IIT JAM Mathematical Statistics

Sample Paper – 3

Instructions

- This paper contains **60 questions** carrying a maximum of **100 marks**. Duration: **3 hours (180 minutes)**.
- **Section A** (**Q1–Q30**, MCQ): Q1–Q10 carry **+1 mark** each; wrong response: $-\frac{1}{3}$ mark. Q11–Q30 carry **+2 marks** each; wrong response: $-\frac{2}{3}$ mark. Choose the **single best** option.
- **Section B** (**Q31–Q40**, MSQ): Each carries **+2 marks**. **No negative marking. One or more** options may be correct; full marks only if *all* correct options (and no incorrect options) are selected.
- **Section C** (**Q41–Q60**, NAT): Q41–Q50 carry **+1 mark**; Q51–Q60 carry **+2 marks**. **No negative marking.** Enter the numerical value using the virtual keypad.
- **Calculator policy:** Personal calculators, log tables, and electronic devices are **strictly prohibited**. A virtual scientific calculator is provided in the CBT interface. Rough sheets are supplied at the centre.
- Once you move to the next section, you cannot return to a previous section.

Section A — Multiple Choice Questions (MCQ)

***Q1–Q10**: 1 mark each ($-\frac{1}{3}$ for wrong). Choose the **single** correct option.*

Q1. Which of the following series is **convergent**?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

(B) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(C) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

Q2. Let $f(x) = x^4 - 4x^3 + 4x^2$. Which of the following statements is **correct**?

(A) f has a local minimum at $x = 1$

(B) f has a global minimum at $x = 0$

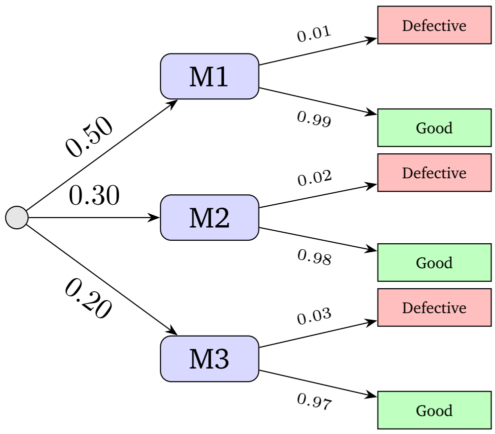
(C) f has no local extrema

(D) f has a local maximum at $x = 1$ with value 1

Q3. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{pmatrix}$. What is the dimension of the **null space** of A?

(A) 2
(B) 1
(C) 3
(D) 0

Q4. Three machines produce 50%, 30%, and 20% of a factory's output with defect rates 1%, 2%, and 3% respectively. A product is chosen at random. What is the overall probability that it is **defective**?



(A) 0.015
(B) 0.017
(C) 0.020
(D) 0.012

Q5. Let X be the number of trials required to obtain the $r = 3$rd success in independent Bernoulli trials with success probability $p = 0.4$. Then $E[X]$ equals:

(A) 6.0
(B) 8.0
(C) 7.5
(D) 5.0

Q6. Let X follow the standard Cauchy distribution with PDF $f(x) = \dfrac{1}{\pi(1+x^2)}$, $x \in \mathbb{R}$. Which statement is **correct**?

(A) $E[X] = 0$
(B) $E[X] = 1$
(C) $E[X] = \pi$
(D) $E[X]$ does not exist for the standard Cauchy distribution

Q7. The value of $\int_0^\infty x\,e^{-x}\,dx$ is:

(A) 1
(B) 0
(C) 2
(D) e

Q8. Consider the system $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$. This system has:

(A) No solution
(B) Infinitely many solutions
(C) A unique solution
(D) Exactly two solutions

Q9. Events A and B are conditionally independent given event C. If $P(A \mid C) = 0.5$ and $P(B \mid C) = 0.4$, then $P(A \cap B \mid C)$ equals:

(A) 0.9
(B) 0.1
(C) 0.45
(D) 0.2

Q10. For a random variable X, if $E[X] = 3$ and $E[X^2] = 13$, then $\mathrm{Var}(X)$ equals:

(A) 2
(B) 9
(C) 4

(D) 13

***Q11–Q30**: 2 marks each ($-\frac{2}{3}$ for wrong). Choose the **single** correct option.*

Q11. Consider the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^2}$. Which statement is **correct**?

(A) The series diverges by comparison with $\sum \frac{1}{n}$
(B) The series converges by the integral test
(C) The series converges by the ratio test (ratio < 1)
(D) The series is a p-series with $p = 2$

Q12. By the Mean Value Theorem for integrals, for $f(x) = \sqrt{x}$ on $[1, 4]$, there exists $c \in (1, 4)$ such that $\displaystyle\int_1^4 f(x)\,dx = f(c)(4 - 1)$. The value of $f(c)$ is:

(A) $\frac{14}{9}$
(B) $\frac{7}{3}$
(C) $\frac{14}{3}$
(D) $\frac{2}{3}$

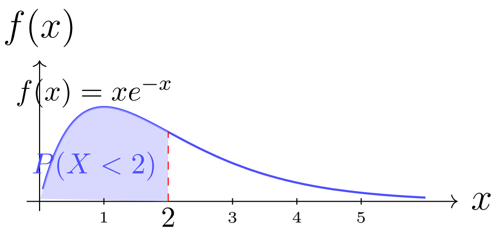
Q13. Consider the matrix $A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$. Which statement is **correct**?

(A) A is not diagonalizable
(B) A has eigenvalues $\{1, 2\}$
(C) A has eigenvalues $\{1, 4\}$
(D) A is diagonalizable with eigenvalues $\{2, 3\}$

Q14. The differential equation $\dfrac{dy}{dx} + y = y^2 e^x$ is a Bernoulli equation. Which substitution v transforms it into a **linear** first-order ODE?

(A) $v = y^2$
(B) $v = e^y$
(C) $v = y^{-1}$
(D) $v = y^3$

Q15. Let $X \sim \text{Gamma}(2, 1)$ with PDF $f(x) = xe^{-x}$ for $x > 0$ (shape $\alpha = 2$, rate $\beta = 1$). The PDF with the shaded region $P(X < 2)$ is shown below. What is $P(X < 2)$?

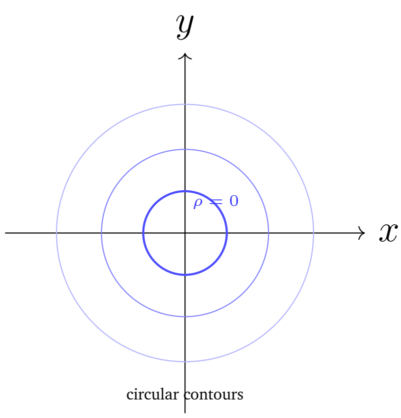


(A) $1 - 4e^{-2}$
(B) $1 - 3e^{-2}$
(C) $1 - 2e^{-2}$
(D) $1 - e^{-2}$

Q16. Let $X_1, \ldots, X_n$ be iid $N(\mu, \sigma^2)$ with σ^2 **known**. By the Lehmann–Scheffé theorem, the UMVUE for μ is:

(A) $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$
(B) The sample median
(C) The sample mode
(D) No UMVUE exists for μ when σ^2 is known

Q17. Let (X, Y) follow a bivariate normal distribution. The circular contour plot below depicts the joint PDF for $\rho = 0$. Which statement is **correct**?



(A) $\rho = 0$ implies X and Y are uncorrelated but *not* independent in general

(B) $\rho = 0$ implies X and Y are negatively correlated
(C) The contours are circular only when $\rho \neq 0$
(D) $\rho = 0$ implies X and Y are independent (unique to bivariate normal)

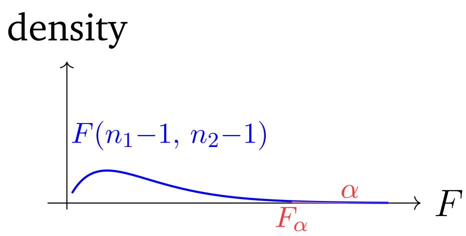
Q18. In the Likelihood Ratio Test for a one-parameter model, let $\Lambda = L(\theta_0)/L(\hat{\theta})$. Under H_0, the asymptotic distribution of $-2\ln\Lambda$ is (Wilks' theorem):

(A) $\chi^2(n-1)$
(B) $F(1, n-1)$
(C) $\chi^2(1)$
(D) $t(n-1)$

Q19. Let $X_1, \dots, X_n$ be iid $N(\mu, \sigma^2)$ with both parameters unknown. The MLE for σ^2 is:

(A) $S^2 = \dfrac{1}{n-1}\displaystyle\sum_{i=1}^{n}(X_i - \bar{X})^2$ (unbiased)
(B) $\hat{\sigma}^2 = \dfrac{1}{n}\displaystyle\sum_{i=1}^{n}(X_i - \bar{X})^2$ (biased)
(C) $\hat{\sigma}^2 = \dfrac{1}{n+1}\displaystyle\sum_{i=1}^{n}(X_i - \bar{X})^2$
(D) $\hat{\sigma}^2 = \bar{X}^2$

Q20. To test $H_0 : \sigma_1^2 = \sigma_2^2$ vs. $H_1 : \sigma_1^2 \neq \sigma_2^2$ using independent samples of sizes n_1 and n_2, the right-tail critical region is shown below. The correct test statistic and its null distribution are:



(A) $F = S_1^2/S_2^2 \sim F(n_1-1,\ n_2-1)$ under H_0
(B) $F = S_1^2/S_2^2 \sim \chi^2(n_1+n_2-2)$ under H_0
(C) $F = S_1^2/S_2^2 \sim t(n_1+n_2-2)$ under H_0
(D) $F = S_1^2/S_2^2 \sim N(0,1)$ under H_0

Q21. The volume of the unit ball $\{(x, y, z) : x^2 + y^2 + z^2 \leq 1\}$ computed via a triple integral equals:

(A) 2π

(B) π

(C) $\frac{2}{3}\pi$

(D) $\frac{4}{3}\pi$

Q22. Let $X \sim N(0, 1)$. Using the moment generating function $M_X(t) = e^{t^2/2}$, the value of $E[e^X]$ is:

(A) $e^0 = 1$

(B) 0

(C) $e^{1/2}$

(D) e

Q23. For random variables X and Y, if $E[X] = 2$, $E[Y] = 3$, and $E[XY] = 10$, then $\text{Cov}(X, Y)$ equals:

(A) 6

(B) 4

(C) 10

(D) 0

Q24. In a regression analysis, $\text{SST} = 100$ and $\text{SSE} = 36$. The coefficient of determination R^2 is:

(A) 0.64

(B) 0.36

(C) 0.50

(D) 0.80

Q25. Let $X_1, X_2, \ldots, X_n$ be iid with $E[X_i] = \mu$ and $\text{Var}(X_i) = \sigma^2 < \infty$. The Central Limit Theorem states that as $n \to \infty$:

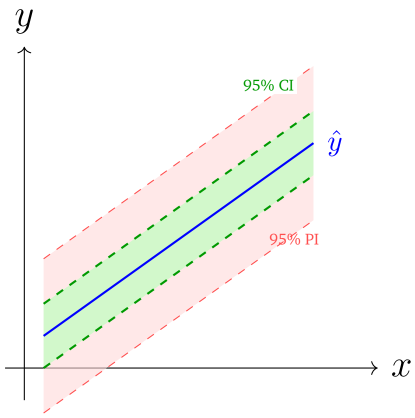
(A) $\dfrac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \xrightarrow{d} N(\mu, \sigma^2)$

(B) $\dfrac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \xrightarrow{d} \text{Exp}(1)$

(C) $\bar{X} \xrightarrow{d} N(0,1)$

(D) $\dfrac{\sqrt{n}(\bar{X}-\mu)}{\sigma} \xrightarrow{d} N(0,1)$

Q26. In simple linear regression, the figure shows the fitted line with the **confidence interval** for $E[Y \mid x_0]$ (green band) and the **prediction interval** for a future Y_0 (red band). Which statement is **correct**?



(A) The prediction interval and confidence interval for $E[Y \mid x_0]$ have the same width

(B) The confidence interval for $E[Y \mid x_0]$ is wider than the prediction interval

(C) The prediction interval is wider than the confidence interval for $E[Y \mid x_0]$

(D) Both intervals have the same width when $x_0 = \bar{x}$

Q27. Let $X_1, \ldots, X_n$ be iid $\text{Exp}(\theta)$ with PDF $f(x;\theta) = \theta e^{-\theta x}$, $x > 0$. By the factorisation theorem and completeness, which statistic is **complete sufficient** for θ?

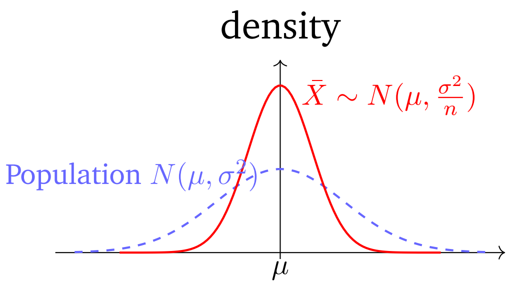
(A) The sample mean $\bar{X}$

(B) $T = \sum_{i=1}^{n} X_i$

(C) The sample minimum $X_{(1)}$

(D) The sample maximum $X_{(n)}$

Q28. The figure below shows the population distribution $N(\mu, \sigma^2)$ (blue dashed, wider) and the sampling distribution of $\bar{X}$ (red solid, narrower) for sample size n. The standard error of $\bar{X}$ is:



(A) $\dfrac{\sigma}{\sqrt{n}}$

(B) $\dfrac{\sigma}{n}$

(C) $\sigma\sqrt{n}$

(D) $\dfrac{\sigma^2}{n}$

Q29. In a chi-square test for independence applied to a 2×3 contingency table (2 rows, 3 columns), the degrees of freedom are:

(A) 4

(B) 6

(C) 3

(D) 2

Q30. For a random sample $X_1, \ldots, X_n$ from $N(\mu, \sigma^2)$, let M_n denote the sample median. The limiting value of $n\,\mathrm{Var}(M_n)$ as $n \to \infty$ is:

(A) σ^2

(B) $2\sigma^2$

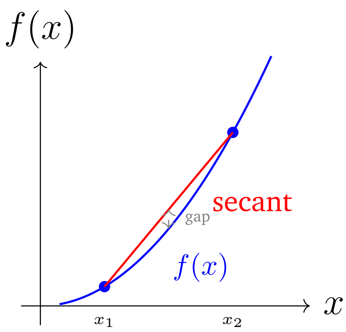
(C) $\dfrac{\pi\sigma^2}{2}$

(D) $\dfrac{\sigma^2}{2}$

Section B — Multiple Select Questions (MSQ)

Q31–Q40: *2 marks each.* ***No negative marking. One or more*** *options may be correct.*

Q31. The figure shows a convex function f with a secant line lying *above* the curve. Which of the following statements about **convex functions** are **correct**?



(A) For a twice-differentiable function, $f''(x) \geq 0$ for all x implies f is convex

(B) Every convex function on $\mathbb{R}$ is differentiable everywhere

(C) e^x is strictly convex on $\mathbb{R}$

(D) For any random variable X, if f is convex then $f(E[X]) \leq E[f(X)]$ (Jensen's inequality)

Q32. Let $\{N(t), t \geq 0\}$ be a Poisson process with rate $\lambda > 0$. Which of the following statements are **correct**?

(A) The inter-arrival times are exponentially distributed with rate $1/\lambda$

(B) The inter-arrival times T_i are iid $\mathrm{Exp}(\lambda)$ (with mean $1/\lambda$)

(C) $P(N(t) = 0) = t\, e^{-\lambda t}$

(D) $E[N(t)] = \lambda t$

Q33. Which of the following statements about **sufficient statistics** are **correct**?

(A) If T is sufficient for θ and g is a one-to-one function, then $g(T)$ is also sufficient for θ

(B) By the Factorisation theorem, $T(\mathbf{X})$ is sufficient iff $f(\mathbf{x};\theta) = g(T(\mathbf{x});\theta)\, h(\mathbf{x})$

(C) Every statistic is a sufficient statistic for θ

(D) For iid $N(\mu, 1)$ observations, $\bar{X}$ is sufficient for μ

Q34. Which of the following statements about continuous distributions are **correct**?

(A) If $X \sim \mathrm{Exp}(1)$ (rate 1), then $2X \sim \mathrm{Exp}(1/2)$ (rate $1/2$)

(B) If $X \sim N(0,1)$, then $X^2 \sim \chi^2(2)$

(C) If $X \sim U(0,1)$, then $-\ln X \sim \mathrm{Exp}(1)$

(D) $X \sim \text{Gamma}(2, 1)$ (shape 2, rate 1) has the same distribution as the sum of two independent $\text{Exp}(2)$ (rate 2) variables

Q35. Which of the following statements about **hypothesis testing** are **correct**?

(A) The size of a test equals $1 -$ power

(B) A uniformly most powerful (UMP) test maximises power uniformly over all $\theta \in \Theta_1$

(C) The Neyman–Pearson lemma gives the most powerful test for simple H_0 vs. simple H_1

(D) The p-value is $\leq \alpha$ if and only if the test rejects H_0 at level α

Q36. In simple linear regression *with an intercept*, which statements about the **OLS estimator** are **correct**?

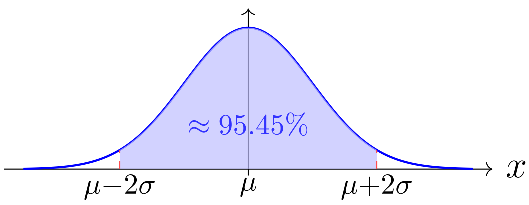
(A) The residuals sum to zero: $\sum_{i=1}^{n} \hat{e}_i = 0$

(B) OLS estimators are always normally distributed regardless of the error distribution

(C) $R^2 = 1 - \text{SSE}/\text{SST}$

(D) The OLS slope estimator is $\hat{\beta}_1 = S_{xy}/S_{yy}$

Q37. The figure shows a normal distribution $N(\mu, \sigma^2)$ with the $\pm 2\sigma$ region shaded. Which statements are **correct**?



(A) $N(\mu, \sigma^2)$ is symmetric about μ

(B) $P(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.9545$

(C) $N(\mu, \sigma^2)$ has positive skewness

(D) If $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ are independent, then $aX + bY \sim N(a\mu_1 + b\mu_2,\ a^2\sigma_1^2 + b^2\sigma_2^2)$

Q38. Let A and B be $n \times n$ matrices. Which of the following statements are **correct**?

(A) $(AB)^\top = A^\top B^\top$
(B) $\det(AB) = \det(A) \cdot \det(B)$
(C) $\det(A^\top) = \det(A)$
(D) If all eigenvalues of A are nonzero, then $\det(A) = 0$

Q39. Which of the following statements about **bivariate distributions** are **correct**?

(A) For a bivariate normal distribution, $\mathrm{Cov}(X, Y) = 0$ implies X and Y are independent
(B) $\mathrm{Cov}(X, Y) = 0$ always implies X and Y are independent (for any distribution)
(C) The correlation coefficient satisfies $\rho(X, Y) = \dfrac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_Y}$
(D) The marginal distributions do not, in general, uniquely determine the joint distribution

Q40. Which of the following statements about **confidence intervals** are **correct**?

(A) A 95% CI means $P(\mu \in \text{CI}) = 0.95$ for any fixed μ
(B) Wider confidence intervals indicate less precision in estimating the parameter
(C) Increasing the sample size n simultaneously decreases the interval width *and* lowers the confidence level
(D) For fixed sample size and distribution, a higher confidence level always requires a wider interval

Section C — Numerical Answer Type (NAT)

Q41–Q50**: 1 mark each. **No negative marking.

Q41. Let $X \sim \mathrm{Gamma}(\alpha = 3,\ \beta = 2)$ where β is the **rate** parameter, so $E[X] = \alpha/\beta = 3/2$ and $\mathrm{Var}(X) = \alpha/\beta^2 = 3/4$. Find $E[X^2]$.

[Enter numerical answer]

Q42. Let $X \sim N(5, 4)$ (mean 5, variance 4). Find $P(X > 7)$, rounded to **4 decimal places**.

[Enter numerical answer]

Q43. Compute $\det(A)$ where $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

[Enter numerical answer]

Q44. $P(A) = 0.4,\ P(B \mid A) = 0.6,\ P(B \mid A^c) = 0.3$. Compute $P(B)$.

[Enter numerical answer]

Q45. $X_1, \ldots, X_6$ are iid $N(\mu, \sigma^2 = 4)$ (variance known). Observed $\bar{x} = 8$. Compute the Z-test statistic for $H_0 : \mu = 7$ vs. $H_1 : \mu \neq 7$, rounded to **2 decimal places**.

[Enter numerical answer]

Q46. Let $X \sim \text{Exp}(\lambda = 2)$ (rate 2). Find the **median** of X, rounded to **3 decimal places**.

[Enter numerical answer]

Q47. Evaluate $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$, rounded to **4 decimal places**.

[Enter numerical answer]

Q48. Let $X \sim \text{Bin}(20, 0.5)$. Using the normal approximation *with continuity correction*, find $P(X = 10)$, rounded to **3 decimal places**.

[Enter numerical answer]

Q49. Let $X \sim N(2, 9)$ and $Y \sim N(3, 4)$ be **independent**. What is $\text{Var}(X + Y)$?

[Enter numerical answer]

Q50. Solve $y'' - 5y' + 6y = 0$ with $y(0) = 0$ and $y'(0) = 1$. Find $y(1)$, rounded to **2 decimal places**.

[Enter numerical answer]

Q51–Q60**: 2 marks each.* ***No negative marking.

Q51. A random sample of size $n = 25$ is drawn from $N(\mu, \sigma^2 = 25)$ (variance known). Observed $\bar{x} = 32$. Compute the **full width** of the 99% confidence interval for μ (use $z_{0.005} = 2.576$), rounded to **3 decimal places**.

[Enter numerical answer]

Q52. In simple linear regression, $\hat{\beta}_0 = 1$ and $\hat{\beta}_1 = 2$. What is the predicted value $\hat{y}$ at $x_0 = 10$?

[Enter numerical answer]

Q53. For iid $\mathrm{Exp}(\lambda)$ observations $X_1, \ldots, X_{10}$ with $\sum X_i = 6$, compute $-2 \ln \Lambda$ for $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$, rounded to **2 decimal places**.

[Enter numerical answer]

Q54. Let $X \sim \chi^2(4)$. It is known that the 95th percentile is $\chi^2_{0.05}(4) = 9.488$. What is $P(X > 9.488)$?

[Enter numerical answer]

Q55. Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find the $(1, 2)$ entry of A^2.

[Enter numerical answer]

Q56. Prior: $\mu \sim N(0, 1)$. Likelihood: $X \mid \mu \sim N(\mu, 1)$. A single observation $X = 3$ is recorded. Compute the **posterior mean** of μ.

[Enter numerical answer]

Q57. Let $X \sim \mathrm{Bin}(5, p)$. Test $H_0 : p = 0.5$ vs. $H_1 : p > 0.5$ with rejection region $\{X = 5\}$. Find the **power** at $p = 0.8$, rounded to **3 decimal places**.

[Enter numerical answer]

Q58. Let $X \sim \mathrm{Bin}(400,\ 0.25)$. Using the normal approximation with continuity correction, compute $P(90 \leq X \leq 110)$, rounded to **3 decimal places**.

[Enter numerical answer]

Q59. Let $X_1, \ldots, X_n$ be iid with $E[X_i] = \mu$ and $\mathrm{Var}(X_i) = \sigma^2 = 16$. Consider $T = (3X_1 + X_2)/4$. What is the MSE of T for estimating μ?

[Enter numerical answer]

Q60. Let $f(x) = c\,x(1-x)^2$ for $0 < x < 1$ (and 0 otherwise) be a valid PDF. Find c.

[Enter numerical answer]

Solutions

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Detailed Solutions

Q1. **Answer: (C)**

Solution

The p-series $\sum_{n=1}^{\infty} 1/n^p$ converges if and only if $p > 1$.
(A) $p = 1$: diverges (harmonic series).
(B) $p = 1/2$: diverges ($p < 1$).
(D) $\sum \ln n/n$: since $\ln n > 1$ for $n \geq 3$, this eventually dominates $\sum 1/n$ and diverges.
(C) $\sum 1/n^{3/2}$: $p = 3/2 > 1$, so this **converges** by the p-series test. ✓

Q2. **Answer: (D)**

Solution

$f(x) = x^4 - 4x^3 + 4x^2$. Differentiate: $f'(x) = 4x^3 - 12x^2 + 8x = 4x(x-1)(x-2)$.
Critical points: $x = 0, 1, 2$.
$f''(x) = 12x^2 - 24x + 8$.
At $x = 1$: $f''(1) = 12 - 24 + 8 = -4 < 0 \Rightarrow$ **local maximum**.
$f(1) = 1 - 4 + 4 = \mathbf{1}$.
So f has a local maximum at $x = 1$ with value 1. ✓

Q3. **Answer: (A)**

Solution

The second row of A equals twice the first row, so $\mathrm{rank}(A) = 1$.
By the rank–nullity theorem:

$$\dim(\mathrm{null}(A)) = \text{number of columns} - \mathrm{rank}(A) = 3 - 1 = \mathbf{2}.$$

Q4. **Answer: (B)**

Solution

Law of Total Probability:

$$P(\text{Defective}) = P(D \mid M_1)P(M_1) + P(D \mid M_2)P(M_2) + P(D \mid M_3)P(M_3)$$

$$= 0.01 \times 0.50 + 0.02 \times 0.30 + 0.03 \times 0.20 = 0.005 + 0.006 + 0.006 = \mathbf{0.017}.$$

Q5. Answer: (C)

Solution

For the Negative Binomial distribution (number of trials to get r successes with probability p):

$$E[X] = \frac{r}{p} = \frac{3}{0.4} = \mathbf{7.5}.$$

Q6. Answer: (D)

Solution

The standard Cauchy PDF is $f(x) = \dfrac{1}{\pi(1+x^2)}$. The integral $\int_{-\infty}^{\infty} |x|\, f(x)\, dx$ diverges (it behaves like $\int 1/|x|\, dx$ for large $|x|$). Hence $E[X]$ **does not exist**. The distribution has no finite moments of order ≥ 1.

Q7. Answer: (A)

Solution

Integration by parts with $u = x$, $dv = e^{-x}dx$:

$$\int_0^{\infty} xe^{-x}\, dx = \Big[-xe^{-x}\Big]_0^{\infty} + \int_0^{\infty} e^{-x}\, dx = 0 + \Big[-e^{-x}\Big]_0^{\infty} = 0 + 1 = \mathbf{1}.$$

Alternatively, this is $\Gamma(2) = 1! = 1$.

Q8. Answer: (B)

Solution

$\det \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} = 4 - 4 = 0$, so the coefficient matrix is singular.

Augmented matrix $\begin{pmatrix} 1 & 2 & | & 3 \\ 2 & 4 & | & 6 \end{pmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{pmatrix} 1 & 2 & | & 3 \\ 0 & 0 & | & 0 \end{pmatrix}$.

The system is consistent with rank $1 < 2$ unknowns, giving **infinitely many solutions**.

Q9. Answer: (D)

Solution

Conditional independence means $P(A \cap B \mid C) = P(A \mid C)\,P(B \mid C)$.

$$P(A \cap B \mid C) = 0.5 \times 0.4 = \mathbf{0.2}.$$

Q10. **Answer: (C)**

Solution

$$\mathrm{Var}(X) = E[X^2] - (E[X])^2 = 13 - 3^2 = 13 - 9 = \mathbf{4}.$$

Q11. **Answer: (B)**

Solution

Let $f(x) = \dfrac{1}{x(\ln x)^2}$, which is positive, continuous, and decreasing for $x \geq 2$.

$$\int_2^\infty \frac{dx}{x(\ln x)^2} = \left[-\frac{1}{\ln x}\right]_2^\infty = 0 + \frac{1}{\ln 2} \approx 1.443 < \infty.$$

Since the integral converges, $\displaystyle\sum_{n=2}^\infty \frac{1}{n(\ln n)^2}$ **converges by the integral test.**

(Ratio test gives limit 1, inconclusive. It is not a simple p-series.)

Q12. **Answer: (A)**

Solution

$$\int_1^4 \sqrt{x}\,dx = \left[\tfrac{2}{3}x^{3/2}\right]_1^4 = \tfrac{2}{3}(8-1) = \tfrac{14}{3}.$$

MVT for integrals: $f(c)(4-1) = \frac{14}{3}$, so $\sqrt{c} = f(c) = \dfrac{14}{9}$.

Thus $c = \left(\frac{14}{9}\right)^2 = \frac{196}{81} \approx 2.42 \in (1,4)$. ✓

Q13. **Answer: (D)**

Solution

$A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$ is upper triangular; its eigenvalues are the diagonal entries: $\lambda_1 = 2$, $\lambda_2 = 3$.

Since the eigenvalues are *distinct*, A is **diagonalizable**.

Eigenvectors: for $\lambda_1 = 2$: $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$; for $\lambda_2 = 3$: $\mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

Q14. **Answer: (C)**

Solution

The equation $dy/dx + y = y^2e^x$ is Bernoulli with $n = 2$. The standard substitution is $v = y^{1-n} = y^{-1}$.
Then $dv/dx = -y^{-2}(dy/dx)$. Divide the ODE by y^2:

$$y^{-2}\frac{dy}{dx} + y^{-1} = e^x \Rightarrow -\frac{dv}{dx} + v = e^x \Rightarrow \frac{dv}{dx} - v = -e^x.$$

This is a **linear** first-order ODE in v. ✓

Q15. **Answer: (B)**

Solution

For $X \sim \text{Gamma}(2,1)$ with PDF $f(x) = xe^{-x}$, integrate by parts ($u = x$, $dv = e^{-x}dx$):

$$P(X < 2) = \int_0^2 xe^{-x}\,dx = \Big[-(x+1)e^{-x}\Big]_0^2 = -(3e^{-2}) + 1 = \mathbf{1 - 3e^{-2}}.$$

Numerically: $1 - 3(0.1353) \approx 0.594$. ✓

Q16. **Answer: (A)**

Solution

For iid $N(\mu, \sigma^2)$ with σ^2 known, $T = \sum X_i$ is a complete sufficient statistic for μ. $\bar{X} = T/n$ is an unbiased function of T. By the **Lehmann–Scheffé theorem**, $\bar{X}$ is the UMVUE for μ. It also attains the Cramér–Rao lower bound σ^2/n.

Q17. **Answer: (D)**

Solution

For the **bivariate normal** distribution, uncorrelatedness ($\rho = 0$) implies **independence** – this is a special property of the normal family. For general bivariate distributions, $\rho = 0$ only guarantees uncorrelatedness, not independence.
The circular contours in the figure confirm $\sigma_X = \sigma_Y$ and $\rho = 0$, making the joint

PDF a product of two independent standard normals (up to scaling).

Q18. **Answer: (C)**

Solution

By **Wilks' theorem**: under H_0 and mild regularity conditions, $-2\ln\Lambda \xrightarrow{d} \chi^2(d)$ as $n \to \infty$, where d equals the number of parameters constrained under H_0. For a one-parameter model, $d = 1$, so $-2\ln\Lambda \xrightarrow{d} \chi^2(\mathbf{1})$.

Q19. **Answer: (B)**

Solution

Maximising the log-likelihood of $N(\mu, \sigma^2)$ with respect to σ^2 yields:

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2 \quad \text{(biased MLE)}.$$

For the sample $\{2, 4, 6, 8, 10\}$: $\bar{X} = 6$, $\hat{\sigma}^2 = \frac{1}{5}(16 + 4 + 0 + 4 + 16) = 8$. The unbiased estimator uses $n - 1 = 4$, giving $S^2 = 10 \neq \hat{\sigma}^2$.

Q20. **Answer: (A)**

Solution

Under $H_0 : \sigma_1^2 = \sigma_2^2$, the ratio of sample variances follows:

$$F = \frac{S_1^2}{S_2^2} \sim F(n_1 - 1,\, n_2 - 1),$$

where $S_k^2 = \sum(X_{ki} - \bar{X}_k)^2/(n_k - 1)$ are the unbiased sample variances. The two-sided critical region (upper and lower tails) is used for $H_1 : \sigma_1^2 \neq \sigma_2^2$.

Q21. **Answer: (D)**

Solution

In spherical coordinates ($x = r\sin\phi\cos\theta$, $y = r\sin\phi\sin\theta$, $z = r\cos\phi$):

$$\iiint_{x^2+y^2+z^2\le 1} dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{1} r^2\sin\phi\, dr\, d\phi\, d\theta = 2\pi \cdot 2 \cdot \frac{1}{3} = \frac{4\pi}{3}.$$

Q22. Answer: (C)

Solution

The moment generating function of $X \sim N(0,1)$ is $M_X(t) = e^{t^2/2}$. Setting $t = 1$:

$$E[e^X] = M_X(1) = e^{1/2}.$$

Verification via Jensen: $E[e^X] \geq e^{E[X]} = e^0 = 1$. Indeed $e^{1/2} \approx 1.649 > 1$. ✓

Q23. Answer: (B)

Solution

$$\mathrm{Cov}(X,Y) = E[XY] - E[X]\,E[Y] = 10 - (2)(3) = 10 - 6 = \mathbf{4}.$$

Q24. Answer: (A)

Solution

$$R^2 = 1 - \frac{\mathrm{SSE}}{\mathrm{SST}} = 1 - \frac{36}{100} = 1 - 0.36 = \mathbf{0.64}.$$

Equivalently, $\mathrm{SSR} = \mathrm{SST} - \mathrm{SSE} = 100 - 36 = 64$, so $R^2 = 64/100 = 0.64$.

Q25. Answer: (D)

Solution

The **Central Limit Theorem** states: if $X_1, \ldots, X_n$ are iid with mean μ and finite variance σ^2, then

$$\frac{\sqrt{n}(\bar{X} - \mu)}{\sigma} \xrightarrow{d} N(0,1) \quad \text{as } n \to \infty.$$

Option (A) is wrong (limit is $N(0,1)$ not $N(\mu, \sigma^2)$); (B) wrong (limit is normal); (C) is missing the standardisation.

Q26. Answer: (C)

Solution

The $100(1-\alpha)\%$ **prediction interval** for a future Y_0 at x_0 uses:

$$\hat{y}_0 \pm t_{\alpha/2,n-2}\,\hat{\sigma}\sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}},$$

while the **confidence interval** for $E[Y \mid x_0]$ uses $\sqrt{\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{S_{xx}}}$ (without the $+1$ term).
The extra 1 under the radical accounts for individual response variability, making the prediction interval **strictly wider**.

Q27. Answer: (B)

Solution

The likelihood for iid $\text{Exp}(\theta)$ is $L(\theta) = \theta^n e^{-\theta \sum x_i}$.
By the **Factorisation theorem**, $T = \sum_{i=1}^n X_i$ is sufficient for θ.
The exponential family has a complete sufficient statistic; hence $T = \sum X_i$ is **complete sufficient**.
Note: $\bar{X} = T/n$ is also sufficient (one-to-one function of T), but the question asks for the canonical form.

Q28. Answer: (A)

Solution

For iid $X_1, \dots, X_n$ with variance σ^2:

$$\text{Var}(\bar{X}) = \frac{\sigma^2}{n} \;\Rightarrow\; \text{SE}(\bar{X}) = \frac{\sigma}{\sqrt{n}}.$$

As shown in the figure, the sampling distribution $N(\mu, \sigma^2/n)$ is narrower than the population $N(\mu, \sigma^2)$ by a factor of $\sqrt{n}$.

Q29. Answer: (D)

Solution

For a $r \times c$ contingency table, degrees of freedom $= (r-1)(c-1)$.
For 2×3: $\text{df} = (2-1)(3-1) = 1 \times 2 = \mathbf{2}$.

Q30. Answer: (C)

Solution

For iid $N(\mu, \sigma^2)$, the sample median M_n satisfies:

$$\sqrt{n}(M_n - \mu) \xrightarrow{d} N\left(0, \frac{1}{4[f(\mu)]^2}\right),$$

where $f(\mu) = \dfrac{1}{\sigma\sqrt{2\pi}}$ is the PDF at the population median μ.

$$\frac{1}{4[f(\mu)]^2} = \frac{1}{4 \cdot \dfrac{1}{2\pi\sigma^2}} = \frac{2\pi\sigma^2}{4} = \frac{\pi\sigma^2}{2}.$$

Hence $n\,\mathrm{Var}(M_n) \to \dfrac{\pi\sigma^2}{2}$. (Compare: for $\bar{X}$, the limit is σ^2; the median is $\pi/2 \approx 1.57$ times less efficient.)

Q31. Answer: (ACD)

Solution

(A) Correct. For a twice-differentiable f: $f''(x) \geq 0$ for all $x \Leftrightarrow f$ is convex. (By Taylor's theorem, the secant lies above the curve.)

(B) Incorrect. $f(x) = |x|$ is convex on $\mathbb{R}$ but is not differentiable at $x = 0$.

(C) Correct. $\dfrac{d^2}{dx^2}e^x = e^x > 0$ for all x, so e^x is **strictly** convex. ✓

(D) Correct. Jensen's inequality: for any convex f and any random variable X with finite mean, $f(E[X]) \leq E[f(X)]$. This follows directly from the definition of convexity. ✓

Q32. Answer: (BD)

Solution

(A) Incorrect. Inter-arrival times are $\mathrm{Exp}(\lambda)$ with *rate* λ (mean $1/\lambda$), *not* rate $1/\lambda$.

(B) Correct. $T_1, T_2, \ldots \overset{iid}{\sim} \mathrm{Exp}(\lambda)$ with mean $1/\lambda$. ✓

(C) Incorrect. $P(N(t) = 0) = e^{-\lambda t}$, not $t\,e^{-\lambda t}$ (the Poisson PMF at $k = 0$).

(D) Correct. $E[N(t)] = \lambda t$ by linearity and the definition of the Poisson process. ✓

Q33. Answer: (ABD)

Solution

(A) Correct. A one-to-one transformation of a sufficient statistic contains identical information, so $g(T)$ is also sufficient. ✓

(B) Correct. This is the Neyman–Fisher Factorisation theorem. ✓

(C) Incorrect. Not every statistic is sufficient; X_1 alone (ignoring $X_2, \ldots, X_n$) is generally not sufficient.

(D) Correct. For iid $N(\mu, 1)$: $L(\mu) \propto \exp\big(-n(\bar{x} - \mu)^2/2\big)$, which depends on the data only through $\bar{X}$, so $\bar{X}$ is sufficient by factorisation. ✓

Q34. **Answer: (AC)**

Solution

(A) Correct. $P(2X > t) = P(X > t/2) = e^{-t/2}$, so $2X \sim \text{Exp}(1/2)$ (rate $1/2$, mean 2). ✓

(B) Incorrect. $X \sim N(0, 1) \Rightarrow X^2 \sim \chi^2(\mathbf{1})$, not $\chi^2(2)$.

(C) Correct. For $X \sim U(0, 1)$: $P(-\ln X > t) = P(X < e^{-t}) = e^{-t}$, confirming $-\ln X \sim \text{Exp}(1)$. ✓

(D) Incorrect. $\text{Gamma}(2, \text{rate} = 1)$ (mean $= 2$) is the sum of two $\text{Exp}(\text{rate} = 1)$ variables, *not* $\text{Exp}(\text{rate} = 2)$ (which has mean $1/2$).

Q35. **Answer: (BCD)**

Solution

(A) Incorrect. Size (Type I error rate) α and power $1 - \beta$ are distinct concepts; size $\neq 1-$power in general.

(B) Correct. A UMP test maximises $\beta(\theta)$ for every $\theta \in \Theta_1$ simultaneously. ✓

(C) Correct. The Neyman–Pearson lemma: reject H_0 when $L(\theta_1)/L(\theta_0) > k$; this gives the most powerful level-α test. ✓

(D) Correct. The p-value is the smallest level at which H_0 is rejected, so $p \leq \alpha \Leftrightarrow$ test rejects at level α. ✓

Q36. **Answer: (AC)**

Solution

(A) Correct. The first normal equation $\partial\text{SSE}/\partial\beta_0 = 0$ gives $\sum_{i=1}^{n} \hat{e}_i = 0$ whenever an intercept is included. ✓

(B) Incorrect. OLS estimators are normally distributed *only if* the error terms are normally distributed (or, asymptotically, by the CLT).

(C) Correct. $R^2 = \text{SSR}/\text{SST} = 1 - \text{SSE}/\text{SST}$ by definition. ✓

(D) Incorrect. The correct formula is $\hat{\beta}_1 = S_{xy}/S_{xx} = \sum(x_i - \bar{x})(y_i - \bar{y})/\sum(x_i - \bar{x})^2$, not S_{xy}/S_{yy}.

Q37. **Answer: (ABD)**

Solution

(A) Correct. The normal PDF $f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}$ is symmetric about μ. ✓

(B) Correct. The empirical rule (68-95-99.7): $P(\mu - 2\sigma < X < \mu + 2\sigma) \approx 0.9545$. ✓

(C) Incorrect. The normal distribution is symmetric, so its skewness is 0, not positive.

(D) Correct. Linear combinations of independent normals are normal (closure under linear operations). ✓

Q38. **Answer: (BC)**

Solution

(A) Incorrect. $(AB)^\top = B^\top A^\top$ (order reverses upon transposition).

(B) Correct. Multiplicativity of the determinant: $\det(AB) = \det(A)\det(B)$. ✓

(C) Correct. $\det(A^\top) = \det(A)$ (transposition does not change the determinant). ✓

(D) Incorrect. If all eigenvalues are nonzero then $\det(A) = \prod_i \lambda_i \neq 0$; the matrix is invertible (non-singular).

Q39. **Answer: (ACD)**

Solution

(A) Correct. This is the unique property of the bivariate normal: zero correlation $\Leftrightarrow$ independence. ✓

(B) Incorrect. In general, $\text{Cov}(X, Y) = 0$ implies only uncorrelatedness. E.g., $X \sim N(0, 1)$, $Y = X^2$: $\text{Cov} = 0$ but Y is a deterministic function of X.

(C) Correct. Definition of Pearson correlation: $\rho = \text{Cov}(X, Y)/(\sigma_X \sigma_Y)$. ✓

(D) Correct. By Sklar's theorem, many copulas can couple the same marginals, so marginals alone do not determine the joint distribution. ✓

Q40. **Answer: (BD)**

Solution

(A) Incorrect. In frequentist inference, μ is a fixed (unknown) constant. A 95% CI means that if the procedure were repeated many times, 95% of resulting intervals would contain μ. It is not a probability statement about a fixed μ.

(B) Correct. A wider interval spans more values of the parameter, providing less precision. ✓

(C) Incorrect. Increasing n narrows the interval (reduces SE) but does *not* change the confidence level, which is set by the practitioner.

(D) Correct. A higher confidence level requires a larger $z_{\alpha/2}$ (e.g., $z_{0.005} = 2.576 > z_{0.025} = 1.96$), widening the interval. ✓

Q41. **Answer: (3)**

Solution

$X \sim \text{Gamma}(\alpha = 3, \beta = 2)$ with rate parameterisation: $E[X] = 3/2$, $\text{Var}(X) = 3/4$.

$$E[X^2] = \text{Var}(X) + (E[X])^2 = \frac{3}{4} + \left(\frac{3}{2}\right)^2 = \frac{3}{4} + \frac{9}{4} = \frac{12}{4} = \mathbf{3}.$$

Q42. **Answer: (0.1587)**

Solution

$X \sim N(5, 4)$, $\sigma = 2$. Standardise:

$$P(X > 7) = P\left(Z > \frac{7-5}{2}\right) = P(Z > 1) = 1 - \Phi(1) \approx 1 - 0.8413 = \mathbf{0.1587}.$$

Q43. **Answer: (12)**

Solution

A is upper triangular; its determinant equals the product of diagonal entries:

$$\det(A) = 2 \times 2 \times 3 = \mathbf{12}.$$

Q44. **Answer: (0.42)**

Solution

Law of Total Probability:

$$P(B) = P(B \mid A)P(A) + P(B \mid A^c)P(A^c) = 0.6 \times 0.4 + 0.3 \times 0.6 = 0.24 + 0.18 = \mathbf{0.42}.$$

Q45. **Answer: (1.22)**

Solution

$\sigma = 2$, $n = 6$. Test statistic:

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{8-7}{2/\sqrt{6}} = \frac{\sqrt{6}}{2} = \frac{2.4495}{2} \approx \mathbf{1.22}.$$

Q46. **Answer: (0.347)**

Solution

For $X \sim \text{Exp}(2)$, the CDF is $F(x) = 1 - e^{-2x}$. The median m satisfies $F(m) = 0.5$:

$$1 - e^{-2m} = 0.5 \;\Rightarrow\; e^{-2m} = 0.5 \;\Rightarrow\; 2m = \ln 2 \;\Rightarrow\; m = \frac{\ln 2}{2} \approx \frac{0.6931}{2} \approx \mathbf{0.347}.$$

Q47. **Answer: (0.3679)**

Solution

Recognise the Maclaurin series $e^x = \displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$. Setting $x = -1$:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} = e^{-1} \approx \mathbf{0.3679}.$$

Q48. **Answer: (0.177)**

Solution

$X \sim \text{Bin}(20, 0.5)$: $\mu = 10$, $\sigma^2 = 5$, $\sigma = \sqrt{5} \approx 2.236$.
Normal approximation with continuity correction:

$$P(X = 10) \approx P(9.5 < Y < 10.5), \quad Y \sim N(10, 5).$$

$$= P\left(\frac{-0.5}{2.236} < Z < \frac{0.5}{2.236}\right) = P(-0.224 < Z < 0.224) \approx 2\Phi(0.224)-1 \approx 2(0.5887)-1 = \mathbf{0.177}.$$

Q49. **Answer: (13)**

Solution

X and Y are independent, so:

$$\text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) = 9 + 4 = \mathbf{13}.$$

Q50. **Answer: (12.70)**

Solution

Characteristic equation: $r^2 - 5r + 6 = 0 \Rightarrow (r-2)(r-3) = 0 \Rightarrow r = 2, 3$.
General solution: $y = C_1 e^{2x} + C_2 e^{3x}$.
Apply ICs: $y(0) = C_1 + C_2 = 0$ and $y'(0) = 2C_1 + 3C_2 = 1$.
Solving: $C_2 = 1$, $C_1 = -1$. Hence $y = e^{3x} - e^{2x}$.
$y(1) = e^3 - e^2 \approx 20.086 - 7.389 = \mathbf{12.70}$.

Q51. **Answer: (5.152)**

Solution

$\sigma = 5$, $n = 25$, so SE $= \sigma/\sqrt{n} = 5/5 = 1$.
The 99% CI for μ: $\bar{x} \pm z_{0.005} \cdot \text{SE} = 32 \pm 2.576 \times 1$.
Full width $= 2 \times 2.576 \times 1 = \mathbf{5.152}$.

Q52. **Answer: (21)**

Solution

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_0 = 1 + 2 \times 10 = \mathbf{21}$.

Q53. **Answer: (2.22)**

Solution

MLE: $\hat{\lambda} = n/\sum X_i = 10/6 = 5/3$.
Log-likelihoods: $\ln L(\lambda) = n \ln \lambda - \lambda \sum x_i$.
$\ln L(1) = 10 \ln 1 - 1 \cdot 6 = 0 - 6 = -6$.

$\ln L(5/3) = 10\ln(5/3) - (5/3)\cdot 6 = 10(0.5108) - 10 = 5.108 - 10 = -4.892.$

$$-2\ln\Lambda = -2\big[\ln L(\lambda_0) - \ln L(\hat{\lambda})\big] = -2(-6+4.892) = -2(-1.108) = \mathbf{2.22}.$$

Q54. **Answer: (0.05)**

Solution

$\chi^2_{0.05}(4) = 9.488$ is the 95th percentile: $P(X \le 9.488) = 0.95$.
Therefore $P(X > 9.488) = 1 - 0.95 = \mathbf{0.05}$.

Q55. **Answer: (10)**

Solution

$$A^2 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1\cdot 1+2\cdot 3 & 1\cdot 2+2\cdot 4 \\ 3\cdot 1+4\cdot 3 & 3\cdot 2+4\cdot 4 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix}.$$

The $(1,2)$ entry is **10**.

Q56. **Answer: (1.5)**

Solution

Prior: $\mu \sim N(\mu_0 = 0, \tau_0^2 = 1)$. Likelihood: $X \mid \mu \sim N(\mu, \sigma^2 = 1)$. Observation $x = 3$.
Posterior precision: $1/\tau_0^2 + n/\sigma^2 = 1 + 1 = 2$, so posterior variance $= 1/2$.
Posterior mean:

$$\frac{\mu_0/\tau_0^2 + x/\sigma^2}{1/\tau_0^2 + 1/\sigma^2} = \frac{0/1 + 3/1}{1+1} = \frac{3}{2} = \mathbf{1.5}.$$

Q57. **Answer: (0.328)**

Solution

Rejection region $\{X = 5\}$. Power at $p = 0.8$:

$$\beta(0.8) = P(X = 5 \mid p = 0.8) = \binom{5}{5}(0.8)^5(0.2)^0 = 0.8^5 = 0.32768 \approx \mathbf{0.328}.$$

Q58. **Answer: (0.774)**

Solution

$X \sim \text{Bin}(400, 0.25)$: $\mu = 100$, $\sigma^2 = 75$, $\sigma = \sqrt{75} \approx 8.660$.
Normal approximation with continuity correction:

$$P(90 \leq X \leq 110) \approx P\left(\frac{89.5 - 100}{8.660} \leq Z \leq \frac{110.5 - 100}{8.660}\right) = P(-1.212 \leq Z \leq 1.212).$$

$\approx 2\Phi(1.21) - 1 \approx 2(0.8869) - 1 = \mathbf{0.774}$.

Q59. $\boxed{\textbf{Answer: (10)}}$

Solution

$T = (3X_1 + X_2)/4$. Since $E[X_i] = \mu$: $E[T] = (3\mu + \mu)/4 = \mu$, so **Bias**$= 0$.

$$\text{Var}(T) = \frac{9\,\text{Var}(X_1) + \text{Var}(X_2)}{16} = \frac{(9+1)\sigma^2}{16} = \frac{10 \times 16}{16} = 10.$$

$$\text{MSE}(T) = \text{Var}(T) + \text{Bias}^2 = 10 + 0 = \mathbf{10}.$$

Q60. $\boxed{\textbf{Answer: (12)}}$

Solution

For f to be a valid PDF:

$$\int_0^1 cx(1-x)^2\,dx = 1.$$

$$\int_0^1 x(1-x)^2\,dx = \int_0^1 (x - 2x^2 + x^3)\,dx = \frac{1}{2} - \frac{2}{3} + \frac{1}{4} = \frac{6 - 8 + 3}{12} = \frac{1}{12}.$$

Therefore $c \times \dfrac{1}{12} = 1 \Rightarrow c = \mathbf{12}$.

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	D	3	A	4	B	5	C
6	D	7	A	8	B	9	D	10	C
11	B	12	A	13	D	14	C	15	B
16	A	17	D	18	C	19	B	20	A
21	D	22	C	23	B	24	A	25	D
26	C	27	B	28	A	29	D	30	C
31	ACD	32	BD	33	ABD	34	AC	35	BCD
36	AC	37	ABD	38	BC	39	ACD	40	BD
41	3	42	0.1587	43	12	44	0.42	45	1.22
46	0.347	47	0.3679	48	0.177	49	13	50	12.70
51	5.152	52	21	53	2.22	54	0.05	55	10
56	1.5	57	0.328	58	0.774	59	10	60	12