GUJCET Mathematics

Sample Paper – 1

Duration: 60 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **GUJCET** entrance.
- Each correct answer carries **+1 mark**. There is a **negative marking of** -0.25 **mark** for each incorrect answer. Unattempted questions carry no penalty.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 12 NCERT (80%) + Class 11 NCERT/GSEB (20%) — Calculus, Algebra, Vectors, 3D Geometry, Probability, Coordinate Geometry, and more.**
- Use of mobile phones, calculators, log tables, or any electronic gadgets is strictly prohibited. Rough work may be done in the space provided on the answer sheet.

Q1. Let $A = \{1, 2, 3\}$ and let $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$ be a relation on A. Then R is:

(A) Symmetric and transitive but not reflexive

(B) Reflexive and symmetric but not transitive

(C) Reflexive and transitive but not symmetric

(D) An equivalence relation

Q2. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 2x + 3$ is:

(A) One-one but not onto

(B) Onto but not one-one

(C) Both one-one and onto (bijective)

(D) Neither one-one nor onto

Q3. The value of $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$ is:

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Q4. Using the identity $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ (when $xy < 1$), the value of $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$ is:

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{4}$

Q5. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then AB equals:

(A) $\begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$

Q6. For any two matrices A and B for which AB is defined, which of the following is **always** true?

(A) $(AB)^T = B^T A^T$

(B) $(AB)^T = A^T B^T$

(C) $(AB)^T = AB^T$

(D) $(AB)^T = A^T B$

Q7. The value of the determinant $\begin{vmatrix} 2 & -3 & 1 \\ 1 & 2 & -1 \\ 3 & 1 & 2 \end{vmatrix}$ is:

(A) 10

(B) 15

(C) 20

(D) 25

Q8. The inverse of the matrix $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ is:

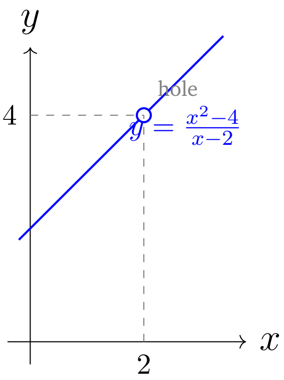
(A) $\begin{pmatrix} 2 & 1 \\ -5 & 3 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & -1 \\ 5 & 3 \end{pmatrix}$

(C) $\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$

(D) $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$

Q9. $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$ is equal to:



(A) 2

(B) 4

(C) 0

(D) Does not exist

Q10. $\lim_{x \to \infty} \dfrac{3x^2 + 2x + 1}{x^2 + 5x - 2}$ equals:

(A) 3

(B) 2

(C) 1

(D) ∞

Q11. The function $f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ is continuous at $x = 3$ if k equals:

(A) 3

(B) 9

(C) 6

(D) 0

Q12. If $y = \sin(x^2)$, then $\dfrac{dy}{dx}$ equals:

(A) $\cos(x^2)$

(B) $2x\cos(x)$

(C) $\sin(2x)$

(D) $2x\cos(x^2)$

Q13. If $x^2 + y^2 = 25$, then $\dfrac{dy}{dx}$ equals:

(A) $-\dfrac{x}{y}$

(B) $\dfrac{x}{y}$

(C) $-\dfrac{y}{x}$

(D) $\dfrac{y}{x}$

Q14. If $y = x^x$ (where $x > 0$), then $\dfrac{dy}{dx}$ equals:

(A) $x^x \ln x$

(B) $x^x(1 + \ln x)$

(C) x^{x-1}

(D) $\dfrac{x^x}{x}$

Q15. If $y = x^3 - 3x^2 + 2x$, then $\dfrac{d^2y}{dx^2}$ equals:

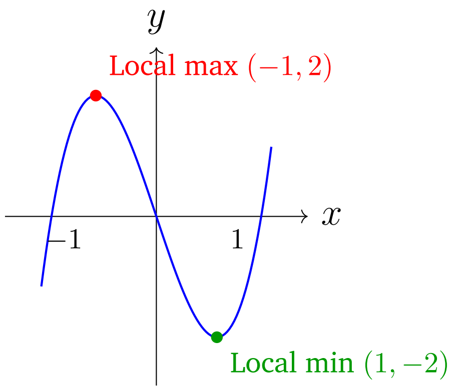
(A) $6x^2 - 6$

(B) $3x - 6$

(C) $6x - 6$

(D) $6x^2 - 3$

Q16. For the function $f(x) = x^3 - 3x$, the local minimum value is:



(A) 2

(B) -1

(C) 1

(D) -2

Q17. The function $f(x) = x^2 - 4x + 3$ is strictly **decreasing** on:

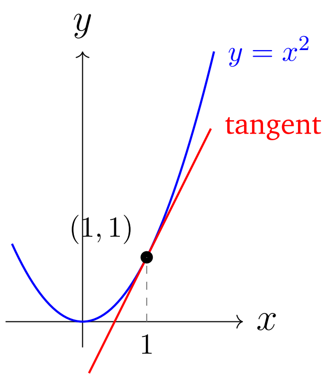
(A) $(-\infty,\ 2)$

(B) $(2,\ \infty)$

(C) $(-\infty,\ -2)$

(D) $(-2,\ 2)$

Q18. The equation of the tangent to the curve $y = x^2$ at the point $(1, 1)$ is:



(A) $y = 2x + 1$

(B) $y = 2x - 1$

(C) $y = x + 1$

(D) $y = x - 1$

Q19. $\int x^5\, dx$ equals:

(A) $5x^4 + C$

(B) $x^6 + C$

(C) $\frac{x^6}{6} + C$

(D) $\frac{x^4}{4} + C$

Q20. $\int x \sin x\, dx$ equals:

(A) $x\cos x + \sin x + C$

(B) $-x\cos x - \sin x + C$

(C) $x\cos x - \sin x + C$

(D) $-x\cos x + \sin x + C$

Q21. $\int \frac{1}{(x+1)(x+2)}\, dx$ equals:

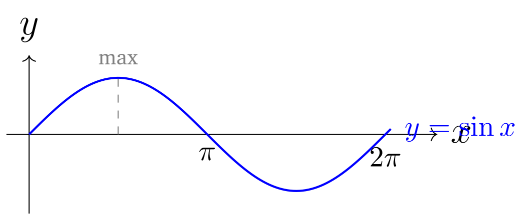
(A) $\ln\left|\frac{x+1}{x+2}\right| + C$

(B) $\ln\left|\frac{x+2}{x+1}\right| + C$

(C) $\ln|(x+1)(x+2)| + C$

(D) $\frac{-1}{(x+1)(x+2)} + C$

Q22. $\int \sin^2 x\, dx$ equals (recall $\sin^2 x = \frac{1-\cos 2x}{2}$):



(A) $-\dfrac{\cos^2 x}{2} + C$

(B) $\dfrac{x}{2} - \dfrac{\sin 2x}{4} + C$

(C) $-\dfrac{\sin 2x}{2} + C$

(D) $x - \dfrac{\sin 2x}{2} + C$

Q23. Using the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, the value of $\int_0^\pi x \sin x\,dx$ is:

(A) 0

(B) $\dfrac{\pi}{2}$

(C) π

(D) 2π

Q24. The value of $\int_0^1 (x^2 + 1)\,dx$ is:

(A) $\dfrac{2}{3}$

(B) 1

(C) $\dfrac{1}{3}$

(D) $\dfrac{4}{3}$

Q25. The value of $\int_{-1}^1 |x|\,dx$ is:

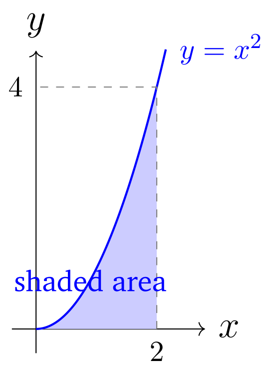
(A) 1

(B) 0

(C) 2

(D) $\dfrac{1}{2}$

Q26. The area of the region bounded by $y = x^2$, the x-axis, $x = 0$ and $x = 2$ is:



(A) $\frac{4}{3}$ sq. units

(B) $\frac{8}{3}$ sq. units

(C) 2 sq. units

(D) 4 sq. units

Q27. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + y = 0$ are respectively:

(A) Order 3, Degree 1

(B) Order 2, Degree 3

(C) Order 2, Degree 1

(D) Order 1, Degree 2

Q28. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is:

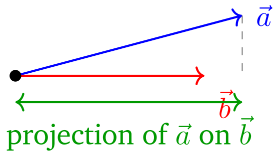
(A) $y = Cx^2$

(B) $y = \frac{C}{x}$

(C) $y = e^x + C$

(D) $y = Cx$

Q29. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$, the projection of $\vec{a}$ on $\vec{b}$ is:



(A) $\dfrac{-1}{\sqrt{6}}$

(B) $\dfrac{1}{\sqrt{6}}$

(C) $-\sqrt{6}$

(D) $\sqrt{6}$

Q30. If $\vec{a}$ and $\vec{b}$ are unit vectors and the angle between them is $\dfrac{\pi}{3}$, then $|\vec{a} \times \vec{b}|$ equals:

(A) $\dfrac{1}{2}$

(B) $\dfrac{\sqrt{3}}{2}$

(C) 1

(D) $\sqrt{3}$

Q31. If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$, the unit vector along $\vec{a} + \vec{b}$ is:

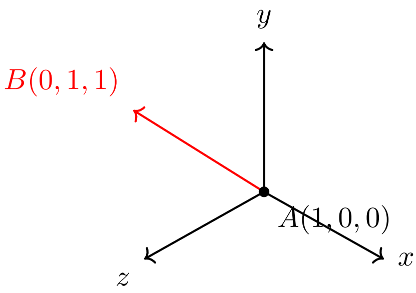
(A) $\dfrac{3\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{27}}$

(B) $\dfrac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$

(C) $\dfrac{3\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{26}}$

(D) $\dfrac{3\hat{i} - \hat{j} + 4\hat{k}}{\sqrt{26}}$

Q32. The direction cosines of the line joining $A(1, 0, 0)$ and $B(0, 1, 1)$ are:



(A) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(B) $\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(C) $\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(D) $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Q33. The midpoint of the line segment joining $A(2, 4, -1)$ and $B(4, 0, 3)$ is:

(A) $(3,\ 2,\ 1)$

(B) $(2,\ 3,\ 1)$

(C) $(3,\ 1,\ 2)$

(D) $(1,\ 2,\ 3)$

Q34. The Cartesian equation of the line passing through the points $(1, 2, 3)$ and $(3, 4, 5)$ is:

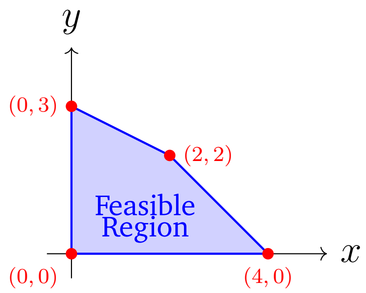
(A) $\frac{x+1}{2} = \frac{y+2}{2} = \frac{z+3}{2}$

(B) $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$

(C) $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{3}$

(D) $\frac{x-3}{1} = \frac{y-4}{1} = \frac{z-5}{2}$

Q35. Maximise $Z = 3x + 2y$ subject to the constraints $x + y \leq 4$, $x + 2y \leq 6$, $x \geq 0$, $y \geq 0$. The maximum value of Z is:



(A) 10

(B) 6

(C) 12

(D) 8

Q36. If $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$, then $P(A \mid B)$ equals:

(A) 0.2

(B) 0.5

(C) 0.8

(D) 0.4

Q37. Bag I contains 3 red and 2 white balls; Bag II contains 2 red and 3 white balls. A bag is chosen at random and one ball is drawn. Given that the ball is white, the probability that it came from Bag II is:

(A) $\frac{3}{5}$

(B) $\frac{2}{5}$

(C) $\frac{1}{2}$

(D) $\frac{3}{10}$

Q38. A fair coin is tossed 5 times. The probability of getting exactly 3 heads is:

(A) $\frac{1}{8}$

(B) $\frac{5}{16}$

(C) $\frac{3}{8}$

(D) $\frac{1}{4}$

Q39. The focus of the parabola $y^2 = 12x$ is:

(A) $(0,\ 3)$

(B) $(0,\ -3)$

(C) $(3,\ 0)$

(D) $(-3,\ 0)$

Q40. The sum of the first 6 terms of a geometric progression with first term $a = 2$ and common ratio $r = 3$ is:

(A) 486

(B) 729

(C) 364

(D) 728

Detailed Solutions

Q1.

Solution

Concept: A relation R on A is *reflexive* if $(a, a) \in R$ for all $a \in A$; *symmetric* if $(a, b) \in R \Rightarrow (b, a) \in R$; *transitive* if $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.

Step 1 — Reflexive check: $(1, 1), (2, 2), (3, 3) \in R$. ✓ Reflexive.

Step 2 — Symmetric check: $(1, 2) \in R$ and $(2, 1) \in R$; $(2, 3) \in R$ and $(3, 2) \in R$. ✓ Symmetric.

Step 3 — Transitive check: $(1, 2) \in R$ and $(2, 3) \in R$, but $(1, 3) \notin R$. $\times$ **Not transitive.**

Why other options are wrong: (A) says not reflexive — false. (C) says not symmetric — false. (D) requires all three properties; since transitivity fails, R is not an equivalence relation.

Answer: (B) Go Back to Q 1

Q2.

Solution

Concept: f is *one-one* if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$; *onto* if every $y \in \mathbb{R}$ has a pre-image.

Step 1 — One-one: Suppose $f(x_1) = f(x_2)$, i.e. $2x_1 + 3 = 2x_2 + 3$. Then $x_1 = x_2$. ✓

Step 2 — Onto: For any $y \in \mathbb{R}$, set $x = \dfrac{y-3}{2} \in \mathbb{R}$. Then $f(x) = 2 \cdot \dfrac{y-3}{2} + 3 = y$. ✓

Conclusion: f is both one-one and onto, hence bijective.

Why other options are wrong: (A) and (B) claim only one property holds; (D) claims neither holds — all contradict the calculations above.

Answer: (C) Go Back to Q 2

Q3.

Solution

Concept: Standard identity: $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$ for all $x \in [-1, 1]$.

Step 1: Apply the identity with $x = \frac{1}{2}$ (which lies in $[-1, 1]$):

$$\sin^{-1}\tfrac{1}{2} + \cos^{-1}\tfrac{1}{2} = \frac{\pi}{2}.$$

Verification: $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$ and $\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$; sum $= \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$. ✓

Why other options are wrong: π (B) is double the correct value; $\pi/3$ (C) is only $\cos^{-1}(1/2)$; $2\pi/3$ (D) is not the sum.

Answer: (A) Go Back to Q 3

Q4.

Solution

Concept: $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ when $xy < 1$.

Step 1: Check condition: $x = \frac{1}{2}$, $y = \frac{1}{3}$; $xy = \frac{1}{6} < 1$. ✓

Step 2: Compute the argument:

$$\frac{x+y}{1-xy} = \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1.$$

Step 3: $\tan^{-1} 1 = \frac{\pi}{4}$.

Why other options are wrong: $\pi/3$ (A) $\Rightarrow \tan(\pi/3) = \sqrt{3} \neq 1$; $\pi/2$ (B) is undefined for $\tan^{-1}$; $\pi/6$ (C) $\Rightarrow \tan(\pi/6) = 1/\sqrt{3} \neq 1$.

Answer: (D) Go Back to Q 4

Q5.

Solution

Concept: Matrix product $(AB)_{ij} = \sum_k A_{ik}B_{kj}$.

Step 1 — First row of AB:

$$(AB)_{11} = 1\cdot 0 + 2\cdot 1 = 2, \quad (AB)_{12} = 1\cdot 1 + 2\cdot 0 = 1.$$

Step 2 — Second row of AB:

$$(AB)_{21} = 3\cdot 0 + 4\cdot 1 = 4, \quad (AB)_{22} = 3\cdot 1 + 4\cdot 0 = 3.$$

Result: $AB = \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}$.

Why other options are wrong: (A) is A itself; (C) is a wrong product; (D) swaps rows.

Answer: (B) Go Back to Q 5

Q6.

Solution

Concept: The reversal law for transposes states $(AB)^T = B^T A^T$ for any matrices A, B for which AB is defined.

Step 1 — Proof sketch: $[(AB)^T]_{ij} = (AB)_{ji} = \sum_k A_{jk}B_{ki} = \sum_k (B^T)_{ik}(A^T)_{kj} = [B^T A^T]_{ij}$. ✓

Why other options are wrong: (B) $A^T B^T$ has dimensions that may not even be compatible; (C) and (D) are not standard identities and fail for non-square matrices.

Answer: (A) Go Back to Q 6

Q7.

Solution

Concept: Expand the 3×3 determinant along the first row using cofactors.

Step 1:

$$\Delta = 2\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} - (-3)\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}.$$

Step 2 — Evaluate each 2×2 determinant:

$$\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix} = 4 + 1 = 5,$$

$$\begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = 2 + 3 = 5,$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1 - 6 = -5.$$

Step 3: $\Delta = 2(5) + 3(5) + 1(-5) = 10 + 15 - 5 = \mathbf{20}$.

Why other options are wrong: 10 (A) ignores the middle and last terms; 15 (B) omits the last term; 25 (D) has an arithmetic error.

Answer: (C) Go Back to Q 7

Q8.

Solution

Concept: For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A^{-1} = \dfrac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Step 1 — Determinant: $\det A = 3 \times 2 - 1 \times 5 = 6 - 5 = 1$.

Step 2 — Adjoint: $\operatorname{adj}(A) = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$.

Step 3: $A^{-1} = \dfrac{1}{1}\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$.

Verification: $AA^{-1} = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. ✓

Why other options are wrong: (A) has $+1$ instead of -1; (B) has $+5$; (C) is the adjoint of a different matrix.

Answer: (D) Go Back to Q 8

Q9.

Solution

Concept: When a limit gives $\frac{0}{0}$, factor and cancel the common factor.

Step 1 — Check the form: At $x = 2$: numerator $= 4 - 4 = 0$, denominator $= 0$. Indeterminate $\frac{0}{0}$.

Step 2 — Factor: $x^2 - 4 = (x-2)(x+2)$, so

$$\frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} = x + 2, \quad x \neq 2.$$

Step 3 — Evaluate: $\lim_{x \to 2}(x+2) = 2 + 2 = \mathbf{4}$.

Why other options are wrong: 2 (A) is just x at $x = 2$ without the $+2$; 0 (C) would require the limit to vanish; "Does not exist" (D) is incorrect since after factoring the limit is well-defined.

Answer: (B) Go Back to Q 9

Q10.

Solution

Concept: For rational functions as $x \to \infty$, divide numerator and denominator by the highest power of x in the denominator.

Step 1: Divide by x^2:

$$\lim_{x\to\infty} \frac{3 + \frac{2}{x} + \frac{1}{x^2}}{1 + \frac{5}{x} - \frac{2}{x^2}}.$$

Step 2: As $x \to \infty$, $\frac{1}{x} \to 0$ and $\frac{1}{x^2} \to 0$, so the limit $= \dfrac{3+0+0}{1+0-0} = \mathbf{3}$.

Why other options are wrong: 2 (B) and 1 (C) do not match the ratio of leading coefficients; ∞ (D) would result only if the degree of numerator exceeded that of denominator.

Answer: (A) Go Back to Q 10

Q11.

Solution

Concept: f is continuous at $x = 3$ iff $\lim_{x\to 3} f(x) = f(3) = k$.

Step 1 — Evaluate the limit: The expression $\dfrac{x^2-9}{x-3}$ is $\frac{0}{0}$ at $x = 3$. Factor:

$$\frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3} = x+3, \quad x \neq 3.$$

Step 2: $\lim_{x\to 3}(x+3) = 3+3 = 6.$

Therefore: $k = 6$.

Why other options are wrong: 3 (A) is x alone at $x = 3$; 9 (B) is $x + 6$; 0 (D) gives a discontinuity.

Answer: (C) Go Back to Q 11

Q12.

Solution

Concept: Chain rule: if $y = f(g(x))$, then $\dfrac{dy}{dx} = f'(g(x)) \cdot g'(x)$.

Step 1: Here $f(u) = \sin u$ and $g(x) = x^2$.

Step 2: $f'(u) = \cos u$ and $g'(x) = 2x$, so

$$\frac{dy}{dx} = \cos(x^2) \cdot 2x = 2x\cos(x^2).$$

Why other options are wrong: (A) forgets the factor $2x$ from the inner derivative; (B) uses $\cos(x)$ instead of $\cos(x^2)$; (C) is the derivative of $-\cos(2x)/2$, which is unrelated.

Answer: (D) Go Back to Q 12

Q13.

Solution

Concept: Implicit differentiation — differentiate both sides with respect to x, treating y as a function of x.

Step 1: $\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(25)$

$$\Rightarrow 2x + 2y\frac{dy}{dx} = 0.$$

Step 2: Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = -\frac{x}{y}.$$

Why other options are wrong: (B) x/y has the wrong sign; (C) $-y/x$ swaps the variables; (D) y/x is doubly wrong.

Answer: (A) Go Back to Q 13

Q14.

Solution

Concept: For $y = x^x$ (a variable base and exponent), take natural logarithm and differentiate implicitly.

Step 1: $\ln y = x \ln x$.

Step 2: Differentiate both sides w.r.t. x:

$$\frac{1}{y}\frac{dy}{dx} = \ln x + x \cdot \frac{1}{x} = \ln x + 1.$$

Step 3: Multiply by $y = x^x$:

$$\frac{dy}{dx} = x^x(1+\ln x).$$

Why other options are wrong: (A) $x^x \ln x$ misses the $+1$; (C) x^{x-1} is the power-rule result valid only for constant exponents; (D) $x^x/x = x^{x-1}$, same error.

Answer: (B) Go Back to Q 14

Q15.

Solution

Concept: Differentiate twice using the power rule.

Step 1 — First derivative:

$$\frac{dy}{dx} = 3x^2 - 6x + 2.$$

Step 2 — Second derivative:

$$\frac{d^2y}{dx^2} = 6x - 6.$$

Why other options are wrong: (A) $6x^2 - 6$ is the first derivative squared — not correct; (B) $3x - 6$ drops a factor of x; (D) $6x^2 - 3$ is a wrong derivative.

Answer: (C) Go Back to Q 15

Q16.

Solution

Concept: Find critical points via $f'(x) = 0$ and use the second derivative test.

Step 1: $f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x-1)(x+1) = 0 \Rightarrow x = 1$ or $x = -1$.

Step 2 — Second derivative test: $f''(x) = 6x$.

- $f''(1) = 6 > 0 \Rightarrow$ local **minimum** at $x = 1$.
- $f''(-1) = -6 < 0 \Rightarrow$ local maximum at $x = -1$.

Step 3: Local minimum value $= f(1) = 1 - 3 = \mathbf{-2}$.

Why other options are wrong: 2 (A) is the local *maximum* value at $x = -1$; -1 (B) and 1 (C) are the x-values of the critical points, not the y-values.

Answer: (D) Go Back to Q 16

Q17.

Solution

Concept: f is strictly decreasing where $f'(x) < 0$.

Step 1: $f'(x) = 2x - 4$.

Step 2: $f'(x) < 0 \Leftrightarrow 2x - 4 < 0 \Leftrightarrow x < 2$, i.e. $x \in (-\infty, 2)$.

Step 3: $f'(x) > 0$ for $x > 2$, so f is increasing on $(2, \infty)$.

Why other options are wrong: (B) $(2, \infty)$ is where f is increasing; (C) $(-\infty, -2)$ confuses the turning point; (D) $(-2, 2)$ is wrong since the vertex is at $x = 2$.

Answer: (A) Go Back to Q 17

Q18.

Solution

Concept: Slope of tangent $= \left.\dfrac{dy}{dx}\right|_{\text{point}}$; equation via point-slope form.

Step 1: $y = x^2 \Rightarrow \dfrac{dy}{dx} = 2x$. At $(1, 1)$: slope $m = 2(1) = 2$.

Step 2 — Tangent equation:

$$y - 1 = 2(x - 1) \Rightarrow y = 2x - 1.$$

Verification: At $x = 1$: $y = 2 - 1 = 1$. ✓

Why other options are wrong: (A) $y = 2x + 1$ has the wrong y-intercept; (C) $y = x + 1$ uses slope 1 instead of 2; (D) $y = x - 1$ uses slope 1 and a different intercept.

Answer: (B) Go Back to Q 18

Q19.

Solution

Concept: Power rule of integration: $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ (for $n \neq -1$).

Step 1: Apply with $n = 5$:

$$\int x^5\,dx = \frac{x^{5+1}}{5+1} + C = \frac{x^6}{6} + C.$$

Why other options are wrong: (A) $5x^4$ is the *derivative* of x^5, not the integral; (B) x^6 forgets the 6 in the denominator; (D) $x^4/4$ decreases the power instead of increasing it.

Answer: (C) Go Back to Q 19

Q20.

Solution

Concept: Integration by parts (IBP): $\int u\,dv = uv - \int v\,du$.

Step 1 — Choose: $u = x$ (algebraic) and $dv = \sin x\,dx$ (trigonometric).

$$\Rightarrow du = dx, \quad v = -\cos x.$$

Step 2 — Apply IBP:

$$\int x\sin x\,dx = x(-\cos x) - \int(-\cos x)\,dx = -x\cos x + \int \cos x\,dx.$$

Step 3:

$$= -x\cos x + \sin x + C.$$

Why other options are wrong: (A) has $+x\cos x$ (sign error in the uv part); (B) subtracts $\sin x$ instead of adding; (C) has $+x\cos x$ with wrong sign.

Answer: (D) Go Back to Q 20

Q21.

Solution

Concept: Partial fractions: write $\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$.

Step 1 — Find A and B:

$$1 = A(x+2) + B(x+1).$$

Set $x = -1$: $1 = A(1) \Rightarrow A = 1$. Set $x = -2$: $1 = B(-1) \Rightarrow B = -1$.

Step 2 — Integrate:

$$\int \frac{dx}{(x+1)(x+2)} = \int \frac{dx}{x+1} - \int \frac{dx}{x+2} = \ln|x+1| - \ln|x+2| + C = \ln\left|\frac{x+1}{x+2}\right| + C.$$

Why other options are wrong: (B) has the fraction inverted; (C) adds the logs instead of subtracting; (D) is not an antiderivative.

Answer: (A) Go Back to Q 21

Q22.

Solution

Concept: Use the half-angle identity $\sin^2 x = \dfrac{1-\cos 2x}{2}$.

Step 1:

$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x\,dx.$$

Step 2:

$$= \frac{x}{2} - \frac{1}{2}\cdot\frac{\sin 2x}{2} + C = \frac{x}{2} - \frac{\sin 2x}{4} + C.$$

Why other options are wrong: (A) $-\cos^2 x/2$ differentiates to $\sin x\cos x = \sin 2x/2 \neq \sin^2 x$; (C) forgets the $x/2$ term; (D) has coefficient errors.

Answer: (B) Go Back to Q 22

Q23.

Solution

Concept: Property: $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

Step 1: Let $I = \displaystyle\int_0^\pi x\sin x\,dx$. Apply the property with $a=\pi$, $f(x) = x\sin x$:

$$I = \int_0^\pi (\pi - x)\sin(\pi - x)\,dx = \int_0^\pi (\pi - x)\sin x\,dx.$$

Step 2 — Add:

$$2I = \int_0^\pi x\sin x\,dx + \int_0^\pi (\pi - x)\sin x\,dx = \pi\int_0^\pi \sin x\,dx.$$

Step 3:

$$2I = \pi\Big[-\cos x\Big]_0^\pi = \pi(-\cos\pi + \cos 0) = \pi(1+1) = 2\pi \Rightarrow I = \pi.$$

Why other options are wrong: 0 (A) would require an odd integrand over a symmetric interval, which is not the case here; $\pi/2$ (B) and 2π (D) are arithmetic errors.

Answer: (C) Go Back to Q 23

Q24.

Solution

Concept: Fundamental Theorem of Calculus: $\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b$.

Step 1:

$$\int_0^1 (x^2+1)\,dx = \left[\frac{x^3}{3}+x\right]_0^1.$$

Step 2:

$$= \left(\frac{1}{3}+1\right) - (0+0) = \frac{1}{3}+\frac{3}{3} = \frac{4}{3}.$$

Why other options are wrong: $2/3$ (A) omits the $+1$ in the integrand; 1 (B) evaluates only $\int_0^1 1\,dx$; $1/3$ (C) evaluates only $\int_0^1 x^2\,dx$.

Answer: (D) Go Back to Q 24

Q25.

Solution

Concept: $|x|$ is an even function, so $\displaystyle\int_{-a}^{a} |x|\,dx = 2\int_0^a x\,dx$.

Step 1: $\displaystyle\int_{-1}^{1} |x|\,dx = 2\int_0^1 x\,dx$.

Step 2: $= 2\left[\frac{x^2}{2}\right]_0^1 = 2\cdot\frac{1}{2} = 1.$

Alternative: Split $\displaystyle\int_{-1}^{0} (-x)\,dx + \int_0^1 x\,dx = \tfrac{1}{2}+\tfrac{1}{2} = 1.$

Why other options are wrong: 0 (B) results from treating $|x|$ as an odd function; 2 (C) doubles the answer; $1/2$ (D) uses only one half of the interval.

Answer: (A) Go Back to Q 25

Q26.

Solution

Concept: Area between $y=f(x)$ and the x-axis from $x=a$ to $x=b$ is $\displaystyle\int_a^b f(x)\,dx$ (when $f(x)\geq 0$).

Step 1: Area $= \displaystyle\int_0^2 x^2\,dx.$

Step 2:

$$= \left[\frac{x^3}{3}\right]_0^2 = \frac{8}{3} - 0 = \frac{8}{3} \text{ sq. units.}$$

Why other options are wrong: $4/3$ (A) corresponds to $\int_0^1 x^2\,dx$; 2 (C) is the length of the interval; 4 (D) is $f(2) = 4$, not the area.

Answer: (B) Go Back to Q 26

Q27.

Solution

Concept: *Order* = highest order derivative present; *Degree* = power of the highest order derivative (when the equation is polynomial in derivatives).

Step 1 — Order: Highest derivative is $\frac{d^2y}{dx^2}$. **Order = 2.**

Step 2 — Degree: The highest order derivative $\frac{d^2y}{dx^2}$ appears with power 1. **Degree = 1.**

(Note: $\left(\frac{dy}{dx}\right)^3$ has order 1, not the highest order.)

Why other options are wrong: (A) mistakes the cube power for order 3; (B) takes the cube power as degree; (D) confuses the exponent of dy/dx with the order.

Answer: (C) Go Back to Q 27

Q28.

Solution

Concept: Variable separable method: separate x and y terms and integrate both sides.

Step 1: $\frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{dy}{y} = \frac{dx}{x}$.

Step 2 — Integrate:

$$\int \frac{dy}{y} = \int \frac{dx}{x} \Rightarrow \ln|y| = \ln|x| + \ln|C|.$$

Step 3:

$$\ln|y| = \ln|Cx| \Rightarrow y = Cx.$$

Why other options are wrong: (A) $y = Cx^2$ would require $dy/dx = 2Cx = 2y/x \neq y/x$; (B) $y = C/x$ gives $dy/dx = -C/x^2 = -y/x \neq y/x$; (C) $e^x + C$ gives

$dy/dx = e^x \neq y/x$.

Answer: (D) Go Back to Q 28

Q29.

Solution

Concept: Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Step 1 — Dot product:

$$\vec{a}\cdot\vec{b} = 2(1) + 1(-1) + (-1)(2) = 2 - 1 - 2 = -1.$$

Step 2 — Magnitude of $\vec{b}$:

$$|\vec{b}| = \sqrt{1^2 + (-1)^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}.$$

Step 3: Projection $= \dfrac{-1}{\sqrt{6}}$.

Why other options are wrong: $+1/\sqrt{6}$ (B) has the wrong sign; $-\sqrt{6}$ (C) is $\vec{a}\cdot\hat{b}\cdot|\vec{b}|$ (wrong formula); $+\sqrt{6}$ (D) has both sign and magnitude errors.

Answer: (A) Go Back to Q 29

Q30.

Solution

Concept: $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$.

Step 1: Both are unit vectors: $|\vec{a}| = |\vec{b}| = 1$. Angle $\theta = \pi/3$.

Step 2:

$$|\vec{a}\times\vec{b}| = 1\cdot 1\cdot \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

Why other options are wrong: $1/2$ (A) is $\sin(\pi/6)$, not $\sin(\pi/3)$; 1 (C) is $\sin(\pi/2)$; $\sqrt{3}$ (D) is $\tan(\pi/3)$.

Answer: (B) Go Back to Q 30

Q31.

Solution

Concept: Unit vector along $\vec{v}$ is $\hat{v} = \dfrac{\vec{v}}{|\vec{v}|}$.

Step 1 — Compute $\vec{a} + \vec{b}$:

$$\vec{a} + \vec{b} = (1+2)\hat{i} + (2-1)\hat{j} + (3+1)\hat{k} = 3\hat{i} + \hat{j} + 4\hat{k}.$$

Step 2 — Magnitude:

$$|\vec{a} + \vec{b}| = \sqrt{3^2 + 1^2 + 4^2} = \sqrt{9 + 1 + 16} = \sqrt{26}.$$

Step 3 — Unit vector:

$$\hat{u} = \frac{3\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{26}}.$$

Why other options are wrong: (A) has $\sqrt{27}$ — wrong denominator; (B) uses $(\hat{i} + \hat{j} + \hat{k})/\sqrt{3}$ — wrong vector; (D) has $-\hat{j}$ from a subtraction error.

Answer: (C) Go Back to Q 31

Q32.

Solution

Concept: Direction cosines of a line $= \dfrac{\text{direction vector}}{|\text{direction vector}|}$ component-wise.

Step 1 — Direction vector:

$$\overrightarrow{AB} = B - A = (0-1,\ 1-0,\ 1-0) = (-1,\ 1,\ 1).$$

Step 2 — Magnitude:

$$|\overrightarrow{AB}| = \sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}.$$

Step 3 — Direction cosines:

$$l = \frac{-1}{\sqrt{3}}, \quad m = \frac{1}{\sqrt{3}}, \quad n = \frac{1}{\sqrt{3}}.$$

Why other options are wrong: (A) takes all positive — wrong sign for l; (B) has m negative; (C) has l positive; only (D) matches.

Answer: (D) Go Back to Q 32

Q33.

Solution

Concept: Midpoint formula in 3D: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.

Step 1:

$$M = \left(\frac{2+4}{2}, \frac{4+0}{2}, \frac{-1+3}{2}\right) = (3,\ 2,\ 1)\,.$$

Why other options are wrong: (B) swaps the x and y coordinates; (C) rearranges the coordinates incorrectly; (D) lists only the components in a different wrong order.

Answer: (A) Go Back to Q 33

Q34.

Solution

Concept: Cartesian form of line through (x_1, y_1, z_1) with direction (a, b, c): $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$.

Step 1 — Direction vector: $(3-1,\ 4-2,\ 5-3) = (2,\ 2,\ 2)$. Simplify to $(1,\ 1,\ 1)$.

Step 2 — Equation (using point $(1, 2, 3)$):

$$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}.$$

Why other options are wrong: (A) uses $(-1, -2, -3)$ as base point — wrong; (C) uses $(2, 2, 3)$ as direction — wrong; (D) uses the second point as base and a wrong direction.

Answer: (B) Go Back to Q 34

Q35.

Solution

Concept: The maximum of the objective function occurs at a corner point of the feasible region.

Step 1 — Corner points:

- $(0, 0)$: $Z = 0$.
- $(4, 0)$: $Z = 3(4) + 2(0) = 12$.
- Intersection of $x + y = 4$ and $x + 2y = 6$: subtract to get $y = 2$, $x = 2$. Point $(2, 2)$: $Z = 3(2) + 2(2) = 10$.

- $(0, 3)$ (from $x + 2y = 6$, $x = 0$): $Z = 2(3) = 6$.

Step 2: Maximum $Z =$ **12** at $(4, 0)$.

Why other options are wrong: 10 (A) is the value at $(2, 2)$; 6 (B) is at $(0, 3)$; 8 (D) is not attained at any corner point.

Answer: (C) Go Back to Q 35

Q36.

Solution

Concept: Conditional probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

Step 1:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5} = 0.4.$$

Why other options are wrong: 0.2 (A) is $P(A \cap B)$ itself; 0.5 (B) is $P(B)$; 0.8 (C) would require $P(A \cap B) = 0.4$, which is not given.

Answer: (D) Go Back to Q 36

Q37.

Solution

Concept: Bayes' theorem: $P(B_i \mid A) = \dfrac{P(A \mid B_i)P(B_i)}{\sum_j P(A \mid B_j)P(B_j)}$.

Step 1 — Prior: $P(\text{Bag I}) = P(\text{Bag II}) = \frac{1}{2}$.

Step 2 — Likelihoods:

$$P(W \mid \text{Bag I}) = \frac{2}{5}, \quad P(W \mid \text{Bag II}) = \frac{3}{5}.$$

Step 3 — Total probability of White:

$$P(W) = \frac{2}{5} \cdot \frac{1}{2} + \frac{3}{5} \cdot \frac{1}{2} = \frac{2}{10} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}.$$

Step 4 — Posterior:

$$P(\text{Bag II} \mid W) = \frac{\frac{3}{5} \cdot \frac{1}{2}}{\frac{1}{2}} = \frac{3}{5}.$$

Why other options are wrong: $2/5$ (B) is the posterior for Bag I; $1/2$ (C) ignores the different likelihoods; $3/10$ (D) is the unnormalised numerator.

Answer: (A) Go Back to Q 37

Q38.

Solution

Concept: Binomial distribution: $P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$.

Step 1: $n=5$, $k=3$, $p=\frac{1}{2}$ (fair coin).

$$P(X=3)=\binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2=10\cdot\frac{1}{32}=\frac{10}{32}=\frac{5}{16}.$$

Step 2: $\binom{5}{3}=\dfrac{5!}{3!\,2!}=10$.

Why other options are wrong: $1/8$ (A) corresponds to $P(X=3)$ with $n=3$; $3/8$ (C) is $\binom{5}{2}(1/2)^5=10/32$ (same error with $k=2$); wait $k=2$: $\binom{5}{2}/32=10/32=5/16$... actually let me note: $3/8=12/32$ which doesn't arise from correct formula; $1/4$ (D) corresponds to $8/32$ with wrong $\binom{n}{k}$.

Answer: (B) Go Back to Q 38

Q39.

Solution

Concept: Standard parabola $y^2=4ax$ has focus at $(a,0)$ and directrix $x=-a$.

Step 1: Compare $y^2=12x$ with $y^2=4ax$:

$$4a=12\Rightarrow a=3.$$

Step 2: Focus $=(a,0)=(3,0)$.

Why other options are wrong: $(0,3)$ (A) and $(0,-3)$ (B) are on the y-axis — that would be the focus of $x^2=4ay$ or $x^2=-4ay$; $(-3,0)$ (D) is the foot of the directrix, not the focus.

Answer: (C) Go Back to Q 39

Q40.

Solution

Concept: Sum of first n terms of a GP: $S_n=\dfrac{a(r^n-1)}{r-1}$ for $r\neq 1$.

Step 1: $a=2$, $r=3$, $n=6$.

$$S_6=\frac{2(3^6-1)}{3-1}=\frac{2(729-1)}{2}=\frac{2\times 728}{2}=728.$$

Step 2: $3^6 = 3^3 \times 3^3 = 27 \times 27 = 729$. ✓

Why other options are wrong: 486 (A) is $2 \times 243 = 2 \times 3^5$, which is just the 6th term, not the sum; 729 (B) is 3^6 without the formula; 364 (C) is $728/2$ (off-by-one error in the formula).

Answer: (D) Go Back to Q 40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	A	4	D	5	B
6	A	7	C	8	D	9	B	10	A
11	C	12	D	13	A	14	B	15	C
16	D	17	A	18	B	19	C	20	D
21	A	22	B	23	C	24	D	25	A
26	B	27	C	28	D	29	A	30	B
31	C	32	D	33	A	34	B	35	C
36	D	37	A	38	B	39	C	40	D