GUJCET Mathematics

Sample Paper – 2

Duration: 60 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **GUJCET** entrance.
- Each correct answer carries **+1 mark**. There is a **negative marking of** -0.25 **mark** for each incorrect answer. Unattempted questions carry no penalty.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 12 NCERT (80%) + Class 11 NCERT/GSEB (20%) — Calculus, Algebra, Vectors, 3D Geometry, Probability, Coordinate Geometry, and more.**
- Use of mobile phones, calculators, log tables, or any electronic gadgets is strictly prohibited. Rough work may be done in the space provided on the answer sheet.

Q1. Let R be a relation on $\mathbb{Z}$ defined by $R = \{(a, b) : 3 \mid (a - b)\}$ (i.e. $a - b$ is divisible by 3). Which of the following is **TRUE**?

(A) R is reflexive but not symmetric

(B) R is symmetric but not transitive

(C) R is an equivalence relation (reflexive, symmetric, and transitive)

(D) R is neither reflexive nor symmetric

Q2. Let $f, g : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x - 1$ and $g(x) = x^2 + 2$. The value of $(g \circ f)(2)$ is:

(A) 14

(B) 25

(C) 22

(D) 27

Q3. Using the identity $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ for $x \in [-1, 1]$, the value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is:

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{3\pi}{4}$

(D) $\frac{5\pi}{6}$

Q4. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$, lying in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, is:

(A) $-\frac{\pi}{3}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $-\frac{2\pi}{3}$

Q5. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$, then $2A + B$ equals:

(A) $\begin{pmatrix} 3 & 4 \\ 4 & 9 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 4 \\ 7 & 9 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 4 \\ 8 & 9 \end{pmatrix}$

(D) $\begin{pmatrix} 2 & 4 \\ 8 & 9 \end{pmatrix}$

Q6. Every square matrix A can be uniquely written as $A = B + C$ where $B = \dfrac{A + A^T}{2}$ is symmetric and $C = \dfrac{A - A^T}{2}$ is skew-symmetric. For $A = \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix}$, the symmetric part B is:

(A) $\begin{pmatrix} 1 & 3 \\ 3 & 7 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 5 \\ 3 & 7 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 4 \\ 4 & 7 \end{pmatrix}$

Q7. Using cofactor expansion or row operations, evaluate $\begin{vmatrix} 2 & 3 & 1 \\ 4 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$:

(A) -25

(B) 15

(C) 25

(D) -15

Q8. For the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 4 \\ 2 & -1 & 5 \end{pmatrix}$, the cofactor C_{12} (of the element in row 1, column 2) is:

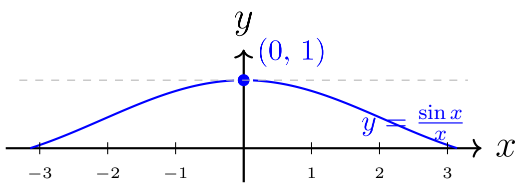
(A) 7

(B) -7

(C) 5

(D) -5

Q9. The graph of $y = \dfrac{\sin x}{x}$ near $x = 0$ is shown below; the limiting value at $x = 0$ is 1:



Using this result, the value of $\lim_{x\to 0} \dfrac{\sin 3x}{5x}$ is:

(A) 3

(B) 5

(C) $\dfrac{3}{5}$

(D) $\dfrac{5}{3}$

Q10. Using the standard result $\lim_{x\to 0} \dfrac{e^x - 1}{x} = 1$, the value of $\lim_{x\to 0} \dfrac{e^{3x} - 1}{2x}$ is:

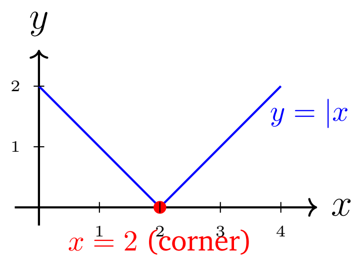
(A) 1

(B) 2

(C) $\dfrac{1}{2}$

(D) $\dfrac{3}{2}$

Q11. The graph of $f(x) = |x - 2|$ is shown below:



At $x = 2$, the function $f(x) = |x - 2|$ is:

(A) Continuous but not differentiable

(B) Differentiable with $f'(2) = 0$

(C) Discontinuous at $x = 2$

(D) Neither continuous nor differentiable

Q12. Using the product rule $\frac{d}{dx}[u \cdot v] = u'v + uv'$, if $f(x) = x^3 \cdot e^x$, then $f'(x)$ equals:

(A) x^3e^x

(B) $e^x(x^3 + 3x^2)$

(C) $3x^2e^x$

(D) $e^x(x^3 - 3x^2)$

Q13. If $y = \frac{x^2+1}{x-1}$, then $\frac{dy}{dx}$ evaluated at $x = 2$ is:

(A) 0

(B) 1

(C) -1

(D) 3

Q14. If $x = a\cos t$ and $y = a\sin t$ are the parametric equations of a curve, then $\frac{dy}{dx}$ equals:

(A) $\tan t$

(B) $-\tan t$

(C) $\cot t$

(D) $-\cot t$

Q15. If $y = \sin^{-1}(x) + \cos^{-1}(x)$ for $x \in [-1, 1]$, then $\frac{dy}{dx}$ equals:

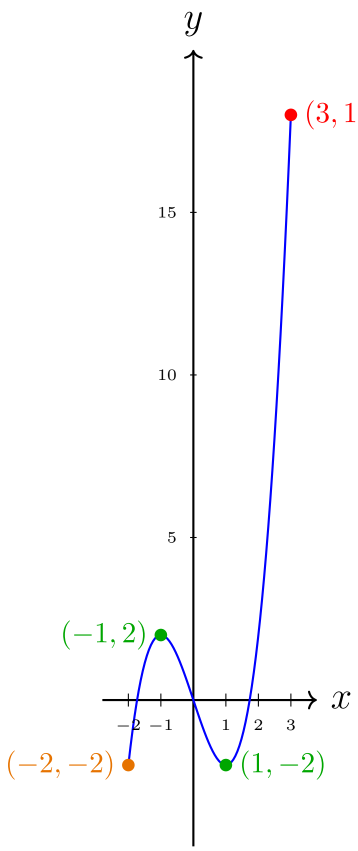
(A) 0

(B) $\frac{1}{\sqrt{1-x^2}}$

(C) $-\frac{1}{\sqrt{1-x^2}}$

(D) $\frac{2}{\sqrt{1-x^2}}$

Q16. The graph of $f(x) = x^3 - 3x$ on $[-2, 3]$ with key values marked:



The **absolute maximum** value of $f(x) = x^3 - 3x$ on the closed interval $[-2, 3]$ is:

(A) 2

(B) 18

(C) -2

(D) 27

Q17. The radius of a sphere is increasing at $3\,\text{cm/s}$. The rate of increase of its volume (cm^3/s) when the radius is $5\,\text{cm}$ is:

(A) 100π

(B) 150π

(C) 300π

(D) 60π

Q18. By Rolle's theorem, if f is continuous on $[a, b]$, differentiable on (a, b), and $f(a) = f(b)$, then $\exists\, c \in (a, b)$ with $f'(c) = 0$. For $f(x) = x(x - 4)$ on $[0,\ 4]$, the value of c is:

(A) 1

(B) 3

(C) 4

(D) 2

Q19. Evaluate $\displaystyle\int \frac{dx}{x^2 + 9}$:

(A) $\dfrac{1}{3}\tan^{-1}\left(\dfrac{x}{3}\right) + C$

(B) $\dfrac{1}{9}\tan^{-1}\left(\dfrac{x}{3}\right) + C$

(C) $\tan^{-1}\left(\dfrac{x}{3}\right) + C$

(D) $\dfrac{1}{3}\tan^{-1}(x) + C$

Q20. The substitution $u = x^2 + 1$, $du = 2x\,dx$ transforms $\displaystyle\int \frac{2x}{x^2 + 1}\,dx$ as illustrated:

$$\begin{tikzcd} \boxed{\int \frac{2x}{x^2+1}\,dx} \arrow[r, bend left, "u = x^2 + 1"] \arrow[r, bend right, "du = 2x\,dx"'] & \boxed{\int \frac{du}{u}} \end{tikzcd}$$

The result of $\displaystyle\int \frac{2x}{x^2 + 1}\,dx$ is:

(A) $2\ln(x^2 + 1) + C$

(B) $\ln(x^2 + 1) + C$

(C) $(x^2 + 1)^2 + C$

(D) $\dfrac{1}{x^2 + 1} + C$

Q21. Using the formula $\int e^x[f(x)+f'(x)]\,dx = e^x f(x)+C$, evaluate $\int e^x(\sin x+\cos x)\,dx$:

(A) $e^x \cos x + C$

(B) $e^x(\sin x - \cos x) + C$

(C) $e^x \sin x + C$

(D) $e^x(\sin x + \cos x) + C$

Q22. Evaluate $\int \frac{dx}{1+\sin x}$ by multiplying numerator and denominator by $(1-\sin x)$:

(A) $\tan x + \sec x + C$

(B) $-\tan x + \sec x + C$

(C) $\sec x + C$

(D) $\tan x - \sec x + C$

Q23. Using the property $\int_0^{\pi/2} f(x)\,dx = \int_0^{\pi/2} f\left(\frac{\pi}{2}-x\right)dx$, the value of $I = \int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x}\,dx$ is:

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{8}$

(D) 1

Q24. The definite integral $\int_1^3 x^2\,dx$, evaluated by the Fundamental Theorem of Calculus (or as a limit of Riemann sums), equals:

(A) 8

(B) $\frac{26}{3}$

(C) $\frac{8}{3}$

(D) 9

Q25. Using the substitution $u = 1 + x^2$, the value of $\int_0^1 \frac{x}{1+x^2}\,dx$ is:

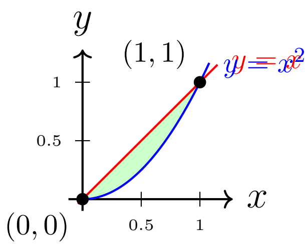
(A) $\ln 2$

(B) $\frac{1}{2}$

(C) $\frac{\ln 2}{2}$

(D) $2\ln 2$

Q26. The shaded region enclosed by $y = x$ (line) and $y = x^2$ (parabola) is shown below:



The area (in sq. units) of the region enclosed between $y = x^2$ and $y = x$ is:

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{1}{6}$

Q27. For the homogeneous ODE $\frac{dy}{dx} = \frac{x+y}{x-y}$, substituting $y = vx$ (so $\frac{dy}{dx} = v + x\frac{dv}{dx}$) reduces the equation to:

(A) $x\,\frac{dv}{dx} = \frac{1+v^2}{1-v}$

(B) $x\,\frac{dv}{dx} = \frac{1-v}{1+v^2}$

(C) $\dfrac{dv}{dx} = \dfrac{1+v^2}{1-v}$

(D) $x\dfrac{dv}{dx} = \dfrac{1-v^2}{1+v}$

Q28. For the linear ODE $\dfrac{dy}{dx} + \dfrac{2}{x}y = x$, the integrating factor $e^{\int P(x)\,dx}$ is:

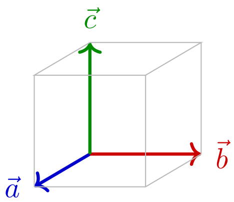
(A) x

(B) x^2

(C) $\dfrac{1}{x}$

(D) $\dfrac{1}{x^2}$

Q29. The volume of a parallelepiped with co-terminal edge-vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ equals $\left|[\vec{a}\,\vec{b}\,\vec{c}]\right|$, as shown:



If $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$, then the scalar triple product $[\vec{a}\,\vec{b}\,\vec{c}]$ equals:

(A) 20

(B) 25

(C) 30

(D) 35

Q30. If $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$, $\vec{c} = \hat{k} + \hat{i}$, then $\vec{a} \times (\vec{b} \times \vec{c})$ equals:

(A) $\hat{i} + \hat{j}$

(B) $\hat{i} - \hat{j}$

(C) $-\hat{i} - \hat{j}$

(D) $-\hat{i} + \hat{j}$

Q31. Using position vectors, the points $A(1, 2, 3)$, $B(3, 4, 5)$, and $C(5, 6, 7)$ are:

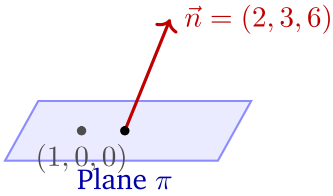
(A) Collinear

(B) Non-collinear, forming a scalene triangle

(C) The vertices of a right-angled triangle

(D) The vertices of an equilateral triangle

Q32. A plane π and its outward unit normal $\vec{n}$ are illustrated below:



The equation of the plane with normal $\vec{n} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ passing through the point $(1, 0, 0)$ is:

(A) $2x + 3y + 6z = 0$

(B) $2x + 3y + 6z = 2$

(C) $x + y + z = 1$

(D) $2x + 3y + 6z = 14$

Q33. The acute angle between two lines with direction ratios $(1,\ 2,\ 2)$ and $(2,\ 1,\ -2)$ is:

(A) 30°

(B) 45°

(C) 90°

(D) 60°

Q34. The perpendicular distance from the point $P(1,\ -1,\ 2)$ to the plane $2x - 3y + 4z = 5$ is:

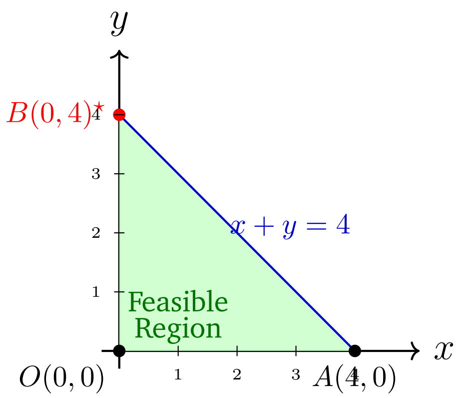
(A) $\dfrac{5}{\sqrt{29}}$

(B) $\dfrac{6}{\sqrt{29}}$

(C) $\dfrac{7}{\sqrt{29}}$

(D) $\dfrac{8}{\sqrt{29}}$

Q35. The feasible region (shaded) for constraints $x + y \le 4$, $x \ge 0$, $y \ge 0$ with corner points is shown below:



Maximise $Z = 3x + 5y$ subject to $x + y \le 4$, $x \ge 0$, $y \ge 0$. The maximum value of Z is:

(A) 20

(B) 12

(C) 16

(D) 15

Q36. Bag I contains 3 red and 2 blue balls; Bag II contains 2 red and 4 blue balls. One ball is drawn at random from *each* bag. The probability that both balls are of the **same colour** is:

(A) $\dfrac{8}{15}$

(B) $\dfrac{7}{15}$

(C) $\dfrac{1}{3}$

(D) $\dfrac{2}{5}$

Q37. Events A and B are independent with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$. Then $P(A \cup B)$ equals:

(A) $\frac{5}{12}$

(B) $\frac{7}{12}$

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

Q38. A binomial random variable X has parameters $n = 12$ and probability of success $p = \frac{1}{3}$. The **mean** $\mu = np$ of this distribution is:

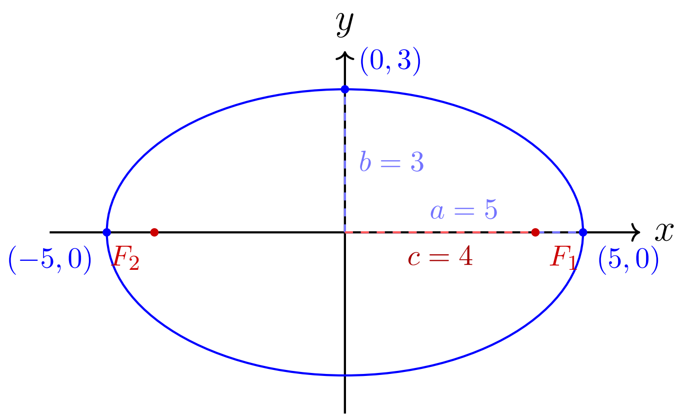
(A) 2

(B) 3

(C) 6

(D) 4

Q39. The ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ with semi-major axis $a = 5$, semi-minor axis $b = 3$, and foci at $(\pm c, 0)$ is shown below:



The eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is:

(A) $\frac{4}{5}$

(B) $\frac{3}{5}$

(C) $\frac{3}{4}$

(D) $\frac{5}{4}$

Q40. The n-th term of an arithmetic progression is $T_n = 3n + 2$. The sum of the first 10 terms is:

(A) 180

(B) 185

(C) 175

(D) 190

Detailed Solutions

Q1.

Solution

Concept: An equivalence relation must be reflexive, symmetric, and transitive.

Step 1 (Reflexive): For any $a \in \mathbb{Z}$, $a - a = 0$ is divisible by 3. So $(a, a) \in R$. ✓

Step 2 (Symmetric): If $3 \mid (a - b)$, then $3 \mid (-(a - b)) = (b - a)$. So $(a, b) \in R \Rightarrow (b, a) \in R$. ✓

Step 3 (Transitive): If $3 \mid (a-b)$ and $3 \mid (b-c)$, then $3 \mid [(a-b)+(b-c)] = (a-c)$. ✓

Conclusion: All three properties hold, so R is an equivalence relation. The equivalence classes are $[0] = \{\ldots, -3, 0, 3, 6, \ldots\}$, $[1] = \{1, 4, 7, \ldots\}$, $[2] = \{2, 5, 8, \ldots\}$.

Why other options are wrong:

- **(A):** R IS symmetric (Step 2), so the claim "not symmetric" is false.
- **(B):** R IS transitive (Step 3), so "not transitive" is false.
- **(D):** R IS reflexive (Step 1), so "neither" is false.

Answer: (C) Go Back to Q 1

Q2.

Solution

Concept: $(g \circ f)(x) = g(f(x))$—evaluate the inner function first, then apply the outer.

Step 1: $f(2) = 3(2) - 1 = 6 - 1 = 5$.

Step 2: $(g \circ f)(2) = g(f(2)) = g(5) = 5^2 + 2 = 25 + 2 = 27$.

Why other options are wrong:

- **(A) 14:** Computes $(f \circ g)$ partially: $g(2) = 6$, then wrong arithmetic.
- **(B) 25:** Computes $g(5) = 25$, omitting the $+2$.
- **(C) 22:** Mixes up the order or uses wrong formula.

Answer: (D) Go Back to Q 2

Q3.

Solution

Concept: $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ for $x \in [-1, 1]$.

Step 1: Standard value: $\cos 60^\circ = \frac{1}{2}$, so $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$.

Step 2: Apply the identity: $\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

Why other options are wrong:

- **(A)** $\pi/3$**:** This is $\cos^{-1}(+1/2)$, ignoring the negative sign.
- **(C)** $3\pi/4$**:** Corresponds to $\cos^{-1}(-1/\sqrt{2})$, not $-1/2$.
- **(D)** $5\pi/6$**:** Corresponds to $\cos^{-1}(-\sqrt{3}/2)$, not $-1/2$.

Answer: (B) Go Back to Q 3

Q4.

Solution

Concept: The principal branch of $\sin^{-1}$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Step 1: We need $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\sin\theta = -\frac{\sqrt{3}}{2}$.

Step 2: $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$, so $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.

Step 3: Since $-\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the principal value is $-\frac{\pi}{3}$.

Why other options are wrong:

- **(B)** $\pi/3$**:** $\sin(\pi/3) = +\sqrt{3}/2 > 0$; wrong sign.
- **(C)** $2\pi/3$**:** Outside the principal range $[-\pi/2, \pi/2]$.
- **(D)** $-2\pi/3$**:** Also outside the principal range.

Answer: (A) Go Back to Q 4

Q5.

Solution

Concept: Scalar multiplication: multiply each entry by the scalar. Matrix addition: add corresponding entries.

Step 1: $2A = 2\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} = \begin{pmatrix}2 & 4\\ 6 & 8\end{pmatrix}$.

Step 2: $2A + B = \begin{pmatrix}2 & 4\\ 6 & 8\end{pmatrix} + \begin{pmatrix}-1 & 0\\ 2 & 1\end{pmatrix} = \begin{pmatrix}2-1 & 4+0\\ 6+2 & 8+1\end{pmatrix} = \begin{pmatrix}1 & 4\\ 8 & 9\end{pmatrix}$.

Why other options are wrong:

- **(A):** $(1,1)$ entry is 3—adds $A+B$ without the factor of 2.
- **(B):** $(2,1)$ entry is 7—arithmetic error $(6+2=7?)$.
- **(D):** $(1,1)$ entry is 2—forgets to add $B_{11} = -1$.

Answer: (C) Go Back to Q 5

Q6.

Solution

Concept: Symmetric part of A is $B = \dfrac{A+A^T}{2}$.

Step 1: $A^T = \begin{pmatrix}1 & 5\\ 3 & 7\end{pmatrix}$.

Step 2: $A + A^T = \begin{pmatrix}1 & 3\\ 5 & 7\end{pmatrix} + \begin{pmatrix}1 & 5\\ 3 & 7\end{pmatrix} = \begin{pmatrix}2 & 8\\ 8 & 14\end{pmatrix}$.

Step 3: $B = \frac{1}{2}\begin{pmatrix}2 & 8\\ 8 & 14\end{pmatrix} = \begin{pmatrix}1 & 4\\ 4 & 7\end{pmatrix}$.

Check: $B^T = \begin{pmatrix}1 & 4\\ 4 & 7\end{pmatrix} = B$. ✓

Why other options are wrong:

- **(A):** Simply mirrors only the lower triangle; gives off-diagonal entry 3 instead of 4.
- **(B):** This is (part of) the skew-symmetric part $C = \frac{A-A^T}{2}$.
- **(C):** Simply transposes A—not the symmetric part.

Answer: (D) Go Back to Q 6

Q7.

Solution

Concept: Cofactor expansion along row 1: $\det A = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$.

Step 1:

$$\begin{vmatrix} 2 & 3 & 1 \\ 4 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 2\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} - 3\begin{vmatrix} 4 & 2 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix}$$

Step 2: Evaluate each 2×2 determinant:

$$= 2(3-4) - 3(12-2) + 1(8-1) = 2(-1) - 3(10) + 7 = -2 - 30 + 7 = -25.$$

Why other options are wrong:

- **(B) 15:** Sign error—adds $3(10)$ instead of subtracting.
- **(C) 25:** Takes the absolute value $|-25|$—ignores the negative sign.
- **(D) -15:** Arithmetic error in one of the 2×2 determinants.

Answer: (A) Go Back to Q 7

Q8.

Solution

Concept: Cofactor $C_{ij} = (-1)^{i+j}M_{ij}$, where M_{ij} is the minor formed by deleting row i and column j.

Step 1: Delete row 1, column 2 to get the 2×2 minor: $\begin{pmatrix} 3 & 4 \\ 2 & 5 \end{pmatrix}$.

Step 2: $M_{12} = \begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix} = 15 - 8 = 7.$

Step 3: $C_{12} = (-1)^{1+2} \cdot M_{12} = (-1)^3 \cdot 7 = -7.$

Why other options are wrong:

- **(A) 7:** Omits the sign factor $(-1)^{1+2} = -1$.
- **(C) 5:** Deletes the wrong row or column.
- **(D) -5:** Wrong minor with the correct sign pattern.

Answer: (B) Go Back to Q 8

Q9.

Solution

Concept: $\lim_{x\to 0}\frac{\sin x}{x} = 1$. For general ax, multiply and divide: $\frac{\sin(ax)}{bx} = \frac{\sin(ax)}{ax}\cdot\frac{a}{b}$.

Step 1: Write $\lim_{x\to 0}\frac{\sin 3x}{5x} = \lim_{x\to 0}\underbrace{\frac{\sin 3x}{3x}}_{\to 1}\cdot\frac{3}{5}$.

Step 2: As $x \to 0$, $3x \to 0$, so $\frac{\sin 3x}{3x} \to 1$.

Step 3: Result $= 1 \times \frac{3}{5} = \frac{3}{5}$.

Why other options are wrong:

- **(A) 3:** Uses only the numerator coefficient, ignoring denominator 5.
- **(B) 5:** Uses only the denominator coefficient.
- **(D) 5/3:** Inverts the ratio $a/b = 3/5$.

Answer: (C) Go Back to Q 9

Q10.

Solution

Concept: $\lim_{x\to 0}\frac{e^x - 1}{x} = 1$. Rewrite to match this form.

Step 1: $\lim_{x\to 0}\frac{e^{3x}-1}{2x} = \lim_{x\to 0}\underbrace{\frac{e^{3x}-1}{3x}}_{\to 1}\cdot\frac{3}{2}$.

Step 2: As $x \to 0$, $3x \to 0$, so $\frac{e^{3x}-1}{3x} \to 1$.

Step 3: Result $= 1 \times \frac{3}{2} = \frac{3}{2}$.

Why other options are wrong:

- **(A) 1:** Treats both coefficients as 1 (ignores 3 in numerator and 2 in denominator).
- **(B) 2:** Uses only the denominator coefficient.
- **(C) 1/2:** Uses $1/2$ from denominator, ignores the factor of 3.

Answer: (D) Go Back to Q 10

Q11.

Solution

Concept: A function is differentiable at a point only if the left and right derivatives agree. At a "V-corner," they do not.

Step 1 (Continuity): $\lim_{x\to 2^-}|x-2|=0$ and $\lim_{x\to 2^+}|x-2|=0=f(2)$. Hence f is *continuous* at $x=2$.

Step 2 (Left derivative): $\lim_{h\to 0^-}\frac{|2+h-2|-0}{h}=\lim_{h\to 0^-}\frac{|h|}{h}=\lim_{h\to 0^-}\frac{-h}{h}=-1$.

Step 3 (Right derivative): $\lim_{h\to 0^+}\frac{|h|}{h}=+1$.

Conclusion: LHD $=-1\neq$ RHD $=+1$, so f is *not differentiable* at $x=2$.

Why other options are wrong:

- **(B):** The derivative does not exist at $x=2$; the value 0 is not achievable.
- **(C):** f is continuous at $x=2$, so "discontinuous" is false.
- **(D):** f IS continuous, making "neither" false.

Answer: (A) Go Back to Q 11

Q12.

Solution

Concept: Product rule: $\frac{d}{dx}[u\cdot v]=u'v+uv'$.

Step 1: Let $u=x^3$, $v=e^x$. Then $u'=3x^2$, $v'=e^x$.

Step 2: $f'(x)=3x^2\cdot e^x+x^3\cdot e^x=e^x(3x^2+x^3)=e^x(x^3+3x^2)$.

Why other options are wrong:

- **(A)** x^3e^x**:** Only computes uv'—omits $u'v=3x^2e^x$.
- **(C)** $3x^2e^x$**:** Only computes $u'v$—omits $uv'=x^3e^x$.
- **(D):** Uses subtraction $u'v-uv'$ instead of addition.

Answer: (B) Go Back to Q 12

Q13.

Solution

Concept: Quotient rule: $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$.

Step 1: $u = x^2 + 1$, $v = x - 1$, $u' = 2x$, $v' = 1$.

$$\frac{dy}{dx} = \frac{2x(x-1) - (x^2+1)\cdot 1}{(x-1)^2} = \frac{2x^2 - 2x - x^2 - 1}{(x-1)^2} = \frac{x^2 - 2x - 1}{(x-1)^2}.$$

Step 2: At $x = 2$:

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{4 - 4 - 1}{(2-1)^2} = \frac{-1}{1} = -1.$$

Why other options are wrong:

- **(A) 0:** Arithmetic error in numerator (gets 0 instead of -1).
- **(B) 1:** Sign error—gets $+1$.
- **(D) 3:** Uses wrong formula or incorrect evaluation.

Answer: (C) Go Back to Q 13

Q14.

Solution

Concept: For parametric equations, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

Step 1: $\frac{dx}{dt} = -a\sin t$; $\frac{dy}{dt} = a\cos t$.

Step 2: $\frac{dy}{dx} = \frac{a\cos t}{-a\sin t} = -\frac{\cos t}{\sin t} = -\cot t$.

Why other options are wrong:

- **(A)** $\tan t$**:** Inverts the ratio $(\sin/\cos)$ and ignores the negative sign.
- **(B)** $-\tan t$**:** Uses $\sin/\cos$ (wrong ratio) with correct sign.
- **(C)** $\cot t$**:** Correct magnitude but wrong sign—misses the $-$ from $dx/dt = -a\sin t$.

Answer: (D) Go Back to Q 14

Q15.

Solution

Concept: Standard identity: $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$ for all $x \in [-1, 1]$.

Step 1: $y = \sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$ (constant).

Step 2: $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{2}\right) = 0.$

Why other options are wrong:

- **(B)** $1/\sqrt{1-x^2}$**:** This is $\frac{d}{dx}\sin^{-1}(x)$ alone (only one term).
- **(C)** $-1/\sqrt{1-x^2}$**:** This is $\frac{d}{dx}\cos^{-1}(x)$ alone.
- **(D)** $2/\sqrt{1-x^2}$**:** Incorrectly adds both derivatives without using the constant identity.

Answer: (A) Go Back to Q 15

Q16.

Solution

Concept: On a closed interval, the absolute maximum is the largest value among all critical points in (a, b) and the endpoints a, b.

Step 1 (Critical points): $f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 0 \Rightarrow x = \pm 1$. Both $\pm 1 \in [-2, 3]$.

Step 2 (Evaluate):

$$f(-2) = (-8)+6 = -2, \quad f(-1) = (-1)+3 = 2, \quad f(1) = 1-3 = -2, \quad f(3) = 27-9 = 18.$$

Step 3: Maximum value among $\{-2,\ 2,\ -2,\ 18\}$ is **18** at $x = 3$.

Why other options are wrong:

- **(A) 2:** This is the local maximum at $x = -1$, not the global one.
- **(C)** -2**:** This is the absolute *minimum*.
- **(D) 27:** $f(3) = 27 - 9 = 18$, not 27. (Forgets the $-3x$ term.)

Answer: (B) Go Back to Q 16

Q17.

Solution

Concept: $V = \frac{4}{3}\pi r^3$. Differentiate both sides with respect to t.

Step 1: $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$.

Step 2: At $r = 5\,\text{cm}$, $\frac{dr}{dt} = 3\,\text{cm/s}$:

$$\frac{dV}{dt} = 4\pi(5)^2(3) = 4\pi \cdot 25 \cdot 3 = 300\pi \text{ cm}^3/\text{s}.$$

Why other options are wrong:

- **(A)** 100π**:** Uses $4\pi r \cdot (dr/dt) = 4\pi \cdot 5 \cdot 3 = 60\pi$—incorrect surface area formula, or other arithmetic.
- **(B)** 150π**:** Uses $2\pi r^2 \cdot (dr/dt)$ instead of $4\pi r^2$.
- **(D)** 60π**:** Uses $4\pi r$ (circumference-like term) instead of $4\pi r^2$.

Answer: (C) Go Back to Q 17

Q18.

Solution

Concept: Rolle's theorem: find $c \in (a, b)$ where $f'(c) = 0$.

Step 1 (Verify conditions): $f(x) = x^2 - 4x$ is a polynomial—continuous on $[0, 4]$, differentiable on $(0, 4)$. $f(0) = 0$ and $f(4) = 16 - 16 = 0$. Conditions satisfied.

Step 2: $f'(x) = 2x - 4 = 0 \Rightarrow x = 2$.

Step 3: $c = 2 \in (0, 4)$. ✓

Why other options are wrong:

- **(A) 1:** $f'(1) = 2 - 4 = -2 \neq 0$.
- **(B) 3:** $f'(3) = 6 - 4 = 2 \neq 0$.
- **(C) 4:** c must be in the *open* interval $(0, 4)$; endpoint excluded.

Answer: (D) Go Back to Q 18

Q19.

Solution

Concept: Standard formula: $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C.$

Step 1: Identify $a^2 = 9 \Rightarrow a = 3.$

Step 2: $\int \frac{dx}{x^2+9} = \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C.$

Verification: $\frac{d}{dx}\left[\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right)\right] = \frac{1}{3}\cdot\frac{1}{1+(x/3)^2}\cdot\frac{1}{3} = \frac{1}{9+x^2}.$ ✓

Why other options are wrong:

- **(B):** Uses $\frac{1}{a^2} = \frac{1}{9}$ instead of $\frac{1}{a} = \frac{1}{3}$.
- **(C):** Omits the $\frac{1}{a}$ factor entirely.
- **(D):** Has $\tan^{-1}(x)$ with $a = 1$, not $\tan^{-1}(x/3)$.

Answer: (A) Go Back to Q 19

Q20.

Solution

Concept: Substitution rule: $u = g(x) \Rightarrow \int f(g(x))g'(x)\,dx = \int f(u)\,du.$

Step 1: Let $u = x^2+1$, $du = 2x\,dx$.

Step 2: $\int \frac{2x}{x^2+1}\,dx = \int \frac{du}{u} = \ln|u| + C = \ln(x^2+1) + C.$

(Since $x^2+1>0$, the absolute value is not needed.)

Why other options are wrong:

- **(A)** $2\ln(x^2+1)+C$**:** The $2x$ numerator IS du, not $2\,du$; extra factor of 2 is wrong.
- **(C):** Treats the integrand as a power rule case—wrong.
- **(D)** $1/(x^2+1)$**:** This is the derivative of $\ln(x^2+1)/2$, not the integral.

Answer: (B) Go Back to Q 20

Q21.

Solution

Concept: $\int e^x[f(x)+f'(x)]\,dx = e^x f(x)+C$. Identify $f(x)$ such that $f'(x)$ matches the second term.

Step 1: Integrand is $e^x(\sin x+\cos x)$. Match with $e^x[f(x)+f'(x)]$: set $f(x)=\sin x$, then $f'(x)=\cos x$.
Check: $f(x)+f'(x)=\sin x+\cos x$. ✓

Step 2: $\int e^x(\sin x+\cos x)\,dx = e^x\sin x + C$.

Why other options are wrong:

- **(A)** $e^x\cos x + C$**:** Would require $f(x)=\cos x$, $f'(x)=-\sin x$; then $f+f' = \cos x - \sin x \neq \sin x + \cos x$.
- **(B):** $e^x(\sin x - \cos x)$ differentiates to $e^x(2\sin x)$—does not match.
- **(D):** $e^x(\sin x + \cos x)$ differentiates to $2e^x\cos x \neq e^x(\sin x+\cos x)$.

Answer: (C) Go Back to Q 21

Q22.

Solution

Concept: Rationalise using $(1-\sin x)$ and the identity $1-\sin^2 x=\cos^2 x$.

Step 1: $\int \frac{dx}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x} = \int \frac{1-\sin x}{\cos^2 x}\,dx = \int(\sec^2 x - \sec x\tan x)\,dx$

Step 2: $=\tan x - \sec x + C$.

Verification: $\frac{d}{dx}(\tan x-\sec x) = \sec^2 x - \sec x\tan x = \frac{1-\sin x}{\cos^2 x} = \frac{1}{1+\sin x}$. ✓

Why other options are wrong:

- **(A):** Sign error—writes $+\sec x$ instead of $-\sec x$.
- **(B):** Flips both signs: $-\tan x + \sec x$.
- **(C):** Only integrates $\sec^2 x$—omits $-\sec x\tan x$ term.

Answer: (D) Go Back to Q 22

Q23.

Solution

Concept: Use $\int_0^{\pi/2} f(x)\,dx = \int_0^{\pi/2} f(\pi/2 - x)\,dx$ and add.

Step 1: Let $I = \displaystyle\int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x}\,dx.$

Step 2: Replace $x \mapsto \pi/2 - x$: $\sin(\pi/2 - x) = \cos x$, $\cos(\pi/2 - x) = \sin x$, so:

$$I = \int_0^{\pi/2} \frac{\cos^5 x}{\cos^5 x + \sin^5 x}\,dx.$$

Step 3: Add the two expressions:

$$2I = \int_0^{\pi/2} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x}\,dx = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2}.$$

Step 4: $I = \dfrac{\pi}{4}.$

Why other options are wrong:

- **(B)** $\pi/2$**:** This is $2I$ before dividing by 2.
- **(C)** $\pi/8$**:** Halves again erroneously.
- **(D) 1:** Does not apply the symmetry property correctly.

Answer: (A) Go Back to Q 23

Q24.

Solution

Concept: $\displaystyle\int_a^b f(x)\,dx = [F(x)]_a^b$ where $F'(x) = f(x)$.

Step 1: $\displaystyle\int x^2\,dx = \frac{x^3}{3} + C.$

Step 2: $\displaystyle\int_1^3 x^2\,dx = \left[\frac{x^3}{3}\right]_1^3 = \frac{27}{3} - \frac{1}{3} = 9 - \frac{1}{3} = \frac{26}{3}.$

Why other options are wrong:

- **(A) 8:** Evaluates $[x^2]_1^3 = 9 - 1 = 8$—applies the function, not the antiderivative.
- **(C) 8/3:** Uses wrong bounds or wrong antiderivative.
- **(D) 9:** Computes $27/3 = 9$ but forgets to subtract $1/3$.

Answer: (B) Go Back to Q 24

Q25.

Solution

Concept: Substitution in a definite integral changes the integrand *and* the limits.

Step 1: Let $u = 1 + x^2$, $du = 2x\,dx$, so $x\,dx = \dfrac{du}{2}$.

Step 2 (New limits): $x = 0 \Rightarrow u = 1; \quad x = 1 \Rightarrow u = 2.$

Step 3: $\displaystyle\int_0^1 \frac{x}{1+x^2}\,dx = \int_1^2 \frac{du/2}{u} = \frac{1}{2}\,[\ln u]_1^2 = \frac{1}{2}(\ln 2 - \ln 1) = \frac{\ln 2}{2}.$

Why other options are wrong:

- **(A)** $\ln 2$**:** Forgets the factor $\frac{1}{2}$ from $x\,dx = \frac{du}{2}$.
- **(B)** $1/2$**:** Evaluates $[\ln u/2]$ at wrong values or confuses $\ln 2$ with 1.
- **(D)** $2\ln 2$**:** Uses factor 2 instead of $\frac{1}{2}$.

Answer: (C) Go Back to Q 25

Q26.

Solution

Concept: Area between two curves $= \displaystyle\int_a^b [\text{upper curve} - \text{lower curve}]\,dx.$

Step 1 (Intersection): $x^2 = x \Rightarrow x(x-1) = 0 \Rightarrow x = 0$ or $x = 1$.

Step 2 (Identify upper/lower on $[0, 1]$**):** At $x = \frac{1}{2}$: $y = \frac{1}{2} > y = \frac{1}{4}$. So $y = x$ is on top.

Step 3: Area $= \displaystyle\int_0^1 (x - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}.$

Why other options are wrong:

- **(A) 1/3:** Evaluates $\int_0^1 x^2\,dx = 1/3$ alone.
- **(B) 1/2:** Evaluates $\int_0^1 x\,dx = 1/2$ alone.
- **(C) 1/4:** Arithmetic error in final subtraction $\frac{1}{2} - \frac{1}{3}$.

Answer: (D) Go Back to Q 26

Q27.

Solution

Concept: Homogeneous substitution $y = vx$ converts $dy/dx = F(y/x)$ into a separable ODE.

Step 1: $\frac{dy}{dx} = v + x\frac{dv}{dx}$ and $\frac{x+y}{x-y} = \frac{1+v}{1-v}$.

Step 2: Substituting: $v + x\frac{dv}{dx} = \frac{1+v}{1-v}$.

Step 3: $x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v(1-v)}{1-v} = \frac{1+v-v+v^2}{1-v} = \frac{1+v^2}{1-v}$.

Why other options are wrong:

- **(B):** This is the inverted (separated) form of the equation.
- **(C):** Missing the crucial factor x; should be $x\,dv/dx$, not dv/dx.
- **(D):** Sign error in numerator: $1 - v^2$ instead of $1 + v^2$.

Answer: (A) Go Back to Q 27

Q28.

Solution

Concept: For $\frac{dy}{dx} + P(x)y = Q(x)$, the integrating factor is $\mu = e^{\int P(x)\,dx}$.

Step 1: Identify $P(x) = \frac{2}{x}$.

Step 2: $\int P\,dx = \int \frac{2}{x}\,dx = 2\ln|x| = \ln(x^2)$.

Step 3: I.F. $= e^{\ln(x^2)} = x^2$.

Why other options are wrong:

- **(A)** x**:** Uses $e^{\int (1/x)dx} = x$—ignores the factor of 2 in $P(x)$.
- **(C)** $1/x$**:** Uses $e^{-\ln x} = 1/x$—wrong sign.
- **(D)** $1/x^2$**:** Uses $e^{-2\ln x} = 1/x^2$—wrong sign applied to P.

Answer: (B) Go Back to Q 28

Q29.

Solution

Concept: $[\vec{a}\,\vec{b}\,\vec{c}] = \det\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$.

Step 1:

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & -2 \end{vmatrix}$$

Step 2: Expand along row 1:

$$= 1\begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} + 3\begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix}$$

Step 3:

$$= 1(2-1) - 2(-4-3) + 3(2+3) = 1\cdot 1 - 2\cdot(-7) + 3\cdot 5 = 1 + 14 + 15 = 30.$$

Why other options are wrong:

- **(A) 20:** Arithmetic error, e.g., $-2(-7)$ computed as $+10$ instead of $+14$.
- **(B) 25:** Computes $3(5) = 10$ instead of 15.
- **(D) 35:** Over-counts one cofactor term.

Answer: (C) Go Back to Q 29

Q30.

Solution

Concept: Compute $\vec{b}\times\vec{c}$ first using the determinant formula, then compute $\vec{a}\times(\vec{b}\times\vec{c})$.

Step 1:

$$\vec{b}\times\vec{c} = (\hat{j}+\hat{k})\times(\hat{k}+\hat{i}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i}(1-0) - \hat{j}(0-1) + \hat{k}(0-1) = \hat{i}+\hat{j}-\hat{k}.$$

Step 2:

$$\vec{a}\times(\vec{b}\times\vec{c}) = (\hat{i}+\hat{j})\times(\hat{i}+\hat{j}-\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \hat{i}(1\cdot(-1)-0\cdot1)-\hat{j}(1\cdot(-1)-0\cdot1)+\hat{k}(1\cdot1-1\cdot1) = -\hat{i}+\hat{j}+0\hat{k} = -\hat{i}+\hat{j}.$$

Why other options are wrong:

- **(A)** $\hat{i}+\hat{j}$**:** Sign error in $\hat{i}$ component.
- **(B)** $\hat{i}-\hat{j}$**:** Both components have wrong signs.
- **(C)** $-\hat{i}-\hat{j}$**:** Sign error in $\hat{j}$ component.

Answer: (D) Go Back to Q 30

Q31.

Solution

Concept: Three points are collinear if $\overrightarrow{AB}$ is a scalar multiple of $\overrightarrow{AC}$ (parallel vectors through a common point).

Step 1: $\overrightarrow{AB} = B - A = (3-1)\hat{i} + (4-2)\hat{j} + (5-3)\hat{k} = 2\hat{i} + 2\hat{j} + 2\hat{k}$.

Step 2: $\overrightarrow{AC} = C - A = (5-1)\hat{i} + (6-2)\hat{j} + (7-3)\hat{k} = 4\hat{i} + 4\hat{j} + 4\hat{k} = 2\,\overrightarrow{AB}$.

Conclusion: $\overrightarrow{AC} = 2\,\overrightarrow{AB}$, so the vectors are parallel. A, B, C are **collinear**.

Why other options are wrong:

- **(B):** Non-collinear is false since $\overrightarrow{AC} = 2\overrightarrow{AB}$.
- **(C) and (D):** Collinear points cannot form any triangle.

Answer: (A) Go Back to Q 31

Q32.

Solution

Concept: Equation of plane with normal $\vec{n} = (a, b, c)$ through $P(x_0, y_0, z_0)$: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.

Step 1: $\vec{n} = (2, 3, 6)$, point $(1, 0, 0)$.

Step 2: $2(x-1)+3(y-0)+6(z-0) = 0 \;\Rightarrow\; 2x-2+3y+6z = 0 \;\Rightarrow\; 2x+3y+6z = 2$.

Why other options are wrong:

- **(A)** $2x + 3y + 6z = 0$**:** This plane passes through the *origin* $(0, 0, 0)$, not $(1, 0, 0)$.
- **(C)** $x + y + z = 1$**:** Uses equal direction ratios—wrong normal vector.
- **(D)** $2x+3y+6z = 14$**:** Incorrectly uses $|\vec{n}| = 7$ as the RHS instead of $\vec{n}\cdot\vec{r}_0 = 2$.

Answer: (B) Go Back to Q 32

Q33.

Solution

Concept: $\cos\theta = \dfrac{|l_1l_2 + m_1m_2 + n_1n_2|}{\sqrt{l_1^2 + m_1^2 + n_1^2} \cdot \sqrt{l_2^2 + m_2^2 + n_2^2}}$.

Step 1 (Dot product): $(1)(2) + (2)(1) + (2)(-2) = 2 + 2 - 4 = 0$.

Step 2: $\cos\theta = \dfrac{|0|}{\sqrt{1+4+4} \cdot \sqrt{4+1+4}} = \dfrac{0}{3 \cdot 3} = 0$.

Step 3: $\theta = 90^\circ$. The lines are perpendicular.

Why other options are wrong:

- **(A) 30°, (B) 45°, (D) 60°:** All require a non-zero dot product. Here the dot product is exactly 0, giving $\cos\theta = 0$ unambiguously.

Answer: (C) Go Back to Q 33

Q34.

Solution

Concept: Distance from (x_1, y_1, z_1) to plane $ax + by + cz = d$: $D = \dfrac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{a^2 + b^2 + c^2}}$.

Step 1: Point $(1, -1, 2)$, plane $2x - 3y + 4z = 5$.

Step 2 (Numerator): $|2(1) - 3(-1) + 4(2) - 5| = |2 + 3 + 8 - 5| = |8| = 8$.

Step 3 (Denominator): $\sqrt{4 + 9 + 16} = \sqrt{29}$.

Step 4: $D = \dfrac{8}{\sqrt{29}}$.

Why other options are wrong:

- **(A) 5/$\sqrt{29}$:** Computes numerator as $|2+3+8| = 13$, then subtracts incorrectly to get 5.
- **(B) 6/$\sqrt{29}$:** Sign error: $-3(-1)$ taken as -3 instead of $+3$.
- **(C) 7/$\sqrt{29}$:** Off-by-one arithmetic error in numerator.

Answer: (D) Go Back to Q 34

Q35.

Solution

Concept: The maximum of a linear objective function on a bounded convex feasible region occurs at a corner (vertex).

Step 1 (Corner points): The constraints $x + y \leq 4$, $x \geq 0$, $y \geq 0$ give vertices: $O(0,0)$, $A(4,0)$, $B(0,4)$.

Step 2 (Evaluate $Z = 3x + 5y$):

$$Z(O) = 0, \quad Z(A) = 3(4) + 5(0) = 12, \quad Z(B) = 3(0) + 5(4) = 20.$$

Step 3: Maximum $Z = \mathbf{20}$ at $B(0,4)$ (marked $\star$ in the graph).

Why other options are wrong:

- **(B) 12:** This is Z at $A(4,0)$—not the maximum because the coefficient of y is larger.
- **(C) 16:** No corner point gives $Z = 16$.
- **(D) 15:** An interior point value; not attained at any vertex.

Answer: (A) Go Back to Q 35

Q36.

Solution

Concept: The two draws are independent. $P(\text{same colour}) = P(\text{both red}) + P(\text{both blue})$.

Step 1: $P(\text{Red from I}) = \frac{3}{5}$, $\quad P(\text{Red from II}) = \frac{2}{6} = \frac{1}{3}$.

$P(\text{Blue from I}) = \frac{2}{5}$, $\quad P(\text{Blue from II}) = \frac{4}{6} = \frac{2}{3}$.

Step 2: $P(\text{both red}) = \frac{3}{5} \cdot \frac{1}{3} = \frac{1}{5} = \frac{3}{15}$.

$P(\text{both blue}) = \frac{2}{5} \cdot \frac{2}{3} = \frac{4}{15}$.

Step 3: $P(\text{same colour}) = \frac{3}{15} + \frac{4}{15} = \frac{7}{15}$.

Why other options are wrong:

- **(A) 8/15:** Errors: e.g., $P(\text{both red}) = 2/15$ (wrong) $+4/15 = 8/15$.
- **(C) 1/3:** Uses only P(both red) with arithmetic error.

- **(D) 2/5:** Uses $P(\text{Blue from I}) = 2/5$ alone.

Answer: (B) Go Back to Q 36

Q37.

Solution

Concept: Independence: $P(A \cap B) = P(A) \cdot P(B)$. Addition: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Step 1: $P(A \cap B) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$.

Step 2: $P(A \cup B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{12} = \frac{4}{12} + \frac{3}{12} - \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$.

Why other options are wrong:

- **(A) 5/12:** Incorrect $P(A \cap B)$—uses $1/12$ correctly but makes fraction error: $7/12 - 2/12 = 5/12$.
- **(B) 7/12:** Simply adds $P(A) + P(B) = 7/12$ without subtracting the intersection.
- **(D) 1/3:** Uses only $P(A)$.

Answer: (C) Go Back to Q 37

Q38.

Solution

Concept: For $X \sim B(n, p)$, the mean is $\mu = np$.

Step 1: $n = 12$, $p = \frac{1}{3}$.

Step 2: $\mu = np = 12 \times \frac{1}{3} = 4$.

Why other options are wrong:

- **(A) 2:** Uses $p = 1/6$ (wrong); $12 \times (1/6) = 2$.
- **(B) 3:** Uses $p = 1/4$ (wrong); $12 \times (1/4) = 3$.
- **(C) 6:** Uses $p = 1/2$ (wrong); $12 \times (1/2) = 6$.

Answer: (D) Go Back to Q 38

Q39.

Solution

Concept: For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b > 0)$: $c^2 = a^2 - b^2$, eccentricity $e = \frac{c}{a}$.

Step 1: $a^2 = 25 \Rightarrow a = 5; \quad b^2 = 9 \Rightarrow b = 3.$

Step 2: $c^2 = 25 - 9 = 16 \Rightarrow c = 4.$

Step 3: $e = \frac{c}{a} = \frac{4}{5}.$

Why other options are wrong:

- **(B) 3/5:** Uses $e = b/a = 3/5$—incorrect definition.
- **(C) 3/4:** Uses $e = b/c = 3/4$—also incorrect.
- **(D) 5/4:** Uses $e = a/c = 5/4 > 1$—impossible for an ellipse (need $e < 1$).

Answer: (A) Go Back to Q 39

Q40.

Solution

Concept: If $T_n = 3n + 2$, then $S_{10} = \sum_{n=1}^{10} T_n = 3\sum_{n=1}^{10} n + 2 \times 10.$

Step 1: $\sum_{n=1}^{10} n = \frac{10 \times 11}{2} = 55.$

Step 2: $S_{10} = 3(55) + 2(10) = 165 + 20 = 185.$

Alternative check: $T_1 = 3(1) + 2 = 5$, $T_{10} = 3(10) + 2 = 32$. $S_{10} = \frac{10}{2}(T_1 + T_{10}) = 5 \times 37 = 185$. ✓

Why other options are wrong:

- **(A) 180:** Uses $T_n = 3n$ (omitting $+2$): $3(55) = 165 \neq 180$.
- **(C) 175:** Subtracts 10 instead of adding: $165 + 10 = 175$—wrong sign on constant.
- **(D) 190:** Over-counts constant term or uses $n = 11$ in sum.

Answer: (B) Go Back to Q 40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	D	3	B	4	A	5	C
6	D	7	A	8	B	9	C	10	D
11	A	12	B	13	C	14	D	15	A
16	B	17	C	18	D	19	A	20	B
21	C	22	D	23	A	24	B	25	C
26	D	27	A	28	B	29	C	30	D
31	A	32	B	33	C	34	D	35	A
36	B	37	C	38	D	39	A	40	B