{41}{24} = \frac{24+41}{24} = \frac{65}{24}.$$

**Exact:** $e^1 = e \approx 2.71828$. $65/24 \approx 2.7083$ (very close).

**Answer: (65/24)** ← *Go back to Q59*

**Q60.**

**Solution**

**Legendre polynomials satisfy $P_n(1) = 1$ for all $n \geq 0$.**

**Direct verification:**

$$P_4(1) = \frac{1}{8}\big(35(1)^4 - 30(1)^2 + 3\big) = \frac{1}{8}(35 - 30 + 3) = \frac{8}{8} = 1.$$

**Answer: (1)** ← *Go back to Q60*

# Answer Key — Sample Paper 9

| Q | Ans | Q | Ans | Q | Ans | Q | Ans | Q | Ans |
|---|---|---|---|---|---|---|---|---|---|
| 1 | D | 2 | C | 3 | B | 4 | A | 5 | D |
| 6 | C | 7 | B | 8 | A | 9 | D | 10 | C |
| 11 | B | 12 | A | 13 | D | 14 | C | 15 | B |
| 16 | A | 17 | D | 18 | C | 19 | B | 20 | A |
| 21 | D | 22 | C | 23 | B | 24 | A | 25 | D |
| 26 | C | 27 | B | 28 | A | 29 | D | 30 | C |
| 31 | AC | 32 | ABD | 33 | BC | 34 | ACD | 35 | BD |
| 36 | AC | 37 | ABD | 38 | BC | 39 | ACD | 40 | BD |
| 41 | 161/72 | 42 | 2.75 | 43 | 8/3 | 44 | 1/4 | 45 | $3e^2 - 2$ |
| 46 | 4 | 47 | 0 | 48 | 4 | 49 | $\pi^2/6$ | 50 | 1 |
| 51 | 1/8 | 52 | 2 | 53 | 0 | 54 | 5/3 | 55 | (1+e)/2 |
| 56 | (3-e)/2 | 57 | -1/2 | 58 | pi/2 | 59 | 65/24 | 60 | 1 |